1 Children's understanding of the abstract logic of counting

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3 Colin Jacobs ^a, Madison Flowers ^a, Julian Jara-Ettinger ^a

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5 ^a Department of Psychology, Yale University

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Abstract

When children learn to count, do they understand its logic independent of the number list that they learned to count with? Here we tested CP-knowers' (ages three to five) understanding of how counting reveals a set's cardinality, even when non-numerical lists are used to count. Participants watched an agent count unobservable objects in two boxes and were asked to identify the larger set. Participants successfully identified the box with more objects when the agent counted using their familiar number list (Experiment 1) and when the agent counted using a non-numeric ordered list, as long as the items in the list were not linguistically used as number words (Experiments 2-3). Additionally, children's performance was strongly influenced by visual cues that helped them link the list's order to representations of magnitude (Experiment 4). Our findings suggest that three- to six-year-olds who can count also understand how counting reveals a set's cardinality, but they require additional time to understand how symbols on any arbitrary ordered list can be used as numerals.

Keywords: number cognition; counting; cognitive development

1. Introduction

Children learn the meaning of number words and counting by going through a systematic set of knowledge stages. At around age two (in a place such as the US), children first learn to recite the number list through rote memorization, but do not understand its meaning (Fuson, 1988; Gelman & Gallistel, 1979; Schaeffer, Eggleston, & Scott, 1974). Children then sequentially learn the meaning associated with the words “one,” “two,” “three,” and sometimes “four,” taking approximately six months to learn the meaning of each word. It is broadly believed that after learning the meaning of these first three or four number words, children undergo a striking change: They suddenly grasp the logic of counting¹ and they can produce sets of the correct size for any cardinality represented in their memorized count list (Carey, 2009; Lee & Sarnecka, 2010; Sarnecka & Lee, 2009; Wynn, 1990, 1992). This pattern, where children first go through the *subset-knower* stages (individually called *one-*, *two-*, *three-*, and *four-knower* stages) before becoming *CP-knowers* (short for *Cardinal Principle knowers*), is universal and independent of the age at which children learn to count (Piantadosi, Jara-Ettinger, & Gibson, 2014; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007; Le Corre, Li, Huang, Jia, & Carey, 2016; Barner, Libenson, Cheung, & Takasaki, 2009).

What do children learn when they master counting? One possibility is that children acquire an abstract understanding of counting that is independent of the number words that they use to count. This type of understanding is evident in adults, as

¹ Throughout, we use the term “logic of counting” to refer to the conceptual knowledge that enables children to count. We adopt this neutral terminology because there are multiple competing theories of how this knowledge is represented (e.g., Gelman & Gallistel, 1979; Carey, 2009; Piantadosi et al., 2012) and the questions in our work do not pre-suppose that any of these proposals is necessarily correct.

we understand that any ordered list can be used to count. Examples of this range from knowing that it is possible to count with number lists from foreign languages, to understanding that non-numeric lists can also be used to determine cardinality. For instance, in one Jewish tradition, members of a *minyan* confirm the presence of ten members by reciting a ten-word verse (with each member reciting one word; Gelman & Gallistel, 1979). This view has been influential in theories of number learning, which often posit that learning to count culminates in a representation of counting that is independent of the count list, and can be applied to any ordered list (Gelman & Gallistel, 1979; Izard, Pica, Spelke, & Dehaene, 2008; Piantadosi, Tenenbaum, & Goodman, 2012). An alternative possibility, however, is that children's early understanding of counting is intrinsically connected to the number words that they learned to count with. After all, children's ability to count co-develops with their knowledge of the count list (Chu, Cheung, Schneider, Sullivan, & Barner, 2020; Davidson, Eng, & Barner, 2012; Cheung, Rubenson, & Barner, 2017). Under this view, children might understand the relationship between counting and cardinality within their count list, but not understand how counting could work using any ordered list.

Making progress on this question is difficult due to a lack of paradigms that probe children's understanding of counting without also requiring fluency with the count list. While substantial research has looked at children's understanding of counting using their language's number words, it remains unknown whether CP-knowers' understanding of the logic of counting is independent of their knowledge of the count list. Here we present a set of studies that begin to answer this question. We introduce a new paradigm, *the hidden cardinality task*, to test children's understanding of counting,

independent of number list knowledge and fluency. Using this paradigm, we test if CP-knowers' understanding of counting is restricted to the use of the number list that they learned to count with, or whether they recognize that the logic of counting is valid using any ordered list. All experiments shared a similar structure and we thus begin by describing the general paradigm (Section 2) before turning to our experiments.

2. General Methods

All procedures, exclusion criteria, and analyses were pre-registered unless noted.

2.1 Inclusion criteria and population characteristics

All experiments focused on CP-knowers, as we expected that success in our task would develop after children have learned to count. Hence, all experiments recruited children three- to five-years-old, but only included CP-knowers (as determined by their performance on the Give-N task; see section 2.2, and Supplemental Materials for details). No participant completed more than one Experiment.

