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**Title: Mutual influences between numerical and non-numerical quantities in
comparison tasks.**

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- 22

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Abstract

24 Humans possess a numerical intuition that allows them to manipulate large non-symbolic
25 quantities. This ability has been broadly assessed with the help of number comparison tasks
26 involving simultaneously displayed arrays. Many authors pointed out that the manipulation (or
27 the lack thereof) of non-numerical features deeply impacts performance in these tasks, but the
28 specific nature of this influence is not clear. The current study investigates the interaction
29 between numerical and non-numerical quantity judgment tasks. Adult participants performed
30 five distinct comparison tasks, each based on a target dimension: numerosity, total area, dot
31 size, convex hull, and mean occupancy. We manipulated the relation between the target and
32 the other dimensions to measure their respective influence on task performance. Results
33 showed that total area and convex hull substantially affected numerosity comparisons. The
34 number of dots conversely acted as an informative dimension when participants had to make a
35 decision based on the total area or the convex hull. Our results illustrate that adults flexibly use
36 non-target dimensions as visual cues to perform comparison judgments. Overall, this suggests
37 that the influence found in numerical comparison tasks is explicit and deliberate rather than
38 due to implicit visual integration processes.

39 Number words: 192.

Mutual influences between numerical and non-numerical quantities in comparison tasks.

Some authors proposed that humans have a *sense of number* enabling them to represent and manipulate numerosity (Dehaene, 2001). It has been assumed that there is a specific cognitive system underlying such *number sense*, referred to as the Approximate Number System (ANS), which allows approximate processing of large numerical quantities (Feigenson, Dehaene, & Spelke, 2004). This approximate numerical processing has been shown to follow the Weber-Fechner law (see Feigenson et al., 2004), so that the distinction between two number magnitudes depends on the ratio between them, rather than on their absolute difference. Subsequently, collections of 10 and 20 visual elements are in that way as discriminable as collections of 100 and 200 similar elements. Such an ability to naturally apprehend and compare numerical quantities has been hypothesized to be crucial for more elaborated number processing such as arithmetic operations (Halberda, Mazocco, & Feigenson, 2008). This led many authors to assess more broadly non-symbolic number abilities and to study their relation to other cognitive abilities (see for instance, Schneider et al., 2017; and Schwenk et al., 2017, for recent reviews).

The most commonly used task to evaluate non-symbolic number abilities is the comparison task, notably following Piazza, Izard, Pinel, Le Bihan, and Dehaene (2004). In such a task, participants are instructed to compare two collections and to determine which one is more numerous. Although some researchers used auditory (e.g. Feigenson, 2011; Kobayashi, Hiraki, & Hasegawa, 2005; Lipton & Spelke, 2003) or haptic stimuli (e.g. Gimbert, Gentaz, Camos, & Mazens, 2016) to elicit number comparisons, the large majority of studies relied on visual presentation (see Dietrich et al., 2016, for a review). In these visual number comparison tasks, arrays are generally shown simultaneously (but see, for instance, Smets, Moors, & Reynvoet,

2016; and Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout, 2013, for sequential presentation), side-by-side, for a relatively short duration to avoid counting strategies. Critically, the numerical ratio (i.e., the ratio between the two quantities) is manipulated to vary the level of difficulty. Small ratios (e.g., 10 vs. 11, ratio of 1.1) are indeed more difficult to compare than large ratios (e.g., 10 vs 16, ratio of 1.6), because magnitude processing follows the Weber-Fechner law (Dehaene, 2003). Interestingly, some individuals can discriminate two quantities with a given ratio while others cannot, indicating individual differences in numerical discrimination thresholds (see, for instance, the anthropological data from Pica, Lemer, Izard, & Dehaene, 2004). Performance in comparing quantities across various ratios is thus assumed to index individual non-symbolic number abilities. Furthermore, some authors suggested computing the Weber fraction, which is the ratio between the amount just noticeably different from a given magnitude and the magnitude itself (w , see Stevens, 1957; van Oeffelen & Vos, 1982), to get a better estimate of individual discrimination thresholds (i.e., the ratio from which reliable comparison between two quantities is possible).

