Factor Score Estimation in Multi-Method Measurement Designs With Planned Missing Data

Mario Lawes and Michael Eid

Freie Universität Berlin, Germany

Postprint, 25 February 2022 (in press, *Psychological Methods*)

© 2022, American Psychological Association. This paper is not the copy of record and may not exactly replicate the final, authoritative version of the article. Please do not copy or cite without authors' permission. The final article will be available, upon publication, via its

DOI: 10.1037/met0000483

Acknowledgements

Mario Lawes is a pre-doctoral fellow of the International Max Planck Research School on the Life Course (LIFE, www.imprs-life.mpg.de). Preliminary results of this work have been presented at the 14th Meeting of the methods and evaluation section of the German Psychological Society, Kiel (Germany), September 16-18, 2019. We are grateful for the fruitful discussions and inspiring feedback during this conference. We further want to thank Katinka Hardt for her feedback on a previous version of this manuscript and Yves Rosseel for sharing valuable insights about factor score estimation in the presence of missing data.

Availability of Data and Materials

All data and materials (simulation and analysis code) are openly available at

<https://osf.io/a85j7/>.

Abstract

Multi-method measurement designs with planned missing data (MMM-PMD) aim at combining cheap proxy methods (e.g., self-reports) with an expensive gold standard method (e.g., biomarker) in order to improve the cost-efficiency of research designs. This article presents a comprehensive simulation study investigating whether accurate factor scores with trustworthy confidence or credible intervals for the gold standard method can be obtained in MMM-PMD designs. The results indicate that the Bartlett-FIML, regression-FIML and fully Bayesian estimator perform equally well in terms of recovering the true rank-order and absolute level of the factor scores for subjects with complete data. However, for subjects with planned missing data the estimated factor scores are considerably biased and cannot be computed with the Bartlett-FIML estimator. The confidence or credible intervals for the estimated factor scores are trustworthy for complete cases. For subjects with planned missing data, the coverage rates vary considerably across conditions. Thus, the present study illustrates that individual scores from MMM-PMD designs should not be used for consequential individual decisions. Still, factor scores from MMM-PMD designs might be useful when used for secondary analyses that rely on group-level statistics like computing propensity scores for matching purposes or for factor score regression. The simulation results underline that multiple reliable proxy methods that are highly correlated with the gold standard method but are not highly correlated with each other should be administered to maximize the accuracy of the estimated factor scores as well as the trustworthiness of the corresponding confidence or credible intervals.

Keywords: planned missing data designs, structural equation modeling, individual scores, factor scores, Bayesian methods

Factor Score Estimation in Multi-Method Measurement Designs With Planned Missing Data

Highly valid assessment methods tend to be expensive, burdensome and sometimes even invasive (Graham et al., 2006). Thus, in order to use resources ethically and efficiently, researchers often cannot administer expensive gold standard methods (e.g., biomarker analysis, experience sampling) in larger studies. Instead, cheaper instruments (e.g., self-reports) that are unfortunately generally systematically biased are often used. Multi-method measurement (MMM) designs with planned missing data (PMD) offer an appealing alternative in these situations, where researchers and practitioners have the choice between rather cheap but flawed measures and more sophisticated but expensive measures. Such MMM-PMD designs are known to optimize specific group-level properties of research designs while keeping the costs and burdens minimal (e.g., Graham et al., 2006). Depending on the number of cheap but biased methods administered, these research designs have been termed two-method-measurement (2MM; Graham et al., 2006) or three-method-measurement (3MM; Lawes et al., 2020) designs. In this article, we will refer to the general class of these research designs as MMM-PMD designs.

The basic rationale of MMM-PMD designs is to administer the cheap proxy method(s) to all subjects while assessing only a randomly drawn subsample additionally with the expensive gold standard. Therefore, the emerging data pattern in MMM-PMD is very specific. On the one side, there are subjects that are assessed with the gold standard method *and* the proxy method(s), which we will call *complete cases*. On the other side, there are subjects that are only assessed with the proxy method(s) and have entirely missing data on the gold standard method. We will call these subjects *planned missingness cases*. Due to the random selection of subjects, who receive the expensive gold standard method, the missingness pattern in MMM-PMD designs is completely random (i.e., missing completely at random [MCAR]). Thus, full information maximum likelihood (FIML) and multiple imputation (MI)

yield unbiased estimates of the structural parameters in MMM-PMD designs (Graham et al., 2006). Simulation studies indicated that MMM-PMD designs are more cost-efficient in terms of the statistical power for predicting a fully observed criterion variable than MMM designs, in which all subjects are assessed using all methods (Graham et al., 2006; Lawes et al., 2020). MMM-PMD designs are especially cost-efficient when multiple highly valid proxy methods are used that have low common systematic variance after controlling for the gold standard method (Lawes et al., 2020).

