A few remarks on stimulus power and interference in visual stimuli

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ABSTRACT

Interference may occur between visual stimuli or between separate elements in visual stimuli. Such interference takes place in the stimuli as these exist independently of vision and visual processing. Interference manifests itself in reduced "stimulus power" which makes this a central concept in this context. The present report examines three potential measures of stimulus power: Amplitude sum, norm, and sum of the absolutes of wavelet coefficients. These are examined with very simple 1-Dimensional stimuli as examples. It is shown how there may be interference with each of these measures. Thus, interference is not only linked to the Fourier Transform. Due to interference a given stimulus may have less stimulus power when presented along with a second stimulus than when presented alone. This has consequences for the concept of stimulus since it makes it possible for the same physical image to have different stimulus power in different contexts.

Key words: Amplitude; Fourier; norm; wavelets; stimuli; masking; crowding; visual search.

INTRODUCTION

Many visual experiments involve determining the effect of altering the stimuli along some dimension. In such cases in order to be able to attribute any perceptual effect specifically to the stimulus dimension in question it is necessary to maintain the stimulus unaltered in other respects. That making changes to a stimulus may alter its stimulus power or the power of one of its critical elements is not widely recognized. Examples of this would be visual masking (Polat & Sagi, 1993; Enns & Di Lollo, 1997, 2000; Breitmeyer & Ogmen, 2000), visual crowding (Levi, 2008; Pelli, 2008; Whitney & Levi, 2011; Manassi & Whitney, 2018) and visual search (Treisman & Gelade, 1980; Wolfe et al., 1992). In these cases subjects are requested to respond to some aspect of a target stimulus in the presence of masking, flanking, or distractor stimuli. Any reduction in performance resulting from the introduction of the masking, flanking, or distractor stimuli have typically been attributed to mechanisms in the visual system. An underlying, although tacit, assumption behind this paradigm is that the target stimulus has the same stimulus power when presented along with masking or flanking stimuli as when it is presented by itself. This assumption need not be valid (Skottun, 2017a, 2017b, 2018b, 2018c).

The reason for this is that there may take place interference between stimuli or between stimulus elements when these are presented together (or in rapid succession)(Skottun, 2017a, 2017b, 2018b, 2018c). This entails that even though a given stimulus, or stimulus element, by itself may be identical when it is presented alone and when it is presented in combination with other stimuli it may have reduced stimulus power in the latter case. Typically, reports on masking and crowding do not discuss stimulus power or interference. The present report explores the concept of "stimulus power" and how it can be measured, particularly so in relation to interference. Basic aspects of this concept is explicated using very simple 1-Dimensional stimuli as examples.

The present analyses are all with regard to the stimuli and do not deal with the responses to them. They, therefore, do not represent attempts at explaining perceptual phenomena (such as, e.g., visual masking and visual crowding) which represent responses to visual stimuli. It would seem that in order to be able to understand responses to visual stimuli we need to first understand the stimuli. The present expositions represent attempts at this as well as an attempt at sketching out a basic framework for such an understanding.

[FOOTNOTE 1]

STIMULUS POWER

Interference manifests itself in a reduction in "stimulus power" (Skottun, 2017a, 2017b, 2018b). This

raises the question of what exactly is to be meant by this term. In many contexts it is required that stimulus power be expressed by a single value. It seems that for any stimulus power operator we might choose we would require that adding of the stimulus to itself would result in twice the power of the stimulus alone. That is, if we denote the stimulus power operator by T and a given stimulus by $\mathbf{a}$ we would require that $T\mathbf{a} + T\mathbf{a} = T(2\mathbf{a}) = 2T\mathbf{a}$. That is to say, we would expect the stimulus power operator to be a linear operator under this specific condition. Lest there be any confusion, this does not mean that the stimulus added to itself would give twice the response since there may exist all kinds of nonlinearities in the stimulus-response relationship. We are here only dealing with the stimuli. (One must not confuse the stimulus and the response.)

AMPLITUDES

One plausible measure of stimulus power would be amplitude sum. Let us consider a simple example. A linear transform, can be described by a matrix. The matrix for a Fourier Transform, F , for a signal 4 elements long is:

$$F = \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & i/2 & -1/2 & -i/2 \\ 1/2 & -1/2 & 1/2 & -1/2 \\ 1/2 & -i/2 & -1/2 & i/2 \end{pmatrix},$$

where i denotes $\sqrt{-1}$, and where each row-vector constitutes a basis function. [FOOTNOTE 2] The Fourier Transform, $\mathbf{A}$, of the stimulus $\mathbf{a} = \{1, 0, 0, 0\}$ thus becomes:

$$\mathbf{A} = F\mathbf{a} = \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & i/2 & -1/2 & -i/2 \\ 1/2 & -1/2 & 1/2 & -1/2 \\ 1/2 & -i/2 & -1/2 & i/2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}.$$

That is, the Fourier Transform of $\mathbf{a}$ is given by the first column-vector of F . (Lower- and upper-case boldface letters are used to denote signals and their Fourier spectra, respectively.)

Since this stimulus has been chosen so as to have a real-valued Fourier spectrum we get that the amplitude spectrum of $\mathbf{a}$, $|\mathbf{A}|$, is also $\{0.5, 0.5, 0.5, 0.5\}$, with $|\cdot|$ denoting the amplitude operator. [FOOTNOTE 3] In the case of $\mathbf{a} + \mathbf{a}$ the signal becomes $\{2, 0, 0, 0\}$ and the Fourier spectrum and the amplitude spectrum both become $\{1, 1, 1, 1\}$. That is, the amplitude spectrum of 2 times $\mathbf{a}$ is two times the amplitude spectrum of $\mathbf{a}$. In other words, $|2\mathbf{A}| = 2|\mathbf{A}|$. In the case of the sum of the amplitudes of $\mathbf{a}$ we get $\sum |\mathbf{A}| = 2$ and in the case of $\mathbf{a} + \mathbf{a}$ we get $\sum |2\mathbf{A}| = 4$. Thus, in the case of the sum of amplitudes we get that the sum of amplitudes of $\mathbf{a} + \mathbf{a}$ is $\sum |\mathbf{A}| + \sum |\mathbf{A}| = \sum |\mathbf{A} + \mathbf{A}| = 2 \sum |\mathbf{A}|$. Which is exactly what we said we wanted at the outset. That is, we want the stimulus power operator to behave as a linear operator in the particular

case where the stimulus is added to itself. [FOOTNOTE 4]