While we did not collect demographic information specific to each participant, here we report summary demographic statistics based on the most recent census data available. 54.6% of participants (n=164) were recruited and tested at a museum in New Haven where on average 19% of adult visitors are Black, 58% are White, 13% are Hispanic or Latino, 3% are Asian, 1% are Native American, and 6% are two or more races (Peabody Museum of Natural History, 2005). 32.3% of participants (n=97) were tested at schools in New Haven County and the surrounding counties, for which we analyzed each source's demographics by zip code. On average, the median income for these areas is \$76, 854 and 11.9% of adults are Black, 73.4% are White, 19.7% are

Hispanic or Latino, 4% are Asian, 0.2% are Native American, 0.02% are Native Hawaiian or Pacific Islander, and 2.9% are two or more races. Additionally, 8.6% of participants (n=26) were tested at a museum in Boston, where 24% of attendees are Black, 47% are White, 17% are Hispanic or Latino, 9% are Asian, and 4% are two or more races. 29% of museum attendees visit on days when there is free or discounted admission. Finally, 5% of participants (n=15) were tested in lab (recruited using a list obtained primarily through public events in the New Haven area). The median household in New Haven income is \$41,142 and on average 33% of adults are Black, 30.3% are White, 30.5% are Hispanic or Latino, 4.7% are Asian, 0.4% are Native American, and 4.3% are two or more races.

2.2 Procedure

The procedure for all four experiments followed the same structure. Each participant first completed the Give-N task to determine whether they qualified for our study, followed by the hidden cardinality task which assessed their understanding of counting.

2.2.1 Give-N Task. The Give-N task served as an inclusion criterion to identify children who already knew how to count. Participants were presented with a bowl with ten bouncy balls and an empty bowl. The task always began with the experimenter requesting participants to move four bouncy balls from one bowl to the other. After each query, all bouncy balls were returned to the first bowl. If the child succeeded in this first trial, the next request was to move five bouncy balls. If the child failed, the next request was to move one bouncy ball. The task then followed a stair-cased procedure: Children

were asked to move $N+1$ bouncy balls if they moved N bouncy balls correctly, and were asked to move $N-1$ bouncy balls otherwise. This procedure had two exceptions: The same request was repeated when children failed at $N=1$ and when they succeeded at $N=8$ (ensuring that moving all bouncy balls was never the correct answer). The task concluded after eight trials, with two exceptions that terminated the task early when children were performing at ceiling or at floor (see Supplemental Materials for details).

In standard Give-N tasks (e.g., Shusterman, Slusser, Halberda, & Odic, 2016; Sarnecka & Carey 2008), children are asked to confirm that they moved the correct number of objects at the end of each query (“Is that N bouncy balls? Can you count them for me please?”) and they are given the opportunity to correct their response when they detect an error (“Can you fix it so there are N bouncy balls in the bowl?”). Pilot data in our task suggested that this standard procedure made the overall task (Give-N task followed by hidden cardinality task; Section 2.2.2) too long, increasing the risk that participants would feel exhausted and become distracted when completing the hidden cardinality task. To minimize this risk, we made two changes to the standard Give-N task. First, when correct, children were not asked to confirm their answers. Second, when incorrect, children were given an opportunity to confirm and correct their answer, but only when the quantity they moved was at most two units away from the requested quantity (e.g., when asked to move 5 objects, children were allowed to confirm their answer whenever it was incorrect and within the range 3-7). The experimenter recorded the original and the revised answers, then used the final answer to determine the next trial. See Supplemental Materials for coding criteria.

2.2.2 Hidden cardinality task. Figure 1 shows the logic of our *hidden cardinality task*, which aimed to evaluate children’s understanding of counting independent of the count list in use. The hidden cardinality task consisted of a warm-up phase, and three test trials.

Figure 1. Logic of the hidden cardinality task, showing the ordered animal list from Experiment 2. a) Warm-up phase. Participants were introduced to an ordered list and watched the researcher count erasers by placing them in one-to-one correspondence with this list (first counting a set of 4 erasers, then counting a set of 8 erasers). b) At test, participants watched an agent use the ordered list to count the number of objects inside two boxes with unobservable contents. After counting the objects in each box, the agent placed a picture of the final item on the front of the box. Participants were then asked which box had more objects in it. Counting order and side of correct answer were counterbalanced. c-e) All participants completed three trials in a random order comparing: two vs. three objects (*parallel individuation* trial), three vs. six objects (*Approximate Number System* trial), and six vs. seven objects (*exact cardinality* trial). The objects are shown here for clarity but were not visible to participants.

In the warm-up phase, children were first presented with a printed ordered list (e.g., a list of animals in Figure 1; see Figures 2-4 for count lists used in our experiments). The experimenter explained that this ordered list was used to count

objects. The experimenter then put four erasers down, one eraser placed next to each of the first four items on the list, and touched each eraser in order while reciting the words out loud (Figure 1a). After counting, the experimenter declared “there are [x] erasers” (e.g., “there are butterfly erasers” in the first warm-up example for the list shown in Figure 1). The experimenter then repeated this display with 8 erasers.

