Induction as a heuristic method based on structures of regularities

Author: Kaoru Takamatsu

Affiliation: Unaffiliated

E-mail: ckeyv910@sutv.zaq.ne.jp

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Abstract

Induction is a heuristic method to guess or to discover regularities (constant patterns that things follow such as a large variety of categories or natural laws). However, the main stream of the study of induction focuses not on the heuristic method but on strength evaluation of given inductive arguments. This article describes an attempt to construct a framework of this heuristic method on the basis of the idea that human beings have acquired through evolution a perceptual-cognitive system with the functions: perception of things and certain primitive regularities in the perceived things, extraction of general structures of the perceived regularities, and application of the acquired knowledge of the structures to guess prospective regularities. A way of this application would be the heuristic method. The constructed framework creates inference that satisfies main properties of induction by regarding strength of inductive arguments as rationality of the inference in the framework. Moreover, the framework includes simple cases of analogy, abduction, and induction about causal relationships between events. Some testable predictions to examine this framework are raised, and the results of this framework is compared with those of existing models of induction.

1. Introduction

Inductive inference is a function of the human perceptual-cognitive system (the PCS) to guess certain unknowns from knowns. The essence of this function is a method to discover regularities in the environment. This method would be one of the heuristic methods or strategies for problem solving adaptive to the environmental structures or conditions peculiar to the problems (Gigerenzer et. al., 2006). We can suppose that human beings (and possibly some kinds of animals) have acquired this function along the course of evolution. Then, to solve the following two problems is essential to understand induction.

1. Why have human beings acquired this heuristic method? In other words, what role does it play in human beings' life? This is the problem of the reason for existence of induction.

2. How does the PCS perform induction? In other words, on what basis is the role feasible? In addition, what mechanism enables the PCS to fill the role? This is the problem of the algorithm for induction.

The solution of Problem 1 provides the computational theory (Marr 1982, Section 1.2) of induction, the theory about the purpose or role of the heuristic method. This theory establishes the basis to work on Problem 2. Naturally, this role should be a part of the PCS's role: Acquisition of useful information about the world from environmental inputs. Typical items of useful information are those about regularities in the world. That is, there exist certain regularities in the world, namely constant patterns that things (objects, properties, or events) follow, such as the existence of various categories of substances, species of living things, their properties, events, and natural laws governing these things. These regularities will be 'uniformities of nature' raised by Mill (Mill 1874, Book III, Chapter 3) as the basis of induction. In this paper, the constant patterns are called *regular patterns*. When the PCS has the information on regular patterns, by applying the information to given situations, the agent with the PCS can guess unknown facts concerning the situations. This could help the agent behave appropriately or advantageously for survival. This advantage can be a cause of the evolution for the PCS to acquire the faculty of guessing regular patterns from their fragmentary data.

Since generalization of data in induction is to guess regular patterns to which the data might conform, induction plays this part of the PCS's role. Thus, this role would be the reason for existence of induction. I adopt this viewpoint in this paper.

Then, Problem 2 is 'On what basis can the PCS acquire the information about regular patterns from input data?' What relates the regular patterns and their instances will be the general structures of the regular patterns that their instances follow. If the PCS has the information about the structures, the PCS could effectively guess certain regular patterns from fragmentary data of the regular patterns following this structure information.

Human beings seem to acquire information about some of these structures without intention. It would be probable that the PCS has a hardware acquired in the course of evolution that extracts certain

regularities in input data. In fact, the eyesight, which perceives input light and extract some regularity in it, is a product of evolution. This hardware would function to analyze inputs according to some basic structures of regular patterns and to find or guess certain elemental regular patterns (e.g. statistical regular patterns, spatio-temporal regular patterns) in inputs. Then, the PCS could extract the structure information that accumulation of regular pattern information reveals. I postulate this faculty to be inborn induction installed in the PCS.

The PCS should also have the faculty to apply acquired information or knowledge to analyze new input. Then, it is plausible that the PCS would create a heuristic method based on the structure information stated above to guess regular patterns efficiently. This way to acquire information should be intentional and inferential part of induction, so to speak, induction acquired based on accumulation of knowledge. According to this conception, I try to construct this heuristic method, presuming the structure information as the background knowledge of induction. The structures of regular patterns are described in Section 2. The conditions necessary to acquire the structure information are described in Section 3. 1.

For the heuristic method to be induction, the method must create inference that shows the well-known properties of induction, and should be natural as human inductive inference. Furthermore, it is desirable for the heuristic method to be relevant to analogy and abduction in principle, because they seem to share a mechanism with induction.

This strategy to investigate induction as a heuristic method is quite different from the traditional one. The traditional strategy mainly focuses on evaluation of strength or confirmation degree of given inductive arguments, i.e. inductive arguments with given premise categories and given conclusion categories (For instance, Hadjichristidis, Sloman, Stevenson, Over 2004, Hayes, Heit, Swendsen 2010, Heit 2000, Kemp, Tenenbaum 2009, Medin, Coley, Storms, Heyes 2003, Osherson, Smith, Wilkie, Lopez, Shafir 1990, Rehder 2009, Sloman 1993). Only Holland et.al. (1986, pp.232) discusses discovery of regularities and proposes an induction scheme: instance based generalization. However, no theoretical basis to support the generalization is offered. The former strategy raises general structures of regular patterns as the basis to

construct a heuristic method. To the best of my knowledge, no literature based on the former strategy has been found. However, the heuristic method for guessing regular patterns should be the essence of induction, hence the principal issue of studying induction.

2. Regular patterns

In general, a regular pattern is a repeat of a pattern within certain things. Then, a regular pattern has degree of regularity based on three attributes: precision of the common pattern, strictness of the pattern repetition, and the range of the pattern repetition. These attributes arrange regular patterns in order. The more precise the pattern is or the stricter the repetition is or wider the range of the repetition is, the stronger its regularity is. Typical regular patterns in the environment are those of categories and natural laws. These regular patterns compose networks, namely, classifications of categories and systems of laws. These regular patterns and their networks have the following structures.