Despite its popularity, the number comparison task has also been the focus of many criticisms. The main issue is the existence of intrinsic correlations between numerical and non-numerical (or continuous) dimensions (see Gebuis, Cohen Kadosh, & Gevers, 2016; and Leibovich, Katzin, Harel, & Henik, 2017, for reviews). In particular, it is not possible to separate number from all continuous dimensions simultaneously (e.g. Gebuis & Reynvoet, 2011, 2012), and it is, therefore, impossible to ascertain that participants only based their decision on number (Gebuis et al., 2016; Leibovich & Henik, 2013; Salti, Katzin, Katzin, Leibovich, & Henik, 2017; Smets, Sasanguie, Szűcs, & Reynvoet, 2015). Some authors thus challenged the claim that number comparison tasks are suitable to evaluate number abilities (e.g. Gilmore, Attridge, & Inglis, 2011; see Guillaume & Van Rinsveld, 2018, for a meta-analysis; e.g. Smets, Gebuis, Defever, & Reynvoet, 2014). Many covarying non-numerical features have indeed been

identified (see for instance the modeling model from DeWind, Adams, Platt, & Brannon, 2015). Among these, we will focus on five distinct dimensions, the first four of which are in continuity with our previous work (Guillaume, Schiltz, & Van Rinsveld, in press). The properties under consideration are the total occupied area, the individual dot size, the extent of the convex hull (i.e., the smallest convex polygon surrounding the array), the mean dot occupancy¹ (i.e., the average space comprising and around each dot, see Allik & Tuulmets, 1991), and the coverage proportion (i.e., the ratio between the covered space and the available space).

Many studies assessed total area (also called cumulative area) influence on number comparison (Abreu-Mendoza, Soto-Alba, & Arias-Trejo, 2013; Guillaume, Nys, Mussolin, & Content, 2013; Hurewitz, Gelman, & Schnitzer, 2006; Lourenco & Bonny, 2017; Lourenco, Bonny, Fernandez, & Rao, 2012; Nys & Content, 2012), but in contrast, there is limited evidence about whether and how other dimensions influence number judgments. This is not a trivial issue since Gilmore, Cragg, Hogan, and Inglis (2016) recently showed that convex hull has a greater impact on number comparison performance than total occupied area. We might consequently expect the impact of each continuous dimension to be substantially different. Furthermore, studies that assessed non-numerical influence on number comparison generally found a bidirectional relation between number and the non-numerical dimension (e.g. Hurewitz et al., 2006; Nys & Content, 2012). In particular, these studies showed that number judgments are influenced by total area, but conversely total area judgments are influenced by numerosity.

¹ Mean occupancy is the ratio of the convex hull area by the number of items. As such, mean occupancy is an index of the average space occupied by each item. It is the inverse of density, the number of items within a given space.

109 Bidirectional effects between number and other visual properties might thus exist; but once
110 again, the strength of these effects might depend on the manipulated visual dimension.

111 The current study aims at extending previous findings, by providing an exhaustive picture of
112 the relationships between numerical and non-numerical dimensions in comparison tasks. The
113 goal is to explore the existence of potentially bidirectional relations between each of these
114 dimensions and number. Young adults had to complete five comparison tasks that involved
115 decisions on five different dimensions. In particular, participants had to make judgments about
116 dot arrays according to their numerosity, the total area occupied by the dots, the individual dot
117 size, the extent of the convex hull, and the mean occupancy within the array. We did not assess
118 coverage proportion in a comparison task because this dimension does not directly covary with
119 number. Nevertheless, we took coverage proportion into account in the construction of the
120 stimuli for the four non-numerical tasks.

121 By using the same construction method to create the arrays for each of the five tasks, we were
122 able to manipulate the non-target features in every task in a similar way. Therefore, we can
123 assess whether the manipulations of non-target dimensions affect judgments on the target
124 dimension. This provides crucial insights not only on whether non-numerical information is
125 used as a cue to compare number quantities but also on whether numerical and non-numerical
126 cues are used to compare continuous dimensions.

