Response time models separate single- and dual-process accounts of memory-based
decisions

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Abstract

Human decisions often deviate from economic rationality and are influenced by cognitive biases. One such bias is the *memory bias* according to which people prefer choice options they have a better memory of—even when the options’ utilities are comparatively low. Although this phenomenon is well supported empirically, its cognitive foundation remains elusive. Here we test two conceivable computational accounts of the memory bias against each other. On the one hand, a single-process account explains the memory bias by assuming a single biased evidence-accumulation process in favor of remembered options. On the contrary, a dual-process account posits that some decisions are driven by a purely memory-driven process and others by a utility-maximizing one. We show that both accounts are indistinguishable based on choices alone as they make similar predictions with respect to the memory bias. However, they make qualitatively different predictions about response times. We tested the qualitative and quantitative predictions of both accounts on behavioral data from a memory-based decision-making task. Our results show that a single-process account provides a better account of the data, both qualitatively and quantitatively. In addition to deepening our understanding of memory-based decision making, our study provides an example of how to rigorously compare single- versus dual-process models using empirical data and hierarchical Bayesian estimation methods.

Introduction

Many decisions in our daily lives, such as where to go on holiday or what to buy in a grocery store, rely at least in part on information from memory. Although the role of memory processes in judgements and decision making has been neglected for a long time, researchers have recently put more emphasis on the relation of these two domains (Shadlen & Shohamy, 2016; Weilbächer & Gluth, 2017). For example, recent studies focused on how memory, by playing a role in the evaluation process, can contribute to violations of standard economic theories of decision making (Weber & Johnson, 2006).

Gluth, Sommer, Rieskamp, and Büchel (2015) showed one of such violations in a decision-making task in which the subjective value (utility) of the options had to be recalled from memory. In this paradigm, individuals first learn to associate different snacks with specific locations. Afterward, they choose between two locations and therefore need to remember to which snacks the two locations were associated with (see Figure 1). The authors reported that participants tended to prefer remembered snacks over forgotten snacks, even when the subjective value of the former was lower than average (and thus more likely to be lower than the forgotten option’s subjective value). The authors referred to this effect as the *memory bias*. Using functional magnetic resonance imaging (fMRI), Gluth et al. (2015) further showed that this tendency was mediated by an increased effective connectivity from the hippocampus to the ventromedial prefrontal cortex. Since these areas are typically associated with memory and value-based decisions, respectively, the fMRI results supported the idea that memory processes exert a biasing influence on valuation and choice processes. Follow-up studies replicated the memory bias and found that it was partly driven by beliefs about the dependency of memory strength on utility (Mechera-Ostrovsky & Gluth, 2018), and that it exhibited typical characteristics of decisions under uncertainty (Weilbächer, Kraemer, & Gluth, 2020).

In the first study on the memory bias, Gluth et al. (2015) proposed a computational model that assumes people to choose between a remembered and a forgotten option by comparing the remembered option’s value against a reference value.

If this reference value is below the average snack value, the model predicts that people are more likely to choose remembered options, leading to the memory bias. Critically, this account assumes that all decisions between a remembered and a forgotten option result from the very same comparison-against-reference-value process. Therefore, it is a *single-process account* of the memory bias. Gluth et al. (2015)’s account of the memory bias is thus in stark contrast to dual-process theories (Evans, 2008; Kahneman & Frederick, 2002). Dual-process models assume that decisions are driven by two independent processes (so-called “Type 1” and “Type 2” processes) (Evans & Stanovich, 2013). Type 1 processes are described as intuitive processes that lead to relatively fast, automatic, and uncontrollable choices. Type 2 processes, on the other hand, are controlled, deliberate processes that lead to slower responses that are closer to normative decisions. Type 2 processes are thus viewed as rational processes (but see Oaksford and Hall, 2016). Such a *dual-process account* explains the memory bias as follows: In some cases, people make a decision based on a Type 1 process which leads them to choose the option they remember better—intuitively and independently of its subjective value. In other cases, they make a decision based on a Type 2 process. The Type 2 process implements an unbiased choice, based on a cognitively demanding decision process that takes the subjective value of the remembered option into account in a rational (i.e., utility-maximizing) way.

Both single- and dual-process accounts can produce the memory bias on choice (see below). Thus, we face a model-selection problem: Two models can account for the same behavioral phenomenon, but the assumed underlying cognitive processes are fundamentally different. To find out which model is more suitable to explain the memory bias, we consider an additional data dimension, namely response times (RTs). The consideration of RTs has a rich tradition in psychological research, since they contain information about the underlying cognitive processes (Luce, 1986). Additionally, RTs can aid model selection (Ballard & McClure, 2019; Gluth & Meiran, 2019; Wilson & Collins, 2019). Critically, we will show that although the single- and dual-process accounts can make similar predictions on choice behavior, they differ with

respect to RTs. Therefore, considering both dimensions, choices and RTs, aids to resolve the present model-selection problem.

Joint modeling of choices and RTs is often done in the framework of sequential-sampling (or evidence-accumulation) models (Bogacz, Brown, Moehlis, Holmes, & Cohen, 2006). A popular sequential-sampling model is the diffusion decision model (DDM) (Ratcliff, 1978; Ratcliff & Rouder, 1998). In a nutshell, the DDM describes a decision between two options as an accumulation of noisy, relative evidence over time (see also Figure 2). Evidence, in the current task, represents the information regarding an option's subjective value that is recalled from memory. The accumulation process ends when the relative evidence for one or the other option surpasses a certain threshold. At that point, the decision is made. The accumulation process can also be biased towards one of the two options already at the beginning of a trial. The rate of evidence accumulation is referred to as the drift-rate of the decision process. The higher the drift-rate towards a specific option, the more often that option is chosen and the faster the decision. On the other side, the threshold controls how cautiously decisions are made. The higher the threshold, the slower and more consistent the decisions are.

