**Cognitive Diagnosis Modelling Incorporating Response Times and Fixation Counts:
Providing Comprehensive Feedback and Accurate Diagnosis**

Peida Zhan*

Department of Psychology, Zhejiang Normal University, China

Kaiwen Man

*Educational Research Program, Educational Studies in Psychology, Research Methodology, and
Counseling, University of Alabama, AL, USA*

Jonathan Malone

*Maryland English Institute Department of Second Language Acquisition, University of
Maryland, MD, USA*

* Corresponding author: Peida Zhan, Email: pdzhan@gmail.com

Cognitive Diagnosis Modelling Incorporating Response Times and Fixation Counts: Providing Comprehensive Feedback and Accurate Diagnosis

Peida Zhan, Kaiwen Man, Jonathan Malone

Abstract

Respondents' problem-solving behaviors comprise behaviors that represent complicated cognitive processes that are frequently systematically tied to one another. Biometric data, such as visual fixation counts (FCs), an important eye-tracking indicator, can be combined with other types of variables reflecting different aspects of problem-solving behavior to quantify this variability in problem-solving behavior. To provide comprehensive feedback and accurate diagnosis when using such multimodal data, the present study proposes a multimodal joint cognitive diagnosis model, accounting for latent attributes, latent ability, processing speed, and visual engagement by including three modals of data simultaneously: response accuracy, response times, and FCs. A set of simulated data is used to test the feasibility of the proposed model. The findings suggest that additionally using FCs could increase the comprehensiveness of feedback on problem-solving-related cognitive characteristics (e.g., cognitive style and fluency) as well as the accuracy of knowledge structure diagnosis. An empirical example is used to demonstrate the applicability and benefits of the proposed model.

Keywords: Cognitive diagnosis, eye tracking, response times, fixation counts, process data, biometric data, problem-solving

1. Introduction

Improving assessment of student problem-solving competence through effective multivariate analysis and modeling has been a frequent topic in the psychological, educational, and behavioral domains. In recent years, cognitive diagnosis (Leighton & Gierl, 2007), which objectively quantifies respondents' mastery or non-mastery of latent attributes (e.g., cognitive skills and knowledge points), has drawn increasing interest. Correspondingly, many cognitive diagnosis models (CDMs) or diagnostic classification models (for review, see M. von Davier & Lee, 2019) have been proposed to provide theoretical support for cognitive diagnosis, such as the most popular deterministic-inputs, noisy "AND" gate (DINA) model (Junker & Sijtsma, 2001) and its generalization (e.g., de la Torre, 2011; Henson et al., 2009).

CDMs are a family of restricted latent class models that model relationships between several discrete latent attributes and observed item responses. A latent attribute can either correspond to a concrete knowledge point or refer to a more abstract latent construct (de la Torre & Chiu, 2016). CDMs have the potential to provide richer diagnostic feedback information than traditional item response theory (IRT) models (Ma & de la Torre, 2020). Diagnostic feedback has been examined for its ability to promote students' learning outcomes (Tang & Zhan, 2021). Teachers can further use diagnostic feedback to design better remedial instruction, with the goal of efficient and targeted teaching interventions for an individual or group of students (e.g., Ren et al., 2021; L. Wang et al., 2020).

Problem-solving behavior consists of activities carried out by respondents, which reflect complex cognitive processes that are often systematically related to one another. The analysis of problem-solving behavior should not be limited to explore respondents' knowledge structure, but also reflect respondents' cognitive/learning styles, cognitive fluency, and other problem-solving related characteristics. For example, according to

the speed and accuracy of problem-solving, respondents can be classified as reflective cognitive style and impulsive cognitive style (Kagan, 1965). The former tend to be thoughtful when solving problems, manifested by using sufficient time to consider the problem, weigh multiple problem-solving solutions, and finally choose a best one; therefore, respondents with reflective cognitive style are slower in solving problem but more accurate. By contrast, the latter tends to quickly test whether the problem-solving solution is effective, and make a hasty decision based on only part of the information extracted from the problem situation; therefore, respondents with impulsive cognitive style solve problem faster but are prone to errors. Another example, according to the extensiveness and intensity of visual attention¹, respondents can be divided into scanners and focusers (Gardner et al., 1959). When solving problems, the scanner has a wide attention extent but low intensity, while the focuser has a narrow attention extent but high intensity. Subsequent studies have also suggested that scanning and focusing are two separable styles (Messick, 1989). Respondents can be divided into narrow and wide scanners according to the extent of attention, and focusers and non-focusers according to the intensity of attention.

Unlike the relatively stable cognitive style, cognitive fluency usually refers to the individual's subjective experience of the difficulty of information processing or problem-solving (Unkelbach, 2006). Thus, fluency may vary depending on the information processed, the problem solved, and the field it belongs to (Biancarosa & Shanley, 2015). Generally, in research on the measurement of fluency (e.g., Wang & Chen, 2020), fluency is regarded as a comprehensive reflection of speed and accuracy. Studies have shown that individuals who reach fluency levels can solve problems quickly and satisfactorily.

However, most of the current CDMs only utilize information on item response

¹ Visual attention in cognitive psychology is broader concept, which is a process that directs a tiny fraction of the information arriving at the visual cortex involved in visual working memory, pattern recognition, and motivation.

accuracy (RA)—a single and traditional data modal, resulting in a limited range of diagnosis. Specifically, they mainly focus on respondents' mastery of cognitive skills or subject knowledge needed to solve problems and only provide the diagnosis of knowledge structure, while ignoring individual differences in other dimensions of problem-solving behavior, such as problem-solving efficiency, which is determined by the processing speed, and visual engagement, which is determined by the intensity of visual attention. In other words, the diagnostic results of most existing CDMs cannot fully reflect the individual differences in problem-solving behaviors, which may further limit the effectiveness of intervention based on existing diagnostic feedback.

For example, see Table 1, to solving a math problem that requires four latent attributes, respondents may not only differ in knowledge structure, but also differ in processing speed and visual attention. Even if some respondents have shown the same mastery pattern among latent attributes (i.e., the same knowledge structure), they may still process knowledge at different speeds and with varying visual engagements. As a result, providing the same diagnostic feedback to them based only on knowledge structure limits the variety and effectiveness of such feedback, given that it could flatten latent behavioral differences in problem-solving which go undetected by current diagnostic assessments. Studies have shown that respondents with different cognitive styles will adopt different problem-solving strategies (Graf & Kinshuk, 2007), and that there are significant differences between typical/normal children and exceptional children (e.g., learning disability and attention deficit hyperactivity disorder) in information processing speed, working memory, attention control ability, and other cognitive processing abilities (e.g., Hitch & McAuley, 1991; Passolunghi et al., 2001; Mayes & Calhoun, 2007). These differences cannot be reflected only by the diagnosis of knowledge structure provided by RA data alone. Given this limitation, it is important to note that the reason these essential latent traits were ignored is at least partially due

to the difficulties in collecting collateral data in the traditional assessment environment (e.g., paper & pencil tests).

Table 1

Example of Same Knowledge Structures with Different Problem-solving Related Characteristics.

Student	Knowledge Structure	Ability	Processing Speed	Visual Engagement
1	(1111)	High	Fast	High
2	(1111)	High	Slow	High
3	(1000)	Low	Fast	Low
4	(1000)	Low	Slow	High
5	(0000)	Low	Slow	Low

Current advances in computer-based or innovative technology-enhanced assessment (TEA) systems (Jiao & Lissitz, 2018) has allowed collateral data collection to become a routine activity in many small- and large-scale educational projects such as Programme for International Student Assessment (PISA). Importantly, the computerized nature of TEA systems makes it possible to automatically and simultaneously capture multimodal data² during the problem-solving activity. For instance, TEA systems can automatically record outcome, process, and biometric data through accuracy scores and biometric information regarding visual gaze patterns, body movements, or neural activity via embedded sensors or laboratory instruments (e.g., eye-trackers). Specifically, outcome data (e.g., RA) are the final responses to items or steps in a task. In CDMs, these are primarily used to obtain current knowledge structure. Process data (e.g., RTs and operating steps) refer to the additional information collected about student interactions with the assessment environment, which can be used to reconstruct the specific behaviors and action sequences a student exhibits within an assessment (Bergner & A. von Davier, 2018). Biometric data (e.g., eye-tracking data, motion tracking data, and neural data) is collected with the aid of

² Multimodal data refers to the complementary and diverse data obtained by multiple instruments, measurement devices or acquisition techniques for the same description object (Lahat et al., 2015).

laboratory instruments, which can be used to provide feedback on how the respondents are interacting with the assessment environment, and perhaps most importantly, whether they are motivated and engaged (Agard & A. von Davier, 2018).

Currently, the eye-tracker is one of the most popular laboratory instruments in psychological, educational, and behavioral studies, and has been used in large-scale assessment programs (e.g., Bos et al., 2005, as cited in Rupp et al., 2010). Eye-tracking indicators can provide evidence of an individual's cognitive process during problem solving (Gorin, 2006). Commonly used eye-tracking indicators include the fixation count (FC, which provides information on the number of times an individual looks at the fixation position),³ the saccade (which provides information on changes in fixation position), the area of interest (which provides information on the focus area), and the regression count (which provides information on the number of times an individual returns the fixation position to a specific target). Among them, the FC is the most commonly used indicator, which can reflect the intensity of attention on the targeted visual area (An et al., 2017; Justice & Lankford, 2002). Specifically, An et al. (2017) reported that when the intensity of attention given to the target increases, the related FCs also increase during the reading task. Similarly, in educational assessments, FCs can be modeled to reveal the level of visual engagement that a student invests in answering items (Man & Harring, 2019; 2020). Specifically, the term “visual engagement” is used to only reflect the visual cognitive process loading amount endorsed by FCs rather than the entire visual attention. In other words, FCs could be used to reflect the visual attention load, and the engagement level depends on the context.

As the technology in collecting and assessing multimodal data continues to

³ Fixations are the number of times our eyes stop scanning the scene at the targeted visual areas. In this study, the fixation counts, namely, the fixations located in the area of interest, are counted as the valid counts for examining the level of visual engagement the respondents put into answering the questions. More details about fixations and an example of the fixations of an item in the empirical example can be found in Section S1 in the online appendix.

improve, data analysis for multimodal data should concurrently develop, and if possible, develop in tandem. As Jeon et al. (2021) noted “data fusion efforts are based on the recognition that human behavior is a product of multiple variables that are systematically related to one another; hence, in order to advance our understanding, those variables should be analyzed in an integrated manner”. Furthermore, collateral data not only give us a more comprehensive picture of individual problem-solving competence, but could also be utilized to improve estimation accuracy (Bolsinova & Tijmstra, 2018; Zhan et al., 2018). Thus, modeling multimodal data would make it possible to provide a more comprehensive and accurate diagnosis than simply relying on outcome data from traditional assessments (see Ercikan & Pellegrino, 2017; He & M. von Davier, 2016; Jiao & Lissitz, 2018; Shute & Venture, 2013).

A psychometric challenge involved in TEA systems is employing applicable methods for analyzing a wide and increasingly-diverse data modal. Relying only on a single measurement model cannot meet the needs of these diversified data modals. Currently, joint-hierarchical modeling (van der Linden, 2007) is one of the most widely adopted psychometric modeling methods in analyzing multimodal data. A series of joint IRT models have been proposed for different test scenarios (for review, see De Boeck & Jeon, 2019). Inspired by the joint IRT models, a few joint CDMs have been proposed to measure latent attributes and processing speed (e.g., S. Wang et al., 2018; Zhan, Jiao et al., 2018). Similar to the limitations of most existing joint IRT models, these joint CDMs only focus on RA and RTs, without including biometric data (e.g., FCs). A substantial gap in these analyses remains in answering the question of how to incorporate all three together in statistically and pedagogically meaningful ways.

To analyse the variety in problem-solving behavior, biometric data can jointly be modeled with other modals of data reflecting different facets of problem-solving distinguishing performance. The realization of joint analysis of multimodal data in

cognitive diagnostic assessments helps to provide students with more comprehensive feedback, also may improve the accuracy of diagnosis. As a result, more personalized remedial instruction could be designed by incorporating multivariate behavioral information related to visual engagement and working efficiency.

To satisfy the need for comprehensive feedback and accurate diagnosis, following the joint-hierarchical cognitive diagnosis modeling (Zhan et al., 2018), this study proposes a multimodal joint-hierarchical CDM, jointly modeling data from three sources: (1) outcome data (RA); (2) process data (RTs); and (3) biometric data (FCs). The remainder of the present work will begin with a review of three measurement models used for modeling RA, RTs, and FCs, respectively. The proposed model is then presented, followed by three simulation studies evaluating the psychometric performance of the proposed model. In addition, an empirical example is analyzed to demonstrate the application of the proposed model, along with a discussion of findings, limitations, and future directions.

2. Multimodal Joint-Hierarchical Cognitive Diagnosis modeling

2.1. The DINA Model

Among existing CDMs, the DINA model is preferable in the present study's analyses of RA, due to its simplicity and straightforward interpretations of parameter estimates. Let Y_{ni} be the observed dichotomous RA for person n ($n = 1, 2, \dots, N$) to item i ($i = 1, 2, \dots, I$), and the DINA model can be expressed as

$$\text{logit}(P(Y_{ni} = 1)) = \beta_i + \delta_i \prod_{k=1}^K \alpha_{nk}^{q_{ik}}, \quad (1)$$

where $\text{logit}(x) = \log(x / (1 - x))$; $\boldsymbol{\alpha}_n = (\alpha_{n1}, \dots, \alpha_{nK})'$ denotes the n th person's attribute pattern, $\alpha_{nk} \in \{0, 1\}$, where $\alpha_{nk} = 1$ if person n masters attribute k ($k = 1, 2, \dots, K$) and $\alpha_{nk} = 0$ if not, where K is the number of required attributes in a test; β_i and δ_i are the intercept and interaction parameters for item i , respectively. The relations between these two

parameters and common guessing, g_i , and slipping, s_i , parameters are $\beta_i = \text{logit}(g_i)$ and $\delta_i = \text{logit}(1 - s_i) - \text{logit}(g_i)$, respectively; The Q-matrix (Tatsuoka, 1983) is an I -by- K confirmatory matrix with element q_{ik} indicating whether the attribute k is required to correctly answer the item i (i.e., $q_{ik} = 1$ if the attribute is required, and $q_{ik} = 0$ otherwise).

Since α_{nk} is a binary variable, the multivariate normal distribution that considers the correlation among person parameters in the joint-hierarchical modeling cannot be used directly. Hence, to facilitate the joint-hierarchical modeling, following Zhan et al. (2018), a higher-order latent structural model (de la Torre & Douglas, 2004) is built upon Equation (1), as

$$\text{logit}(P(\alpha_{nk} = 1)) = \gamma_k \theta_n - \lambda_k, \quad (2)$$

where θ_n is the higher-order latent ability of person n , and is assumed to follow the standard normal distribution for model identification; γ_k and λ_k are the slope and difficulty parameters for attribute k , respectively.