127 We expect judgments in the number comparison task to be influenced by some covarying visual
128 dimensions such as total occupied area and convex hull, as observed in previous studies
129 (Gilmore et al., 2016; Nys & Content, 2012). Also, covariation of number should affect
130 judgments of total area in the continuity of previous findings (Guillaume et al., 2013; Hurewitz
131 et al., 2006; Lourenco & Bonny, 2017; Lourenco et al., 2012; Nys & Content, 2012), and we
132 additionally expect covariation of number to similarly impact judgments on the other

continuous dimensions manipulated. By systematically examining judgments on the five dimensions considered, we aim at delivering a more complete and more detailed picture of the impacts and eventual bidirectional relations between them.

Method

Participants

Fifty-eight undergraduate students participated in this experiment for course credit. Six participants were excluded for poor performance in an arithmetic test (TTR cut-off = 100, see section *Tasks and procedure*), and one reported a developmental disorder. We additionally had to exclude one participant for a technical reason and another one because of large fluctuations of performance. The final sample consisted of 49 young adults (11 males) with a mean age of 23.65 years (*Standard Deviation* (SD) = 2.44, range = 19-29). The study was approved by the Faculty Ethical Committee (identification number: 005/2015) before the start of data acquisition.

Tasks and procedure

Participants completed five computerized dot comparison tasks. They were instructed to compare dot collections as accurately as possible according to a target dimension: in the *Number (NU) task*, they had to choose the most numerous array; in the *Total Area (TA) task*, they had to choose the array with the largest occupied area; in the *Individual Size (IS) task*, they had to choose the array with the biggest dot size; in the *Convex Hull (CH) task*, they had to choose the array with the largest convex hull; and in the *Mean Occupancy (MO) task*, they had to choose the array with the largest mean occupancy (i.e., the largest averaged space between the dots).

Participants first received four trials with feedback. Each trial began with a fixation cross at the center of the screen for a duration of 250ms. The arrays were then displayed for 1 second on each side of the screen within two visible circles (radius: 300 pixels) that were present during the entire experiment. A dynamic mask was presented after the arrays to suppress aftereffects. This mask consisted of colored pixels, randomly changing RGB code with each screen refresh. The mask remained on the screen until participants responded. If they responded before the onset of the mask, it still appeared for a minimal duration of 50ms (see Figure 1). The Interstimulus Interval was 250ms. Participants had to press the leftmost button of the button box to select the left array, and the rightmost button to select the right array. They were instructed to respond as accurately as possible. The order of the trials and the position of the larger array were randomized. The order of the five tasks was also randomized across participants. The experiment took approximately 50 minutes to complete.

167

168 **Figure 1. Illustration of the steps of the experiment.**

Before taking part in the experiment, participants signed a written informed consent form. They completed the Tempo-Test Rekenen (TTR; De Vos, 1992), a math fluency test consisting of five columns of 40 calculations of increasing difficulty for the four main operations (additions, subtractions, multiplications, divisions, and a final mixed column). Participants also responded to a questionnaire about the eventual presence of developmental disorders. The experiment

took place in a quiet room. Five participants were tested at the same time, but the experiment was individual since they were separated by large panels.

The tasks were run on a Mac minicomputer (OS X 10.10) with a 19-inch screen (1280 × 1024 pixels resolution). The experiment script was written in Python (v.3.6) and ran on Psychopy (v.3.0.1. ; Peirce et al., 2019). We used an ioLab Systems Button box to collect responses.

Stimuli

The dot arrays (dimension: 600 × 600 pixels) consisted of dark blue (RGB: 3-37-82) dots on a light blue (RGB: 188-185-255) background. They were generated with NASCO (Guillaume et al., in press). This program creates dot arrays varying in number, controlling for the four dimensions considered in the present study. In NASCO, all four dimensions are expressed in number of pixels. In the number comparison task, we kept Total Area (TA) constant across the array pairs for half the trials, and we kept Individual Size (IS) constant for the other half (as in Dehaene, Izard, & Piazza, 2005). Additionally, we kept Convex Hull area (CH) constant across the array pairs for half of the trials, and we kept Mean Occupancy (MO) constant for the other half. By crossing both manipulations, we thus created four item categories in which two visual properties are equalized: TA and CH constant, TA and MO constant, IS and CH constant, and finally IS and MO constant (see Figure 2.A). It is crucial to emphasize that dimensions that are not equalized necessarily vary with the target dimension (see Supplementary Materials for further explanations). For instance, Individual Size and Mean Occupancy systematically change with Number in the first set in which TA and CH are constant. This means that the TA-CH constant set could alternatively be described as the condition in which IS and MO covary with Number.

