Same model, different conclusions: An identifiability issue in the linear ballistic accumulator model of decision-making

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Abstract

Evidence accumulation models (EAMs) – the dominant modelling framework for speeded decision-making – have become an important tool for *model application*. Model application involves using specific model to estimate parameter values that relate to different components of the cognitive process, and how these values differ over experimental conditions and/or between groups of participants. In this context, researchers are often agnostic to the specific theoretical assumptions made by different EAM variants, and simply desire a model that will provide them with an accurate measurement of the parameters that they are interested in. However, recent research has suggested that the two most commonly applied EAMs – the diffusion model and the linear ballistic accumulator (LBA) – come to fundamentally different conclusions when applied to the same empirical data. The current study provides an in-depth assessment of the measurement properties of the two models, as well as the mapping between, using two large scale simulation studies and a reanalysis of Evans (2020a). Importantly, the findings indicate that there is a major identifiability issue within the standard LBA, where differences in decision threshold between conditions are practically unidentifiable, which appears to be caused by a tradeoff between the threshold parameter and the overall drift rate across the different accumulators. While this issue can be remedied by placing some constraint on the overall drift rate across the different accumulators – such as constraining the average drift rate or the drift rate of one accumulator to have the same value in each condition – these constraints can qualitatively change the conclusions of the LBA regarding other constructs, such as non-decision time. Furthermore, all LBA variants considered in the current study still provide qualitatively different conclusions to the diffusion model. Importantly, the current findings suggest that researchers should not use the unconstrained version of the LBA for *model application*, and bring into question the conclusions of previous studies using the unconstrained LBA.

Introduction

Cognitive modelling has become a powerful approach for understanding the processes that underlie performance in a range of different cognitive tasks (Stone, 1960; Nosofsky, 1986; Shiffrin & Steyvers, 1997; Osth & Dennis, 2015). Cognitive models are theories of cognitive processes that are formalized into mathematical equations, providing precise quantitative predictions about empirical data. Importantly, cognitive models allow researchers to directly assess how well the predictions of the theory match the trends observed in empirical data, and compare different competing theories about how the underlying cognitive process operates. Within the speeded decision-making literature, evidence accumulation models (EAMs; Stone, 1960) have become the dominant framework of cognitive models. EAMs propose that decisions are made through accumulating evidence from the environment for each alternative, until the evidence for one alternative reaches some threshold amount to trigger a decision. Importantly, EAMs have been shown to successfully account for a range of empirical data trends (Stone, 1960; Ratcliff, 1978; Ratcliff & Rouder, 1998; Usher & McClelland, 2001; Ratcliff & Starns, 2009; Teodorescu & Usher, 2013; Smith, 2016; Evans, Dutilh, Wagenmakers, & van der Maas, 2020), which has led to their dominance in the speeded decision-making literature (Evans & Wagenmakers, 2020).

One of the primary uses of EAMs has been for *model application* (Crüwell, Stefan, & Evans, 2019). In model application, researchers assume that a specific model provides an accurate account of the underlying cognitive process and use the model to estimate parameter values – which relate to different components of the cognitive process – and how they differ over experimental conditions and/or between groups of participants (see Donkin, Averell, Brown, & Heathcote, 2009 for a tutorial within EAMs). For EAMs, these parameters are (1) the “drift rate”, which is the rate of evidence accumulation for a specific alternative, and reflects task difficulty and participant ability, (2) the “decision threshold”,

which is the amount of evidence required to trigger a decision, and reflects task caution, and (3) the “non-decision time”, which is the amount of time dedicated to processes that are not directly part of the decision process, and reflects the speed of perceptual encoding and motor responding. EAMs have been used as the measurement tool in model application to answer a range of different research questions in a range of different research areas, such as intelligence (van Ravenzwaaij, Brown, & Wagenmakers, 2011; Lerche et al., 2020), performance optimality (Starns & Ratcliff, 2012; Evans & Brown, 2017; Evans, Bennett, & Brown, 2019), cognitive workload (Ratcliff & Strayer, 2014; Tillman, Strayer, Eidels, & Heathcote, 2017; Howard, Evans, Innes, Brown, & Eidels, 2020), ageing (Ratcliff, Thapar, & McKoon, 2001; Servant & Evans, 2020), emotion processing (Lerche, Bucher, & Voss, 2019), alcohol consumption (van Ravenzwaaij, Dutilh, & Wagenmakers, 2012), and ego depletion (Lin, Saunders, Frieze, Evans, & Inzlicht, 2020), just to name a few.

Several specific models have been proposed within the broader EAM framework (Stone, 1960; Ratcliff, 1978; Busemeyer & Townsend, 1993; Ratcliff & Rouder, 1998; Usher & McClelland, 2001; Brown & Heathcote, 2005, 2008; Verdonck & Tuerlinckx, 2014; Tillman, Van Zandt, & Logan, 2020). While all variants share the same basic underlying assumptions, where evidence accumulates for the different alternatives until the evidence for one reaches a threshold, each variant makes a different proposal about the exact dynamics of the decision-making process (Evans, Dutilh, et al., 2020). For example, two of the most commonly applied EAMs – the diffusion model (Stone, 1960; Ratcliff, 1978) and the linear ballistic accumulator (LBA; Brown & Heathcote, 2008) – propose underlying processes that differ in several respects. The diffusion model proposes that information processing is subject to moment-to-moment noise, which is reflected in the model by the Wiener process, and results in moment-to-moment variability in the amount of evidence accumulated. In contrast, the LBA proposes that information processing is noiseless, and

that the only source of noise in evidence accumulation is the variation between trials in the rate of accumulation; a process that is sometimes (Ratcliff, 1978), though not always (see van Ravenzwaaij & Oberauer, 2009; Lerche & Voss, 2016; Boehm et al., 2018; Evans, Tillman, & Wagenmakers, 2020 for studies showing the issues with including this source of variability within the diffusion model), included in the diffusion model. Furthermore, the LBA assumes that evidence is accumulated independently for each alternative, whereas the diffusion model assumes that evidence for one alternative is also evidence against the other alternative.

The development of different models within the EAM framework is fundamental for understanding the decision-making process, as each variant reflects a different specific theory about how the decision-making process operates. Comparing these models allows researchers to adjudicate between different explanations of the decision-making process, and determine which specific decision-making dynamics seem most likely given the available data (Navarro, 2019; Evans, 2020b). However, in the case of *model application* – where the model only serves as a measurement tool, providing researchers with the ability to estimate parameter values that reflect the different components of the decision-making process – researchers are often agnostic to the more specific dynamics of the model. That is, researchers simply desire a model that will provide them with an accurate measurement of the decision-making components that they are interested in, and are indifferent to the specific theoretical proposal of each model, such as whether evidence accumulation is independent or dependent between alternatives. Given that EAMs are commonly used as measurement tools, it seems important to establish whether different EAMs come to similar conclusions, or whether the choice of EAM can influence the conclusions of the study. Furthermore, if the choice of EAM can influence the conclusions of the study, then it is important to determine why these differences occur, and which EAM researchers should

use.

Previous research has generally concluded that the theoretical differences between EAM variants are of little practical consequence. For example, when assessing the ability of the models to account for response time and response choice data, previous research has suggested that most EAM variants show strong mimicry of one another (Teodorescu & Usher, 2013; Evans, Dutilh, et al., 2020), making it difficult to distinguish between their predictions about the data. More importantly, in model application contexts, previous research has suggested that different EAMs generally come to the same conclusions about psychological mechanisms, meaning that the choice of EAM is unlikely to influence the conclusions of the study (Donkin, Brown, Heathcote, & Wagenmakers, 2011; Rae, Heathcote, Donkin, Averell, & Brown, 2014). However, some recent studies have questioned the practical agreement between the diffusion model and the LBA in their conclusions about psychological mechanisms (Osth, Bora, Dennis, & Heathcote, 2017; Dutilh et al., 2019; Evans, 2020a). Two notable examples are the many-labs study of Dutilh et al. (2019) and the large-scale selective influence reanalysis of Evans (2020a).

Dutilh et al. (2019) involved 17 teams of experts providing inferences on 14 different empirical “pseudo-experiments” (see Dutilh et al., 2019; Evans, 2019a, 2020a for an in-depth explanation), with each pseudo-experiment having two within-subjects conditions. Each team was required to determine which components of the decision-making process had been experimentally manipulated across the within-subjects conditions, while being blind to the experimental manipulations used, and were able to use any model(s) and method(s) to make these inferences. Most notably, the agreement in inferences was much higher between teams of experts who chose the same model, compared to teams of experts who chose different models. Specifically, teams of experts who chose the diffusion model were more likely to infer that non-decision time differed across experimental conditions

than teams of experts who chose the LBA, and less likely to infer that drift rate differed across experimental conditions. However, given the relatively small number of teams and the many differences between them in other aspects of their methodology, it is unclear whether these patterns of results are due to systematic differences between the models, or differences between other aspects of their methodology.

Evans (2020a) provided a comprehensive reassessment of which parameters are influenced by experimental manipulations of urgency emphasis, as previous studies had either suggested that this manipulation selectively influences decision threshold (Ratcliff, 2002; Dutilh et al., 2019), influences both decision threshold and drift rate (Rae et al., 2014), or influences both decision threshold and non-decision time (Voss, Rothermund, & Voss, 2004; Lerche & Voss, 2017). The reassessment applied both the diffusion model and the LBA – allowing for the assessment of consistency between the models – to 6 previous data sets that implemented an urgency emphasis manipulation – allowing for the assessment of consistency across experiments – using a common statistical framework to control for potential differences due to methodology. Most notably, Evans (2020a) found that the diffusion model generally concluded that both decision threshold and non-decision time decreased under urgency emphasis, whereas the LBA generally concluded that both decision threshold and drift rate decreased under urgency emphasis. Importantly, this suggests that the two different models come to fundamentally different conclusions about the underlying cognitive process, which could partially explain the differences in conclusions between previous studies.