For this study, the DINA model was selected only as an example to illustrate the conceptualization of the proposed modeling approach; there is no theoretical challenge in extending it to other CDMs. For example, for generalization purposes, the DINA model can be replaced by generalized CDMs (e.g., de la Torre, 2011; Henson et al., 2009). Noting that the main reason why we did not consider using generalized models in the current study is that generalized models always need a large sample size to obtain a robust parameter estimate (Jiang & Ma, 2018). It is difficult to meet such a requirement in the current small-scale TEA system with laboratory instruments.

2.2. The Lognormal RT Model

RTs are always non-negative and the distributions of RTs are positively skewed in most cases, which makes a lognormal distribution-based model as the primary choice to fit RT data. In this study, the lognormal RT model (van der Linden, 2006) was chosen for RT analysis. Let T_{ni} be the observed RTs of person n to item i , and the lognormal RT

model can be expressed as

$$T_{ni} \sim f(t_{ni}; \tau_n, \omega_i, \xi_i) = \frac{\omega_i}{t_{ni} \sqrt{2\pi}} \exp\left(-\frac{\omega_i^2}{2} (\log t_{ni} - (\xi_i - \tau_n))^2\right), \quad (3)$$

where $\log t_{ni}$ is the logarithm of RTs used to transform the positively-skewed distribution of RT into a more symmetrical shape; τ_n is the latent processing speed of person n when completing a test and is assumed to be normally distributed with a mean of zero and a variance, σ_τ^2 ; ξ_i is the time-intensity of item i , or the average time needed to complete item i ; ω_i is the reciprocal of the standard deviation of the error term, which is here treated as a time-precision parameter. For simplicity, Equation 3 can also be expressed as

$$\log(T_{ni}) \sim N(\xi_i - \tau_n, \omega_i^{-2}). \quad (4)$$

It is important to note that useful extensions of the base lognormal RT model have been proposed (e.g., Fox & Mariani, 2016; C. Wang, et al., 2013; Zhan et al., 2021). However, the lognormal RT model was still chosen to fit the RTs in the current study due to its ease of implementation in practice and general applicability.

2.3. The Negative Binomial Fixation Model

Recently, a negative binomial fixation (NBF) model (Man & Harring, 2019) was proposed to analyze discrete FCs, and is the preferred measurement model for FCs in this study. The advantages of using the NBF model are twofold: (1) it incorporates individualized latent person parameters, which can reveal each student's level of visual engagement; and (2) it can show item-level characteristics simultaneously, which reflects the extent of visual effort required from respondents to finish answering an item, along with its discriminating power among them. Let V_{ni} be the observed FC of person n to item i , and the probability mass function of the NBF model can be expressed as

$$V_{ni} \sim f(v_{ni}; \varepsilon_n, h_i, m_i) = \frac{\Gamma(v_{ni} + h_i)}{v_{ni}! \Gamma(h_i)} \left(\frac{h_i}{\exp(\varepsilon_n + m_i) + h_i} \right)^{h_i} \left(\frac{\exp(\varepsilon_n + m_i)}{\exp(\varepsilon_n + m_i) + h_i} \right)^{v_{ni}}, \quad (5)$$

where ε_n is the latent visual engagement of person n , which represents the overall visual engagement level paid to the test; m_i is the visual-intensity of item i , which represents the averaged amount of visual engagement needed to complete an item; h_i is the dispersion or shape parameter of the expectation of the random variable V_{ni} , where $E(V_{ni}) = \mu_{ni} = \exp(\varepsilon_n + m_i)$. In cases where a given item has a fixed m_i , persons with a higher level of visual engagement would result in a proportional increase in FCs. Conversely, for a given person with fixed ε_n , items requiring a higher amount of visual-intensity would also result in higher FCs. Furthermore, a visual-discrimination parameter for item i is defined as $d_i = 1/\sqrt{\mu_i + \mu_i^2/h_i}$, where $\mu_i = \sum_{n=1}^I \mu_{ni}/I$; it reflects the overall dispersion of the FCs on item i : larger values lead to steeper slopes of the probability mass function of the negative binomial distribution, while smaller values correspond to shallower slopes. For simplicity, Equation 5 can also be expressed as

$$V_{ni} \sim NB(\exp(\varepsilon_n + m_i), d_i^{-2}). \quad (6)^4$$

An important underlying assumption of applying the NBF model is that the latent visual engagement of each person is assumed to be constant throughout the test. This assumption is the same as the assumption of having a constant latent processing speed for a student across items in the lognormal RT model. Although it is possible for the constant assumption to be relaxed (see Man & Harring, 2019), the NBF model is used in its current form in this study for simplicity.

2.4. The Multimodal Joint-Hierarchical DINA Model

In recent years, there has been an increasing interest in combining information provided by multimodal data to provide a unified explanation of psychological phenomena of interest. In fact, in TEA systems, RA, RT, and FC data are collected

⁴ Another alternative parametrization can be expressed as $V_{ni} \sim NB(h_i, \exp(\varepsilon_n + m_i))$, which is often used in ecology.

almost simultaneously, and they provide parallel information about respondents' responses to the same items (e.g., it takes 30 seconds and 40 fixation counts for a respondent to response an item correctly), so some researchers have referred to such multimodal data as parallel data (Jeon et al., 2021). If such parallel information can be analyzed and used mutually, not only can the relationship between different latent variables be directly analyzed, but also the accuracy of parameter estimation of the respective measurement models may be improved.

Following the joint-hierarchical cognitive diagnosis modeling, a new multimodal joint-hierarchical DINA (MJ-DINA) model is proposed. In the MJ-DINA model, Y_{ni} , $\log(T_{ni})$, and V_{ni} are separately modeled at the first level; two variance and covariance latent structures are modeled at the second level to consider the correlations among item parameters and person parameters, respectively.

The DINA model (Equation 1), the lognormal RT model (Equation 4), and the NBF model (Equation 6) are here used as the measurement models for RA, RTs, and FCs, respectively. Item parameters of the MJ-DINA model are assumed to follow a multivariate normal distribution as follows:

$$\Psi_i = \begin{pmatrix} \beta_i \\ \delta_i \\ \xi_i \\ m_i \end{pmatrix} \sim MVN_4 \left(\begin{pmatrix} \mu_\beta \\ \mu_\delta \\ \mu_\xi \\ \mu_m \end{pmatrix}, \Sigma_{\text{item}} \right), \quad \Sigma_{\text{item}} = \begin{pmatrix} \sigma_\beta^2 & \sigma_{\beta\delta} & \sigma_{\beta\xi} & \sigma_{\beta m} \\ \sigma_\delta^2 & \sigma_{\delta\xi} & \sigma_{\delta m} \\ \sigma_\xi^2 & \sigma_{\xi m} \\ \sigma_m^2 \end{pmatrix}, \quad (7)$$

where $\mu_{\text{item}} = (\mu_\beta, \mu_\delta, \mu_\xi, \mu_m)'$ is the mean vector of item parameters; additionally, Σ_{item} is the variance and covariance matrix of item parameters. To reduce the computational burden, only these four-item parameters whose interrelationships can be easily explained were included in the multivariate normal distribution. Other item parameters (e.g., ω_i and h_i) are assumed to be independently distributed (Man & Harring, 2019; Zhan et al., 2018), and are thereby are not included in Ψ_i .

Likewise, person parameters of the MJ-DINA are assumed to follow a trivariate

normal distribution as follows:

$$\Theta_n = \begin{pmatrix} \theta_n \\ \tau_n \\ \varepsilon_n \end{pmatrix} \sim MVN_3 \left(\begin{pmatrix} \mu_\theta \\ \mu_\tau \\ \mu_\varepsilon \end{pmatrix}, \Sigma_{\text{person}} \right), \quad \Sigma_{\text{person}} = \begin{pmatrix} \sigma_\theta^2 & \sigma_{\theta\tau} & \sigma_{\theta\varepsilon} \\ \sigma_{\theta\tau} & \sigma_\tau^2 & \sigma_{\tau\varepsilon} \\ \sigma_{\theta\varepsilon} & \sigma_{\tau\varepsilon} & \sigma_\varepsilon^2 \end{pmatrix}, \quad (8)$$

where $\mu_{\text{person}} = (\mu_\theta, \mu_\tau, \mu_\varepsilon)'$ is the mean vector of person parameters; Σ_{person} is the variance and covariance matrix of person parameters.

In sum, equations (1-8) together constitute the MJ-DINA. A graphical representation of the MJ-DINA is given in Figure 1. In brief, the proposed model can be seen as an extension of the joint responses and times DINA (JRT-DINA) model (Zhan et al., 2018), which was designed to include both outcome, process, and biometric data. Noting that the choice between the MJ-DINA model and the JRT-DINA model is determined by the number of models of data to be collected and analyzed; if only two modalities of data (e.g., RA + RTs) are collected, it is natural to consider using the JRT-DINA model. However, from the perspective of providing more comprehensive feedback, more modalities of data could be collected and analyzed. In such cases, the proposed model is one of the alternative data analysis models.

Figure 1

A Graphical Representation of the MJ-DINA.

2.5. Monotonicity, Scale Identifiability, and Conditional Independence Assumptions

In addition to the model construction, several additional constraints on the model were applied, including monotonicity, scale identifiability, and conditional independence assumptions. First, following Zhan et al. (2018), we constrain μ_δ to be positive, which means that at least half of the items are constrained to satisfy the monotonicity requirement (i.e., additional mastery of the required attributes increases the correct response probability).

Second, we set $\mu_\theta = \mu_\tau = \mu_\varepsilon = 0$ and $\sigma_\theta^2 = 1$ to ensure scale identifiability. In such cases, there is no need to impose other constraints on the mean and variance of the item parameters or the variance of the person parameters.

Third, following the joint-hierarchical modeling, in the MJ-DINA model, there are at least eight conditional independence assumptions are made: (1) α_{nk} s are conditionally independent given θ_n , which is the basic assumption of the higher-order latent structural model (Equation 2); (2) Y_{ni} s are conditionally independent given α_n , which is the basic assumption of the DINA model (Equation 1); (3) $\log(T_{ni})$ s are conditionally independent given τ_n , which is the basic assumption of the lognormal RT model (Equation 4); (4) V_{ni} s are conditionally independent given ε_n , which is the basic assumption of the NBF model (Equation 6); (5) Y_{ni} , $\log(T_{ni})$, and V_{ni} on a single item i are conditionally independent given α_n , θ_n , τ_n and ω_n , which assumes that the observed variables are affected by only the person parameters concerned in this study; (6) Y_{ni} is independent of τ_n and ω_n conditional on α_n ; (7) $\log(T_{ni})$ is independent of α_n and ε_n conditional on τ_n ; and (8) V_{ni} is independent of α_n and τ_n conditional on ε_n . The last three assumptions indicate that the proposed model follows simple-structure hierarchical modelling without cross-loading (see Bolsinova & Tijmstra, 2018). Within this modeling, separate measurement models are used for the RA, for the RT, and for the FC, where RA is taken to be fully explained by latent attributes, the RT is taken to

be fully explained by the latent processing speed, and the FC is taken to be fully explained by the latent visual engagement.

2.6. Knowledge Structure Diagnosis and Cognitive Characteristic Inference

The relative advantage of the MJ-DINA model is that it provides students with feedback about problem-solving-related cognitive characteristics while still providing a diagnosis about knowledge structure. The estimated attribute pattern can directly reflect the knowledge structure. However, unlike the attributes which are categorical variables, the latent ability, latent processing speed, and latent visual engagement are continuous variables. Therefore, it is impossible to directly classify students' latent ability, latent processing speed, and latent visual engagement. As a result, it cannot directly achieve the diagnosis of individuals' cognitive characteristics.

A relatively straightforward classification approach is to use the average as the cut-point (i.e., 0 in this study): when the estimated value of the student is greater than the average, it indicates that the student belongs to a category with relatively good latent ability, latent processing speed, or latent visual engagement, and vice versa. Theoretically, further combinations of those three person parameters can obtain eight categories of cognitive characteristics. Table 2 presents the broad categories of these eight cognitive characteristics and their possible causes or behaviors, which can be used to infer the students' cognitive characteristics combined with their person parameter estimates. Of course, it should be emphasized that this classification method is relatively rough and is suitable for rough inference of individual cognitive characteristics rather than an accurate measurement or diagnosis.

Table 2*Problem-solving Related Characteristics and Corresponding Inferences.*

Ability	Processing Speed	Visual Engagement	Corresponding Inference	Possible Cause or Behavior
+	+	+	Fluency + Focuser	Able to extract key information from questions comprehensively
+	+	–	Fluency + Non-focuser	Able to eliminate distractions and extract rare key information from items
+	–	+	Reflective + Focuser	Deliberate on possible solutions based on the key information extracted
+	–	–	Reflective + Non-focuser	Able to eliminate distractions and deliberate based on a few key information
–	+	+	Impulsive + Focuser	Quickly extract some information and quickly validate the solution
–	+	–	Impulsive + Non-focuser	Under-extracting information and tending to make rapid-guessing
–	–	+	Non-fluency + Focuser	Motivated to response but has low cognitive ability
–	–	–	Non-fluency + Non-focuser	Unmotivated

Note, fluency and reflective-impulsive style are inferred from speed and accuracy; Focuser-Non-focuser style is inferred from the level of visual engagement; “+” means higher than the average; “–” means lower than the average.

2.7. Bayesian Parameter Estimation

Similar to the JRT-DINA model, the parameters of the MJ-DINA model can be estimated using the Bayesian approach with the Markov chain Monte Carlo (MCMC) method. In this study, the Just Another Gibbs Sampler (JAGS) software (Plummer, 2015) was used to estimate parameters. The sample code and toy data for the MJ-DINA model is available from <https://.....> Further details about how JAGS is used for Bayesian CDM estimation can be found in Zhan, Jiao, Man, et al. (2019).

In this study, weakly but not non-informative priors are preferentially used to increase the generalizability of our codes by imposing vague prior beliefs on estimating parameters. The setting of priors refers to those in Zhan et al. (2018) and Man and Harring (2019; 2020).