This, it should be emphasized, does not mean that the stimulus power operator needs to be a linear operator in general. On the contrary, if $\mathbf{a}$ and $\mathbf{b}$ are two different signals with amplitude spectra denoted as $|\mathbf{A}|$ and $|\mathbf{B}|$, respectively, we do not in general get that $|\mathbf{A}| + |\mathbf{B}| = |\mathbf{A} + \mathbf{B}|$. Consider the case were $\mathbf{a} = \{1, 0, 0, 0\}$, as above, and $\mathbf{b} = \{0, 0, 1, 0\}$. In this case the Fourier Transform of $\mathbf{a}$ remains $\mathbf{A} = \{0.5, 0.5, 0.5, 0.5\}$. The Fourier Transform of $\mathbf{b}$ which is given by the third column-vector of F is $\mathbf{B} = \{0.5, -0.5, 0.5, -0.5\}$. In both cases the amplitude spectra are: $|\mathbf{A}| = |\mathbf{B}| = \{0.5, 0.5, 0.5, 0.5\}$ which sums to 2. In the case of adding the stimuli we get Stimulus $\mathbf{a}$ plus Stimulus $\mathbf{b}$ is $\{1, 0, 1, 0\}$ the Fourier Transform of which corresponds to the first column-vector of F plus the third column-vector of F which add to $\{1, 0, 1, 0\}$. Since this Fourier spectrum is real-valued the amplitude spectrum equals the Fourier spectrum. Consequently, the sum of $\sum |\mathbf{A} + \mathbf{B}| = \sum \{1, 0, 1, 0\} = 2$. However, as we just saw, the amplitude sum of both $\mathbf{A}$ and $\mathbf{B}$ are 2 making the sum of the amplitudes in $\mathbf{A}$ plus the sum of amplitudes in $\mathbf{B}$ equal to 4. Thus, $\sum |\mathbf{A}| + \sum |\mathbf{B}| = 4 \neq \sum |\mathbf{A} + \mathbf{B}| = 2$. The difference between $\sum |\mathbf{A}| + \sum |\mathbf{B}|$ on the one hand and $\sum |\mathbf{A} + \mathbf{B}|$ on the other is the result of interference.

One requirement for an operator T to be linear is that $T\mathbf{a} + T\mathbf{b} = T(\mathbf{a} + \mathbf{b})$, where $\mathbf{a}$ and $\mathbf{b}$ are two entities upon which the operator operates. The Fourier Transform fulfills this requirement. That is, the transform of a signal into its *Fourier spectrum* is a linear operation. However, the transform of a signal into its *amplitude spectrum* is generally not. This transform is linear only when the two signals have identical phase spectra (as is the case when adding the signal to itself).

INTERFERENCE

The explanation for the inequality $\sum |\mathbf{A}| + \sum |\mathbf{B}| = 4 \neq \sum |\mathbf{A} + \mathbf{B}| = 2$ is interference caused by differences in the phase spectra. Interference can be understood by considering the summation of two sine functions. Let us consider the two functions $\sin(x - \theta_1)$ and $\sin(x - \theta_2)$ where θ_1 and θ_2 denote the phase values. If $\theta_1 \neq \theta_2$ these functions will differ only in their phase values. As an example let us set $\theta_1 = 0$ and $\theta_2 = \pi/2$. (Since $\pi/2 = 90$ degrees this makes the two functions 90 degrees out of phase.)

In Fig. 1A are shown these two functions (solid lines) along with their sum (dashed line). As can be seen, the sum is a another sine function. However, while the two sine functions depicted with solid lines each have an amplitude of 1 the amplitude of their sum is not 2. That is, the amplitude of the sum is not the sum of the amplitudes. In fact, its is considerably less, namely $\sqrt{2} \approx 1.41$. What makes the amplitude

Fig. 1. (A) Solid lines represent sine functions given by $\sin(x+0)$ (marked "0") and $\sin(x-\pi/2)$ (marked $\pi/2$). Their sum is indicated with a dashed line. As ought to be apparent, the amplitude of the sum is smaller than the sum of the amplitudes. (B) The addition of the functions in Panel A represented as vector addition. Since the vectors marked "0" and " $\pi/2$ " both have unit length and are orthogonal we get that the sum (marked " $0 + \pi/2$ ") becomes $\sqrt{1+1} = \sqrt{2} \approx 1.41$.

of the sum smaller than the sum of the amplitudes is the difference in phase angle, i.e. $\theta_2 - \theta_1 = \pi/2$. Had there been no difference in phase (i.e., had $\theta_2 - \theta_1 = 0$) the amplitude of the sum would have been 2. That is, the amplitude of the sum would have been equal to the sum of the amplitudes. On the other hand, had the difference between θ_1 and θ_2 been π the amplitude of the sum would have been zero (see Fig. 1 of Skottun, 2017a). Generally, $|\sin(x-\theta_1) + \sin(x-\theta_2)| < |\sin(x-\theta_1)| + |\sin(x-\theta_2)|$ when $\theta_1 - \theta_2 \neq 2n\pi$, where n is an integer (including 0).

A sine function may be represented by a vector of which the length represents the amplitude and the

direction represents the phase. The summation of the two sine functions in Fig. 1A is shown as vector addition in Fig. 1B. Since the two functions both have the amplitude of 1 and are at 90 degrees to each other (i.e., $|\theta_1 - \theta_2| = \pi/2$) we get that the length of the vector representing the sum is $\sqrt{1+1} = \sqrt{2} \approx 1.41$.

This holds for the particular case where there are two sine functions of identical frequency and unit amplitude with a phase difference of $\pi/2$. In the more general case we get that the amplitude of the sum of two sine functions of equal frequency is $\sqrt{(g_1 \cos(\theta_1) + g_2 \cos(\theta_2))^2 + (g_1 \sin(\theta_1) + g_2 \sin(\theta_2))^2}$, where g_1 and g_2 represent the amplitudes of the two functions and θ_1 and θ_2 give their phase angles. For the amplitude of the sum of multiple such functions we have $\sqrt{(\sum_i g_i \cos(\theta_i))^2 + (\sum_i g_i \sin(\theta_i))^2}$. A consequence of this is that the amplitude of the sum of two (or more) sine functions need not equal the sum of the amplitudes. The amplitude of the sum can be smaller than the sum of amplitudes but not larger. How much smaller depends on the phase relationship between the functions that are being summed. Critical in the present context is that when the phases differ the amplitude of the sum will be smaller than the sum of the amplitudes. This, most importantly, entails that when the phases differ the full amplitudes of all the summed functions cannot all be fully contained in the amplitude of the sum.

These considerations, it should be emphasized, are all for sine functions having the same frequency. In the present context that is all we are concerned with since the adding of Fourier spectra and amplitude spectra always involves adding components of the same frequency.