For the four non-numerical comparison tasks, we applied the same construction method by systematically keeping two dimensions constant at a time (and letting the other two vary with the target dimension). We thus constructed four different stimulus categories for each task. It is important to note that we had to introduce a fifth dimension to get four independent properties in these tasks. Since all previous dimensions were mediated by number, it was not possible to get independent relations between these properties. We thus needed an extra dimension that was totally independent from number. For this reason, we introduced Coverage Proportion (CP), which represents the ratio between the area covered by the dots, Total Area, and the available space, the area of the Convex Hull, hence $CP = \frac{TA}{CH}$. It should be noted that Coverage Proportion could also be computed by dividing Individual Size by Mean occupancy, $CP = \frac{IS}{MO}$. Critically, this dimension has no unit and does not directly depend on number. We made use of this dimension in every non-numerical comparison task (see Figure 2).

For each task, we used six ratios (ranging from 1.1 to 1.6, with an incremental step of 0.1) to determine the values of the dimension under consideration within the pairs. We thus constructed eight pairs for each ratio and each category, for a total of 192 items per task. The average value for the standard ratio was 23 for the NU task (range: 10-30), 755 pixels (range: 254-1662) for the IS task, 12901 pixels (range: 6030-25140) for the TA task, 104508 pixels (range: 60000-195440) for the CH task, 4794 pixels (range: 3000-7560) for the MO task.

Incidentally, visual properties that were not equalized in a given category necessarily covaried with the target dimension in the same proportion (see Guillaume et al., in press, for a detailed explanation). This implies that ratios of the varying non-target visual features were exactly the same as the target ratios. We used the same ratios for all comparison tasks.

Figure 2. Illustration of all array pair categories, highlighting the crossing of two manipulated relations between the visual features, for (A) the Number (NU) task, (B) the Total Area (TA) task, (C) the Individual Size (IS) task, (D) the Convex Hull (CH) task, and (E) the Mean Occupancy (MO) task.

Statistical analyses

We used Pingouin (Vallat, 2018), Seaborn and Matplotlib packages from Python (v.3.7.4) on JupyterLab (0.35.4), and RStudio (v.1.2.1335) running on R (v. 3.6.1) to conduct statistical analyses. In a first step, we filtered outliers by cut-off with a range of 150-2500ms. We report the Greenhouse-Geisser corrected p-value if sphericity was not assumed.

Results

Since instructions focused on accuracy, we only report here results relative to correct response rates. Analyses conducted on response times showed a similar pattern and are available in Supplementary Materials.

General performance

Overall, participants reached 85.0% ($SD = 4.7$) of correct responses in the Number task, 85.2% ($SD = 5.8$) in the Total Area task, 87.8% ($SD = 6.7$) in the Individual Size task, 89.3% ($SD = 5.5$) in the Convex Hull task, and 68.7% ($SD = 7.4$) in the Mean Occupancy task. We conducted a repeated-measures analysis of variance (ANOVA) on correct responses with Task (5 levels) and Ratio (6 levels) as within-subject factors (see Figure 3). The analysis revealed that the effect of ratio was significant, $F(5, 240) = 572.68$, $p < .001$, with larger ratios associated with higher accuracy (1.6, 91.5% , $SD = 4.2$) than smaller ratios (1.1, 67.4%, $SD = 5.0$) across all tasks. More critically, accuracy was significantly affected by the task, $F(4, 192) = 142.83$, $p < .001$, and the interaction between the two factors was significant, $F(20, 960) = 9.94$, $p < .001$.