2. 1 Categories

There exists variety kinds of objects in the world. The objects of the same kind cause physical or chemical effects specific to them, which approximate or are similar to each other. The effects (e.g. light) have some features (e.g. wavelength, intensity) that fix the effects. Then, the feature distribution connected by a similarity relationship forms a cluster relatively apart from the features of other effects. The chain of the similarity that connects the features of the effects in the cluster is a repetition of the similarity. Thus, this similarity is a regular pattern, though there can be clear or vague parts of the regular pattern in a cluster. The cluster of the features of the effects, which follow the regular pattern, is a property of the objects that cause the effects. There can be a property for each kind of features of the effects. The set of the objects with an effect in a property (properties) is called *an object category* determined by the property (properties). In general, the property of an object category of a natural kind includes plural component properties (For example, the property of a species of living things is a synthesis of properties of shapes, physiology, behaviors and

so on.). The component properties vary in degree of importance for the property, namely essential or subordinate component of the property.

Events have structures composed of objects and properties. Then, according to object categories and their properties, events make clusters such as oxidation of metals, growth of plants, or sports games. The basic structure of events would be ‘An object x in an object category A has an feature y in a property B ’. This is a regular pattern of events. Such a cluster of events is called *an event category*.

There exist only specific kinds of objects, which have only specific properties. These properties are classified according to equivalence-difference relationships between their component properties and inclusion relationships (general-special relationships) based on the equivalence-difference relationships. These relationships make classification of categories, such as the taxonomy of living things, which have levels of generality.

2. 2 Laws

There are natural laws of the type ‘Every object x in an object category A has a property B (a feature y in B)’. These laws are regular patterns of facts about object categories. The relationship between A (or the property P^A to determine A) and B is implication: If an object has the property P^A , then, the object has the property B . When A and B are fixed, this implication constitutes a network between elements of P^A (features in P^A) and those of B . In this implication, P^A is the antecedent or the sufficient condition of B and B is the consequence or the necessary condition of P^A .

Some laws of this type are instances of a superordinate law. Let any category X in its superordinate category α imply some category Y in its superordinate category β . Then, laws with the form: X implies Y , connect α and β , and make up a network based on general-special relationships between categories in the laws. Category α in this implication is called an *A-dimension* and β is called a *C-dimension*. For example, the set of the laws about electrical conductivity of all kinds of metals forms the superordinate law, ‘Metal has high electrical conductivity’. This is a regular pattern of the set of the laws about electrical conductivity of every kind of metals. The laws in this set line up according to the

properties that determine the kinds of metals (e.g., iron-hood, copper-hood), which constitute the A-dimension of the implication. Then, the values of the electrical conductivity constitute the C-dimension. The simplest C-dimension for electrical conductivity can be made of two values: ‘high electrical conductivity’ and ‘low electrical conductivity’. The laws about all kinds of metals connect the two dimensions. Let B be the ‘electrical conductivity’ dimension. Then, Category D : concentration of free electrons is also an A-dimension of B , because Category D implies Category B . Since every kind of metals has a value in D , the laws between elements in ‘metal’ and the elements in B hold via D .

Another example of general-special relationship between laws is the network of laws made of ‘motion equation’, ‘motion equation under a gravitation field’, and ‘motion equation under the earth’s gravity’. In this case, subordinate laws are restrictions of variables in its superordinate law. Such a network of laws constitute levels of generality.

The discussion stated above applies not only to properties but also to events. When any event in an event category, $E2$, follows an event in an event category, $E1$, then, $E1$ implies $E2$, where the implication relationship between them is a regular pattern. For example, a food chain is a regular pattern between variation of the headcount of animals in a species and the headcount in certain other animal species. The event ‘Increase of the headcount of herbivorous animals’ implies the event ‘increase of the headcount of carnivorous animals’. The category of the latter events constitutes the C-dimension, and the category of the former events constitutes the A-dimension of the regular pattern of the food chain.

There are laws about basis-derivative relationships between categories. Some categories are bases of certain other categories, which are compounds or derivative of the bases (e.g. ‘the elemental particles’ - ‘atoms’ - ‘chemical compounds’ or ‘movement of molecules in liquid’ - temperature of the liquid’). This basis-derivative relationship is an implication. Let a category A be the basis of a category B , and let $d(x)$ be the derivation from x in A to y in B . Then, if x is a member of A , $d(x)$ is a member of B . These relationships are laws and make up a chain of categories. Thus, such a chain of the basis-derivative relationships

generates levels of categories. Using these relationships, laws about the derivative categories are drawn from laws about the basic categories. Then, the connection between the former laws and the latter laws is also basis-derivative relationships. For example, the laws about chemical compounds composed of atoms depend on the laws about elemental particles that compose atoms and the basis-derivative relationships between the particles and chemical compounds. Thus, the basis-derivative relationships set up levels among laws.

General-special and basis-derivative relationships between categories (or laws) constitute category networks (or law networks) composed of levels of basis on which levels of generality lies. The level of a category (or a law) is relevant to the range that the category (or the law) holds. The more basic and general the level is, the more universal the category (or the law) is. Thus, category networks and law networks constitute a system of regularities of the world as a whole

There is an inner structure of categories. Each of the members of an object category A determined by Property P^A has P^A and certain other properties. Then, there may be a property that the majority of the members have. These members are typical instances of A , e.g. birds that can fly. This typicality is an inner structure of object categories. This typicality can be derived from relevance of the sufficient condition of the property specific to the typical objects (i.e. the factor in the objects that causes the objects to have the property) and the sufficient condition of P^A in the following way.

When an A 's member has a property Q in addition to P^A , Q may be typical or atypical in A . Let S_q be the sufficient condition of Q in A and S_p be that of P^A . If S_q has no relevance to S_p , Q is independent of P^A and the typicality of Q is chance. If S_q has relevance to S_p , P^A and Q share some components S_c in their sufficient conditions, or S_q has some affinity to S_p (tendency of S_q to follow S_p without S_c). For instance, let P^A be 'bird-hood', and let Q be 'flying faculty', then the S_c includes 'having a pair of wings' and S_q is S_c plus 'strong muscle' and 'relatively lightweight', and so on, which are not in S_p . When Q is 'less than 1 Kg in weight', S_p tends to be followed by S_q but wouldn't have S_c (affinity between S_p and S_q). Therefore, that Q is a typical property in A implies existence of S_c that has strong effect on (great importance for) Q or

strong affinity between their sufficient conditions. On the other hand, that Q is an atypical property in A implies no relevance of S_p and S_q or existence of S_c with weak effect on Q . Thus, the properties of a member of A have their own typicality proportional to strength of the relevance of their S_q to S_p .

There is a case where typicality of a member of A depends on appearance degree of P^A . Appearance of P^A in members of A may vary in degree (e.g. acidity of acids). Then, members that have P^A definitely are also called typical. This typicality can be caused by degree for the members to have S_p for P^A (e.g. potential of hydrogen for acidity).