Relative Amplitude Sum. Based on the sum of amplitudes we may calculate the *Relative Amplitude Sum* for the addition of two functions as $\sum |\mathbf{A} + \mathbf{B}| / (\sum |\mathbf{A}| + \sum |\mathbf{B}|)$ where $|\mathbf{A}|$, $|\mathbf{B}|$, and $|\mathbf{A} + \mathbf{B}|$, denote the amplitude spectra of Stimulus **a**, Stimulus **b**, and Stimuli **a** and **b**, combined. This provides a measure of the sum of the amplitudes in the two stimuli combined relative to the sum of their amplitudes determined separately. A Relative Amplitude Sum of less than 1.0 indicates that the sum of amplitudes in the combined stimulus is smaller than the sum of the amplitudes in the two stimuli measured separately. This would represent evidence for interference. The Relative Amplitude Sum may be considered as (1 - Interference) which gives Interference as (1 - Relative Amplitude Sum). (The Relative Amplitude Sum can be expanded to apply to combinations of more than two stimuli. In which case we have: $\sum |\mathbf{A} + \mathbf{B} + \mathbf{C} \dots \mathbf{N}| / (\sum |\mathbf{A}| + \sum |\mathbf{B}| + \sum |\mathbf{C}| \dots \sum |\mathbf{N}|)$.)

That a stimulus need not have the same stimulus power when it is presented together with other stimuli as when presented by itself is illustrated in Fig. 2. In this figure the same stimulus (solid line) is shown in combination with different other stimuli (dashed lines). The stimuli were modeled as arrays of 64 elements

(which may represent either space or time). The stimulus indicated with the solid line had a value of 1 for

Fig. 2

Fig. 2. Four examples of the combination of two stimuli. Both stimuli have an intensity = 1. One stimulus has a fixed extent of 16 elements (solid line). The second stimulus (dashed line) surrounds the first leaving a gap between the two stimuli on either side of 10 elements in A, 4 elements in B, 2 elements in C, and 0 elements in D. The Relative Amplitude Sum for each addition (indicated as "RAS = ...") were calculated for the dc value in the Fourier spectra set equal to 0. (The RAS values computed with the dc values included were 0.65, 0.59, 0.53, and 0.21, for separations of 10, 4, 2, and 0 elements, respectively.) The three traces in each of the four panels show the central stimulus (top), the surrounding stimulus (middle) and the addition of the two (bottom trace). The total extent of the stimuli were 64 elements. A thing to note is that the central stimulus (the solid line) is the same in all four panels yet its stimulus power is obviously vastly different.

16 elements. That is, 16 of its 64 elements were set to 1 while the rest were set to 0. In the case of the stimulus

shown with dashed lines these were also of intensity 1. However the length of the portion in which the values were 1 was varied so that when combined with the first stimulus (lower trace in each panel) the separation between the stimuli was varied. Four different stimulus configurations were explored. The separations between the first stimulus and the second stimulus were 10, 4, 2, and 0 elements in Panels A, B, C, and D, respectively (these values are for the gaps on either side of the first stimulus). These stimulus configurations give Relative Amplitude Sums (with the dc value set to zero, see comments on *Average luminance* below for discussion) of 0.58, 0.5, 0.42, and 0.0, for the four configurations. (The Relative Amplitude Sums for the four configurations with the dc values unaltered were, respectively, 0.65, 0.59, 0.53, and 0.21.) Since the stimulus with the solid line is the same in all four constellations these illustrations make clear that, as measured by the Relative Amplitude Sum, a given stimulus need not have the same stimulus power when presented together with other stimuli as when it is presented by itself.

As was pointed out above, any difference in phase spectra will make the amplitude of the sum smaller than the sum of the amplitudes but can never make it larger. A consequence of this is that when combining two visual stimuli any difference in their phase spectra will cause the sum of the amplitudes in the combined stimulus to be smaller than the sum of the amplitudes in the first stimulus plus the sum of amplitudes in the second. Consequently, any phase difference will cause the Relative Amplitude Sum to fall below 1.0.

THE NORM

It was above proposed that the sum of amplitudes may serve as a measure of stimulus power. An alternative measure would be the norm. The norm of a vector is its length. The length of a vector $\mathbf{h}$ is given as $\|\mathbf{h}\| = \sqrt{\sum_i (\mathbf{h}_i)^2}$, where $\mathbf{h}_i$ denotes the i th element of $\mathbf{h}$. This equation applies to real valued vectors. In the case of complex valued vectors, such as a Fourier spectrum, the vector needs to be multiplied with its complex conjugate. In this case we get: $\|\mathbf{h}\| = \sqrt{\sum_i (\mathbf{h}_i \overline{\mathbf{h}_i})}$, where the overline denotes the complex conjugate. (For a complex number, $z = x + y i$, where x and y are real numbers and $i = \sqrt{-1}$, the complex conjugate is $\bar{z} = x - y i$. That is, the sign of the imaginary part is flipped.)

In the case of the example above where $\mathbf{a} = \{1, 0, 0, 0\}$ with a Fourier spectrum of $\mathbf{A} = \{0.5, 0.5, 0.5, 0.5\}$ we get that $\|\mathbf{A}\| = 1$. Likewise for $\mathbf{b} = \{0, 0, 1, 0\}$ we get $\|\mathbf{B}\| = 1$. For the sum of $\mathbf{a} + \mathbf{b} = \{1, 0, 1, 0\}$ the norm is $\|\mathbf{A} + \mathbf{B}\| = \sqrt{2}$. Thus, we have that $\|\mathbf{A}\| + \|\mathbf{B}\| = 2$ which means that $\|\mathbf{A}\| + \|\mathbf{B}\| = 2 \neq \|\mathbf{A} + \mathbf{B}\| = \sqrt{2}$. Based on the norms we may calculate what is here called the *Relative Norm* as $\|\mathbf{A} + \mathbf{B}\| / (\|\mathbf{A}\| + \|\mathbf{B}\|)$. In the current example this would be $\sqrt{2}/2 \approx 0.71$. This shows that interference can also be demonstrated with

the norms.

Important in this context is that the norm of a Fourier transform of a given signal is equal to the norm of the signal. That is, $\|\mathbf{a}\| = \|\mathbf{A}\|$ and $\|\mathbf{b}\| = \|\mathbf{B}\|$. In order to make use of the complex conjugate we will demonstrate this with a signal, $\mathbf{c} = \{0, 1, 0, 0\}$, which has a complex-valued Fourier Transform $\mathbf{C} = \{0.5, 0.5i, -0.5, -0.5i\}$. The norm of $\mathbf{C}$ then becomes $\sqrt{\sum_i \mathbf{C}_i \overline{\mathbf{C}}_i}$. Since $\overline{\mathbf{C}} = \{0.5, -0.5i, -0.5, 0.5i\}$ we get

$$\|\mathbf{C}\| = \sqrt{(0.5 \ 0.5) + (0.5i \ -0.5i) + (-0.5 \ -0.5) + (-0.5i \ 0.5i)} = \sqrt{0.25 + 0.25 + 0.25 + 0.25} = \sqrt{1} = 1.$$

(Keep in mind that since $i = \sqrt{-1}$ we get $i^2 = -1$. The function of the complex conjugate is to make each product $\mathbf{C}_i \overline{\mathbf{C}}_i$ have a positive value.) For the norm of $\mathbf{c}$ we get $\|\mathbf{c}\| = \sqrt{\{0 + 1 + 0 + 0\}} = \sqrt{1} = 1$. Thus, $\|\mathbf{c}\| = \|\mathbf{C}\|$.