MMM Designs as a Confirmatory Factor Analysis Model

2MM and 3MM designs have generally been specified using specific multi-method confirmatory factor analysis (CFA) models (see Figure 1C). Identical CFA models have been discussed as *correlated trait–correlated method minus one (CTC[M-1])* models in the multitrait-multimethod context (Eid et al., 2003, 2016) or as *bifactor-(S-1)* models in the context of bifactor models (Eid, 2020; Eid et al., 2017, 2018). The indicators of the gold standard and the proxy method(s) load on the latent variable termed *GS*, which can be interpreted as the true score variable of the construct of interest (e.g., smoking behavior, ability, affective well-being) measured by the gold standard method (Eid et al., 2003; Graham et al., 2006). All indicators belonging to the same cheap proxy method additionally load on a second latent factor, which represents reliable systematic deviations of all indicators of the proxy method that cannot be predicted by the gold standard method (Eid et al., 2003; Lawes et al., 2020). In the context of multitrait-multimethod research and bifactor modeling these factors are often of substantive interest and are referred to as *method factors* or *specific factors* (Eid et al., 2003, 2017). In the existing literature on MMM-PMD designs (Graham, 2012; Graham et al., 2006), these factors have been called *bias factors* and were generally not of focal substantive interest. In order to be consistent with previous work on MMM-PMD designs, we will refer to these factors as *bias factors* in this article. It is, however, important to note that in this context the term *bias* does not refer to estimation bias in a statistical sense but

is rather used as a descriptive label for different response biases of the proxy methods (see Reynolds & Suzuki, 2013). In 2MM designs, only one proxy method is available, thus only one bias factor is modeled (Graham et al., 2006). In 3MM designs, two proxy methods are administered and two bias factors are specified (Lawes et al., 2020). The bias factors are by definition uncorrelated with the factor of the gold standard method but they can be correlated with each other indicating a similar deviation of the proxy methods from the gold standard method (Eid et al., 2003; Lawes et al., 2020).

Figure 1 illustrates three different data-equivalent parametrizations of 3MM designs with three indicators (e.g., scales or items) per method. Figure 1A depicts the traditional correlated factors model, which allows defining one factor for each assessment method. However, it is not possible to model the bias factors in the correlated factors model. In order to obtain the bias factors, the correlated factors model has to be re-parameterized. In particular, the factors corresponding to the proxy methods need to be regressed onto the factor corresponding to the gold standard method. The residual variances of the two proxy factors correspond to the bias factors (see Figure 1B). Moreover, by imposing specific restrictions on the factor loadings, a 3MM model that is data-equivalent to the correlated factors model can be specified (see Figure 1C). The model in Figure 1C corresponds to the restricted CTC(M-1) model (see Geiser et al., 2012). 2MM models can be specified in the same way (see Figure S1 in online supplements).

Individual Scores in MMM-PMD Designs

Existing research has focused on investigating the performance of MMM-PMD designs in terms of group-level parameters (e.g., Graham et al., 2006; Lawes et al., 2020). However, researchers and practitioners are often also interested in deriving individual scores that can be related to norm-populations (e.g., score belongs to the highest 10 %), earlier scores of the same individual (e.g., score increased by 5 points from last year's test) or external criteria (e.g., score is above a certain cut-off value). Moreover, the use of individual scores

has recently gained relevance due to several methodological advances. In particular, individual scores in structural equation models (SEM) can be used if it is computationally impossible to estimate the measurement and structural models simultaneously (Curran et al., 2014) or if the structural model is prone to misspecifications, which might lead to biased estimates in the measurement model (Devlieger & Rosseel, 2017; Hoshino & Bentler, 2013). Individual scores from latent variable models can further be used for obtaining propensity scores from fallible covariates (Raykov, 2012) or for conducting integrative data analyses, in which several studies that measure the same construct using different measures are combined (Curran & Hussong, 2009).

