On the Multitude of Mathematics Skills – Spatial Numerical Associations and Geometry Skill?

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Cipora K., Schroeder P.A., & Nuerk H.-C. (2018). On the multitude of mathematics skills – Spatial-Numerical Associations and geometry skill?. In: Mix K., Battista M. (eds) *Visualizing Mathematics. Research in Mathematics Education* (pp. 361-371). Springer, Cham. https://doi.org/10.1007/978-3-319-98767-5_18

Version 3 of the preprint. Adjusted title and information on citation.

Battista et al. rightly point out the importance of the relationship between spatial and geometric reasoning. They are also working towards more fine-grained analyses considering different aspects of both spatial and mathematical reasoning, and postulate that analyses should examine particular skills, rather than these very general constructs. The authors support their claim with the results of a series of well-designed and carefully conducted studies, using one-on-one interviews, one-on-one teaching experiments, and case studies. They conclude that the ability to visualize objects and build accurate mental models thereof is linked to property-based spatial reasoning.

Attempting to relate correlational psychological results to the perspective by Battista et al., we can identify important gaps in the literature and between disciplines. Of particular interest, the detailed skills required for geometric problem solving could partially substantiate explicit and implicit SNAs as introduced in our taxonomy, with the former also being reflected in the introspective reports of mental models by their participants. Overall, these observations invoke us to consider three distinctions:

- (i) variations in math skill operationalization
- (ii) variations in spatial dimensions (which can be combinations of extension and the different directionalities) and processing (e.g., spatial imagery, spatial reasoning)
- (iii) the links between (i) and (ii), which are in turn dependent on the choice of math and space operationalization, but also perhaps on individual and instructional differences.

Variation in math skill operationalization

We agree with Battista et al. in general, but in this comment, we would like to point out that their interesting contribution about geometry underlines the necessity of considering a broader range of mathematical skills when looking at potential correlates of SNA. To date, the vast majority of studies have focused on the potential relationship between SNA and arithmetic skill, which diverges from geometry skill (e.g., Lourenco, Bonny, Fernandez, & Rao, 2012). For an overview, Table 1 summarizes studies reporting a relationship between the SNARC effect (perhaps the most prominent SNA) and measures of math skill.

Table 1. Arithmetic skill and directional implicit space-number associations in adults (top rows) and children (bottom rows), in chronological order.

Study	Age	N	Impairments / special skills	SNA task and effect	Math skill assessment	Correlation	Group comparison	Direction	Bayesian or Power testing (or just non-significance)
ADULTS									
Dehaene et al. (1993), Exp 1	Adults	20	10 literary vs. 10 science students	Parity Judgment: interaction term (SNARC effect)	field of study	-	$ps < .10^{\dagger}$	No significant difference	-
Fischer & Rottmann (2005)	Adults	20	10 psychology vs 10 math students	Parity Judgment b (SNARC effect) for positive & negative integers	field of study	-	$p = .28$	No significant difference	-
Bonato et al. (2007), Exp. 1	Adults	20	10 psychology vs. 10 engineering students	Fraction magnitude comparison	field of study	-	$p = .65$	No significant difference	-
Bull et al. (2013), Exp. 2	Adults	40	-	Go-Nogo Colour Judgment: b (SNARC)	Numerical operations subtest (WIAT-II-UK)	$r = -.07$	-	No significant correlation	-
Cipora & Nuerk (2013)	Adults	71	Math-related (18) and math-unrelated studies (53)	Parity Judgment: b (SNARC effect)	Equation verification	$r = .14$	$p = .58$	No significant correlation or difference	88 % posterior probability of Null
Hoffmann et al. (2014)	Adults	95	38 literary-related vs. 38 science students vs. 19 students with math difficulties (self-identified)	Parity Judgment: b and β (SNARC effect)	Arithmetic tasks (Arith & FastMath)	$r = .28$	$p < .05$	SNARC effect was increased with decreased math proficiency	-