The significance of this is that since the norms of the Fourier spectra of the stimuli provide evidence for interference and the norms of the Fourier spectra are the same as the space/time domain representations of the stimuli it follows that interference can also be demonstrated using the norms of the space/time domain signals. Consequently, interference is not specific to the Fourier Transform but can also be demonstrated in the space/time domain signals by using the norm as measure of stimulus power.

In this connection it should be emphasized that since the norm is the length of a vector the norm of a signal added to itself will necessarily be twice the original norm. (One can calculate this from the x- and y-components of a vector the length/norm of which is $\sqrt{x^2 + y^2}$. Adding this vector to itself we get $\sqrt{(x+x)^2 + (y+y)^2} = \sqrt{(2x)^2 + (2y)^2} = 2\sqrt{x^2 + y^2}$.) This means that the norm fulfills the requirement we set out for a measure of stimulus power, namely that the stimulus power of the stimulus added to itself should be twice the stimulus power of the stimulus.

WAVELET TRANSFORM

An alternative method for representing stimuli is provided by Wavelets. The Wavelet Transform (like the Fourier Transform) is based on orthonormal basis functions [FOOTNOTE 5]. The simplest Wavelet is the Haar Wavelet. (For a simple introduction to Haar Wavelets see Stollnitz et al., 1996. For more general discussions of Wavelets see Chui, 1992; Meyer, 1992, 1993; Daubechies, 1992; Mallat, 1998). As was pointed out above, any linear transformation may be represented by multiplying a signal with a matrix. (Where each row-vector in the matrix represents a basis function.) The matrix giving the basis functions for a Haar Wavelet Transform for 1-D stimuli having a length of four elements is:

$$\Psi = \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}.$$

For stimulus $\mathbf{a} = \{1, 0, 0, 0\}$ we then get the Haar Wavelet Transform given by the coefficients:

$$\mathbf{c}_\mathbf{a} = \Psi \mathbf{a} = \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \\ 0 \end{pmatrix},$$

which corresponds to the first column-vector of Ψ . For the stimulus $\mathbf{b} = \{0, 0, 1, 0\}$ we get $\mathbf{c}_\mathbf{b} = \Psi \mathbf{b} = \{1/2, -1/2, 0, 1/\sqrt{2}\}$, which is equal to the third column-vector of Ψ .

In order to obtain a measure of stimulus power based on wavelet coefficients it is reasonable to base this on the absolute of the coefficients. The reason for this is that coefficients from the Haar Transform can be both positive and negative. It is therefore possible that positive and negative coefficients may cancel each other. This may cause the power of the stimuli to be underestimated were it to be based on summing wavelet coefficients.

For instance, were we to add coefficients we would get the sum of coefficients for $\mathbf{a}$ is $\sum\{1/2, 1/2, 1/\sqrt{2}, 0\} = 1 + 1/\sqrt{2}$ whereas the sum of amplitudes for $\mathbf{b}$ is $\sum\{1/2, -1/2, 1/\sqrt{2}, 0\} = 1/\sqrt{2}$. Thus, the stimulus power of these two stimuli would be quite unequal which is unreasonable given that the stimuli differ only by one being shifted relative to the other. On the other hand, were we to sum the absolutes of the coefficients we get that both stimuli have a stimulus power of $1 + 1/\sqrt{2}$.

In the case of the sum of $\mathbf{a}$ and $\mathbf{b}$ we get $\mathbf{a} + \mathbf{b} = \{1, 0, 1, 0\}$. The Wavelet coefficients of this stimulus is the first column-vector of Ψ plus the third column-vector of Ψ giving $\mathbf{c}_{\mathbf{a}+\mathbf{b}} = \{1, 0, 1/\sqrt{2}, 1/\sqrt{2}\}$ which sums to $\sum \mathbf{c}_{\mathbf{a}+\mathbf{b}} = 1 + \sqrt{2}$. This is equal to $\sum \mathbf{c}_\mathbf{a} + \sum \mathbf{c}_\mathbf{b} = \sum\{1/2, 1/2, 1/\sqrt{2}, 0\} + \sum\{1/2, -1/2, 1/\sqrt{2}, 0\} = 1 + \sqrt{2}$. This means that $\sum \mathbf{c}_{\mathbf{a}+\mathbf{b}} / (\sum \mathbf{c}_\mathbf{a} + \sum \mathbf{c}_\mathbf{b}) = 1$. In other words, summing the coefficient would give no evidence for interference. The problem, as was just pointed out, is that the sum of coefficients give very different stimulus powers for $\mathbf{a}$ and $\mathbf{b}$ (i.e., $1 + 1/\sqrt{2}$ and $1/\sqrt{2}$, respectively) which is unreasonable.

On the other hand, by taking the absolutes of the coefficients, which as we just demonstrated is a reasonable measure of stimulus power, we get: $\sum |\mathbf{c}_\mathbf{a}| = \sum |\{1/2, 1/2, 1/\sqrt{2}, 0\}| = 1 + 1/\sqrt{2}$ and $\sum |\mathbf{c}_\mathbf{b}| = \sum |\{1/2, -1/2, 1/\sqrt{2}, 0\}| = \sum\{1/2, 1/2, 1/\sqrt{2}, 0\} = 1 + 1/\sqrt{2}$. For $\mathbf{c}_{\mathbf{a}+\mathbf{b}}$ we get $\sum |\mathbf{c}_{\mathbf{a}+\mathbf{b}}| = \sum |\{1, 0, 1/\sqrt{2}, 1/\sqrt{2}\}| = 1 + \sqrt{2}$. This gives $\sum |\mathbf{c}_{\mathbf{a}+\mathbf{b}}| / (\sum |\mathbf{c}_\mathbf{a}| + \sum |\mathbf{c}_\mathbf{b}|) = (1 + \sqrt{2}) / (1 + 1/\sqrt{2} + 1 + 1/\sqrt{2}) = (1 + \sqrt{2}) / (2 + \sqrt{2}) \approx 0.71$. That is, when using the sum of the absolutes of the wavelet coefficients we find evidence for interference.

Thus, in the case of Haar Wavelets it would be possible to avoid interference between **a** and **b** but this would require the adoption of an unreasonable measure of stimulus power. However, when employing the sum of the absolutes of the coefficients, which represents a reasonable measure of stimulus power, interference is evident.

