Title

Influence of Task Characteristics on Eye-Movement Patterns Related to Numerical and
Textual Information in Arithmetic Word Problems

Running title

Eye-Movements in Word Problems

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Abstract

Both linguistic and arithmetic task characteristics contribute to the difficulty of a word problem. However, the role of these characteristics and the exact cognitive processes underlying arithmetic word problems are often not clear, but they might be detectable by analysing eye-movement patterns. Not much is known about how eye-movements change under different linguistic and arithmetic task characteristics in regard to the whole problem as well as to specific parts of it (numerical and textual elements). This study examined the effects of linguistic and arithmetic task characteristics on the word problem-solving ability of children aged 10-13 years while their eye-movements were monitored. We manipulated the task characteristics independently of each other, using the mathematical factor operation (addition/subtraction) and the linguistic factors consistency (consistent/inconsistent) and nominalization (verbalized/nominalized). The results showed that eye-movements generally increase with increasing linguistic (i.e., nominalization) or arithmetic (i.e., operation) difficulty. However, specific parts of the text were differentially affected based on task characteristics: In general, increasing arithmetic difficulty shifts eye-movements towards numerical elements and increasing linguistic difficulty shifts eye-movements towards textual elements. However, the increase of difficulty in the arithmetic domain can also affect processing in the linguistic domain. For instance, as textual parts of the word problem were more frequently attended to when arithmetic difficulty increased but not vice versa. This indicates that in the process of word problem-solving, text comprehension and calculation are not sequential independent processes, but partially rely on the same processing components, such as working memory resources.

Keywords

eye tracking, arithmetic word problem, nominalization, lexical consistency, operation

Introduction

Behavioural studies have shown that both linguistic and arithmetic task characteristics contribute to the difficulty of a word problem (Barbu & Beal, 2010; Boonen, van der Schoot, van Wesel, de Vries, & Jolles, 2013; Daroczy, Wolska, Meurers, & Nuerk, 2015; Davis-Dorsey, Ross, & Morrison, 1991; De Corte, 1988; De Corte & Verschaffel, 1987; De Corte, Verschaffel, & De Win, 1985; De Corte, Verschaffel, & Pauwels, 1990; Lewis & Mayer, 1987; Orrantia & Múñez, 2013; Orrantia, Múñez, San Romualdo, & Verschaffel, 2015). However, relatively little is known about the exact relationship between arithmetic and linguistic task characteristics and the process of word problem-solving. The key question is how people read (initial reading process), understand and create a mathematical model (i.e., integration and transition process which results in a mental model) and choose a calculation, which in the end they carry out (calculation process) for different types of word problems.

In research and modelling of word problem-solving there is a debate about the existence or non-existence of common processing stages in the problem-solving processes: some suggest a (sequential) separation of the cognitive processes of reading and mathematical problem-solving (Fuchs et al., 2008; Fuchs, Gilbert, Fuchs, Seethaler, & Martin, 2018; Powell & Fuchs, 2014), while others argue for joint processing stages between the processes (Bergqvist & Österholm, 2010; Dehaene & Cohen, 1995). Whether the problem-solving processes are distinct or rely on joint processes can be reflected in the interaction of arithmetic and linguistic task characteristics (Daroczy, Meurers, Heller, Wolska, & Nürk, 2020).

In behavioural studies not all interaction effects between arithmetic and linguistic task characteristics can be detected because reaction times may be the same for different factors but do not reflect which parts of the text required greater attention, and thus play an important part in creating the mathematical mental model. Even if there is an interaction, we do not know

whether it comes from arithmetic influences on linguistic factors and/or vice versa. On the other hand, with other methods, like eye tracking, we can get a deeper insight into the processes underlying linguistic and arithmetic difficulties and about the word problem-solving process (Hegarty, Mayer, & Green, 1992; Van der Schoot, Arkema, Horsley, & van Lieshout, 2009). For example, it is assumed that the elements in a word problem that are looked at longer are central to the construction of the representation (i.e., mental model) and production of the solution (Van der Schoot et al., 2009). Monitoring eye-movements can provide this information. Therefore, the aim of the current study was to investigate the contribution of linguistic and arithmetic task characteristics using eye tracking in order to understand the direct influence of the task characteristics on the process of word problem-solving as well as the possible common processing phases.

The direct influence of textual difficulty has already been shown to be visible in eye-movement data. Namely, increasingly difficult word problems elicit greater and more prolonged attention as well as re-readings of the overall problem (Hegarty, Mayer, & Monk, 1995; Strohmaier, Lehner, Beitlich, & Reiss, 2019). Additionally, linguistically complex word problems seem to increase the frequency and length of regressions to the number, relational term and variable names, as compared to linguistically less complex problems (Van der Schoot et al., 2009; Verschaffel, De Corte, & Pauwels, 1992). In this study, we expected to replicate these results.

Less is known about how task characteristics influence specific parts of a text. It has already been shown that highly capable students differ in looking at the numbers and keywords; and other part of the text compared to students who solve word problems with less accuracy (De Corte et al., 1990; Hegarty et al., 1992; Van der Schoot et al., 2009; Verschaffel et al., 1992). Nevertheless, students who display less accuracy were also able to apply successful

strategies to solve word problems with low semantic difficulty and showed similar eye-movements for specific parts of the problems as compared with highly capable students (Van der Schoot et al., 2009). This might indicate that task characteristics elicit an attentional shift in distinct eye-movement patterns targeting specific parts of the word problem in general; however, this has never been investigated for change problems, that is, problems that usually start with an initial quantity and require an action that causes either an increase or decrease in that quantity (Riley, Greeno, & Heller, 1983).

Lastly, this study also aimed to explore how the interaction of task characteristics manifests in eye-movement behaviour with the objective of investigating the separability of linguistic and arithmetic processes. Existing word problem-solving models – the propositional theory (e.g., Kintsch & Greeno, 1985) and the cyclic model (e.g., Bergqvist & Österholm, 2010) – focus heavily on how the language (especially the first reading process) influences mathematical problem-solving. Yet, the models do not address if and how arithmetic processes affect the mental model. Eye-movements over numbers were not analysed separately from eye-movements over keywords in the majority of the previous studies, especially in those investigating the process of problem-solving and the creation of the mental model (Hegarty et al., 1992; Hegarty et al., 1995; Van der Schoot et al., 2009). In order to understand the relation of calculation and comprehension processes in word problem-solving, it is especially important to separate areas of interest for numbers, keywords and other textual elements. Our hypothesis was that if linguistic and arithmetic processes are fully separable, increased linguistic complexity should lead to greater attention to the linguistic part of the problem only and increased arithmetic complexity should lead to greater attention to only the arithmetic part (i.e., the numbers) of the problem. On the other hand, if there is an interaction the linguistic/arithmetic complexity should lead to different eye-movement patterns for both arithmetic and linguistic parts of the problem. The interaction could be uni- or bidirectional, for example,

linguistic complexity could influence arithmetic processing or arithmetic complexity could influence linguistic processing.

In sum, very little is known about how task characteristics – i.e., different levels of linguistic and arithmetic difficulty – influence attention on word problems as indexed by eye-movement patterns as regards the whole problem or important parts of the text. This is very important because eye-movement analysis can detect both direct influences and interactions of the problem-solving processes. In the following paragraphs, we elaborate on the important linguistic and arithmetic task characteristics of word problems and lay out which ones we chose to examine in this eye tracking study with children.

Arithmetic Task Characteristics of Word Problems

Arithmetic factors influencing the difficulty of word problems are for instance, arithmetic operation (Terao et al., 2004), the types of numbers and calculation (De Corte, 1988), the number of calculations (Terao et al., 2004) and the magnitude of the numbers (Thevenot & Oakhill, 2005). Often, such arithmetic factors are neglected in eye tracking studies on word problems, but when they are systematically manipulated they usually influence performance. An arithmetic factor, which changes the underlying mathematical structure of a word problem, is arithmetic operation. For instance, subtraction is considered to be more difficult than addition (Artemenko, Pixner, Moeller, & Nuerk, 2018) and can be solved by different strategies (e.g., by indirect addition or direct subtraction Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009). Operation is a relevant factor for investigating arithmetic processes and their relation to other problem-solving processes.

Linguistic Task Characteristics of Word Problems

Linguistic factors, just like arithmetic factors, can influence the difficulty of a word problem.

Lexical consistency concerns specific keywords in the text (Hinsley, Hayes, & Simon, 1977).