After the warm-up phase, the experimenter removed the erasers and left the count list in front of the participant so that they could reference it at any time. The experimenter explained that the participant would watch “some videos of my friend counting blocks, and then you are going to tell me which box has more blocks in it.” The experimenter then showed three videotaped test trials on an iPad. Each trial began with an agent sitting behind a table with two black boxes, one on each side (Figure 1b). The video started with the agent looking at the camera with their hands resting on their lap. The agent then looked into one of the boxes and used one of their hands to point inside the box repeatedly while reciting the count list in order (Figure 1b). Importantly, participants were unable to see either set of objects within the boxes at any time. After counting the objects in the first box, the agent announced “There are [N] blocks in this box”, where [N] was an item in the ordered list, and then attached a picture of the corresponding symbol to the front of the box. The agent then proceeded to count the blocks in the second box in the same way. The actor counted with a neutral tone, such that children could not infer the right answer by relying on prosody (see online repository for stimuli; Section 2.3). When finished, the agent put their hands in their lap again and stared straight at the camera. When the video concluded, children were shown a picture of the two boxes, each with the corresponding symbol attached to the

front. Participants were then asked “Can you tell me which box has more blocks in it? Can you point?” The picture was always positioned behind the laminated count list.

All participants completed three trials. Trial order, position of the box with more objects, and which box the agent counted first were all counterbalanced across participants. In the *parallel individuation* trial (2 vs. 3; Figure 1c), the agent counted two objects in one box and three objects in the other. Because even infants can represent exact quantities of up to three using their *parallel individuation system* (Cheung & LeCorre, 2018; Feigenson, Carey, & Hauser, 2002; Feigenson & Carey, 2003, 2005), participants should be able to track how many words the agent uttered while counting and determine which box had more objects. In the *approximate number* trial (3 vs. 6; Figure 1d), the agent counted three objects in one box and six objects in the other box. We expected participants to succeed using their *Approximate Number System*, which even infants can use to discriminate relative quantities at a 2:1 ratio (both visually and auditorily; Xu & Spelke, 2000; Lipton & Spelke, 2004). Note that for both of these trials, children could also succeed simply by tracking the amount of time that the agent spent counting the objects in each box without having to track the discrete utterances.

Finally, in the *exact cardinality* trial (6 vs. 7; Figure 1d), the agent counted six objects in one box and seven objects in the other. In this critical trial, neither the *parallel individuation system* nor the *Approximate Number System* can help children distinguish six from seven objects or sounds (nor the difference between the time it takes to count six and seven objects). Here, if children realize that the agent is using the ordered list to count, they should recognize that the items in the list represent different cardinal values. Equipped with this understanding, children are in a position to infer that symbols that

come later in the list represent greater cardinalities (i.e., inferring that the agent's counting behavior creates a linear relationship between symbol positions and cardinal values, referred to as the *later-greater principle*; Le Corre, 2014; Davidson, Eng, & Barner, 2012). Therefore, children's success on this trial would reveal that children are attributing their familiar count list's order-magnitude relationship to the list that the agent is counting with.

2.3 Materials, pre-registration, and statistical approach

All studies were pre-registered. All stimuli, data, pre-registrations, and analyses are available at: https://osf.io/jn9ur/?view_only=70038b03e7ba42888ec9489c10fff640.

Due to conceptual and methodological limitations with null-hypothesis significance testing (Cohen, 1994; Cumming 2013, 2014), our main method of analyses focused on estimating effect sizes. We thus compute 95% bootstrapped confidence intervals for our key statistics. We interpret an effect size as reliable if chance performance is not included in the confidence interval, and as unreliable otherwise. Our only exception to this approach is in our use of pre-registered logistic regressions to test for developmental changes, where we also report the associated p-values. Throughout, we analyze the data of all children as a single group, but effects broken down by age group can be found in Supplemental Materials.

3. Experiment 1: Counting with Arabic Numerals

In Experiment 1 we began by testing if CP-knowers could determine which of two boxes has more hidden objects by watching an agent count using their familiar count list. Our

goal was to establish baseline performance on the hidden cardinality task, which then enables us to interpret successes and failures in later studies.

3.1 Methods

3.1.1 Participants. 60 CP-knowers, as determined by the Give-N task (Wynn, 1992; Carey, 2009; Sarnecka & Lee, 2009; Lee & Sarnecka, 2010) were recruited for this study (mean age: 4.65; range = 3.03-5.93). 41 additional children were recruited but not included in the study because they were not CP-knowers (as determined by our pre-registered criteria; n=31 participants), because they declined to complete the task (n=4 participants), because of an experimental error (n=4 participants), or because they had already participated in a related previous study (n=2 participants).

3.1.2 Stimuli. Stimuli for the hidden cardinality task (see General Methods) consisted of a printed ordered list of Arabic numerals (Figure 2a), and videos of the agent counting using English number words.

3.1.3 Procedure. After completing the Give-N task, participants completed the hidden cardinality task using the Arabic count list (Figure 2a). During the warm-up phase, the experimenter explained “I’m going to show you how I like to count using this number line,” then proceeded as described in the General Methods section.