Over the years, the DDM has been applied successfully to processes of memory retrieval and perceptual decision making (Ratcliff, Smith, Brown, & Mckoon, 2016), but researchers also adapted it to value-based decisions (for recent overviews see Busmeyer, Gluth, Rieskamp, and Turner (2019) and Clithero (2018)). Importantly, Gluth et al. (2015) used the DDM as the basis of their single-process model of memory-based decisions. Within their model, the memory bias results from a biased reference value for forgotten options, which influences the drift-rate in the decision process. In recent years, novel evidence-accumulation models were developed to reflect ideas from dual-process theories of decision making (Alós-Ferrer, 2018; Diederich & Trueblood, 2018). In particular, the dual-process diffusion model by Alós-Ferrer provided a suitable account of a range of phenomena from judgment and decision-making tasks. The model assumes two independent decision processes: a Type 1-like heuristic process and a Type 2-like utility-maximizing process. Since both the single- and dual-process accounts

operate within the same framework (i.e., sequential sampling modeling), it is possible to rigorously evaluate which of the two accounts better explains the memory bias.

In the present study, we first derived choice and RT predictions of a single- and a dual-process DDM in the context of memory-based decisions. Next, we compared the different qualitative predictions of these models with the actual RT data from a comparatively large sample of participants which performed the remember-and-decide task (Mechera-Ostrovsky & Gluth, 2018). Finally, we performed quantitative model comparison within an hierarchical Bayesian parameter estimation framework. Both our qualitative and quantitative comparisons lend unequivocal support for the single-process account, thus strengthening our knowledge of the computational cognitive basis of value-based decisions from memory.

Methods

Participants

We analyzed data from a previously published study (Mechera-Ostrovsky & Gluth, 2018). In total, 96 participants (67 female, mean age = 23.5, age range: [19, 35]) took part in that study. Due to early termination and age restrictions, the data from 6 participants were not analyzed. The participants were students who took part in the study for course credits. The procedure was approved by the ethics committee of the University of Basel and all participants gave written informed consent.

Procedures

The experimental procedures of this experiment are described in Mechera-Ostrovsky and Gluth (2018). Here, we summarize the procedures relevant to the present research question. Participants were required to fast for four hours before the experiment started. They familiarized themselves with a set of 48 snacks. For each snack, they learned intuitive three-letter abbreviations (e.g., “sni” for “Snickers”) until they reached 100% accuracy. Participants’ subjective valuation of each snack was assessed on a continuous rating scale (Figure 1). This evaluation was incentivized by

selecting two snacks randomly at the end of the experiment and giving the higher-rated snack to the participant to eat.

After eliciting subjective valuations, participants faced the remember-and-decide task which consisted of the four periods encoding, distraction, decision, and recall (in that order). During encoding, participants saw empty squares at six different locations on the screen. One after another, each location was highlighted and a snack image appeared in the respective square. Participants had to associate and remember which snack was located in which square. During the distraction period, participants performed a 2-back working memory task that prevented them from rehearsing the information obtained in the encoding phase. During the crucial decision period, the six squares were presented again and, in each trial, two squares were highlighted. Participants picked one of the two snacks hidden behind the empty squares. Since the snack identity was not visually accessible, they had to retrieve it from memory to make an informed choice. During recall, the snack-location associations were probed to assess memory strength for each snack location.

Data preprocessing

First, we excluded trials that were unlikely to originate from a deliberate process: In particular, we excluded trials in which no choice was made (2.0%) and in which RTs were lower than 200 ms (1.4%). We then excluded trials that do not help to discriminate between single- and dual-process accounts: In particular, both accounts make the same predictions for behavior in trials in which both options are remembered or both options are forgotten (see Appendix A1 2 for more information). Therefore, we restricted our analyses to trials where one snack was remembered and the other was forgotten. This resulted in a total of 8031 trials (on average 89.2 trials per participant, $SD = 15.80$, range: [39, 118]).

Cognitive models

The diffusion decision model. In its original form, the DDM predicts choices and RTs using four parameters (Ratcliff, 1978). First, the boundary separation a

Figure 1. Experiment on memory-based decision making. During task 1, participants rated their subjective value for all snacks. Task 2 was a remember-and-decide task which comprised four phases: encoding, distraction, decision and recall. Here, we show one example trial per phase. During encoding, participants associated snacks with individual locations. The distraction phase contained a 2-back task with integers. In the decision phase, they retrieved snack-location associations from memory and made preferential choices between snacks. During recall, participants indicated which snack they associated with each location. Note that this is a simplified depiction of the experiment. For a full overview, see Mechera-Ostrovsky and Gluth (2018).

determines the (relative) amount of evidence required to terminate the deliberation process. This parameter is responsible for speed-accuracy tradeoffs. Second, the starting-point bias z determines the amount of relative evidence at the beginning of the deliberation process. This parameter reflects prior information or a bias in favor of one of the options. Third, the drift-rate ν determines the speed of evidence accumulation. In value-based decision making, the drift-rate is often directly proportional to the value difference between the two available options (e.g., Kragbich, Armel, and Rangel, 2010). The stronger the value difference, the higher the drift-rate, making choices both faster and more frequently in favor of the option the drift-rate is directed to. Analogously, small differences of values imply a low drift-rate and thus higher RTs and less frequent choices in the direction of the drift-rate. Finally, the non-decision time T_{er} absorbs every process that is not part of the deliberation process, such as the time it takes to execute the button press or to visually encode the stimuli.

Single-process account. According to Gluth et al. (2015)’s model of the memory bias, participants compare the subjective value of the remembered snack with a reference value. Thus, the drift-rate depends on a comparison process such that

$$\nu_{SPDM} = V_{rem} - \gamma, \quad (1)$$

where ν_{SPDM} is the drift-rate, V_{rem} is the subjective value of the remembered option and γ is the reference value. Here, a single evaluation process gives rise to the memory bias in every trial. This is why we refer to the model as a single-process diffusion model (SPDM). An example of a diffusion process is depicted in Figure 2A.

Importantly, Gluth et al. (2015) argued that—assuming that memory strength is independent of value—this reference value should be unbiased (i.e., equal to the mean of all options) in order to maximize utility. When estimating it as a free parameter, however, the reference value was found to be biased such that remembered options appeared to be more valuable, even if it was comparatively unattractive.