These keywords can be either consistent or inconsistent with the operation required for the correct solution. To address any incongruency between the text and the correct solution (i.e., for inconsistent problems), there is a need for different strategies than for consistent problems (Boonen, de Koning, Jolles, & van der Schoot, 2016). There is empirical evidence for the consistency effect (i.e., longer reaction times and higher reversal error rates and errors resulting from the choice of the opposite operation) for word problems containing inconsistent compared to consistent keywords (Hegarty et al., 1992; Hegarty et al., 1995; Pape, 2003; Van der Schoot et al., 2009; Verschaffel et al., 1992). One main goal of eye tracking research and word problems has been to locate the origin of the consistency effect (Hegarty et al., 1995; Verschaffel et al., 1992) and to postulate in which phase of the solution process the consistency effect occurs. It is hypothesised that the additional processing required for inconsistent problems, as compared to consistent problems, occurs not in the first reading period but during later phases of the solution process. Also, interactions between consistency and other factors play an important role. For instance, it has been shown that consistency interacts with operation, likely due to semantic complexity (Van der Schoot et al., 2009). In eye tracking studies involving word problems one way to increase the semantic complexity of (inconsistent) compare problems is to manipulate the relational term – i.e. markedness. Markedness refers to a central concept of linguistics in which one linguistic element is more distinctively identified (marked) than another (unmarked) element; however, the linguistic factor markedness cannot be independently manipulated from the factor operation (Van der Schoot et al., 2009). Because the factors markedness and operation are confounded, the independent analysis of eye-movement patterns for these factors as well as for their interaction is not possible. Therefore,

in this study we needed a factor that increased semantic complexity independently of lexical consistency.

Even simple linguistic changes to the text can increase the difficulty of problem comprehension (Boonen et al., 2013; Van der Schoot et al., 2009). Contrary to markedness, some of the above-mentioned factors can be manipulated independently from operation and lexical consistency. Here, we use the factor **nominalization**, (a syntactic form that turns verbs into nouns, e.g., the Nominalized form of the verb “to earn” is “the earning”) which leads to longer reading times (Gibson & Warren, 2004). Nominalization has been shown to influence the reading difficulty of word problems (Prediger, Wilhelm, Büchter, Gürsoy, & Benholz, 2015) but does not affect the underlying arithmetic structure of the problem. As mentioned before nominalization, contrary to markedness, serves to increase semantic difficulty. Since nominalization is related to reading difficulty it therefore can be associated more with the initial reading phase.

There is agreement in the literature that both linguistic and arithmetic attributes contribute to word problem difficulty (for a review see Daroczy et al., 2015) but the studies provide contradictory evidence concerning the relation between linguistic and arithmetic factors. While the interaction between linguistic and arithmetic factors has been shown to manifest in the solution process (De Corte et al., 1990; Hegarty et al., 1992; Van der Schoot et al., 2009; Verschaffel et al., 1992), it has not yet been shown in eye-movement behaviour.

Eye tracking in Word Problem Research

A general assumption in eye tracking research and mathematics is that more numerous and difficult stimuli lead to longer fixations (Knoblich, Ohlsson, & Raney, 2001). Fixation time reflects the sum of the gaze time devoted to the areas of interest. This has also been observed

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in the case of word problems: students spent more fixation time on the whole problem for complex problems than for simple problems (De Corte et al., 1990). Another important measure for the duration of eye-movements, which has been applied in the domain of word problems, is regression time, where people look back at previously fixated areas (Frisson & Pickering, 1999; Rayner, 1998; Van der Schoot et al., 2009). Refixations and regressions are believed to often result from comprehension issues because additional processing for more difficult problems is assumed to occur during phases after the first reading phase (De Corte et al., 1990; Hegarty et al., 1992; Kintsch & Greeno, 1985; Verschaffel et al., 1992), and is associated with putting more effort into building a problem model (Strohmaier et al., 2019). Additionally, in the study of Strohmaier et al. (2019) global measures, for example mean fixation time, while considering the problem as one text, were associated both with task difficulty and abilities of the reader. Therefore, we would expect that more difficult task characteristics elicit longer fixation times on the whole problem and a greater frequency of regressions to previously seen areas of the problem.

Much less is known about how linguistic and arithmetic task characteristics generally guide attention to various parts of the text. The assumptions made by studies on reading also hold for studies on arithmetic word problem-solving: the elements in a word problem that are fixated upon longer are assumed to be more deeply processed and, consequently, central to the construction of the representation and production of the solution (Van der Schoot et al., 2009). For example, linguistically more complex word problems are hypothesised to induce longer and more numerous regressions to the numbers, relational terms and variable names than linguistically less complex problems (Van der Schoot et al., 2009; Verschaffel et al., 1992). Additionally, Nuerk, Moeller, Klein, Willmes, and Fischer (2011) found that carry problems were associated with a longer total reading time and a greater number of regressions than non-carry problems. Therefore, in this current study we expected increased linguistic difficulty to

shift eye-movements mostly towards the (difficult) textual elements, and increased arithmetic difficulty to shift movements mostly towards the (difficult) numerical elements, as these parts require more attention. This hypothesis has also been proposed to explain the results of previous behavioural studies (e.g., Daroczy et al., 2015, for a review), but whether or not it is true (as indexed by eye-movements) has never been tested before.

If the cognitive processes of reading and calculation are fully separable for word problems, increased linguistic difficulty should lead to increased attention towards the textual part of the problem only and arithmetic processes on the numerical part of the problem only. Nevertheless, if linguistic and arithmetic difficulty interact or rely on a common processing phase, they should lead to a different eye-movement pattern also in those areas which correspond to the other part of the problem associated with the manipulation (i.e., linguistic elements for greater arithmetic difficulty and numerical elements for greater linguistic difficulty).

Linguistic factors do not only influence linguistic difficulty, but also interact with arithmetic factors. One reason why linguistic difficulty should affect numerical areas is that the linguistic attributes of word problems can influence the difficulty of the calculation. For instance, Bassok, Chase, and Martin (1998) and Boonen et al. (2016) have shown that functionally connected objects (e.g., tulips–vases) are easier to add up in word problems than categorically related objects (e.g., tulips–daisies). This suggests that depending on problem formulation the calculation process might be different. Such an interaction should be especially pronounced in the case of lexical consistency (Verschaffel et al., 1992). If we see increased fixation time on numbers for inconsistent problems, it is evidence that in some cases complex linguistic factors make the calculation more difficult.

Arithmetic factors not only influence mathematical difficulty, but also interact with linguistic factors. As concepts of addition and subtraction (e.g., giving, taking) are evoked in everyday language (Carraher, Schliemann, & Carraher, 1988), they can be easily connected to linguistic features in word problems (Lave, 1992). Calculation failures are mostly hypothesised to result from the improper construction of a mental model for the word problem, this would indicate that operation is involved in the building of a mental model, which, furthermore, is associated with greater attention and re-readings of the textual elements of the word problem (Hegarty et al., 1992). This indicates that not only the numbers, but also the textual elements of a word problem could be affected by a change in the eye-movement pattern when the arithmetic difficulty increases.

Underlying Cognitive Processes. Investigating eye-movement behaviour can shed light on underlying cognitive processing (De Corte et al., 1990; Hegarty et al., 1992; Hegarty et al., 1995; Verschaffel et al., 1992). Many studies rely on the model of Kintsch and Greeno (1985) and assume that word problem-solving requires four distinct phases (Mayer, 1984): an initial reading phase (translation of the text), integration (mental representation and the construction of the problem model), planning (generating a solution plan) and solution execution (calculation). If participants construct a model, they frequently focus on specific parts of the text while reading through the problem (De Corte et al., 1990; Hegarty et al., 1992; Van der Schoot et al., 2009; Verschaffel et al., 1992). The proposed models for word problem-solving suggest differences in terms of the relationship between problem-solving phases. Some suggest a separation of the cognitive processes of reading and calculation (Fuchs et al., 2008; Fuchs et al., 2018; Powell & Fuchs, 2014), while others argue for an interaction (Bergqvist & Österholm, 2010; Dehaene & Cohen, 1995). According to the propositional theory model, the textual difficulty (here: nominalization) affects only the initial text comprehension processing and, also, arithmetic processing does not interfere with processing phases other than the calculation

phase. However, other types of models contradict these assumptions and suggest common or interactive processing for linguistic and numerical information: For instance, Bergqvist and Österholm (2010) suggest that text formulation affects not only the first reading but also other phases in the solution process and that problem comprehension and computational processes interact. This means that the model is based on the assumption that we cannot separate the reading and solving of arithmetic tasks. The first reading creates a version of the mental representation. The authors also found that carrying out a calculation adds to the mental representation, while re-reading actually changes the existing mental representation.