First, Y_{ni} , $\log(T_{ni})$, V_{ni} and α_{nk} are assumed to be distributed as:

$$\begin{aligned} Y_{ni} &\sim \text{Bernoulli}(P(Y_{ni} = 1)), \\ \log(T_{ni}) &\sim N(\xi_i - \tau_n, \omega_i^{-2}), \\ V_{ni} &\sim NB(\exp(\epsilon_n + m_i), d_i^{-2}), \\ \alpha_{nk} &\sim \text{Bernoulli}(P(\alpha_{ni} = 1)). \end{aligned}$$

Then, the priors of the four main item parameters are assumed from a multivariate normal distribution as

$$\begin{pmatrix} \beta_i \\ \delta_i \\ \xi_i \\ m_i \end{pmatrix} \sim MVN_4 \left(\begin{pmatrix} \mu_\beta \\ \mu_\delta \\ \mu_\xi \\ \mu_m \end{pmatrix}, \Sigma_{\text{item}} \right),$$

Furthermore, hyper priors are specified as

$$\begin{aligned} \mu_\beta &\sim N(-1.096, 2), \\ \mu_\delta &\sim N(2.192, 2) \text{ I}(\mu_\delta > 0), \\ \mu_\xi &\sim N(4, 2), \\ \mu_m &\sim N(4, 2), \\ \Sigma_{\text{item}} &\sim \text{InvWishart}(\mathbf{R}, 4), \end{aligned}$$

where $\mathbf{R}$ is a four-dimensional identity matrix. For another two item parameters, the priors are set as

$$\begin{aligned}\omega_i^{-2} &\sim \text{InvGamma}(1, 1), \\ h_i &\sim \text{InvGamma}(1, 1).\end{aligned}$$

In addition, the priors of person parameters are set as

$$\begin{pmatrix} \theta_n \\ \tau_n \\ \varepsilon_n \end{pmatrix} \sim MVN_3 \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \Sigma_{\text{person}} \right).$$

As suggested by Zhan et al. (2018), the Cholesky decomposition of the Σ_{person} is used because the variance of θ_n is set to 1 for identification. Then $\Sigma_{\text{person}} = \Delta_{\text{person}} \Delta'_{\text{person}}$, where

$$\Delta_{\text{person}} = \begin{pmatrix} 1 & & \\ \varphi_{12} & \psi_2 & \\ \varphi_{13} & \varphi_{23} & \psi_3 \end{pmatrix},$$

is a low triangular matrix with positive entries on the diagonal and unrestricted entries below the diagonal, Δ'_{person} is the conjugate transposition of Δ_{person} . The priors of the elements in Δ_{person} are specified as

$$\begin{aligned}\varphi_{12} &\sim N(0, 1), \\ \varphi_{13} &\sim N(0, 1), \\ \varphi_{23} &\sim N(0, 1), \\ \psi_2 &\sim \text{Gamma}(1, 1), \\ \psi_3 &\sim \text{Gamma}(1, 1).\end{aligned}$$

Then, the priors of the higher-order latent structure parameters are specified as

$$\lambda_k \sim N(0, 4), \gamma_k \sim N(0, 4) \text{I}(\gamma_k > 0).$$

where $\text{I}(\cdot)$ is an indicator function.

In addition, the robustness of model parameters to non-informative prior distributions were presented in Section S2 in the online appendix.

3. Simulation Studies

Three simulation studies were conducted. In the first simulation study, model parameter recovery in the MJ-DINA model was examined by using a proposed Bayesian estimation algorithm. Data were generated from the MJ-DINA model across planned simulation conditions, and then refitted to the MJ-DINA model itself for evaluating parameter recovery. The second simulation study aimed to evaluate the consequences of ignoring FC data in the joint modeling, when data were simulated from the MJ-DINA model but analyzed by both the MJ-DINA and JRT-DINA models. Obviously, multimodal data are always be present in practical tests, and the important question is whether recording and analyzing them adds to the analytical process. Comparing these models both with and without biometric data allows for that direct comparison of the relative contribution of biometric data over and above existing data sources. Furthermore, the third simulation study investigated the need for joint modeling when analyzing multimodal data, with data simulated from the MJ-DINA model; however, it was analyzed by both the MJ-DINA model and a so-called separate model (i.e., fitting three modals of data with three measurement models separately).

3.1. Simulation Study 1

3.1.1. Design and Data Generation

Three factors were manipulated to evaluate parameter recovery in the MJ-DINA model: (1) sample size; (2) test length; and (3) correlation matrices. Considering the possible scale of adoption of eye-tracking technology in practical usage (typically, less than 100 participants) and the possibilities for the future, sample sizes were set at $N = 100, 300, \text{ and } 500$. Test lengths were set at $I = 15 \text{ and } 30$, as in previous simulation studies (e.g., Zhan, Liao et al., 2018; Zhan et al., 2021). A theoretical assumption of the joint-hierarchical modeling is that the information provided by each observed variable (e.g., RA and RT) is transmitted through the correlation coefficient between the person

parameters: the higher the degree of correlation, the higher the mutual availability of information and the more it contributes to the accuracy of parameter estimation, and vice versa (Bolsinova & Tijmstra, 2018; Ranger, 2013). Thus, we want to explore how the degree of correlation between person parameters affects the estimation accuracy of the proposed model. Correlation matrices among person parameters are listed as follows:

$$\Phi_{\text{person}} = \begin{pmatrix} 1 & & \\ \rho_{\theta\tau} & 1 & \\ \rho_{\theta\epsilon} & \rho_{\tau\epsilon} & 1 \end{pmatrix} = \begin{pmatrix} 1 & & \\ -0.2 & 1 & \\ 0.2 & -0.2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & & \\ -0.4 & 1 & \\ 0.4 & -0.4 & 1 \end{pmatrix}, \begin{pmatrix} 1 & & \\ -0.8 & 1 & \\ 0.8 & -0.8 & 1 \end{pmatrix}.$$

Such settings, ($\rho_{\theta\tau} < 0$, $\rho_{\theta\epsilon} > 0$, and $\rho_{\tau\epsilon} < 0$) are based on previous studies (e.g., Zhan et al., 2018) and assumes that the higher the student's problem-solving ability, the slower the processing speed and the higher the visual engagement level (i.e., reflective cognitive style); at the same time, the lower the student's problem-solving ability, the faster the processing speed and the lower the visual engagement level (i.e., reflective cognitive style).⁵ To match planned correlations among person-side parameters listed above, examinee trait parameters can be generated from the variance-covariance matrix according to Equation 8 as

$$\begin{pmatrix} \theta_n \\ \tau_n \\ \epsilon_n \end{pmatrix} \sim MVN_3 \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & & \\ 0.5\rho_{\theta\tau} & 0.25 & \\ 0.5\rho_{\theta\epsilon} & 0.25\rho_{\tau\epsilon} & 0.25 \end{pmatrix} \right).$$

Additionally, five attributes ($K = 5$) were measured, and the Q-matrices are presented in Figure 2. One or two identity matrices were contained in the Q-matrix and each attribute was required by more than three items to make the Q-matrix complete (Chiu, 2013), and to make the model identifiable (Gu & Xu, 2021).

Item parameters were generated from a four-dimensional normal distribution

⁵ In practice, this assumption is not always correct. The relationships among the three latent traits depend on actual situations.

$$\begin{pmatrix} \beta_i \\ \delta_i \\ \xi_i \\ m_i \end{pmatrix} \sim MVN_4 \left(\begin{pmatrix} -2.197 \\ 4.394 \\ 4 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 & & & \\ 0.5 & 1 & & \\ -0.25 & -0.25 & 0.25 & \\ -0.25 & -0.25 & 0.125 & 0.25 \end{pmatrix} \right),$$

where the correlation coefficients between items parameters are $\rho_{\beta\delta} = .5$, $\rho_{\beta\xi} = -.5$, $\rho_{\beta m} = -.5$, $\rho_{\delta\xi} = -.5$, $\rho_{\delta m} = -.5$, and $\rho_{\xi m} = .5$, respectively. Such settings are partly due to the results of real data analysis in Zhan et al. (2018) and Man & Harring (2019), in which items with higher intercepts (i.e., guessing probability) tend to be less time- and visual-intensive.⁶ Four main item parameters were then generated according to Equation 8. The item time-discrimination parameter ω_i was generated from a uniform distribution, $U(1.25, 2)$, and item dispersion parameters, h_i , were generated from an inverse-Gamma distribution, $\text{InvGamma}(2, 6)$.

Higher-order latent structural parameters γ_{ks} were all set at 1.5, with $\lambda_k = (-2, -1, 0, 1, 2)$ for the five attributes. Finally, Y_{ni} , $\log(T_{ni})$, V_{ni} , and α_{nk} were generated from Equations 1, 4, 6, and 2, respectively. Fifty datasets were generated in each simulated condition. In each replication, only the higher-order latent structural parameters were fixed, and all other item and person parameters were generated from the previously defined distributions.

Figure 2

K-by-*I* Q' Matrix for Simulation Study 1.

	1*	2*	3*	4*	5*	6	7	8	9	10	11*	12	13	14*	15*	16	17	18*	19	20*	21*	22	23*	24	25	26*	27*	28	29	30*
α_1																														
α_2																														
α_3																														
α_4																														
α_5																														

Note: blank means "0", gray means "1"; "*" denotes items used in the $I = 15$ conditions.

⁶ Again, in practice, this assumption is not always correct. The relationships among the four item parameters depend on the actual situation.

3.1.2. Analysis

Fifty replications were implemented in each condition. In each replicate, two Markov chains with random starting points were used, and each chain ran 35,000 iterations, with the first 25,000 iterations in each chain as burn-in. The remaining 10,000 iterations per chain (20,000 in total) were used for model parameter inferences. The potential scale reduction factor (PSRF; Brooks & Gelman, 1998) was computed to assess the convergence of each parameter. In each replication, all PSRFs were less than 1.2, and around 99% PSRFs were less than 1.05, indicating good convergence in the specified setting.

To evaluate parameter recovery, the bias and root mean square error (RMSE) were computed: $\text{bias}(\hat{x}) = \sum_{r=1}^R \hat{x}_r - x / R$, $\text{RMSE}(\hat{x}) = \sqrt{\sum_{r=1}^R (\hat{x}_r - x)^2 / R}$, where $\hat{x}$ is the estimated value of the model parameter in r th replication and x is the true value of the corresponding model parameter, respectively; R is the total number of replications. The attribute and pattern correct classification rate (i.e., ACCR and PCCR) were computed to evaluate the classification accuracy of individual attributes and patterns: $\text{ACCR} = \sum_{r=1}^R \sum_{n=1}^N I(\hat{a}_{nr} = a_{nr}) / NR$, $\text{PCCR} = \sum_{r=1}^R \sum_{n=1}^N I(\hat{a}_{nr} = a_{nr}) / NR$, where $I(\cdot)$ is an indicator function.

3.1.3. Results

Table 2 presents the recovery of item parameters. Across the board, all item parameters were well recovered in the proposed sample size conditions. The recovery of time-related parameters (i.e., ξ_i and ω_i) and that of visual-related parameters (i.e., m_i and d_i) were both stronger than that of accuracy-related parameters (i.e., β_i and δ_i). Unsurprisingly, increasing the sample size yielded better recovery of all item parameters. Having a longer test improved the recovery of accuracy-related parameters, but had little effect on the rest of the item parameters. It seems that correlations among person parameters had no systematic impact on the recovery of item parameters.

Table 3 summarizes classification accuracy for each attribute and corresponding attribute pattern. ACCRs were higher than 0.9 across all conditions. The lowest PCCR was recorded at nearly 0.7. ACCR and PCCR are only affected by test length; the longer the test, the higher the classification accuracy. Increasing the correlations among person parameters leads to higher classification accuracy. However, having different sample sizes had no systematic effect on classification accuracy.

Table 4 shows the recovery of person parameters. The recovery of latent processing speed and latent visual engagement were both better than the recovery of higher-order ability. This is because only a limited number of attributes provide information directly related to the higher-order ability. The longer test yielded a better recovery of all three person parameters. The sample size had almost no effect on the recovery of person parameters. Moreover, stronger correlations among person parameters resulted in better recovery of all person parameters, particularly in higher-order ability. This indicates that both RTs and FCs provided useful information in facilitating the estimation of the higher-order ability.

Table 3*Recovery of Item Parameters in the Simulation Study 1.*

N	I	Φ	Mean Bias						Mean RMSE						Cor					
			β	δ	ξ	m	ω	d	β	δ	ξ	m	ω	d	β	δ	ξ	m	ω	d
100	15	0.2	-0.005	-0.164	0.000	0.000	-0.011	0.000	0.431	0.736	0.062	0.061	0.115	0.004	0.915	0.711	0.993	0.993	0.852	0.986
		0.4	0.020	-0.197	0.002	-0.003	-0.019	0.000	0.431	0.748	0.068	0.059	0.120	0.004	0.906	0.740	0.992	0.993	0.842	0.983
		0.8	0.033	-0.125	-0.005	-0.005	-0.028	0.000	0.438	0.743	0.059	0.062	0.114	0.004	0.912	0.720	0.993	0.993	0.880	0.987
	30	0.2	0.013	-0.055	0.002	-0.003	-0.028	0.000	0.384	0.691	0.064	0.059	0.114	0.004	0.926	0.763	0.992	0.994	0.875	0.988
		0.4	-0.029	-0.059	-0.006	-0.003	-0.026	0.000	0.382	0.678	0.064	0.061	0.115	0.004	0.929	0.756	0.992	0.993	0.875	0.987
		0.8	-0.005	-0.062	0.001	0.000	-0.024	0.000	0.364	0.681	0.066	0.058	0.114	0.004	0.936	0.759	0.992	0.994	0.885	0.990
	300	0.2	0.003	-0.051	-0.002	0.001	-0.005	0.000	0.328	0.650	0.037	0.036	0.067	0.002	0.945	0.792	0.997	0.998	0.947	0.996
		0.4	0.011	-0.069	0.001	0.005	-0.010	0.000	0.324	0.653	0.034	0.034	0.073	0.002	0.948	0.812	0.998	0.998	0.947	0.995
		0.8	0.009	-0.075	0.002	-0.002	-0.009	0.000	0.313	0.684	0.033	0.036	0.066	0.002	0.948	0.818	0.998	0.998	0.951	0.994
500	15	0.2	-0.003	-0.007	-0.002	-0.002	-0.006	0.000	0.280	0.577	0.029	0.029	0.053	0.002	0.959	0.843	0.998	0.998	0.969	0.997
		0.4	0.026	-0.063	0.000	0.000	0.004	0.000	0.275	0.564	0.029	0.026	0.059	0.002	0.958	0.843	0.998	0.999	0.961	0.997
		0.8	0.010	-0.074	0.003	0.001	-0.008	0.000	0.272	0.546	0.026	0.029	0.055	0.002	0.961	0.849	0.999	0.999	0.965	0.997
	30	0.2	-0.009	-0.007	-0.002	-0.001	-0.002	0.000	0.237	0.473	0.029	0.029	0.053	0.002	0.970	0.889	0.998	0.998	0.971	0.997
		0.4	0.010	-0.044	-0.001	-0.002	-0.003	0.000	0.225	0.473	0.028	0.028	0.052	0.002	0.973	0.888	0.998	0.999	0.972	0.997
		0.8	-0.004	-0.021	-0.001	-0.002	-0.003	0.000	0.221	0.455	0.029	0.026	0.054	0.002	0.974	0.896	0.998	0.999	0.971	0.997

Note: N = sample size; I = test length; Φ = correlation matrix among person parameters; β = item intercept; δ = item interaction; ξ = item time-intensity; m = item visual-intensity; ω = item time-discrimination; d = item visual-discrimination; Mean Bias = mean bias across all items; Mean RMSE = mean root mean squared error across all items; Cor = correlation between true and generated values.