It was initially stated as a requirement that for any measure of stimulus power the stimulus power of the stimulus added to itself would equal twice the stimulus power (i.e., $T\mathbf{a} + T\mathbf{a} = T2\mathbf{a} = 2T\mathbf{a}$, where **a** is a stimulus and T denotes the stimulus power operator). For the sake of completeness it will be demonstrated that the sum of the absolutes of the Haar Wavelet coefficients fulfills this requirement. For $\mathbf{a} = \{1, 0, 0, 0\}$ we have $\Psi\mathbf{a} = \{1/2, 1/2, 1/\sqrt{2}, 0\}$ which gives $\sum |\Psi\mathbf{a}| = 1 + 1/\sqrt{2}$. For $2\mathbf{a}$ we get $2\mathbf{a} = \{2, 0, 0, 0\}$ which has the Wavelet coefficients of $\Psi2\mathbf{a} = \{1, 1, \sqrt{2}, 0\}$ the absolutes of which sum to $\sum |\Psi2\mathbf{a}| = 2 + \sqrt{2}$. Since $\sum |\Psi\mathbf{a}| = 1 + 1/\sqrt{2}$ we get that $\sum |\Psi\mathbf{a}| + \sum |\Psi\mathbf{a}|$ becomes $2 + 2/\sqrt{2} = 2 + \sqrt{2}$. Thus, $\sum |\Psi\mathbf{a}| + \sum |\Psi\mathbf{a}| = \sum |\Psi2\mathbf{a}|$. [FOOTNOTE 6]

SOME ISSUES

Based on these general and basic remarks on stimulus power and interference we may turn our attention to some particular issues.

The combining of stimuli. When adding two stimuli it is possible for both of the stimuli to be fully contained in the combined stimulus while their stimulus powers are not. In other words, when combining two or more stimuli it is possible for the physical images of the stimuli to be rendered accurately without any degradation or reduction while their stimulus powers may be reduced. This is illustrated in Fig. 3 where it can be seen that Stimulus **a** and Stimulus **b** are non-overlapping in the sense that their non-zero values are at different locations (at position 1 in Stimulus **a** and in position 3 in Stimulus **b**) and when added both stimuli are represented accurately in Stimulus **a + b**.

Both Stimulus **a** and Stimulus **b** have an amplitude sum of 2 (see above) and a norm of 1 (again, see above). Yet, when they are combined the amplitude sum of the combined stimulus remains 2 and the norm of the combined stimulus becomes $\sqrt{2}$. Thus, in both cases is there evidence for interference. In the case of the amplitude sums we have $2 + 2 = 4$ for the amplitude sum of Stimulus **a** plus the amplitude sum of Stimulus **b** which is twice that of the combined stimulus which is also 2 and which gives a Relative Amplitude Sum = 0.5. In the case of the norms we have that the Relative Norm is $\sqrt{2}/(1 + 1) = \sqrt{2}/2 \approx 0.71$ which also provides evidence for interference. In the case of Haar Wavelets we have that both Stimulus **a** and Stimulus

b have sums of the absolutes of the coefficients of $1 + 1/\sqrt{2} \approx 1.71$ but that the sum for the combined

Fig. 3

Fig. 3. Stimulus **a** = {1, 0, 0, 0} and Stimulus **b** = {0, 0, 1, 0} and their sum Stimulus **a + b** = {1, 0, 1, 0}. As can be seen, both Stimulus **a** and **b** are fully preserved in Signal **a + b**. The amplitude sum of Signal **a** and Signal **b** is both 2, yet the amplitude sum of Stimulus **a + b** is also 2. Thus, the Relative Amplitude Sum is 0.5 (i.e., $2/(2 + 2) = 0.5$). Therefore, the full amplitudes of both Stimulus **a** and Stimulus **b** cannot both be contained in the amplitudes of Stimulus **a + b**. The figure also demonstrates that stimuli do not need to overlap (in space or in time) in order to interfere (see also Fig. 4).

stimulus (**a + b**) is $1 + \sqrt{2} \approx 2.41$ which is smaller than $(1 + 1/\sqrt{2}) + (1 + 1/\sqrt{2}) \approx 3.41$. We have, therefore, what may seem paradoxical, namely that while both Stimulus **a** and Stimulus **b** are fully and accurately represented in Stimulus (**a + b**) their stimulus powers are reduced. This, it would seem, has consequences for what we mean by the term "stimulus". For instance, can we speak of the "same stimulus" when the physical image is the same but the stimulus power is not?

Overlap. A further lesson to be drawn from Fig. 3 is that in order for there to be interference between two stimuli the stimuli need not overlap in the space/time domain. What is required is that their Fourier spectra

overlap. (It is also required that the phase spectra differ.) This is illustrated in Fig. 4 in which is shown two non-overlapping Gabor functions having carriers with identical frequency. The Relative Amplitude Sum

Fig. 4

Fig. 4. Interference between non-overlapping of Gabor functions. (A) Two non-overlapping Gabor functions with carriers having the same frequency. The Relative Amplitude Sum in this case is 0.63. In other words, a considerable amount of interference is present. (B) Two non-overlapping Gabor functions in which the frequency of the carrier of the one on the right is twice that of the one on the left. The Relative Amplitude Sum in this case is 0.99. That is, there is essentially no interference. (C) Amplitude spectra of the two functions in A. One spectrum that is shown with a dashed line has been displaced slightly in the vertical direction in order to separate the two functions. (D) The amplitude spectra of the two functions in B. The spectrum of the function with the higher frequency carrier is shown with a dashed line.

in this case is 0.63 which entails the existence of a fair amount of interference. In Fig. 4B is shown two non-overlapping Gabor functions in which the carrier of the function on the right has twice the frequency of the one on the left. The Relative Amplitude Sum in this case is 0.99. That is, there is essentially no interference. In Fig. 4C and 4D are shown the amplitude spectra of the Gabor functions in Fig. 4A and 4B, respectively. As ought to be apparent (and as was to be expected), the two functions in Fig. 4A have identical amplitude spectra (Fig. 4C) whereas the two functions in Fig. 4B have essentially non-overlapping

spectra (Fig. 4D). This illustrates the fact that in order for interference to occur the amplitude spectra need to overlap. That is, for two stimuli to interfere their amplitude spectra need to have at least one common component with amplitude larger than zero in both spectra (and for this component to have different phase values in the two spectra). In regard to the space/time domain locations what is required for there to be interference between two stimuli is that both stimuli be located within the space/time region from which the stimulation is being integrated.