There is a phenomenon about typicality: Some properties of the typical members of a category tend to be also properties of the other members (e.g. a property ‘having a pair of wings’ of birds that can fly is a property of birds in general). This tendency can be derived from the inner structure of categories stated above in the following way. Let Q' be another property that coexists with a typical property Q in A . Let $S_{q'}$ be the sufficient condition of Q' . Then, $S_{q'}$ would be included in S_p or in S_q . Then $S_{q'}$ tends to be included in S_p or S_c , because of the typicality of Q in A . Accordingly, every member of A tends to have Q' . In the above example of Q : ‘flying faculty of birds’, properties such as ‘having a pair of wings’ can be Q' . Then, $S_{q'}$ is in S_c , and Q' coexists with P^A . The property ‘living in the sky’ can also be Q' . Then, $S_{q'}$ is in S_q except S_c and Q' coexists with Q but not with P^A . The more typical Q is in A , the larger the effect of S_c on Q becomes, and the smaller the effect of S_q except S_c on Q does. Then, the probability that Q' does not coexist with P^A becomes lower. When some affinity between S_q and S_p causes typicality of Q in A and $S_{q'}$ is in S_q , Q' will not expand to A beyond the typical part in A because of the absence of S_c .

Typicality caused by degree for the members to have S_p causes the same effect. When the typical members in this case have a common property Q' , the more dominant the typical members are, the more probable it is that the S_p includes $S_{q'}$. Then, atypical member also tends to have the property.

There is a tendency about similarity: In general, the more similar two objects (or two subordinate categories) of a category A are each other, the more likely it is that they share some other properties. This

tendency is also derived from that inner structure of categories in the following way.

When two objects in a category have properties p_1 and p_2 respectively, where p_1 and p_2 have common components in their sufficient conditions, the two properties could be equivalent to a degree, (i.e. similar) proportional to the importance of the common components for the properties. This similarity based on the importance of their common components implies that the more similar the two properties of objects in a category are to each other, the stronger the probability of the common components important for the properties is. Then, similarity of object properties affects the probability for the objects to share some properties, because the common components for the properties similar to each other may be the sufficient condition of the shared properties. It would be typical that the common component of the sufficient conditions of properties is a cause of similarity of the properties. Therefore, this cause underlies the tendency about similarity stated above.

2.3 Isomorphisms

Things (objects, properties or events) are typically composed of parts that are connected by a network of relationships to make up the things. Such networks of relationships are structures, a kind of property of the things. When a mapping from the parts of a thing to the parts of another thing is an isomorphism, (i.e. the mapping that preserves their structures), their parts are related in the same manner. Thus, this mapping is a regular pattern in the set of the ordered pairs of the related parts.

There are categories of things each of which have a structure that relates the elements in the category, such as 'natural numbers', whose members are ordered. Then, the mapping between the categories that preserves their structures is an isomorphism. For instance, both the mass and the weight of objects have an order relationship, and the relationship between the mass and the weight of objects is a direct proportion. That is, an object with x kg of mass has nearly $2x$ pounds in weight. This regular pattern preserves the order relationship of the values within 'Weight' dimension and that of the values within 'Mass' dimension

respectively. This is an isomorphism i.e. a regular pattern between the two dimensions or categories.

3. A heuristic method for guessing regularities

3. 1 Background knowledge for the heuristic method

As stated in Section 1, the PCS needs a hardware to acquire information about certain regular patterns and their structures stated in Section 2. This information would be the foundation of the heuristic method to guess regular patterns from inputs efficiently, namely, the background knowledge to support the heuristic method. Then, the study of the heuristic method must begin with looking at the background knowledge and the conditions for the PCS to acquire this background knowledge.

(1) Information about categories.

To perceive regular patterns, the PCS must have mechanisms to classify and to relate environmental inputs following their physical or chemical features. Sense organs connected with a nerve center are these mechanisms. They analyze inputs (e.g. light) into some kinds of component features (e.g. wavelength, intensity) and extract certain features in a distribution of inputs (e.g. contours or spatio-temporal variations). The items of information about each kinds of features can be related according to similarity or approximation relationships between the features. They compose a distribution corresponding to the distribution of the features. Then, each component feature of inputs that belong to a property of a category would form a cluster of items of feature information in the distribution (e.g. shapes of birds, colors of flowers). The regular pattern that determines this cluster would approximately correspond to the property. The information about this regular pattern is acquired by extracting the constant relationship between the elements of the cluster and evaluating the range of the regularity and combining the two. Then, the synthesis of the items of information about the regular patterns of the kinds of component features should be the information about the category, a conjunction of the items of information about properties of the category. Therefore, the

mechanism to acquire information about categories, inborn induction, needs to carry out the operations stated above. It should be noted that since such regular patterns are statistical tendencies, the items of information about them are probabilistic.

According to a category network, items of information about categories would be related. The PCS can acquire this information by evaluating general-special relationships between categories. The general-special relationships consists of equivalence-difference relationships between category components, and inclusion relationships between the categories based on the equivalence-difference relationships (e.g. taxonomy or the periodic table). Then, information about the network structures can be extracted from this information about the networks. This extraction is possible by eliminating the items of information about categories from those about the networks; the elimination leaves the framework of the general-special relationships. Thus, the PCS needs to carry out these operations.

(2) Information about laws.

In the same way as the information about categories, information about laws can be acquired from accumulation of items of information about facts. Typical elemental laws are implication relationships between the objects in an object category and a property. The information of such a law is acquired by evaluating the commonality of the properties that the objects in a category have. Furthermore, accumulation of items of information about laws composes networks of them following classification (general-special relationships) information about the categories in the laws. The networks are information about law networks in which the properties that determine the object categories (e.g. copper-hood, silver-hood, iron-hood etc.) are subordinates of a property that determines a superordinate object category (e.g. metal-hood). This superordinate property is called an *A-dimension*. The properties (e.g. values of electrical conductivity) are also those of a superordinate property category (e.g. electrical conductivity). This superordinate category is called a *C-dimension*. The operations necessary to acquire this network information are evaluation of the commonality (equivalence-difference) and inclusion relationships between items of

acquired information about laws, and elimination of variable components from the information, the same operations as those necessary to acquire category information.