3.2 Results

Figure 2b shows the results from the experiment. Participants succeeded in all trials, confirming that CP-knowers can identify which set has more hidden objects after watching an agent count them. Of the 60 CP-knowers included in the study, 95% of participants (N=56; 95% CI: 90-100) correctly identified the box with more objects in the 2 vs. 3 trial, 93.33% of participants (N=56; 95% CI: 88.33-100) correctly identified the box with more objects in the 3 vs. 6 trial, and 83.33% of participants (N=50; 95% CI: 75-93.33) identified the box with more objects in the 6 vs. 7 trial.

To test for any developmental change, we ran three mixed effects logistic regressions predicting children's response on each trial as a function of age (as continuous variable), with trial order as a random intercept.² Children's performance did not improve as a function of age on any of the three trials (all $\beta < 1.24$ all $p > 0.163$ See Figure 5).

3.3 Discussion

Experiment 1 showed that CP-knowers can identify which of two boxes has more unobservable objects by watching an agent count their contents. How did children succeed in our task?

A first possibility is that CP-knowers relied solely on their knowledge of numerals, without reasoning about the logic of counting per se. That is, children could have succeeded by simply knowing the meaning of different number words (e.g., knowing that the word "seven" represents a higher cardinality than "six") or by recognizing the

² Our pre-registrations stated that we would compute bootstrapped confidence interval for the beta age estimates, but this is an error in the pre-registration and we thus do not include those.

Arabic numerals on the boxes (e.g., recognizing that the symbol 7 represents a higher cardinality than 6). This account predicts a strong developmental effect, because young CP-knowers struggle to identify which number words denote greater quantities (particularly with quantities above four; Davidson et al., 2012; Le Corre, 2014), and children between three and six years old undergo a sharp transition in their ability to recognize Arabic numerals between 1-9 (while four-year-olds perform at chance, five-year-olds show over 80% success; Knudsen, Fischer, & Aschersleben, 2015; Purpura & Lonigan, 2015; Bartelet, Vaessen, Blomert, & Ansari, 2014). These expected developmental effects contrast with our findings, where CP-knowers succeeded on all trials, independent of age (with even three-year-olds performing near ceiling; Figure 5). This suggests that children in our task did not rely on their knowledge of numerals alone.

Given that our findings cannot be explained by appealing to children's understanding of numerals alone, an alternate possibility is that children successfully identified which box had more objects by attending to the agent's counting behavior. For instance, children could have realized that, when the agent counted the larger set, they recited the smaller set's cardinality and continued reciting additional number words (or conversely, when the agent counted the smaller set, they never recited the number word associated with the larger set). If this is the case, then children should also succeed in this task when the Arabic numerals are replaced with a non-numeric ordered list. We test this prediction in Experiment 2.

4. Experiment 2: Counting with Color-ordered Animals

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280 In Experiment 2 we test if CP-knowers can infer which of two boxes has more objects
281 when an agent counts using a non-numeric ordered list. To achieve this, we replaced
282 the list of Arabic numerals from Experiment 1 with a non-numeric list of animals ordered
283 by color (Figure 2c). If CP-knowers recognize how counting reveals cardinality,
284 independent of the list in use, we expected them to succeed in all trials.

285

286 **4.1 Methods**

287 **4.1.1 Participants.** 60 CP-knowers, as determined by the Give-N Task, were
288 recruited for this study (mean age: 4.66 years; range = 3.01-5.99). 20 additional children
289 were recruited but not included in the study because they were not CP-knowers (as
290 determined by our pre-registered criteria; n=17 participants), due to an experimental
291 error (n=1 participant), because they were not fluent in English (n=1 participant), or
292 because they chose to stop participation (n=1 participant).

293

294 **4.1.2 Stimuli.** Stimuli were identical to those in Experiment 1 with one exception.
295 The count list for this study consisted of eight different animals, ordered by color: a
296 green alligator, a blue frog, a purple octopus, a magenta butterfly, a pink flamingo, a red
297 lobster, an orange fox, and a yellow duck. All animals were matched in size (Figure 2c).

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299 **4.1.3 Procedure.** Experiment 2 used the same procedure as Experiment 1 with
300 the difference that the hidden cardinality task now used an animal list ordered by color
301 (see Figure 1 for procedure). To introduce this count list, the experimenter explained

that the animals were “ordered like the rainbow,” and that they “count using these animals.” Throughout the hidden cardinality task, the name of each animal on the list was used to count (“Alligator, frog, octopus, butterfly. There are butterfly erasers,” and “Alligator, frog. There are frog blocks in this box”).

4.2 Results

Figure 2. a) Numeral list used in Experiment 1. b) Results from Experiment 1. The x axis shows the trial and the y axis shows the percentage of participants who correctly identified the box with more objects. Vertical lines show 95% bootstrapped confidence intervals and the horizontal dotted line represents chance. Overall, participants were able to identify which box had more objects in all three trials. c) Animal list used in Experiment 2. d) Results from Experiment 2.

Figure 2d shows the results from the experiment. Similar to Experiment 1 (Counting with Arabic Numerals), participants once again succeeded in the *parallel individuation* (2 vs. 3) and the *Approximate Number System* (3 vs. 6) trials. Of the 60

CP-knowers included in the study, 81.67% of participants (N=49; 95% CI: 71.67-91.67) correctly identified the box with more objects in the 2 vs. 3 trial, and 80.0% of participants (N=48; 95% CI: 70-90) correctly identified the box with more objects in the 3 vs. 6 trial. In contrast to Experiment 1, however, only 55.0% of participants (N=33; 95% CI: 43.33-66.67) identified the box with seven objects as the larger set in the *exact cardinality* trial (6 vs. 7).