The results of Evans (2020a) may seem surprising given the findings of Donkin et al. (2011), who directly investigated the relationship between the diffusion model and the LBA in both simulated and empirical settings, and found that the models came to the same conclusions in the majority of cases. One potential explanation for the contrasting

conclusions between Evans (2020a) and Donkin et al. (2011) is the exact specification of the LBA used in each study. Specifically, Donkin et al. (2011) imposed a constraint on the drift rate parameters for each decision alternative, where the drift rate parameters of the two accumulators had to sum to the fixed value of 1 – often referred to as the *sum-to-1* constraint. Although some previous studies using the LBA have implemented the sum-to-1 constraint (Donkin, Little, & Hout, 2014; Osth et al., 2017), the more common practice is to separately estimate the drift rate for each accumulator, allowing for more flexibility in the model (Brown & Heathcote, 2008; Rae et al., 2014; Weigard, Huang-Pollock, Heathcote, Hawk, & Schliez, 2018; Dutilh et al., 2019; Evans, 2019a; Castro, Strayer, Matzke, & Heathcote, 2019; Palada, Neal, Strayer, Ballard, & Heathcote, 2019; Evans, 2020a; Howard et al., 2020). Importantly, Evans (2020a) – as well as several other relevant studies (Rae et al., 2014; Dutilh et al., 2019) – separately estimated the drift rate for each accumulator, meaning that the differences between studies – and whether the conclusions of the diffusion model and LBA agree or disagree – may be caused by this difference in the definition of the LBA.

There are two potential reasons why the sum-to-1 constraint might result in the LBA providing closer agreement with the diffusion model. The first potential reason is that the sum-to-1 constraint may result in the LBA becoming more similar to the diffusion model, which in turn could result in their inferences becoming more similar. Specifically, the sum-to-1 constraint provides a dependence between the mean drift rates of the two accumulators, where increasing the mean drift rate of one accumulator results in an equivalent decrease in the mean drift rate of the other accumulator. Although a dependence in mean drift rates is not equivalent to the moment-to-moment dependence in accumulated evidence contained within the diffusion model, it is possible that the level of dependence is a practically important assumption of EAMs that influences the parameter estimates, and therefore,

the conclusions drawn. If the level evidence accumulation dependence is an important practical factor in EAMs, then it is possible that introducing more dependence into the LBA framework results in conclusions that are increasingly similar the diffusion model. However, it should be noted that the diffusion model and sum-to-1 LBA do not always come to the same conclusions (Osth et al., 2017), showing that there are still practical differences between the sum-to-1 LBA and the diffusion model. The second potential reason is that the LBA may be underconstrained when the drift rates for each accumulator are estimated separately, and that the sum-to-1 definition may provide the constraint necessary for robust estimation. Specifically, the additional flexibility of separately estimated drift rates may cause identifiability issues within the LBA, which are solved by adding additional constraint, such as forcing the drift rates to sum to a fixed value. Although previous studies have suggested that the standard LBA possesses robust measurement properties, and may even be overconstrained (Brown & Heathcote, 2008; Donkin, Brown, & Heathcote, 2009), these studies have not provided an in-depth assessment of the measurement properties of the LBA in commonly applied contexts. If the standard LBA is underconstrained and has identifiability issues, then it is possible that the sum-to-1 LBA and the diffusion model often come to similar conclusions as they provide reliable inferences, and that the standard LBA and the diffusion model often come to different conclusions as the standard LBA provides unreliable inferences.

The current study aims to provide a comprehensive investigation into the differences in findings between Donkin et al. (2011) and Evans (2020a). Specifically, the current study aims to determine whether (1) constraining the mean drift rate of the two accumulators in the LBA to sum to a fixed value – such as the *sum-to-1* constraint – leads to inferences that more closely match the those of the diffusion model, and (2) whether the standard LBA – where the mean drift rates for both accumulators are estimated separately – provides suf-

ficient constraint to robustly estimate the components of the underlying decision-making process, or whether further constraints – such as the *sum-to-1* constraint – are needed to provide robust estimation. The current study uses two large-scale simulation studies based on two different practical situations that researchers commonly investigate: estimating the parameters of the model for a single experimental condition – as is commonly done when assessing whether parameters differ between groups of participants – and estimating the parameters of the model across two experimental conditions – as is commonly done when assessing whether parameters differ based on an experimental manipulation. Each simulation study focuses on the standard LBA, more constrained versions of the LBA, and the diffusion model, assessing the *parameter recovery* of the models – which is how accurately each model can estimate the data generating parameters from its own simulated data – as well as the *cross-fit* between the models – which is how closely that same psychological constructs in different models map to one another, assessed through how accurately each model can estimate the data generating parameters from simulated data of other models. The current study also provides a reanalysis of Evans (2020a) with several more constrained versions of the LBA, assessing whether the conclusions of these versions of the LBA agree or disagree with the conclusions of the standard LBA and the diffusion model implemented in the original study.

Simulation study 1: Estimating the parameters for a single condition

The first simulation study assesses the measurement properties of three models – the unconstrained LBA, the constrained LBA, and the simple diffusion model (see the *Models* subsection) – for a single experimental condition, which is a common context when assessing whether estimated parameters differ between groups of participants. The current

simulation study follows the following structure: (1) a description of the simulation and fitting details, (2) definitions for all three models considered, (3) the results of the parameter recovery and cross-fitting assessments, and (4) a discussion of the results and what they mean for *model application* studies using these models.

Simulation and fitting methods

The simulation study required generating a large number of data sets from each of the three model of interest, with each of the models of interest detailed below in the *Models* subsection. The recovery assessment involved fitting each model to the data sets that it generated, and assessing whether the estimated parameters matched the data generating parameters. The cross-fitting assessment involved fitting each model to the data sets that were generated by the other models, and assessing the mapping between the generating parameters of one model and the estimated parameters of another model, focusing only on the relationship between parameters that relate to the same cognitive construct.

Generating parameter sets were selected using a Latin hypercube sampling design (McKay, Beckman, & Conover, 1979; Evans, Trueblood, & Holmes, 2020), which provides the most efficient method of sampling values from a specified, continuous, multidimensional space. The parameter space differed between models, as each model contains different parameter values with different plausible ranges, which can be found in the *Models* subsection under each model. 4,000 parameter sets were drawn from the hypercube for each data generating model, with each parameter set used to generate a single data set of 1,000 trials using the method and framework of Evans (2019b). However, data sets were only included in the recovery assessment if they resembled realistic choice response time distributions, in order to restrict the recovery assessment to realistic situations¹ (Evans, Trueblood, &

¹It should be noted that defining “realistic” data sets is an inherently subjective process, particularly if the specific trends that are deemed to make data “realistic” are inconsistent between empirical data sets. Importantly, if the parameter recovery assessment focuses on an overly narrow region of the parameter

Holmes, 2020). Therefore, although the same number of parameter sets were drawn from each hypercube, the number of included data sets differed between generating models based on the number of unrealistic data sets that were discarded for each model. The exact number of included data sets can be found in the *Models* subsection under each model. The inclusion criteria for the data sets were:

$$0.4 < \textit{Mean RT} < 2.5$$

$$0.4 < \textit{Median RT} < 2.5$$

$$0.1 < \textit{IQR of RT} < 2$$

$$\textit{Minimum RT} < 1.5$$

$$0.5 < \textit{Maximum RT}$$

$$0.4 < \textit{Accuracy} < 1$$

The models were fit to the data using Bayesian methods of parameter estimation, which involves estimating the joint probability distributions of the parameter values given the data (i.e., the joint posterior distributions). Specifically, the estimation procedure was performed using Differential Evolution Markov Chain Monte Carlo (DE-MCMC; Ter Braak, 2006; Turner, Sederberg, Brown, & Steyvers, 2013), which is specifically designed to sample from posterior distributions with highly correlated dimensions using several interacting chains (see also Evans & Annis, 2019 for an extension to marginal likelihood estimation). The DE-MCMC algorithm was implemented using $3k$ chains – where k is the number of free parameters in the model² – which were each run for 3,000 iterations, with the first

space, then the conclusions of the study could indicate that the identifiability of the model is better than it actually is. However, parameter identifiability issues cannot be fixed by extending the region of space assessed in the recovery.

²Note that if $3k$ was less than 12, then 12 chains were used instead, based on my own past experiences

1,000 iterations discarded as burn-in. The priors distributions differed between parameters and models, though were only mildly informative to prevent extreme values from being estimated, and can be found in the “Models” subsection under each model. Estimated parameter values were obtained by taking the mean of the estimated posterior distribution for each parameter.

The parameter recovery assessment consisted of qualitative and quantitative analyses of how well the estimated parameter values matched the data generating parameter values. Specifically, the match was qualitatively evaluated via a scatterplot of the generated and estimated parameter values of each included data set, and quantitatively evaluated via the correlation between the generated and estimated parameter values. The qualitative assessment involved evaluating how closely the generated and estimated values fell on the line of perfect recovery (i.e., $x = y$), both in terms of the bias (i.e., whether the points were centred on the line) and precision (i.e., the dispersion of the points) of the estimates. The quantitative assessment involved calculating Pearson’s correlation coefficient, which provides a complementary quantitative assessment of the precision of the estimates.

The cross-fit assessment also consisted of qualitative analyses via scatterplots and quantitative analyses via correlations. However, for the cross-fit assessment, the focus was on whether the parameter values estimated from one model provided a one-to-one mapping with the parameter values used to generate the data from another model, allowing for inferences about how closely the same constructs in different models map onto one another. Importantly, as the parameter values for different models may show a one-to-one mapping while being on different scales, the line of perfect recovery (i.e., $x = y$) was not a key focus³. Instead, the qualitative and quantitative assessments focused on the precision of the estimates, in order to determine whether the generated and estimated parameter values

with DE-MCMC samplers failing when using 9 or less chains.

³Note that this line is still included within the scatterplots as a point of reference.

displayed any clear, linear mapping.

Models

The first simulation study implemented three models: the standard LBA, which I refer to as the *unconstrained LBA*; the LBA with the sum of the drift rates fixed to a set value (i.e., the *sum-to-1* constraint, though with a different fixed value), which I refer to as the *constrained LBA*, and the four parameter diffusion model. Here, I provide an explanation of each model, as well as the hypercube dimensions and priors used for the simulation study.