A basis-derivative relationship between categories or laws is a production or constitution relationship between the categories or laws, a kind of regular patterns. Thus, the PCS should have the capacity to evaluate these relationships. The relationships between laws that make up a law network (i.e. relationships between laws existing in the world) correspond to truth relationships between items of information about the laws. Thus, these items of information are logically connected to make up the information about the law network. Then, the information about a law network would have levels that correspond to levels of basis-derivative relationships; each of these information levels is a ground of levels that correspond to levels of general-special relationships of categories or laws. Thus, these levels are logically connected to constitute a network of information, which should correspond to the structure of the law network.

The heuristic method to guess regular patterns based on the information stated above does not ensure the reality or probability of the conclusions of the method. When an agent intend to use information of a regular pattern guessed using a heuristic method, the agent needs to evaluate credibility of the information. Since conclusions of the heuristic method are acquired from some basic information, the criteria to evaluate credibility of information depend on some properties of the basic information and the procedure in the method as follows: (1) degree of regularity that the basic information represent, (2) causality of the regular pattern that the basic information represents, (3) universality of the network to which the basic information belongs, i.e. generality and basis-ness, (4) logicity of the procedures performed in the heuristic method. The credibility of an item of information or inference to derive the information based on these criteria is called *rationality* of the information or inference.

The background knowledge necessary for the heuristic method is, in principle, possible when the PCS is equipped with the faculties that carry out the operations stated above. In this paper, it is assumed that the PCS acquired these faculties in the form of a hardware in the course of

evolution. Then, the background knowledge precedes the heuristic method and forms the basis of the heuristic method.

It should be noted that since certain accumulation of background knowledge is necessary to acquire the heuristic method, the method would function more clearly as people grow older. Gelman and Davidson (2013), and Sloutsky, Deng, Fisher and Kloos (2015), provide evidence consistent with this assumption: Induction develops from similarity based to category based, and conceptual influence on induction follows growth.

3. 2 A heuristic method

A heuristic method to guess regular patterns of the type ‘Every object x in a category A determined by a property P^A has a property Q ’ is constructed in the following way. The relationship between the regular pattern and its instance ‘An object a in A has Q ’ is situated in the structure of the law network stated in Section 2.2. P^A is a sufficient condition of Q , and this implication relates the dimension including P^A (A-dimension) and the dimension including Q (C-dimension). Since a has P^A and Q , this instance is a clue to look for the position of the regular pattern in this network structure made of the two dimensions.

To regress the regular pattern from an instance according to this structure, the PCS has to look for the two dimensions and the property in the A-dimension that determines a prospect of A . The steps for the regression are illustrated with an instance. In the following discussion, the notation $[e]$ denotes the item of information that an expression ‘ e ’ carries. Let the premise be [A material that is a piece of copper has high electrical conductivity.] and let the background knowledge include only vague information about electrical conductivity of substances.

1. Retrieve from the background knowledge the knowledge of the C-dimension that is a superordinate category of the premise property Q . Let the knowledge be [The C-dimension consists of two values: ‘high electrical conductivity’ and ‘low electrical conductivity’].
2. Look for in the background knowledge the knowledge of categories of properties that might have an implication relationship with properties in the C-dimension, prospective A-dimensions, which is denoted by A^p -dimension. In general, there would be plural A^p -dimensions for a C-

dimension. For instance, '(Kinds of substances)-hood' would be an A^P -dimension, for the above C-dimension. The knowledge of this A^P -dimension can be acquired statistically using inborn induction when the background knowledge includes data of electrical conductivity about some kinds of substances. In this case, the PCS might have plural classifications of substances, e.g. 'elements', 'metal-nonmetal', or 'solid-liquid', etc. The PCS might have a theory about qualitative relationships between the categories. For instance, 'concentration of free electrons' is also prospective one when the background knowledge includes a knowledge of a qualitative relationship between electrical conductivity and concentration of free electrons. Items of knowledge about these plural A^P -dimensions would be arranged in order following their rationality degrees.

3. Look for the value of the premise category a in each A^P -dimension, the category to which the a 's property in this dimension belongs. This value is a prospect of P^A . The knowledge of the properties of copper in these A^P -dimensions would be [copper-hood], [metal-hood], [solid-hood] or [having high concentration of free electrons]. Each of these values determines a prospective conclusion category. That is, the results are [Copper has high electrical conductivity], [Metals have high electrical conductivity], [Solids have high electrical conductivity] and [Objects with high concentration of free electrons have high electrical conductivity], respectively.

Such information about a C-dimension and A^P -dimensions serves for finding out P^A according to the structures of law networks stated in Section 2. 2. *ID* denotes this procedure 1~ 3. The object category determined by each of the value (property) is a candidate for A . *ID* is a heuristic method to guess regular patterns of the form, 'Every member x in A has Q ' from a premise [a has Q] and knowledge of a 's properties.

There is a derivative of *ID*, (*IDd*), that makes a guess about a 's property in an A^P -dimension, when a 's property is unknown but the regular pattern is premised (e.g. the background knowledge does not include [Copper is a metal], and the regular pattern, [Metals have high electrical conductivity], is premised). *IDd* follows Step 1.

- 2'. Make the dimension related with the C-dimension in the premised regular pattern, the A^P -dimension. (make Category {metal-hood, nonmetal-hood} be the A^P -dimension).
- 3'. Look for the value in the A^P -dimension by applying the premised regular pattern to Q . (The value in this dimension derived from Property 'high electrical conductivity' using the premised regular pattern is 'metal-hood').
- 4'. Attribute this value (property) to a , i.e. guess that a has this property (attribute the value, 'metal-hood' to [copper]).

Thus, the guess is [Copper has metal-hood] or [Copper is metal].

3.3 Rationality of guessing

The results of ID applied to a datum are arranged in order according to the degree of rationality of performance in Step 2 and Step 3.

In the example of 'electrical conductivity', [A^P -dimension 'kinds of substances'] can be drawn by means of inborn induction from statistical knowledge of laws about electrical conductivity of some kinds of substances. The inference is performed on the premise that the relationship between this A^P -dimension and the C-dimension is not inconsistent with the background knowledge. When the background knowledge includes knowledge of a law about electrical conductivity, this knowledge may also support the implication relationship between the two dimensions. For instance, [A^P -dimension 'concentration of free electron'] is drawn logically from [The cause of electrical conductivity is concentration of free electrons]. When the PCS has this knowledge, this leads to the knowledge of implication relationship between the A^P -dimension and the C-dimension. In the two prospective A^P -dimensions stated above, the latter is more rational than the former. This is because the latter A^P -dimension is causally related to the C-dimension, which is a basis level of the C-dimension. To the contrary, the former A^P -dimension is not the basis level of the C-dimension, a less universal level.

