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Studying Developmental Processes in Accelerated Cohort-Sequential Designs with
Discrete- and Continuous-Time Latent Change Score Models

Eduardo Estrada

Emilio Ferrer

University of California, Davis

Author Note: Correspondence should be sent to Eduardo Estrada. Department of Psychology.

University of California, Davis. eduardo.estrada.rs@gmail.com

Computer code for the statistical analyses can be found at:

<https://github.com/EduardoEstradaRs/PsychMeths2019-Developmental-Processes-ALD-DT-CT-LCS>

Abstract

Studying the time-related course of psychological processes is a challenging endeavor, particularly over long developmental periods. Accelerated longitudinal designs (ALD) allow capturing such periods with a limited number of assessments in a much shorter time framework. In ALDs, participants from different cohorts are measured repeatedly but the measures provided by each participant cover only a fraction of the time range of the study. It is then assumed that the common trajectory can be studied by aggregating the information provided by the different converging cohorts. We conducted a Monte Carlo study to evaluate the practical relevance of using discrete- and continuous-time latent change score models for recovering the trajectories of a developmental process from ALD data under different sampling conditions. We focused on exponential trajectories typically found in the development of cognitive abilities from childhood to early adulthood. The results support the appropriateness of ALD designs to study such processes under various conditions of sampling. When all cohorts are drawn from the same population, both discrete- and continuous-time models are able to recover the parameters defining the underlying developmental process. However, discrete-time models yield biased estimates when time lags between observations are not constant. When cohorts are not from the same population and, thus, lack convergence, both types of models show bias in various parameters. We discuss the findings in the context of developmental methodology, encourage researchers to adopt continuous time models to analyze data from ALDs, and provide recommendations about how to implement such research designs.

Keywords: longitudinal data analysis; continuous time modeling; accelerated longitudinal design; latent change score models; state-space models; SEM

Translational abstract

It is difficult to study psychological processes over time, particularly for long developmental periods. Accelerated longitudinal designs (ALD) make it possible to assess these periods in a more condensed time framework. With ALDs, participants from different cohorts are measured repeatedly, although only for a limited portion of the time range of the study. Combining information across converging cohorts allows an assessment of the common developmental process. Data from ALDs have often been analyzed with latent change score models that are specified in discrete time (DT) with measurement occasion as the time metric. Newer continuous time (CT) models allow studying latent change while considering time explicitly. This study compared the performance of both DT and CT models in conditions that are frequent in empirical developmental research. We focused on patterns typically found in the development of cognitive abilities from childhood to early adulthood. The results support the use of ALD: When all cohorts are drawn from the same population, both DT and CT models accurately map an underlying developmental process. However, DT models yield biased estimates when time lags between observations are not constant. In contrast, when cohorts are not from the same population and, thus, lack convergence, both DT and CT models show bias. We encourage researchers to use CT models when analyzing data from ALDs and discuss the findings in the context of developmental methodology. Finally, we offer recommendations on how to apply such research designs.

Studying Developmental Processes in Accelerated Cohort-Sequential Designs with Discrete- and Continuous-Time Latent Change Score Models

Many psychological phenomena unfold over long periods of time. Capturing such processes and mapping them onto their time course is challenging at best because it requires repeated assessments across many years. Consider, for example, the development of cognitive abilities from childhood to early adulthood. Researchers interested in understanding this development would need to take repeated measures from participants covering a wide age range from, say, early childhood until they are in their early 20s. These lengthy studies are rarely feasible. Even under the best funding circumstances, they entail multiple methodological problems, such as cumulative testing effects, participant attrition and consequent selection effects –not all potential participants are willing to be measured over a long period.

One solution proposed to address this issue was the *accelerated longitudinal design* (ALD; Bell, 1953, 1954; S. C. Duncan, Duncan, & Hops, 1996; McArdle, 1994; also called *cohort-sequential design*, Nesselroade & Baltes, 1979; or *cross-sequential design*, Schaie, 1965). In an ALD, participants are measured repeatedly in the variables of interest, but the measures provided by each participant cover only a fraction of the time range of the study. Consider, for example, a study examining the development of reasoning from childhood to early adulthood, with an intended target age range of 5 to 20 years. At the initial occasion, the researcher selects a sample of participants ranging from 5 to 18 years of age, and each participant is assessed once a year during three consecutive years. Although each person has three assessments only, spanning three years, the entire target age range is covered because participants have different ages at the first measurement occasion –i.e., they come from different *cohorts*– resulting in temporally overlapping measurements of the various age groups (S. C. Duncan et al., 1996). Therefore, in this example, the youngest participants are five when they are first measured, and seven by the end of the study, whereas the oldest participants are 18 in their first measure and 20 by the end of

the study (see S. C. Duncan et al., 1996 for further details).

This approach assumes that all the overlapping cohorts share a common longitudinal trajectory in the variables of interest, and this trajectory can be studied by aggregating the information provided by the different cohorts –i.e., through the “convergence” analysis of the longitudinal and cross-sectional information (Bell, 1953, 1954; McArdle & Bell, 2000). Some authors have referred to these types of sampling schedules as *planned missing data* or *incomplete data* designs (Graham, Taylor, & Cumsille, 2001; Hamagami & McArdle, 2001; McArdle & Woodcock, 1997; Mistler & Enders, 2012; Rhemtulla & Hancock, 2016; Rhemtulla, Jia, Wu, & Little, 2014).

ALDs have been widely used in psychological research, perhaps most extensively in studies of cognitive and socio-emotional development, especially during childhood and adolescence. For example, Duncan, Duncan & Hops (1994) applied one such design for studying adolescent alcohol use. Using latent growth curve models, they found evidence for a shared trajectory between cohorts, and for effects of family cohesion and peer encouragement on alcohol consumption. Also using a latent growth model, Watt (2008) analyzed data from an ALD and found different trajectories for adolescent boys and girls in the perception and values associated with English and math. Poteat, O’Dwyer, & Mereish (2012) used an ALD to study changes in use of homophobic epithets by adolescent and also found different trajectories for boys and girls. McArdle & Woodcock (1997) proposed an ALD involving only two measures per case, with different time lags for different individuals, to study the development of memory and reading ability through latent growth models. This research was later extended by McArdle, Ferrer, Hamagami & Woodcock (2002), who used a large sample from an ALD consisting of two measurement occasions to examine the growth and decline of multiple cognitive abilities over the life span.

In addition to helping create a wide age span for a study, ALDs are particularly useful

when the variables of interest are expensive to measure. This is particularly the case with brain imaging measures. As such, neuroimaging studies of brain development have employed ALDs to capture the desired age ranges. One such study is the large scale NIH MRI study of normal brain development (Evans, 2006). Data from this study have led to important findings relating brain structure development to various cognitive abilities (c.f., Karama et al., 2009; Román et al., 2018). Other studies have also adopted ALDs with the goal of identifying developmental sequences linking brain structure and functioning to cognitive abilities such as fluid reasoning (e.g., Ferrer, 2019; Ferrer, O'Hare, & Bunge, 2009; Green, Bunge, Briones Chiongbian, Barrow, & Ferrer, 2017; Wendelken et al., 2017), or memory (Fandakova et al., 2017), to name a few.

In the remainder of the paper, we examine the extent to which varying ALDs can provide information necessary to detect trajectories of developmental processes. For this, we employ longitudinal models using discrete- or continuous-time, and evaluate their performance under different simulated sampling conditions. In the next sections, we describe our modeling approach and the simulation conditions. Next, we report the results from the simulation analyses, and put them in the context of current knowledge about developmental methodology. We conclude the paper offering a number of recommendations regarding the design of longitudinal studies based on ALDs as well as the analyses of resulting data from such designs.

Examining developmental sequences with latent change score models

In this study, we focus on models for analyzing data that follow the typical functions found in the development of intellectual abilities from childhood to early adulthood. Most cognitive abilities show a fast growth during the first years of life followed by a gradual deceleration, until an maximum point somewhere between 20 and 30 years –the exact age depends on the specific ability and the specific individual (McArdle et al., 2002). After this point, some abilities –e.g., processing speed, working memory capacity, fluid reasoning – slowly

decrease (Ferrer & McArdle, 2004; Kail, 1991; Kail & Park, 1992; Kail & Salthouse, 1994; Salthouse & Kail, 1983), whereas others –such as crystallized intelligence– continue to rise at a very low rate (McArdle et al., 2002).

One model suited to capture this type of exponential function is the Latent Change Score model (LCS, Ferrer & McArdle, 2010; McArdle, 2001, 2009; McArdle & Hamagami, 2001). An LCS model represents the process of interest as a dynamical system in which the changes, instead of the levels, are the focus and are modeled as latent variables. A path diagram of this model is depicted in the top panel of Figure 1.

INSERT FIGURE 1 HERE

At each repeated occasion t , a latent variable representing changes (Δy_t) in a latent process (y_t) is specified. Thus, at each occasion, the latent process is a function of the initial unobserved level, y_{t0} , plus the accumulation of changes up to that occasion. One general specification to capture such changes is the so-called *dual LCS* model, in which changes are a function of: a) an additive linear effect captured by the latent variable y_s , and b) a proportional effect from the latent level of the process at the previous occasion –captured by the self-feedback parameter β . The means of the latent initial level and slope (μ_{t0} and μ_s) capture, respectively, the mean level in the process at the first occasion, and the average additive component at every repeated occasion. The variances of these latent variables (σ_{t0}^2 and σ_s^2) denote individual differences in such initial level and additive change. These two components are typically allowed to be correlated ($\sigma_{t0,s}$ expressed as a covariance, or $\rho_{t0,s}$ as a correlation). The variance of the measurement error, or observed variability not due to the process, is captured by the parameter σ_e^2 .

When modeling data representing cognitive development or achievement from childhood to early adulthood, the linear effect is typically positive. This entails an overall positive trend – i.e., growth– in the scores over time. In turn, the proportional effect β is negative, representing a

damping effect –i.e., the process tends to an equilibrium point or asymptote. The combined effect of the linear and proportional components of change leads to a non-linear trajectory with less overall gains over time. For more information on LCS models and their application to developmental change, see McArdle (2001), McArdle & Hamagami (2001), Ferrer & McArdle (2004), Ferrer et al. (2007), Ferrer & McArdle (2010), and Kievit et al. (2018).

In the classic specification of a LCS, time is considered discrete (LCS-DT). All individuals within the sample are expected to be measured at the same fixed time points –i.e., the time lag between those points is assumed to be constant across participants and occasions. Participants without data for one specific occasion are handled through conventional missing data procedures such as Full Information Maximum Likelihood. If none of the participants are measured at one time point, a *phantom variable* is typically used to keep the time lag constant (Ferrer, Hamagami, & McArdle, 2004; Ferrer & McArdle, 2004; Oud & Voelkle, 2014).

In empirical studies, however, participants are rarely measured at exactly the same age. For example, if the time metric underlying the sampling is age in years and each participant is measured yearly, one participant can be measured at ages 5.45, 6.31, 7.52, 8.41, etc., whereas another participant is measured at ages 5.14, 6.27, 7.09, 8.08, and so on. In other words, the exact time lag between measures is not constant between participants or between occasions for each participant. This problem is typically addressed by creating age bins –e.g., age 5, 6, 7, 8, etc.– and rounding the participant's age to the closest age bin (Ferrer, 2019; Ferrer & McArdle, 2004; McArdle & Bell, 2000). Some studies have investigated the effect of this binning procedure on the results from latent growth curve models (McArdle & Bell, 2000; Miller & Ferrer, 2017), but, to the best of our knowledge, no study has formally investigated the effect on the estimates from LCS models, assessing potential bias, accuracy, or efficiency.