The propositional, sequential and cyclic models suggest different eye-movement patterns, which we investigated in the present study. First, if the propositional theory was true, we expected a clear difference between the measures regression time and fixation time for word problems, because fixation time also involves the initial reading phase, and regression time is associated with the phases after the initial reading period. This would mean that nominalization, which may be associated with the initial reading phase, should increase the total fixation time but not the regression time on the text or the numbers because it would only increase the reading difficulty. On the other hand, consistency, which is associated with the mental model building phase (Verschaffel et al., 1992), should affect the regression times. This is supported by findings that lexical consistency elicited more regressions on the other parts of the text than the keyword of the texts (Hegarty et al., 1992). According to the propositional model, only the factor operation should affect the numbers because it is involved in the calculation process. According to models which argue for the non-sequentiality of problem-solving, nominalization should also affect the numerical areas. This suggests that the initial reading process influences another phase of problem-solving and is part of the mental representation phase. Similarly, if operation influences eye-movement behaviour over the text, text comprehension would also be confounded with the calculation process during word

problem-solving when the difficulty of the operation increases. In this way, eye-movements enable a deeper understanding of the (possibly interactive) influence of task characteristics on the process of word problem-solving.

The Present Study

In this study, we aimed to investigate how linguistic and arithmetic difficulty affect the processing of the whole problem as well as specific parts of (i.e., numerical and textual elements). In this within-subject design we included one arithmetic factor and two linguistic factors orthogonally. The arithmetic factor was operation (addition vs. subtraction) and the linguistic factors were lexical consistency (consistency vs. inconsistency) and nominalization (verbalization vs. nominalization).

The first hypothesis was that longer fixation and regression times and lower accuracy could be expected for the whole problem for generally complex items (subtraction, inconsistency, nominalization). This hypothesis is a conceptual replication of previous eye tracking studies using word problems and serves as a validity test. The second more specific hypothesis not previously tested as such was that with the increasing difficulty of arithmetic information eye-movements are shifted towards numerical elements and that with the increasing difficulty of linguistic information eye-movements are shifted towards textual elements. Moreover, we expected an interaction of linguistic and arithmetic factors on the total fixation time as well as on regression times on specific areas of areas of interests. Namely, for linguistically complex word problems (lexical inconsistency, nominalization), we expected longer fixations not only on the textual elements but also on the numerical elements of the word problem. For arithmetically complex word problems we expected greater fixation time not only on the numerical elements but also the textual elements.

Methods

Participants

In total, 42 children between 10 and 13 years old were tested. All children were native German speakers with normal or corrected-to-normal (only with soft contact lenses) vision and had no history of neurological or psychological disorders. The study was approved by the local ethics committee of the Medical Faculty of the University of Tuebingen and all procedures were in accordance with the latest version of the Declaration of Helsinki.

Out of the 42 children, 33 children (19 boys, 14 girls; age: $M = 11.48$, $SD = 0.66$ years) were included in the analysis. Participants were excluded because of technical problems ($n = 4$), incomplete data ($n = 2$), or an overall error rate above 40% ($n = 3$).

Apparatus

We used a remote version of the SR Research EyeLink 1000 eye tracker (from SR Research Ltd, Ottawa, Ontario) that does not require a forehead/chin rest and uses a sampling rate of 500 Hz. The system tracks eye-movements by measuring changes in pupil position in a video image. The best eye to record was automatically selected during a nine-point calibration procedure. Calibration is the process used to set the eye tracking software to accurately track eye-movements. Calibration was accepted when the worst error in gaze position was smaller than 1.5° and the average error was smaller than 1.0° . Participants in the experiment were seated such that the distance between their eyes and the monitor (51 cm diagonally with a resolution of 1025 x 768), on which the word problems were displayed, was approximately 70 cm. The height of the chair was adjusted based on the stature of each child so that the line of sight was standardized across all participants.

Stimuli and Design

The study had a 2 x 2 x 2 within-subject design with the factors operation (addition/subtraction), consistency (consistent/inconsistent) and nominalization (verbalized/nominalized; see Table 2). Forty-eight simple one-step arithmetic word problems in German were designed for the study (see Supplementary Material 1), each consisting of 4 sentences. The first sentence was to be used as a reference point (e.g., “*A market woman is selling apples at the market.*”). The second and the third sentences contained the two numbers necessary for the mathematical operation (e.g., “*The market woman arrived with 42 apples. She sold 17 apples.*”). The fourth and final sentence contained the question (e.g., “*How many apples did she have at the end?*”).

The problems were constructed in such a way that manipulation of one factor did not affect the other factors (i.e., the factors were manipulated orthogonally). All word problems belonged to the type *Change* according to the categorization of Riley et al. (1983). We also ensured that each task could be clearly solved with one calculation, that the text was not ambiguous and that there was only one correct solution. Finally, all sentences were constructed using active verb forms because the active voice helps people to better understand word problems (Abedi, Courtney, Mirocha, Leon, & Goldberg, 2005; Shaftel, Belton-Kocher, Glasnapp, & Poggio, 2006). Unnecessary information was also avoided because the presence of extraneous information reduces student solution accuracy (Muth, 1992).

All problems included two two-digit numbers in Arabic notation as operands constituting addition or subtraction as the arithmetic operation. Overall problem size and parity were matched between subtraction and addition. Pure decades (e.g., 50), tie numbers (e.g., 66), unit ties (e.g., 23 - 13), decade ties (e.g., 41 + 43), and mirror numbers (e.g., 24 - 42) were excluded to prevent any automatic mental retrieval. We kept carry operations constant (i.e., all addition tasks needed carrying and all subtraction tasks needed borrowing), because we wanted not only

text understanding, but also arithmetic computations to be demanding (see: Artemenko et al., 2018; Nuerk et al., 2011).

Lexical consistency was manipulated by systematically varying both the keyword and the required arithmetic operation: The second or third sentence contained the keyword, which evoked an operation (e.g., “sell”/“selling” for subtraction, “buy”/“buying” for addition). The word problem, however, was constructed such that both operations, addition and subtraction, were equally likely for each keyword type, resulting in trials that were 50% lexically consistent and 50% inconsistent. The linguistic factor nominalization was manipulated independently of consistency and operation: The type of nominalization in the text was formulated so that the words lost their verbal characteristics and behaved like real nouns (Hamm & van Lambalgen, 2002). All nominalized forms originated from the same lexical root as the corresponding verbs, illustrated by the following example: Nominalized form: “*He **earns** 15 euros.*” Verbalized form: “*He is happy about **the earning** of 15 euros.*”

Procedure

The experiment took place in a laboratory of the University of Tuebingen. All participants and caregivers were first informed about the nature and structure of the experiment. After signing the informed consent, the participants filled out questionnaires for neuropsychological measurements: arithmetic ability, intelligence and reading comprehension. Arithmetic ability was assessed by two speed calculation tests on addition and subtraction (Huber, Fischer, Moeller, & Nuerk, 2013) that looked at the number of correctly solved items in 90 s for two-digit addition and two-digit subtraction. Intelligence was assessed by the subtest Matrices from the HAWIK (Petermann & Petermann, 2008). Reading ability was assessed by the LGVT test (Schneider, Schlagmüller, & Ennemoser, 2007) with the sub-scales reading speed (i.e., the number of read characters) and reading comprehension (i.e., correctly solved reading

comprehension items). Verbal working memory was assessed with letter span forward and letter span backward (Individual Characteristics and Eye-Movement Patterns: see Supplementary Material 4).

For the experiment, subjects were instructed that only mental calculations were allowed and that no other calculation support (such as finger movements) should be used. To ensure that all participants started to encode the addition problem at the first summand, a fixation point to the left of this summand was used (x/y coordinates: 112/384 pixel). Vocal response was used to detect the reaction time. The Voice Key (Creative EMU 0202, USB audio interface, Scotts Valley, USA) was used as a trigger, consisting of the interface and a headset with a microphone. Voice Key continuously monitors the voice volume and triggers a recording as soon as a user-defined limit is exceeded. Past research has shown, however, that there may be a conditional bias in recording louder sounds using Voice Key so, for instance, phonemes in the beginning might be registered with a delay (Kessler, Treiman, & Mullennix, 2002; Tyler, Tyler, & Burnham, 2005). For the Voice Key calibration, 32 two-digit numbers were each presented centrally on the screen in two blocks, and participants had to loudly read the numbers. Following Voice Key calibration, the main experiment started. In the eye tracking experiment, there was a fixation point at the beginning of each trial.