Potential Applications of MMM-PMD Designs

There are numerous potential applications for MMM-PMD designs in medical and psychological research (see Table S1 in the online supplement for a detailed list of potential applications). In the medical domain, MMM-PMD designs can, for example, be used to measure smoking behavior with highly valid biochemical indicators (e.g., carbon monoxide in breath, cotinine in saliva) as well as cheaper, but biased self- or peer report questionnaires (Graham et al., 2006; Palmer et al., 1994). Zawadzki et al. (2012) applied MMM-PMD designs to the measurement of blood pressure and combined blood pressure assessments in the clinic, which are cheap but biased (e.g., white coat hypertension), with highly valid ambulatory blood pressure monitoring. Further, MMM-PMD designs can be useful in the domain of ability testing in educational psychology. Time-intensive and expensive test batteries could, for example, only be administered to a random subsample, whereas cheaper alternatives (e.g., teacher reports) are administered to all students (Little & Rhemtulla, 2013). Additionally, MMM-PMD designs can be used to cost-efficiently measure affective well-being. The gold standard method for measuring affect is the experience sampling method (ESM; Hektner et al., 2007), in which subjects are asked about their mood several times throughout the day via a pager or smartphone. Within a MMM-PMD design, ESM

measurements can be combined with retrospective assessments of affective well-being like the day reconstruction method (DRM; Kahneman et al., 2004). The DRM is highly correlated with ESM measurements but is influenced by recall bias and expectations (Lucas et al., 2021).

All described applications have in common that the gold standard method is highly expensive, time-consuming, tiresome or even invasive. In these cases, MMM-PMD designs can help to minimize the costs as well as the burdens for participants and test administrators. Thus, if it were possible to derive trustworthy individual scores from MMM-PMD designs, these designs could be a highly attractive way to cost-efficiently obtain individual scores for diagnostic purposes or secondary data analyses. However, up to now it is an entirely open question, how valid individual scores obtained from MMM-PMD designs are.

Factor Score Estimation Methods

In the context of CFA, individual scores are generally called *factor scores* and several methods to estimate or predict these factor scores are available (Grice, 2001). Three general approaches to estimating factor scores from fitted CFA can be differentiated: maximum likelihood estimation, empirical Bayes estimation, and fully Bayesian estimation. Maximum likelihood and empirical Bayes factor score estimation is a two-step procedure, where the structural parameters (e.g., factor loadings and (co)variances) are estimated first (e.g., using FIML). Then, these estimated structural parameters are used as fixed parameters to estimate the factor scores. Within the fully Bayesian framework, the structural parameters and the factor scores are estimated simultaneously in a one-step procedure.

Maximum Likelihood Estimator for Factor Scores

In the maximum likelihood framework, factor scores are conceptualized as fixed entities that can be obtained by maximizing the likelihood of the observed data given the factor score estimates (Estabrook & Neale, 2013; Loncke et al., 2018; Skrondal & Rabe-Hesketh, 2004). The maximum likelihood estimator yields conditionally unbiased factor

1 scores that correspond to the traditional Bartlett scores in CFA models (Skrondal & Rabe-
 2 Hesketh, 2004). Bartlett scores for a subject j are defined as

$$\widehat{FS}_{B_j} = (\Lambda' \Theta^{-1} \Lambda)^{-1} \Lambda' \Theta^{-1} y_j, \quad (1)$$

3 where Λ is the factor loading matrix, Θ is the residual covariance matrix of the
 4 manifest variables and y_j is the response vector of subject j (Bartlett, 1937; Hardt et al.,
 5 2019). Often, the residual covariance matrix Θ is replaced with the model implied
 6 covariance matrix, which yields identical results (Bentler & Yuan, 1997; Loncke et al., 2018).
 7 Standard errors for the Bartlett scores can be computed by taking the square root of the
 8 diagonal elements of the estimation error variance-covariance matrix P_B (Hardt et al., 2020;
 9 Lawley & Maxwell, 1971), which is defined as

$$P_B = (\Lambda' \Theta^{-1} \Lambda)^{-1}. \quad (2)$$

10 When missing data are present, the Bartlett scores with the corresponding standard
 11 errors can be separately estimated for each missingness pattern in the data by adjusting the
 12 residual covariance matrix Θ and the factor loading matrix Λ in a way that they only contain
 13 the rows and columns of the non-missing indicators (Loncke et al., 2018). Analogous to
 14 Loncke et al. (2018) we will refer to this method as the Bartlett-FIML estimator. The standard
 15 errors of Bartlett-FIML scores are identical for all subjects within each missingness pattern.
 16 Crucially, for planned missingness cases in MMM-PMD designs, all elements of the adjusted
 17 factor loading matrix Λ_{adj} that correspond to the factor of the gold standard method equal
 18 zero. Thus, in MMM-PMD designs the Bartlett-FIML estimator always yields estimated
 19 factor scores for the gold standard method that equal zero for subjects with planned missing
 20 data. Therefore, the Bartlett-FIML estimator is not suited for MMM-PMD designs.