In recent years, various authors have proposed the use of continuous time (CT) models for characterizing psychological processes that evolve over time (Boker, 2001; Boker, Neale, & Rausch, 2004; de Haan-Rietdijk, Voelkle, Keijsers, & Hamaker, 2017; Deboeck & Preacher, 2016; Ji & Chow, 2019; Oravecz, Tuerlinckx, & Vandekerckhove, 2009, 2011; Oud & Jansen, 2000; van Montfort, Oud, & Voelkle, in press; Voelkle & Oud, 2012, 2015; Voelkle, Oud, Davidov, & Schmidt, 2012). These models typically use differential equations to describe the longitudinal trajectory of the variable of interest, which is assumed to unfold continuously. Every time-specific measure is considered a discrete realization of that continuous process.

Proponents of these models have emphasized a number of practical and theoretical advantages over conventional discrete-time models such as auto-regressive cross-lagged models (Deboeck & Preacher, 2016; Voelkle et al., 2012), or the LCS-DT described above. CT models can capture features such as latent change, self-feedback and cross-lagged regressions. However, one of their practical advantages is that time is considered explicitly and, therefore, it is easier to compare parameters that were estimated based on different time intervals (Voelkle et al., 2012). Indeed, CT models yield estimates that are independent of the time lag and can be transformed to any specific time interval (Voelkle & Oud, 2015; Voelkle et al., 2012). Another advantage is their ability to naturally account for different time lags between assessments, across time points and across individuals, without using artificial time bins or phantom variables. In fact, from the continuous time perspective, there are no missing data¹. The underlying process is assumed to develop continuously over time, but is only observed at some selected discrete time points (Oud & Voelkle, 2014).

From a theoretical standpoint, even when researchers fit a DT model to empirical data, they work under the assumption that the process of interest does not cease to exist between

¹ Multivariate continuous-time models can have partially missing data (e.g., if only one of two variables is observed at a particular time point).

observations. In other words, almost all, if not all, psychological processes are assumed to unfold in continuous time. Therefore, a CT model is proposed as a more accurate representation of those processes, as it explicitly captures this feature (Oud & Delsing, 2010). Indeed, the LCS-DT model described in the previous section can be considered as a specific case of an underlying CT model. A CT model contains the same information as the DT model and more. The former accounts for the order of measurement occasions, but also for the time interval between them. Therefore, the parameters from the CT model contain all the necessary information to reconstruct the DT parameters for any specific time length, whereas the opposite is not true (Voelkle & Oud, 2015; Voelkle et al., 2012). One of the goals of this study is to test whether a CT model is more effective than the analogue DT model for recovering the information describing a developmental process (see more details below).

State-space models

Within the psychology literature, there are two prevalent frameworks for estimating CT models. One is based on SEM (Driver, Oud, & Voelkle, 2017; Oud & Jansen, 2000; Voelkle et al., 2012); the other uses state-space models based on filtering techniques (Hunter, 2018; Ou, Hunter, & Chow, 2017; Oud & Singer, 2008). These two frameworks come from different traditions and have different historical development, but are mathematically related (Chow, Ho, Hamaker, & Dolan, 2010). In this study, we estimated a univariate continuous time state-space model (SSM-CT). This model is an alternative parameterization to the classic LCS-DT described above and includes the same number of parameters. However, the interpretation of some of these parameters is different between the two models. Here, we describe this SSM-CT model and point out its mathematical and conceptual relations to the alternative LCS-DT (for more information about these models, their differences and equivalences, see Hunter, 2018; Ji & Chow, 2019; Voelkle & Oud, 2015).

The first difference between both models is the specification of the self-feedback². In the traditional LCS-DT, the parameter β captures the influence from the latent level of y in $t-1$ on the latent change Δy in t . This can be re-specified as a vector auto-regressive (VAR) model by eliminating the latent change variables and allowing the latent level of y to be predicted by itself at the previous occasion (Usami, Hayes, & McArdle, 2015). This version includes the same seven parameters, with the exact same interpretation for six of them. Only the self-feedback changes. It becomes an auto-regression coefficient with a new value $1+\beta$. Note that in the traditional LCS-DT, this parameter typically has a value $-1 \leq \beta \leq 0$, therefore $0 \leq (1+\beta) \leq 1$ in the VAR model.

The second difference is the definition of the latent linear component y_s . In an SSM-DT, this variable is specified at each repeated measure. In other words, the model includes two latent variables that unfold over time: one is the latent level y_l —equivalent to the latent intercept at time zero—, and the other is the latent linear component y_s . The relation between them, and their variance-covariance structure—either stationary or time-varying—can be defined in different ways depending on the researcher's hypothesis. The following SSM -DT specification is mathematically equivalent to the LCS-DT model.

$$\begin{bmatrix} y_{l,it} \\ y_{s,it} \end{bmatrix} = \begin{bmatrix} 1+\beta & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{l,it-1} \\ y_{s,it-1} \end{bmatrix} \quad [1]$$

The first matrix in the right-hand side of equation 1 is often called the *transition* matrix—conceptually related to the *drift* matrix in a continuous time model. As defined in equation 1, the latent linear component y_s is stationary—i.e., its mean and variance do not change over time—and influences the latent level y_l , but it is not influenced by it. The mean vector for the two latent

² In multivariate systems, the time-lagged effects (both auto-regressions/self-feedbacks and cross-influences between variables) are typically specified in a square “dynamic”, “transition”, or “drift” matrix $\mathbf{A}$. However, our study includes one variable and therefore this matrix is reduced to a single self-feedback parameter β (we decided to keep the notation from the LCS framework).

variables is defined at time zero as $\boldsymbol{\mu} = [\mu_{l0} \ \mu_s]$, and their covariance matrix is

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{l0}^2 & \sigma_{l0,s} \\ \sigma_{l0,s} & \sigma_s^2 \end{bmatrix} \quad [2]$$

The two latent variables are typically assumed to follow a bivariate normal distribution. At any given time t , the manifest scores are given by $Y_{it} = y_{l,it} + e_{it}$, where the measurement error $e_{it} \sim N(0, \text{diag}[\sigma_e])$. Note that a) the latent variable y_s is not linked to any manifest observation, b) the measurement error e is stationary, with mean zero, constant variance and no auto-correlation, and c) no dynamic error is assumed –i.e., there is no random *diffusion* in the process, and the innovations do not carry over the future states of the system.³

The model described above is defined in discrete time. Estimating an SSM-CT involves a third step: specifying an ordinary differential equation system. In the SSM-CT, the initial level of y is given by the latent variable y_{l0} and, at any time t , the latent scores in y are given by the initial level plus the accumulation of changes up to that time. The latter is defined by the differential equation:

$$d \begin{bmatrix} y_{l,i} \\ y_{s,i} \end{bmatrix} / dt = \begin{bmatrix} \beta & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{l,i} \\ y_{s,i} \end{bmatrix} \quad [3]$$

In Equation 3, the variables y_l and y_s represent the latent level and latent linear component, respectively. This equation specifies their derivative with respect to time (dt), that is, the dynamics describing how these variables change over time. The parameters in the drift matrix –first matrix in the right hand side of Equation 3– imply that y_l changes as a function of itself (self-feedback β), and y_s (with a loading of 1). In contrast, y_s neither changes nor receives any influence from y_l , and thus is invariant (see Figure 1, bottom; and also the section *Data Analysis*

³ Innovation or *diffusion* in the latent process can be incorporated in the model (Oravecz et al., 2011; Voelkle & Oud, 2015; Voelkle et al., 2012). In this study, we did not include it because one of our main goals is to compare the LCS-DT and SSM-CT models, and the specification of diffusion is very uncommon in the LCS-DT framework.

for a formal description of the full SSM-CT).

The rest of the model matrices are the same in SSM-DT and SSM-CT, and so is the time metric –e.g., a one-point increase represents one year. However, the interpretation and scale of the parameters β , μ_s , σ^2_s , and $\sigma_{I0,s}$ differs between models. In the SSM-DT, they capture the change in the system for a specific time lag (typically $\Delta t=1$), whereas in the SSM-CT they capture the change for an infinitesimally brief time lag (dt). If the CT values are known, the DT values can be computed. In contrast, a particular set of DT values does not provide enough information for computing the analogue CT values, as the former can lead to non-invertible or non-unique solutions. Table 1 reports the relations between the parameters in both models. For a more general description of these transformations, and the extension to multivariate systems, see Voelkle & Oud (2015) and Voelkle et al. (2012). The bottom panel of Figure 1 depicts our SSM-CT model.

Purpose of the study

As described previously, the defining feature of an ALD is the fact that different individuals are intentionally measured at different ages, and the age range of the sample is wider than the age range for any individual. In this scenario, applying an LCS-DT within the SEM framework implies working with a highly sparse dataset, and very low coverage for some sections of the covariance matrix. Furthermore, in empirical settings, the time interval between measures is likely to vary across both time points and participants. This is not captured by an LCS-DT, which assumes that all the individuals are measured at the exact same time points. In contrast, these design features are not an issue in a CT model, and, thus, this modeling approach seems like an optimal choice for data from ALDs.

In this study we sought to evaluate the practical use of discrete and continuous time models for analyzing data from ALDs. Specifically, we investigated the performance of two

analogue model specifications for examining data commonly found in studies of developmental processes from childhood to early adulthood under different sampling schemes. Our goal was to examine the sampling conditions under which these models can recover population parameters both accurately and efficiently. With this information, we hope to make recommendations about data collection strategies and analytic procedures for studying such processes when data are incomplete by design.

Methods

Monte Carlo study

We generated repeated measures for a latent process y which unfolds in continuous time. The process is defined by the SSM-CT model described previously. The parameters of this model are reported in Table 1, and were based on empirical estimates using data from the Woodcock-Johnson reading cluster, computed in a large and representative US sample (Ferrer et al., 2007; Ferrer, Shaywitz, Holahan, Marchione, & Shaywitz, 2010; Shaywitz, Shaywitz, Fletcher, & Escobar, 1990). Figure 2 depicts 200 individual trajectories of the latent process y across the hypothetical ages 5 to 19.

INSERT TABLE 1 HERE

INSERT FIGURE 2 HERE

The first factor manipulated in our study was the sampling schedule. For this, we sampled the continuous latent process y according to seven ALDs. In each design, each discrete measurement yielded a value for the observed variable Y . Table 2 reports the sampling scheme and the number of measures per case in each ALD. Figure 3 depicts the latent and observed trajectories resulting from each design.

INSERT TABLE 2 HERE

INSERT FIGURE 3 HERE

In designs one and two all cases are measured at all the scheduled time points. Thus, these designs do not involve cohorts. In design one, all cases are measured once a year during the duration of the study; in design two, all cases are measured every other year. In design three, all cases are measured at two occasions only, sampled two years apart. Design four includes three measures per case, sampled in three consecutive years. Design five also includes three measures, but separated by two years apart each –i.e., spanning 5 years for each case. In design six, all cases are measured in four consecutive years. Finally, design seven includes a different number of measures depending on the age at first measure: the four younger cohorts are measured at four consecutive years, the three middle cohorts are measured three times, separated by two years each (as in design 5), and the four older cohorts are measured at two occasions, separated by two years (as in design 3).