Mixed-effects logistic regressions identical to the ones used in Experiment 1 revealed a significant effect of age on success in the *parallel individuation* trial only ($\beta=1.425$, $p<0.01$; See Figure 5). As Figure 5 shows, this effect appears to capture a change where children of all ages succeed in the task, but only the oldest participants reached near-ceiling performance.

4.3 Discussion

Children's failure in Experiment 2's *exact cardinality* trial is surprising considering that (i) all participants knew how to count; (ii) they learned that the non-numeric list would be used to count and watched the experimenter do this twice during the warm-up stage; (iii) the task had minimal memory demands (the animal list was always in front of them, and there were pictures of animals attached to the front of each box indicating the final item the agent reached when counting); (iv) success involved simply selecting the animal that came later in the list.

These results suggest that young CP-knowers either failed to understand that the animals in the list represented cardinalities, or that they did not realize that animals that appear later in the list represent greater cardinalities (i.e., linking animals to

cardinalities, but not recognizing the later-greater relationship). Without this understanding, children could only solve the *parallel individuation* trial (2 vs. 3) and the *Approximate Number System* trial (3 vs. 6) by tracking how many sounds the agent produced when counting the objects in each box (or the total duration of time spent counting each box), which does not require treating any animal as representing an exact cardinality.

One possible interpretation of these results is that young CP-knowers' ability to reason about cardinalities is intrinsically tied to the number list that they learned to count with. Under this view, children may have not understood how the animal list was relevant to the task, or how the agent's behavior using this novel list could be considered counting. Alternatively, a second possibility is that young CP-knowers failed due to task demands. Beyond requiring children to conceptually represent animals as cardinalities, our task also required children to *linguistically* represent animal names as number words (e.g., "there are *octopus* blocks in this box"), and children may have struggled with these unusual linguistic constructions. We test this possibility in Experiment 3 through a variation of Experiment 2 where we removed linguistic task demands.

5. Experiment 3: Feeding Animals

In Experiment 3 we test a modified version of the hidden cardinality task where animal names are no longer used as number words (i.e., when they are not used linguistically as words that denote a cardinality; e.g., "There are octopus blocks in this

box”). Experiment 3 followed the same logic as Experiments 1 and 2, with one critical change. The counting agent was now introduced as a farmer who always fed a list of animals in the same order. To determine how much food was in each box, they listed the animal names in order, then highlighted the last animal that they would be able to feed with the amount of food in that box. If children’s previous failure was due to trouble treating animal names as number words, they should now succeed in the task.

5.1 Methods

5.1.1 Participants. 60 CP-knowers, as determined by the Give-N Task, were recruited for this study (mean age: 4.90 years; range = 3.58-5.98). 18 additional children were recruited but not included in the study because they were not CP-knowers (n=15 participants), due to family interference (n=1 participants), or because they declined to complete the task (n=2 participants).

5.1.2 Stimuli. Stimuli were identical to those in Experiment 2 with two exceptions. First, the counting list for this study consisted of eight ordered farm animals: a rooster, a horse, a pig, a cow, a goat, a sheep, a duck, and a donkey. All animals were standardized in size (Figure 3a). Additionally, the warm-up phase used 10 small brown paper circles to symbolize food pellets (rather than 10 erasers).

5.1.3 Procedure. The procedure was similar to Experiment 1-2 with the difference that we changed the script to avoid using animal names directly as number

words. During the warm-up phase, the experimenter told the children about “a farmer that likes to feed his animals in the exact same order every time.” The warm-up then proceeded with the experimenter placing four food pellets, one in front of each of the first four animals, and saying “This is how I know how many animals the farmer can feed.” The experimenter then used a finger to touch each animal as they recited each animal name “rooster, horse, pig, cow.” The experimenter then concluded “The farmer has enough food to feed all the way to cow.” This process was then repeated with all eight animals on the list.

The test trials were identical to the ones used in Experiment 1-2, with the difference that, for each video of the agent counting hidden sets, the final declaration “there are N blocks in this box,” was changed to “I can feed all the way to [final animal] with the food in this box.” To remain congruent with the change of language, the test question was changed to ask “Which box has more food in it?”

5.2 Results

Figure 3b shows the results from the experiment. Participants succeeded in all trials, showing that they recognized how the agent’s counting behavior revealed which set was larger. Of the 60 CP-knowers included in the study, 73.33% of participants (N=44; 95% CI: 63.33-85) correctly identified the box with more objects in the 2 vs. 3 trial, and 80.0% of participants (N=48; 95% CI: 70-90) correctly identified the box with more objects in the 3 vs. 6 trial. Most importantly, participants now succeeded in the critical 6 vs. 7 trial; 71.66% of participants (N=43; CI: 60-83.33) correctly identified the box with more objects.