To further investigate the differences in performance between the tasks across the six ratios, we computed Weber Fractions (w) for every task. To do so, for each participant and task, we adjusted a Gaussian cumulative probability distribution using nonlinear regression, based on the Levenberg-Marquardt-Fletcher nonlinear least-square iterative method (as in Guillaume et

al., 2013). The parameter w in the expression below, as a function of ratios (r), is the Weber fraction.

$$1 - \frac{1}{2} \operatorname{erfc}\left(\frac{1}{w\sqrt{2(r^2+1)/(r-1)^2}}\right).$$

Average w_{NU} was 0.17 (SD = 0.05), 0.17 (SD = 0.06) for w_{TA} , 0.15 (SD = 0.06) for w_{IS} , 0.13 (SD = 0.07) for w_{CH} , and 0.60 (SD = 0.72) for w_{MO} . We conducted a repeated-measures ANOVA on Weber fraction values with Task (5 levels) as a within-subject factor. The effect of task was significant, $F(4, 192) = 19.06$, $p < .001$, confirming the difference of performance between tasks. Bonferroni-corrected tests confirmed that w_{CH} was significantly lower than in the other tasks, all $ps < .05$, and that w_{MO} was significantly larger than in the other tasks, all $ps < .05$. Post-hoc analyses also revealed that w_{NU} did not significantly differ from w_{TA} value, but it did differ from the value computed in the IS task, $p = .01$. Also, w_{NU} significantly correlated with every other w (w_{TA} : $\tau = .38$, w_{IS} : $\tau = .40$, w_{CH} : $\tau = .29$, w_{MO} : $\tau = .24$), all $ps < .05$. The complete correlation matrix is available in Supplementary Materials.

Figure 3. Percentage of correct responses for each task as a function of ratio. Vertical lines depict 95% confidence interval.

Specific performance

To examine the influence of covarying dimensions, we conducted separate ANOVAs on accuracy. The results are summarized in Table 1.

Number comparison task

A within-subject ANOVA with Ratio (6 Levels), and the two covariation relations (Total Area and Individual Size, TA-IS; Convex Hull and Mean Occupancy, CH-MO) showed a significant effect of Ratio, $F(5, 240) = 152.14$, $p < .001$, with larger ratios (1.6, 95.1%, $SD = 4.6$) being easier to compare than smaller ones (1.1, 67.8%, $SD = 8.7$). The TA-IS relation had a significant effect, $F(1, 48) = 58.67$, $p < .001$, which showed that performance was significantly better when Total Area covaried with number (88.3%, $SD = 4.5$) than when IS covaried with number (81.7%, $SD = 6.4$). The effect of the CH-MO relation was also significant, $F(1, 48) = 30.46$, $p < .001$, with facilitation when Convex Hull was varying with number (87.1%, $SD = 4.9$) compared to when Mean Occupancy was covarying (82.9%, $SD = 5.7$). The interaction between the two relations was significant, $F(1, 48) = 5.53$, $p = .023$, and we also observed a significant interaction between ratio and the TA-IS relation, $F(5, 240) = 3.35$, $p = .006$. Other interactions did not reach the significance level.

Total Area comparison task

Performance in the Total Area task was analysed as a function of Ratio, and the two covariation relations (Number and Individual Size, NU-IS; Convex Hull and Coverage Proportion, CH-CP). The ratio had a significant effect on performance, $F(5, 240) = 105.08$, $p < .001$, with larger ratios (1.6, 93.4%, $SD = 5.9$) being easier to compare than smaller ones (1.1, 69.6%, $SD =$

10.5). In this task, Number and Convex Hull seemed to be particularly helpful, since performance was better when these two dimensions covaried with Total Area (respectively 86.5%, SD = 6.2, and 89.5%, SD = 5.8) than when Individual Size and Coverage Proportion were the covarying dimensions (respectively 83.9%, SD = 7.2, and 81%, SD = 9.3). Both factors had a significant impact on performance, respectively $F(1,48) = 6.86$, $p = .01$ and $F(1, 48) = 35.48$, $p < .001$. Further, the interaction between these relations did not reach significance, $F(1, 48) = 3.94$, $p = .052$, but the three-way interaction effect was significant, $F(5, 240) = 3.5$, $p = .004$. We also found a significant interaction effect between the NU-IS relation and the ratio, $F(5, 240) = 3.5$, $p = .004$. Other interactions were not significant.