For all designs with cohorts (three to seven), the percentage of cases per cohort was not the same. This is because a uniform distribution of cases in cohorts would lead to lower data density at the beginning and end of the trajectory. However, previous research has shown that the developmental trajectories of cognitive abilities –such as y – are better estimated when the information provided by the sample is as complete as possible at the region with greatest curvature (Mistler & Enders, 2012; Rhemtulla & Hancock, 2016). Because we wanted to include realistic data sampling schemes, designs 3 to 7 ensured that the data density at the section with greatest curvature was not lower than at other sections of the trajectory. The same rationale guided the choice of design seven: we included it to test whether a higher data density in the first years lead to higher accuracy and efficiency in the parameter estimation

The second design feature that we manipulated in our study was the time interval between assessments. Here, we focused on whether the time lag between measures were *fixed* or *random*. In the condition with fixed lags, all individuals were sampled the first week of the year, for all years. In the condition with random lags, data points for each individual were sampled

randomly at any given week within the corresponding year. Figure 4 depicts these two conditions for research design 2. The top panels depict the latent and observed trajectories when the process is sampled at fixed time points (the first week of the year); the bottom panels depict the same trajectories sampled at one random week in the year.

INSERT FIGURE 4 HERE

The third design feature was whether convergence existed among cohorts. In the *convergence* condition, all the cohorts – as well as cases within cohorts – were drawn from a population with trajectories defined by the parameters in Table 1. In the *non-convergence* condition, the cohorts were not drawn from the same population. Rather, only the cases in the oldest cohort –i.e., cohort 13, entering the study at older age– were drawn from the population defined in Table 1. In contrast, the youngest cases –cohort 1, which would be born later in an empirical study – were drawn from a population in which the mean value for the linear additive component μ_s was one standard deviation above the value in cohort 13. In other words, the oldest cohort had $\mu_{s,CT} = 8.001$, whereas the youngest cohort had $\mu_{s,CT} = 8.001 + .859 = 8.860$. Each intermediate cohort had an increase of $.859/12 = .072$ with respect to the immediately older cohort. The remaining parameters did not vary between cohorts. This implied that entering the study at a younger age was related with a higher mean additive value. In consequence, younger cohorts were expected to achieve higher mean levels –because the asymptote for the exponential trajectory is $y_{As} = \mu_s / (-\beta)$, and β did not change between cohorts.

For each of the 28 combinations of *research design*, *type of lag*, and *convergence*, we generated 100 samples of size 200 and 100 samples of size 500.⁴

Summary of simulation conditions (28 in total)

1. Research design: Seven sampling schemes (see Figure 3)

⁴ Note that research designs 1 and 2 included one cohort only. Therefore, the results for these designs were identical in the conditions with and without convergence.

2. Fixed vs. Random time lags between measures (see Figure 4)
3. Convergence vs. non-convergence between cohorts.
4. Sample size: 200 or 500 cases

Data analysis

For each sample in each condition, we estimated the two statistical models described in our introduction:

a) Latent Change Score model in discrete time (LCS-DT). SEM specification, using *OpenMx* in *R* (RAM parameterization estimated with Full Information Maximum Likelihood; c.f., Ghisletta & McArdle, 2012; Neale et al., 2016). As explained previously, this specification requires a discrete measurement occasion, and not actual age, as the time signature. In the random lag conditions, the age bins were created in the center of each year –i.e., at ages 5.5, 6.5, 7.5, etc. In consequence, in those conditions, the variability in time lags between occasions is not specifically accounted for in the model. Using discrete measurement occasions also entails creating very sparse data sets and very low coverage –or even zero– for some of the covariances in the covariance matrix. For the research design 2, there are no data points available for the even years. Therefore, for this design, the even years were modeled as phantom variables.

b) State Space Model in continuous time (SSM-CT). This model is analogous to the LCS-DT and includes the same number of parameters (see Table 1). It is composed of two equations⁵:

$$dy_t / dt = Ay_t + Bu_t + q_t \quad [4]$$

$$Y_t = Cy_t + Du_t + r_t \quad [5]$$

Equation 4 is called the *state equation* and describes how the latent states change over time with a first-order linear differential equation: y_t is an $l \times 1$ vector of latent states, t is time –

⁵ We adapt here the notation presented in Hunter (2018), also used in *OpenMx*. To highlight the similarities with the common LCS-DT specification, we changed the notation of the latent states (y) and observed variables (Y).

or an equivalent index. The derivative of y_t with respect to t is dy_t/dt . u_t is an $m \times 1$ vector of covariates or exogenous variables, and q_t is an $l \times 1$ vector of dynamic noise with mean zero and covariance Q . A is an $l \times l$ matrix of autoregressive dynamics –i.e., drift matrix–, and B is an $l \times m$ matrix of covariate effects on the latent states. Equation 5 is called the *output equation*. It is identical to the measurement model in the SEM framework (Chow et al., 2010; Hunter, 2018) and describes how the latent states relate to the observed variables at a single point in time: Y_t is a $n \times 1$ vector of observed variables or outputs at time t , r_t is an $n \times 1$ vector of observation noise (or measurement error) with mean zero and covariance R . C is an $n \times l$ matrix of factor loadings, and D is an $n \times m$ matrix of covariate effects on the observed variables. Note that, because our model did not include covariates or dynamic noise, Equation 4 is a re-expression of Equation 3.

Estimation is carried out through a set of recursive algorithms called hybrid Kalman Filter (Boker et al., 2018; Chow et al., 2010; Hunter, 2018). This procedure iterates in cycles of one prediction step (using a Kalman-Bucy filter) and one correction step (using a Kalman filter). For a given time t , the prediction step makes a forecast for (the factor scores in) the state vector y_t and the state covariance matrix, based on the initial state of the system at t_0 , the time interval between t and t_0 , and the dynamics of the system (A , B , Q and u_t). The update –or correction– step uses the observed data and the measurement model to correct the forecast from the previous step. The algorithm is designed to iteratively reduce the prediction error by adjusting the parameter estimates through Maximum Likelihood prediction error decomposition (for further details, see Boker et al., 2018; Chow et al., 2010; Hunter, 2018; Kalman, 1960; Kalman & Bucy, 1961).

We estimated this model through the functions *mxExpectationStateSpaceContinuousTime* and *mxFitFunctionML* in *OpenMx* (Boker et al., 2018; Hunter, 2018; Neale et al., 2016). In this implementation, a multiple group model is estimated in which each person is a group with a time series of observations, and all free parameters are constrained to be equal across “groups”. This

specification uses only the available data points and the exact time –i.e., age– at which they were measured. Therefore, potential “missing data” is not an issue. The *R* code for estimating both the LCS-DT and the SSM-CT is available at:

<https://github.com/EduardoEstradaRs/PsychMeths2019-Developmental-Processes-ALD-DT-CT-LCS>

The time metric was the same for both models: a one-point increase represented one year. However, the interpretation and scale of the parameters β , μ_s , $\sigma^2_{s,s}$, and $\sigma_{I0,s}$ (but not $\rho_{I0,s}$) differs between models (Voelkle & Oud, 2015; Voelkle et al., 2012). In the DT model they describe the change with $\Delta t=1$, whereas in the CT model they do so for an interval $\Delta t \rightarrow 0$. Table 1 reports the parameters from the generating model in both discrete and continuous time, and the equivalences between them. After fitting both models to all samples, we obtained the parameter estimates and their standard errors.

In any empirical situation, and especially when alternative statistical models are being compared, it is highly recommended to compute various fit indices for the models and assess the extent to which the data are adequately represented by each of them. In our study, however, the fit indices most suitable for comparing LCS-DT and SSM-CT – $-2\text{Log}L$, *AIC* and *BIC* – are completely dependent on the number of data points available in the dataset. Consequently, both models are expected to achieve the exact same fit. In fact, their mean fit value was exactly the same in all conditions. The variability –i.e., standard deviation– in model fit was also fairly similar between LCS-DT and SSM-CT. Thus, we did not include the fit results in the paper. This information, along with the standard error for the relative biases, is available from the authors upon request.

Results

Both model specifications converged for all samples in all conditions. We evaluated the accuracy and variability of the parameter estimates from both models.

Convergence between cohorts.

Accuracy. For estimating accuracy, we computed the relative bias for each of the parameters in the model: $RB = (\bar{\theta}_{est} - \theta) / \theta$, where θ is the true parameter value and $\bar{\theta}_{est}$ is the average estimated value of the parameter across all replications in a given condition⁶. Figure 5 (top section) reports the relative bias (RB) for all seven model parameters across the conditions with convergent cohorts.

INSERT FIGURE 5 HERE⁷

Because the covariance between the latent level and additive components ($\sigma_{l0,s}$) is highly dependent on the metric –i.e., variance– of both of them, we report the correlation ($\rho_{l0,s}$) instead of the covariance in top section of Figure 5. This allows an easier comparison between conditions. Values of RB closer to zero imply unbiased estimates, positive values imply overestimation, and negative values imply underestimation. Previous literature has considered estimates to be substantially biased when $|RB| > .10$ (Flora & Curran, 2004; Rhemtulla, Brosseau-Liard, & Savalei, 2012).

Based on the results depicted in the top section of Figure 5, the SSM-CT led to unbiased estimates for all seven parameters across all conditions, regardless of research design, presence of random lags, or sample size. The LCS-DT also led to unbiased estimates in all conditions with fixed lags between measurement occasions. Interestingly, in the conditions with random lags, LCS-DT yielded somewhat biased estimates, particularly for designs 3 and 4. Specifically, the variance of the measurement error σ_e^2 was overestimated in most conditions (mean RB across conditions with random lags = .108). The variance of the latent linear component σ_s^2 and,

⁶ Due to age binning, when the LCS-DT model was applied to the conditions with random lags, the first measurement occasion was at 5.5 years. Thus, the latent level had different expected values for its mean (14.38), variance (22.25), and correlation with the latent additive component (.759). In these conditions, we compared $\bar{\theta}_{est}$ with these expected values, not the ones reported in Table 1.

⁷ For clarity, we report our results in figures in the body of the manuscript. See the Appendix for tables with all the numerical results.

particularly, the latent correlation $\rho_{l0,s}$ also tended to be biased in research designs 3 and 4. The variance was overestimated, and the correlation was underestimated, with a much larger bias than in the fixed lag conditions. The latent correlation was also underestimated in designs 5, 6 and 7, although the bias was lower in these three conditions.

We computed the total relative bias for all parameters as $RMSRB = \sqrt{\sum_{k=1}^K RB_k^2 / K}$,

where $K=7$ is the number of parameters in the model. The bottom section of Figure 5 depicts the $RMSRB$ for both models across all the conditions. For the SSM-CT model, the total RB was low in all conditions. The highest $RMSRB$ was found in for design three, with 200 cases and fixed lags ($RMSRB_{SSM-CT} = .029$). The LCS-DT model yielded nearly identically accurate estimates in conditions with fixed lags. However, it showed higher $RMSRB$ in some conditions with random lags: the mean value of $RMSRB$ was .102 in designs 3 and 4, and .058 in the remaining designs. The highest values of $RMSRB$ were found in conditions that combined random lags, designs 3 and 4, and LCS-DT model. In these conditions, larger sample size led to slightly higher bias.