Mixed-effects logistic regressions identical to the ones used in Experiments 1-2 revealed an age effect on the approximate number trial only ($\beta=1.27$, $p<0.05$). As Figure 5 shows, this effect captures the fact that older CP-knowers performed nearly at ceiling, while young CP-knowers were more likely to fail in the task.

Figure 3. a) Animal list used in Experiment 3. b) Results from Experiment 3. The x axis shows the trial and the y axis shows the percentage of participants who correctly identified the box with more objects. Vertical lines show 95% bootstrapped confidence intervals and the horizontal dotted line represents chance. Overall, participants were able to identify which box had more objects in all three trials.

5.3 Discussion

Our results revealed that CP-knowers can identify which of two boxes has more objects by watching an agent count with a non-numeric list. This shows that, when an agent counts with a non-numeric list, children can treat the list symbols as cardinalities,

and they understand that symbols appearing later in the list denote greater cardinal values.

These results also suggest that children's failure on Experiment 2 (Counting with Color-Ordered Animals) was caused by a difficulty in treating animal names as number words. As described previously, the unusual linguistic constructions (e.g., "There are octopus blocks in this box") may interfere with children's ability to understand the agent's behavior. Could children overcome this challenge when the list contains stronger visual cues that help them associate animal words with numerical order? We test this in Experiment 4.

6. Experiment 4: Counting with Size-Ordered Animals

In Experiment 4 we test whether children's ability to treat non-numeric words as representing number concepts is affected by visual cues that highlight the structure of the non-numeric list. Experiment 4 was identical to Experiment 2 (Counting with Color-Ordered Animals) with the difference that the animals now varied in size to emphasize the list's order. In Experiment 4a, the animal list was presented in ascending order, while in Experiment 4b, it was presented in descending order.

6.1 Methods

6.1.1 Participants. 120 CP-knowers (n=60 in Exp 4a and 4b, respectively), as determined by the Give-N Task, were recruited for this study (mean age: 4.65 years;

range = 3.32-5.99). 44 additional participants were recruited but not included in the study because they were not CP-knowers (n=27 participants in Experiment 4a, and n=9 in Experiment 4b), or because they declined to complete the study (n=5 in Experiment 4a and n=3 in Experiment 4b).

6.1.2 Stimuli. Stimuli were identical to the one used in Experiment 2, with the difference that we now used a list of eight animals ordered in size: an ant, a mouse, a cat, a pig, a cow, a bear, an elephant, and a giraffe. The list was presented in ascending order in Experiment 4a, and in descending order in Experiment 4b (Figure 4a and Figure 4c, respectively).

6.1.3 Procedure. The procedure was identical to Experiment 2 (Counting with Color-Ordered Animals).

6.2 Results

6.2.1 Experiment 4a: Ascending animal list

Figure 4b shows the results from the experiment. Participants succeeded in all trials. Of the 60 CP-knowers included in the study: 86.66% (95% CI: 78.33- 96.67; N=52 participants) correctly identified the box with more objects in the 2 vs. 3 trial, 80.00% of participants (N=48; 95% CI: 70.00-90.00) correctly identified the box with more objects in the 3 vs. 6 trial, and 86.67% of participants (N=52; 95% CI: 78.33-96.97) correctly identified the box with more objects in the 6 vs. 7 trial. Together, these results show that

CP-knowers were able to use a congruent magnitude cue to overcome the failure observed in Experiment 2 (Counting with Color-Ordered Animals). Mixed-effects logistic regressions identical to the ones used in Experiments 1-3 revealed age effects on all trials (all $\beta > 1.53$, all $p < 0.01$; See Figure 5), showing that children's performance improved as a function of age.

6.2.2 Experiment 4b: Descending animal list

Figure 4d shows the results from the experiment. When the animal list was ordered in a descending order, participants failed to identify the larger set of objects in all three trials. Of the 60 CP-knowers included in the study, only 45% (N=27; 95% CI: 31.67- 56.67) correctly identified the box with more objects in the 2 vs. 3 trial, 41.67% of participants (N=25; 95% CI: 28.33-53.33) correctly identified the box with more objects in the 3 vs. 6 trial, and 38.33% of participants (N=23; 95% CI: 25-50) correctly identified the box with more objects in the 6 vs. 7 trial. These results show that the inverse relationship between size and quantity in a non-numeric list now hindered participant performance, even on trials where the larger set can be identified through the *parallel individuation system* (2 vs. 3) and *Approximate Number System* (3 vs. 6). Mixed-effects logistic regressions identical to the ones used in Experiments 1-3 revealed no age effects (all $\beta < 0.445$, all $p > 0.184$; See Figure 5).

Figure 4. a) Animal list used in Experiment 4a. b) Results from Experiment 4a. The x axis shows the trial and the y axis shows the percentage of participants who correctly identified the box with more objects. Vertical lines show 95% bootstrapped confidence intervals and the horizontal dotted line represents chance. Overall, participants were able to identify which box had more objects in all three trials. c) Animal list used in Experiment 4b. d) Results from Experiment 4b. The x axis shows the trial and the y axis shows the percentage of participants who correctly identified the box with more objects. Vertical lines show 95% bootstrapped confidence intervals and the horizontal dotted line represents chance. Overall, participants were unable to identify which box had more objects in all three trials.