Table 4*Recovery of Attributes in the Simulation Study 1.*

N	I	Φ	ACCR					PCCR
			α_1	α_2	α_3	α_4	α_5	
100	15	0.2	0.912	0.917	0.911	0.898	0.960	0.696
		0.4	0.909	0.918	0.912	0.899	0.960	0.698
		0.8	0.910	0.926	0.921	0.917	0.961	0.704
	30	0.2	0.959	0.953	0.965	0.980	0.989	0.871
		0.4	0.962	0.968	0.967	0.983	0.990	0.885
		0.8	0.959	0.960	0.967	0.985	0.989	0.889
	15	0.2	0.917	0.921	0.916	0.914	0.958	0.706
		0.4	0.914	0.923	0.915	0.919	0.960	0.706
		0.8	0.912	0.927	0.918	0.920	0.963	0.708
300	30	0.2	0.962	0.963	0.966	0.980	0.986	0.875
		0.4	0.963	0.963	0.968	0.982	0.989	0.878
		0.8	0.963	0.962	0.976	0.982	0.989	0.888
	15	0.2	0.912	0.927	0.917	0.910	0.960	0.707
		0.4	0.920	0.920	0.920	0.914	0.958	0.710
		0.8	0.919	0.920	0.922	0.915	0.964	0.714
	30	0.2	0.951	0.960	0.972	0.979	0.990	0.873
		0.4	0.953	0.963	0.970	0.980	0.988	0.879
		0.8	0.954	0.965	0.970	0.982	0.988	0.886

Note, N = sample size; I = test length; Φ = correlation matrix among person parameters; ACCR = attribute correct classification rate; PCCR = pattern correct classification rate.

Table 5*Recovery of Person Parameters in the Simulation Study 1.*

N	I	Φ	Mean Bias			Mean RMSE			Cor		
			θ	τ	ε	θ	τ	ε	θ	τ	ε
100	15	0.2	-0.003	0.003	0.000	0.691	0.149	0.120	0.709	0.955	0.970
		0.4	0.003	-0.001	0.001	0.657	0.147	0.120	0.746	0.954	0.970
		0.8	0.003	-0.002	0.003	0.507	0.142	0.123	0.860	0.958	0.969
	30	0.2	0.004	0.001	0.003	0.660	0.111	0.088	0.743	0.975	0.984
		0.4	0.005	-0.002	0.004	0.608	0.108	0.088	0.788	0.976	0.985
		0.8	-0.001	0.002	-0.001	0.474	0.104	0.080	0.877	0.978	0.987
	15	0.2	-0.001	0.000	-0.001	0.668	0.150	0.121	0.738	0.953	0.970
		0.4	-0.001	0.000	0.000	0.651	0.149	0.119	0.754	0.954	0.971
		0.8	0.002	-0.001	0.001	0.491	0.139	0.117	0.869	0.960	0.972

500	30	0.2	0.001	0.001	0.001	0.633	0.108	0.086	0.768	0.976	0.985
		0.4	0.001	0.000	0.002	0.606	0.110	0.083	0.790	0.975	0.986
		0.8	-0.003	0.002	-0.002	0.465	0.104	0.081	0.883	0.978	0.987
	15	0.2	0.000	-0.001	0.002	0.675	0.150	0.118	0.731	0.953	0.971
		0.4	0.000	0.000	0.000	0.647	0.147	0.117	0.757	0.955	0.972
		0.8	0.000	0.000	0.000	0.486	0.140	0.118	0.872	0.960	0.972
	30	0.2	0.000	-0.001	0.000	0.632	0.108	0.084	0.769	0.976	0.986
		0.4	0.002	-0.002	0.002	0.607	0.107	0.083	0.790	0.976	0.986
		0.8	0.003	-0.001	0.001	0.462	0.104	0.084	0.885	0.978	0.986

Note, N = sample size; I = test length; Φ = correlation matrix among person parameters; θ = higher-order ability; τ = latent processing speed; ε = latent visual engagement; Mean Bias = mean bias across all persons; Mean RMSE = mean root mean squared error across all persons; Cor = correlation between true and generated values.

3.2. Simulation Study 2

3.2.1. Design, Data Generation, and Analysis

In simulation study 2, three factors were manipulated: (1) analytical models; (2) Q-matrices; and (3) the correlation matrix among person parameters. Two analytical models, the MJ-DINA model and the JRT-DINA were fitted separately to the data simulated from the MJ-DINA model, allowing for comparison of respective performance in parameter recovery. This comparison is particularly useful in assessing the consequences (if any) of ignoring the information of FCs in the analysis, since JRT-DINA does not include biometric data. Two types of Q-matrices were compared, in order to evaluate the effectiveness of utilizing auxiliary information to improve the estimation precision of model parameter estimates. One Q-matrix was generated in a relatively simple form, and labeled completeness (see Q1 in Figure 3). The other was generated in a more complex form, and labeled incompleteness (see Q2 in Figure 3). Based on findings from Zhan et al. (2018) that auxiliary information could improve the parameter recovery more when the Q-matrix is incomplete, we hypothesized that there would be a comparative advantage in using the MJ-DINA to estimate parameter precision over the JRT-DINA when the incomplete Q-matrix (i.e., Q2) appears.

The correlation matrix among person parameters, Φ_{person} , was also considered in this comparison. We manipulated the correlation between higher-order ability and latent visual engagement (i.e., $\rho_{\theta\epsilon}$) and between latent processing speed and the visual engagement (i.e., $\rho_{\tau\epsilon}$). As aforementioned, a theoretical assumption of joint-hierarchical modeling is that latent ability estimation can only be improved (compared to only modeling RA) if the latent ability is not orthogonal to other latent traits (e.g., latent processing speed). Thus, we hypothesize that the comparative advantage of the MJ-DINA model over the JRT-DINA model is expected to increase with stronger correlations in $\rho_{\theta\epsilon}$ and $\rho_{\tau\epsilon}$. In this way, three levels of Φ_{person} were manipulated as

$$\Phi_{\text{person}} = \begin{pmatrix} 1 & & \\ \rho_{\theta\tau} & 1 & \\ \rho_{\theta\epsilon} & \rho_{\tau\epsilon} & 1 \end{pmatrix} = \begin{pmatrix} 1 & & \\ -0.4 & 1 & \\ 0.2 & -0.2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & & \\ -0.4 & 1 & \\ 0.4 & -0.4 & 1 \end{pmatrix}, \text{ or } \begin{pmatrix} 1 & & \\ -0.4 & 1 & \\ 0.8 & -0.8 & 1 \end{pmatrix}.$$

The correlation coefficient between higher-order ability and latent processing speed was fixed at a moderate level (i.e., $\rho_{\theta\tau} = -.4$). The test length and sample size were fixed at 15 and 100 (i.e., a condition with limited items and small sample size), respectively. The settings of other parameters (e.g., item parameters and latent structural parameters) were consistent with those in simulation study 1. Finally, the MJ-DINA was used as the true model to generate data. Fifty datasets were generated in each simulated condition.

Analytical processes for evaluating model recovery were identical to those in simulation study 1. Convergence was again strong according to the PSRF. Posterior predictive model checking (PPMC; Gelman et al., 2014) was used to evaluate model-data fit. A posterior predictive probability (*ppp*) value near .5 indicates that there are no systematic differences between the realized and predictive values, and thus an adequate fit of the model. For simplicity, the PPMC using test statistics (i.e., only the differences between realized data and predictive data are compared, without model entities) (Levy & Mislevy, 2016) was employed.⁷ In simulation study 2, the PPMC of the

⁷ The PPMC using discrepancy measures, which focuses on a feature of the data and the model, rather than a feature

measurement model to RA and RTs was considered and evaluated separately, since the JRT-DINA does not include the measurement model for FCs. Both test- and item-level ppp values were provided. Additionally, the log conditional predictive ordinate (LCPO) (Levy & Mislevy, 2016, p. 247) was computed for model selection as

$$LCPO_{ni} = \log \left(\frac{1}{Z} \sum_{z=1}^Z \frac{1}{p(u_{ni} | \Omega_z)} \right)^{-1},$$

where Z is the number of samples from the Markov chain and $p(u_{ni} | \Omega_z)$ is the likelihood of the data for person n and item i (u_{ni}) based on the sampled item and person parameter values at z -th sampling (Ω_z). A test-level LCPO for a model can be computed by the summation of the values of $LCPO_{ni}$ across all persons and all items: $LCPO = \sum_{n=1}^N \sum_{i=1}^I LCPO_{ni}$. The preferred model is the model with the larger LCPO value. Similarly, the LCPO of the measurement model to RA and RTs was considered and evaluated separately.

In order to assess the advantages of the MJ-DINA over the JRT-DINA, the absolute increment (i.e., quantitative improvement) and the relative increment (i.e., advantage ratio) were computed and reported. Specifically, for RMSE, the absolute increment is equal to $(x_{JRT} - x_{MJ})$, and the relative increment is equal to $(x_{JRT} - x_{MJ}) / x_{JRT}$; this is because the smaller the RMSE, the better the parameter recovery. In contrast, for Cor and PCCR, the absolute increment is equal to $(x_{MJ} - x_{JRT})$, and the relative increment is equal to $(x_{MJ} - x_{JRT}) / x_{MJ}$; similarly, this is because the larger the Cor and PCCR, the better the parameter recovery. x_{MJ} and x_{JRT} are the index value (e.g., RMSE or Cor) of the MJ-DINA and the JRT-DINA, respectively.

of the data alone, can also be used (see Bolsinova & Tijmstra, 2018; Zhan et al., 2018).

Figure 3*K*-by-*I* Q' Matrices for Stimulation Study 2.

	Q1															Q2														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
α_1	■					■	■				■	■	■			■	■	■				■	■	■				■	■	■
α_2		■						■				■		■				■	■			■	■		■		■	■		■
α_3			■						■		■			■					■	■		■			■	■		■	■	■
α_4				■					■	■			■	■	■					■	■			■	■		■	■	■	■
α_5					■		■			■		■	■		■	■	■				■		■	■		■		■	■	■

Note, blank means "0", gray means "1"; Q1 = complete Q-matrix; Q2 = incomplete Q-matrix.

3.2.2. Results

Table 5 presents the PPMC. Across all conditions, the *ppp* values were close to .5 for both measurement models of RA and RTs, indicating (1) adequate fit of both models to the RA and RTs, and (2) that ignoring FCs would not cause misfits of joint modeling of the RA and RTs. Further, Table A2 (in the online appendix) presents the improvement of the MJ-DINA over the JRT-DINA in LCPO of RA and RTs. When $\varrho_{\theta\epsilon}$ and $\varrho_{\tau\epsilon}$ were low (0.2), the MJ-DINA fit the data a little worse than the JRT-DINA; however, when $\varrho_{\theta\epsilon}$ and $\varrho_{\tau\epsilon}$ were moderate (0.4) and high (0.8), the MJ-DINA fit the data better than the JRT-DINA. When values of $\varrho_{\theta\epsilon}$ and $\varrho_{\tau\epsilon}$ were higher, the MJ-DINA model demonstrated clear advantages over JRT-DINA in LCPO of RA and RTs.

Table 6 presents the improvement of the MJ-DINA over the JRT-DINA in the recovery of latent attributes (and that of item and person parameters can be found in Tables A3 to A4 in the online appendix). To avoid redundancy, we do not elaborate on the results in detail. First, higher values of $\varrho_{\theta\epsilon}$ and $\varrho_{\tau\epsilon}$ result in clearly superior recovery of model parameters by the MJ-DINA model, compared with JRT-DINA. In addition, in recovery of attributes, the maximum relative increment of PCCR is 5.49%, and in most conditions, the relative increment of PCCR is less than 1%. This phenomenon is not unique to this study; similarly, Ranger (2013) has also pointed out that the gains for model parameter accuracy are limited within the joint-hierarchical modeling generally.

Such results meet predicted expectations for a number of reasons. When the test situation is ideal enough (i.e., with a complete Q-matrix and a sufficient number of items) and RA data alone provides sufficient information for accurate latent attribute estimates, the incorporating of collateral information (e.g., RTs or FCs) would not provide additional help in providing precision in estimation (Zhan et al., 2018). By contrast, there is a comparative advantage of using MJ-DINA over JRT-DINA in parameter recovery when the Q-matrix is incomplete. Thus, when the test scenario is not ideal due to an inappropriately-specified Q matrix, incorporating additional data sources is conducive to obtaining accurate parameter estimates.

In general, the results of simulation study 2 give evidence for our hypotheses regarding the analytic superiority of the MJ-DINA model, both in parameter estimates and in its flexibility.

Table 6

Posterior Predictive Model Checking (PPMC) in Simulation Study 2.

Q	Φ	Model	PPMC_RA	PPMC_RT
Q1	0.2	MJ	0.521	0.502
		JRT	0.520	0.502
	0.4	MJ	0.521	0.501
		JRT	0.521	0.500
	0.8	MJ	0.520	0.502
		JRT	0.520	0.502
Q2	0.2	MJ	0.520	0.501
		JRT	0.521	0.499
	0.4	MJ	0.518	0.501
		JRT	0.518	0.500
	0.8	MJ	0.519	0.501
		JRT	0.521	0.499

Note, Q = Q-matrix; Φ = correlation matrix among person parameters; MJ = the multimodal joint DINA model; JRT = the joint responses and times DINA model; RA = response accuracy; RT = response time.

Table 7*Improvement in the Recovery of Attributes in the Simulation Study 2 (PCCR only).*

Q	Φ	Absolute Increment	Relative Increment
Q1	0.2	0.001	0.15%
	0.4	0.002	0.24%
	0.8	0.011	1.58%
Q2	0.2	0.003	0.63%
	0.4	0.004	0.68%
	0.8	0.030	5.49%

Note, N = sample size; I = test length; Φ = correlation matrix among person parameters; PCCR = pattern correct classification rate; Absolute Increment = $(x_{MJ} - x_{JRT})$; Relative Increment = $(x_{MJ} - x_{JRT}) / x_{MJ}$; x_{MJ} and x_{JRT} is the index value of the MJ-DINA and the JRT-DINA, respectively.