Individual Size comparison task

We conducted a within-subject ANOVA with Ratio and the two covariation relations (Number and Total Area, NU-TA; Mean Occupancy and Coverage Proportion, MO-CP). Once again, the ratio had a significant effect on performance, $F(5, 240) = 174.26$, $p < .001$, with larger ratios (1.6, 97.4%, SD = 4.0) leading to better scores than smaller ones (1.1, 66.7%, SD = 12.3). In this task, participants actually performed worse when Number covaried with individual dot sizes (85.3%, SD = 8.8) than when Total Area was covarying with individual dot sizes (90.3%, SD = 5.3), $F(1,48) = 36.68$, $p < .001$. The MO-CP relation did not have any significant effect on performance, $F(1, 48) = 1.58$, $p = .21$, and there was no significant interaction.

Convex Hull comparison task

The within-subject ANOVA with Ratio and the two covariation relations (Number and Coverage Proportion, NU-CP; Mean Occupancy and Total Area, MO-TA) was conducted. The ratio effect was also significant, $F(5, 240) = 147.02$, $p < .001$, with larger ratios (1.6, 95.7%, SD = 4.9) being easier to compare than closer ratios (1.1, 71.7%, SD = 8.2). In this task, Number and Coverage Proportion seemed to be particularly helpful, since performance was

better when these two dimensions covaried with Convex Hull (respectively 91.5%, SD = 5.6, and 90.1%, SD = 6.5) than when Mean Occupancy and Total Area were covarying with the target dimension (respectively 87.1%, SD = 6.4, and 88.5%, SD = 5.4). Both factors had a significant impact on performance, respectively $F(1,48) = 39.17$, $p = .01$ and $F(1, 48) = 5.40$, $p = .024$, with a significant interaction, $F(1, 48) = 9.45$, $p = .003$. We also found a significant interaction effect between the TA-CP relation and the ratio, $F(5, 240) = 3.61$, $p = .003$, and a significant triple interaction, $F(1, 48) = 9.45$, $p = .003$. Other interactions did not reach the significance level.

Mean Occupancy comparison task

Lastly, performance in the Mean Occupancy task was analysed as function of Ratio and the two covariation relations (Number and Convex Hull, NU-CH; Individual Size and Coverage Proportion, IS-CP). The ratio also had a significant effect on participants' judgments, $F(5, 240) = 25.17$, $p < .001$, with larger ratios (1.6, 75.9%, SD = 11.5) being easier to compare than closer ratios (1.1, 61.1%, SD = 7.9). We found no significant difference between the NU-CH relation, $F < 1$, but performance was better when Coverage Proportion varied with Mean Occupancy (76%, SD = 8.1) than when it was the Individual Size of dots (60.9%, SD = 10.2), $F(1, 48) = 98.13$, $p < .001$, and the interaction between the two relations was significant, $F(1, 48) = 8.94$, $p = .004$. Moreover, the interaction between the second dimension (IS-CP) and ratio, $F(5, 240) = 8.76$, $p < .001$, and the triple interaction, $F(5, 240) = 3.15$, $p = .009$ reached significance.

Table 1. Mean Percentages of correct responses (and corresponding Standard Deviations) for each task according to the properties that were kept constant and varied with the target dimension.

Task	Accuracy in percent (and <i>SD</i>)			
Numerosity	TA varied (<i>IS equal</i>)	IS varied (<i>TA equal</i>)	Total	
	MO varied (<i>CH equal</i>)	85.6 (5.8)	80.3 (7.8)	82.9 (5.7)
	CH varied (<i>MO equal</i>)	91.1 (4.6)	83.1 (7.2)	87.1 (4.9)
	Total	88.3 (4.5)	81.7 (6.4)	85.0 (4.7)
Total Area	NU varied (<i>IS equal</i>)	IS varied (<i>NU equal</i>)	Total	
	CP varied (<i>CH equal</i>)	83.0 (10.3)	79.0 (10.6)	81.0 (9.3)
	CH varied (<i>CP equal</i>)	90.1 (5.7)	88.8 (7.1)	89.2 (5.8)
	Total	86.5 (6.2)	83.9 (7.2)	85.2 (5.8)
Individual Size	NU varied (<i>TA equal</i>)	TA varied (<i>NU equal</i>)	Total	
	CP varied (<i>MO equal</i>)	85.6 (8.7)	90.8 (5.7)	88.2 (6.6)
	MO varied (<i>CP equal</i>)	85.1 (10.1)	89.7 (6.5)	87.4 (7.6)
	Total	85.3 (8.8)	90.3 (5.3)	87.8 (6.7)
Convex Hull	NU varied (<i>MO equal</i>)	MO varied (<i>NU equal</i>)	Total	
	CP varied (<i>TA equal</i>)	91.4 (6.9)	88.8 (7.3)	90.1 (6.5)
	TA varied (<i>CP equal</i>)	91.6 (5.6)	85.4 (7.2)	88.5 (5.4)
	Total	91.5 (5.6)	87.1 (6.4)	89.3 (5.5)
Mean Occupancy	NU varied (<i>CH equal</i>)	CH varied (<i>NU equal</i>)	Total	
	CP varied (<i>IS equal</i>)	73.8 (12.7)	79.4 (10.9)	76 (8.1)
	IS varied (<i>CP equal</i>)	61.6 (13.3)	60.2 (18.1)	60.9 (10.2)
	Total	67.7 (11.8)	69.8 (12.6)	68.7 (7.4)