Dual-process account. To model the dual-process account, we adopted the dual-process diffusion model (DPDM) by Alós-Ferrer (2018). The DPDM assumes that people vary between two types of choice strategies across trials (see Figure 2B). In some

Figure 2. Diffusion Decision Models. **A** Diffusion process of the single-process account. Evidence for an option accumulates over time with a rate of ν_{SPDM} until the threshold (boundary) for one choice option (remembered or forgotten) is reached. The boundary separation depends on the parameter a and T_{er} depicts the non-decision time. **B** The dual-process account assumes two diffusion processes with different drift-rates ($\nu_{utility}$ and $\nu_{heuristic}$). A Bernoulli trial based on the parameter Δ selects either the utility or the heuristic diffusion process.

trials, people use a *utility process* which captures “computational-normative aspects of decision making” (p. 203). In other trials, a *heuristic process* favors “intuitive-affective attributes” (ibid) of a choice option with a relatively high drift-rate, leading to fast and more consistent responses. The selection of a process in a given trial is supposedly governed by the central executive. It selects the utility process with the probability Δ and the heuristic process with $1 - \Delta$. The drift-rate of the DPDM in a given trial is given by

$$\nu_{DPDM} = \begin{cases} \nu_{utility}, & \text{if } k = 1 \\ \nu_{heuristic}, & \text{if } k = 0 \end{cases} \quad (2)$$

where k is the outcome of a Bernoulli trial $\mathcal{B}(\Delta)$.

In the context of memory-based decisions in the remember-and-decide task, we propose that $\nu_{utility}$ reflects the utility-maximizing process in which V_{rem} is compared to the (unbiased) average snack value of all possible snacks V_{avg} . Thus, the cognitively

demanding utility process tends to select the option with the higher subjective utility without a bias towards the remembered option. If individuals do not follow the utility-maximizing process, they rely on a simple decision rule to make a choice. In the context of memory-based decisions, they can use recognition as a cue for value, much in line with the recognition heuristic in judgment tasks (Goldstein & Gigerenzer, 2002) (see also Discussion). In the diffusion-model framework, $\nu_{heuristic}$ reflects the drift-rate of the heuristic process that favors the choice of a snack, because it can be recalled correctly—independently of its subjective value.

Qualitative predictions

As mentioned above, the SPDM and DPDM can make very similar predictions with respect to choices. The SPDM assumes that the drift-rate depends on V_{rem} and γ (see Equation 1). The choice probability of a diffusion process relates to the drift-rate as

$$P(\nu, a, \sigma) = 1 - \frac{1}{1 + e^{\frac{2\nu a}{\sigma^2}}}, \quad (3)$$

where ν is the drift-rate, a is the boundary separation and σ is the noise of the drift (e.g., Bogacz et al., 2006). The probability of choosing the remembered option is a sigmoidal function that depends on $V_{rem} - \gamma$. If γ is positive, V_{rem} must be negative at the point where participants are indifferent between the remembered and forgotten option (i.e., $P(\text{choose remembered}) = .5$). Thus, the sigmoidal choice curve is shifted to the left, making it more likely to choose the remembered option, even if its value is comparatively unattractive (Figure 3B).

The DPDM is as a mixture of two processes, each with their own drift-rate. The drift-rate of the utility-based process depends on the difference of V_{rem} and V_{avg} . Such a choice curve is illustrated in Figure 3A as the blue dashed line. The heuristic choices in this example result from a high drift-rate that causes a preference in favor of the remembered snack. Moreover, the heuristic process selects the remembered snack without computing its actual value and is thus independent of V_{rem} (see Figure 3A, red dotted line). Since the dual-process account assumes that a strategy selection parameter Δ selects among the heuristic and utility process in a given trial, the final

Figure 3. Qualitative predictions. **A** Expected choice curves of the utility process (blue, dashed) and the heuristic process (red, dotted) as a function of the value of the remembered snack V_{rem} . **B** The weighted average of the two curves in **A** results in the expected choice curve of the DPDM (purple, dashed-dotted), which is depicted along with the expected curve for the SPDM (cyan, solid). **C** Expected RT curves for the utility and heuristic process. **D** Expected RT curves for the SPDM and DPDM.

choice curve over all trials would be equivalent to a weighted average of both curves (Figure 3B). Hence, both the SPDM and DPDM can produce choice curves which describe a shift of the indifference point and can mimic each other very closely.

Critically, however, the qualitative predictions of the SPDM and DPDM differ with respect to RTs. The expected RTs can be calculated as

$$\begin{aligned} RT &= T_{er} + DT, \\ DT(\nu, a, \sigma) &= \frac{a}{\nu} \tanh\left(\frac{\nu a}{\sigma^2}\right), \end{aligned} \tag{4}$$

where T_{er} is the non-decision time and DT is the mean decision time as a function of the diffusion parameters (Bogacz et al., 2006). The DDM typically predicts a U-shaped curve of expected RT as a function of the drift-rate. RT will be highest when the drift-rate is lowest (Clithero, 2018; Ratcliff & Rouder, 1998). For the utility process, this means that the drift-rate is lowest and expected RT are highest when $V_{rem} = V_{avg}$ (Figure 3C). Since the drift-rate of the heuristic process is relatively high, it produces short RTs. More importantly, the expected RT of the heuristic process are independent of V_{rem} . Taking the weighted average of these curves, the total expected RT of the DPDM is maximal as $V_{rem} = V_{avg}$. For the SPDM on the contrary, the lowest drift-rate is when $V_{rem} = \gamma$. Thus the SPDM predicts a shift in the RT curve to the left, while the DPDM expects the RT curve to be centered around V_{avg} . From these theoretical considerations, we can see that both models make different predictions with respect to the RT curve (Figure 3D). See Appendix A2 for an analytical derivation of RT predictions.

Testing qualitative predictions

As outlined in the previous section, single- and dual-process accounts make qualitatively different predictions with respect to response times. To test these general patterns that are independent of additional, more specific model assumptions, we fitted a quadratic function to the RT data of each participant:

$$RT(V_{rem}) = \beta_0 + \beta_1 \times (V_{rem} + \beta_2)^2 \tag{5}$$

In this regression model, β_2 is the parameter that shifts the indifference point of the remembered option V_{rem} . If β_2 equals zero, the curve is symmetrical around V_{avg} which matches the RT predictions of the DPDM. If β_2 is larger than 0, the RT curve is shifted to the left, in line with the SPDM.