Unconstrained LBA. The LBA proposes that the evidence accumulation for each alternative is independent, where the amount of evidence in favour of one alternative is unrelated to the amount of evidence in favour of the other alternative (Brown & Heathcote, 2008). Furthermore, the LBA proposes that there is no moment-to-moment noise in evidence accumulation, and that the only sources of noise within the accumulation process are between different decisions, with drift rate varying between trials according to a normal distribution, and the initial evidence varying between trials according to a uniform distribution (Brown & Heathcote, 2005). These simplifying assumptions mean that the accumulation process of the LBA can be expressed through 5 parameters for each accumulator: the mean drift rate over trials (v), the standard deviation in drift rate over trials (s), the decision threshold (b), the range of the uniform distribution of initial evidence (A), and the non-decision time (t_0).

However, additional simplifying assumptions are generally implemented to further constrain the model. First and foremost, the s parameter for one accumulator is fixed to 1 to solve a scaling issue within the model (Brown & Heathcote, 2008), and the s parameter for the other accumulator is generally fixed to 1 to further constrain the model (though

see Donkin, Brown, & Heathcote, 2009). Second, the A and t_0 parameters are generally constrained to have the same values for both accumulators, again further constraining the model. Finally, the b parameter is commonly constrained to have the same value for both accumulators in cases where researchers can assume that any response biases are minimal (Evans, Rae, Bushmakin, Rubin, & Brown, 2017), which is the case in the current simulation study as all data sets are generated without response bias. The current study makes these simplifying assumptions, meaning that only the v parameter is estimated separately for each accumulator: v_1 and v_2 . Therefore, the unconstrained LBA contained 5 parameters within this simulation study: v_1 , v_2 , b , A , and t_0 . However, for the purposes of the data simulation and recovery assessment, the mean drift rate parameters were instead expressed as the midpoint of the two drift rates (v_{mean} ; $\frac{v_1+v_2}{2}$), and the separation between the two drift rates (v_{diff} ; $v_1 - v_2$). Also note that b was always parameterized as $B = b - A$, in order to ensure that the initial evidence was always less than the decision threshold.

The range of values for each parameter within the Latin hypercube were:

$$\begin{aligned} v_{mean} &= [2.5, 5.5] \\ v_{diff} &= [0.05, 2] \\ t_0 &= [0.1, 0.6] \\ B &= [0.4, 3] \end{aligned}$$

with A fixed to 1. Based on the inclusion criteria, this resulted in 3,150 data sets being included in the analysis. The priors for each parameter in model, used for the Bayesian parameter estimation on the data generated by all models, were:

$$v_1 \sim TN(3, 1, 0, \infty)$$

$$v_2 \sim TN(1, 1, 0, \infty)$$

$$t_0 \sim TN(0.3, 0.1, 0, \infty)$$

$$B \sim TN(1, 0.5, 0, \infty)$$

$$A \sim TN(1, 0.5, 0, \infty)$$

where TN is the truncated normal distribution with parameters mean, standard deviation, lower truncation bound, and upper truncation bound.

Constrained LBA. The constrained LBA implemented within this simulation study is identical to the unconstrained LBA, though with one additional restriction: that the mean drift rates of both accumulators (i.e., v_1 and v_2) must sum to a fixed value. This introduces a dependence between the values of the mean drift rates of each accumulator, though the evidence accumulation process itself remains independent, where in each decision the amount of evidence in favour of one alternative is unrelated to the amount of evidence in favour of the other alternative. The fixed value was set at 7.5, which was the median of the sum of the generating drift rates values for the included data sets of the unconstrained LBA. Therefore, the constrained LBA contained 4 parameters within this simulation study: v_1 , b , A , and t_0 , with v_2 being fixed at $7.5 - v_1$. However, for the purposes of the data simulation and recovery assessment, the mean drift rate parameters were instead expressed as the midpoint of the two drift rates ($v_{mean}; \frac{v_1+v_2}{2}$), and the separation between the two drift rates ($v_{diff}; v_1 - v_2$). Also note that b was always parameterized as $B = b - A$, in order to ensure that the initial evidence was always less than the decision threshold.

The range of values for each parameter within the Latin hypercube were:

$$\begin{aligned}
v_{diff} &= [0.05, 2] \\
t_0 &= [0.1, 0.6] \\
B &= [0.4, 3]
\end{aligned}$$

with A fixed to 1 and v_{mean} fixed to 3.75 (i.e., $\frac{7.5}{2}$). Based on the inclusion criteria, this resulted in 3,830 data sets being included in the analysis. The priors for each parameter in model, used for the Bayesian parameter estimation on the data generated by all models, were:

$$\begin{aligned}
v_1 &\sim TN(3, 1, 0, \infty) \\
t_0 &\sim TN(0.3, 0.1, 0, \infty) \\
B &\sim TN(1, 0.5, 0, \infty) \\
A &\sim TN(1, 0.5, 0, \infty)
\end{aligned}$$

where TN is the truncated normal distribution with parameters mean, standard deviation, lower truncation bound, and upper truncation bound.

Diffusion model. The diffusion model proposes that the evidence accumulation for one alternative is dependent on the other alternative, where evidence for one alternative counts as evidence against the other alternative (Stone, 1960). Furthermore, the diffusion model proposes that evidence accumulation is subject to moment-to-moment noise, with the accumulation varying between time points within each trial according to a normal distribution. These assumptions mean that the accumulation process can be expressed as

the relative evidence between the two alternative through 5 parameters: the difference in drift rate between the two alternatives (v ; similar to the v_{diff} parameter in the LBA), the standard deviation of the accumulation noise (s), the difference between the two thresholds (a), the difference in initial evidence between the two alternatives (z), and the non-decision time (t_{er} ; similar to the t_0 parameter in the LBA). However, the s parameter is generally fixed to 1 (or 0.1) to solve a scaling issue within the model (Ratcliff, 1978). Furthermore, the z parameter is commonly fixed to be the midpoint of the threshold separation in cases where researchers can assume that any response biases are minimal (Ulrich, Schröter, Leuthold, & Birngruber, 2015; Evans & Servant, 2020), which is the case in the current simulation study as all data sets are generated without response bias. The current study makes these simplifying assumptions, meaning that diffusion model contained 3 parameters within this simulation study: v , a , and t_{er} . It should be noted that the diffusion model is also often extended to include between-trial variability in several components (drift rate, s_v , Ratcliff, 1978; initial evidence, s_z , Ratcliff & Rouder, 1998; non-decision time, s_{ter} , Ratcliff & Tuerlinckx, 2002), though the current study does not include these additional terms as they are already known to create parameter recovery issues (van Ravenzwaaij & Oberauer, 2009; Lerche & Voss, 2016; Boehm et al., 2018; Evans, Tillman, & Wagenmakers, 2020).

The range of values for each parameter within the Latin hypercube were:

$$\begin{aligned} v &= [0.1, 4] \\ t_{er} &= [0.1, 0.6] \\ a &= [0.45, 1.75] \end{aligned}$$

Based on the inclusion criteria, this resulted in 2,866 data sets being included in the anal-

ysis. The priors for each parameter in model, used for the Bayesian parameter estimation on the data generated by all models, were:

$$\begin{aligned} v &\sim TN(3, 1, 0, \infty) \\ t_{er} &\sim TN(0.3, 0.1, 0, \infty) \\ a &\sim TN(2, 1, 0, \infty) \end{aligned}$$

where TN is the truncated normal distribution with parameters mean, standard deviation, lower truncation bound, and upper truncation bound.

Results and Discussion

Here, I discuss the parameter recovery results for each model separately, before discussing the cross-fitting results together for all models. Note that each figure displays the estimation of a specific model to all data sets (i.e., all generating models), with different figures for the estimation of different models. In all cases, the top row of the figure shows the parameter recovery of the model (i.e., the data sets where the estimated model is the data generating model), and the lower rows show the cross-fits. Note that the red line of perfect recovery is included in all plots, though it is primarily relevant for the parameter recovery assessment.

Unconstrained LBA recovery. The top row of Figure 1 displays the parameter recovery for the unconstrained LBA. Interestingly, the recovery seems to differ a great deal between the different parameters. For the v_{diff} parameter the estimates fall directly along the red line with only minor variability, resulting in a near-perfect correlation. However, the other parameters show poorer recovery, ranging from slightly worse to substantially

worse. For the t_0 parameter the estimates mostly fall along the red line, though with some tendency towards an overestimation of small generating values. Furthermore, the variability in the estimates is much larger for t_0 than v_{diff} , resulting in a poorer (though still reasonably high; 0.94) correlation. For the v_{mean} parameter the recovery is worse again, having similar variability in the estimates to non-decision time (and a similar correlation; 0.93), though with some problematic tendencies under higher generating values, with the estimates flattening out and showing a tendency towards underestimation. Finally, for the B parameter the recovery is the worst, having the highest variability in the estimates and lowest correlation (0.87), and a more extreme tendency than the v_{mean} parameter for the underestimation of high generating values.

One interesting trend in the recoveries is the similar patterns of estimation problems for the B and v_{mean} parameters, where both show unbiased – though variable – estimation under lower generating values, though show tendencies towards underestimation under higher generating values. These similar patterns of bias and imprecise estimation suggest a possible tradeoff between these parameters, where decreases in B can be compensated for by equivalent decreases in v_{mean} . This potential tradeoff would make sense within the unconstrained LBA, as decreasing B and increasing v_{mean} both result in decreases in response time and decreases in accuracy. Importantly, if a tradeoff between B and v_{mean} is the cause of their respective recovery issues, then some combination of these two parameters – such as the average time to the threshold (i.e., $\frac{B}{v_{mean}}$) – might show substantially better recovery. The top right panel of Figure 1 displays the recovery of this new ratio of parameters – the average time to the threshold, $\frac{B}{v_{mean}}$ – which shows better recovery than either parameter alone, with the recovery being drastically improved from the B parameter, though still imperfect (variable estimates with a slight underestimation of high generating values).

Figure 1. Displays the parameter recovery (top row) of the unconstrained LBA for simulation study 1, as well as the cross-fit to data generated by the constrained LBA (middle row) and the diffusion model (bottom row). The columns display the different parameters of the model, with the x-axis displaying the generated values and the y-axis displaying the estimated values. In each plot different points reflect different data sets, and the red line reflects perfect parameter recovery (i.e., $x = y$).