In Step 3, the value of the premise category a in an A^P -dimension is a category that has a degree for a to conform to the regular pattern that determines the category (i.e. typicality of a in the category). When a conforms to the regular pattern well, the probability that a has the sufficient condition of the regular pattern would be strong. Then, this

sufficient condition is supposed to be that of the premise property, Q , because of the implication relationships between the dimensions. Thus, the probability that the conclusion category determined by the value has the premise property is strong. Logicality of this derivation of the category from a depends on its validity i.e. the conformity degree. Therefore, conformity degree of a to the category is rationality degree of the derivation.

For instance, let an acquired datum be [A piece of newspaper burns when heated]. The C-dimension including ‘combustion’ consists of properties caused by heating, which would include properties such as ‘combustion’, ‘evaporation’, ‘melting’, and ‘expansion’. Let the PCS have the knowledge of this C-dimension. Then, the A^P -dimensions supported by the background knowledge may be ‘(kind of objects)-hood’, ‘(kind of chemical compound)-hood’, etc. The information of the value of ‘a piece of newspaper’ in these A^P -dimensions would be [paper-hood] or [(organic compound)-hood], etc. This leads to Conclusion [Paper (or organic compound) burns when heated]. Let the information that ‘a piece of newspaper’ is more typical in Category ‘paper’ than in Category ‘organic compound’ be in the background knowledge. Then, the choice of [paper] is more rational than choice of [organic compound].

The above consideration shows that ID’s conclusions are ordered according to the two kinds of rationality: rationality to draw an A^P -dimension from the C-dimension in question, and rationality to choose a category on the basis of conformity of the premise category in the value of Step 3. Thus, ID needs evaluation of these two kinds of rationality to choose the most credible one among the guesses. However, it is an open question how these two kinds of rationality are combined to determine the rationality of ID’s conclusion.

3.4 Extension of A^P -dimensions

A category relevant to an A^P -dimension may also be an A^P -dimension for the same C-dimension. For instance, ‘species’ compose an A^P -dimension for the C-dimension consisting of ‘traits’. In general, an ancestor of a species has a causal relationship with the species with respect to their traits. Thus, ‘ancestors of the species’ also compose an A^P -dimension for the C-dimension. Prey-predator relation would also

preserve implication with respect to some diseases, because of their transmission. There are other kinds of relationships between A^P -dimensions. For instance, when the C-dimension is ‘melt of kinds of substances’, ‘temperature’ is an A^P -dimension. Then, the dimension composed of properties defined as ‘higher temperature than a temperature’ is also an A^P -dimension for the C-dimension derived from the temperature dimension. That is, when the elements in an A^P -dimension are ordered with respect to the elements in the C-dimension, the dimension whose elements are sets of the ordered elements beginning from the elements in the A^P -dimension may also be an A^P -dimension for the C-dimension. The dimension composed of properties defined as ‘properties similar to a property in an A^P -dimension’ may also be an A^P -dimension for the C-dimension as stated in Section 2.1. When the PCS has such a background knowledge, the information about such derivative A^P -dimensions is available for Step 2 in ID.

3.5 Application of ID

(1) Causal regular patterns

ID is applicable to guess causal regular patterns between events. This is illustrated in the following example. When Datum, [Event E : That Ken is obese follows Event C : that Ken has a sweet tooth.], is acquired, C and E in this order can make a premise for ID under some condition such as ‘being all other things equal’; a condition that could make the PCS notice C and E as an ordered pair. Let the background knowledge include the knowledge that an event category including E is the category of events with the regular pattern, ‘A human being x is obese’, and the knowledge that the C-dimension including this event category consists of the event categories with the regular pattern, ‘A human being x has a figure y ’. Then, the relationship between the event categories and their causes is implication, accordingly the causes constitute an A-dimension. The knowledge of the categories of probable causes (A^P -dimensions) for this C-dimension in the background knowledge would be the knowledge of event categories with the regular pattern ‘A human being x has a habit z ’, or ‘A human being x has heredity u ,’ etc.; ‘a human being x ’ in these dimensions are identical with the ‘human being x ’ in the C-dimension. The value of C in the A^P -dimension ‘A human being x has a habit z ’

would be the event category having the regular pattern: ‘A human being x has a sweet tooth’. Let the background knowledge include this information about the value. Then, the guess of the regular pattern to which the datum might conform is [That a human being has sweet tooth implies that the human being is obese].

It should be noted that the category or the individual in the consequent event is, in general, a function of, or something in relation to the category or individual in the antecedent event (e.g. When E is ‘Ken’s father is obese’ then the relation is ‘Ken’ in $C \rightarrow$ ‘Ken’s father’ in E). In the above example, the function is the identity function, which is represented by the same variable x .

When C is unknown, but the causal regular pattern that leads from ‘A human being x has a habit z ’ to ‘A human being x has a figure y .’ is known, IDd is available to guess C . The steps of the guess are as follows:

1. The C-dimension is the event category with the regular pattern, ‘A human being x has a figure y ’. Let the background knowledge include this knowledge.
 - 2’. Make ‘The human being x has a habit z ’ the A-dimension for the C-dimension according to the premised regular pattern.
 - 3’. Derive the information about the value in this A^p -dimension from [A human being x is obese] using the premised causal regular pattern. Let the information about the value be [A human being x has a sweet tooth].
 4. Attribute this value to [Ken], the same information as that in E .
- The conclusion is Guess: [Ken is obese because Ken has a sweet tooth].

(2) Guess of isomorphisms

As stated in Section 2.3, an isomorphism of two things is a regular pattern that relates the two things preserving their structures. The steps of ID to guess an isomorphism of two things, A and B , based on an isomorphism of a part of A , a , and a part of B , b , is as follows.