We used maximum likelihood estimation to find the best-fitting parameters, separately for each participant. The details of the procedure can be found in Appendix A3. As the hypothesis test, we ran a one-sample, t -test on the β_2 parameters of the individuals. In addition, we sought to test whether treating β_2 as a free parameter allows to predict the RT data significantly better. Thereto, we performed a model comparison between the more general RT-Model_{SPDM} with β_2 being a free parameter and the RT-Model_{DPDM} with β_2 being fixed to zero. We performed a likelihood-ratio test to assess whether the higher complexity of the more general model is statistically justified by the data on an individual level. We fitted both models to the individual RT data and tested whether the improvement of fit was significant.

Importantly, the SPDM does not only predict a shift to the left in both the choice and the RT curves (see Figure B and D), but it also assumes that these shifts arise from the same process. Therefore, these two shifts of choice and RT curves should be related to each other. To test this relationship, we also quantified the *memory bias on choice* by fitting a logistic regression model to the choice data of individual participants. The intercept parameter of the logistic regression model quantifies the shift of the choice curve to the left, and thus the memory bias on choice. Finally, we tested for a positive correlation between this memory bias on choice and the β_2 parameter of the RT-Model, which quantifies the *memory bias on response times*.

Quantitative model comparison via hierarchical Bayesian modeling

In addition to comparing their qualitative predictions, we performed rigorous model comparison using a hierarchical Bayesian approach. Notably, the quantitative model comparison offers a more precise evaluation of the validity of the assumed cognitive processes. To the best of our knowledge, the DPDM (Alós-Ferrer, 2018) has only been used to derive qualitative predictions so far. Therefore, we consider our model comparison as providing a principled way of gauging the DPDM's quantitative adequacy.

SPDM. In the SPDM, choice and RT data come from a diffusion process that results in a Wiener distribution:

$$y \sim Wiener(a, z, T_{er}, \nu). \quad (6)$$

In the hierarchical model, the subject-specific parameters (denoted by subscript s) are drawn from normal group-level distributions with respective group-level parameters μ and σ (hyper priors are listed in Appendix B1). Boundary separation a_s , starting point bias z_s and non-decision time T_{er_s} were estimated as

$$\begin{aligned} a_s &\sim e^{\mathcal{N}(\mu_a, \sigma_a)}, \\ z_s &\sim \Phi(\mathcal{N}(\mu_z, \sigma_z)), \\ T_{er_s} &\sim e^{\mathcal{N}(\mu_{T_{er}}, \sigma_{T_{er}})}, \end{aligned} \quad (7)$$

where Φ denotes the cumulative distribution function of the standard normal. The drift-rate parameter ν varies from trial to trial (subscript t) as follows:

$$\begin{aligned} \nu_{s,t} &= d_{SPDM_s} [Rem_{right_{s,t}} \times (V_{right_{s,t}} - \gamma_s) - Rem_{left_{s,t}} \times (V_{left_{s,t}} - \gamma_s)], \\ d_{SPDM_s} &\sim e^{\mathcal{N}(\mu_{d_{SPDM}}, \sigma_{d_{SPDM}})}, \\ \gamma_s &\sim \mathcal{N}(\mu_\gamma, \sigma_\gamma), \end{aligned} \quad (8)$$

where $Rem_{right_{s,t}}$ ($Rem_{left_{s,t}}$) are dummy variables, indicating whether the right (left) snack was remembered, and $V_{right_{s,t}}$ ($V_{left_{s,t}}$) indicating the subjective value of the right (left) snack. d_{SPDM_s} is a free parameter which scales value differences to the speed of evidence accumulation. γ was the parameter which acts as reference value, indicating the biased value comparison in the SPDM.

DPDM. The DPDM is a mixture model, where the choices and RTs come from two different diffusion processes:

$$\begin{aligned} y &\sim \Delta \times Wiener(a, z, T_{er}, \nu_{Utility}) + (1 - \Delta) \times Wiener(a, z, T_{er}, \nu_{Heuristic}), \\ \Delta_s &\sim \Phi(\mathcal{N}(\mu_\Delta, \sigma_\Delta)). \end{aligned} \quad (9)$$

The mixing parameter Δ indicates the proportion of trials in which the response was generated by the utility process.

While both processes share the same parameters a , z and T_{er} , they differ with respect to their drift-rates $\nu_{Utility}$ and $\nu_{Heuristic}$. $\nu_{Utility}$ is the same as in the SPDM but compares V_{rem} to the average snack value (V_{avg_s}) instead of γ .

$$\begin{aligned}\nu_{Utility_{s,t}} &= d_{Utility_s} [Rem_{right_{s,t}} \times (V_{right_{s,t}} - V_{avg_s}) - Rem_{left_{s,t}} \times (V_{left_{s,t}} - V_{avg_s})], \\ d_{Utility_s} &\sim e^{\mathcal{N}(\mu_{dUtility}, \sigma_{dUtility})}.\end{aligned}\tag{10}$$

The drift-rate of the heuristic process is a free parameter:

$$\begin{aligned}\nu_{Heuristic_{s,t}} &= d_{Heuristic_s} [Rem_{right_{s,t}} - Rem_{left_{s,t}}], \\ d_{Heuristic_s} &\sim e^{\mathcal{N}(\mu_{dHeuristic}, \sigma_{dHeuristic})}.\end{aligned}\tag{11}$$

Note that the exponential transformation enforces a positive drift-rate in the direction of the remembered option.

Model fitting and model comparison. We estimated the parameters of both hierarchical models with *Stan* (Stan-Development-Team, 2018) using a No-U-Turn sampler (Hoffman & Gelman, 2014). Each model was estimated with four chains of 10,000 iterations each (50% of which were warm-up iterations that were discarded). To ensure model convergence using the $\hat{R}$ statistic (Gelman & Rubin, 1992), we checked that $\hat{R} < 1.01$ for all parameters. We compared the penalized-for-complexity fit of both models using the widely applicable information criterion (WAIC; Watanabe, 2013).