In summary, the accuracy results indicate that the SSM-CT was very robust for recovering the parameters from the true generating model and yielded unbiased estimates for all conditions in the study. The LCS-DT led to unbiased estimates in most conditions, but it achieved lower accuracy for some parameters in conditions with random time lags between measurement occasions, particularly designs 3 and 4. We discuss the reasons for this in the discussion section.

Standard deviation of the relative bias. For quantifying the variability of the parameter estimates, we computed the standard deviation of the relative bias across all the samples in each condition. Because each parameter has a different metric, we computed $SDRB$ as $SD[(\theta_{est} - \theta) / \theta]$.⁸ This index allowed expressing the estimation inefficiency in the same scale for all

⁸ When comparing different models, studies on incomplete data often use a measure of efficiency based on the ratio

parameters. This index is always positive, and values closer to zero imply less variability of the estimates in any given condition –i.e., more efficiency. Figure 6 (top section) depicts the *SDRB* for all parameters in both models in all the conditions.

INSERT FIGURE 6 HERE

Unlike the relative bias, there are no clear criteria for determining substantive variability (i.e., inefficiency) of the estimates. Values closer to zero indicate that the estimates have similar values across samples and, therefore, the standard deviation of θ_{est} is low. The values in the top section of Figure 6 indicate that, in general, the parameters can be divided in two groups with relatively high and low variability. The parameters defining the mean trajectory (self-feedback β , mean level μ_{l0} , and mean linear component μ_s) were fairly efficient for both models in all conditions. The *SDRB* range was .006-.065 for β , .016-.053 for μ_{l0} , and .006-.054 for μ_s . In contrast, the parameters accounting for the variance in the individual trajectories (variance of the latent level σ^2_{l0} and latent linear component σ^2_s , and correlation between them $\rho_{l0,s}$) showed higher variability. The *SDRB* range was .070-.230 for σ^2_{l0} , .074-.249 for σ^2_s , and .031-.304 for $\rho_{l0,s}$. The measurement error variance σ^2_e was in between these two groups, with *SDRB* range .017-.140. The three parameters with higher *SDRB* showed worse performance in the research designs three and four, especially in the conditions with $n=200$. Both specifications yielded similar values.

The bottom section of Figure 6 reports the general *SDRB* of each model, computed as the mean of all seven parameters. These results were fairly similar across both specifications and sample sizes –although the largest n achieved slightly lower values. The average *SDRB* for designs one to seven was .042, .045, .120, .105, .079, .081, and .088, respectively. Thus, designs

[variance(estimates from model₀) / variance (estimates from model₁)], where model₀ is the baseline model, expected to be more efficient –i.e., achieve lower variance of parameter estimates–, and model₁ is an alternative model expected to show higher variability –i.e., lower efficiency. We did not use this measure of efficiency because there is no baseline model common for all the simulation conditions. Instead, we compared the variability with the true parameter value, which was the only invariant criterion across conditions.

three and four showed higher variability. As expected, design one –with highest data density– showed the lowest estimate variability. Interestingly, the difference between designs one and two was minimal on average and across all conditions. This implies that taking 8 measures per case is almost as efficient as taking 15, in terms of variability in parameter estimation. It is also interesting to compare the variability of designs four, five and seven, because they have a similar number of measures per case –three measures in designs four and five, and 3.15 measures on average in design seven. Among them, design (3 points per person, spaced every other year) five was the most efficient. Design 6 uses four measures per case. However, it led to higher variability than design 5 (three measures) for all conditions except for fixed lags and $n=500$.

In summary, the findings on this section indicate that: a) In most scenarios, covering a wide age range for each individual is preferable over including additional assessments; b) There does not seem to be large differences between both specifications in terms of estimate variability.

Coverage. This index is computed as the proportion of 95% confidence intervals around the estimated parameter value that include the true parameter value⁹. As such, 95% is the optimal value of coverage, and coverage below 90% is considered inadequate (Collins, Schafer, & Kam, 2001; Enders & Peugh, 2004). We computed the coverage for all the parameters across all the conditions. The results are summarized in Figure 7.

INSERT FIGURE 7 HERE

The coverage rates were mostly explained by the degree to which the parameter estimates were biased. Thus, all parameters had excellent coverage across most conditions. However, the LCS-DT model fitted in the conditions with random lags yielded very low rates of coverage for the measurement error variance σ_e^2 –except for the research designs 3 and 5, probably due to high standard errors leading to wider confidence intervals in such designs. The latent covariance

⁹ Created using standard error estimates. These are Wald-type confidence intervals based on asymptotic normal theory, not profile likelihood confidence intervals.

between intercept and additive component also showed poor coverage with LCS-DT, random lags and $n=500$.

Together with the results in Figure 5, the coverage results indicate that estimating an LCS-DT model in a situation with non-fixed time lags leads to substantial bias in the variance of the measurement error, and in the latent correlation/covariance between level and linear additive component. The rest of parameter estimates appear to be accurate in both specifications.

Non-convergence between cohorts.

We discuss here the results from the research designs including different cohorts –i.e., research designs 1 and 2 are included only for reference, as they involve a single cohort. Figure 8 depicts the observed values in one convergent and one non-convergent sample of $n=500$, with random lags, from research design 5. Interestingly, the trajectories in Figure 8 appear to be quite similar in the convergent (left) and non-convergent (right) samples.

INSERT FIGURE 8 HERE

Mere visual inspection of Figure 8 may lead to the conclusion that cohort effects are not present or relevant. However, non-convergence had a large impact on some parameter estimates. Figure 9 depicts the relative bias (RB) across all non-convergence conditions. We only report the values for the five parameters with identical values across different cohorts –i.e., the mean and variance of the latent additive component are not discussed because the former varies across cohorts and the latter is expected to be higher than the generating value for any sample comprised of different cohorts.

INSERT FIGURE 9 HERE

Figure 9 (top section) shows that the model parameters were substantially biased in the conditions including different cohorts. The most consistent pattern was found for the self-feedback β : it was substantially overestimated in all conditions, particularly for designs 3 and 4,

irrespective of the model, type of lags or sample size. The measurement error variance σ_e^2 followed a pattern similar to the conditions with cohort convergence: it was overestimated in the LCS-DT model with random lags. The variance of the latent intercept $\sigma_{\mu_0}^2$ showed no relevant differences relative to convergence conditions. The mean of the latent intercept μ_{μ_0} was somewhat underestimated in most conditions. The rest of parameters followed different patterns depending on the condition, but none of them appeared to be robust to non-convergence. The values for *RMSRB* (Figure 9, bottom section) indicate that, in all conditions, the non-convergence led to an increase in the total model bias. This increase was higher for the LCS-DT model in the conditions with random lags and for research designs 3 and 4.

Figure 10 depicts the parameter coverage for the parameters with invariant values across cohorts. The results in Figure 10 indicate that even relatively small bias in the parameter estimates led to a substantial loss of coverage. Specifically, the coverage for the self-feedback β ranged between 0 and .43 (*mean*=.37), and between .13 and .94 (*mean*=.69) for the mean of the latent intercept μ_{μ_0} . The remaining parameters showed a pattern similar to the conditions with convergence: the latent intercept variance had excellent coverage in all conditions, whereas the coverage for the measurement error variance σ_e^2 was inadequate only in the LCS-DT model in the conditions with random lags.

INSERT FIGURE 10 HERE

In summary, the between-cohorts differences in the mean of the latent additive component μ_s affected all the parameters, except the variances of the latent intercept and measurement error. These results suggest that differences among cohorts lead to nontrivial model misspecification. Such misspecification affected both the SSM-CT and the LCS-DT models, and this was the case irrespective of the research design.

Summary of findings

In this study, we sought to evaluate the practical relevance of using discrete- and continuous-time models for recovering the trajectories of a developmental process from data obtained through different ALDs. For this, we used a simulation study with varying conditions of sampling design, sample size, and cohort convergence. Several findings from our analyses are worth noting.

The first relevant finding concerns convergence. When the different cohorts in the study come from the same population –i.e., convergence is met–, and the time intervals between measures are constant across cases and measurement occasions, both discrete- and continuous-time models can adequately recover the parameters defining the underlying developmental process using ALDs. This implies that such trajectories can be recovered without having to measure all the sample cases at all the ages in the study. Under many conditions, both models achieved excellent results in terms of convergence rates, parameter accuracy, efficiency and coverage. However, the discrete-time models had substantially worse performance in some scenarios that are common in empirical research. When the time lags were not constant –i.e., random lags conditions–, the LCS-DT models yielded estimates with marked bias. These models had an especially poor performance in research designs with two measures per case separated by two years each (D3), and with three measures per case taken in consecutive years (D4). In the rest of conditions with cohort convergence, the results from both types of models were fairly similar.

Regarding sample size, we did not find a substantial effect. The accuracy and coverage for the parameters were similar for the two sample sizes tested ($n=200$ and $n=500$). As expected, the estimates showed lower variability with larger samples, but our results suggest that other design and model features –such as the specific parameter and the sampling schedule– have a stronger effect on variability.

Another important finding from our analyses is related to the sampling schedules. This finding is particularly relevant given its potential implication for the design of longitudinal studies. The first two designs in our study (D1 and D2) implied following a single cohort during a period of 15 years, from age 5 to age 19. D1 involves 15 assessments per case (one every year), whereas D2 involves 8 assessments (only the odd years). Interestingly, both designs led to similar results: we found only minimal differences between them regarding convergence rate, parameter accuracy, variability, and coverage. Therefore, at least in the conditions simulated here and for the models under consideration, D2 appears to be a better choice than D1 because it achieves similar performance using 53.3% of the information.

If cohort convergence holds, a comparison between D2 and D5 (three assessments per case, in alternate years) is worth reporting. We found that model performance for D2 was slightly better for most indicators. When considering the practical implications of the design, however, we found that D5 was a more efficient choice because D2 requires following participants during 15 years instead of five and because D5 requires only $3/8=37.5\%$ of the data points in D2. Another important comparison is D2 with $n=200$ (1600 measures in total) and D5 with $n=500$ (1500 measures). Overall, we found that the results between both designs are fairly similar in all indicators. Both conditions require a similar number of data points but the former implies a much longer duration of the study. Of course, these comparisons are reasonable only if the assumption of cohort convergence holds –note that D2 involves only one cohort and thus it is not affected by non-convergence.

Design 5 also achieved better results than the two other designs with three measures per case, D4 and D7 (3.15 measures on average). Moreover, D5 achieved better overall results than D6, which indicates that covering a wider age range per case is preferable over having more measures and a narrower age span. These findings –and the relative superiority of D5– were consistent in conditions of cohort convergence. We must note, however, that it is unclear whether

these findings can be generalized to different ALDs or other statistical models not tested here.

Importantly, in the conditions with non-convergent cohorts –i.e., each cohort is drawn from a population with slightly different values for the mean latent additive component–, none of the research designs including cohorts yielded adequate results. We found marked bias and low coverage rates even for parameters that were invariant across cohorts. Therefore, we hold that using ALDs is problematic unless cohort effects are detected and controlled for (see below).