6.3 Discussion

Experiment 4 shows that young CP-knowers readily associated space and number in our non-numeric count lists, and that this association had a big influence on their performance (and we return to this point in the General Discussion). Note, however, that our results do not imply that children were responding through a size-

based cue alone. If they had, children should have significantly below chance rather than at chance in Experiment 4b.

Figure 5. Participant responses in all experiments. Each dot represents a participant's answer. The x-axis shows their age, and the y axis shows whether they successfully identified the box with more blocks in it. Data is jittered slightly on the y-axis for visibility purposes but was not jittered on the x-axis. Color indicates the trial type. The lines show logistic regressions, with the shaded areas indicating the standard error for each regression.

7. General Discussion

Children's mastery of counting constitutes a major milestone towards their understanding of number concepts. Here we presented a first step in testing if Cardinality Principle-knowers (or *CP-knowers*—children who understand that the size of

a set of objects corresponds to the final number used in the count list) understand counting independently of the number words that they learned to count with. To achieve this, we introduced the *hidden cardinality task*, where children had to infer which of two boxes had more hidden objects by watching an agent count using a novel ordered list. The results from our study are summarized in Table 1. Our work first confirmed that CP-knowers could determine the relative cardinality of hidden sets by watching another agent count with English number words (Experiment 1: Counting with Arabic Numerals; Figure 2). This capacity, however, broke down when we replaced the count list with an ordered list of animals (Experiment 2: Counting with Color-Ordered Animals; Figure 2). Here, CP-knowers still succeeded in the *parallel individuation* and the *Approximate Number System* trials by tracking the number of sounds that the agent uttered (or the time spent) when counting each set. However, CP-knowers now performed at chance in the *exact cardinality* trial, which required them to understand that the animals represented cardinal values, and that animals later in the list represent greater set sizes.

Why did children fail the hidden cardinality task when using a novel count list? Experiment 3 (Feeding Animals; Figure 3) provided evidence that Experiment 2 (Counting with Color-Ordered Animals) presented children with a linguistic challenge: using animal names (nouns) as number words (adjectives; e.g., “there are *octopus* blocks in this box”), since children succeeded in a variation where the agent stopped using animal names as number words (e.g. switching “there are *duck* blocks in the box” to “I can feed all the way to *duck*”). Experiment 4 (Counting with Size-Ordered Animals) further provided evidence that children also struggled in linking the novel list to

cardinality. Children’s performance improved when the size cues on the animal list were congruent with list order (i.e., larger animals representing greater cardinalities; Experiment 4a), but hindered children when the size cues were incongruent with list order (i.e., smaller animals representing greater cardinalities; Experiment 4b). Together, these results suggests that CP-knowers understand that it is possible to count with a novel ordered list, but that this understanding is brittle, as it can break down by simple linguistic demands (treating animal names as number words in Experiment 2) and by the use of incongruent visual cues (Ordering animals in descending size order in Experiment 4b).

Experiment	Behavior	PI (2 vs. 3)	ANS (3 vs. 6)	EC (6 vs. 7)
Exp 1: Counting with Arabic Numerals	Counting with Arabic numerals.	✓	✓	✓
Exp 2: Counting with Color-Ordered Animals	Counting with animal list, ordered by color.	✓	✓	✗
Exp 3: Feeding Animals	Counting with animal list, but animal names are no longer used as number words.	✓	✓	✓
Exp 4a: Counting with Size-Ordered Animals (Ascending)	Counting with animal list, ordered by size (ascending)	✓	✓	✓
Exp 4b: Counting with Size-Ordered Animals (Descending)	Counting with animal list, ordered by size (descending)	✗	✗	✗
Table 1. Summary of experiments and outcomes. Labels on the last three columns stand for “Parallel Individuation” trial (PI); “Approximate Number System” trial (ANS); and “Exact Cardinality” trial (EC).				

Our work adds to a growing body of work revealing that, while children’s mastery of counting is a major milestone (Carey, 2009; Piantadosi et al., 2012), their understanding of cardinalities and number continues to develop after this achievement. Indeed, related research has found that young CP-knowers often fail to identify which

number words denote greater cardinalities (e.g., “which is more, three or five?” Davidson et al., 2012; see also Experiment 1 Discussion for the implications of our study on this point), do not understand that adjacent words in the number list represent adjacent cardinalities (Spaepen, Gunderson, Gibson, Goldin-Meadow, & Levine, 2018), do not appreciate that number words outside their memorized count list have a successor (Cheung et al., 2017), and have difficulty tracking numerical changes in sets (Jara-Ettinger, Piantadosi, Spelke, Levy, & Gibson, 2017). Our work adds to this growing literature by showing that, while young CP-knowers understand that an agent can count with an arbitrary ordered list, their ability to treat these lists as representing a set of ordered cardinalities remains fragile. In the below sections (8.1 & 8.2), we consider why children failed in particular variations of our tasks.