As all model comparison procedures are relative measures of fit (i.e., they can only assess the model performance relative to other models, not in absolute terms), we generated posterior predictive distributions for choice rates and RTs as a function of V_{rem} to evaluate absolute model performance (i.e., the degree to which they are able to capture quantitative and qualitative patterns in the empirical data). To do so, we simulated 500 experiments using virtual agents who behaved according to the model equations. Each agent's parameter vector was drawn randomly from the posterior distribution obtained during model fitting. We aggregated the choices and RTs into 8 bins and calculated the respective 95% highest-density interval (HDI; Kruschke, 2015). Using parameter- and model-recovery analyses we confirmed that both models were able to recover data-generating parameters and that the models make different

predictions with respect to behavior such that a data-generating model can be correctly identified (see Appendix B2 for details).

Results

Qualitative results

Our first approach to compare a single- and a dual-process account of memory-based decisions making was to evaluate the qualitative predictions of the respective approaches by fitting a regression model (Equation 5) to the individual RT data. As outlined in the Method section, only the single-process account predicts that the typical U-shape of the RT curve (as a function of value difference between the options) is shifted such that the peak lies to the left of the average value of V_{rem} . This shift is quantified by the parameter β_2 (see Equation 5). In line with this prediction, parameter estimates of β_2 were positive and significantly different from zero ($M = .48, SD = 1.15, p < .001, d = .41$). To illustrate this, we computed RT curves under the fitted models for all participants. The mean RT curve and its 95% confidence intervals are depicted in Figure 4A.

As a model comparison among different versions of the RT-Model, we performed a likelihood-ratio test on the improvement of deviances between RT-Model_{SPDM} and RT-Model_{DPDM}. The individual improvements are depicted in Figure 4B, showing significant improvement in fit for 14 of 90 participants. On the group level, we obtained a significant difference of the deviances ($\chi^2(df = 90) = 183.85, p < .001$). This indicates that the increased complexity of the RT-Model_{SPDM}, which treats β_2 as a free parameter, is justified by a statistically better fit to the data, thus lending further support to the single-process account.

Because the single-process account predicts that the memory bias on RT and the memory bias on choice arise from the same underlying process, we also fitted a logistic regression model to the individual choice data and correlated the intercept parameters (which reflect the memory bias on choices) with the β_2 parameters of the RT-model (which reflect the memory bias on RT). The intercept estimates were significantly

Figure 4. Qualitative results. **A** Average RTs depending on V_{rem} in black. Error bars depict the empirical 95% CIs. The orange curve yields the average RT curves from the RT-Model. The dotted lines indicate the 95% confidence interval for the average curve. **B** Improvement of deviance between the RT-Model_{DPDM} and RT-Model_{SPDM} for every participant. The grey horizontal line indicates the critical χ^2 value which indicates a significant improvement of model fit on the subject level. **C** Empirical choices and the estimated mean choice curve. **D** Correlation of the individual estimates of memory bias on choice, and the memory bias on RT.

positive ($M = .38, SD = .55, p < .001, d = .68$), which is in line with a memory bias on choice and reproduces the results of Mechera-Ostrovsky and Gluth (2018). The mean choice curves and their 95% CIs are plotted in Figure 4C. As predicted, the individual intercepts from the regression model correlated positively with the β_2 estimates from the RT-Model ($r = 0.32, p = 0.002$, Fig. 4D). Thus, the shift in the RT curve is associated with the shift in the choice curve, suggesting a common underlying mechanism.

Quantitative results

In addition to the qualitative analysis, we compared the SPDM and the DPDM on a quantitative level by a model comparison within a hierarchical Bayesian framework. Summary statistics of the posterior distributions of all group-level parameters are provided in Table 1.

We relied on the WAIC for model comparison. The SPDM had a lower WAIC than the DPDM (17,394 vs. 17,468), with a difference in WAICs of 74.34 ($SE = 24.74$), resulting in a strong standardized effect size of $\frac{\Delta WAIC}{SE} = 3.01$.

We performed posterior predictive checks to assess absolute model performance. Both models are capable to produce a shift in the choice curve (see Figure 5A). However, whereas the SPDM predicts most data points well (i.e., within the 95% HDI), the empirical data often lie outside the 95% HDI of the DPDM, particularly when V_{rem} is low. With respect to RTs, this difference is even stronger (Figure 5B). Specifically, when V_{rem} was below average, the DPDM underestimated the RTs. When it was above average, it overestimated them. This result is in line with the notion that the DPDM is unable to account for a shifted U-shaped RT curve, as it is forced to predict a curve that is symmetrical around zero. In contrast, the SPDM provides an accurate account of the empirical RT curve. Taken together, both the relative and the absolute model comparisons confirm the qualitative results and provide additional support for a single- and against a dual-process account of memory-based decisions.

Table 1

Group-level parameter estimates for the single- and dual-process diffusion models.

Model	Parameter	M	95% HDI
SPDM	μ_a	1.77	[1.68, 1.88]
	σ_a	1.30	[1.25, 1.35]
	μ_{Ter}	.29	[.27, .32]
	σ_{Ter}	1.46	[1.38, 1.57]
	μ_z	.52	[.51, .52]
	σ_z	1.05	[1.03, 1.08]
	$\mu_{d_{SPDM}}$	.36	[.31, .41]
	$\sigma_{d_{SPDM}}$	1.70	[1.51, 1.93]
	μ_γ	-.54	[-.70, -.39]
	σ_γ	1.82	[1.58, 2.10]
DPDM	μ_a	1.82	[1.73, 1.93]
	σ_a	1.31	[1.26, 1.36]
	μ_{Ter}	.29	[.26, .31]
	σ_{Ter}	1.48	[1.38, 1.57]
	μ_z	.52	[.51, .52]
	σ_z	1.06	[1.04, 1.09]
	$\mu_{d_{Utility}}$	.70	[.55, .85]
	$\sigma_{d_{Utility}}$	1.54	[1.30, 1.86]
	$\mu_{d_{Heuristic}}$	.29	[.17, .48]
	$\sigma_{d_{Heuristic}}$	3.03	[2.03, 4.62]
	μ_Δ	.55	[.44, .66]
	σ_Δ	1.84	[1.49, 2.32]

Note. M represents the posterior mean. The 95%-HDI depicts the boundaries of the 95% highest-density interval. Note that these values are transformed with respective transformation functions (see main text).

Figure 5. Posterior predictives. **A** shows the 95%-HDI of the means of simulated data sets, based on the estimated posterior parameter distributions of the SPDM (cyan) and DPDM (purple). The black dots indicate the empirical means. **B** shows the same for response times.