Interpretation of parameters in the SSM-CT model

Because continuous time models may be relatively new to some readers, here we provide a brief interpretation of the SSM-CT parameters. All parameters from these models are interpretable in continuous-time metric, although some of them may be more informative for specific research questions after being transformed to a specific DT metric –e.g., if the researcher is interested in a particular time lag, the parameters can be transformed to that specific metric. All parameters capturing a discrete time feature have the exact same interpretation as in the LCS-DT model: a) The mean and variance of the latent intercept (μ_{t0} and σ^2_{t0}) capture the mean and variability of the latent process y when $t=0$. Note that, in our data, $t=0$ was the first measurement occasion in the sample, but the time scale can be arbitrarily modified so that $t=0$ refers to a different time point. b) The measurement error variance σ^2_e refers to the variability introduced in the observed scores whenever the process is measured. This variability does not affect the latent scores and is not carried over subsequent states of the system. c) The latent correlation between intercept and additive component (ρ_{t0-s}) captures the linear relation between both latent variables. Because a correlation has a standardized metric, it does not change when the metric of such latent variables change.

On the other hand, the parameters defining how the process changes over time have a metric that is dependent on the time interval in which they are defined. These parameters are the

mean and variance of the additive component, μ_s and σ^2_s , the self-feedback β , and the covariance between intercept and additive component σ_{I0-s} . In our CT model, these are part of a differential equation system and, therefore, refer to an infinitesimally small lapse of time. In our univariate specification, the mean μ_s and self-feedback β define an exponential trajectory. If $\beta < 0$, the trajectory approximate the asymptote given by $y_{As} = y_s / (-\beta)$. This trajectory is increasing if $y_{As} > y_{I0}$, and decreasing if $y_{As} < y_{I0}$. If $\beta > 0$, the system becomes volatile and tends to positive or negative infinity. These four parameters can be transformed to any arbitrary time interval with the equations provided in Table 1. The different values for different time lags have important substantive implications and can account for inconsistent findings across studies using different time lags (for more details see Voelkle et al., 2012).

Theoretical and methodological considerations

Despite recent computational developments, continuous time models are underused in psychological research. Several reasons for this appear plausible. First, they may appear harder to understand and use. This could be due to the understanding of calculus required for using them. Second, they are relatively novel, compared to better-known longitudinal approaches (such as SEM or multilevel models). In spite of these issues, the SSM-CT model in this study is actually more convenient to implement with empirical data than the alternative LCS-DT. Due to its treatment of the studied processes as continuous entities, SSM-CT provides built-in mechanisms for handling the time lag variability across different participants and occasions within participants. Hence, it does not require dummy or phantom variables nor specific procedures to deal with data incompleteness (in univariate systems). Both approaches (DT and CT) can be implemented in the open source widely available *R* framework, and both require similar programming skills (the code for the two specifications in this paper can be found at:

<https://github.com/EduardoEstradaRs/PsychMeths2019-Developmental-Processes-ALD-DT-CT->

[LCS](#)).

It is reasonable to think that the SSM-CT model will yield estimates at least as good as the LCS-DT with ALD data for at least two reasons: 1) Age binning involves losing relevant between-individual variability about the exact time at which information is gathered for each case. At the same time, binning adds additional non-relevant variance –i.e., error– in the trajectories –see Figure 11–; 2) The LCS-DT model is a particular –constrained– case of the SSM-CT. Any data structure that can be modeled through the LCS-DT can be modeled through the SSM-CT at least equally well. However, our findings are relevant for several reasons. First, virtually all the studies analyzing data from ALD have used random lags and LCS-DT models. This combination of conditions yielded the worst results in our simulations, even when the convergence assumption was met. Second, our results provide empirical evidence for the idea that LCS-DT actually has an adequate performance under some conditions: with fixed time lags and cohort convergence, the differences between discrete- and continuous-time models were minimal.

In the conditions with cohort convergence, LCS-DT had adequate performance as long as the time lags were constant. However, it had poor performance with random lags: the variance of the measurement error and, in some conditions, the latent correlation between level and additive component were substantially biased. Random lags also led to poor coverage of these parameters, as reported in Figure 7. It appears that using measurement occasion as the time metric had an adverse effect on the model estimates: ignoring the differences in time lags causes the individual trajectories to be “ragged”. This causes overestimation of the measurement error variance. Figure 11 depicts this problem by comparing the trajectories in one sample with random lags. The top panels in Figure 11 represent the latent and observed trajectories for y when age was used as time signature. The bottom panels represent the same trajectories from the same sample, but using the measurement occasion (age bin) as the time signature. Note that even

the latent trajectories in the bottom left panel are ragged.

INSERT FIGURE 11 HERE

This last finding is highly relevant because the vast majority of ALDs conducted to present date had varying time intervals. Our results show that, unless every participant is measured at the same age every year, the measurement error variance will likely be overestimated. These results are consistent with previous research on the effects of sampling-time variation on parameter estimation in latent growth curves (Miller & Ferrer, 2017). In that study, time binning led to overestimation of the measurement error variance.

One finding worth highlighting regards designs with two measurement occasions. In our study, D3 entails two measures per case, each taken two years apart. Considering a hypothetical budget of time and money, this was our least expensive schedule: it required the fewest data points and the shortest study duration. Interestingly, even with this sparse design, the SSM-CT yielded unbiased estimates and excellent coverage for the conditions with cohort convergence in our study. Regarding estimation variability, the parameters defining the mean trajectory (means of latent level and additive component, and self-feedback) were estimated very efficiently, whereas the rest of the parameters, capturing the variability in the individual trajectories, showed higher variability. These findings conflict with the still common conception in the literature that two time points are insufficient for studying change (Rogosa, 1995; Rogosa, Brandt, & Zimowski, 1982; Rogosa & Willett, 1983). In fact, our results show that, when using the right sampling scheme and an appropriate statistical model (in CT), two measures per case *do* provide enough information for recovering the generating process without bias. It is true that D3 had the highest estimation variability, particularly for some of the parameters. But, overall, our results support the applicability of this design, as long as the cohorts are convergent, and especially if funding or other problems make it unfeasible to collect a third measure (D5).

However, in conditions without cohort convergence, all the ALDs (D3 to D7) yielded

biased parameters. The self-feedback was particularly affected, as it was overestimated in all conditions. This structural misfit was not corrected by a larger sample size or by a CT model. One likely reason for this is that, because the mean trajectory for the whole sample is given by the self-feedback and the latent additive component, the cohort-related increase in the mean additive component forced the models to overestimate the self-feedback as well. This finding has important implications for developmental research. It shows that dynamic parameters in an LCS/SSM model will likely be biased if there is no convergence between the cohorts. It is reasonable to expect that the effects of this misspecification might be even more relevant in multivariate systems.

The procedure of simulating full trajectories from a developmental dynamic process, and then “degrading” them by creating data sparsity according to different schemes was used by Hamagami & McArdle (2001) to study the performance of LCS models. They found that the parameters in their model could be recovered even in two-occasion and three-occasion data sets, involving >85% sparsity. The present study could be considered as a continuation and expansion of this previous research, using modern CT models and with a focus on the best practices for longitudinal research design.

Some authors have pointed out that using unequally spaced measurements is more efficient for gathering information about longitudinal process (de Haan-Rietdijk et al., 2017; Voelkle & Oud, 2012). Our results do not fully support that claim. In fact, for the SSM-CT model, we did not find any substantial effect of having fixed or random lags in accuracy, coverage, or estimation variability. The biases of the standard errors were not identical in these two conditions, but the results in Figure 7 do not indicate a clear advantage of using random lags, especially if SSM-CT is used.

We used the state-space modeling routines implemented in *OpenMx* (Hunter, 2018). This approach is very convenient when the dynamic function defining change over time is linear –

e.g., in our data, dy/dt was a linear function of y_l and y_s . However, other longitudinal processes may require specifying different functions, including nonlinear rates of instantaneous change, oscillatory processes, non-stationary features, thresholds (Hamaker, Zhang, & Maas, 2009), or multiple regimes. In such scenarios, researchers can resort to other software packages such as *dynr* (Ou et al., 2017) and *ctsem* (Driver et al., 2017). Similarly, functional data analysis is an alternative non-parametric framework for modeling longitudinal processes in continuous time that allows answering different questions about the data (Levitin, Nuzzo, Vines, & Ramsay, 2007; Ramsay & Silverman, 2007; Ullah & Finch, 2013).

Limitations and future directions

In this study, we focused on recovering the dynamics of a system composed of one exponential process only. LCS and SSM models are particularly useful for capturing the dynamics of such systems, and these dynamics are more complex and interesting when the system includes more than one variable or when influenced by external factors.

Future research could extend this work by examining the performance of LCS-DT and SSM-CT models in ALDs with multivariate systems involving two or more processes. In particular, it would be interesting to evaluate how the different designs studied here perform with respect to the coupling parameters in a bivariate LCS model. Indeed, researchers are often interested in bivariate LCS applications. However, it is reasonable to start the investigation in a relatively simple scenario to fully understand its underlying mechanism before moving to more complex extensions. To the best of our knowledge, our study is the first systematic comparison of the performance of DT and CT LCS with data from ALDs, and the findings reported here are relevant for a wide variety of research domains. Similarly, trajectories other than those described by exponential functions are worth pursuing. These could include complex dynamics systems such as oscillators (McKee, Rappaport, Boker, Moskowitz, & Neale, 2018; Voelkle & Oud,

2012), as well as trajectories with discrete change points (McArdle & Wang, 2008).

Another aspect that deserves further investigation is the extension of the described model to include both time invariant and time varying covariates. Time invariant covariates can capture between-person differences in psychological or demographical features –e.g., gender, personality traits, SES, type of school, to name only a few (S. C. Duncan et al., 1996; Raudenbush & Chan, 1992). For example, researchers may be interested in evaluating whether the latent intercept, and additive component, are different between boys and girls, or whether they can be predicted by conscientiousness, or other personality traits. In contrast, time varying covariates can capture the influence of traits that change over time, as well as the occurrence of important life events. For example, the general level of anxiety, or the application of a psychological intervention program, or the change from middle to high school, could affect cognitive development during adolescence.

In empirical settings, the assumption that participants from different cohorts come from the same population is not always tenable. Indeed, social influences such as changes in the school curriculum can have important effects on the development of some cognitive abilities such as reading or mathematics. Our results show that even relatively small cohort effects can lead to substantial model misspecification. Miyazaki & Raudenbush (2000) investigated how to include cohorts effect in latent growth curve models, but to the best of our knowledge, there are no general methods for detecting and controlling for cohort differences in LCS/SSM, or across different models. In principle, the specific type of non-convergence in our data could be controlled by adding *age of birth* as a time invariant covariate in the model, and allowing it to predict the latent additive component. That way, the differences between cohorts would be accounted for. However, this solution requires knowing the specific part of the model that differs across cohorts. In empirical work, researchers do not know whether convergence assumptions are met in their data, and do not always have clear hypotheses about which aspects of the

trajectories may or may not be constant across cohorts. Moreover, cohort differences in the self-feedback parameter cannot be captured by means of a covariate in the traditional LCS model. Therefore, future research should investigate this important problem and determine how to test for different forms of non-convergence. It is reasonable to think that research designs involving larger cohort overlap would be better to detect cohort differences.

We note that, in the context of developmental processes, we selected a number of research designs that are often found in empirical developmental research on cognitive development. Of course, our findings apply to such designs, and generalization to other sampling schemes should be done very carefully.