3.3. Simulation Study 3

3.3.1. Design, Data Generation, and Analysis

Many previous joint modelling studies have explored the advantages of using joint modelling for multimodal data analysis. However, most existing studies only focus on the joint modeling of RA and RTs data. As aforementioned, the main theoretical advantage of joint modeling lies in modelling the correlations between the person parameters. Thus, in this study, we focus on the impact of the degree of correlation among latent ability, latent processing speed, and latent visual engagement on the necessity of joint modeling for RA, RTs, and FCs. The correlation matrices among the three person parameters are the same as in simulation study 1, including $\Phi_{\text{person}} = 0.2$, 0.4, and 0.8, indicating low, medium, and high degrees of correlation, respectively. Another two factors were manipulated: (1) sample size, $N = 100$ and 500; and (2) test length, $I = 15$ and 30. The settings of other parameters were consistent with those in simulation study 1. Finally, the MJ-DINA was used as the true model to generate the data. Fifty datasets were generated in each simulated condition. The MJ-DINA model

and a separate model were used to fit the data in each simulated condition.⁸ Analytical processes were identical to those in simulation study 1.

3.3.2. Results

Figure 4 displays the recovery of item parameters (details can be found in Table A5 in the online appendix). Dots in various colors demonstrate the correlations between the item parameter estimates and its corresponding true values. For instance, the blue dot in the condition of $I = 15$ and $N = 100$ shows that the correlation between the β estimates and their true values is around 0.9. In addition, the short colorful lines represent the differences regarding RMSEs between the MJ-DINA model and the separate model. For instance, the blue line in the condition of $I = 15$ and $N = 100$ indicates that the RMSE value of β is lower for the MJ-DINA model than the separate model. Several findings across all conditions were summarized as follows: First, the main difference between the two models (i.e., whether joint modeling is considered or not) was reflected in the recovery of parameters of the measurement model for RA (i.e., item intercept and item interaction). The introduction of joint modeling improves the recovery of item intercept and item interaction, which are relatively the most difficult to estimate accurately among all item parameters. Second, it seems that correlations among person parameters had no systematic impact on the recovery of item parameters, which is consistent with the conclusion of simulation study 1. Third, sample size and test length do not appear to have a systematic effect on the difference between the two models.

⁸ The sample code for the separate model is also available from <https://.....>

Figure 4*Recovery of Item Parameters in the Simulation Study 3.*

Note, MJ-DINA = multimodal joint DINA model; Separate = a separate model (i.e., fitting three modals of data with three measurement models separately); N = sample size; I = test length; Φ = correlation matrix among person parameters; β = item intercept; δ = item interaction; ξ = item time-intensity; m = item visual-intensity; ω = item time-discrimination; d = item visual-discrimination; RMSE = mean root mean squared error across all items; Cor = correlation between true and generated values.

Figure 5 displays the recovery of person parameters (details can be found in Table A6 in the online appendix). First, the main difference between the two models was reflected in the recovery of latent ability, with little effect on latent processing speed and latent visual engagement. Specifically, the joint modeling improves the recovery of latent ability, which is relatively the most difficult to estimate accurately among all person parameters. Second, the higher the correlation among person parameters, the greater the relative advantage of the MJ-DINA model over the separate model. Third, sample size and test length do not appear to have a systematic effect on the difference between the two models.

Figure 5*Recovery of Person Parameters in the Simulation Study 3.*

Note, MJ-DINA = multimodal joint DINA model; Separate = a separate model (i.e., fitting three modals of data with three measurement models separately); N = sample size; I = test length; Φ = correlation matrix among person parameters; θ = higher-order ability; τ = latent processing speed; ε = latent visual engagement; RMSE = mean root mean squared error across all persons; Cor = correlation between true and generated values.

Figure 6 displays the recovery of attributes. There is less variability in the performance of the two models; namely, there is less improvement in the recovery of attributes with the introduction of joint modeling. It can be found that when the correlation among person parameters is high (i.e., $\Phi_{\text{person}} = 0.8$), the PCCR of the MJ-DINA model is around 2% higher than that of the separate model. Of course, that 2% increase is also significant if you agree with the idea of *No Child Left Behind*. Sample size and test length do not appear to have a systematic effect on the difference between the two models.

Overall, the results of Simulation Study 3 indicated that introducing joint modeling

when analyzing multimodal data could help to improve parameter recoveries and classification accuracies, which is in line with the findings of previous joint modelling works.

Figure 6

Recovery of Attributes in the Simulation Study 3.

Note, MJ-DINA = multimodal joint DINA model; Separate = a separate model (i.e., fitting three modals of data with three measurement models separately); N = sample size; I = test length; Φ = correlation matrix among person parameters; ACCR = attribute correct classification rate; PCCR = pattern correct classification rate.

4. Empirical Example

4.1. Data Description and Analysis

A multimodal data set for a real-world mathematics test will now be used to illustrate the application of the proposed MJ-DINA model. Data were collected in an eye-tracking lab setting at a large university on the East Coast of the United States with the institutional review board approval (Man & Harring, 2019), in which a total of $N = 93$ university students with normal or corrected vision were asked to take a test

consisting of $I = 10$ mathematics items. $K = 4$ attributes were included in the test: (α_1) arithmetic, (α_2) algebra, (α_3) geometry, and (α_4) data analysis. The corresponding Q-matrix is presented in Figure 7. The data contain three data modals, RA, RTs, and FCs, which were simultaneously collected.⁹

In this study, the eye-tracker (i.e., Gazepoint; the tutorials can be found at <https://www.gazept.com/tutorials/>) generates the FCs automatically when the respondent starts looking at an item presented on the screen. Typically, FCs are the sum of the number of fixations as defined by the fixation filter, which determines what constitutes a valid fixation. In this study, the position-variance technique, which is based on visual displacement was used (Jacob, 1995), which is the default algorithm for detecting fixations in Gazepoint software. The gaze data clustered in a local gaze-focused region will be counted as a fixation count; the gaze data outside of this region is determined as the start of a new fixation. The device allows the fixation center to drift as subsequent gaze data is collected and added to the current fixation data. Figure 8 displays a box plot representing an aggregate of FCs for the ten items. Further details about data collection can be found in Man and Harring (2019).¹⁰

The MJ-DINA and JRT-DINA models were fitted to the data, with analytical processes identical to those used in the simulation studies. The performance of these two models in test scenarios with small sample sizes has been evaluated in above simulation studies. Convergence was well-achieved according to the PSRF.

Figure 7

K-by-I Q' Matrix for Real Data Example.

	1	2	3	4	5	6	7	8	9	10
α_1										
α_2										
α_3										
α_4										

Note, blank means "0", gray means "1".

⁹ This dataset is not publicly available, as this study contained sensitive information regarding reliability and validity of a high-stakes exam. However, it is available by reasonable request from the corresponding author.

¹⁰ It should be noted that their study used verbal reasoning data, while this study used mathematics data.

Figure 8*Box Plot of Fixation Counts of 10 Items.*

4.2. Results

Table 8 presents the test-level and item-level PPMC. Across all items in all conditions, the *ppp* values were near .5, indicating adequate fit of both models to the real data. Table 9 presents test- and item-level LCPO. Although JRT-DINA was superior to MJ-DINA for certain individual items, from a whole-test perspective, MJ-DINA still fitted the data better than JRT-DINA. Thus, the MJ-DINA provided more accurate diagnostic results on a holistic level than the JRT-DINA, and as a result, only results pertaining to MJ-DINA are discussed below.

Figure 9 presents estimated item parameters. No item interaction parameter (δ_i) is

negative, which means that all items satisfy the monotonicity assumption: $g_i < 1 - s_i$ (Culpepper, 2015). The item intercept parameter (β_i) for the first two items is positive, which indicates that the guessing probability of these two items was higher than .5. A high value of guessing probability indicates that omissive attributes (or additional factors) might contribute to these item responses. Additionally, it can be seen that ξ_i and m_i had similar patterns of change with items. A similar change pattern also can be found between δ_i and β_i .

To better understand the overall relationship between item parameters, the item mean vector and variance and covariance matrix were estimated (see Table A7 in the online appendix). $Q_{\xi m}$ was estimated to be .590, which means that items with higher time-intensity tended to be higher in visual-intensity. Similarly, a moderate positive correlation was revealed between δ_i and β_i , following the trend shown in Figure 5. Additionally, $Q_{\beta \xi}$ and $Q_{\beta m}$ were estimated to be -0.238 and -0.267 , which means that items with higher intercepts (i.e., guessing probability) tended to both involve less time and visual intensity. These findings are consistent with behavioral predictions used to generate item parameters in the simulation studies.

Table 10 presents the estimated person variance and covariance matrix. $Q_{\theta \tau}$ and $Q_{\theta \epsilon}$ were estimated to be -0.125 and 0.182 , respectively, which means that a low negative correlation existed between higher-order ability and processing speed, and a low positive association was revealed between higher-order ability and visual engagement on the test. In addition, processing speed and visual engagement on the test were highly negatively correlated, quite logically indicating that a person with high processing speed tended to spend less visual engagement to the test, and vice versa. It is important to note that a strong correlation between the two variables does not indicate that they reflect the same cognitive or behavioral trait. Height and weight, for example, are highly correlated, but reflect two separate traits.

Figure A2 in the online appendix displays the posterior mixing proportions of all 16 attribute patterns. Attribute pattern (1111) was the most prevalent with a proportion of 46.24%; the next two most prevalent attribute patterns were (1010) and (1000), with a proportion of 12.90% and 11.83, respectively. Furthermore, Table A8 in the online appendix presents the estimates (i.e., posterior means) of all the person parameters of all students. It can be found that the more attributes the students mastered, the higher their latent abilities were. But even so, the latent ability, latent processing speed, and latent visual engagement differed across students within the same attribute pattern (i.e., knowledge structure). The results do not only show that the differences between students are not limited to the differences in knowledge structure, but also reflect the practical application value of the proposed model.

More specifically, Table 11 presents a sample of individual-level feedback of the first five students, including four types of latent variables: latent attribute pattern, latent ability, latent processing speed, and latent visual engagement. Combining the inferences of cognitive characteristics in Table 2, students 1 and 3, for example, have the same knowledge structure, but they have different processing speed and visually engaged in different levels; this suggests that even if they have the same knowledge structure, they may still diverge in cognitive characteristics such as cognitive style and fluency. Moreover, for students with deficits in knowledge structure, targeted interventions should take into account the cognitive characteristics of different individuals in addition to the attributes of the deficits. For example, students 2 and 5 were deficient in attributes 2 and 4, but different targeted interventions may need to be adopted due to their different cognitive characteristics. For student 2 (impulsive + unfocused), since she or he tends to make hasty decisions only based on partial information extracted from the problem situation, in addition to the missing attributes, it can also try to cultivate her or his concentration, and encourage him to examine the

items and answer them carefully. By contrast, student 5 (non-fluency + focuser) seemed to have the motivation or desire to solve the problem, but due to his limited ability, she or he could not extract the critical information in the problem even with a high level of visual engagement. Therefore, for this student, the focus should be on remedying the missing attributes.

Table 8

Posterior Predictive Model Checking (PPMC) in Real Data Example.

Items	MJ-DINA			JRT-DINA	
	PPMC_RA	PPMC_RT	PPMC_FC	PPMC_RA	PPMC_RT
1	0.443	0.494	0.530	0.463	0.513
2	0.450	0.501	0.442	0.447	0.487
3	0.601	0.539	0.553	0.580	0.564
4	0.538	0.516	0.429	0.546	0.502
5	0.552	0.509	0.495	0.538	0.518
6	0.572	0.524	0.522	0.551	0.533
7	0.532	0.472	0.551	0.540	0.474
8	0.603	0.470	0.520	0.600	0.468
9	0.492	0.497	0.552	0.490	0.489
10	0.578	0.501	0.481	0.581	0.466
Test-level	0.511	0.509	0.537	0.499	0.501

Note, RA = response accuracy; RT = response time; FC = fixation count; MJ-DINA = the multimodal joint DINA model; JRT-DINA = joint responses and times DINA model.

Table 9

Log Conditional Predictive Ordinate (LCPO) in Real Data Example.

Items	RA		RT	
	MJ-DINA	JRT-DINA	MJ-DINA	JRT-DINA
1	-27.410	-27.305	-305.263	-304.811
2	-32.595	-32.432	-362.894	-369.066
3	-61.423	-61.525	-315.184	-316.470
4	-52.782	-52.625	-358.382	-365.326
5	-56.214	-56.557	-368.646	-372.546
6	-35.674	-35.511	-376.940	-378.723
7	-57.082	-57.159	-420.540	-418.276
8	-50.745	-50.828	-435.531	-434.952

9	-65.542	-65.735	-430.508	-426.946
10	-41.864	-42.036	-484.484	-479.969
Test-level	-481.331	-481.713	-3858.372	-3867.084

Note, RA = response accuracy; RT = response time; MJ-DINA = the multimodal joint DINA model; JRT-DINA = joint responses and times DINA model; Bold letter indicate larger value and better fit.

Figure 9.

Estimated Item Parameters in Real Data Example.

Note, β = item intercept; δ = item interaction; ξ = item time-intensity; m = item visual-intensity.

Table 10

Person Variance and Covariance Matrix Estimates in Real Data Example.

	θ	τ	ε
θ	1.000	-0.125	0.182
τ	-0.016 (0.033)	0.017 (0.005)	-0.959
ε	0.023 (0.033)	-0.016 (0.004)	0.018 (0.006)

Note, Covariance in lower triangular matrix and correlation coefficient in upper triangular matrix respectively; standard error (standard deviation of posterior distribution) is in parentheses; θ = higher-order latent ability; τ = latent processing speed; ε = latent visual engagement.

Table 11*Example of Individual-Level Diagnostic Results in Real Data Example.*

ID	Attribute Pattern	θ	τ	ε	Corresponding Inference
1	1 1 1 1	0.682	-0.092	0.104	Reflective + Focuser
2	1 0 1 0	-0.584	0.123	-0.127	Impulsive + Non-Focuser
3	1 1 1 1	0.687	0.027	-0.028	Fluency + Non-Focuser
4	1 1 1 1	0.799	-0.175	0.176	Reflective + Focuser
5	1 0 1 0	-0.302	-0.141	0.140	Non-Fluency + Focuser

Note, θ = higher-order latent ability; τ = latent processing speed; ε = latent visual engagement.

5. Conclusion and Discussion

To utilize the multimodal information in the TEA systems, this study proposed a multimodal joint-hierarchical CDM that can jointly analyze RA, RTs, and FCs. The proposed model can provide respondents with a more comprehensive feedback and a more accurate diagnosis, thereby helping provide researchers with a fuller picture of the problem-solving competence of individual respondents. This type of cognitive diagnosis via multimodal data will be increasingly available with the growth of TEA systems. Compared with existing joint CDMs, the proposed model accounts for more modals of data from TEA systems, without losing the analytical power of previous models.