1. Let the background knowledge include the knowledge, [The C-dimension consists of two values: isomorphic, not isomorphic].
2. Since an isomorphism of two things a , b is a mapping that preserves the structure common to them, the isomorphism is a property of the ordered pair of them, $\langle a, b \rangle$. Thus, ordered pairs of the structures of

things are properties that have implication with the properties in the C-dimension. That is, b in $\langle a, b \rangle$ is isomorphic to a iff the structures of a and b are identical. Therefore, the ordered structure pairs compose an A-dimension for the C-dimension. There also is an extension of this A-dimension. In general, there are things that include a thing a as a part, things extended from a . The things extended from a are the values of a function aX , where X is the variable for the parts connecting to a . Corresponding to an ordered thing pair, $\langle a, b \rangle$, the category of ordered extended thing pairs determined by the function pair $\langle aX, bY \rangle$ is constructed. This ordered pair may have implication with a value in the C-dimension; it is not clear that $\langle xX, yY \rangle$ implies a value in the C-dimension. Thus the set of the ordered pairs, $\langle xX, yY \rangle$ makes up an extension of the A-dimension stated above. This is an A^p -dimension.

3. The value of the ordered pair $\langle a, b \rangle$ in this extended A^p -dimension is $\langle aX, bY \rangle$, which determines the category of the ordered pair of structures.

The conclusion is a guess [Every ordered pair of things with an ordered pair of structures in the category is isomorphic to each other]. Then, [A is isomorphic to B .] is deduced from this guess.

When A 's structure is in the premise, B 's structure is guessed from the A 's structure according to the isomorphism. For instance, the anatomical structure of a familiar animal can be projected to that of an unfamiliar animal when its partial structure is found to be isomorphic to a partial structure of the known animal.

The steps stated above can be applied to event categories. When an event category is an isomorphic relationship between two categories, this isomorphism can be guessed by applying ID to some instances of the isomorphism. For instance, let the premise be Data: [a_1 kg of mass causes b_1 N of gravity], [a_2 kg of mass causes b_2 N of gravity], [a_3 kg of mass causes b_3 N of gravity]. This premise would generate a new premise: [The function $y_i = gx_i$, where g is a constant and x_i is an element in the set $a = \{a_1, a_2, a_3\}$ and y_i is an element in the set $b = \{b_1, b_2, b_3\}$, preserves the order relationships in quantities within both a and b .]. Let the background knowledge include the knowledge that a is a part of mass dimension M , and b is a part of gravity dimension G .

The steps of ID applied to this new premise is as follows.

Let the background knowledge include the knowledge necessary in the following steps.

1. The two values ‘isomorphic’ and ‘not isomorphic’ compose a C-dimension.
2. The set of ordered pairs of structures composes an A-dimension. The category of the ordered pairs of structures extended in the same way as that stated above is an A^P -dimension.
3. The value of the ordered pair $\langle a, b \rangle$ in this A^P -dimension is ‘the ordered pairs of structures that are the extensions of the ordered pair of the structures of $\langle a, b \rangle$ ’: The ordered pair of the structure of $\langle aA, bY \rangle$.

The above steps results in the guess [Every ordered pair of things including $\langle a, b \rangle$ as its part is isomorphic]. From this, [G is isomorphic to M , and $y = gx$ is an isomorphism of M and G .] is deduced.

This result implies that the generalizing the isomorphism from $\langle a, b \rangle$ to $\langle M, G \rangle$ is equivalent to making a guess of the existence of $\langle M, G \rangle$. When M is known but G except b is unknown, G can be guessed from M by applying the isomorphism.

These applications of ID to isomorphisms is a simple case of analogy.

4. Induction phenomena

There are well-known induction phenomena about strength of inductive inference, such as typicality effects, similarity effects and so on. They are touchstones for induction models. In these phenomena, the strength seems to be reasonableness of inference based on the background knowledge rather than certainty of the inference results. The rationality of inference introduced in Section 3. 1 consists of basic factors of the reasonableness of inference. When the strength of inductive inference is identified as the rationality, ID accounts for many of these phenomena as follows.

(1) Typicality effects: The more typical the premise category is, with respect to the conclusion category in an inductive inference, the stronger the inference is. For example, Inference ‘Dogs are tamable $\rightarrow$ Pets are tamable’ is stronger than Inference ‘Birds are tamable $\rightarrow$ Pets are

tamable' (presuming that the background knowledge includes [dogs are pets more typical than birds.]).

Let [A category a has a property Q] be a premise. Let A be the conclusion category determined by a property P^A that is a result of ID. As stated in Section 2. 2, the more typical a is in A , the stronger the probability that Q coexists with P^A . Therefore, the more typical a is in A , the stronger the probability that every member of A has Q is. We can postulate that the PCS can acquire the information on this general tendency. Thus, induction from a typical premise category is more rational than that from an atypical one. Thus, ID explains typicality effects when identifying inference strength as rationality of the inference.

From the conclusion by typicality effects, its instances are deduced, e, g., [A category b has Q] when b is a member of A . The rationality to deduce an instance from a conclusion is roughly equivalent to that to derive the conclusion. This leads to the asymmetry of inference strength between the inference from a typical premise category to an atypical instance category and the inference from an atypical premise category to a typical instance category. For example, the inductive inference, 'tamable-ness of dogs $\rightarrow$ tamable-ness of birds', is stronger than the inference, 'tamable-ness of birds $\rightarrow$ tamable-ness of dogs'. Both inferences are carried out via the general conclusion, 'tamable-ness of pets'. Then, the former inference is more rational than the latter to infer this medium general conclusion, and the both conclusions are deductions from the general conclusion.

Inference by ID can also solve the problem of inclusion fallacy: Though the probability of a specific conclusion should be stronger than that of the general conclusion, some inductive inferences that draw general conclusions are stronger than those that draw specific ones. As stated above, since a specific conclusion is deduced from a general conclusion, the rationality degree of the latter is roughly equivalent to that of the former. However, the additional steps of inference would expand uncertainty of typicality effects, hence could reduce the rationality of the specific conclusion slightly (See Hampton and Cannon 2004).

(2) Similarity effects: The more similar the premise category is to the conclusion category, the stronger the inductive inference is. For example, Inference ‘Lions are carnivorous $\rightarrow$ Tigers are carnivorous’ is stronger than ‘Lions are carnivorous $\rightarrow$ Cats are carnivorous.’ As stated in Section 3.4, a dimension composed of categories defined by similarity to the values in an A^p -dimension can be another A^p -dimension. As stated in Section 2. 2, similarity of properties of objects in an object category typically depends on commonality of a component in the sufficient conditions of properties similar to each other’. This component gives support to the projection of the premise property to the conclusion category of ID. The strength of this support is proportional to similarity degree, which is identical with typicality degree in this case. A conclusion category (e.g. tigers or cats) similar to the premise category (e.g. lions) is a member of the category (e.g. beasts similar to lions) that is the value of the premise category in this A^p -dimension. The more typical the conclusion category is in the value, the more rational the inference is. This accounts for similarity effects.