8.1 *Why did CP-knowers fail the exact cardinality trial in Experiment 2: Counting with Color-Ordered Animals?*

In Experiment 2 (Counting with Color-Ordered Animals), CP-knowers succeeded on the *parallel individuation* and *Approximate Number System* trials, suggesting that they understood that the box associated with more sounds (or time spent counting) had more objects. However, they did not realize that the last animals named when counting each box revealed the sets’ relative cardinality (as they failed on the *exact cardinality* trial). There are two reasons why this could be the case. One possibility is that children found the original language of Experiment 2 difficult to understand, because the agent used nouns as numerals (e.g., “There are *frog* blocks in this box”). A second possibility, however, is that children’s failures in Experiment 2 were specifically tied to our choice to

use animal names. Although by age four children already understand that words can have multiple meanings, this understanding is biased towards an expectation that words encode related meanings (Floyd & Goldberg, 2020; Floyd, Goldberg, & Lew-Williams, 2020; Srinivasan, Berner, & Rabagliati, 2019). This assumption could have hindered children's ability to treat animal names simultaneously as biological categories and exact cardinalities. Under this account, children might be able to succeed in a variation of Experiment 2 that uses unfamiliar words (such as number words in a foreign language).

Conversely, it is also possible that children in Experiment 2 struggled because they were unfamiliar with the animal names. Under this account, children couldn't map the sounds that the agent produced to the animal pictures on the list. For example, while watching the agent count, they may have struggled to recall which animal in the list was the "lobster." It is therefore possible that having to map sounds to pictures created an additional task demand. If this is the case, then children may find it even more difficult to succeed in a variation that uses unfamiliar words (e.g., another language's number words). Though this is possible, we think it is unlikely for two reasons. First, our task arguably does not require children to know the animal names. This is because the test questions were purely visual, and children only needed to identify which of two animal pictures appeared later in the list. And second, because related research has found that preschool-aged children can produce nearly 100 different animal names (Gleason et al., 2009), suggesting that children in our task were familiar with the animals that we used. That said, it is possible that children are more familiar with some animal names, particularly the ones that we used in Experiment 3

(where, in contrast to the remaining Experiments, we used farm animals), and this influence is an open question for future research.

8.2 *Why did CP-knowers fail on all trials of Experiment 4b: Counting with Size-Ordered Animals (Descending)?*

The results of Experiment 4 (Counting with Size-Ordered Animals) revealed that CP-knowers had a fragile association between symbols in the list and cardinal values, as children's performance was strongly affected by size cues. When the list was ordered by size, children performed reliably above chance on all trials. By contrast, the list's incongruent size cues in Experiment 4b (Descending size order) impeded children's ability to answer correctly across all trials.

These results are consistent with research documenting a strong relationship between representations of space and number, classically referred to as the Spatial Numerical Association of Response Codes (SNARC for short; Dehaene 2011). This work has found that, in places such as the US, young children expect magnitudes to be organized from left to right. Further, from infancy we associate magnitudes across different dimensions including size and cardinality (Lourenco & Longo, 2011). SNARC being elicited with a non-numeric list would be consistent with research demonstrating that these associations are flexible and vary based on the child's experience (Bull, Marschark, & Blatto-Vallee, 2005; Shaki & Fischer, 2008). More research is needed to determine how children's spatial-numeric association can be elicited with non-numeric count lists, and when children may have the competence to overcome incongruent associations.

8.3 Future directions and conclusions.

Our work opens several questions for future work. First, our studies used animal lists to test children's understanding of the logic of counting. We do not know how children would perform in the hidden cardinality task with a completely novel list with no pre-associated meanings (e.g., using a list of unfamiliar characters or a list of novel objects). This is a direction that we hope to explore in future work. A related open question is whether children's performance in the hidden cardinality task is modulated by their exposure to multiple counting systems. Related research has shown that conceptual transitions in counting are synchronized across languages in bilingual children (such as becoming a CP-knower; Wagner, Kimura, Cheung, & Barner, 2015; Kimura, Wagner, & Barner, 2013; Marchand, Wade, Sullivan, & Barner, 2020). Similarly, it is possible that children with exposure to multiple number systems may have an earlier appreciation of how counting can be done with any ordered list.

Finally, children's performance in the exact cardinality trial with reduced linguistic demands (Experiment 3: Feeding Animals study; Figure 5) did not reveal any effects of age: even the youngest CP-knowers in our sample succeeded on all trials. These results open the possibility that children can recognize the validity of counting with non-numeric lists as soon as they have mastered counting, and perhaps even before children can successfully count themselves (such that children first understand the logic of counting, but require additional practice to execute it correctly). We hope to explore this question in future work.

Overall, our work shows that at the point in development when we reach one of the most significant milestones in number cognition—the mastery of counting—children’s understanding of counting is already, at least in part, independent from their knowledge of number words. Yet this understanding is brittle, breaking down when non-numeric words are used as number words, and when supporting cues to associate non-numeric symbols with ordered cardinalities aren’t available. Our results add to a growing body of work suggesting that children’s mastery of counting includes an abstract understanding of how this process reveals a set’s cardinality, but that it is nonetheless an intermediate step in children’s arduous path towards understanding number words, number concepts, counting, and the rich relations between them.

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