Discussion

The present study compared a single- with a dual-process account of memory-based decisions. The single-process account assumes that memory affects the valuation of options, such that better-remembered options are perceived as more valuable. In contrast, the dual-process account assumes that each decision is made by one of two processes, where a rational process competes with a heuristic-based process which ignores value and uses memory-strength information only. While the

single-process account was already tested before Gluth et al. (2015), the dual-process account, implemented on the basis of the DPDM as proposed by Alós-Ferrer (2018), has never been tested in the current context. This study thus provides a first empirical test of these opposing theories on memory-based decisions. We found that both models can make similar predictions with respect to choices but differ regarding their predictions of RTs. Using previously published data from a memory-based choice task (Mechera-Ostrovsky & Gluth, 2018), we found unequivocal support for the single-process account in both qualitative and quantitative analyses.

Our results bear strong analogies to a debate on the use of the recognition heuristic in inference tasks. The recognition heuristic states that people judge recognized items as being more important / frequent / larger than unrecognized items (Goldstein & Gigerenzer, 2002). Originally, the heuristic was not conceptualized in the framework of dual-process accounts. However, to account for the fact that people do not always go with the recognized cue, it has been argued that the heuristic is applied in some but not all trials (Pachur, 2011). Yet, this proposal was refuted by a recent study that—similar to our approach—relied on RT data to dissociate between the recognition heuristic and competing theories of inferential judgements (Heck & Erdfelder, 2017).

Despite being very popular in judgment and decision making and other psychological disciplines (Evans, 2008), dual-process theories often became target of fundamental criticism for conceptual issues but also for the lack of empirical support (Keren & Schul, 2009; Kruglanski & Gigerenzer, 2011; Melnikoff & Bargh, 2018). One shortcoming of many dual-process theories is their poor formalization which impedes quantitative model comparison (Diederich & Trueblood, 2018). This is especially true, when between-trial dynamics which account for choice and RT differences are ignored (Krajbich, Bartling, Hare, & Fehr, 2015). In this study, we outlined a principled way to test a dual-process against a single-process account by means of quantitative and qualitative model comparison. Our approach is based on a recently developed formal model of a dual-process account (Alós-Ferrer, 2018) with suitable assumptions for testing our particular hypothesis. Apart from model comparison, estimation of the

model also offers a deeper understanding of the underlying processes, such as the relative proportion of the two presumed processes (e.g., Δ), or their within-trial dynamics (e.g., drift-rates). We believe the field of judgment and decision making is well advised to formalize the proposed dual-process models and to test their empirical content. This approach has the potential to move the debate on dual-process models forward by adhering to empirical findings and methodological rigor.

Within the dual-process framework, there are two types of conceptualization which specify how the two processes can be implemented (Evans, 2008). According to the *parallel-competitive* structure, Type 1 and Type 2 processes run in parallel and a potential conflict between them has to be resolved to determine which process is applied in a given decision. The DPDM of Alós-Ferrer (2018) can be assigned to this group of models. The second influential dual-process architecture comprises the *default-interventionist* models. These models assume that a Type 1 process is active to generate an intuitive default response but may be overcome by the reflective Type 2 process. From a diffusion model perspective, such a process could be reflected in a starting point bias in favor of the intuitive option (e.g., Chen and Krajbich, 2018). Hence, we also tested whether the assumption of a starting point bias towards the remembered option could account for the present data, but found that it cannot (see Appendix C3). An alternative implementation of a default-interventionist model was proposed by Diederich and Trueblood (2018), who investigated risky choices, drawing on prospect theory (Kahneman & Tversky, 1979) and expected utility theory (von Neumann & Morgenstern, 1953). In our study, we were interested in the domain of value-based decision making and preferential choice, which is directly covered by the DPDM of Alós-Ferrer (2018) but not by Diederich and Trueblood (2018).

In sum, our results clearly indicate that a single decision process in which the evaluation process is biased by memory describes the memory bias better than a dual-process account that assumed two independent processes (i.e., a memory-heuristic and a utility process). We fit both models in a hierarchical Bayesian modeling framework and outlined a rigorous procedure to empirically test single- and

dual-process accounts.

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Open Practices

The data and code are available at (<https://osf.io/kbyas/>). The experiment was preregistered but was conducted for a different purpose.

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Appendices

A1 Trial types

In the remember-and-decide task, there are three types of trials, based on the number of snacks remembered (i.e., both remembered; both forgotten; one remembered, one forgotten). For this study, only the last trial type was of interest, as the memory bias (i.e., the tendency to prefer remembered over forgotten options) can only be tested with these kinds of trials. Table 2 summarizes the trial numbers for each trial type.

Table 2

Numbers of different trial types.

Trial type	Total	M	SD	Range
Both forgotten	5651	62.8	32.9	13-163
One remembered, one forgotten	8031	89.2	15.8	39-118
Both remembered	5468	60.8	30.0	7-142

Note. Total: Number of trials in the data set; M : mean trial number per participant; SD : standard deviation; Range: range of trials between participants.

A2 Analytical derivation of expected mean response times

In the main text, we illustrate the RT predictions of the DPDM (and the SPDM) using simulations. Predictions for the expected RT (and for the symmetry of the predicted RT curve around 0; see Figure 1) can also be derived analytically. Here, we report these derivations.

The DPDM describes the expected RTs (eRT) as the weighted sum of mean RTs of the heuristic and the utility process. It can be described as

$$\begin{aligned}
 eRT_{\Delta V} = & p(H) \times p(R|H) \times eRT(R|H) + p(H) \times [1 - p(R|H)] \times eRT(F|H) \\
 & + [1 - p(H)] \times p(R|U, \Delta V, \gamma) \times eRT(R|U, \Delta V, \gamma) \\
 & + [1 - p(H)] \times [1 - p(R|U, \Delta V, \gamma)] \times eRT(F|U, \Delta V, \gamma)
 \end{aligned} \tag{12}$$

where $eRT_{\Delta V}$ is the expected RT for the value difference ΔV , $p(H)$ is the probability of the heuristic process, $p(R)$ [$p(F)$] is the probability of remembering [forgetting] a snack,

and $eRT(R)$ [$eRT(F)$] is the mean RT for remembered [forgotten] snacks. The probability of a utility process is $1 - p(H)$, the probability and mean RT of remembered and forgotten choices in utility trials depend on ΔV and the reference value γ which is equal to V_{avg} in the DPDM.

