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**Reasons to avoid z-scores
in graphs displaying profile or group differences**

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Abstract

This article summarizes reasons why z-standardized scores should be avoided in graphs that display profiles or mean score differences between groups. It provides examples showing why and how the use of z-scores in such group comparisons can be misleading and concludes with proposing alternative methods that avoid such risks of misinterpretation.

The reasons why z-standardized scores should be avoided when displaying differences between groups of profiles are:

- (1) The ratio of the difference between two groups is distorted in z-scores.
- (2) The ratio of the difference between two variables is distorted in z-scores.
- (3) Information about item endorsement and item rejection is lost.
- (4) The psychological meaning a given z-score cannot be compared across samples and variables.
- (5) People may end up in a group they do not belong to if z-scores are used to assign individuals to groups.
- (6) The group size and group frequency may be affected if z-scores instead of raw scores are used to assign individuals to groups.
- (7) Group differences in further outcome variables can change if z-scores instead of raw scores are used to assign individuals to groups.
- (8) Alternative transformations perform better than z-standardization in cluster analyses.

Alternatives to using z-scores in graphs displaying profiles and group differences are using raw scores or using scale transformations that do not lead to the aforementioned problems, such as the POMS transformation. The latter can be combined with the formula used to calculate the item difficulty to relate to larger audiences, or with outlier removal techniques to make it less susceptible to the influence of outliers.

Keywords. z-standardization, profiles, group comparisons, cluster analysis, misinterpretations.

Reasons to avoid z-scores in graphs displaying differences between profiles or groups

This article summarizes reasons why z-standardized scores should be avoided in graphs displaying profiles or mean score differences between groups. It provides examples showing why using z-scores in group comparisons can be misleading, and concludes with proposing alternative methods that avoid such risks of misinterpretation.

1. What is z-standardization and why is it used to compare profiles and groups?

Z-standardization is used to transform raw scores into z-scores. A raw score is z-transformed by subtracting the mean score of the sample from the raw score, and dividing the result by the standard deviation of the sample. The resulting distribution of z-scores has a mean of 0 and a standard deviation of 1.

Oftentimes, researchers compare the z-scores of several variables within and across unobserved groups of individuals (clusters or profiles resulting from cluster or latent profile analyses; e.g., Kahn et al., 2013). In other cases, researchers compare observed groups, such as males and females, in terms of their average z-scores in relevant outcome variables, such as gender differences in anxiety (e.g., Dere et al., 2014).

One function of z-scores in group and profile comparisons is to bring different variables to the same metric, to make them comparable. That can be useful when inspecting profiles of variables measured on different response scales. A second reason why z-scores are used in group and profile comparisons is the researcher's wish to emphasize differences between groups, which often appear larger in z-scores than in raw scores. Third, standardized scores are used to optimize the process of identifying unobserved groups in cluster analyses. For instance, the statistic program SPSS automatically z-standardized continuous variables before performing a twostep cluster analyses.

These approaches can be problematic and even achieve the opposite of what they intended for reasons summarized below.

2. Reasons to avoid z-scores in group or profile comparisons

(1) The ratio of the difference between two groups is distorted in z-scores

In group comparisons, it often matters how large a difference between two groups is. It may affect a study's conclusion whether individuals in group A have on average twice as many versus four times as many children as group B, or if country A reported twice as many versus twenty times as many deaths as country B during a pandemic.

Z-scores distort these relative ratios in the comparisons between groups, compared to raw scores. For example, compare the graphs for Italy and the UK in Figure 1, which show the reported deaths due to Covid-19 infections for three countries on April 12th (Johns Hopkins University, 2020). The left graph shows raw scores, the right graph the corresponding z-scores, based on the mean and standard deviation of the three countries included in the graph.

Figure 1: Reported deaths due to Covid-19 infections by country on April 12th, 8:29 p.m. (based on data from the Johns Hopkins University, 2020)

The raw scores suggest that Italy reported roughly twice (1.87 times) as many deaths as the UK due to Covid-19 at that date, whereas the z-scores suggest that Italy reported 23.78 times as many deaths as the UK.

This distortion of the size of the difference between two groups can affect the effect size.

The effect size d for instance would differ if calculated with the mean score and standard deviation for raw scores, compared to d being calculated with the mean and standard deviation of the z-scores. Maybe not many researchers would opt to calculate d using z-scores, but every researcher should have the opportunity to calculate an effect size for a group difference based on the reported results, and if only z-scores were reported, then readers cannot calculate a realistic effect size, due to the distortions described above.