Average luminance. When adding luminance uniformly to a stimulus this alters the dc level in the Fourier spectrum but leaves the other components unchanged. Thus, adding luminance causes the sum of amplitudes to change. This may influence the Relative Amplitude Sum. Consider the hypothetical case where two stimuli each have an amplitude sum of 1.0 and the combined stimulus has an amplitude sum of 1.5. This would give a Relative Amplitude Sum of $1.5/(1 + 1) = 1.5/2 = 0.75$. Were we to add a dc value of 1 to each of the stimuli we would get a Relative Amplitude Sum of $(1.5+1)/((1+1) + (1+1)) = 2.5/4 = 0.625$. [FOOTNOTE 7] Were we to add a dc value of 2 instead of 1 we would get $(1.5+2)/((1+2) + (1+2)) = 3.5/6 = 0.583$. Thus, the Relative Amplitude Sum will depend upon the dc level.

Complicating this issue further is the fact that the dc value does not only depend on the mean luminance but also upon the area over which the stimuli are being sampled. Thus, for a stimulus in which the mean luminance is above 0 including more space into the analyses will tend to increase the dc value.

In the examples above involving Stimuli **a** and **b** (illustrated in Fig. 3) interference was calculated without setting the dc value or the mean luminance to zero. Setting the dc value to zero we get the Relative Amplitude Sum to be 0.33 for $\sum |\mathbf{A} + \mathbf{B}|/(\sum |\mathbf{A}| + \sum |\mathbf{B}|)$ (as opposed to 0.5 for spectra with the dc value intact). (For the Relative Norm based on Fourier spectra with the dc value set to 0 we get a value of ≈ 0.58 . As opposed to ≈ 0.71 for Relative Norm with dc value unchanged). In the case of wavelets the first basis function (i.e., the first row-vector in Ψ) is responsible for the first coefficient in the Wavelet Transform which corresponds to the dc value in a Fourier spectrum. If we set this coefficient to 0 we get $\sum |\mathbf{c}_{a+b}|/(\sum |\mathbf{c}_a| + \sum |\mathbf{c}_b|) \approx 0.58$ (as opposed to ≈ 0.71 for when the first coefficient is left unaltered in the Wavelet Transform).

This may not represent a major issue when the stimuli do not have large dc values, i.e., do not have large average luminance. However, for accurate quantitative assessments of interference this issue will need to be taken into account. Generally, including dc values will tend to push the Relative Amplitude Sum towards 0.5 which in most cases will represent an overestimation of the interference. This problem can be avoided

by either setting the dc value equal to zero or by subtracting out the mean luminance before carrying out any analyses.

On ascribing interference to a particular stimulus. In tests of visual masking and visual crowding the task facing the subject is generally linked to the target stimulus. A relevant question then becomes whether or not one can determine the effect of interference specifically upon this stimulus. Since interference can be detected by using the norms of the space/time domain stimuli it may be thought that it would be possible to determine the interference effect upon a target stimulus based on the norms. The problem in this case is that the norm is a non-linear operator: Generally $\|\mathbf{a}\| + \|\mathbf{b}\| \neq \|\mathbf{a}+\mathbf{b}\|$. (It is in this regard similar to the transform of a signal into its amplitude spectrum, but is unlike the transform of a signal into its Fourier spectrum which is a linear operation.)

To give a concrete example, consider the space domain stimuli $\mathbf{a} = \{1, 0, 0, 0\}$ and $\mathbf{b} = \{0, 0, 1, 0\}$. Since the norm is the square root of the inner product of the stimulus with itself we get $\|\mathbf{a}\| = \sqrt{\sum_i a_i a_i} = \sqrt{1+0+0+0} = \sqrt{1} = 1$. Similarly for $\mathbf{b}$ we get $\|\mathbf{b}\| = \sqrt{0+0+1+0} = \sqrt{1} = 1$. Thus, $\|\mathbf{a}\| + \|\mathbf{b}\| = 1 + 1 = 2$. For the space/time domain sum of $\mathbf{a}$ and $\mathbf{b}$ we get $\mathbf{a} + \mathbf{b} = \{1, 0, 1, 0\}$. The norm of which is $\sqrt{1+0+1+0} = \sqrt{2} \approx 1.41$. Consequently, we have $\|\mathbf{a}+\mathbf{b}\| = 1.41$ whereas $\|\mathbf{a}\| + \|\mathbf{b}\| = 2$. Therefore, in this case the norm of the sum is not the sum of the norms (just like the amplitude of the sum of two functions is generally not the sum of their amplitudes). The critical question now becomes: Could one based on this in some way determine the interference specifically in regard to each of the two stimuli? Since $\|\mathbf{a}+\mathbf{b}\| \approx 1.41$ this means that the combined stimulus power of $\mathbf{a}$ and $\mathbf{b}$ was reduced when these were combined. However, the stimulus $\mathbf{a}+\mathbf{b} = \{1, 0, 1, 0\}$ which means that both $\mathbf{a}$ and $\mathbf{b}$ are fully represented in the combined stimulus (cf. Fig. 3) but the sum of their norms is not. Were one to try to determine the stimulus power of $\mathbf{a}$ in the $\mathbf{a}+\mathbf{b}$ combination one might attempt to do this subtracting $\mathbf{b}$ from $\mathbf{a}+\mathbf{b}$. What we then would have get was $\|\mathbf{a}+\mathbf{b}-\mathbf{b}\| = \|\mathbf{a}\|$. This would therefore simply return us to the norm of $\mathbf{a}$ alone not to "a in the $\mathbf{a} + \mathbf{b}$ combination".

Another way to illustrate this issue is in terms of vector addition. The norm is the length of a vector. That is, $\|\mathbf{a}\|$ is the length of the vector $\mathbf{a}$ and $\|\mathbf{b}\|$ is the length of the vector $\mathbf{b}$. When adding vectors $\mathbf{a}$ and $\mathbf{b}$ this gives us the the vector $\mathbf{a}+\mathbf{b}$. The length of $\mathbf{a}+\mathbf{b} < \text{length of } \mathbf{a} \text{ plus the length of } \mathbf{b}$ unless the two vectors have the same direction. What we would have wanted to know is, in the case where the directions differ, and the length of $\mathbf{a}+\mathbf{b} < \text{the length of } \mathbf{a} \text{ plus the length of } \mathbf{b}$, how large is the contribution of $\mathbf{a}$

to $\mathbf{a}+\mathbf{b}$ and how large is the contribution of $\mathbf{b}$ to $\mathbf{a}+\mathbf{b}$. Unfortunately there is no simple answer to this question. In fact, it is not even clear that the question is meaningful.