Since we assume that the RTs come from a diffusion process, such that $eRT(chooseR) = eRT(chooseF)$, Equation 12 can be simplified to

$$eRT_{\Delta V} = p(H) \times eRT(R|H) + [1 - p(H)] \times eRT(R|U, \Delta V, \gamma). \quad (13)$$

Because the utility process is the only one that depends on V_{rem} and assumes the lowest drift-rate when $V_{rem} = V_{avg}$, the predicted peak of the RT is at V_{avg} .

The same logic and equation can be applied to the SPDM by setting $p(H)$ to 0 and allowing the reference value γ to be a free parameter. Then, the predicted peak RT in the single-process account is at γ .

A3: Maximum likelihood estimation details

We used a SLSQP algorithm to minimize the deviance between the choice curve and the RT data of the individual participants. The parameter bounds and initial values of the estimation procedure can be found in Table 3.

The logistic regression on the choice data was performed with a SLSQP algorithm. The initializing values were randomly drawn from continuous uniform distributions (intercept $\sim \mathcal{U}(\min(V_{rem_s}), \max(V_{rem_s}))$; slope $\sim \mathcal{U}(-2, 2)$).

Table 3

Parameter	Lower bound	Upper bound	Initial values
β_0	0	∞	$[0, 5000]$
β_1	∞	∞	$[-2, 2]$
β_2	$\min(V_{rem_s})$	$\max(V_{rem_s})$	$[\min(V_{rem_s}), \max(V_{rem_s})]$
SD	0	∞	$[1000, 5000]$

Note: β_0 represents the intercept, β_1 the slope, β_2 the shifting parameter of the quadratic function. SD is the standard deviation which is fitted in the course of the maximum likelihood procedure. The subscript s in V_{rem_s} represents the subject-specific snack values. The columns show the lower and upper bounds of the parameters of the quadratic function, as well as the intervals $[a, b]$ from which the initial values were uniformly drawn.

B1 Prior distributions of group parameters

In our hierarchical models, the individual model parameter were all drawn from normal distributions at the group level. Thus, for each model parameter, we estimated a group mean and a group standard deviation, on top of the individual parameters themselves. All group means were given a weakly-informative normal prior, and all group standard deviations were given a weakly-informative half-normal prior. The prior distributions for the group-level parameters are listed below.

Table 4

Prior distributions for group parameters.

Group parameter	SPDM	DPDM
μ_a	$\mathcal{N}(0, 3)$	$\mathcal{N}(0, 3)$
σ_a	$\mathcal{HN}(0, 3)$	$\mathcal{HN}(0, 3)$
μ_z	$\mathcal{N}(0, 1)$	$\mathcal{N}(0, 1)$
σ_z	$\mathcal{HN}(0, 1)$	$\mathcal{HN}(0, 1)$
μ_{Ter}	$\mathcal{N}(-1, 1)$	$\mathcal{N}(-1, 1)$
σ_{Ter}	$\mathcal{HN}(0, 3)$	$\mathcal{HN}(0, 3)$
$\mu_{d_{SPDM}}$	$\mathcal{N}(0, 3)$	—
$\sigma_{d_{SPDM}}$	$\mathcal{HN}(0, 3)$	—
μ_γ	$\mathcal{N}(0, 1)$	—
σ_γ	$\mathcal{HN}(0, 3)$	—
$\mu_{d_{Utility}}$	—	$\mathcal{N}(0, 3)$
$\sigma_{d_{Utility}}$	—	$\mathcal{HN}(0, 2)$
$\mu_{d_{Heuristic}}$	—	$\mathcal{N}(0, 3)$
$\sigma_{d_{Heuristic}}$	—	$\mathcal{HN}(0, 2)$
μ_Δ	—	$\mathcal{N}(0, 3)$
σ_Δ	—	$\mathcal{HN}(0, 3)$

B2 Parameter and model recovery analyses

Parameter recovery. We performed a parameter-recovery analysis for both models. For each model, we chose the 10 group-level parameter estimates from the respective models' posteriors that had the highest log-likelihood. From these group-level distributions, we drew 90 independent samples that determined individual-level behavior of the artificial agents. As inputs for the agents, we used the same inputs that we used for model fitting. For the DPDM, each trial was simulated to be either a heuristic or utility trial, based on independent Bernoulli trials $\mathcal{B}(\Delta_s)$. We re-fitted each

of these generated datasets with the respective models to assess the degree to which parameters could be recovered.

Following an analysis approach implemented in Fontanesi, Gluth, Spektor, and Rieskamp (2019), we addressed three questions in this analysis: 1) Do parameter values trade off during the estimation procedure (i.e., to which degree do models suffer from sloppiness)? 2) Are the models able to correctly identify the group-level parameters? 3) Are the models able to correctly identify the individual-level parameters?

To address question 1, we calculated Pearson correlation coefficients between the posterior samples of all group parameters of each of the 10 fitted artificial data sets. Subsequently, the 10 correlation coefficients were Fisher-transformed and subsequently averaged and their standard distribution was calculated. Note that group parameters were drawn independently from each other. Therefore, there should be no correlation between the posterior samples. Figure 6 shows the mean correlations between the parameters. Overall, we only observed weak correlations between μ_{dSPDM} and σ_{dSPDM} , and μ_{dSPDM} and μ_{γ} . The DPDM suffered from stronger sloppiness, especially between $\mu_{dUtility}$, $\mu_{dHeuristic}$ and μ_{Δ} .

Figure 6. Correlations of group parameter posterior samples of the SPDM. Each cell shows the mean correlation coefficients (using Fisher’s transformation) with the corresponding standard deviation in brackets.

To address question 2, we calculated the 95% HDIs of the posterior group-level parameter distributions. If the parameters are identifiable, the 95% HDI of should include the true data-generating parameter value (see also Spektor & Kellen, 2018). We observed good identifiability of the group-level parameters for both models (Figs. 8; 9).