(3) Diversity effects: Greater diversity of premises promote induction.

Let [An individual a_1 in a category A has a property P] and [An individual a_2 in A has P] be two premises. The sufficient condition of P that a_1 has may be identical with one that a_2 has. Let S_p be this identical sufficient condition. ID applied to the premise about a_1 would raise some possible sufficient conditions of P (the values of a_1 in an A^p -dimensions). From the premise about a_2 , possible sufficient conditions of P (the values of a_2 in the A^p -dimensions) would be acquired. The sufficient condition of P consistent with the both premises is S_p . The properties common to a_1 and a_2 in the A^p -dimension are P^A or P^A ’s sufficient condition or the sufficient condition that makes a_1 and a_2 similar to each other. S_p should be one of them. Then, the difference between a_1 and a_2 tends to exclude the sufficient condition common to a_1 and a_2 that makes a_1 and a_2 similar to each other. That is, the greater the difference between a_1 and a_2 is in A , the stronger the probability that S_p is P^A or the sufficient condition of P^A , resulting in increase of the rationality to guess the conclusion based on the two premises, ‘The members in the category determined by P^A (i.e. A) has P ’.

When a category is homogeneous, every member of the category has the same value in the A^P -dimensions. Moreover, similarity between the members can be neglected. Thus, it is rational to guess from one instance in the category that the value is a sufficient condition of P (homogeneous effects). That is, every member in the category would have P . This accounts for homogeneous effects.

(4) Monotonicity effects: greater number of premises promotes induction.

Greater number of premises could increase statistical certainty, which is regularity of the basic information of the inference. Therefore, the rationality of the value of Step 3 increases. The more certain the foundation of the inference is, the more rational the inference is. This accounts for monotonicity effects.

(5) Nonmonotonicity effects: There are cases that show the opposite effect of monotonicity effects.

For example, from Premise [Dogs are tamable], ID can lead to a conclusion, [Mammals sharing the living environment with human beings are tamable]. The A^P -dimension for this conclusion is '(kinds of animals)-hood'. From this conclusion, [Horses are tamable] is deduced. On the other hand, from the premises about Categories 'dog' and 'dolphins', ID leads to a conclusion, e.g., [Mammals are tamable] (diversity effects), from which [Horses are tamable] is also deduced. However, the former inference is more rational than the latter in spite of its fewer premises. This is because '(mammal sharing the living environment with human beings)-hood' is a property in which 'dog-hood' is more typical than in 'mammal-hood'. This difference of typicality of the property in the different categories accounts for nonmonotonicity effects. This explanation agrees with that given by Osherson, Smith, Wilkie, Lopetz and Shafir (1990).

(6) Property effects: There are cases where the property in the premise affects inductive inference.

Property effects are straightforward for ID because of its procedure based on the premise property. A well-known example is the induction

with the premise having a behavioral property ‘Tigers prefer to eat at night’ (Heit 2000). Application of ID to this premise is the following. The C-dimension of Property ‘prefer to eat at night’ is ‘diet behavior’, and the prospective A^P -dimensions for this C-dimension are, for instance, ‘(kinds of animals)-hood’, ‘habits of animals’, or ‘physiological characters’, etc. Let the PCS have knowledge of the relevance between these categories. Then, the value of ‘tiger-hood’ in the A^P -dimension ‘(kinds of animals)-hood’ is Category ‘(cat family)-hood’. However, a more inclusive value ‘mammal-hood’ in the A^P -dimension would be excluded because of the inconsistency of the conclusion with the background knowledge. The value of ‘tiger’ in the A^P -dimension ‘habits of animals’ is Category ‘nocturnal’. The conclusion adopting the latter value is [Every nocturnal animal prefers to eat at night]. From this conclusion, for example, [Owls prefer to eat at night] is deduced. This conclusion would not be acquired without the relevance of those dimensions via the premise property.

Functional properties are another kind of examples. Application of ID to the premise with a functional property [Poodle can bite through barbed wire] (Heit 2000) is the following. The C-dimension is ‘faculty of dogs (or animals)’. An A^P -dimension is ‘(species of dogs)-hood’. Then, ‘(the sets of species of dogs similar to a species of dog)-hood’ is an extension of the A^P -dimension as stated in Section 3.4. The value of ‘poodle’ in this derivative A^P -dimension is ‘(species of dogs similar to poodle)-hood’. There is also another extension of the ‘(species of dogs)-hood’ dimension. Species of dogs approximately line up in order according to their power. Then, each species of dogs defines a set of species of dogs that are more powerful than the species. These sets could also have implication relevance to the C-dimension (causality in this case), thus, make an A^P -dimension. The value of ‘poodle’ in this derivative A^P -dimension is ‘(species of dogs more powerful than poodle)-hood’. Then, when the background knowledge includes the knowledge of the causal relevance, the conclusion using this knowledge is [The species of dogs more powerful than poodle can bite through barbed wire]. This conclusion is more rational than the former, because a causality based extension of an A^P -dimension is more rational than a similarity based one.

(7) Default of induction: Induction can operate on premises with blank properties (unknown properties), for example, [Robins have Property X] → [Sparrows have Property X].

In this case, the conclusion categories are typically derived via a super ordinate category of the premise categories based on the taxonomy, namely classifications of the categories. Category based induction by Osherson, Smith, Wilkie, Lopetz and Shafir (1990) discusses induction with blank properties. This effect would be natural, because the classifications of the categories are highly systematized bases of the background knowledge in the PCS. Many properties, especially attributes of categories are arranged in or related to the classifications. Thus, we tend to assume a blank property to be one of the properties in the classifications. Therefore, the classifications of categories can be regarded as the default background knowledge for ID when a premise property is blank.

5. Predictions

ID, as a hypothesis about induction, must be tested by examining predictions made by ID. The followings are basic predictions.

1. ID is a heuristic method using background knowledge to guess the regularity to which a premise might conform. Therefore, in general, by specifying agents' background knowledge relevant to a premise, we would be able to compare the conclusion derived using ID from the premise with human induction. Then, the strength of inductive arguments would be consistent with the rationality of the inference.
2. Because the rationale of ID depends on relationships between regular patterns and their instances in those general structures stated in Section 2, ID would not operate well on things that have poor regularity. Therefore, induction would also not operate well on such things.
3. ID works via the premise property. When the premise property is blank, we tend to assume a property organized in a classification as the default. If the premise property is entirely unknown and assumption of the default is forbidden, ID does not work. Therefore, induction would also yield no conclusion under this condition.