Norm versus sum of energies. The norm of a vector, $\mathbf{d}$, is given as $\sqrt{\sum_i \mathbf{d}_i^2}$, where $\mathbf{d}_i$ denotes the i th element of $\mathbf{d}$. This means that the norm is the square root of the sum of the energies. This raises the question why one would not simply use the sum of the energies. This suggestion faces a problem in regard to the need for the measure of stimulus power to be doubled when the stimulus is added to itself. Such doubling holds for the sum of amplitudes and for norms (as was explained above) but not for the sum of energies. For the sum of the energies of the vector $\mathbf{d}$ we get $\sum_i \mathbf{d}_i^2 = \mathbf{d}_1^2 + \mathbf{d}_2^2 + \mathbf{d}_3^2 \dots \mathbf{d}_n^2$. Let us consider the simple example where only the first element has a non-zero value. In this case we get $\sum_i \mathbf{d}_i^2 = \mathbf{d}_1^2 + 0^2 + 0^2 \dots 0^2 = \mathbf{d}_1^2$ for the stimulus alone. For the case where the stimulus is added to itself we get $\sum_i (\mathbf{d}_i + \mathbf{d}_i)^2 = \sum_i (2\mathbf{d}_i)^2 = (2\mathbf{d}_1)^2$ which is $4\mathbf{d}_1^2$. This is four times the value for $\mathbf{d}$ by itself (i.e., four times $\mathbf{d}_1^2$) and not two times as we wanted. (In the case of the norm, on the other hand, we get $\|\mathbf{2 a}\| = \sqrt{(2\mathbf{d}_1)^2} = \sqrt{4\mathbf{d}_1^2} = 2\mathbf{d}_1$, which is precisely what we wanted.) Thus, it would seem the sum of energies would not be an inappropriate measure of stimulus power. Also, using the energies would not eliminate the issue of interference. For instance, $\sin(x)$ and $\sin(x - \pi)$ which for a non-zero interval of x both have energies larger than zero, yet the combined function, $\sin(x) + \sin(x - \pi)$, has zero energies over the same interval.

Summing of luminance. For the sake of completeness it may be useful to consider the case of adding only dc components. Consider the signals $\mathbf{a} = \{1, 0, 0, 0\}$, $\mathbf{b} = \{0, 0, 1, 0\}$, $\mathbf{c} = \{0, 1, 0, 0\}$, and $\mathbf{d} = \{0, 0, 0, 1\}$. Adding these gives $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = \{1, 1, 1, 1\}$. The Fourier spectrum of this signal is $\{2, 0, 0, 0\}$ giving an amplitude sum of 2. The sums of amplitudes in $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$ are all 2. Thus, the sum of amplitudes in all these stimuli would be 8 which is clearly different from the sum of amplitudes in $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}$ together which is 2. On the other hand, since the dc components in $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$ are all 0.5 these would add to 2 which equals the dc component of $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}$ combined. Although this measure of stimulus power will not give interference it would be a highly unsatisfactory measure since it would ignore any variations (be they spatial or temporal) in the stimuli. In which case $\{1, 1, 1, 1\}$ would have the same stimulus power as $\{1, 0, 2, 1\}$. In terms of amplitude sums the two signals are clearly different with the former having an amplitude sum of 2 whereas the latter has an amplitude sum of $3 + \sqrt{2} \approx 4.41$. (In the case of the norms we get $\sqrt{4} = 2$ and $\sqrt{6} = 2.45$ for $\{1, 1, 1, 1\}$ and $\{1, 0, 2, 1\}$, respectively.)

DISCUSSION

The present analyses have examined several candidate measures of stimulus power. It was found that the sum of Fourier coefficients, the sum of energies, and the sum of wavelet coefficients all face problems. On the other hand the amplitude sum, the norm, and the sum of the absolutes of wavelet coefficients all provide plausible measures. This should not be taken to mean that there cannot be other reasonable measures of stimulus power.

In the case of the amplitude sum, the norm, and the sum of the absolutes of wavelet coefficients all these three measures are susceptible to interference which manifests itself in that the stimulus power of a combined stimulus can be smaller than the sum of the stimulus powers in the individual stimuli (i.e., it is possible for $T(\mathbf{a} + \mathbf{b}) < T\mathbf{a} + T\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ denote two stimuli and T represents the stimulus power operator). In other words, the stimulus power of the sum may be smaller than the sum of the stimulus powers. When this occurs the full stimulus powers of the individual stimuli cannot all be contained in the stimulus power of the combined stimulus. Consequently, the stimulus powers of the individual stimuli will be reduced when the stimuli are combined. Interference is a consequence of the adding/combining of stimuli or stimulus elements. It is not the result of the operation of some "mechanism".

Important to note in this connection is the fact that for all the three measures of stimulus power a stimulus may have the same physical appearance but still have reduced stimulus power depending on the context. This raises the question of what we are to mean by "stimulus". For instance, in the case of visual crowding if we were to place a printed character between two other printed characters should we say it constituted "the same stimulus" even when its stimulus power is reduced?

The interference effects described here *are in the stimuli*. That is, the interference takes place irrespective of whether or not the stimuli are being seen or activate any visual system. Interference is a property of the stimuli and, like other stimulus properties, it persists when the subject closes his/her eyes. Since the interference is a stimulus property it is not something which can be tested by psychophysical experiments. (In being a stimulus property interference is on par with, say, the spatial extension of the stimulus. One would presumably not carry out an psychophysical experiment in order to determine if the stimulus subtends 25 degrees of visual angle.) This, however, should not be taken to mean that interference is not in a position to influence psychophysical results. On the contrary, like any other stimulus property we have to expect interference to be able to affect psychophysical performance.

It is clearly possible to create stimuli in which interference will not be generated. For instance as was

shown in Fig. 4, stimuli with non-overlapping amplitude spectra will not interfere. For this reason, also having stimuli of different orientations may avoid or reduce interference (Skottun, 2018b). It is in this connection of interest to note that visual masking effects are generally spatial frequency specific and require the spatial frequencies of the stimuli to be similar (see pages 193-195 of De Valois & De Valois, 1990, for a brief overview). Also relevant seems to be the finding of Ishikawa et al. (2006) that metacontrast masking requires target and masking stimuli to have similar orientations. It therefore appears that visual masking effects are found under conditions which are conducive to interference.

A number of authors (Breitmeyer & Ogmen, 2000; Hermens et al., 2008; Manassi et al., 2016; Shin et al., 2017) have described the effect upon psychophysical performance of adding a second stimulus as the result of inhibitory interactions in the visual system. The present account of interference does not exclude such possibilities. In fact, it does not exclude the possibility of any interactions in the visual system. Interference in the stimuli and interactions in the visual system are not mutually exclusive. What the present account does is to provide a description of visual stimuli in the cases when these involve combinations of several stimulus elements. It represents an attempt at understanding the kinds of stimuli used in tests of visual masking and visual crowding. Therefore, it represents a starting point for the forming of theories of these phenomena.