The stimuli were presented on a 19-inch monitor in white on a black background (Times New Roman font, font size 24, bold, all four sentences at an equal distance from one another). To reduce potential measurement inaccuracy or at least to keep it constant, all participants were asked to say loudly and clearly "is" before pronouncing a number (e.g., "ist 45") in both the Voice Key calibration and the main experiment. As soon as the participants responded the trial disappeared, and the screen turned black for 360 ms. There was a time limit of 90 s for response. Participant responses were recorded by the experimenter. The 48 trials were divided into two

blocks with 24 trials each with a break in between. The trials were presented separately in a pseudo-randomized order, so that in each block all 8 conditions were included, the conditions were mixed, and a single condition was presented no more than three times in a row.

Data Analysis

Behavioural Data Analysis. Response time (RT) was defined as the time between the start of the word problem presentation on the screen and the trigger of the participant's response sent by the Voice Key. For RT and accuracy (ACC) analyses, some responses were excluded due to errors in the Voice Key: (1) Voice Key was not triggered, so that the solution had to be repeated, (2) the subject corrected the answer after the first response, (3) other sounds triggered the Voice Key (e.g., throat clearing). For RT analysis, incorrectly solved trials were excluded. Additionally, all RTs shorter than 3,000 ms and longer than 30,000 ms were excluded. Furthermore, a trimming procedure eliminated all RTs falling below or above three *SDs* of an individual participant's mean. For statistical analysis of RT and ACC in word problems, 2 x 2 x 2 ANOVAs were conducted with the factors: operation (addition/subtraction), consistency (consistent/inconsistent) and nominalization (verbalized/nominalized).

Eye-Movement Analysis. Following the method of Hegarty et al. (1995) and Van der Schoot et al. (2009), regression times and fixation time were calculated. Fixation time was defined as the total fixation time on an area. The regression times were defined as the total fixation time on an area minus the first pass time (Frisson & Pickering, 1999; Rayner, 1998). This includes all regressions after the first pass fixation period. First pass duration on the target areas of interest was defined as the sum of the duration of all fixations on the areas of interest from the first fixation on the areas of interest until the first time that the reader left the areas of interest. There are also other eye tracking measures such as fixation and regression count, but since, for example, the pattern of the regression count corresponds to the pattern of regression time in

word problem research (Van der Schoot et al., 2009), we focused our analyses only on fixation time and regression time.

The eye-movement measures fixation time and regression time were calculated for the following areas in each word problem (see Figure 1¹): the total text (I0), the numbers (I1), the total text minus the numbers (I2) and the keyword (I3).

Insert Figure 1 about here

In order to handle these discontinuous areas, we calculated combined areas of interests (CAOIs). The basis of the calculations were fixations on individual words in the areas I0-I3. In the technical sense each word has its own area of interest (AOI). The first pass time for the larger CAOIs are the sum of the first pass time of the word AOIs belonging to the CAOI. The regression time for the larger CAOIs are the sum of the total fixation time of the word AOIs minus the sum of the first pass time of the word AOIs belonging to the CAOI.

Additionally, we have chosen absolute measures (mean fixation time, mean regression time) instead of relative measures (fixation time per number of words, percentage of regression time) for two reasons. The first reason is that we aligned ourselves with studies, for instance, Van der Schoot et al. (2009), that used absolute measures. The second reason is that our stimuli were comparable in length, word count and frequency, contrary, for instance, to studies like that of Strohmaier et al. (2019), where the stimuli differed more and relative measures were

¹ Throughout the paper, for presentation purposes we show English word-by-word glosses of the German example stimuli that were used. The original German stimuli can be found in the Supplementary Material 1.

necessary. Table 1 provides the average word count, character length, syllable count, character count per word and syllable count per word for each condition as well as frequency measures. Frequencies were calculated using the COW German Corpus (Schäfer, 2015; Schäfer & Bildhauer, 2012).

Insert Table 1 about here

Contrary to some eye tracking studies (Hegarty et al., 1992; Hegarty et al., 1995; Van der Schoot et al., 2009; Verschaffel et al., 1992) that chose to analyse only the words referring to one of the quantities in the problem, we decided to include all of the words of the word problem in the analysis. Because, according to Hegarty et al. (1992) students using successful solution strategies pay attention to all words in the problem and not only to the single words which refer to one of the quantities.

Insert Figure 2 about here

For statistical analysis of the areas of interests, 2 x 2 x 2 ANOVAs were conducted with the factors operation (addition/subtraction), consistency (consistent/inconsistent) and nominalization (verbalized/nominalized) separately for the following CAOIs: whole problem (I0), numbers (I1), whole problem without numbers (I2) and keyword (I3). To understand the consistency effect, we also examined the keyword separately from the numbers, because according to Experiment 3, Verschaffel et al. (1992), there was a significant effect of language consistency only for the second sentence which included the relational statement. These results indicate that the consistency effect manifests in the sentence where the keyword is included.

Finally, as numbers and keywords were considered areas of interest in most previous studies (e.g., Hegarty et al., 1992), an additional analysis of the areas of interest including keyword and numbers is included in the Supplementary Material 2. Furthermore, because the areas of interest including keyword and numbers and the areas of interest including the whole problem without keyword and numbers are associated with specific problem-solving strategies, the analyses of these areas can be found in the Supplementary Material 3.

Results

Behavioural Measures

Table 2 shows the statistical data for RT and ACC.

 Insert Table 2 about here

Table 3 summarises the results from the ANOVAs of RT and ACC.

 Insert Table 3 about here

Reaction Time. For RT ($M = 23.93$, $SD = 8.43$), there was a significant main effect of operation, indicating that subtraction problems took longer to solve (26.03 s) than addition problems (22.26 s). There was also a significant main effect of nominalization, indicating that word problems with the nominalized form took longer to solve (24.97 s) than problems with the verbalized form (23.32 s). All other effects and interactions were not significant.

Accuracy. For ACC ($M = 84.41$, $SD = 7.00$), there were significant main effects of operation and consistency. Moreover, there was a significant interaction of *consistency and*

nominalization, indicating a consistency effect (reduced accuracy for inconsistent problems (76.5%) compared to consistent problems (89.1%) only in nominalized sentences, $t(32) = 4.57$, $p < .001$, but not in verbalized sentences; cf. Figure 2). Additionally, there was a significant interaction of *nominalization and operation*, indicating an operation effect (reduced accuracy for subtraction (79.04%) compared to addition (86.61%) only in sentences with the nominalized form, $t(32) = 2.83$, $p = .008$, but not in sentences with the Verbalized form; cf. Figure 2). All other effects and interactions were not significant.

Insert Figure 2 about here

Summary of Results. For RT, effects for operation and nominalization were found. For ACC, effects of consistency and operation were found only for sentences with the nominalized form (i.e., semantically more complex sentences) but not for easier sentences with the Verbalized form.

Eye Tracking Measures

Table 4 shows the statistical data for fixation time and regression time.

Insert Table 4 about here

Table 5 shows the ANOVAs about fixation time and regression time on the defined areas of interests.

Insert Table 5 about here

General Combined Area of Interest: The whole problem (I0). In the case of *total fixation time* ($M = 20.42$, $SD = 7.38$), there were significant main effects of operation and nominalization. These indicate that subtraction problems took longer to solve (21.99 s) than addition problems (18.96 s), and problems with the nominalized form (20.92 s) took longer to solve than problems with the verbalized form (19.92 s). In the case of *total regression time* ($M = 14.24$, $SD = 6.51$), there was a significant main effect of operation, indicating that subtraction problems (6.68 s) elicited more regression time on the total text than addition problems (5.50 s). Thus, the general eye tracking data is consistent with earlier studies.

Specific Combined Area of Interest: The two numbers (I1). For *fixation time looking at the numbers*, there was a significant main effect of operation, indicating that for subtraction participants had a longer fixation time (8.18 s) than for addition (6.33 s). For *regression time*, there was a significant main effect of operation. This indicates that for subtraction participants displayed more regression time (7.34 s) than for addition (5.43 s). No linguistic manipulation (consistency, nominalization) led to any effects regarding the fixation patterns on numbers.

Specific Combined Area of Interest: Whole problem without numbers (I2). For *fixation time*, there were significant main effects of operation, indicating that subtraction leads to a longer fixation time (13.84 s) than addition (12.69 s), and of nominalization, indicating that nominalization (13.98 s) leads to longer fixations than verbalization (12.55 s).

For *regression time*, there was a significant main effect of operation, indicating that subtraction elicited a longer regression time (8.43 s) than addition (7.35 s). Therefore, not only linguistic, but also numerical factors influenced eye-movement effects in the text without numbers.