Despite such distortions in the ratios of differences between groups, some researchers purposefully apply z-scores to emphasize differences between two groups, which may look smaller or imperceptible if raw scores are displayed instead. As a rule of thumb, differences between groups tend to look larger in z-scores than in raw scores, because using z-scores to display group differences resembles the use of a truncated y-axis, which we are taught to avoid. Readers should always be given the information about the theoretically possible and empirically observed ranges of a distribution, to decide for themselves whether they consider the distance between two observations, two groups, or two profiles, large or small. Graphs displaying raw scores typically display the information about the range and the theoretically possible minimum and maximum of a distribution on the Y-axis, unless the Y-axis is truncated, whereas z-scores obfuscate that information.

(2) The ratio of the difference between two variables is distorted in z-scores.

Z-scores distort the ratios between the scores of two variables, compared to raw scores. This implies that the form of profiles may change dramatically when z-scores instead of raw scores are used to display profiles. It also implies that it is not possible to determine whether a score in one variable was originally larger or smaller than the score of another variable in a profile. That can have dramatic consequences for the interpretation and conclusion of a study. It is, for example, possible that z-scores suggest that variable A was higher than variable B in a profile or observed

group, while on the contrary the raw scores of variable A were lower than the raw scores of variable B.

For an example, consider the research on the motivational construct of passion, which often examines harmonious passion (HP) and obsessive passion (OP), which are measured with the same response scale (1 = *do not agree at all* to 7 = *completely agree*; Vallerand et al., 2003). When individuals' profiles of HP and OP were examined, the raw scores revealed that all profiles showed higher HP than OP, while the z-scores had suggested that one group showed higher OP than HP (Moeller, Keiner, & Grassinger, 2015).

(3) Information about item endorsement is lost

Many studies determine the agreement or disagreement of participants with items by using bound scale (e.g., 0 = *don't agree at all* to 7 = *totally agree*, or 0 = *does not apply at all* to 10 = *fully applies*). In such scales, the scale midpoint is often a meaningful threshold at which the item rejection (minimal score to scale midpoint) turns into item endorsement (scale midpoint to maximal score). Whether participants rather agree or disagree, or whether their score exceeds a theoretically relevant cut-off is often a crucial information. This information, however, is lost in z-scores, in which the empirical distribution instead of the original response scale becomes the frame of reference. Consequently, a z-score can appear to be "high" (e.g., above the mean score of $z = 0$), while representing an item rejection. Whether a z-score represents an item endorsement or rejection can differ for the same variable across samples, or for the same sample between variables measured on the same response scale.

(4) The psychological meaning of z-scores depends on the sample's and variable's distribution

Since z-scores depend on the distribution of the variable in the sample, the same z-score of a variable can have an entirely different psychological meaning in one sample versus another. For example, the average z-score of 0 of an item assessing anxiety likely indicates a normal level of anxiety in a representative sample but a clinically relevant high level of anxiety in a population of

anxiety disorder patients in a closed psychiatry ward. Likewise, the same z-score of two variables measured on the same response scale in the same sample can have two entirely different psychological meanings, because each variable has its own frame of reference for the z-standardization (its own mean and standard deviation).

These sample- and variable-specific characteristics of z-scores make it very difficult to determine the psychological meaning of z-scores in graphs displaying multiple variables' z-scores in one or multiple groups, and in comparisons of results across samples.

(5) People may end up in a group they do not belong to if z-scores are used to assign individuals to groups.

Some studies not only compare z-scores across variables or groups, but use z-scores to classify individuals into distinct groups before comparing these groups' z-scores in these or other variables. One risk resulting from that approach is that people may end up in a group they do not belong to according to the applied definition. For example, some studies on passion use z-scores of HP and OP to classify individuals into a “mainly obsessively passionate” group ($z_{OP} > z_{HP}$) and a “mainly harmoniously passionate” group ($z_{OP} < z_{HP}$; e.g., Philippe, Vallerand, & Lavigne, 2009). However, because of the different item difficulties of HP and OP, most individuals classified as “mainly obsessively passionate” actually show higher HP than OP, which conflicts with the definition of the group.

(6) The group size and group frequency may be affected if z-scores instead of raw scores are used to assign individuals to groups.