Constrained LBA recovery. The top row of Figure 2 displays the parameter recovery for the constrained LBA model. The recovery for the v_{diff} parameter appears to be near identical to the unconstrained LBA, with the estimates falling directly along the red line with only minor variability, resulting in a near-perfect correlation. The recovery for the t_0 parameter is also similar to the unconstrained LBA, though the estimates from the constrained LBA show slightly lower variability, resulting in a higher correlation (0.96). The differences in recovery between the models are most pronounced when assessing the B parameter. The constrained LBA provides a substantially better recovery of the B

parameter than the unconstrained LBA, with the estimates mostly falling along the red line, though with some tendency towards an underestimation of high generating values, resulting in a relatively high correlation (0.95).

Figure 2. Displays the parameter recovery (top row) of the constrained LBA for simulation study 1, as well as the cross-fit to data generated by the unconstrained LBA (middle row) and the diffusion model (bottom row). The columns display the different parameters of the model, with the x-axis displaying the generated values and the y-axis displaying the estimated values. In each plot different points reflect different data sets, and the red line reflects perfect parameter recovery (i.e., $x = y$).

Diffusion model recovery. The top row of Figure 3 displays the parameter recovery for the diffusion model. Note that the column titles use the LBA notation from the parameters, meaning that v is v_{diff} , a is B , and t_{er} is t_0 . The recovery for all parameters is near-perfect, with the estimates falling directly along the red line with only minor variability, resulting in

a near-perfect (or rounded to perfect) correlation. Therefore, the parameters of the simple diffusion model can all be recovered with extreme precision, which shows an improvement over the constrained LBA, and a major improvement over the unconstrained LBA.

Figure 3. Displays the parameter recovery (top row) of the diffusion model for simulation study 1, as well as the cross-fit to data generated by the constrained LBA (middle row) and the unconstrained LBA (bottom row). The columns display the different parameters of the model (note that these use the LBA notation), with the x-axis displaying the generated values and the y-axis displaying the estimated values. In each plot different points reflect different data sets, and the red line reflects perfect parameter recovery (i.e., $x = y$).

Cross-fits. The middle and bottom rows of Figure 1 display the estimates of the unconstrained LBA from data generated by the constrained LBA and the diffusion model, respectively. The patterns of estimates for the data generated by the constrained LBA

(middle row) are very similar to those produced in the recovery assessment of the unconstrained LBA, which perhaps is not surprising given that the constrained LBA is nested within the unconstrained LBA. In contrast, the patterns of estimates for the data generated by the diffusion model (bottom row) generally fails to show a one-to-one mapping between the same constructs. The estimates appear to provide the closest mapping for drift rate, though are still quite poor, showing (1) high variability, (2) a poor correlation (0.85), and (3) an inability to distinguish between higher generating values. These problems become worse in the estimates for the other parameters, with the threshold estimates showing extremely high variability, resulting in a very poor correlation (0.77), and the non-decision time estimates showing a problematic tendency of being any value between zero and the true generating value. While the estimates for threshold are improved when looking at the estimated $\frac{B}{v_{mean}}$ ratio, as the variability is decreased, the variability is still too high to show any clear one-to-one mapping between the threshold values generated by the diffusion model and the $\frac{B}{v_{mean}}$ ratio estimated by the unconstrained LBA.

The middle and bottom rows of Figure 2 display the estimates of the constrained LBA from data generated by the unconstrained LBA and the diffusion model, respectively. Unlike the unconstrained LBA estimates, the patterns of estimates for the data generated by the unconstrained LBA (middle row) are much poorer than those produced in the recovery assessment of the constrained LBA. Although drift rate remains well estimated, suggesting that both versions of the LBA have a perfect one-to-one mapping for the drift rate construct, the estimates for non-decision time are much more variable, and the estimates of threshold are sufficiently variable to make any mapping impossible. However, the estimates of threshold become substantially less variable when compared to the generated $\frac{B}{v_{mean}}$ ratio, suggesting that threshold estimate in the constrained LBA more closely maps to the ratio between threshold and the midpoint of the drift rates across accumulators.

The patterns of estimates for the data generated by the diffusion model (bottom row) are poorer versions of the patterns of unconstrained LBA estimates to the data generated by the diffusion model (Figure 1, bottom row). Specifically, the patterns of estimates for the two LBA versions are similar in nature, though the estimates from the constrained LBA are substantially more variable, even causing the mapping for drift rate to become unclear.

The middle and bottom rows of Figure 3 display the estimates of the diffusion model from data generated by the constrained LBA and the unconstrained LBA, respectively. Unlike the estimates of the two LBA versions for the data generated by the diffusion model, the diffusion model estimates provide a reasonable mapping to most of the data generating LBA parameters. For the data generated by the constrained LBA (middle row), the diffusion model estimates for both drift rate and threshold have low variability that is fairly constant across the parameter space, suggesting a clear one-to-one mapping with the generating values for these constructs. However, the estimates of non-decision time show high variability, suggesting that there is no clear one-to-one mapping between the non-decision time parameters in the diffusion model and the LBA. The patterns of estimates for the data generated by the unconstrained LBA (bottom row) are similar to those produced for the data generated by the constrained LBA, with the exception of threshold, which is sufficiently variable to make any mapping impossible when estimated from data generated by the unconstrained LBA. However, the diffusion model estimates of threshold show a one-to-one – though somewhat variable – mapping with the generating $\frac{B}{v_{mean}}$ ratio, suggesting that threshold estimate in the diffusion model more closely maps to the ratio between threshold and the midpoint of the drift rates across accumulators in the unconstrained LBA.

Discussion. The current simulation study assessed the measurement properties of the unconstrained LBA, the constrained LBA, and the simple diffusion model in the context

of a single experimental condition, which provided some interesting results. First, both the recovery and cross-fit assessments showcased the robust measurement properties of the simple diffusion model. The diffusion model provided a near-perfect recovery for every parameter, supporting previous conclusions that the diffusion model is identifiable even when applied to small sample sizes (Lerche, Voss, & Nagler, 2017). Furthermore, the diffusion model estimates provided a one-to-one mapping with the generating drift rate and threshold value for the constrained LBA, as well as the drift rate and $\frac{\text{threshold}}{\text{drift rate midpoint}}$ ratio (in place of threshold) for the unconstrained LBA. Importantly, these findings suggest that there are minimal problematic degeneracies in the simple diffusion model, meaning that it may be best EAM for use in *model application* contexts.

Second, and perhaps most importantly, the measurement properties of the unconstrained LBA were much poorer than those of the constrained LBA. Although both versions recovered drift rate exceptionally well, and the unconstrained LBA only provided a slightly worse recovery of non-decision time, the unconstrained LBA recovered threshold substantially worse than the constrained LBA. Specifically, the threshold estimates of the unconstrained LBA showed both high variability and a problematic tendency for underestimation under higher generating values. This was similar to the pattern in midpoint of the drift rates, suggesting a potential tradeoff between the two parameters⁴. Importantly, if a tradeoff between threshold and the midpoint of the drift rates was the cause of their respective recovery issues, then some combination of these two parameters – such as the average time to the threshold (i.e., $\frac{\text{threshold}}{\text{drift rate midpoint}}$) – might show substantially better

⁴It should be noted that the problematic tendency of underestimating high generating values could partially be due to the priors on the B and v_{mean} parameters, as these higher generating values are at the edge of what is plausible under the priors. However, the priors on these parameters were no more restrictive than those on some of the diffusion model parameters (e.g., the drift rate), suggesting that even if the priors did play some role, this was only due to the degeneracy in the model caused by the tradeoff between B and v_{mean} . Furthermore, the priors on the threshold parameter were identical for the constrained and unconstrained LBA, meaning that the poorer performance of the unconstrained LBA could only be due to poorer measurement properties.

recovery. Interestingly, the average time to the threshold showed substantially better recovery than the threshold parameter in the unconstrained LBA, and provided a much better mapping to the threshold parameters in the constrained LBA and the diffusion model, suggesting that the $\frac{\text{threshold}}{\text{drift rate midpoint}}$ ratio may be the more robust way of estimating caution within the unconstrained LBA.

Finally, the current simulation study does not appear to show a clearer mapping between the constrained LBA – as opposed to the unconstrained LBA – and the diffusion model. When estimating both versions of the LBA on data generated by the diffusion model, the patterns of estimates were quite similar in nature. However, the estimates from the constrained LBA were substantially more variable than those of the unconstrained LBA, causing the already questionable mappings to become even more unclear. Importantly, this suggests that the constrained LBA is not necessarily more similar to the diffusion model, as the unconstrained LBA produces estimates that more closely map with the data generating parameters of the diffusion model. At a broader level, this result – when combined with the poorer measurement properties of the unconstrained LBA – suggests that the differences in conclusions between the standard LBA and the diffusion model may be the result of the poor measurement properties of the unconstrained LBA, rather than the constrained LBA being “diffusion-like” than the unconstrained LBA.

Simulation study 2: Estimating the change in parameters across two conditions

The previous simulation study provided insight into the measurement properties of the unconstrained LBA, the constrained LBA, and the simple diffusion model in the context of a single experimental condition, which is a common context when assessing whether estimated parameters differ between groups of participants. However, the context most

commonly of interest to researchers is where participants complete multiple experimental conditions, and researchers assess whether the parameters of the model – specifically, threshold, drift rate, and non-decision time – differ between conditions (Brown & Heathcote, 2008; Rae et al., 2014; Lerche & Voss, 2017; Weigard et al., 2018; Lerche et al., 2019; Dutilh et al., 2019; Evans, 2019a; Castro et al., 2019; Palada et al., 2019; Evans, 2020a; Howard et al., 2020). The second simulation study assesses the measurement properties of four models – the unconstrained LBA, two versions of the constrained LBA, and the simple diffusion model (see the *Models* subsection) – in the context of whether parameters differ between two experiment conditions. The current simulation study also includes the average time to threshold (i.e., the $\frac{B}{v_{mean}}$ ratio) within the unconstrained LBA assessments, which appeared to provide a more robust measurement of caution than the threshold parameter. The current simulation study follows the following structure: (1) a description of the simulation and fitting details that differ from the first simulation study, (2) definitions for how the models were extended to the context of two experimental conditions, (3) the results of the parameter recovery and cross-fitting assessments, and (4) a discussion of the results and what they mean for *model application* studies using these models.