Figure 7. Correlations of group parameter posterior samples of the DPDM. Each cell shows the mean correlation coefficients (using Fisher’s transformation) with the corresponding standard deviation in brackets.

To address question 3, we correlated the true data-generating parameters on the individual level with the estimated means of the posterior distributions. For this analysis, the means were transformed back to the space of the group-level parameters. For both models, we observed very good recovery of the a and T_{er} parameters. The recoverability of z was lower but still fair which may be due to the low explanatory power of this parameter. The parameters which specified the drift-rate were recovered well for the SPDM, and fairly well for the DPDM. Overall, we judge the recoverability to be good for the SPDM and relatively good for the DPDM, and sufficient for both

Figure 8. Identifiability of the SPDM group parameters. Horizontal black lines indicate the generating parameter values for the 10 simulations. Vertical lines indicate the 95%-HDI of the estimated parameter distributions.

models.

Figure 9. Identifiability of the DPDM group parameters. Horizontal black lines indicate the generating parameter values for the 10 simulations. Vertical lines indicate the 95%-HDI of the estimated parameter distributions.

Model recovery. We performed a model-recovery analysis to assess whether both models could in principle be identified as the winning model, given that the data are generating by the respective model. From the parameter recovery analysis, we used the 20 simulated set of data of which 10 were generated by each of the models. We fitted both models to each set of data separately and calculated the WAICs of the fits. The critical test for the model recovery was whether a model from which the data was generated wins the comparison against the competing model on that data. In other

Figure 10. Identifiability of the SPDM individual level parameters, pooled over all 10 simulations. The position on the X-axes indicate generating parameter value, the Y-position show the estimated posterior mean. Values around the dashed grey identity line yield good recovery. The Pearson's correlation coefficients of generated and estimated parameters is indicated on the lower left for each parameter.

words, given the data were generated with the SPDM, the WAIC of the SPDM should be lower than the WAIC of the DPDM (and vice versa for the DPDM). The SPDM outperformed the DPDM on data which was generated under the SPDM (Figure 12). The DPDM, on the other hand, won the model comparison against the SPDM in all sets of data that came from the DPDM, showing good model recovery.

Figure 11. Identifiability of the DPDM individual level parameters, pooled over all 10 simulations. The position on the X-axes indicate generating parameter value, the Y-position show the estimated posterior mean. Values around the dashed grey identity line yield good recovery. The Pearson's correlation coefficients of generated and estimated parameters is indicated on the lower left for each parameter.

C1 Does a default-interventionist process explain the memory bias?

Apart from the parallel-competitive dual-process account of Alós-Ferrer (2018), it is conceivable that a default-interventionist account could explain the memory bias. Such an approach assumes that during each decision, rapid pre-conscious processes either approve of the Type 1 decision, or intervene by initiating the rational Type 2 process (Evans, 2008). From a diffusion model perspective, a default-interventionist

Figure 12. Model recovery. The plot shows the WAIC values for the parameter fits of the SPDM (cyan rectangles), and the DPDM (purple triangles) for each simulated data set. The left panel shows the datasets generated from the SPDM, the right panel shows those from the DPDM.

account resembles a biased starting point z (Chen & Krajbich, 2018). More specifically, assuming that the default response would be implemented as a starting-point bias, the diffusion process would be biased so that the intuitive option would be selected quicker and more often than the non-intuitive one. Technically, this would still be a single-process account because the default option cannot finalize the decision process (there is some deliberation necessary), but its predictions resemble a default-interventionist dual-process model (e.g., fast errors).

With respect to the remember-and-decide task, this means that the starting point bias should favor the remembered option. If a biased starting point provided a better explanation for the memory bias, the memory bias parameter should be affected when a model is estimated with a free starting point which can favor the remembered option.

To evaluate this, we fitted another diffusion model to the data. In order to fit the starting-point bias parameter, we reparameterized the DDM so that the upper boundary would represent choices in favor of the remembered option, and the lower

boundary would represent choices against it. Other than that, the model was parameterized similarly to the SPDM (see Eq 7)

$$\begin{aligned}\nu_{s,t} &= d_s(V_{rem_{s,t}} - \gamma_s), \\ d_s &\sim e^{\mathcal{N}(\mu_{d_{SPDM}}, \sigma_{d_{SPDM}})}, \\ \gamma_s &\sim \mathcal{N}(\mu_\gamma, \sigma_\gamma),\end{aligned}\tag{14}$$

where $V_{rem_{s,t}}$ is the value of the remembered option, d_s is the scale parameter and γ_s the memory bias parameter.

Note that the starting point bias $f(z)$ is scaled between zero and one. A value of .5 yields that there is no starting point bias. A value $> .5$ indicates that there was a bias in favor of the remembered option, a value $< .5$ indicates that the starting point bias was favoring the forgotten option.

After fitting the model, the posterior distributions revealed that the starting point bias parameter $f(\mu_z)$ did not bias decisions in favor of the remembered option (see Table 5). From the posterior estimates it is evident that, if anything, there was a small starting point bias in favor of the forgotten option.

The model yielded a WAIC of 17,425 which was higher than the WAIC of the SPDM.

Table 5

Group parameter estimates

Parameter	M	95%-HDI	$f(M)$	$f(95\text{-HDI})$
μ_a	0.57	[0.52, 0.63]	1.77	[1.68, 1.88]
σ_a	0.26	[0.22, 0.30]	1.30	[1.25, 1.35]
$\mu_{T_{er}}$	-1.24	[-1.32, -1.16]	0.29	[0.27, 0.31]
$\sigma_{T_{er}}$	0.38	[0.32, 0.45]	1.46	[1.38, 1.57]
μ_z	-0.03	[-0.05, -0.01]	0.49	[0.48, 0.50]
σ_z	0.05	[0.00, 0.08]	1.05	[1.00, 1.08]
$\mu_{d_{SPDM}}$	-1.02	[-1.16, -0.89]	0.36	[0.31, 0.41]
$\sigma_{d_{SPDM}}$	0.53	[0.41, 0.66]	1.70	[1.51, 1.93]
μ_γ	-0.62	[-0.78, -0.45]	-0.62	[-0.78, -0.45]
σ_γ	0.60	[0.46, 0.74]	1.82	[1.58, 2.10]