Göbel et al. (2015) ^a	Adults	114	British and Arab adults	Parity judgment: β (SNARC effect)	WRAT Arithmetics	$r = .02$	-	No significant correlation for either country	-
Cipora et al. (2016)	Adults	44	Doctoral students of mathematics (14), engineering (15), humanities and social sciences (15)	Parity Judgment: b and β (SNARC effect)	field of study	-	$p = .03^*$	Smaller SNARC in mathematicians vs. humanities students	65-67 % posterior probability of Null for the other group comparisons
CHILDREN									
Bachot et al. (2005)	7-12-year olds	32	Visuospatial disability VSD (16), matched controls (16)	Magnitude Comparison: b (SNARC effect)	Group comparison	-	$p < .05^*$	No SNARC in VSD	-
Schneider et al. (2009), Exp. 2	5.-6. grade	110	-	Parity Judgment: β (SNARC effect)	Mathematics mark	$r = .08$	-	No significant correlation	-
Hoffmann et al. (2013)	kindergarten children	84	-	Magnitude Judgment: b (SNARC effect)	Verbal counting strategies	$r = -.30^*$	-	SNARC emerges with transition to magnitude classification without counting	-
					Arabic digit writing	$r = -.40^{**}$		Stronger SNARC when Arabic digit writing was better	
Crollen et al. (2015)	6-13-year-olds	30	Poor visuospatial ability NVLD (15), matched controls (15)	Magnitude Comparison: b (SNARC effect) NLE, Line Bisection	Group comparison	-	$p < .05^*$	No SNARC in LVLD in hands-crossed posture	-
Crollen & Noël, (2015)	4. grade	70	Weak vs. high visuospatial groups (each 25, 35/65 th percentile split)	Magnitude Comparison: by category (SNARC effect)	Group comparison	-	$p = .20$	No significant differences between groups and postures	-

Gibson & Maurer (2016)	6-8-year olds	60	-	Magnitude Comparison: compatibility ratio (SNARC effect)	Test of Early Mathematics Ability (TEMA-3)	$r_p = .13$	-	No significant correlation	-
Georges et al. (2017b)	3.-4. grade	55	-	Parity Judgment: b (SNARC effect)	Heidelberg Mathematics Test HRT 1-4	$r = -.28^*$	-	better math with stronger SNARC	-

Note. b = indicates unstandardized regression coefficient, β = standardized regression coefficient, *by category*: refers to comparison of dRTs between small and large magnitude numbers. PAE = percentage of absolute error, * significant correlation / comparison / interaction term at $p < .05$ or ** $p < .01$ or † trend for significance at $p < .10$. ^a Correlation results from the study were reported at the 5th Workshop on Numerical Cognition in Tübingen, Germany, 2017.

Inspection of Table 1 shows that math skill was operationalized diversely across the different studies. In adult studies, it was either determined by field of study or by standardized arithmetic tasks. In children studies, math skill was operationalized according to scores in arithmetic tasks or in other tasks involving the use of numbers in Arabic notation. Three studies compared groups that were formed based on visuospatial skill level. One study considered school grades in math. To sum up, one can say that these operationalizations differ considerably. Obviously, this can be a problem, especially if construct validities between different operationalizations of math skills are mediocre or low. For instance, Lourenco et al. (2012) report that arithmetic performance in a task with and without time pressure correlated only moderately ($r = .57$). Another measure of math skill - geometry performance only weakly correlated with timed arithmetic task performance ($r = .25$), and moderately with untimed one ($r = .56$). All these correlations account for no more than 33% of variance. In such cases, the choice of “math” assessment itself contributes to the outcome of whether space and math are found to be related.

Regarding adult studies, the field of study can be a proxy measure of general math skill. However, it also may reflect a variety of other factors leading to the choice of a field of study, such as interests, attitudes, learning motivation or anxiety (Maloney & Beilock, 2012). As regards using calculation tasks, its difficulty level covers skills that are formally required from all students at the level of secondary school (Cipora & Nuerk, 2013). Therefore, the differences in performance might originate from the use of inefficient strategies, working memory overload, math anxiety, or decreased performance due to time pressure. Similar inconsistencies can be also found in children studies. Math grade can be a kind of proxy for general math performance, but apart from the actual performance in different tasks it is affected by individuals’ motivation, attitudes, anxiety, etc. Therefore, such measures seem to be largely heterogeneous and the construct validity of the dependent variable is at least questionable.

Keeping all that in mind, one might argue that despite the disadvantages listed above, (mixed) calculation scores are relatively valid measures of math skill. Nevertheless, even within (speeded) arithmetic operations, there are considerable differences in cognitive processes needed to solve problems. The score might reflect different math skills, but may also register the automaticity of processing. Multiplication, especially, has been attributed to fast fact retrieval (Grabner et al., 2009), which supposedly has different underlying neurocognitive components than other arithmetic operations. In case of multiplications of multi-digit numbers a considerable working memory load appears. On the other hand, addition and subtraction usually require more advanced cognitive processing than fact retrieval because different procedural steps have to be chosen and applied. Importantly, different operations seem to engage different brain regions both in adults (Arsalidou & Taylor, 2011; Prado et al., 2011), and in children (Prado, Mutreja, & Booth, 2014). The evidence on division is least conclusive, but it seems that easy problems are solved by means of inverse multiplication (Huber, Fischer, Moeller, & Nuerk, 2013), which is not necessarily the case in complex division.