Specific Combined Area of Interest: Keyword (I3). For *fixation time*, there was a significant interaction of nominalization and operation, indicating that there was a significant difference between the addition (0.73 s) and subtraction (0.82 s) sentences in the case of nominalization,

$t(32) = -2.13, p = .004$, but not verbalization. Additionally, there was a significant interaction of nominalization and consistency, indicating a significant difference between inconsistent (0.87 s) and consistent sentences (0.69 s) in the case of nominalization, $t(32) = -5.44, p < .001$, but not verbalization. Finally, there was a significant interaction of consistency and operation, indicating longer fixation times for consistent sentences (0.89 s) than inconsistent sentences (0.75 s) for subtraction, $t(32) = 2.56, p = .015$, but longer fixation times for inconsistent sentences (0.95 s) than consistent sentences (0.73 s) in the case of addition, $t(32) = -3.73, p = .001$ (cf. Figure 3).

Insert Figure 3 about here

For *regression time*, there was a significant interaction of nominalization and consistency, indicating a significant difference between inconsistent (0.53 s) and consistent sentences (0.39 s) in the case of nominalization, $t(32) = -4.45, p < .001$, but not verbalisation (cf. Figure 4). Furthermore, there was a significant interaction between consistency and operation, indicating a significant difference between consistent (0.42 s) and inconsistent (0.58 s) sentences in the case of addition, $t(32) = -2.97, p = .006$, but not in the case of subtraction (cf. Figure 4).

Insert Figure 4 about here

In sum, both linguistic and numerical factors as well as their interaction influenced eye-movement patterns over keywords.

Summary of Results. For the whole problem including numbers (I0), effects for operation and nominalization were found. Fixation and regression time on numbers (I1) provided effects for operation, i.e. only for numerical manipulations. In contrast, fixation and regression time on text without numbers (I2) provided an operation effect as well as a nominalization effect for fixation time only. Thus, eye-movement behaviour over the text *without numbers* is not only influenced by linguistic complexity, but also by numerical complexity. This holds even for eye tracking data for one single word, the so-called keyword. Fixation and regression time on the keyword (I3) provided an interaction of the linguistic factors, namely, a consistency effect only in case of nominalization. However, it also provided evidence for interaction of linguistic and numerical factors, namely between nominalization and operation as well as consistency and operation. The interaction on the keyword (I3) between consistency and operation was disordinal.

Discussion

In this study we systematically investigated the relationship between arithmetic and linguistic task characteristics and the process of word problem-solving. For this we used eye tracking (fixation and regression time spent on the whole problem and on the text, numbers and keyword) in order to understand its direct influence as well as the possible common processing phases. The direct influence of difficulty could be conceptually replicated: The difficulty of word problems was increased by arithmetic factors in terms of the arithmetic operation, as well as by linguistic factors in terms of lexical consistency and nominalization. The results speak to an interaction between the initial reading process and the other problem-solving processes. Furthermore, task characteristics elicited an attentional shift in distinct eye-movement patterns targeting specific parts of the word problem. Finally, we found interactions in the eye-movement behaviour: arithmetic complexity alone led to an increased eye-movement pattern on the linguistic part of the problem. This indicates that arithmetic processes are not fully

separable from other problem-solving processes. How specific task characteristics influence problem-solving and how they are related to problem-solving processes will be further discussed in the following section.

Increased Difficulty of Word Problems: Behavioural Results

Behavioural studies have shown that both linguistic and arithmetic task characteristics contribute (directly or via interaction) to the difficulty of a word problem. Our results replicated these findings.

Operation Effect. We found a general increase in reaction time for subtraction as compared to addition. This is in line with the finding that subtraction elicits greater response times (Orrantia, Rodríguez, Múñez, & Vicente, 2012) than addition for arithmetic word problems. Although, subtraction is generally more difficult (Artemenko, Moeller, Huber, & Klein, 2015) in this study, in terms of accuracy, the effects of operation were found only for sentences with the nominalized form (i.e., semantically more complex sentences) but not for sentences with the Verbalized form. This can be explained by the fact that some types of simple word problem language modification, without changing the arithmetic structure, might lead to higher success rates (Cummins, Kintsch, Reusser, & Weimer, 1988; Davis-Dorsey et al., 1991; Vicente, Orrantia, & Verschaffel, 2007). However, these studies did not investigate the general influence of rewording on the arithmetic operation and our results suggest that the complexity of operation can be influenced by a simple rewording of the text.

Nominalization Effect. Regarding the linguistic difficulty of the word problems, we found a general increase in the reaction time for nominalization. This extends previous findings of other studies that found nominalizations to be a source of difficulty (Abedi et al., 2005; Prediger et al., 2015). Additionally, contrary to studies that found effects of nominalization in complex

multi-step word problems only with a higher density of nominalization as compared to lower density of nominalization (Schlager, Kaulvers, & Büchter, 2017), our study shows that even small changes can lead to a difference in the solution process, suggesting that simply rewording a problem into a nominalized form influences problem complexity.

For accuracy, there was no main effect of nominalization, but we found interactions with consistency and operation only in nominalized sentences. This shows that when the text is easy to read and comprehend children are able to solve word problems which require a more complex mental representation. However, when the text has the same underlying arithmetic structure but is more difficult to read, the additional difficulty of consistency and operation increase the probability of errors (see also Vicente, Orrantia, & Verschaffel, 2008). suggests that in these cases it gets more difficult to create an appropriate mental representation. This is also supported by the finding that even inept problem solvers are able to solve difficult word problems when the text is semantically easy (Van der Schoot et al., 2009).

Consistency Effect. Regarding the linguistic difficulty of the word problems, we found a general increase in the reaction time for nominalization, but not for lexical consistency. This result is contrary to some previous studies (Hegarty et al., 1992; Hegarty et al., 1995; Pape, 2003; Van der Schoot et al., 2009; Verschaffel et al., 1992), where word problems of inconsistent form elicited longer reaction times as compared to problems of consistent form, but in line with the study of Verschaffel et al. (1992) who found no consistency effect in adults. However, we found the consistency effect only for nominalized word problems, suggesting, as described above, that the presence of the consistency effect might depend on the formulation of the text, so that the difficulty of creating a problem model increases. This might also explain why some studies find the consistency effect (e.g., Van der Schoot et al., 2009) and others do not (e.g., Verschaffel et al., 1992).

In sum, our behavioural results show a direct influence (but no interactions) in the case of response time. On the other hand, the results on accuracy show that common processing stages are possible: we found effects of operation and consistency only in the case of the semantically more complex, nominalized sentences. The behavioural results also speak to an overlap of the initial reading phase and the other phases of problem-solving. Namely, for accuracy the effects of consistency and operation were found only for nominalized sentences, but not for verbalized sentences, suggesting a common processing stage. As nominalization could be associated more with the initial reading phase this means that in case of children it is hard to clearly distinguish between the initial reading process, especially when the reading demand increases, and other problem-solving processes. This indicates that the difficulty of the text formulation influences the creation of the problem model, i.e., a simple rewording can make it difficult to find the correct arithmetic mental model. The interaction between operation and nominalization indicates that in the case of children arithmetic processes interact with the initial reading processes. However, behavioural studies do not reflect which parts of the text play an important part in creating the arithmetic mental model and even in the case of an interaction we do not know whether it comes from arithmetic or linguistic difficulties.

Increased Difficulty of Word Problems: General Eye-Movement

In this study we aimed to replicate the direct influence of textual difficulty on word problem-solving performance (Hegarty et al., 1995; Strohmaier et al., 2019). Additionally, we hypothesised that task characteristics should elicit an attentional shift in the corresponding distinct eye-movement patterns targeting specific parts of the word problem in general.

Eye-Movement Pattern Over the Whole Problem. It has been already shown that numerous and difficult stimuli in mathematics lead to longer fixations and regression times (Knoblich et al., 2001). For word problems in particular, there is a relationship between increased task

difficulty and increases in eye-movement measures related to the whole text (Strohmaier et al., 2019). Here, we found similar results, namely that subtraction problems elicited a longer total fixation time as well as longer regression on the whole problem than addition problems. This is in line with De Corte et al. (1990), who found that students spent more fixation time on the whole word problem when it involved subtraction as compared to addition. As regressions are hypothesised to represent additional processing, this result suggests that subtraction requires extra processing and that the calculation process is probably extensively present. Additionally, we also showed that nominalized sentences elicit more total fixation time on the whole problem compared to verbalization. This demonstrates that even small textual changes influence the eye-movement behaviour. Finally, in the conclusion of the study of Strohmaier et al. (2019) higher mean fixation time was interpreted as reflecting either the direct translation strategy or struggling with the word problem, and relatively short mean fixation as reflecting the problem model strategy. In our analysis only the correct items were included, so a correct mental representation can be assumed. Therefore, the observation that linguistically and arithmetically more complex items elicited longer mean fixation time show that the item characteristics of word problems play an essential role in the eye-movement behaviour.