Imagine we stick with the aforementioned definition of the “mainly obsessive passionate” group as individuals with higher OP than HP, and use raw scores instead of the problematic z-scores to compare HP with OP. With that approach, the “mainly obsessive passionate” individuals are extremely rare, because almost no participant shows higher raw OP than HP (Moeller et al., 2015). Likewise, due to possible misclassification problems described below under point 8, the size

of clusters in cluster analyses may change when the variables are z-standardized before performing a cluster analyses on them.

(7) Group differences in further outcome variables can change if z-scores instead of raw scores are used to assign individuals to groups.

Since the group composition (person-group assignment) may change if raw scores instead of z-scores are used to define groups (see the example of HP and OP above), a logical consequence is that group differences in outcome variables may look very different.

(8) Other scale transformations perform better than z-standardization in cluster analyses.

Although many studies and statistics programs, such as SPSS, z-standardize continuous variables by default before using them in cluster analyses, other standardization techniques using the range (max – min) instead of the standard deviation as divisor (see equation 1 below) were found to perform much better in the cluster identification in simulation studies (e.g., Milligan & Cooper, 1988; Schaffer & Green, 2010; Everitt, Landau, & Leese, 2001). In the cited simulation studies, the true group membership is typically known, and different standardization techniques were compared in terms of their performance in identifying and distinguishing these known groups as clusters. Based on the finding that the z-standardized variables lead to inefficient cluster classifications, we can deduct that the clusters identified with z-standardized scores may differ in size and composition from the true groups in the population that the cluster analyses aims to identify. Consequently, all problems described above under the points 5-7 may apply automatically, no matter whether the profiles are displayed in graphs using the raw scores or the z-scores. If the differences between the clusters were displayed in z-scores, then additionally the points 1-4 may apply.

3. Alternatives to standardization in group comparisons

As alternatives to z-standardization, graphs displaying profiles and groups should use raw scores or alternative scale transformations that do not change the ratios between two variables or two (observed or unobserved) groups.

Raw scores have the advantage that they are easy to understand. They can easily be compared across samples and studies if the same measures and response scales are used. To understand their psychological meaning, readers do not have to translate the reported scores into the frame of reference that participants saw when responding to a question. Raw scores indicate whether a variable was endorsed by a participant, or whether the average score of a group or variable was above a relevant threshold (e.g., clinical cut-off). Disadvantages of raw scores are that they make it difficult to compare variables that were measured with different response scales, or with unbound response scales.

An alternative scale transformation not changing the ratios in scores between variables or groups is a procedure sometimes called the Proportion of Maximum Scaling Transformation (POMS, see Cohen, Cohen, Aiken, & West, 1999; Gower, 1971; Sneath & Sokal, 1973; Little, 2013). It is a linear scale transformation bringing all variables to a range from 0 to 1 with the following formula:

$$\text{POMS-transformed score} = (\text{observed score} - \text{minimum}) / (\text{maximum} - \text{minimum}) \quad [\text{Equation 1}]$$

POMS-transformed scores have the advantage that they provide one universal metric (from 0 to 1), so that scores can be compared across variables and samples, without any of the problems affecting z-scores mentioned above. The ratios in the differences between groups and between variables are unaffected by the transformation.

A disadvantage of the POMS transformation is the comparably larger effect of outliers on the min and max scores used in the POMS transformation, compared to the relatively smaller outlier

effect on the mean and standard deviation used in z-standardization. This is likely to affect unbound scales (e.g., number of sex partners, income) stronger, because only for those the POMS formula uses the empirically observed minimum and maximum, whereas for bound scales (e.g., 1 = *not at all* to 5 = *very much*), the pre-defined min and max values are used so that the difference between outliers and non-outliers tends to be smaller than for unbound scales, depending on the phenomenon being assessed and the response options used. A way to minimize the influence of outliers on the POMS transformation for unbound scales is using more robust estimators for the sample's min and max values, e.g., by removing outliers by trimming or winsorizing (see e.g., Wilcox, 2010).

In sum, standardization techniques using the range, such as the one described above, avoid many of the problems that affect z-scores. Since there are many different methods available for standardization (or normalization, as it is often called), it is not entirely clear why the particularly problematic z-standardization is so frequently used in the Psychological literature. Further problems and risks of misinterpretations occur when z-standardization is used on longitudinal data (see e.g., Moeller, 2015), and those problems can come in combination with the ones described here, if longitudinal data and group or profile comparisons are combined. It seems that there may be few good reasons to prefer z-standardization to standardization methods using the range or raw scores in the comparison of observed or unobserved groups.

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