Two simulation studies were conducted to explore the psychometric performance of the proposed model. The results of simulation study 1 mainly indicated that the proposed Bayesian MCMC estimation algorithm recovered model parameters well and that stronger associations between multiple latent traits resulted in better model parameter recovery. The results of simulation study 2 mainly indicated that incorporating FCs in addition to RTs in data analysis can further improve accuracy of model parameters estimation. Although the benefit in terms of estimation accuracy is

limited, the incorporation of FCs can additionally explore the level of latent visual engagement, which could help to obtain a more comprehensive understanding of the individual's problem-solving behavior. An empirical example was also employed to illustrate applications of the proposed model: how to diagnose individual knowledge structure and infer other cognitive characteristics (e.g., cognitive style) based on the results of data analysis.

For practitioners, using the information from multimodal data to infer particular problem-solving-related cognitive characteristics would undoubtedly lead to a more comprehensive understanding of individual differences in the problem-solving competence of respondents. In addition, this understanding can be taken into consideration for individualizing instruction to promote learning outcomes. For instance, based on the results in Table 11, researchers can implement targeted intervention based on the specific attribute pattern as well as determine the best intervention for each student given her or his processing speed and visual engagement level. In short, it allows for much more information than the classical correct/incorrect dichotomy inherent in RA data.

Overall, the proposed model extends the application scope of the joint-hierarchical cognitive diagnosis modeling and indicates that it is feasible to analyze the data other than RA and RTs in diagnostic assessments; meanwhile, it also brings the hope for further use of other modal data to achieve more comprehensive feedback and accurate diagnoses.

Despite promising results, there are still some limitations worthy of further study. First, due to the limitation of hardware conditions (i.e., there are no mass eye-trackers), all explorations in the current study are limited to small-scale assessments. For example, there were only 93 participants in the empirical study, and the measurement models used were as simple as possible. Only the DINA model, the lognormal RT model, and

the NBF model were used as templates to illustrate the conceptualization of the proposed model. With the popularization of portable and low-cost laboratory instruments, we expect and have reason to believe that we can break through the limitation of hardware conditions in the future, and realize the acquisition and analysis of multimodal data in large-scale assessments. At that time, the generalized model may have a greater application prospect. For example, general CDMs (e.g., de la Torre, 2011; Henson et al., 2009), general RT models (e.g., Fox & Marianti, 2016), and general FC models (e.g., Man & Harring, 2019) can be used to increase the generality of the proposed model.

Second, only one modal of data, namely, FCs, was additionally analyzed in this study compared with the existing studies that jointly analyzed RA and RTs. Given the “plug-and-play” nature of the joint-hierarchical modeling framework, the proposed model can be extended by introducing various measurement models for different modals of data. For example, the sequential CDM (Ma & de la Torre, 2016) can be adopted to handle polytomous outcome data; the Rasch Poisson counts model can be adapted to handle item revisit behavior (Bezirhan et al., 2021); the Markov model (Shu et al., 2017) or the multilevel mixture model (Liu et al., 2018) can be used to handle operating steps data. Additionally, other kinds of data sources, such as neural data (Jeon et al., 2021) and the data collected in collaborative problem-solving tasks (A. von Davier & Halpin, 2013) could be incorporated within the same framework. Of course, the psychometric performance of these innovative joint models will need to be tested further across differing categories of data.

Third, this study only focuses on inferring limited cognitive characteristics, such as reflective-impulsive cognitive style, focuser cognitive style, and cognitive fluency. In fact, many other kinds of cognitive characteristics maybe worthy of exploring. Taking the cognitive style as an example, there are also styles such as field-independent vs.

field-dependent, visual vs. verbal, converging thinking vs. diverging thinking, and so on (Riding & Rayner, 2012). From the perspective of comprehensive feedback, whether it is possible to use more modals of data to infer more cognitive characteristics in the future, or even identify some non-cognitive factors (such as motivation, attitude, and belief), is a practical concern.

Fourth, as mentioned in Section 2.6, based on the proposed model, individual cognitive characteristics associated with problem-solving can only be inferred, but not measured or diagnosed. In the future, we can try to directly construct the person parameters of specific cognitive characteristics through categorical variables to realize the measurement of individual cognitive characteristics instead of inference (e.g., Wang & Chen, 2020).

Fifth, the proposed model follows the simple-structure joint-hierarchical modeling framework. Within this modeling framework separate measurement models are used for the RA, for the RT, and for the FC, where RA is taken to be fully explained by latent attributes, RT is taken to be fully explained by the latent processing speed, and FC is taken to be fully explained by the latent visual engagement; see conditional independence assumptions (6) to (8) in the section 2.5. In recent years, to release these conditional independence assumptions, some studies proposed the cross-loading modeling framework (e.g., Bolsinova & Tijmstra, 2018; Molenaar et al., 2015), which attempts to improve the estimation precision of ability by directly extracting the information from collateral data. The cross-loading modeling framework also can be try to introduced into the proposed model in further studies. Another possible advantage of using cross-loading modeling over hierarchical modeling could be: the cross-loading modelling no longer requires a multivariate normal distribution to describe the associations among person parameters. It can also directly exploit the information provided by the collateral data through cross-loading to improve the classification

accuracy of latent attributes. In such a case, the higher-order latent structural model can be removed from the current proposed model.

Sixth, in Bayesian estimation, the prior distribution reflects the beliefs of the data analyst. This study used specific prior distributions based on existing data analysis experience and existing research results (Man & Harring, 2019; 2020; Zhan et al., 2018). Although the results of the robustness analysis show that the proposed model is robust to non-informative priors, it does not mean that the prior distributions used in this study are applicable to all test situations. In subsequent practical applications, data analysts could use hype-priors to explore the appropriate prior distribution for new empirical data.

Last but not least, at present, in comparison with its use in cognitive and experimental psychological research, eye-tracking studies of educational and psychological measurements have been relatively scarce. "This is likely due to the relatively high cost of the equipment as well as the limited knowledge of the technology outside of cognitive psychology and science" (Gorin, 2006). This study is an attempt to investigate the role of eye-tracking data (FCs, specifically) from the perspective of latent variable modeling. This attempt to introduce eye-tracking data into CDMs provides a new perspective for further introducing laboratory instrument-based measures or quantitative methods from cognitive and experimental psychology into traditional cognitive diagnostic assessments, and to some extent expands the paradigm of cognitive diagnosis. However, as a kind of collateral data, eye-tracking data may be more than that. For example, first, only one type of eye-tracking data (i.e., FCs) was considered in the current study. To better comprehensively assess individual differences in terms of cognitive functionalities in responding to test items, more types of eye-tracking data (e.g., the number of saccades and the number of regressions) could be considered. In addition, eye-tracking data can

identify different cognitive processes or strategies for solving problems (Gorin, 2006). It can also be used to verify whether students extract the key information needed to correctly solve the problem from the problem situation (Gardner et al., 1959) and judge the attractiveness of options in multiple-choice items. Overall, the study of eye-tracking or other biometric data in educational and psychological measurements is still in its infancy. Although many issues remain to be explored, psychometricians should be inclusive and open-minded in studying and applying biometric data.

References

- An, L., Wang, Y., & Sun, Y. (2017). Reading words or pictures: Eye movement patterns in adults and children differ by age group and receptive language ability. *Frontiers in Psychology, 8*, <https://doi.org/10.3389/fpsyg.2017.00791>
- Agard, C., & von Davier, A. (2018). *The virtual world and reality of testing: Building virtual assessments*. In Jiao, H., & Lissitz, R. (Eds.), *Technology enhanced innovative assessment: Development, modeling, and scoring from an interdisciplinary perspective* (pp. 1–30). Charlotte, NC: Information Age Publishing
- Bergner, Y., & von Davier, A. (2018). Process data in NEAP: Past, present, and future. *Journal of Educational and Behavioral Statistics*. <http://doi.org/10.3102/1076998618784700>
- Bezirhan, U., von Davier, M., & Grabovsky, I. (2021). Modeling item revisit behavior: The hierarchical speed-accuracy-revisits model. *Educational and Psychological Measurement, 81*(2), 363–387.
- Biancarosa, G., & Shanley, L. (2015). What is fluency? (p.1-18). Cummings, K. D., & Petscher, Y. (Eds.). *The fluency construct: Curriculum-based measurement concepts and applications*. Springer.
- Bolsinova, M., & Tijmstra, J. (2018). Improving precision of ability estimation: Getting more from response times. *British Journal of Mathematical and Statistical Psychology, 71*, 13–38.
- Brooks, S. P., & Gelman, A. (1998). General methods for monitoring convergence of iterative simulations. *Journal of Computational and Graphical Statistics, 7*, 434–455. <https://doi.org/10.2307/1390675>
- Chiu, C.-Y. (2013). Statistical refinement of the Q-matrix in cognitive diagnosis. *Applied Psychological*

- Measurement*, 37, 598–618. <https://doi.org/10.1177/0146621613488436>
- Culpepper, S. A. (2015). Bayesian estimation of the DINA model with Gibbs Sampling. *Journal of Educational and Behavioral Statistics*, 40, 454–476.
- De Boeck, P., & Jeon, M. (2019). An overview of models for response times and processes in cognitive tests. *Frontiers in Psychology*, 10:102.
- de la Torre, J. (2011). The generalized DINA model framework. *Psychometrika*, 76, 179–199.
- de la Torre, J., & Chiu, C.-Y. (2016). A general method of empirical Q-matrix validation. *Psychometrika*, 81(2), 252-273.
- de la Torre, J., & Douglas, J. (2004). Higher-order latent trait models for cognitive diagnosis. *Psychometrika*, 69, 333–353. <https://doi.org/10.1007/BF02295640>
- Ercikan, K., & Pellegrino, J. W. (2017). *Validation of score meaning for the next generation of assessments: The use of response processes*. New York, NY: Taylor & Francis.
- Fox, J.-P., & Marianti, S. (2016). Joint modeling of ability and differential speed using responses and response times. *Multivariate Behavioral Research*, 51, 540–553.
<https://doi.org/10.1080/00273171.2016.1171128>
- Gardner, R. W., Holzman, P. S., Klein, G. S., Linton, H. B., & Spence, D. (1959). Cognitive control: A study of individual consistencies in cognitive behavior. *Psychological Issues*, 1, Monograph 4.
- Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., & Rubin, D. B. (2014). *Bayesian data analysis*. Boca Raton: CRC Press.
- Gorin, J. S. (2006). Test design with cognition in mind. *Educational measurement: Issues and practice*, 25(4), 21-35.
- Graf, S., & Kinshuk. (2007). *Providing adaptive courses in learning management systems with respect to learning styles* (p. 2576-2583). G. Richards. (Eds.) *Proceedings of the World Conference on E-Learning in Corporate, Government, Health care, and Higher Education (e-Learn)*. Chesapeake,VA:AACE Press, 2576-2583.
- Gu, Y., & Xu, G. (2021). Sufficient and necessary conditions for the identifiability of the Q-matrix. *Statistica Sinica*, 31, 449-472.
- He, Q., and Von Davier, M. (2016). Analyzing process data from problem-solving items with n-grams: insights from a computer-based large-scale assessment. in *Handbook of Research on Technology Tools for Real-World Skill Development*, eds Y. Rosen, S. Ferrara, and M. Mosharraf

- (New York, NY: Chapman and Hall/CRC Press), 749–776. doi: 10.4018/978-1-4666-9441-5.ch029
- Henson, R., Templin, J., & Willse, J. (2009). Defining a family of cognitive diagnosis models using loglinear models with latent variables. *Psychometrika*, 74, 191–210.
<https://doi.org/10.1007/s11336-008-9089-5>
- Hitch, G. J., & McAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology*, 82, 375–386
- Jacob, R. J. K. (1995). Eye tracking in advanced interface design. In W. Barfield & T. A. Furness (Eds.), *Virtual environments and advanced interface design* (pp. 258–288). New York, NY: Oxford University Press.
- Jiang, Z., & Ma, W. (2018). Integrating differential evolution optimization to cognitive diagnostic model estimation. *Frontiers in Psychology*, 9, 2142. <https://doi.org/10.3389/fpsyg.2018.02142>
- Jeon, M., De Boeck, P., Luo, J., Li, X., & Lu, Z.-L. (2021). Modeling within-item dependencies in parallel data on test responses and brain activation. *Psychometrika*. Advanced Online.
<https://doi.org/10.1007/s11336-020-09741-2>
- Jiao, H., & Roberts, W. L. (2018). *Technology enhanced innovative assessment: Development, modeling, and scoring from an interdisciplinary perspective*. Charlotte, NC: Information Age Publishing.
- Junker, B. W., & Sijtsma, K. (2001). Cognitive assessment models with few assumptions, and connections with nonparametric item response theory. *Applied Psychological Measurement*, 25, 258–272.
- Justice, M., & Lankford, C. (2002). Pilot findings. *Communication Disorders Quarterly*, 1, 11–21.
- Kagan, J. (1965). Reflection-impulsivity and reading ability in primary grade children. *Child Development*, 36, 609–628.
- Klein Entink, R. H., van der Linden, W. J., & Fox, J.-P., (2009). A box-cox normal model for response times. *British Journal of Mathematical and Statistical Psychology*, 62, 621–640.
<https://doi.org/10.1348/000711008X374126>
- Lahat, D., Adah, T., & Jutten, C. (2015). Multimodal data fusion: An overview of methods, challenges and prospects. *Proceedings of the IEEE*, 103(9), 1449–1477.
- Leighton, J. P., & Gierl, M. (2007). *Cognitive diagnostic assessment for education: Theory and applications*. Cambridge University Press.
- Leighton, J. P., Gierl, M. J., & Hunka, S. M. (2004). The attribute hierarchy method for cognitive

- assessment: A variation on Tatsuoaka's rule-space approach. *Journal of Educational Measurement*, 41, 205-237.
- Levy, R., & Mislevy, R. J. (2016). *Bayesian psychometric modeling*. Boca Raton, FL: CRC Press.
- Liu, H., Liu Y., & Li, M. (2018) Analysis of process data of PISA 2012 computer-based problem solving: Application of the modified multilevel mixture IRT model. *Frontiers in Psychology*, 9:1372.
- Ma, W., & de la Torre, J. (2020). Choosing between CDM and unidimensional IRT: The proportional reasoning test case. *Measurement Interdisciplinary Research and Perspectives*, 18(2):87-96.
- Maris, E.(1999). Estimating multiple classification latent class models. *Psychometrika*, 64, 187–212.
<https://doi.org/10.1007/BF02294535>
- Man, K., & Harring, J. R. (2019). Negative binomial models for visual fixation counts on test items. *Educational and Psychological Measurement*, 79(4), 617-635.
<http://doi.org/0.1177/0013164418824148>
- Man, K., & Harring, J. R. (2020). Assessing pre-knowledge cheating via innovative measures: A multiple-group analysis of jointly modeling item responses, response times, and visual fixation counts. *Educational and Psychological Measurement*. Advanced Online.
<https://doi.org/10.1177/0013164420968630>
- Mayes, S. D., & Calhoun, S. L. (2007). Learning, attention, writing, and processing speed in typical children and children with ADHD, autism, anxiety, depression, and oppositional-defiant disorder. *Child Neuropsychology*, 13:6, 469-493. <https://doi.org/10.1080/09297040601112773>
- Messick, S. (1989). *Cognitive style and personality: Scanning and orientation toward affect*. ETS Research Report Series, RR-89-16. <https://doi.org/10.1002/j.2330-8516.1989.tb00342.x>
- Molenaar, D., Tuerlinckx, F., & van der Maas, H. L. (2015). A generalized linear factor model approach to the hierarchical framework for responses and response times. *British Journal of Mathematical and Statistical Psychology*, 68(2), 197-219. <https://doi.org/10.1111/bmsp.12042>
- Passolunghi, M .C., & Siegel, L. S. (2001). Short-term memory, working memory and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of Experimental Child Psychology*, 80, 44-51.
- Plummer, M. (2015). *Jags: Just another gibbs sampler*. Retrieved from <http://mcmc-jags.sourceforge.net/> (version 4.0.0)