4. Application of ID to event categories would also generate induction phenomena discussed in Section 4. If this is the case, induction would also generate phenomena corresponding to these effects on events.

6. The comparison of ID and existing induction models

The fundamental difference between ID and the well-known existing induction models lies in their purposes, namely a heuristic method to guess regular patterns from premises vs. evaluation of strength (confirmation) of given inductive arguments. The directions of the studies diverges according to these purposes. For construction of the heuristic method, we need to make clear the reason for existence of induction or the role of induction first. Then, we proceed to find a method to fill the role and examine if it works as induction. For evaluation of inductive argument strength, we need to find some laws in induction phenomena about the strength of inductive arguments and to examine if they work as criteria of the evaluation and account for induction phenomena.

For instance, Similarity coverage model by Osherson, Smith, Wilkie Lopez, and Shafir (1990) and Sloman's Feature Based Induction (1993) adopt similarity between the premise category and the conclusion category in an inductive argument as the basis of argument strength. The more similar the categories are to each other, the stronger the inductive arguments are. These models can account for main phenomena of inductive arguments except property effects. However, the problem of why the similarity is the criterion of argument strength is left untouched. Relevance theory by Medin, Coley, Storms, and Heyes (2003) adopts relevance between the premise category and the conclusion category in an inductive argument to evaluate induction strength. The stronger the relevance is, the stronger the inductive argument is. The same problem as that about similarity also applies to the relevance. In ID, the background knowledge on structures of regular patterns and rationality of the inference accounts for this problem.

Bayesian model by Heit (2000) and its extension, statistical structured model by Kemp and Tenenbaum (2009) adopt probability of induction conclusions as the basis for the strength of inductive

arguments. These models calculate the probability of the conclusion (the posteriori probability) of an inductive inference by applying Bayes's theorem to the premise and certain background knowledge or beliefs (the prior probability). This probability calculation is strength evaluation for inductive arguments in these models. On the other hand, in ID, inference depends on general structures of regular patterns, and the strength of the inference is regarded as rationality to carry out the inference. This rationality includes a different criterion from probability to evaluate induction. A result of this difference appears in explanation of inclusion fallacy and non-monotonicity effects, which contradict inference based on probability. ID accounts for them naturally as stated in Section 4, whereas Structured statistical model has difficulties in accounting for them. This model needs to interpret those arguments into revised probabilistic representations to account for them (Kemp and Tenenbaum 2009). Since rationality supports credibility and credibility is relevant to probability, the relationships between the probability and rationality is an interesting future problem.

The induction scheme of 'instance based generalization' discussed by Holland et, al., (1986. pp232) takes statistical tendencies of data into account. Since the tendencies are parts of structure information, the scheme is similar to ID. The following is an example of the scheme.

- (1) An instance: *a* is an unknown kind (F) of bird and is blue.
 - (2) Background knowledge: Every known kinds of bird has a color.
- Then, the scheme generates a confident generalization of the instance: Every F is blue.

Thus, the relationship between 'kinds of birds' and 'colors' is the implication relationship between an A^P -dimension and the C-dimension in ID. However, the scheme has no basis to support the relationship between the generalization of instances and the statistical tendencies of data (background knowledge) concerning the instances. A theoretical basis of the relationship is necessary for systematic construction of induction scheme, which establishes rationality of induction. Thus, the scheme is not well developed, and has difficulties in deriving induction phenomena. On the other hand, ID is constructed on the basis: the general structures of regularities, as stated in the paper.

Connection between A^P -dimensions and a C-dimension in background knowledge consists of local facts that hold in a particular domain. Norton (2003) also emphasizes necessity of this knowledge for inductive inference. However, he does not develop the inductive scheme based on the background knowledge.

7. Summary and Discussion

In this study, I tried to construct inductive inference as a heuristic method to guess regular patterns in the world from known data, applying the knowledge about structures of the regular patterns. At first, the role or the purpose of induction was drawn from the principle of evolution. That is, the role of induction is to acquire information of regular patterns in the world, because of its usefulness for survival; this usefulness can be the basis on which the PCS has acquired a hardware to play a basic part of the role in the course of evolution. I have postulated this hardware in the PCS and the faculty of the PCS to apply acquired information to guess regular patterns. Then, it is natural that this faculty will include creation of a heuristic method to guess regular patterns efficiently based on acquired information about general structures of regular patterns. We can find the general structures in scientific knowledge. The condition for the PCS to acquire the information about the general structure were raised as the foundation of the method.

According to this strategy, Heuristic method ID was constructed. The structures that ID depends on are structures of categories, category networks and law networks. In ID, the knowledge on the structure of the networks leads the guesses of conclusions from premises. Then, rationality of the performance in ID supports the guesses. Thus, the degree of the rationality is regarded as the strength degree to support the guesses (inference).

The results of ID are as follows.

1. ID creates inference that satisfies main properties of induction, typicality effect, similarity effect, diversity effect, and so on.
2. ID is applicable to guess regular patterns of causal relationships between events.

3. ID is applicable to guess isomorphism between objects or existence of an object isomorphic to a premised object. This application is analogy.
4. A derivative of ID, IDd, is a simple case of abduction.

This study has not looked at closely the problem of how to find the A^P -dimensions relevant to a C-dimension from the background knowledge. To solve this problem, we need to make clear a criterion to choose the A^P -dimension under the given condition. This criterion would depend on the background knowledge about category networks, law networks and the direction or purpose of the inductive inference, that is, for what information the inference is carried out under the given condition. To clarify this criterion will be an important future problem for the basis of induction.

Quantitative evaluation of rationality degree is another problem this study has left open. This problem is also relevant to the background knowledge about structures of regular patterns. It is certain that the typicality in a category and the levels of basis-derivative and general-special relationships in these networks are the basic factors of the rationality of inference. Qualitative evaluation of the rationality will be possible on the basis of the typicality and the levels. However, it is not clear how the typicality and the levels work together to make up the rationality of inference. This is also a future problem.

We can examine whether ID is a proper model of induction by testing the predictions raised in Section 5, with the reservation of the unsettled problems stated above. Furthermore, application of ID to related domains will be effective confirmation. ID has two prospective domains. Application of ID to isomorphisms seems to be relevant to analogy, and the derivative of ID, IDd, does to abduction. Whether ID can be extended to analogy and abduction in general or not is of great interest.

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