Simulation and fitting methods

The simulation and fitting methods were identical to the previous simulation study with two exceptions. First, as the parameter space for the current simulation study was much larger than the previous simulation study, 8,000 parameter sets were drawn from the hypercube for each data generating model, as opposed to the 4,000 in the previous simulation study. Second, as the current simulation study uses two simulated experimental conditions, 1,000 trials were simulated per experimental condition (i.e., 2,000 total per data set). All generating procedures, inclusion criteria, fitting methods, and methods of parameter recovery and cross-fit assessment were otherwise identical to the previous

simulation study. Also, it should be noted that the current simulation study focused solely on the differences in parameters across conditions, as this is the key extension of the current simulation study from the previous simulation study. However, figures that display the recovery of the midpoint of the parameter values across conditions can be found on the OSF page for this article (<https://osf.io/hxsk8/>), along with the relevant code for each section of the article.

Models

All model definitions were near-identical to the previous simulation study, with the only differences being due to the extra simulated experimental condition. The diffusion model and unconstrained LBA remained identical, though with all parameters estimated for both experimental conditions, and the generating hypercubes were defined in terms of the midpoint of the parameter values over both conditions, and the difference in the parameter values between conditions. However, the constrained LBA was split into two different versions. Importantly, as there are now two experimental conditions, the v_{mean} parameter does not need to be fixed to a specific constant, and instead can be constrained over experimental conditions. However, there are two ways that researchers can constrain the mean drift rate. The first is to estimate a single value of the v_{mean} parameter for both conditions, and to estimate the v_{diff} parameter for each condition, directly constraining the v_{mean} parameter. I refer to this first constraint as the “mean constrained LBA”. The second is to estimate a single value for the drift rate of the error accumulator (i.e., v_2) for both conditions, and to estimate the drift rate of the correct accumulator (i.e., v_1) for each conditions, allowing the v_{mean} parameter to vary between conditions, but only as a function of the v_{diff} parameters. I refer to this second constraint as the “error constrained LBA”. Each of these models serve as a more constrained version of the LBA that is similar to the *sum-to-1* constraint, though allow the additional flexibility in different parts of the model.

Below I list the generating hypercubes and the prior distributions for the parameters of each model.

Unconstrained LBA. The range of values for each parameter within the Latin hypercube were:

Midpoint over conditions

$$v_{mean} = [2.5, 5.5]$$

$$v_{diff} = [0.05, 2]$$

$$t_0 = [0.1, 0.6]$$

$$B = [0.4, 3]$$

Difference between conditions

$$v_{mean} = [-3, 3]$$

$$v_{diff} = [-2, 2]$$

$$t_0 = [-0.2, 0.2]$$

$$B = [-0.8, 0.8]$$

with A fixed to 1 in both conditions. Note that the midpoint generating ranges are identical to the ranges used in the previous simulation study. Based on the inclusion criteria, this resulted in 7,360 data sets being included in the analysis. The priors for each parameter in model, used for the Bayesian parameter estimation on the data generated by all models, were:

$$v_{1,i} \sim TN(3, 1, 0, \infty)$$

$$v_{2,i} \sim TN(1, 1, 0, \infty)$$

$$t_{0,i} \sim TN(0.3, 0.1, 0, \infty)$$

$$B_i \sim TN(1, 0.5, 0, \infty)$$

$$A \sim TN(1, 0.5, 0, \infty)$$

where i indexes the experimental condition.

Mean constrained LBA. The range of values for each parameter within the Latin hypercube were:

Midpoint over conditions

$$v_{mean} = [2.5, 5.5]$$

$$v_{diff} = [0.05, 2]$$

$$t_0 = [0.1, 0.6]$$

$$B = [0.4, 3]$$

Difference between conditions

$$v_{diff} = [-2, 2]$$

$$t_0 = [-0.2, 0.2]$$

$$B = [-0.8, 0.8]$$

with A fixed to 1 in both conditions. Note that the midpoint generating ranges are identical to the ranges used in the previous simulation study. Based on the inclusion criteria, this resulted in 7,284 data sets being included in the analysis. The priors for each parameter in model, used for the Bayesian parameter estimation on the data generated by all models, were:

$$\begin{aligned}
 v_{mean} &\sim TN(3, 1, 0, \infty) \\
 v_{diff,i} &\sim TN(1, 1, 0, \infty) \\
 t_{0,i} &\sim TN(0.3, 0.1, 0, \infty) \\
 B_i &\sim TN(1, 0.5, 0, \infty) \\
 A &\sim TN(1, 0.5, 0, \infty)
 \end{aligned}$$

where i indexes the experimental condition.

Error constrained LBA. The range of values for each parameter within the Latin hypercube were:

Midpoint over conditions

$$v_{mean} = [2.5, 5.5]$$

$$v_{diff} = [0.05, 2]$$

$$t_0 = [0.1, 0.6]$$

$$B = [0.4, 3]$$

Difference between conditions

$$v_1 = [-2, 2]$$

$$t_0 = [-0.2, 0.2]$$

$$B = [-0.8, 0.8]$$

with A fixed to 1 in both conditions. Note that the midpoint generating ranges are identical to the ranges used in the previous simulation study. Based on the inclusion criteria, this resulted in 7,301 data sets being included in the analysis. The priors for each parameter in model, used for the Bayesian parameter estimation on the data generated by all models, were:

$$\begin{aligned}
v_{1,i} &\sim TN(3, 1, 0, \infty) \\
v_2 &\sim TN(1, 1, 0, \infty) \\
t_{0,i} &\sim TN(0.3, 0.1, 0, \infty) \\
B_i &\sim TN(1, 0.5, 0, \infty) \\
A &\sim TN(1, 0.5, 0, \infty)
\end{aligned}$$

where i indexes the experimental condition.

Diffusion model. The range of values for each parameter within the Latin hypercube were:

Midpoint over conditions

$$\begin{aligned}
v &= [0.1, 4] \\
t_{er} &= [0.1, 0.6] \\
a &= [0.45, 1.75]
\end{aligned}$$

Difference between conditions

$$\begin{aligned}
v &= [-2, 2] \\
t_{er} &= [-0.2, 0.2] \\
a &= [-0.8, 0.8]
\end{aligned}$$

Note that the midpoint generating ranges are identical to the ranges used in the previous

simulation study. Based on the inclusion criteria, this resulted in 6,575 data sets being included in the analysis. The priors for each parameter in model, used for the Bayesian parameter estimation on the data generated by all models, were:

$$\begin{aligned} v_i &\sim TN(3, 1, 0, \infty) \\ t_{er,i} &\sim TN(0.3, 0.1, 0, \infty) \\ a_i &\sim TN(2, 1, 0, \infty) \end{aligned}$$

where i indexes the experimental condition.

Results and Discussion

Here, I discuss the parameter recovery results for each model separately, before discussing the cross-fitting results together for all models. Note that each figure displays the estimation of a specific model to all data sets (i.e., all generating models), with different figures for the estimation of different models. In all cases, the top row of the figure shows the parameter recovery of the model (i.e., the data sets where the estimated model is the data generating model), and the lower rows show the cross-fits. Note that the red line of perfect recovery is included in all plots, though it is primarily relevant for the parameter recovery assessment.

Unconstrained LBA recovery. The top row of Figure 4 displays the parameter recovery for the unconstrained LBA. The recovery of the differences between conditions is fairly robust for most parameters in the model. As with the estimates of v_{diff} in the single condition context, the estimates of the differences in the v_{diff} parameter fall directly along the red line with only minor variability, resulting in a near-perfect correlation. The

estimates of the differences in the v_{mean} parameter appear to mostly fall along the red line, though with some tendency towards the underestimation of the absolute magnitude of the difference between conditions. The estimates of the differences in the t_0 parameter are similar to the estimates of t_0 in the single condition context, where the estimates mostly fall along the red line, but the variability in the estimates is reasonably large. However, the recovery is extremely poor for differences in the B parameter between conditions. Specifically, the variability in the estimates is extremely high and show a tendency towards the underestimation of the absolute magnitude of the difference between conditions, resulting in an extremely low correlation (0.63). Importantly, the combination of these issues mean that even data generated with a large positive difference in B between conditions can be estimated as a large negative difference in B (and vice versa), making differences between experimental conditions in threshold practically unidentifiable. However, the recovery of the differences in $\frac{B}{v_{mean}}$ (i.e., the average time to the threshold) is excellent, with the estimates falling directly along the red line with only minor variability, resulting in a near-perfect correlation.

Figure 4. Displays the parameter recovery (top row) of the unconstrained LBA for simulation study 2, as well as the cross-fit to data generated by the error constrained LBA (second row), the mean constrained LBA (third row), and the diffusion model (bottom row). The columns display the different parameters of the model, with the x-axis displaying the generated values and the y-axis displaying the estimated values. In each plot different points reflect different data sets, and the red line reflects perfect parameter recovery (i.e., $x = y$).

Mean constrained LBA recovery. The top row of Figure 5 displays the parameter recovery for the mean constrained LBA. The recovery of the differences between conditions is fairly robust for all parameters in the model. Similar to the unconstrained LBA, the estimates of the differences in the v_{diff} parameter fall directly along the red line with only minor variability for most of the parameter space, resulting in a high correlation (0.98).

However, the estimates also show some higher variability for more extreme generating values, with a tendency towards the underestimation of the absolute magnitude of the difference. The estimates of the differences in the t_0 parameter are greatly improved from the unconstrained LBA, with the estimates mostly fall along the red line with greatly reduced variability in the estimates. Perhaps most notably, the recovery for differences in the B parameter between conditions is immensely improved from the unconstrained LBA. Specifically, the estimates mostly fall along the red line with moderate variability, resulting in a reasonably high correlation (0.96). Importantly, this drastic decrease in variability allows the identifiability of differences between experimental conditions in threshold with reasonable accuracy, meaning that constraining the v_{mean} parameter to take the same value over conditions remedies the identifiability issues of the unconstrained LBA. The recovery further improved when instead assessing the estimates of the differences in $\frac{B}{v_{mean}}$, where the variability in the estimates are further reduced.