- Poole, A., Ball, L. J., & Phillips, P. (2004). In search of salience: A response-time and eye movement analysis of bookmark recognition. In S. Fincher, P. Markopoulos, D. Moore & R. Ruddle (Eds.), *People and computers xviii: Design for life*. London, England: Springer.
- Ranger, J. (2013). A note on the hierarchical model for responses and response times in tests of van der Linden (2007). *Psychometrika*, 78, 3, 538-544.
- Riding, R., & Rayner, S. (2012). *Cognitive styles and learning strategies: Understanding style differences in learning and behavior*. Routledge, New York.
- Ren, H., Xu, N., Lin, Y., Zhang, S., & Yang, T. (2021). Remedial teaching and learning from a cognitive diagnostic model perspective: Taking the data distribution characteristics as an example. *Frontiers in Psychology*, 12:628607. <https://doi.org/10.3389/fpsyg.2021.628607>
- Shu, Z., Bergner, Y., Zhu, M., Hao, J., von Davier, A. (2017). An item response theory analysis of problem-solving processes in scenario-based tasks. *Psychological Test and Assessment Modeling*, 59(1), 109–131.
- Shute, V. J., & Venture, M. (2013). *Measuring and supporting learning in games: Stealth assessment*. Cambridge, MA: The MIT Press.
- Tatsuoka, K. K. (1983). Rule space: An approach for dealing with misconceptions based on item response theory. *Journal of Educational Measurement*, 20, 345–354.
<https://doi.org/10.1111/j.1745-3984.1983.tb00212.x>
- Tang, F., & Zhan, P. (2021). *Does diagnostic feedback promote learning? Evidence from a longitudinal cognitive diagnostic assessment*. ResearchGate Preprint.
<https://doi.org/10.13140/RG.2.2.23511.19365>
- Unkelbach, C. (2006). The learned interpretation of cognitive fluency. *Psychological Science*, 17(4), 339-345.
- van der Linden, W. J. (2006). A lognormal model for response times on test items. *Journal of Educational and Behavioral Statistics*, 31(2), 181-204.
- van der Linden, W. J. (2007). A hierarchical framework for modeling speed and accuracy on test items. *Psychometrika*, 72(3), 287-308.
- von Davier, A. & Halpin, P. F. (2013). *Collaborative problem solving and the assessment of cognitive skills: Psychometric considerations*. (RR-13-41). ETS research reports.
- von Davier, M., & Lee, Y.-S. (2019). *Handbook of diagnostic classification models: Models and model*

- extensions, applications, software packages*. New York, NY: Springer.
- Wang, C., Chang, H., & Douglas, J. (2013). The linear transformation model with frailties for the analysis of item response times. *British Journal of Mathematical and Statistical Psychology*, 66, 144–168. <http://dx.doi.org/10.1111/j.2044-8317.2012.02045.x>
- Wang, C., & Xu, G. (2015). A mixture hierarchical model for response times and response accuracy. *British Journal of Mathematical and Statistical Psychology*, 68, 456–477. <http://dx.doi.org/10.1111/bmsp.12054>
- Wang, S., & Chen, Y. (2020). Using response times and response accuracy to measure fluency within cognitive diagnosis models. *Psychometrika*, 85, 600–629.
- Wang, S., Zhang, S., Douglas, J., & Culpepper, S. (2018). Using response times to assess learning progress: A joint model for responses and response times. *Measurement: Interdisciplinary Research and Perspectives*, 16(1), 45–58.
- Zhan, P., Jiao, H., & Liao, D. (2018). Cognitive diagnosis modelling incorporating item response times. *British Journal of Mathematical and Statistical Psychology*, 71, 262–286.
- Zhan, P., Jiao, H., Man, K., & Wang, L. (2019). Using JAGS for Bayesian cognitive diagnosis modeling: A tutorial. *Journal of Educational and Behavioral Statistics*, 44, 473–503.
- Zhan, P., Jiao, H., Man, K., Wang, W.-C., & He, K. (2021). Variable speed across dimensions of ability in the joint model for responses and response times. *Frontiers in Psychology*, 12: 469196.

Online Appendix

S1. Fixations

Fixations are the most common feature of looking that eye-tracking researchers analyze to make inferences about cognitive processes or states that they are interested in probing. Fixations are the number of times when our eyes essentially stop scanning the scene at the targeted visual areas so that the visual system can take in detailed information about what is being looked at. In this study, the fixations located in the Area of Interests (AOIs) are counted as the valid counts for examining the level of visual engagement the test-takers put into answering the quantitative questions. The AOIs demonstrated in Figure A1 below in blue and red are defined based on the screen size, font size, and the structure of items after in-depth discussions with scholars from the Educational Neuroscience program. For instance, the red and blue areas cover the parts of an item, one for the item stem, and another for the item options. The total number of fixations (i.e., fixation counts) for this item was the summation of the dots in both areas. The ones that fell outside of the AOIs, caused by unexpected distractions (e.g., surrounding noise), were not counted towards the total number of fixations for answering questions. By eliminating the ones located outside of the AOIs, the measurement errors associated with the fixations could be reduced.

Figure A1

An Example of the Fixations of An Item in the Empirical Example.

S2. Robustness of Model Parameter Estimates to Non-Informative Priors

S.2.1. Data Generation

In Bayesian estimation, the posterior distribution of model parameters will be affected by their prior distribution, especially for a test scenario with small sample size and a limited number of items. The used weakly informative priori distributions in the main text are set by reference to existing studies and authors' experience. A small-scale simulation study was conducted to further explore the robustness of the proposed model to non-informative priors.

One of the simulated conditions in the simulation study 1 was used here. Specifically, $I = 15$ and $N = 100$ (i.e., a condition with limited items and small sample size), respectively. $K = 5$ attributes were measured, and the Q-matrix is given in Figure 2 in the main text. In addition, person parameters were generated from a three-dimensional normal distribution

$$\begin{pmatrix} \theta_n \\ \tau_n \\ \varepsilon_n \end{pmatrix} \sim MVN_3 \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & & \\ 0.5 \times (-0.4) & 0.25 & \\ 0.5 \times (0.4) & 0.25 \times (-0.4) & 0.25 \end{pmatrix} \right).$$

The settings of other parameters (e.g., item parameters and latent structural parameters) were consistent with those in simulation study 1. Finally, the MJ-DINA was used as the true model to generate fifty data sets.

S.2.2. Non-informative Priors

Corresponding to those weakly informative prior distributions, the non-informative prior distributions were set as

$$\begin{aligned} \mu_\beta &\sim N(-1.096, 10), \mu_\delta \sim N(2.192, 10) \text{ I}(\mu_\delta > 0), \mu_\xi \sim N(0, 10), \mu_m \sim N(0, 10), \\ \omega_i^{-2} &\sim \text{InvGamma}(10, 10), h_i \sim \text{InvGamma}(10, 10), \\ \varphi_{12} &\sim N(0, 10), \varphi_{13} \sim N(0, 10), \varphi_{23} \sim N(0, 10), \psi_2 \sim \text{Gamma}(10, 10), \psi_3 \sim \text{Gamma}(10, 10), \\ \lambda_k &\sim N(0, 10), \gamma_k \sim N(0, 10) \text{ I}(\gamma_k > 0). \end{aligned}$$

S.2.3. Analysis and Results

For continuous variables (e.g., item parameters and person parameters), the consistent bias ($\text{cbias}(\hat{v}) = \sum_{r=1}^R \hat{v}_r - \hat{v}_r^* / R$, where $\hat{v}_r$ and $\hat{v}_r^*$ are the parameter estimates under the weakly informative priors and non-informative priors in r th replication, respectively) and the correlation coefficient (Cor) between the parameter estimates of two sets of priors were computed. For latent attributes, the classification consistency index (CCI) was used to evaluate the degree of classification consistency of the two sets of priors:

$$\text{CCI} = \frac{\sum_{r=1}^R \sum_{n=1}^N I(\hat{\mathbf{a}}_{nr} = \hat{\mathbf{a}}_{nr}^*)}{NR},$$

where $\hat{\mathbf{a}}_{nr}$ and $\hat{\mathbf{a}}_{nr}^*$ are the estimated attribute pattern of person n under the weakly

informative priors and non-informative priors in r th replication, respectively.

Table A1 presents the model parameter estimation consistency between two sets of priors. For both item parameters and person parameters, the parameter estimates under the two sets of priors have a high degree of consistency. In addition, the CCI was 0.948, indicating a there is a quite high degree of classification consistency between the two sets of priors. Overall, the results shown that the proposed model is robust to different priors even in the condition with limited items and small sample size.

Table A1

Estimation Consistency between Weakly Informative Priors and Non-informative Priors.

Parameters		Mean cbias	Cor
Item Parameter	β	0.004	0.999
	δ	0.065	0.998
	ξ	0.000	1.000
	ω	-0.108	0.996
	m	0.002	1.000
	d	-0.003	0.996
Person Parameter	θ	0.014	0.993
	τ	0.003	0.999
	ε	0.002	1.000

Note, β = item intercept; δ = item interaction; ξ = item time-intensity; ω = item time-discrimination; m = item visual-intensity; d = item visual-discrimination; Mean cbias = mean consistent bias across all items or across all persons; Cor = correlation between true and generated values.

Table A2*Improvement in Log Conditional Predictive Ordinate (LCPO) in Simulation Study 2.*

Q	Φ	RA		RT	
		$LCPO_{MJ} > LCPO_{JRT}$	Absolute Increment	$LCPO_{MJ} > LCPO_{JRT}$	Absolute Increment
Q1	0.2	FALSE	-0.121	FALSE	-0.083
	0.4	TRUE	0.462	TRUE	0.163
	0.8	TRUE	4.842	TRUE	3.677
Q2	0.2	FALSE	-0.144	FALSE	-0.125
	0.4	TRUE	0.290	TRUE	0.355
	0.8	TRUE	4.714	TRUE	4.145

Note, Q = Q-matrix; Φ = correlation matrix among person parameters; MJ = the multisource joint DINA model; JRT = the joint responses and times DINA model; Absolute Increment = $(x_{MJ} - x_{JRT})$, where x_{MJ} and x_{JRT} is the index value of the MJ-DINA and the JRT-DINA, respectively.

Table A3*Improvement in the Recovery of Item Parameters in the Simulation Study 2.*

Q	Φ	Absolute Increment								Relative Increment							
		β		δ		ξ		ω		β		δ		ξ		ω	
		RMSE	Cor	RMSE	Cor	RMSE	Cor	RMSE	Cor	RMSE	Cor	RMSE	Cor	RMSE	Cor	RMSE	Cor
Q1	0.2	-0.001	-0.002	0.005	-0.003	-0.001	0.000	0.000	0.000	-0.23%	-0.24%	0.67%	-0.39%	-1.59%	0.00%	0.00%	-0.01%
	0.4	0.003	-0.001	0.002	0.003	0.000	0.000	0.000	0.000	0.64%	-0.15%	0.25%	0.36%	0.00%	-0.01%	0.00%	-0.01%
	0.8	0.004	-0.002	0.008	0.014	-0.001	0.000	0.000	0.001	0.86%	-0.25%	1.05%	2.00%	-1.69%	0.00%	0.00%	0.07%
Q2	0.2	-0.003	-0.001	0.004	0.005	-0.001	0.000	0.000	0.000	-0.78%	-0.07%	0.51%	0.53%	-1.64%	-0.01%	0.00%	0.00%
	0.4	0.000	-0.002	0.016	0.006	0.000	0.000	0.000	0.001	0.00%	-0.18%	1.97%	0.78%	0.00%	0.00%	0.00%	0.14%
	0.8	0.002	-0.001	0.002	0.010	-0.001	0.000	0.000	0.001	0.50%	-0.09%	0.25%	1.42%	-1.64%	0.00%	0.00%	0.09%

Note, Q = Q-matrix; Φ = correlation matrix among person parameters; β = item intercept; δ = item interaction; ξ = item time-intensity; ω = item time-discrimination; RMSE = root mean squared error across all items; Cor = correlation between true and generated values; Absolute Increment = $(x_{JRT} - x_{MJ})$ for RMSE and $(x_{MJ} - x_{JRT})$ for Cor, respectively; Relative Increment = $(x_{JRT} - x_{MJ}) / x_{JRT}$ and $(x_{MJ} - x_{JRT}) / x_{MJ}$ for RMSE and Cor, respectively; x_{MJ} and x_{JRT} is the index value of the MJ-DINA and the JRT-DINA, respectively.

Table A4*Improvement in the Recovery of Person Parameters in the Simulation Study 2.*

Q	Φ	Absolute Increment				Relative Increment			
		θ		τ		θ		τ	
		RMSE	Cor	RMSE	Cor	RMSE	Cor	RMSE	Cor
Q1	0.2	-0.001	0.000	0.000	0.000	-0.15%	0.00%	0.00%	0.00%
	0.4	0.006	0.005	0.000	0.000	0.91%	0.67%	0.00%	0.04%
	0.8	0.175	0.130	0.009	0.006	26.40%	14.91%	6.08%	0.61%
Q2	0.2	-0.011	-0.010	0.000	0.000	-1.55%	-1.46%	0.00%	0.00%
	0.4	0.010	0.008	0.000	0.000	1.42%	1.17%	0.00%	0.04%
	0.8	0.201	0.159	0.015	0.009	28.51%	18.51%	9.87%	0.95%

Note, Q = Q-matrix; Φ = correlation matrix among person parameters; θ = higher-order ability; τ = latent speed; RMSE = root mean squared error across all persons; Cor = correlation between true and generated values; Absolute Increment = $(x_{JRT} - x_{MJ})$ for RMSE and $(x_{MJ} - x_{JRT})$ for Cor, respectively; Relative Increment = $(x_{JRT} - x_{MJ}) / x_{JRT}$ and $(x_{MJ} - x_{JRT}) / x_{MJ}$ for RMSE and Cor, respectively; x_{MJ} and x_{JRT} is the index value of the MJ-DINA and the JRT-DINA, respectively.