Note. NU = Number; TA = Total Area; IS = Individual Size; CH = Convex Hull; MO = Mean Occupancy; CP = Coverage Proportion.

Discussion

In the present study, we aimed at evaluating performance in five comparison tasks and at measuring the influence of irrelevant numerical or non-numerical covarying dimensions on such performance. We considered the following dimensions as targets in our comparison tasks: Number, Total Area, Individual Size, Convex Hull, and Mean Occupancy. We used the same construction method to design every stimulus set across the tasks, to identify which dimensions were used as informative cues to judge the target dimension and to determine whether the same cues were systematically used across all tasks. In particular, we not only aimed at highlighting the non-numerical dimensions affecting Number comparisons but also, conversely, at assessing whether number influences non-numerical judgments.

As expected, we found ratio effects in all tasks; participants performed substantially better for large ratios than for small ones. This pattern characterizes the Weber-like property of magnitude comparison processes (Dehaene, 2003; Feigenson et al., 2004). Visual inspection of Figure 3 clearly illustrates this property for every condition, except for mean occupancy. The dissimilar pattern for mean occupancy corroborates the idea that specific texture-density mechanisms that do not strictly follow Weber's law are involved in the perception of occupancy (Anobile, Cicchini, & Burr, 2014; Anobile, Turi, Cicchini, & Burr, 2015). Regarding the other conditions, our findings are in line with the hypothesis that there are some common grounds in magnitude comparison processes across various perceptual dimensions (see also Leibovich et al., 2017; Walsh, 2003).

Overall, participants were highly accurate in every task, except for Mean Occupancy. Such a poor performance suggests that deliberate decisions on occupancy (or density) are difficult. Occupancy may be more difficult to extract and/or manipulate than other dimensions because it relies on texture-like mechanisms depending on factors such as eccentricity, which we did

not manipulate (Anobile et al., 2015). Comparing occupancies could also be a less familiar task than comparing salient features such as object size. Indeed, participants performed best at comparing convex hulls, which is relatively similar to comparing the size of arrays as whole objects (as a Gestalt, see for instance Pomerantz, 1983). As such, comparing array sizes consists of a fast and easily available decision (Foltz, Poltrock, & Potts, 1984), leading to the highest accuracy. In the same vein, participants were also significantly better at comparing the size of individual dots than they were for numerosity. Interestingly, the comparison of dot size, however fast and easy, was slightly less efficient than the comparison of array size (i.e., convex hull). This may be due to the involvement of selective attention to focus on local/detailed information (see Johnston & Dark, 1986, for a review). Lastly, performance in the Total Area task was similar to the one from the Number comparison task (as in Odic & Halberda, 2015).

We also investigated the potential interactions between dimensions. In the Number task, participants produced more accurate numerical judgments when number covaried with total area and, with to an even larger extent, when it covaried with convex hull. This finding is in line with previous studies showing that Number judgments are affected by total area (Abreu-Mendoza et al., 2013; Guillaume et al., 2013; Hurewitz et al., 2006; Lourenco & Bonny, 2017; Lourenco et al., 2012; Nys & Content, 2012). They also support recent observations suggesting that convex hull is even more influential than total area on Number comparison (Gilmore et al., 2016; Guillaume et al., in press).