Arithmetic Difficulty and the Mathematical Part of the Problem. Despite the major role of arithmetic properties in mathematical cognition, arithmetic difficulty has not been investigated extensively for word problem tasks with eye tracking (Fuchs et al., 2010). Nevertheless, we found that arithmetic difficulty increased the eye-movements towards the numbers in the word problem in terms of both total fixation time and regression time. Namely, subtraction elicited more fixation time on the numbers than addition. This is in line with De Corte and Verschaffel (1986) who observed a relatively strong focus on the numbers in word problems.

Linguistic Difficulty and the Textual Part of the Problem. Linguistic difficulty increased fixation time and regression time on the text in the word problem. For instance, the linguistic factor nominalization elicited greater attention towards the whole text without numbers in the case of total fixation time but not in the case of regression. Eye-movements towards the keyword showed a consistency effect, only in case of nominalization, on both fixation and regression time. Additionally, the consistency effect was found only for addition and an inverted consistency effect was found for subtraction problems. This finding is partly supported by previous research (Pape, 2003; Van der Schoot et al., 2009; Verschaffel et al., 1992) reporting a consistency effect only for addition and not subtraction. This shows that even a slight increase in the text difficulty (without changing the semantic congruency of the problem) elicits changes in the eye-movement behaviour. Interestingly, we did not observe the consistency effect in terms of total fixation on the whole word problem but only on a specific textual part: the keyword. This indicates that the consistency effect does not manifest itself in the whole problem, but rather on a specific part of the problem and a specific phase of the solution process. This is in line with many studies that observed the consistency effect only for the second sentence containing the keyword (De Corte et al., 1990) or only in the second phase of problem-solving (e.g., Hegarty et al., 1992). Additionally, we found an interaction of nominalization in the fixation time on the overall problem and with consistency also in the regression time on the keyword. This means that the factor nominalization did not only influence the initial reading phase, but also that the comprehension of word problems possibly depends on the wording. This indicates that the process of reading is not completely separable from the process of solving and text formulation affects not only the first reading but also the other phases of the solution process, which is supported by the cyclic model (e.g., Bergqvist & Österholm, 2010).

In sum, word problems are not generally more difficult compared to other mathematical tasks but differ in the linguistic and arithmetic difficulty: Increasing arithmetic difficulty shifts eye-movements towards numerical elements and increasing linguistic difficulty shifts eye-movements towards the textual elements of word problems.

Linguistic and Arithmetic Factors Interact in Word Problem-solving

In this study we hypothesised that if the cognitive processes of reading and calculation are fully separable in terms of word problems, increased linguistic difficulty should lead to increased cognitive effort and attention on only the textual part of the problem (as indicated by eye tracking data) and mathematical processes should lead to increased cognitive effort and attention on only the numerical part of the problem. This was only partially true: linguistic difficulty did not affect the numerical areas, but arithmetic difficulty did affect the textual areas.

Linguistic Difficulty and Numerical Areas. Contrary to what was expected, increased linguistic difficulty (lexical consistency or nominalization) did not increase the attention paid to the numbers. One explanation could be that lexical consistency affects other specific parts of the text, focusing more on the keyword. Another explanation could be that it depends on the language proficiency of the problem solver. For example, for non-native speakers linguistic difficulty has a significant influence on student perception of the arithmetic difficulty of problems (Barbu & Beal, 2010).

Arithmetic Difficulty and Textual Areas. As the number of areas of interest were affected only by the factor operation this suggests at first glance that the arithmetic processing might not interfere with the other solution phases as the number of areas of interests were affected only by the factor operation. Nevertheless, for arithmetically more complex problems, an increase in the duration of fixation and regression on the textual elements was observed, which means

that with increasing arithmetic difficulty the children looked at the text more. This can be interpreted to mean that arithmetic processes might also influence another process of problem-solving and subtraction makes it more difficult to create a representation of a word problem and therefore people struggle not only with the numbers but also with the text. Bergqvist and Österholm (2010) also said that carrying out a calculation adds to the mental representation but does not change it, and they associated re-reading with changes to the existing mental representation. This also supports the idea that arithmetic processes do not only add to the mental model but change it. The explanation may be, that since subtraction is more difficult than addition (Artemenko et al., 2018) and there are less readily available strategies than in the case of addition, a larger portion of the cognitive resources will be gradually occupied as word problems work as a dual task, where the participant needs to simultaneously process both mathematics and language. If they allocate more resources for one task (math), there are less resources for the other (language). Therefore, there are probably less resources available to keep the non-arithmetic text in mind and the participant needs to spend a longer amount of time reading/looking at the text, while keeping and manipulating the subtraction problem in their mind.

Limitations & Future Directions

It is important to point out that in eye tracking research there is no ideal way of choosing a correct measure. For example, eye tracking studies usually use word level analysis, which is a controversial measure (Rayner, 1998). For instance, De Corte and Verschaffel (1986) showed that many people did not even look at important areas of the text problems and yet they answered correctly. This means that word level analysis does not necessarily contain all of the information about the processing. Many words are skipped, so the definition of fixation on all the words is not always enough. On the other hand, gaze duration (i.e., the sum of all fixation

time) contains not only the reading but also the text integration processes. Last but not least we should be careful with the interpretation of fixation and regression times as well. On the one hand, the difficulty of the items generally increases fixation and regression times, but on the other hand it has also been shown that people with more expertise in a specific subject needed fewer eye fixations and regressions to process information in specific areas. Therefore, it would be preferable to investigate other eye-movement behaviours, such as the sequence of reading.

Conclusion

We conclude that eye-movement behaviour depends on the linguistic and arithmetic task characteristics. Complex and simple word problems generally result in different strategies, which can be detected in eye-movements. For instance, more complex problems reinforce longer fixations and regression times and are associated with lower performance. Even for children, word problems are not generally more difficult, but differ in the linguistic and arithmetic difficulty: Increasing arithmetic difficulty shifts eye-movements towards numerical elements and increasing linguistic difficulty shifts eye-movements towards the textual elements of a word problem. However, the influence of linguistic and arithmetic factors is not reduced to main effects as interactions can be observed in specific parts of word problems. For instance, in arithmetically complex problems (e.g., with subtraction) children spend more time on the textual parts of the word problem. However, how long children look at the numbers in the word problem does not seem to be affected by linguistic difficulty.

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References

- Abedi, J., Courtney, M., Mirocha, J., Leon, S., & Goldberg, J. (2005). *Language Accommodations for English Language Learners in Large-Scale Assessments: Bilingual Dictionaries and Linguistic Modification*. CSE Report 666. Retrieved from <https://files.eric.ed.gov/fulltext/ED518681.pdf>
- Artemenko, C., Moeller, K., Huber, S., & Klein, E. (2015). Differential influences of unilateral tDCS over the intraparietal cortex on numerical cognition. *Frontiers in Human Neuroscience*, 9, 1-8.
- Artemenko, C., Pixner, S., Moeller, K., & Nuerk, H. C. (2018). Longitudinal development of subtraction performance in elementary school. *British Journal of Developmental Psychology*, 36(2), 188-205.
- Barbu, O. C., & Beal, C. R. (2010). Effects of linguistic complexity and math difficulty on word problem solving by English learners. *International Journal of Education*, 2(2), 1-19.
- Bassok, M., Chase, V. M., & Martin, S. A. (1998). Adding apples and oranges: Alignment of semantic and formal knowledge. *Cognitive Psychology*, 35(2), 99-134.
- Bergqvist, E., & Österholm, M. (2010, January). *A theoretical model of the connection between the process of reading and the process of solving mathematical tasks*. Paper presented at the MADIF 7, The Seventh Swedish Mathematics Education Research Seminar, Stockholm.
- Boonen, A. J., de Koning, B. B., Jolles, J., & van der Schoot, M. (2016). Word problem solving in contemporary math education: A plea for reading comprehension skills training. *Frontiers in Psychology*, 7, 191.
- Boonen, A. J., van der Schoot, M., van Wesel, F., de Vries, M. H., & Jolles, J. (2013). What underlies successful word problem solving? A path analysis in sixth grade students. *Contemporary Educational Psychology*, 38(3), 271-279.