Table A5*Recovery of Item Parameters in Simulation Study 3.*

N	I	Φ	Model	Bias						RMSE						Cor					
				β	δ	ξ	m	ω	d	β	δ	ξ	m	ω	d	β	δ	ξ	m	ω	d
100	15	0.2	MJ-DINA	-0.005	-0.164	0.000	0.000	-0.011	0.000	0.431	0.736	0.062	0.061	0.115	0.004	0.915	0.711	0.993	0.993	0.852	0.986
			Separate	-0.034	-0.138	-0.002	0.000	-0.010	0.000	0.443	0.778	0.061	0.060	0.115	0.004	0.909	0.659	0.993	0.993	0.852	0.986
	0.4	MJ-DINA	0.020	-0.197	0.002	-0.003	-0.019	0.000	0.431	0.748	0.068	0.059	0.120	0.004	0.906	0.740	0.992	0.993	0.842	0.983	
		Separate	-0.016	-0.176	0.003	-0.001	-0.019	0.000	0.438	0.773	0.067	0.059	0.120	0.004	0.902	0.702	0.992	0.993	0.842	0.984	
	0.8	MJ-DINA	0.033	-0.125	-0.005	-0.005	-0.028	0.000	0.438	0.743	0.059	0.062	0.114	0.004	0.912	0.720	0.993	0.993	0.880	0.987	
		Separate	-0.014	-0.046	-0.003	-0.002	-0.027	0.000	0.460	0.786	0.057	0.059	0.114	0.004	0.901	0.694	0.993	0.993	0.878	0.987	
30	0.2	MJ-DINA	0.013	-0.055	0.002	-0.003	-0.028	0.000	0.384	0.691	0.064	0.059	0.114	0.004	0.926	0.763	0.992	0.994	0.875	0.988	
		Separate	-0.021	-0.050	0.001	0.000	-0.028	0.000	0.389	0.722	0.063	0.057	0.114	0.004	0.926	0.738	0.992	0.994	0.875	0.987	
	0.4	MJ-DINA	-0.029	-0.059	-0.006	-0.003	-0.026	0.000	0.382	0.678	0.064	0.061	0.115	0.004	0.929	0.756	0.992	0.993	0.875	0.987	
		Separate	-0.074	-0.019	-0.002	0.001	-0.025	0.000	0.399	0.713	0.063	0.058	0.115	0.004	0.926	0.719	0.992	0.993	0.875	0.987	
	0.8	MJ-DINA	-0.005	-0.062	0.001	0.000	-0.024	0.000	0.364	0.681	0.066	0.058	0.114	0.004	0.936	0.759	0.992	0.994	0.885	0.990	
		Separate	-0.043	-0.042	0.000	-0.002	-0.024	0.000	0.378	0.716	0.065	0.057	0.114	0.004	0.935	0.721	0.992	0.993	0.884	0.990	
500	15	0.2	MJ-DINA	-0.003	-0.007	-0.002	-0.002	-0.006	0.000	0.280	0.577	0.029	0.029	0.053	0.002	0.959	0.843	0.998	0.998	0.969	0.997
			Separate	-0.023	-0.018	-0.001	-0.001	-0.006	0.000	0.290	0.578	0.028	0.029	0.053	0.002	0.957	0.844	0.998	0.998	0.969	0.997
	0.4	MJ-DINA	0.026	-0.063	0.000	0.000	0.004	0.000	0.275	0.564	0.029	0.026	0.059	0.002	0.958	0.843	0.998	0.999	0.961	0.997	
		Separate	0.000	-0.059	-0.001	-0.001	0.004	0.000	0.280	0.560	0.028	0.025	0.059	0.002	0.958	0.841	0.998	0.999	0.960	0.997	
	0.8	MJ-DINA	0.010	-0.074	0.003	0.001	-0.008	0.000	0.272	0.546	0.026	0.029	0.055	0.002	0.961	0.849	0.999	0.999	0.965	0.997	
		Separate	-0.017	-0.094	0.002	0.002	-0.008	0.000	0.274	0.551	0.026	0.028	0.056	0.002	0.962	0.844	0.999	0.999	0.964	0.997	
30	0.2	MJ-DINA	-0.009	-0.007	-0.002	-0.001	-0.002	0.000	0.237	0.473	0.029	0.029	0.053	0.002	0.970	0.889	0.998	0.998	0.971	0.997	
			Separate	-0.034	-0.017	-0.001	0.000	-0.002	0.000	0.247	0.489	0.028	0.028	0.053	0.002	0.970	0.877	0.998	0.998	0.971	0.997
	0.4	MJ-DINA	0.010	-0.044	-0.001	-0.002	-0.003	0.000	0.225	0.473	0.028	0.028	0.052	0.002	0.973	0.888	0.998	0.999	0.972	0.997	
		Separate	-0.009	-0.065	0.000	-0.001	-0.003	0.000	0.222	0.494	0.028	0.027	0.052	0.002	0.974	0.873	0.998	0.999	0.972	0.997	
	0.8	MJ-DINA	-0.004	-0.021	-0.001	-0.002	-0.003	0.000	0.221	0.455	0.029	0.026	0.054	0.002	0.974	0.896	0.998	0.999	0.971	0.997	
		Separate	-0.025	-0.037	0.001	-0.001	-0.003	0.000	0.231	0.474	0.028	0.026	0.054	0.002	0.972	0.884	0.998	0.999	0.970	0.997	

Table A6*Recovery of Person Parameters in Simulation Study 3.*

N	I	Φ	Model	Bias			RMSE			Cor		
				θ	τ	ε	θ	τ	ε	θ	τ	ε
100	15	0.2	MJ-DINA	-0.003	0.003	0.000	0.691	0.149	0.120	0.709	0.955	0.971
			Separate	0.000	0.000	0.000	0.695	0.149	0.120	0.714	0.955	0.970
		0.4	MJ-DINA	0.003	-0.001	0.001	0.657	0.147	0.120	0.746	0.954	0.970
			Separate	-0.001	0.000	-0.001	0.692	0.150	0.120	0.713	0.953	0.970
		0.8	MJ-DINA	0.003	-0.002	0.003	0.507	0.142	0.123	0.860	0.958	0.969
			Separate	-0.002	0.000	0.000	0.698	0.150	0.128	0.706	0.953	0.966
	30	0.2	MJ-DINA	0.004	0.001	0.003	0.657	0.111	0.088	0.743	0.975	0.984
			Separate	0.003	0.000	0.001	0.660	0.111	0.088	0.745	0.975	0.984
		0.4	MJ-DINA	0.005	-0.002	0.004	0.608	0.108	0.088	0.788	0.976	0.985
			Separate	0.003	0.001	0.000	0.634	0.108	0.087	0.766	0.976	0.985
		0.8	MJ-DINA	-0.001	0.002	-0.001	0.474	0.104	0.080	0.877	0.978	0.987
			Separate	-0.001	0.001	0.002	0.641	0.108	0.081	0.761	0.976	0.987
500	15	0.2	MJ-DINA	0.000	-0.001	0.002	0.675	0.150	0.118	0.731	0.953	0.971
			Separate	0.000	0.000	0.000	0.686	0.150	0.118	0.721	0.953	0.971
		0.4	MJ-DINA	0.000	0.000	0.000	0.647	0.147	0.117	0.757	0.955	0.972
			Separate	0.000	0.000	0.001	0.686	0.148	0.118	0.720	0.954	0.971
		0.8	MJ-DINA	0.000	0.000	0.000	0.486	0.140	0.118	0.872	0.960	0.972
			Separate	0.000	0.000	0.000	0.686	0.150	0.124	0.721	0.953	0.968
	30	0.2	MJ-DINA	0.000	-0.001	0.000	0.632	0.108	0.084	0.769	0.976	0.986
			Separate	0.000	0.000	0.000	0.640	0.108	0.084	0.763	0.976	0.986
		0.4	MJ-DINA	0.002	-0.002	0.002	0.607	0.107	0.083	0.790	0.976	0.986
			Separate	0.000	-0.001	0.000	0.642	0.108	0.083	0.761	0.976	0.986
		0.8	MJ-DINA	0.003	-0.001	0.001	0.462	0.104	0.084	0.885	0.978	0.986
			Separate	0.000	0.000	0.000	0.643	0.108	0.086	0.761	0.976	0.985

Table A7*Item Mean Vector and Variance and Covariance Matrix Estimates in Empirical Example.*

μ_{item}		Σ_{item}				
		β	δ	ξ	m	
μ_{β}	-0.598 (0.454)	β	2.007 (1.297)	0.610	-0.238	-0.267
μ_{δ}	1.967 (0.520)	δ	1.145 (1.147)	1.868 (2.089)	-0.118	-0.154
μ_{ξ}	3.344 (0.174)	ξ	-0.199 (0.324)	-0.092 (0.332)	0.324 (0.181)	0.590
μ_m	4.729 (0.178)	m	-0.229 (0.335)	-0.121 (0.341)	0.202 (0.150)	0.340 (0.340)

Note, In Σ_{item} , covariance in lower triangular matrix and correlation coefficient in upper triangular matrix respectively; standard error (standard deviation of posterior distribution) is in parentheses; β = item intercept; δ = item interaction; ξ = item time-intensity; m = item visual-intensity

Figure A2

Posterior Mixing Proportions of Attribute Patterns in Empirical Example.

Table A8*Estimates of All Person Parameters of All Students in Empirical Example.*

Attribute Pattern	θ	τ	ε	Attribute Pattern	θ	τ	ε
1 1 1 1	0.682	-0.092	0.104	1 1 1 0	-0.033	-0.122	0.123
1 1 1 1	0.687	0.027	-0.028	1 1 1 0	-0.116	0.031	-0.031
1 1 1 1	0.799	-0.175	0.176	1 1 1 0	-0.244	0.063	-0.072
1 1 1 1	0.734	-0.090	0.100	1 1 1 0	-0.190	-0.064	0.060
1 1 1 1	0.736	-0.043	0.049	1 1 1 0	-0.299	0.190	-0.193
1 1 1 1	0.720	-0.021	0.031	1 1 1 0	-0.057	0.009	-0.013
1 1 1 1	0.542	0.108	-0.106	1 1 1 0	-0.220	0.013	-0.016
1 1 1 1	0.484	0.092	-0.092	1 1 0 0	-0.090	0.095	-0.102
1 1 1 1	0.604	-0.081	0.090	1 1 0 0	-0.312	-0.041	0.038
1 1 1 1	0.374	-0.068	0.064	1 1 0 0	-0.049	-0.121	0.128
1 1 1 1	0.478	0.091	-0.086	1 1 0 0	-0.057	-0.094	0.096
1 1 1 1	0.541	-0.075	0.080	1 1 0 0	-0.551	0.144	-0.155
1 1 1 1	0.269	0.037	-0.030	1 1 0 0	-0.163	-0.101	0.098
1 1 1 1	0.408	-0.155	0.162	1 0 1 1	-0.127	-0.021	0.020
1 1 1 1	0.322	-0.001	0.002	1 0 1 1	-0.160	0.002	-0.006
1 1 1 1	0.239	0.057	-0.060	1 0 1 1	-0.144	0.072	-0.075
1 1 1 1	0.670	-0.024	0.028	1 0 1 0	-0.584	0.123	-0.127
1 1 1 1	0.252	0.041	-0.035	1 0 1 0	-0.302	-0.141	0.140
1 1 1 1	0.024	0.254	-0.251	1 0 1 0	-0.336	0.056	-0.062
1 1 1 1	0.312	0.021	-0.026	1 0 1 0	-0.676	0.096	-0.101
1 1 1 1	0.768	-0.020	0.021	1 0 1 0	-0.562	-0.030	0.025
1 1 1 1	0.395	0.036	-0.033	1 0 1 0	-0.616	0.092	-0.091
1 1 1 1	0.486	0.079	-0.071	1 0 1 0	-0.100	-0.036	0.038
1 1 1 1	0.492	-0.005	0.013	1 0 1 0	-0.737	-0.036	0.034
1 1 1 1	0.707	0.088	-0.084	1 0 1 0	-0.550	-0.083	0.075
1 1 1 1	0.644	-0.078	0.084	1 0 1 0	-0.489	0.121	-0.128
1 1 1 1	0.926	-0.130	0.141	1 0 1 0	-0.191	-0.111	0.110
1 1 1 1	0.718	-0.016	0.021	1 0 1 0	-0.639	-0.017	0.011
1 1 1 1	0.434	0.194	-0.197	1 0 0 1	-0.177	-0.080	0.085
1 1 1 1	0.322	-0.028	0.023	1 0 0 1	-0.334	0.190	-0.200
1 1 1 1	0.811	0.013	-0.011	1 0 0 0	-0.974	0.139	-0.151
1 1 1 1	0.748	-0.089	0.097	1 0 0 0	-0.608	0.056	-0.058
1 1 1 1	0.462	-0.123	0.123	1 0 0 0	-0.589	-0.134	0.131
1 1 1 1	0.217	0.129	-0.129	1 0 0 0	-0.613	0.116	-0.124
1 1 1 1	0.595	-0.019	0.027	1 0 0 0	-0.405	-0.008	0.002
1 1 1 1	0.904	-0.137	0.148	1 0 0 0	-0.771	0.036	-0.044
1 1 1 1	0.632	-0.171	0.177	1 0 0 0	-0.414	0.022	-0.021
1 1 1 1	0.795	-0.072	0.083	1 0 0 0	-0.535	0.078	-0.088
1 1 1 1	0.472	0.038	-0.037	1 0 0 0	-0.344	-0.103	0.098
1 1 1 1	0.609	0.039	-0.035	1 0 0 0	-0.777	0.056	-0.065
1 1 1 1	0.600	0.087	-0.086	1 0 0 0	-0.725	-0.151	0.141
1 1 1 1	0.573	0.029	-0.025	0 0 1 0	-0.753	0.047	-0.056
1 1 1 1	0.895	-0.092	0.102	0 0 0 1	-0.502	0.162	-0.177
				0 0 0 0	-0.578	-0.201	0.204
				0 0 0 0	-0.536	-0.126	0.125
				0 0 0 0	-0.991	0.060	-0.069
				0 0 0 0	-1.118	0.174	-0.187
				0 0 0 0	-0.760	-0.130	0.135
				0 0 0 0	-0.900	-0.067	0.060
				0 0 0 0	-0.887	0.016	-0.027

Note, sort by attribute pattern; θ = latent ability; τ = latent processing speed; ε = latent visual engagement.