Figure 5. Displays the parameter recovery (top row) of the mean constrained LBA for simulation study 2, as well as the cross-fit to data generated by the error constrained LBA (second row), the unconstrained LBA (third row), and the diffusion model (bottom row). The columns display the different parameters of the model, with the x-axis displaying the generated values and the y-axis displaying the estimated values. In each plot different points reflect different data sets, and the red line reflects perfect parameter recovery (i.e., $x = y$).

Error constrained LBA recovery. The top row of Figure 6 displays the parameter recovery for the mean constrained LBA. The recovery of the differences between conditions

is fairly robust for all parameters in the model. The recovery of the differences in the v_{diff} , t_0 , B , and $\frac{B}{v_{mean}}$ parameters are near identical to the mean constrained LBA, meaning that constraining the v_2 parameter (i.e., the drift rate of the error accumulator) to take the same value over conditions remedies the identifiability issues of the unconstrained LBA. However, unlike the mean constrained LBA, the error constrained LBA also allows for the estimation of the differences in the v_{mean} parameter over conditions. These estimates fall directly along the red line with only minor variability for most of the parameter space, resulting in a high correlation (0.98). The recovery of the differences in the v_{mean} parameter over conditions looks near-identical to the recovery of the differences in the v_{diff} parameter over conditions, which is perhaps not surprising given that they are tightly connected within the error constrained LBA.

Figure 6. Displays the parameter recovery (top row) of the error constrained LBA for simulation study 2, as well as the cross-fit to data generated by the unconstrained LBA (second row), the mean constrained LBA (third row), and the diffusion model (bottom row). The columns display the different parameters of the model, with the x-axis displaying the generated values and the y-axis displaying the estimated values. In each plot different points reflect different data sets, and the red line reflects perfect parameter recovery (i.e., $x = y$).

Diffusion model recovery. The top row of Figure 7 displays the parameter recovery for the diffusion model. Note that the column titles use the LBA notation from the parameters, meaning that v is v_{diff} , a is B , and t_{er} is t_0 . As with the recovery in the single condition context, the diffusion model provides more precise estimates than all versions of the LBA, with the estimates of the differences in the v_{diff} , t_0 , and B parameters all falling directly

along the red line with only minor variability, resulting in a near-perfect correlations.

Figure 7. Displays the parameter recovery (top row) of the diffusion for simulation study 2, as well as the cross-fit to data generated by the mean constrained LBA (second row), the error constrained LBA (third row), and the unconstrained LBA (bottom row). The columns display the different parameters of the model (note that these use the LBA notation), with the x-axis displaying the generated values and the y-axis displaying the estimated values. In each plot different points reflect different data sets, and the red line reflects perfect parameter recovery (i.e., $x = y$).

Cross-fit. The second, third, and bottom rows of Figure 4 display the estimates of the unconstrained LBA from data generated by the error constrained LBA, the mean constrained LBA, and the diffusion model, respectively. The patterns of estimates for the data generated by the error constrained LBA (second row) are near identical to those produced for the data generated by the mean constrained LBA (third row), and hence are discussed together. The estimates for the differences in the v_{diff} , t_0 , and B parameters are similar to those for the parameter recovery of the unconstrained LBA, showing a close mapping for v_{diff} , a good (though somewhat variable) mapping for t_0 , and no clear mapping for B due to the extremely high variability. However, unlike the parameter recovery of the unconstrained LBA, the mapping between the differences in the $\frac{B}{v_{mean}}$ ratio is much poorer, showing higher variability, and resulting in a lower correlations (0.91 and 0.9). The reason for the poorer mapping to the differences in the $\frac{B}{v_{mean}}$ ratio seems to be reflected in the mapping between the differences in the v_{mean} parameter for the data generated by the error constrained LBA, which shows no clear mapping and a very poor correlation (0.6) due to the extremely high variability. Therefore, it appears that the differences between conditions in the v_{mean} parameter generated by the constrained LBA versions do not map onto the estimates of the unconstrained LBA, which is perhaps not surprising given that the primary difference between the models is their constraint on the differences between conditions in v_{mean} . The patterns of estimates for the data generated by the diffusion model (bottom row) show a much poorer mapping than to the data generated by the other LBA versions. The estimates for the differences in the t_0 parameter show an extremely poor mapping, with the variability in the estimates being sufficiently high to prevent any clear mapping between the data generating and estimated values. The estimates for the differences in the v_{diff} , B , and $\frac{B}{v_{mean}}$ show some improvement, though are still far too variable to provide a clear one-to-one mapping across the entire parameter space. However, the reasonable

correlations and patterns of estimates suggest that there is some basic mapping, where generated negative differences are mostly estimated as negative differences and generated positive differences are mostly estimated as positive differences, though the magnitude of the generating and estimated differences do not seem to map onto one another.

The second, third, and bottom rows of Figure 5 display the estimates of the mean constrained LBA from data generated by the error constrained LBA, the unconstrained LBA, and the diffusion model, respectively. The patterns of estimates for the data generated by the error constrained LBA (second row) are similar to the patterns of estimates of the unconstrained LBA for these same data (Figure 4, second row), showing a close mapping for v_{diff} , good (though somewhat variable) mappings for t_0 and the $\frac{B}{v_{mean}}$ ratio, and no clear mapping for B . The patterns of estimates for the data generated by the unconstrained LBA (third row) are poorer overall. While the mapping remains close for v_{diff} , and becomes close for the $\frac{B}{v_{mean}}$ ratio (though with a slight tendency for overestimating the magnitude of the difference), there is no clear mapping for the t_0 and B parameters due to the extremely high variability in the estimates. The patterns of estimates for the data generated by the diffusion model (bottom row) are near-identical to the patterns of estimates of the unconstrained LBA for these same data (Figure 4, bottom row), showing no clear mapping for differences in t_0 , and only the basic mapping of the direction of the differences for v_{diff} , B , and $\frac{B}{v_{mean}}$.

The second, third, and bottom rows of Figure 6 display the estimates of the error constrained LBA from data generated by the unconstrained LBA, the mean constrained LBA, and the diffusion model, respectively. The patterns of estimates for the data generated by the unconstrained LBA (second row) are near-identical to the patterns of estimates of the mean constrained LBA for these same data (Figure 5, third row), showing a clear mapping between the differences in v_{diff} and $\frac{B}{v_{mean}}$, and no clear mapping between the differences

in t_0 and B . The patterns of estimates for the data generated by the mean constrained LBA (third row) are poorer overall. While the mapping remains close for differences in the v_{diff} , the variability remains reasonably high for differences in the t_0 parameter, becomes high for differences in the $\frac{B}{v_{mean}}$ ratio, and remains very high for differences in the B parameter. Interestingly, this means that the estimates of the mean constrained LBA to the data generated by the error constrained LBA (Figure 5, second row) seem to more closely map to one another than the estimates of the error constrained LBA to the data generated by mean constrained LBA (Figure 6, third row). The patterns of estimates for the data generated by the diffusion model (bottom row) are near-identical to the patterns of estimates of the unconstrained LBA (Figure 4, bottom row) and mean constrained LBA (Figure 5, bottom) for these same data, showing no clear mapping for differences in t_0 , and only the basic mapping of the direction of the differences for the differences in v_{diff} , B , and $\frac{B}{v_{mean}}$. Importantly, the similar, poor mapping between data generated by the diffusion model and each of the LBA versions suggests that (1) the LBA and diffusion model do not closely map onto one another, regardless of the specific constraints applied to the LBA, and (2) that the constrained LBA does not provide a closer mapping than the unconstrained LBA to the diffusion model.

The second, third, and bottom rows of Figure 7 display the estimates of the diffusion model from data generated by the mean constrained LBA, the error constrained LBA, and the unconstrained LBA, respectively. In all cases, the mapping between the generating difference in v_{diff} and the diffusion model estimates are fairly close, particularly when the data were generated by a constrained LBA. This is in contrast to the LBA estimates of drift rate for data generated by the diffusion model, which at best showed a basic mapping between positive and negative differences (bottom rows of Figures 4, 5, and 6). There is also some mapping between the generating difference in t_0 for each of the

constrained LBA versions and the diffusion model estimates, though the variability is somewhat high, preventing any clear one-to-one mapping. This variability increases when generating with the unconstrained LBA, making the mapping even less clear. For the estimates of the differences in B , there is some mapping when the data are generated from the error constrained LBA (correlation of 0.85; similar to the t_0 mapping) though the variability in the estimates is substantial when the data are generated by the mean constrained LBA or the unconstrained LBA, making any mapping impossible. Finally, the generating difference in $\frac{B}{v_{mean}}$ ratio for the unconstrained LBA provides a much better mapping than the differences in B to the diffusion model, showing a clear mapping between the direction of the generating differences and the estimated differences, though less of a mapping in the magnitude of the differences.

Discussion. The current simulation study assessed the measurement properties of the unconstrained LBA, the mean constrained LBA, the error constrained LBA, and the simple diffusion model in the context of the differences in parameter values across two experimental conditions. Importantly, the current simulation study both reinforced several findings of the previous simulation study, and provided some interesting new results. First, the recovery and cross-fit assessments again both showcased the robust measurement properties of the simple diffusion model. The diffusion model provided a precise, accurate recovery for the differences between conditions in every parameter, showing that it can robustly estimate changes in the underlying psychological process caused by experimental manipulations. Furthermore, the diffusion model estimates provided a one-to-one mapping with the generating differences in drift rate in all LBA versions, suggesting that the diffusion model can accurately estimate changes in drift rate between experimental conditions regardless of the generating model. The diffusion model estimates also provided a reasonable mapping to the generating differences in non-decision time in the constrained versions

of the LBA, to the generating differences in threshold in the error constrained LBA, and to the generating differences in the average time to threshold in the unconstrained LBA. Interestingly, the diffusion model estimates did not provide any clear mapping to the generating differences in threshold in the mean constrained LBA, again suggesting that constraining the sum of the drift rates in the LBA does not necessarily make the LBA more similar to the diffusion model. Importantly, these findings also suggest that there are minimal problematic degeneracies in the simple diffusion model, meaning that it may be the best EAM for use in *model application* contexts.