Interference has here been discussed in terms of how it may reduce the stimulus power of stimulus elements. There is evidence to indicate that interference, by selectively reducing stimulus power, may also have the effect of shifting the distribution of stimulus power along some stimulus dimension (e.g., orientation). This, it has been suggested, may alter the appearance of stimuli in other ways than by merely lower their stimulus power. For instance, it has been proposed that interference may cause the orientational distribution of the stimulus power to be shifted so as to induce illusory tilt (Skottun, 2018a).

A number of authors have used the term "interference" within the framework of visual masking to characterize the effect of a masking stimulus upon a target (e.g., Di Lollo et al., 1974; Legge & Foley 1980; Enns, 2004; Louie et al., 2007). In these cases the term is used in a non-technical meaning to simply indicate that the presence of the masking stimulus affects the perception of the target. It is not clear if this "interference" is supposed to be in the stimuli, in the nervous system, if it is a matter of perception, or of neurophysiology. The present account, by contrast, uses the term "interference" in a highly specific sense, namely to denote the destructive interaction between stimulus elements. This destructive effect takes place in the stimuli and can be understood most easily in terms of differences in the phase spectra of the elements.

Stimulus power has here been examined in relation to interference between stimuli or stimulus elements. This should not be taken to mean that stimulus power is only relevant in this context. It is relevant when any stimulus manipulation is carried out. For instance, in the case of studying the effect of filtering stimuli it is important that the filtering does not alter the stimulus power. Also, in the case of visual discrimination of two stimuli differing along some stimulus dimension it is important that the stimuli have the same stimulus power otherwise it might be that the discrimination is based on the stimulus power rather than the intended dimension. The importance of stimulus power in connection with discrimination may not merely consist in the possibility that stimuli are discriminated directly on the basis of the difference in stimulus power it may also be that the stimuli with higher stimulus power contains more information upon which to carry out the discrimination. For instance, it has been documented that the appearance of images is determined more by their phase spectra than by their amplitude spectra (Oppenheim & Lim, 1981; Piotrowski & Campbell, 1982). It would seem that a stimulus with higher stimulus power could provide more accurate phase information than a stimulus with smaller stimulus power. This could either be because high stimulus power has amplitudes at more components or because the amplitudes are larger. In either case, the larger stimulus power might carry more information. A further possibility to be considered, would be that stimuli with smaller stimulus power may be more susceptible to noise. If the noise contains a fixed component this component could be larger relative to a smaller stimulus power than to a large one.

It would appear, therefore, that "stimulus power" needs to be treated as a fundamental aspect of visual stimuli. In the case of masking, crowding, and visual search it may be suggested that it may be possible to provide a measure of stimulus power with which combinations of stimuli do not lead to interference. However, this would not eliminate the facts that amplitude sums, norms, and the sum of the absolutes of wavelet coefficients are susceptible to interference. It would seem, therefore, that in order to interpret results from experiments on visual masking, crowding, and visual search one will need to take account of the possibility of interference.

FOOTNOTES

1. An example of how physiological results can be understood by understanding the stimuli is provided by Skottun et al. (1994). This paper demonstrates the importance of selecting the appropriate basis functions for describing the stimuli.
2. The Fourier Transform is not usually expressed as a matrix. Rather, it is given as $\int f(x)e^{ikx}$ where

$f(x)$ is the signal to be transformed, $i = \sqrt{-1}$, and k is a constant which determines the frequency. For discrete signals the Fourier Transform becomes $\sum f[x]e^{ikx}$, with square brackets denoting discrete sampling. The basis functions are e^{ikx} for $k = 0, 1, 2, \dots$, which for a signal with a length of four elements has to be evaluated at $x = 0, \pi/2, \pi$, and $3\pi/2$ with $k = 0, 1, 2, 3$. For $k = 0$ this gives us $\{1, 1, 1, 1\}$, for $k = 1$ we get $\{1, i, -1, -i\}$, for $k = 2$ we get $\{1, -1, 1, -1\}$, and for $k = 3$ this gives $\{1, -i, -1, i\}$. These functions then need to be normalized (i.e. made to have unit norm). This is achieved by multiplying them with $1/\sqrt{n}$ where n is the length of the signal ($n = 4$ in the present case) making $1/\sqrt{n} = 1/2$. These four vectors constitute the basis functions for the Fourier Transform and are given as row-vectors in the matrix F . (Note also that the four basis functions are orthogonal. For explanation of orthogonality of two functions see FOOTNOTE 5.)

3. In general each element in a Fourier spectrum is given by a complex number, z , of the form $z = c + d i$ where c and d are real numbers and $i = \sqrt{-1}$. The amplitude of z , $|z|$, is given by the positive root of $\sqrt{c^2 + d^2}$.

4. It may perhaps be suggested that one ought to use the sum of the Fourier spectra (rather than the sum of the amplitude spectra) as measure of stimulus power. In the case of $\mathbf{a} = \{1, 0, 0, 0\}$ and $\mathbf{b} = \{0, 0, 1, 0\}$ with Fourier Transforms of $\mathbf{A} = \{0.5, 0.5, 0.5, 0.5\}$ and $\mathbf{B} = \{0.5, -0.5, 0.5, -0.5\}$, respectively. We would then have $\sum \mathbf{A} = 2$ and $\sum \mathbf{B} = 0$. This is clearly unreasonable given that $\mathbf{a}$ and $\mathbf{b}$ are identical stimuli with the only exception that one is shifted relative to the other. It seems we would wish the stimulus power of a given stimulus to remain unchanged when the stimulus is placed in a different position. (On the other hand, $\sum |\mathbf{A}| = \sum |\mathbf{B}| = 2$.)

5. "Orthonormal" means that the functions are orthogonal and have a norm of 1. In order for two functions f_i and f_j to be orthogonal it is required that $\int f_i(x)f_j(x) = 0$ when $i \neq j$. For them to be normalized with norm of 1 we need $\sqrt{\int f_i(x)f_i(x)} = 1$ when $i = j$ (which entails that $\int f_i(x)f_i(x) = 1$). In the case of discretely sampled functions we would express this as $\sum f_i[x]f_j[x] = 0$ when $i \neq j$ and for $\sqrt{\sum f_i[x]f_i[x]} = 1$ when $i = j$, with square brackets indicating discrete sampling. (For complex valued functions the inner products become $\int f_i(x)\overline{f_j(x)}$ and $\sum f_i[x]\overline{f_j[x]}$.)

6. Some may object that Haar Wavelets are physiologically implausible or unrealistic. In that connection it

should be pointed out that they are not intended here to simulate a visual system but are here only made use of to analyze stimuli.

7. It may be objected that this does not describe the summing of stimuli since adding 1 to the dc level in Stimulus **a** and to that of Stimulus **b** would add 2 to Stimulus **a + b**. However, in actual vision experiments the average luminance is sought to be held constant when a masking stimulus is added to a target. Thus, the dc level in the combined stimulus would typically be (approximately) equal to that of Stimulus **a** and Stimulus **b**.

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