- Carraher, T. N., Schliemann, A. D., & Carraher, D. W. (1988). Mathematical concepts in everyday life. *New Directions for Child and Adolescent Development*, 1988(41), 71-87.
- Cummins, D. D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive Psychology*, 20(4), 405-438.
- Daroczy, G., Meurers, D., Heller, J., Wolska, M., & Nürk, H.-C. (2020). The interaction of linguistic and arithmetic factors affects adult performance on arithmetic word problems. *Cognitive Processing*, 21(1), 105-125.
- Daroczy, G., Wolska, M., Meurers, W. D., & Nuerk, H.-C. (2015). Word problems: a review of linguistic and numerical factors contributing to their difficulty. *Frontiers in Psychology*, 6.
- Davis-Dorsey, J., Ross, S. M., & Morrison, G. R. (1991). The role of rewording and context personalization in the solving of mathematical word problems. *Journal of Educational Psychology*, 83(1), 61-68.
- De Corte, E. (1988). Influence of Number Size, Problem Structure and Response Mode on Children's Solutions of Multiplication Word Problems. *The Journal of Mathematical Behavior*, 7(3), 197-216.
- De Corte, E., & Verschaffel, L. (1986). *Eye-Movement Data as Access to Solution Processes of Elementary Addition and Subtraction Problems*. Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco.
- De Corte, E., & Verschaffel, L. (1987). The effect of semantic structure on first graders' strategies for solving addition and subtraction word problems. *Journal for Research in Mathematics Education*, 18(5), 363-381.
- De Corte, E., Verschaffel, L., & De Win, L. (1985). Influence of rewording verbal problems on children's problem representations and solutions. *Journal of Educational Psychology*, 77(4), 460-470.

- De Corte, E., Verschaffel, L., & Pauwels, A. (1990). Influence of the semantic structure of word problems on second graders' eye movements. *Journal of Educational Psychology*, 82(2), 359-365.
- Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, 1(1), 83-120.
- Frisson, S., & Pickering, M. J. (1999). The processing of metonymy: Evidence from eye movements. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 25(6), 1366.
- Fuchs, L. S., Fuchs, D., Stuebing, K., Fletcher, J. M., Hamlett, C. L., & Lambert, W. (2008). Problem solving and computational skill: Are they shared or distinct aspects of mathematical cognition? *Journal of Educational Psychology*, 100(1), 30-47.
- Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., Seethaler, P. M., . . . Schatschneider, C. (2010). Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? *Developmental Psychology*, 46(6), 1731.
- Fuchs, L. S., Gilbert, J. K., Fuchs, D., Seethaler, P. M., & Martin, B. N. (2018). Text comprehension and oral language as predictors of word-problem solving: Insights into word-problem solving as a form of text comprehension. *Scientific Studies of Reading*, 22(2), 152-166.
- Gibson, E., & Warren, T. (2004). Reading-time evidence for intermediate linguistic structure in long-distance dependencies. *Syntax*, 7(1), 55-78.
- Hamm, F., & van Lambalgen, M. (2002). Formal foundations for semantic theories of nominalisation. *ZAS Papers in Linguistics*, 27, 1-21.

- Hegarty, M., Mayer, R. E., & Green, C. E. (1992). Comprehension of arithmetic word problems: Evidence from students' eye fixations. *Journal of Educational Psychology*, 84(1), 76-84.
- Hegarty, M., Mayer, R. E., & Monk, C. A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology*, 87(1), 18-32.
- Hinsley, D. A., Hayes, J. R., & Simon, H. A. (1977). From words to equations: Meaning and representation in algebra word problems. In P. A. Carpenter & M. A. Just (Eds.), *Cognitive Processes in Comprehension* (pp. 89-106). Hillsdale, New York: Lawrence Erlbaum Associates.
- Kessler, B., Treiman, R., & Mullennix, J. (2002). Phonetic biases in voice key response time measurements. *Journal of Memory and Language*, 47(1), 145-171.
- Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review*, 92(1), 109-129.
- Knoblich, G., Ohlsson, S., & Raney, G. E. (2001). An eye movement study of insight problem solving. *Memory & Cognition*, 29(7), 1000-1009.
- Lave, J. (1992). Word problems: A microcosm of theories of learning. In P. Ligth & G. Butterworth (Eds.), *Context and cognition: Ways of learning and knowing* (pp. 74-92). Harvester-Wheatsheaf: Hemel Hempstead.
- Lewis, A. B., & Mayer, R. E. (1987). Students' miscomprehension of relational statements in arithmetic word problems. *Journal of Educational Psychology*, 79(4), 363-371.
- Mayer, R. E. (1984). A cognitive analysis of mathematical problem-solving ability. In R. J. Sternberg (Ed.), *Advances in the Psychology of Human Intelligence* (pp. 231-273). New York: Lawrence Erlbaum Associates.

- Muth, K. D. (1992). Extraneous information and extra steps in arithmetic word problems. *Contemporary Educational Psychology*, 17(3), 278-285.
- Nuerk, H.-C., Moeller, K., Klein, E., Willmes, K., & Fischer, M. H. (2011). Extending the mental number line. *Zeitschrift für Psychologie*, 219(1), 3-22.
- Orrantia, J., & Múñez, D. (2013). Arithmetic word problem solving: evidence for a magnitude-based mental representation. *Memory & Cognition*, 41(1), 98-108.
- Orrantia, J., Múñez, D., San Romualdo, S., & Verschaffel, L. (2015). Effects of numerical surface form in arithmetic word problems. *Psicológica*, 36(2), 265-281.
- Orrantia, J., Rodríguez, L., Múñez, D., & Vicente, S. (2012). Inverse reference in subtraction performance: An analysis from arithmetic word problems. *The Quarterly Journal of Experimental Psychology*, 65(4), 725-738.
- Pape, S. J. (2003). Compare word problems: Consistency hypothesis revisited. *Contemporary Educational Psychology*, 28(3), 396-421.
- Petermann, F., & Petermann, U. (2008). HAWIK-IV. *Kindheit und Entwicklung*, 17(2), 71-75.
- Powell, S. R., & Fuchs, L. S. (2014). Does early algebraic reasoning differ as a function of students' difficulty with calculations versus word problems? *Learning Disabilities Research & Practice*, 29(3), 106-116.
- Prediger, S., Wilhelm, N., Büchter, A., Gürsoy, E., & Benholz, C. (2015). Sprachkompetenz und Mathematikleistung—Empirische Untersuchung sprachlich bedingter Hürden in den Zentralen Prüfungen 10. *Journal für Mathematik-Didaktik*, 36(1), 77-104.
- Rayner, K. (1998). Eye movements in reading and information processing: 20 years of research. *Psychological Bulletin*, 124(3), 372.
- Riley, M. S., Greeno, J., & Heller, J. (1983). Development of children's problem-solving ability in arithmetic. *The Development of Mathematical Thinking*, 1, 153-196.

- Schäfer, R. (2015, July). *Processing and querying large web corpora with the COW14 architecture*. Paper presented at the 3rd Workshop on Challenges in the Management of Large Corpora (CMLC-3), Lanchaster.
- Schäfer, R., & Bildhauer, F. (2012, May). *Building large corpora from the web using a new efficient tool chain*. Paper presented at the Eighth International Conference on Language Resources and Evaluation (LREC 12).
- Schlager, S., Kaulvers, J., & Büchter, A. (2017, February). *Effects of linguistic variations of word problems on the achievement in high stakes tests*. Paper presented at the Tenth Congress of the European Society for Research in Mathematics Education, Dublin, Ireland.
- Schneider, W., Schlagmüller, M., & Ennemoser, M. (2007). *LGVT 6-12: Lesegeschwindigkeits- und-verständnistest für die Klassen 6-12*: Hogrefe Göttingen.
- Shaftel, J., Belton-Kocher, E., Glasnapp, D., & Poggio, J. (2006). The impact of language characteristics in mathematics test items on the performance of English language learners and students with disabilities. *Educational Assessment, 11*(2), 105-126.
- Strohmaier, A. R., Lehner, M. C., Beitlich, J. T., & Reiss, K. M. (2019). Eye movements during mathematical word problem solving—global measures and individual differences. *Journal für Mathematik-Didaktik, 40*(2), 255-287.
- Terao, A., Koedinger, K. R., Sohn, M.-H., Qin, Y., Anderson, J. R., & Carter, C. S. (2004). *An fMRI study of the interplay of symbolic and visuo-spatial systems in mathematical reasoning*. Paper presented at the Proceedings of the Annual Meeting of the Cognitive Science Society, Chicago.
- Thevenot, C., & Oakhill, J. (2005). The strategic use of alternative representations in arithmetic word problem solving. *The Quarterly Journal of Experimental Psychology Section A, 58*(7), 1311-1323.