Second, and perhaps most importantly, the measurement properties of the unconstrained LBA were substantially poorer than both of the constrained LBA versions. While all versions showed a near-perfect recovery of the differences in drift rate, the unconstrained LBA provided a poorer recovery of differences in non-decision time than both of the constrained LBAs. Most notably, generating differences in threshold between conditions were unidentifiable in the unconstrained LBA, bringing into question previous studies that have used the unconstrained LBA to infer changes in the underlying psychological process caused by experimental manipulations. However, differences in the average time to threshold showed excellent recovery, remedying this issue. When combined with the fact that the threshold estimates of the diffusion model provided a much closer mapping with the generating average time to threshold in the unconstrained LBA than the threshold parameter in any version of the LBA, these findings suggest that researchers should consider using the average time to threshold as the measurement of caution in the unconstrained LBA, rather than the threshold itself. Furthermore, the robust measurement properties of the mean constrained LBA and the error constrained LBA suggest that researchers should consider using these constrained versions of the LBA in place of the unconstrained LBA.

Finally, none of the LBA versions appear to show a close mapping with the diffusion

model. While the diffusion model estimates provide a good or reasonable mapping with some of the generating differences in each of the LBA versions – the best overall being with the differences generated by the error constrained LBA – the mapping is far from perfect. This mapping was particularly poor for the mean constrained LBA – the version of the LBA implemented that most closely resembles the *sum-to-1* constraint – again suggesting that directly constraining the sum of the drift rates does not necessarily make the LBA more “diffusion-like”. Furthermore, the estimates of each of the LBA variants generally provided a poor mapping with the differences generated by the diffusion model, showing either a basic mapping – where estimated positive/negative differences correctly mapped to the generated positive/negative differences, but the magnitude of the difference was difficult to identify – or no mapping, where the variability was too high to show any coherent mapping between the estimates and the generating values. Importantly, this result – when combined with the results of the previous simulation study – suggest that the inferences of the LBA and the diffusion model do not closely map to one another in realistic application contexts, meaning that these models are not interchangeable for use in *model application*.

Reanalysis of Evans (2020a)

As discussed in the introduction, one notable example of the diffusion model and unconstrained LBA coming to qualitatively different conclusions was the recent large-scale selective influence reanalysis of Evans (2020a), who provided a comprehensive reassessment of which parameters are influenced by experimental manipulations of urgency emphasis. Manipulations of urgency emphasis involve multiple experimental conditions – usually two – where participants are instructed to perform the task as quickly as possible (without resorting to guessing) in one condition, and as accurately as possible (without spending long periods of time on each decision) in the other condition. Specifically, Evans (2020a)

applied both the diffusion model and the LBA to 6 existing data sets with identical fitting and inference methods, finding that the diffusion model analysis showed evidence in favour of a decrease in threshold and non-decision time under urgency emphasis, whereas the LBA analysis showed evidence in favour of a decrease in threshold and drift rate under urgency emphasis. Furthermore, while the diffusion model analysis consistently found a decrease in threshold under urgency emphasis across studies, showing a general trend towards the effect in all studies, the findings of the LBA analysis were more mixed, only showing a general trend towards the effect in half of the studies.

Importantly, the results of the second simulation study show that unconstrained LBA cannot identify differences in threshold over conditions, potentially explaining part of the discrepancy between the diffusion model and LBA, as the mixed evidence for decreases in threshold in the LBA may have been due to the problematic measurement properties of the unconstrained model. However, the inability of the unconstrained LBA to identify differences in threshold cannot explain the differences between the models in their conclusions about drift rate and non-decision, which based on the cross-fit assessments in the current study, may be a result of the poor mapping between the diffusion model and the LBA, regardless of the constraints applied to the LBA. In order to determine whether constraining the LBA would (1) lead to clearer inferences about the decrease in threshold under urgency emphasis, and (2) still result in qualitatively different conclusions to the diffusion model regarding whether or not drift rate and non-decision time decrease under urgency emphasis, I performed a reanalysis of Evans (2020a) using the three more constrained versions of the LBA proposed within the second simulation study: (1) the mean constrained LBA, (2) the error constrained LBA, and (3) the $\frac{\text{threshold}}{\text{drift rate midpoint}}$ constrained LBA. For brevity, I only provide brief details about the data sets, models, and fitting methods, and interested readers are referred to Evans (2020a) for specifics.

Method

As with the original study, the reanalysis uses a Bayesian model selection approach (Jeffreys, 1961; Kass & Raftery, 1995; Evans & Brown, 2018; Annis, Evans, Miller, & Palmeri, 2019) – performed separately for each participant to allow for individual differences in effects – to assess whether emphasizing urgency influences drift rate, decision threshold, and/or non-decision time. Specifically, model selection was performed using the marginal likelihood probability weight assigned to each model, with the marginal likelihoods calculated via bridge sampling (Gronau et al., 2017). DE-MCMC was again used to sample from the posterior distributions, with $3k$ parallel chains that were each run for 11,000 iterations, with the first 1,000 iterations discarded as burn-in. A different model was defined for each potential combinations of decreases, increases, or no changes in drift rate, threshold, and non-decision time, resulting in 27 different potential models. All three versions of the LBA were identical to the unconstrained LBA in Evans (2020a) – including the prior distributions placed on the parameters – apart from their specific constraint on the overall drift rate. The constraints for the mean constrained LBA and the error constrained LBA were defined in an identical manner as the second simulation study, and the $\frac{\text{threshold}}{\text{drift rate midpoint}}$ constrained LBA assessed whether the ratio between threshold and mean drift rate differed between conditions, rather than threshold itself. It should also be noted that all models used a contamination process (Ratcliff & Tuerlinckx, 2002), where the implemented probability density function for each model was made up of a mixture of two distributions: the actual probability density function of the model ($1 - p_c$), and a uniform contaminant distribution that spanned the range of the included data (p_c).

The reanalysis used all 6 speed-accuracy tradeoff data sets from Evans (2020a): the 32 participant random dot motion task of Dutilh, Forstmann, Vandekerckhove, and Wagenmakers (2013) (Experiment 1), the 20 participant random dot motion task of Dutilh

et al. (2019), the 49 participant (with 2 excluded based on performance) brightness discrimination task of Evans et al. (2017) (Experiment 1), the 38 participant figural task of Lerche and Voss (2017) (Experiment 1), the 49 participant (with 14 excluded based on performance) brightness discrimination task Rae et al. (2014) (Experiment 1), and the 17 participant lexical decision-making task of Wagenmakers, Ratcliff, Gomez, and McKoon (2008) (Experiment 1). In order to compress the information provided by the comparison of 27 different models for each participant across 6 studies, the probability weights are displayed as cumulative bar plots, with different colours and different plots within a row referring to different models. Each row of plots in the figures (i.e., Figure 8, 9, and 10) present a different study, whereas different plots within each row show the probability weights for models where threshold decreases (left), is unchanged (middle), or increases (right) when urgency is emphasized. Within each plot, the 9 combinations of models where drift rate and non-decision time decrease, are unchanged, or increase are represented by 9 different colours, which can be seen in Figure 11. Each bar within a plot represents a different participant within the study, and the corresponding bar for each of the different plots within each row represent the same participant, meaning that all corresponding bars within a row of plots sum to a total probability of 1. A more detailed explanation of the analysis method can be found in Evans (2020a).

Results and Discussion

In Evans (2020a), the diffusion model analysis suggested a decrease in threshold under urgency emphasis in all 6 data sets, a decrease in drift rate under urgency emphasis in 2 data sets (Dutilh et al., 2013; Rae et al., 2014), and a decrease in non-decision time under urgency emphasis in 5 data sets (Dutilh et al., 2019; Evans et al., 2017; Lerche & Voss, 2017; Rae et al., 2014; Wagenmakers et al., 2008). In contrast, the LBA analysis suggested a decrease in threshold under urgency emphasis in 3 data sets (Dutilh et al.,

2013; Lerche & Voss, 2017; Wagenmakers et al., 2008), a decrease in drift rate under urgency emphasis in 3 data sets (Dutilh et al., 2013, 2019; Wagenmakers et al., 2008), and a decrease in non-decision time under urgency emphasis in 0 data sets. For brevity, I only provide a summary of the results for each of the three different LBA version with a focus on whether the general patterns of results agree with those in Evans (2020a), rather than detailed discussions of the results of each model in each study as in Evans (2020a).

Figure 8 shows the results for the mean constrained LBA. Across all data sets, the trends in drift rate are very similar to those in Evans (2020a): the same 3 data sets (Dutilh et al., 2013, 2019; Wagenmakers et al., 2008) appear to show evidence for a decrease in drift rate under urgency emphasis (i.e., the purple, red, and cream segments of the bars), and the other data sets again do not. However, unlike Evans (2020a), almost every data sets (Dutilh et al., 2013 is somewhat mixed) shows clear evidence for a decrease in threshold under urgency emphasis (i.e., almost all segments of the cumulative bars in the leftmost panel). Furthermore, while the LBA analysis of Evans (2020a) showed no change in non-decision time, the mean constrained LBA suggests increases in non-decision time under urgency emphasis in the Evans et al. (2017) and Wagenmakers et al. (2008) data sets (i.e., the cream, green, and yellow segments of the bars), with some participants in the Dutilh et al. (2019) and Rae et al. (2014) data sets also showing this increase. Therefore, although the mean constrained LBA rectifies the under-detection of threshold effects seen in the unconstrained LBA analysis of Evans (2020a), making the results somewhat more similar to the diffusion model, the mean constrained LBA still finds a decrease in drift rate in the same data sets as the unconstrained LBA, and an increase in non-decision time in several cases that is directly in contrast to the decrease in non-decision time found by the diffusion model.