Conclusion

Studying the time-related sequences of psychological processes is a challenging endeavor. Accelerated longitudinal designs allow reducing the duration of empirical studies by combining data from multiple cohorts. Our results support the appropriateness of these designs, in combination with LCS models, to study developmental processes, under particular sampling conditions, and as long as cohort convergence can be assumed (importantly, when the different cohorts cannot be assumed to be convergent, both DT and CT models are expected to yield biased estimates and poor coverage rates). Specifically, based on our results, we make the following recommendations¹⁰:

1. If a single cohort can be followed over a long period of time, taking one measure per year does not add much additional information over taking one measure every other year. Therefore, design 2 is recommended over design 1.
2. If multiple convergent cohorts are available, taking three measures per case in alternate

¹⁰ Of course, these recommendations apply only to the conditions tested in our simulation.

years (design 5) appears to be an optimal strategy, relative to the sampling schemes evaluated here.

3. If the observations cannot be equally spaced –as is the case in most empirical studies, the SSM-CT model should be used to avoid over-estimation of the measurement error variance (i.e., applying LCS-DT models with age binning is tenable only if the time lags between measures are constant across cases and time points).
4. If a sample of around 200 cases is available, including additional measures per case is preferable over including additional cases.
5. If two measures per case, separated by two years each, are available (design 3), adding a third measure (design 5) is recommendable. If this is not possible, using the SSM-CT model to analyze the data is the optimal approach.

Based on our findings, we encourage researchers to adopt continuous time models to analyze data from ALDs, as they achieved excellent accuracy, coverage, and overall efficiency in all simulated conditions with convergent cohorts. We hope the findings in this study provide researchers with useful information about study designs and data analyses for studying processes that unfold over time.

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Table 1. True parameter values in discrete and continuous time.

Parameter	Value in DT ($\Delta t=1$)	Value in CT	Transformation
β Self-feedback	-.228	-.259	$\beta_{DT} = e^{\beta_{CT} \cdot \Delta t} - 1$
μ_{I0} Intercept mean	12.096	12.096	
μ_s Additive component mean	7.050	8.001	$\mu_{s,DT} = \mu_{s,CT} \cdot \frac{e^{\beta_{CT} \cdot \Delta t} - 1}{\beta_{CT}}$
σ^2_{I0} Intercept variance	25.353	25.353	
σ^2_s Additive component variance	.573	.738	$\sigma^2_{s,DT} = \sigma^2_{s,CT} \cdot \left(\frac{e^{\beta_{CT} \cdot \Delta t} - 1}{\beta_{CT}} \right)^2$
σ_{I0-s} Intercept-Additive component covariance	2.736	3.105	$\sigma_{I0-s,DT} = \sigma_{I0-s,CT} \cdot \frac{e^{\beta_{CT} \cdot \Delta t} - 1}{\beta_{CT}}$
ρ_{I0-s} Intercept- Additive component correlation (this is just a transformation of σ_{I0-s})	.718	.718	
σ^2_e Measurement error variance	1.892	1.892	

Table 2. Sampling scheme for each research design. Shaded cells represent one measurement occasion for all cohort members during the corresponding year.

		Age in years															% of cases entering in each cohort	
	DESIGN	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19		
All cases measured at all occasions	D1	(1 cohort)															100.0	
	D2																100.0	
Two occasions per case	D3	13 cohorts																
																		13.3
																		13.3
																		6.7
																		6.7
																		6.7
																		6.7
																		6.7
																		6.7
																		6.7
																		6.7
																		6.7
																		6.7
Three occasions per case	D4	13 cohorts																
																		19.4
																		9.7
																		6.5
																		6.5
																		6.5
																		6.5
																		6.5
																		6.5
																		6.5
																		6.5
																		6.5
																		6.5
Three occasions per case	D5	11 cohorts																
																		18.8
																		18.8
																		9.4
																		9.4
																		6.3
																		6.3
																		6.3
																		6.3
																		6.3
																		6.3
																		6.3
																		6.3
Four occasions per case	D6	12 cohorts																
																		24.5
																		12.2
																		8.2
																		6.1
																		6.1
																		6.1
																		6.1
																		6.1
																		6.1
																		6.1
																		6.1
																		6.1
Younger cohort measured at more occasions	D7	13 cohorts																
																		19.4
																		9.7
																		6.5
																		4.8
																		4.8
																		6.5
																		6.5
																		6.5
																		9.7
																		6.5
																		6.5
																		6.5
																6.5		

Figure 1. Path diagram of a Latent Change Score model (top) and diagram of a SSM-CT model (bottom)

Figure 2. Individual trajectories in continuous time for the latent variable y ($n = 200$)

Figure 3. Latent and observed trajectories in each research design

Figure 4. Trajectories resulting from measuring the process y at fixed or random time lags, according to research design 2.

Note: In the fixed lags conditions, the individuals are measured the first week of the year. In the random lags conditions, they are measure one random week during the year.

Figure 5. Relative bias across all conditions with cohort convergence.

Note: Top section depicts *Relative Bias*, $RB = [mean(\theta_{est} - \theta)] / \theta$ for each parameter. Bottom section depicts the *(Total) Root Mean Square* $RME(RB) = \sqrt{\sum_{k=1}^K RB_k^2 / K}$, where $K=7$ is the number of parameters, for the whole model in all conditions.

Figure 6. Standard Deviation of Relative Bias (*SDRB*) across all conditions with cohort convergence.

Note: Top section depicts $SDRB = SD[(\theta_{est} - \theta) / \theta]$ for each parameter. Bottom section depicts the mean of the *SDRB* values for all seven parameters in each condition.

Figure 7. Rates of 95% Confidence Interval (CI) coverage for all parameters across all conditions with cohort convergence.

Figure 8. Observed trajectories of one convergent and one non-convergent sample with research design 5 and fixed lags ($n=500$).

Figure 9. Relative bias across all conditions without cohort convergence

Note: Only the parameters with invariant value across cohorts are depicted. The Total Relative Bias is computed based on such parameters.

Figure 10. Rates of 95% Confidence Interval coverage for all parameters across all conditions without cohort convergence.

Note: Only the parameters with invariant value across cohorts are depicted.

Figure 11. Latent and observed trajectories of one sample using age (top panels) and age bin (bottom panels) as the time metric ($n=200$)

Note: The age bins are created at the center of each year –i.e., 5.5, 6.5, 7.5, etc.

Appendix. Tables with numerical results

Table A1. Relative Bias with cohort convergence

design	lags	n	model	β	μ_0	μ_s	σ_0	σ_s	$\sigma_{0,s}$	σ_e	$\rho_{0,s}$	$RMS(RB)$
1	fixed	200	LCS-DT	-.001	.004	-.001	-.015	-.007	-.014	-.003	-.004	.007
1	fixed	500	LCS-DT	.000	-.002	-.001	-.011	-.003	-.008	.004	-.002	.005
2	fixed	200	LCS-DT	.003	.003	.002	-.016	.003	-.005	-.004	.001	.007
2	fixed	500	LCS-DT	.000	-.001	-.001	-.008	-.004	-.008	-.004	-.002	.004
3	fixed	200	LCS-DT	.009	.001	.006	-.046	.019	.022	-.001	.061	.030
3	fixed	500	LCS-DT	.001	.003	-.001	-.010	.000	-.007	.001	.007	.005
4	fixed	200	LCS-DT	.005	.003	.004	-.034	.005	-.005	-.001	.028	.017
4	fixed	500	LCS-DT	.006	-.001	.004	-.016	.004	.014	.009	.028	.013
5	fixed	200	LCS-DT	.003	.002	.003	-.020	.001	-.008	-.001	.006	.008
5	fixed	500	LCS-DT	-.001	-.002	-.002	-.015	-.011	.000	-.012	.016	.011
6	fixed	200	LCS-DT	.000	.002	.001	-.030	-.013	-.017	.003	.011	.013
6	fixed	500	LCS-DT	.005	-.002	.002	-.007	.007	-.002	-.002	.000	.004
7	fixed	200	LCS-DT	-.002	.003	-.002	-.024	-.010	-.019	-.001	.005	.010
7	fixed	500	LCS-DT	.000	.001	-.002	-.010	-.010	.005	-.001	.019	.009
1	random	200	LCS-DT	.001	-.007	.001	.012	-.003	-.017	.112	-.023	.044
1	random	500	LCS-DT	.001	-.012	.000	.022	-.003	-.010	.116	-.019	.045
2	random	200	LCS-DT	.000	-.007	.000	.045	-.012	-.014	.097	-.031	.043
2	random	500	LCS-DT	.002	-.010	.000	.027	-.004	-.018	.093	-.030	.039
3	random	200	LCS-DT	-.002	-.003	-.003	.057	.078	-.190	.068	-.212	.092
3	random	500	LCS-DT	.000	-.008	-.002	.110	.080	-.218	.055	-.273	.117
4	random	200	LCS-DT	.003	-.007	.001	.044	.065	-.164	.118	-.193	.091
4	random	500	LCS-DT	.004	-.009	.001	.058	.090	-.187	.124	-.235	.108
5	random	200	LCS-DT	.006	-.007	.004	.042	.016	-.069	.099	-.088	.053
5	random	500	LCS-DT	.002	-.013	.001	.072	.019	-.076	.081	-.115	.061
6	random	200	LCS-DT	.002	-.008	.001	.028	.026	-.091	.144	-.106	.069
6	random	500	LCS-DT	.004	-.010	.001	.054	.045	-.111	.126	-.150	.079
7	random	200	LCS-DT	.006	-.005	.004	.006	.042	-.113	.122	-.122	.067
7	random	500	LCS-DT	.003	-.010	.001	.037	.028	-.090	.161	-.114	.077
1	fixed	200	SSM-CT	-.002	-.004	-.001	-.009	-.007	-.013	-.003	-.006	.005
1	fixed	500	SSM-CT	-.001	-.009	-.001	-.006	-.003	-.008	.004	-.004	.005
2	fixed	200	SSM-CT	.002	-.005	.002	-.011	.004	-.004	-.004	-.001	.005
2	fixed	500	SSM-CT	-.001	-.009	-.001	-.003	-.004	-.007	-.004	-.005	.004
3	fixed	200	SSM-CT	.010	-.007	.008	-.041	.023	.025	-.001	.060	.029
3	fixed	500	SSM-CT	.000	-.004	-.001	-.005	.002	-.006	.001	.004	.003
4	fixed	200	SSM-CT	.005	-.005	.005	-.029	.009	-.002	-.001	.026	.015
4	fixed	500	SSM-CT	.006	-.009	.005	-.010	.007	.015	.009	.026	.012
5	fixed	200	SSM-CT	.003	-.005	.004	-.015	.002	-.007	-.001	.003	.006
5	fixed	500	SSM-CT	-.002	-.010	-.002	-.010	-.011	.000	-.012	.014	.010
6	fixed	200	SSM-CT	.000	-.006	.001	-.025	-.011	-.015	.003	.009	.011
6	fixed	500	SSM-CT	.004	-.010	.003	-.002	.008	-.001	-.002	-.002	.005
7	fixed	200	SSM-CT	-.003	-.005	-.002	-.019	-.009	-.018	-.001	.003	.008
7	fixed	500	SSM-CT	-.001	-.007	-.001	-.005	-.009	.005	-.001	.017	.008
1	random	200	SSM-CT	-.001	-.003	.000	-.006	-.011	-.017	-.008	-.011	.007
1	random	500	SSM-CT	.000	-.009	-.001	-.003	-.004	-.006	.003	-.003	.004
2	random	200	SSM-CT	-.002	-.004	-.001	.000	-.013	-.008	-.002	-.003	.005
2	random	500	SSM-CT	.001	-.009	.001	-.009	-.003	-.007	-.005	-.003	.005