Considering abovementioned dissociations, one can expect that the relationship between SNAs and math skill may be different within these operations, so that treating them as equivalent operationalizations of one construct called “math” or “arithmetic” might in fact lead to confusing and contradictory results. One might speculate that SNAs relate more strongly to performance in tasks which do not solely depend on fact retrieval (like simple multiplications), but rather on magnitude manipulation (such as additions, subtractions, and possibly divisions). Nevertheless, this requires further investigation.

So far, we have largely referred to school mathematics like arithmetic operations. However, the picture gets even more complicated if one considers more complex math reasoning, including algebra, conducting proofs, or calculus. In such occasions the processes required to successfully solve the problem comprise operating on symbols, transforming them according to certain rules, and finding complex relationships between them. This set of operations approaches the definition criteria for fluid intelligence (see Cipora et al., 2016 for arguments), whereas the arithmetic calculations themselves play a secondary role. The multitude of mental operations required to succeed in math gets ever broader if one considers geometry, to which we wish to draw reader's attention in this commentary.

Variation in visuo-spatial skills operationalization

In the literature one can find evidence that SNAs are related to visuospatial skills. Such results can be observed for different SNA categories, e.g., Number Line Estimation (Sella, Sader, Lolliot, & Cohen Kadosh, 2016). Unsurprisingly, individuals characterized with higher visuospatial skills perform the number line estimation task more accurately. There is also evidence indicating that the SNARC effect depends on individual differences in visuospatial skills. Namely, a stronger effect of mental rotation (i.e., larger increase in the time to decide on object identity depending on the degree to which it is rotated) relates to more pronounced SNARC (Viarge, Hubbard, & McCandliss, 2014). In a more recent study it was shown that an individual's visualization profile moderates the relationship between magnitude and parity SNARC (Georges, Hoffmann, & Schiltz, 2017a). Importantly, visuospatial skills themselves are also correlated with different types of math skill at several stages of development (Mix et al., 2016). Like math skills, visuo-spatial skills and processes must also be differentiated when their relation to math is investigated (see Mix et al., 2016).

Keeping these developments in mind, one can postulate that the branch of math, which quantifies physical space by means of mathematical concepts, that is, geometry, may be related to SNAs. Some spatial mapping of the magnitude of geometrical entities has already been demonstrated. Specifically, angle magnitude is spatially mapped (Fumarola et al., 2016): large angles are responded to faster with the left hand and small angles with the right hand. This effect was only present in engineering students, but not in psychology students. The authors' interpretation is that education and practice shape this kind of spatial association. All these findings together suggest that investigating the relationship between SNAs and geometry skill will be an interesting new avenue of research.

Links between math skills and visuospatial skills.

If one wishes to investigate the relationship between math skill and spatial skills, it is important to keep in mind that both these constructs are very heterogeneous. Therefore, the links between those skills, namely between space and number are also rather heterogeneous. As we have already expressed in our contribution to this volume (Chapter 4), SNAs should not be treated as part of the same melting pot, but rather as a family of phenomena. They share some common traits (numbers are somehow related to space), but are also largely differentiated, and there is no strong common denominator of all SNA. They can rather be characterized by Wittgenstein's family resemblance (Wittgenstein, 1953), that is, their similarity comes from multiple and partly overlapping features and not from a core set of features shared by all SNA instances.

Conclusions

When investigating the relationship between number and space, the heterogeneity on all sides, concerning math skill, visuo-spatial skills and their relations should be thoroughly considered. In particular, a multitude of different math skills should be acknowledged as well as the fact that

different members of the “SNA family” can particularly strongly relate to certain members of the “math skill family”. Such strong relationships need to be investigated, and this can be far more instructive than attempts to capture the very blurred and inaccurate general picture. Finally, the relations between the space-number association, math skills and spatial skills need to be investigated in a differentiated way. As pointed out by Battista et al, there is compelling evidence for a relationship between visuospatial skill and math skill. This observation encourages us to investigate the relationship between the “SNA family” and the relatively neglected member of the “math skill family”, that is the geometry skill.

Acknowledgments

KC is supported by a DFG grant [NU 265/3-1] to HCN. KC and HCN are further supported by the LEAD Graduate School & Research Network [GSC1028], which is funded within the framework of the Excellence Initiative of the German federal and state governments. We thank Julianne Skinner for proofreading the manuscript.

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