Figure 9 shows the results for the error constrained LBA. Again, the trends in drift

rate are very similar to those in Evans (2020a), though the overall trend is weaker in those 3 data sets (Dutilh et al., 2013, 2019; Wagenmakers et al., 2008), with less participants showing strong evidence for a decrease in drift rate under urgency emphasis. As with the mean constrained LBA, almost every data set (Dutilh et al., 2013 is again somewhat mixed, though less so than with the mean constrained LBA) shows clear evidence for a decrease in threshold under urgency emphasis. Furthermore, there again appears to be increases in non-decision time under urgency emphasis in the Evans et al. (2017) and Wagenmakers et al. (2008) data sets, with some participants in the Dutilh et al. (2019) and Rae et al. (2014) data sets also showing this increase. Overall, the inferences of the error constrained LBA are extremely similar to the mean constrained LBA, though the evidence for a decrease in drift rate under urgency emphasis is somewhat weaker.

Figure 10 shows the results for the $\frac{\text{threshold}}{\text{drift rate midpoint}}$ constrained LBA. As with the mean constrained and error constrained LBA versions, the trends in drift rate are very similar to those in Evans (2020a), with the same 3 data sets (Dutilh et al., 2013, 2019; Wagenmakers et al., 2008) appearing to show evidence for a decrease in drift rate under urgency emphasis. Furthermore, almost every data set (Dutilh et al., 2013 is again somewhat mixed) shows clear evidence for a decrease in threshold under urgency emphasis. However, like the unconstrained LBA analysis of Evans (2020a), all data sets show evidence for no change in non-decision time, as opposed to the theoretically strange increases in non-decision time found by the mean constrained LBA and error constrained LBA. Therefore, the $\frac{\text{threshold}}{\text{drift rate midpoint}}$ constrained LBA still comes to qualitatively different conclusions than the diffusion model in some respects, though the overall conclusions are more similar than any other LBA variant to the diffusion model.

Figure 8. Displays the probability weights assigned to each of the 27 different model for the mean constrained LBA analysis. More details can be found in the *Method* subsection of the *Reanalysis of Evans (2020a)* section of the main text, and a legend for the colours can be seen in Figure 11.

Figure 9. Displays the probability weights assigned to each of the 27 different model for the error constrained LBA analysis. More details can be found in the *Method* subsection of the *Reanalysis of Evans (2020a)* section of the main text, and a legend for the colours can be seen in Figure 11.

Figure 10. Displays the probability weights assigned to each of the 27 different model for the $\frac{\text{threshold}}{\text{drift rate midpoint}}$ constrained LBA analysis. More details can be found in the *Method* subsection of the *Reanalysis of Evans (2020a)* section of the main text, and a legend for the colours can be seen in Figure 11.

Figure 11. Displays the colours used for the model probability weights in Figures 8, 9, and 10, and the drift rate and non-decision time effects that they correspond to. Note that threshold effects are reflected by the position of the bar in the rows of plots, rather than a colour, for ease of visualisation.

General Discussion

The current study aimed to provide a comprehensive investigation into why the diffusion model and the LBA came to different conclusions in Evans (2020a) and Dutilh et al. (2019), yet came to similar conclusions in Donkin et al. (2011). Importantly, Donkin et al. (2011) implemented a more constrained version of the LBA, where the drift rates of

both accumulators were constrained to sum to a fixed value (i.e., 1), which differs from the unconstrained version of the model that most researchers apply to empirical data in most situations (e.g., Brown & Heathcote, 2008; Rae et al., 2014; Weigard et al., 2018; Dutilh et al., 2019; Evans, 2019a; Castro et al., 2019; Palada et al., 2019; Evans, 2020a; Howard et al., 2020). Specifically, the current study aimed to determine whether constraining the drift rates of the LBA leads to inferences that more closely match those of the diffusion model, and whether the unconstrained LBA provides robust measurement properties. Two large-scale simulation studies were used to determine the measurement properties of the unconstrained LBA, more constrained versions of the LBA, and the simple diffusion model, as well as how closely the same constructs in the different models mapped onto one another. The current study also provided a reanalysis of Evans (2020a) with three more constrained versions of the LBA – the mean constrained LBA, the error constrained LBA, and the average time to threshold LBA (i.e., the ratio of threshold and mean drift rate) – in order to assess whether these more constrained versions of the LBA came to qualitatively similar conclusions as the diffusion model, or provided similar conclusions to the unconstrained LBA.

First and foremost, the current study revealed the poor measurement properties of the standard, unconstrained LBA. While the model showed several problematic tendencies, the most notable one was the inability to identify changes in threshold between experimental conditions, which is a common context of interest for *model application* with EAMs. The inability to identify changes in threshold appears to be caused by a strong trade off between threshold and the overall drift rate of both accumulators, where either decreasing threshold or increasing the drift rate of both accumulators result in both a decrease in accuracy and faster response times, which appears to be a major degeneracy in the model (i.e., the “sloppy models” problem; Holmes, 2015; Gutenkunst et al., 2007). However, the

inability to identify changes in threshold can be remedied by implementing one of many possible constraints that prevent the mean drift rate and threshold parameters from trading off with one another. The current study assessed three of these potential constraints, which each made changes in threshold between experimental conditions identifiable: (1) constraining the midpoint of the mean drift rates of the two accumulators to have the same value in both conditions, (2) constraining the mean drift rate of the error accumulator to have the same value in both conditions, or (3) using the ratio between threshold and the midpoint of the mean drift rates (i.e., $\frac{\text{threshold}}{\text{drift rate midpoint}}$) of the two accumulators as a proxy to “response caution” instead of threshold itself. Therefore, the findings of the current study suggest that researchers should not use the unconstrained LBA as a measurement tool, and should instead use either a more constrained version of the LBA or the diffusion model.

The current study also further showcased the qualitative differences between the LBA and the diffusion model in their inferences about the underlying decision-making process. The cross fitting assessments showed that none of the LBA variants – regardless of the constraint applied to the LBA – provided a clear one-for-one mapping between their estimated parameters and those used to generate the data from the diffusion model, suggesting fundamental differences between the models resulting from their different underlying architectures. Furthermore, the reanalysis of Evans (2020a) – using the three more constrained variants of the LBA assessed within the current study – demonstrated that even with further constraints the LBA does not come to the same qualitative conclusions as the diffusion model. Specifically, all LBA variants found a general trend for a decrease in drift rate under urgency emphasis, and that non-decision time was either unchanged or increasing under urgency emphasis, which is in contrast to the findings of the diffusion model. Importantly, these findings mean that the choice of EAM can have practical impli-

cations on the conclusions drawn from the data, and that researchers should (1) carefully justify their choice of EAM for *model application* studies, and/or (2) apply multiple EAMs to assess the consistency of their conclusions under the assumptions of different models (Heathcote & Hayes, 2012; Osth et al., 2017; Evans, 2020a).

Although the current study displayed the importance of adequately constraining the LBA, it does not provide any clear indication of which LBA constraint is the best one to implement. In general, the mean and error constrained LBA variants came to quite similar conclusions – particularly, in terms of (1) parameter recovery, (2) cross-fits to data generated by the unconstrained LBA, and (3) their conclusions in the reanalysis of Evans (2020a) – suggesting that there is little practical difference between these variants, and that researchers should be able to use these two variants somewhat interchangeably. In contrast, assessing the ratio between threshold and the midpoint of the mean drift rates (i.e., $\frac{\text{threshold}}{\text{drift rate midpoint}}$) within the unconstrained LBA provided slightly poorer measurement properties, due to the higher variability in the recovery of non-decision time. However, the $\frac{\text{threshold}}{\text{drift rate midpoint}}$ constraint also came to more sensible conclusions in the reanalysis of Evans (2020a), finding no change in non-decision time under urgency emphasis, rather than the theoretically strange increase in non-decision time under urgency emphasis in some data sets found by the mean and error constrained LBA variants. Furthermore, while using the average time to threshold as a proxy to “response caution” may also seem theoretically strange, there is a precedence for similar constraints in models of response time where threshold is mathematically unidentifiable (Heathcote & Love, 2012; Rouder, Province, Morey, Gomez, & Heathcote, 2015). There are unique advantages to using either the mean/error constrained LBA or the $\frac{\text{threshold}}{\text{drift rate midpoint}}$ constrained LBA for *model application* studies, and future research should provide a more in depth investigation into which constraint is most suitable.

While the current study has clear implications for parameter estimation approaches to *model application*, the implications for model selection approaches is less clear. Importantly, the fact that changes in threshold over conditions cannot be identified in the unconstrained LBA does not mean that a model with no change in the threshold parameter between conditions cannot be distinguished from a model that allows a change in the threshold parameter between conditions. Specifically, this means that while the current study has shown that parameter identifiability does not hold in this context, the less strict criterion of *model identifiability* may still hold. However, the reanalysis of Evans (2020a) suggested that the parameter identifiability problems in the unconstrained LBA may also cause a problem for model selection, as the unconstrained LBA was the only model that failed to show a strong general trend for a decrease in threshold under urgency emphasis. Future research should explore this issue in more depth, and determine whether the unconstrained LBA is still suitable for use in model selection approaches.

One final point of consideration is the distinction between *practical identifiability* and *mathematical identifiability*. The current study provided a simulation based assessment of the practical identifiability (i.e., parameter recovery) of the unconstrained LBA with data that reflected a large, but realistic, sample (i.e., 1,000 trials per condition). However, the inability to practically identify changes in threshold between conditions in the unconstrained LBA does not necessarily mean that these changes are mathematically unidentifiable. Provided that the threshold and the midpoint of the mean drift rates of the two accumulators do not perfectly scale with one another (i.e., the scaling problem; see Donkin, Brown, & Heathcote, 2009 for an explanation), changes in threshold between conditions should be identifiable with infinitely large samples of data, as the subtle differences between the parameters should become more apparent as the amount of data increases. The mathematical identifiability of the unconstrained LBA is supported by the observed

pattern of parameter recovery (i.e., the top row of Figure 4) for the threshold parameter, where the generating and estimated values clearly have a linear relationship with one another (correlation of 0.63), though this relationship is sufficiently noisy that negative and positive generating values cannot be separated from one another in estimation. Therefore, the unconstrained LBA is likely identifiable in extremely large sample sizes, though in the practical sample sizes that researchers commonly collect, changes in the threshold parameter between conditions are unidentifiable.

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