3	random	200	SSM-CT	.013	-.005	.011	-.014	.022	-.007	-.013	.012	.014
3	random	500	SSM-CT	.001	-.005	.000	-.002	-.008	.004	.006	.025	.010
4	random	200	SSM-CT	.010	-.003	.008	-.021	.007	-.010	-.015	.013	.012
4	random	500	SSM-CT	.001	-.005	.000	-.011	.009	-.018	-.001	-.006	.006
5	random	200	SSM-CT	.006	-.005	.005	-.035	.002	-.007	.011	.018	.016
5	random	500	SSM-CT	.004	-.010	.003	.004	-.001	.000	-.003	.001	.005
6	random	200	SSM-CT	.002	-.004	.002	-.028	-.013	.008	.013	.038	.019
6	random	500	SSM-CT	.003	-.009	.002	.005	.000	-.011	-.002	-.010	.006
7	random	200	SSM-CT	.007	-.003	.005	-.040	.011	-.030	-.019	.001	.017
7	random	500	SSM-CT	.003	-.008	.001	-.012	.000	.004	.010	.013	.008

Table A2. Standard deviation of the relative bias (*SDRB*) with cohort convergence

design	lags	n	model	β	μ_0	μ_s	σ_0	σ_s	$\sigma_{0,s}$	σ_e	$\rho_{0,s}$	Mean SDRB
1	fixed	200	LCS-DT	.010	.030	.011	.107	.111	.130	.027	.055	.050
1	fixed	500	LCS-DT	.006	.019	.006	.071	.079	.088	.018	.033	.033
2	fixed	200	LCS-DT	.014	.030	.015	.108	.116	.136	.043	.058	.055
2	fixed	500	LCS-DT	.009	.019	.009	.070	.081	.083	.027	.031	.035
3	fixed	200	LCS-DT	.057	.048	.046	.182	.215	.278	.128	.302	.140
3	fixed	500	LCS-DT	.035	.029	.028	.120	.144	.179	.079	.182	.088
4	fixed	200	LCS-DT	.055	.052	.043	.189	.210	.276	.076	.274	.128
4	fixed	500	LCS-DT	.032	.030	.025	.123	.127	.146	.051	.152	.077
5	fixed	200	LCS-DT	.039	.043	.031	.148	.134	.192	.090	.155	.092
5	fixed	500	LCS-DT	.028	.023	.023	.104	.112	.117	.054	.097	.063
6	fixed	200	LCS-DT	.040	.040	.031	.168	.150	.200	.063	.165	.094
6	fixed	500	LCS-DT	.027	.025	.021	.104	.110	.122	.041	.094	.060
7	fixed	200	LCS-DT	.040	.048	.031	.186	.148	.213	.081	.178	.102
7	fixed	500	LCS-DT	.027	.028	.022	.123	.107	.123	.055	.105	.067
1	random	200	LCS-DT	.013	.025	.012	.104	.108	.126	.030	.051	.049
1	random	500	LCS-DT	.009	.016	.009	.081	.074	.087	.020	.031	.034
2	random	200	LCS-DT	.016	.024	.016	.106	.111	.126	.044	.052	.053
2	random	500	LCS-DT	.012	.016	.012	.077	.077	.088	.027	.034	.036
3	random	200	LCS-DT	.062	.044	.052	.230	.249	.263	.140	.281	.151
3	random	500	LCS-DT	.046	.025	.039	.147	.178	.169	.101	.181	.102
4	random	200	LCS-DT	.059	.041	.049	.197	.218	.266	.087	.251	.129
4	random	500	LCS-DT	.041	.027	.033	.126	.162	.156	.055	.156	.086
5	random	200	LCS-DT	.046	.032	.037	.174	.168	.154	.080	.107	.092
5	random	500	LCS-DT	.031	.021	.024	.124	.104	.126	.056	.088	.064
6	random	200	LCS-DT	.050	.036	.040	.163	.180	.200	.078	.176	.103
6	random	500	LCS-DT	.030	.022	.023	.119	.111	.131	.044	.111	.066
7	random	200	LCS-DT	.045	.038	.037	.188	.180	.190	.085	.180	.108
7	random	500	LCS-DT	.034	.023	.029	.147	.127	.134	.055	.108	.075
1	fixed	200	SSM-CT	.011	.030	.012	.108	.112	.131	.027	.055	.051
1	fixed	500	SSM-CT	.006	.019	.007	.071	.079	.088	.018	.033	.033
2	fixed	200	SSM-CT	.016	.030	.017	.109	.117	.137	.043	.058	.056
2	fixed	500	SSM-CT	.010	.019	.010	.070	.082	.083	.027	.031	.036
3	fixed	200	SSM-CT	.065	.049	.054	.184	.226	.284	.128	.304	.144
3	fixed	500	SSM-CT	.040	.029	.033	.121	.151	.182	.079	.183	.091
4	fixed	200	SSM-CT	.062	.052	.051	.190	.220	.281	.076	.275	.132
4	fixed	500	SSM-CT	.037	.030	.029	.124	.132	.148	.051	.152	.079
5	fixed	200	SSM-CT	.045	.043	.037	.149	.139	.194	.090	.156	.094
5	fixed	500	SSM-CT	.032	.023	.027	.105	.117	.119	.054	.097	.065
6	fixed	200	SSM-CT	.046	.040	.036	.169	.156	.204	.063	.166	.097
6	fixed	500	SSM-CT	.031	.025	.025	.105	.113	.124	.041	.095	.062
7	fixed	200	SSM-CT	.045	.048	.037	.187	.154	.217	.081	.179	.104
7	fixed	500	SSM-CT	.030	.028	.026	.124	.110	.124	.055	.105	.068
1	random	200	SSM-CT	.014	.031	.013	.102	.107	.135	.029	.065	.052
1	random	500	SSM-CT	.009	.020	.009	.076	.076	.088	.017	.034	.035
2	random	200	SSM-CT	.016	.031	.016	.108	.108	.131	.043	.061	.054
2	random	500	SSM-CT	.012	.021	.012	.075	.079	.090	.026	.037	.037
3	random	200	SSM-CT	.062	.053	.050	.223	.198	.262	.114	.269	.139
3	random	500	SSM-CT	.047	.034	.040	.147	.157	.195	.090	.224	.106
4	random	200	SSM-CT	.061	.053	.049	.198	.197	.246	.085	.235	.125
4	random	500	SSM-CT	.038	.036	.031	.126	.143	.171	.045	.188	.087
5	random	200	SSM-CT	.051	.042	.042	.171	.175	.174	.073	.124	.097
5	random	500	SSM-CT	.033	.028	.027	.116	.106	.141	.053	.111	.068
6	random	200	SSM-CT	.055	.045	.044	.157	.172	.214	.064	.188	.103

6	random	500	SSM-CT	.031	.026	.025	.118	.105	.140	.038	.123	.066
7	random	200	SSM-CT	.048	.050	.040	.194	.174	.194	.075	.193	.111
7	random	500	SSM-CT	.033	.029	.029	.143	.118	.147	.054	.108	.073

Table A3. Parameter Coverage with cohort convergence

design	lags	n	model	β	μ_0	μ_s	σ_0	σ_s	$\sigma_{0,s}$	σ_e	Mean cover.
1	fixed	200	LCS-DT	.910	.960	.980	.920	.940	.930	.950	.941
1	fixed	500	LCS-DT	.980	.960	.980	.920	.910	.900	.940	.941
2	fixed	200	LCS-DT	.920	.970	.910	.920	.960	.940	.920	.934
2	fixed	500	LCS-DT	.920	.970	.930	.940	.890	.920	.950	.931
3	fixed	200	LCS-DT	.885	.923	.923	.962	1.000	.962	.769	.918
3	fixed	500	LCS-DT	1.000	.955	1.000	1.000	1.000	.864	.818	.948
4	fixed	200	LCS-DT	.905	.905	.905	.952	.976	.952	.976	.939
4	fixed	500	LCS-DT	.921	.947	.921	.895	.974	.947	.868	.925
5	fixed	200	LCS-DT	.970	.929	.970	.939	.949	.919	.929	.944
5	fixed	500	LCS-DT	.960	.960	.950	.920	.910	.910	.950	.937
6	fixed	200	LCS-DT	.960	.960	.940	.900	.960	.940	.950	.944
6	fixed	500	LCS-DT	.960	.939	.970	.919	.939	.949	.949	.947
7	fixed	200	LCS-DT	.979	.915	.957	.904	.936	.947	.926	.938
7	fixed	500	LCS-DT	.949	.949	.939	.888	.908	.949	.898	.926
1	random	200	LCS-DT	.910	.950	.990	.960	.950	.930	.100	.827
1	random	500	LCS-DT	.900	.830	.930	.890	.890	.900	.000	.763
2	random	200	LCS-DT	.980	.940	.960	.940	.960	.940	.400	.874
2	random	500	LCS-DT	.800	.850	.910	.920	.930	.890	.090	.770
3	random	200	LCS-DT	.986	.946	.986	.919	.946	.878	.959	.946
3	random	500	LCS-DT	.957	.986	.957	.914	.871	.729	.914	.904
4	random	200	LCS-DT	.948	.974	.961	.948	.974	.857	.857	.931
4	random	500	LCS-DT	.908	.920	.920	.966	.874	.759	.448	.828
5	random	200	LCS-DT	.960	.970	.970	.930	.930	.950	.890	.943
5	random	500	LCS-DT	.940	.870	.960	.880	.960	.860	.770	.891
6	random	200	LCS-DT	.950	.920	.960	.950	.960	.920	.460	.874
6	random	500	LCS-DT	.960	.920	.980	.930	.940	.800	.170	.814
7	random	200	LCS-DT	.959	.959	.959	.929	.949	.888	.806	.921
7	random	500	LCS-DT	.970	.949	.939	.879	.919	.889	.152	.814
1	fixed	200	SSM-CT	.960	.950	.980	.920	.940	.930	.950	.947
1	fixed	500	SSM-CT	.980	.910	.980	.930	.910	.900	.940	.936
2	fixed	200	SSM-CT	.940	.980	.920	.930	.960	.940	.920	.941
2	fixed	500	SSM-CT	.940	.940	.930	.930	.890	.930	.950	.930
3	fixed	200	SSM-CT	.960	.950	.950	.940	.970	.940	.930	.949
3	fixed	500	SSM-CT	.950	.970	.960	.950	.940	.940	.920	.947
4	fixed	200	SSM-CT	.900	.920	.900	.890	.920	.920	.990	.920
4	fixed	500	SSM-CT	.940	.970	.950	.920	.950	.940	.930	.943
5	fixed	200	SSM-CT	.980	.920	.970	.960	.960	.920	.930	.949
5	fixed	500	SSM-CT	.960	.930	.960	.910	.920	.920	.950	.936
6	fixed	200	SSM-CT	.970	.950	.940	.900	.970	.940	.950	.946
6	fixed	500	SSM-CT	.970	.960	.970	.930	.950	.950	.950	.954
7	fixed	200	SSM-CT	.970	.920	.970	.900	.940	.950	.930	.940
7	fixed	500	SSM-CT	.950	.950	.940	.880	.910	.960	.900	.927
1	random	200	SSM-CT	.930	.950	.990	.950	.960	.920	.920	.946
1	random	500	SSM-CT	.960	.930	.960	.890	.880	.910	.940	.924
2	random	200	SSM-CT	.990	.960	.960	.970	.970	.940	.940	.961
2	random	500	SSM-CT	.890	.900	.890	.930	.930	.910	.920	.910
3	random	200	SSM-CT	.980	.960	.980	.920	.980	.970	.970	.966
3	random	500	SSM-CT	.970	.960	.960	.950	.890	.940	.900	.939
4	random	200	SSM-CT	.960	.950	.960	.930	.940	.970	.920	.947
4	random	500	SSM-CT	.950	.910	.960	.950	.910	.920	.990	.941
5	random	200	SSM-CT	.950	.970	.970	.910	.920	.960	1.000	.954
5	random	500	SSM-CT	.950	.890	.970	.930	.950	.900	.930	.931
6	random	200	SSM-CT	.950	.940	.960	.920	.950	.950	.960	.947