- Torbeyns, J., De Smedt, B., Ghesquière, P., & Verschaffel, L. (2009). Jump or compensate? Strategy flexibility in the number domain up to 100. *ZDM*, 41(5), 581-590.
- Tyler, M. D., Tyler, L., & Burnham, D. K. (2005). The delayed trigger voice key: An improved analogue voice key for psycholinguistic research. *Behavior Research Methods*, 37(1), 139-147.
- Van der Schoot, M., Arkema, A. H. B., Horsley, T. M., & van Lieshout, E. C. (2009). The consistency effect depends on markedness in less successful but not successful problem solvers: An eye movement study in primary school children. *Contemporary Educational Psychology*, 34(1), 58-66.
- Verschaffel, L., De Corte, E., & Pauwels, A. (1992). Solving compare problems: An eye movement test of Lewis and Mayer's consistency hypothesis. *Journal of Educational Psychology*, 84(1), 85-94.
- Vicente, S., Orrantia, J., & Verschaffel, L. (2007). Influence of situational and conceptual rewording on word problem solving. *British Journal of Educational Psychology*, 77(4), 829-848.
- Vicente, S., Orrantia, J., & Verschaffel, L. (2008). Influence of situational and mathematical information on situationally difficult word problems. *Studia Psychologica*, 50(4), 337-356.

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Appendices

Figure 1. Combined Areas of Interest (CAOIs) for numbers (I1), text without numbers (I2) and keyword (I3).

Figure 2. Two-Way Interactions in Accuracy: Operation × Nominalization, Consistency × Nominalization.

Figure 3. Two-way Interactions in Fixation Time on Keyword (I3): Consistency $\times$ Nominalization, Consistency $\times$ Operation, Operation $\times$ Nominalization.

Figure 4. Interactions in Regression Time on Keyword (I3): Consistency $\times$ Operation, Consistency $\times$ Nominalization.

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Table 1.

Linguistics Measures Across Conditions.

	Word Count		Character Length		Syllable Count		Character Count / Word		f_logpermil_10		band	
	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
Addition												
Consistent Form												
Nominalized Form	30.17	4.07	144.67	27.04	47.33	9.73	4.64	0.35	121.30	10.81	7.13	0.42
Verbalized Form	24.67	2.73	117.33	18.64	38.33	7.28	4.58	0.34	115.71	10.17	7.13	0.53
Inconsistent Form												
Nominalized Form	29.00	4.20	144.33	29.19	46.50	10.13	4.84	0.39	120.42	9.55	7.37	0.27
Verbalized Form	25.50	3.33	126.50	25.26	40.17	9.33	4.77	0.39	114.11	8.43	7.43	0.50
Subtraction												
Consistent Form												
Nominalized Form	29.50	2.35	147.50	15.85	45.67	5.35	4.92	0.35	120.62	7.98	7.42	0.22
Verbalized Form	26.00	1.67	128.17	15.04	41.00	4.94	4.76	0.34	117.58	6.63	7.33	0.52
Inconsistent Form												
Nominalized Form	30.17	2.64	144.00	16.04	45.83	5.91	4.77	0.38	122.18	11.87	7.31	0.59

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Verbalized Form	26.50	2.59	130.67	21.41	43.00	8.07	4.76	0.51	115.83	11.10	7.29	0.54
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Table 2.

Descriptive Data for Behavioral Measures.

	Verbalized Form								Nominalized Form							
	Consistent Form				Inconsistent Form				Consistent Form				Inconsistent Form			
	Addition		Subtraction		Addition		Subtraction		Addition		Subtraction		Addition		Subtraction	
	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
Reaction Time (s)	21.00	1.55	24.65	1.54	21.75	1.65	25.89	1.77	23.76	1.61	26.56	1.98	22.54	1.32	27.03	1.91
Accuracy (Percentage)	88.38	1.98	84.34	2.61	85.86	2.91	85.35	2.69	91.41	1.94	86.87	2.69	81.82	2.75	71.21	3.50

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Table 3.

Analysis of Variance for Behavioural Data.

	Reaction Time			Accuracy		
	<i>F</i>	<i>p</i>	η_p^2	<i>F</i>	<i>p</i>	η_p^2
Nominalization	9.43	.004	.23	2.94	.096	.08
Consistency	0.43	.519	.01	11.17	.002	.26
Operation	21.85	<.001	.41	4.87	.035	.13
Nominalization x Consistency	1.95	.172	.06	12.90	.001	.29
Nominalization x Operation	0.68	.796	.00	4.47	.042	.12
Consistency x Operation	1.21	.279	.04	0.10	.755	.00
Nominalization x Consistency x Operation	0.47	.496	.01	3.57	.068	.10

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Table 4.

Descriptive Data for Eye-Movement Measures (Means and SD in s).

			Verbalized Form								Nominalized Form							
			Consistent Form				Inconsistent Form				Consistent Form				Inconsistent Form			
			Addition		Subtraction		Addition		Subtraction		Addition		Subtraction		Addition		Subtraction	
			<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Fixation Time	Whole Problem	(I0)	17.89	7.86	20.99	7.67	18.86	8.56	21.94	8.71	19.99	7.59	22.3	9.65	18.95	6.05	22.85	9.03
	Numbers	(I1)	6.54	4.21	8.1	3.56	6.5	3.08	8.33	4.17	6.39	3.74	7.9	3.68	5.5	2.39	8.38	4.57
	Text without	Numbers (I2)	11.35	4.27	12.89	5.08	12.36	6.40	13.61	6.35	13.6	5.44	14.40	7.43	13.45	4.69	14.47	6.21
	Keyword	(I3)	0.87	0.44	0.97	0.48	1.01	0.93	0.67	0.37	0.60	0.23	0.80	0.32	0.89	0.35	0.86	0.36
Regression Time	Whole Problem	(I0)	1.24	7.05	14.97	7.14	13.15	7.62	15.91	7.99	13.25	6.44	15.73	8.84	12.33	5.10	16.45	8.26
	Numbers	(I1)	5.79	4.05	7.27	3.54	5.67	2.97	7.48	4.06	5.57	3.62	7.01	3.56	4.71	2.31	7.60	4.53
	Text without	Numbers (I2)	6.62	3.55	7.70	4.62	7.48	5.59	8.43	5.60	7.68	4.33	8.72	6.64	7.63	3.82	8.85	5.38
	Keyword	(I3)	0.52	0.42	0.59	0.44	0.63	0.89	0.40	0.35	0.33	0.18	0.46	0.25	0.52	0.29	0.55	0.32

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Table 5.

Analysis of Variance for Eye-Movement Measures.

	Whole Problem (I0)			Numbers (I1)			Text without Numbers (I2)			Keyword (I3)		
	<i>F</i>	<i>p</i>	η_p^2	<i>F</i>	<i>p</i>	η_p^2	<i>F</i>	<i>p</i>	η_p^2	<i>F</i>	<i>p</i>	η_p^2
Fixation Time												
Nominalization	5.23	.029	.14	1.76	.194	.05	13.33	.001	.29	1.66	.207	.05
Consistency	0.73	.400	.02	0.06	.811	.00	1.36	.253	.04	1.57	.219	.05
Operation	23.13	< .001	.42	20.32	< .001	.39	11.06	.002	.26	0.12	.729	.00
Nominalization x Consistency	1.64	.209	.05	0.64	.430	.02	1.05	.313	.03	9.27	.005	.22
Nominalization x Operation	0.00	.988	.00	1.3	.264	.04	0.59	.447	.02	5.27	.028	.14
Consistency x Operation	0.92	.346	.03	2.28	.141	.07	0.00	.960	.00	13.96	.001	.30
Nominalization x Consistency x Operation	0.97	.332	.03	1.44	.239	.04	0.18	.675	.01	1.25	.272	.04
Regression Time												
Nominalization	0.51	.481	.02	1.78	.192	.05	3.51	.070	.10	0.99	.327	.03
Consistency	0.87	.357	.03	0.04	.835	.00	1.71	.990	.05	2.3	.139	.07
Operation	21.54	< .001	.40	18.94	< .001	.37	10.28	.003	.24	0.00	.956	.00
Nominalization x Consistency	1.16	.290	.03	0.22	.639	.01	0.82	.373	.03	4.64	.039	.13

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Nominalization x Operation	0.60	.444	.02	1.43	.241	.04	0.04	.843	.01	2.76	.107	.08
Consistency x Operation	1.30	.263	.04	2.63	.114	.08	0.00	.967	.00	6.21	.018	.16
Nominalization x Consistency x Operation	0.88	.356	.03	1.58	.218	.05	0.08	.784	.02	1.32	.259	.04