In the Total Area task, the performance was facilitated by the covariation with number and convex hull. In the Convex Hull task, we found better accuracy when number covaried with the target dimension. In the Individual Size task, covariation of total area improved performance, but interestingly, covariation of number decreased performance. The latter observation actually corresponds to the fact that number and item size are naturally related in

the opposite direction: large quantities generally consist of small elements (see Guillaume et al., in press). Lastly, in the Mean Occupancy task, the covariation of coverage proportion improved performance.

In three out of four comparison tasks, number – which was irrelevant to the task – had a significant positive or negative impact on accuracy. This supports and extends previous observations showing that number can serve as a cue to make non-numerical decisions (Guillaume et al., 2013; Nys & Content, 2012). Note that number did not affect occupancy judgments, but these led to poor performance overall. Such a poor performance might have hindered the influence of numerical information in the decision process. It is therefore possible that we failed to observe an impact from number on occupancy judgments simply due to the low-performance level. Alternatively, one could even argue that performance in this task was low because participants did not use all available information to respond. This is of course hypothetical and this claim will need to be further investigated in future studies.

It was previously assumed that the influence of numerical and non-numerical dimensions is bidirectional (e.g. Hurewitz et al., 2006; Lourenco et al., 2012) but the nature of this relation is still unclear. In our data, we did not find such a systematic mutual influence between all non-numerical dimensions. For instance, participants benefited from covariation of convex hull in the Total Area task, but they did not benefit from the covariation of total area in the Convex Hull task. Number was the only dimension for which such bidirectional relationships were always found. The impact of numerosity on non-numerical dimensions could be due to the salience of number. Recent studies indeed revealed that humans spontaneously and preferentially process number over other dimensions (Anobile et al., 2019; Cicchini, Anobile, & Burr, 2016; Ferrigno, Jara-Ettinger, Piantadosi, & Cantlon, 2017). In this respect, covariations of number could serve as strong and easy cues to make non-numerical judgments.

Furthermore, we did not find any systematic interference or facilitation in our results. For instance, participants obtained the best performance in the Convex Hull comparison task, which may be one of the most salient dimensions. Despite this, covariations of convex hull did not affect all comparison tasks. This lack of systematicity suggests that the nature of the influences is not related to automatic (perceptual) processes. Rather it supports the view that the facilitation relies on higher-level processes and indexes the strategic use of covarying dimensions to help decision making. In other words, the lack of systematic mutual influences would reflect the deliberate choice and strategic use of these cues, depending on the task to be performed. This corroborates the claim that adults can flexibly and adaptively use all available information to perform numerical comparison judgments (Roquet & Lemaire, 2019). Performance in comparison tasks is thus deeply affected by irrelevant covarying cues, irrespective of the numerical aspect. Regarding numerosity, some authors have argued that number extraction is not independent of the processing of other visual dimensions (Gebuis et al., 2016; Leibovich et al., 2017), but recent electrophysiological data showed that numerosity elicits strong responses in the visual cortex independently of other visual dimensions (Van Rinsveld et al., 2020), reinforcing the view that the influence of others dimensions stems from later processing stages. This conclusion also suggests that the interpretation of individual differences in the Number comparison task should take into consideration the possible impact of executive functions. The recruitment of such cognitive processes into decision-making mechanisms requires further investigation in future studies.

To conclude, the five comparison tasks proposed, as well as the method for creating the stimuli, highlight that the number can differently be used as a cue just as the non-numerical dimensions (e.g. Total Area, Convex Hull) depending on the comparison task. We also highlight that these cues, numerical or non-numerical, could be used differently depending on the dimension under consideration, suggesting the recruitment of decision-making mechanisms. This challenges

431 once again the assumption that comparison tasks are suitable to assess specific cognitive
432 processes.

433

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435

Conflict of Interest Statement

436 The authors declare that there is no conflict of interest.

437

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441

Authors' Contribution

442 Mathieu Guillaume, Amandine Van Rinsveld, and Alain Content designed the study; Charlotte

443 Hendryckx and Anthony Beuel created the stimuli and the display software; Charlotte

444 Hendryckx collected and analyzed the data; Charlotte Hendryckx and Mathieu Guillaume

445 wrote the first draft of the manuscript; Amandine Van Rinsveld and Alain Content revised the

446 manuscript.

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