6	random	500	SSM-CT	.960	.960	.960	.920	.970	.910	.960	.949
7	random	200	SSM-CT	.960	.960	.960	.910	.930	.970	.920	.944
7	random	500	SSM-CT	.970	.970	.960	.880	.900	.930	.920	.933

Table A4. Relative Bias without cohort convergence

design	lags	n	model	β	μ_0	σ_0	$\sigma_{0,s}$	σ_e	$\rho_{0,s}$	<i>RMS(RB)</i>
1	fixed	200	LCS-DT	-.001	.004	-.015	-.014	-.003	-.004	.007
1	fixed	500	LCS-DT	.000	-.002	-.011	-.008	.004	-.002	.005
2	fixed	200	LCS-DT	.003	.003	-.016	-.005	-.004	.001	.007
2	fixed	500	LCS-DT	.000	-.001	-.008	-.008	-.004	-.002	.004
3	fixed	200	LCS-DT	.150	-.039	-.004	.300	.013	.157	.163
3	fixed	500	LCS-DT	.164	-.045	.022	.264	.017	.071	.174
4	fixed	200	LCS-DT	.146	-.033	-.033	.240	.009	.129	.150
4	fixed	500	LCS-DT	.151	-.043	.014	.281	.008	.114	.155
5	fixed	200	LCS-DT	.095	-.063	.000	.153	.006	.053	.109
5	fixed	500	LCS-DT	.083	-.062	.025	.156	.008	.041	.104
6	fixed	200	LCS-DT	.111	-.067	.002	.229	.004	.094	.133
6	fixed	500	LCS-DT	.110	-.064	.025	.222	.004	.075	.127
7	fixed	200	LCS-DT	.118	-.068	-.009	.179	.015	.064	.125
7	fixed	500	LCS-DT	.113	-.071	.019	.201	.009	.058	.129
1	random	200	LCS-DT	.001	-.007	.012	-.017	.112	-.023	.044
1	random	500	LCS-DT	.001	-.012	.022	-.010	.116	-.019	.045
2	random	200	LCS-DT	.000	-.007	.045	-.014	.097	-.031	.043
2	random	500	LCS-DT	.002	-.010	.027	-.018	.093	-.030	.039
3	random	200	LCS-DT	.187	-.022	.187	.029	.088	-.221	.252
3	random	500	LCS-DT	.182	-.021	.142	-.019	.084	-.249	.250
4	random	200	LCS-DT	.176	-.011	.088	.009	.153	-.190	.229
4	random	500	LCS-DT	.179	-.018	.107	.041	.167	-.188	.239
5	random	200	LCS-DT	.101	-.039	.114	.092	.110	-.080	.145
5	random	500	LCS-DT	.098	-.038	.116	.067	.125	-.101	.144
6	random	200	LCS-DT	.131	-.036	.073	.087	.186	-.092	.178
6	random	500	LCS-DT	.134	-.034	.066	.081	.175	-.103	.182
7	random	200	LCS-DT	.124	-.041	.052	.062	.183	-.089	.163
7	random	500	LCS-DT	.124	-.033	.068	.087	.204	-.082	.167
1	fixed	200	SSM-CT	-.002	-.004	-.009	-.013	-.003	-.006	.005
1	fixed	500	SSM-CT	-.001	-.009	-.006	-.008	.004	-.004	.005
2	fixed	200	SSM-CT	.002	-.005	-.011	-.004	-.004	-.001	.005
2	fixed	500	SSM-CT	-.001	-.009	-.003	-.007	-.004	-.005	.004
3	fixed	200	SSM-CT	.175	-.048	.001	.331	.013	.156	.188
3	fixed	500	SSM-CT	.191	-.055	.028	.296	.017	.069	.204
4	fixed	200	SSM-CT	.170	-.043	-.027	.269	.009	.127	.175
4	fixed	500	SSM-CT	.176	-.052	.020	.311	.008	.113	.180
5	fixed	200	SSM-CT	.109	-.073	.005	.170	.006	.051	.124
5	fixed	500	SSM-CT	.095	-.071	.031	.170	.008	.039	.117
6	fixed	200	SSM-CT	.128	-.076	.007	.251	.004	.092	.151
6	fixed	500	SSM-CT	.127	-.073	.031	.243	.004	.073	.145
7	fixed	200	SSM-CT	.136	-.078	-.003	.201	.015	.062	.145
7	fixed	500	SSM-CT	.130	-.080	.025	.222	.009	.056	.147
1	random	200	SSM-CT	-.001	-.003	-.006	-.017	-.008	-.011	.007
1	random	500	SSM-CT	.000	-.009	-.003	-.006	.003	-.003	.004
2	random	200	SSM-CT	-.002	-.004	.000	-.008	-.002	-.003	.005
2	random	500	SSM-CT	.001	-.009	-.009	-.007	-.005	-.003	.005
3	random	200	SSM-CT	.212	-.076	.034	.373	.026	.141	.222
3	random	500	SSM-CT	.203	-.072	-.016	.316	.030	.110	.214
4	random	200	SSM-CT	.191	-.061	-.012	.315	.010	.132	.199
4	random	500	SSM-CT	.193	-.070	.024	.329	.006	.103	.203

5	random	200	SSM-CT	.107	-.091	.041	.200	-.004	.052	.132
5	random	500	SSM-CT	.103	-.087	.029	.182	.019	.046	.125
6	random	200	SSM-CT	.138	-.089	.006	.229	.010	.069	.158
6	random	500	SSM-CT	.142	-.087	.009	.231	.002	.061	.162
7	random	200	SSM-CT	.136	-.102	.022	.201	-.016	.047	.152
7	random	500	SSM-CT	.131	-.087	.008	.226	.010	.070	.148

Table A5. Parameter Coverage without cohort convergence

design	lags	n	model	β	μ_0	σ_0	$\sigma_{0,s}$	σ_e	Mean cover.
1	fixed	200	LCS-DT	.960	.960	.920	.930	.950	.949
1	fixed	500	LCS-DT	.980	.960	.920	.900	.940	.941
2	fixed	200	LCS-DT	.940	.970	.920	.940	.920	.937
2	fixed	500	LCS-DT	.940	.970	.940	.920	.950	.934
3	fixed	200	LCS-DT	.210	.870	.920	.820	.950	.651
3	fixed	500	LCS-DT	.000	.740	.950	.730	.960	.523
4	fixed	200	LCS-DT	.160	.930	.890	.880	.970	.660
4	fixed	500	LCS-DT	.000	.670	.930	.620	1.000	.511
5	fixed	200	LCS-DT	.380	.630	.950	.910	.960	.660
5	fixed	500	LCS-DT	.060	.330	.980	.760	.950	.514
6	fixed	200	LCS-DT	.270	.620	.940	.750	.950	.601
6	fixed	500	LCS-DT	.010	.350	.950	.570	.890	.447
7	fixed	200	LCS-DT	.160	.700	.940	.850	.960	.617
7	fixed	500	LCS-DT	.000	.280	.980	.670	.930	.447
1	random	200	LCS-DT	.960	.940	.950	.930	.040	.826
1	random	500	LCS-DT	.910	.830	.890	.900	.000	.764
2	random	200	LCS-DT	.970	.940	.940	.940	.400	.873
2	random	500	LCS-DT	.920	.850	.920	.890	.090	.787
3	random	200	LCS-DT	.260	.940	.940	.940	.970	.659
3	random	500	LCS-DT	.000	.920	.910	.960	.860	.536
4	random	200	LCS-DT	.220	.920	.970	.940	.640	.621
4	random	500	LCS-DT	.010	.940	.940	.930	.130	.440
5	random	200	LCS-DT	.420	.810	.910	.930	.820	.667
5	random	500	LCS-DT	.130	.590	.900	.920	.450	.473
6	random	200	LCS-DT	.210	.850	.930	.940	.320	.556
6	random	500	LCS-DT	.000	.640	.950	.950	.010	.387
7	random	200	LCS-DT	.290	.740	.950	.940	.490	.581
7	random	500	LCS-DT	.030	.710	.970	.900	.020	.410
1	fixed	200	SSM-CT	.960	.950	.920	.930	.950	.947
1	fixed	500	SSM-CT	.970	.910	.930	.900	.940	.934
2	fixed	200	SSM-CT	.940	.980	.930	.940	.920	.941
2	fixed	500	SSM-CT	.940	.940	.930	.930	.950	.930
3	fixed	200	SSM-CT	.220	.810	.920	.810	.950	.646
3	fixed	500	SSM-CT	.010	.580	.950	.710	.960	.483
4	fixed	200	SSM-CT	.190	.900	.900	.860	.970	.656
4	fixed	500	SSM-CT	.000	.630	.930	.600	1.000	.487
5	fixed	200	SSM-CT	.380	.540	.950	.900	.960	.644
5	fixed	500	SSM-CT	.080	.190	.980	.730	.950	.484
6	fixed	200	SSM-CT	.270	.480	.940	.730	.950	.573
6	fixed	500	SSM-CT	.010	.200	.940	.550	.890	.410
7	fixed	200	SSM-CT	.180	.630	.930	.850	.960	.604
7	fixed	500	SSM-CT	.000	.220	.990	.660	.930	.431
1	random	200	SSM-CT	.930	.950	.950	.920	.920	.946
1	random	500	SSM-CT	.960	.930	.890	.910	.940	.924
2	random	200	SSM-CT	.990	.960	.970	.940	.940	.961
2	random	500	SSM-CT	.890	.900	.930	.910	.920	.910
3	random	200	SSM-CT	.170	.770	.940	.810	.980	.623
3	random	500	SSM-CT	.000	.460	.930	.670	.970	.461
4	random	200	SSM-CT	.150	.790	.950	.780	.960	.624
4	random	500	SSM-CT	.010	.430	.990	.630	.950	.454
5	random	200	SSM-CT	.430	.510	.940	.880	.960	.650
5	random	500	SSM-CT	.120	.150	.960	.690	.930	.477
6	random	200	SSM-CT	.170	.460	.950	.860	.950	.587

6	random	500	SSM-CT	.000	.130	.960	.600	.970	.410
7	random	200	SSM-CT	.290	.460	.940	.870	.980	.604
7	random	500	SSM-CT	.010	.190	.960	.650	.980	.439