21

22

Empirical Bayes Estimator

In the empirical Bayes framework (sometimes also called expected a posteriori [EAP]) factor scores are assumed to be random parameters (Skrondal & Rabe-Hesketh, 2004). The empirical Bayes approach minimizes the mean squared prediction error of the estimated factor scores but yields asymptotically biased factor score estimates (Lawley & Maxwell, 1971; Skrondal & Rabe-Hesketh, 2004). Specifically, factor scores are obtained by maximizing the likelihood of the factor score estimates given the observed data, $L(\widehat{FS}_j | y_j)$, which is proportional to the product of the likelihood of the data given the factor score estimates, $L(y_j | \widehat{FS}_j)$, and the likelihood of the factor score estimate, $L(\widehat{FS}_j)$ (Estabrook & Neale, 2013; Loncke et al., 2018; Skrondal & Rabe-Hesketh, 2004):

$$L(\widehat{FS}_j | y_j) \propto L(\widehat{FS}_j)L(y_j | \widehat{FS}_j). \quad (3)$$

Because $L(\widehat{FS}_j | y_j)$ is a multivariate normal distribution in CFA models the empirical Bayes approach is identical to the empirical Bayes modal (EBM) estimator, which is sometimes also called Bayes modal or modal a posteriori (MAP) estimator (Skrondal & Rabe-Hesketh, 2004). Crucially, in CFA models the traditional regression method for obtaining factor scores (Thomson, 1938; Thurstone, 1934) corresponds to the empirical Bayes estimator (Skrondal & Rabe-Hesketh, 2004). Regression scores of a subject j are defined as

$$\widehat{FS}_{R_j} = \Phi \Lambda' \Sigma^{-1} y_j, \quad (4)$$

where Φ is the covariance matrix of the factors, Λ is the factor loading matrix, Σ is the model-implied covariance matrix of the observed variables and y_j is the response vector of subject j (Hardt et al., 2019; Thomson, 1938; Thurstone, 1934). Standard errors for the estimated regression scores can be computed by taking the square root of the diagonal elements of the estimation error variance-covariance matrix P_R (Hardt et al., 2020; Lawley & Maxwell, 1971), which is defined as

$$P_R = \Phi(I + (\Lambda' \Theta^{-1} \Lambda) \Phi)^{-1}. \quad (5)$$

Analogous to the Bartlett-FIML method, the regression scores and corresponding standard errors can be separately estimated for different missingness pattern by adjusting the residual covariance matrix Θ and the factor loading matrix Λ so that they only contain the rows and columns of the non-missing indicators (Loncke et al., 2018; B. O. Muthén & Muthén, 2004). Following Loncke et al. (2018) we will refer to this method as the regression-FIML estimator. Importantly, the regression-FIML estimator is suited to estimate factor scores of the gold standard method for subjects with planned missing data in MMM-PMD designs. As shown in equation (4), the regression method incorporates the covariance matrix of the factors, Φ , when estimating the factor scores, which allows estimating the factor scores for all subjects in MMM-PMD designs. Conceptually, Φ acts as a prior to identify the factor scores (Estabrook & Neale, 2013). Moreover, standard errors of the estimated factor scores are also defined for all subjects in MMM-PMD designs when using the regression-FIML method.

Fully Bayesian Factor Score Estimator

In a fully Bayesian framework, the structural parameters (e.g., factor loadings and (co)variances) as well as the factor scores are treated as random variables and their posterior distributions are estimated simultaneously (Skron dal & Rabe-Hesketh, 2004). In order to derive the posterior distributions, researchers have to define priors for each model parameter. These priors can be more or less informative depending on how much prior knowledge a researcher has about the parameters. The joint posterior distribution of all parameters can be obtained with a Markov chain Monte Carlo (MCMC) sampler (e.g., Gibbs sampler). The mean, mode or median of the conditional posterior distribution of the factor scores can be used as a point estimate of the factor score. In each iteration of the MCMC sampler, values for the posterior distribution of the missing observed variables are drawn by using the current

values of the other parameters. Thus, missing values are treated like any other random variable in the model (McElreath, 2015). When the missingness in the data is MCAR, the fully Bayesian approach minimizes the mean squared prediction error of the factor score estimates but yields asymptotically biased factor score estimates (Skrondal & Rabe-Hesketh, 2004). As the factor scores are computed by using the posterior distributions of the estimated factor scores, the uncertainty in the estimated structural parameters is also reflected in the uncertainty of the factor scores (Asparouhov & Muthén, 2010). Thus, the posterior variances of the estimated factor scores in the fully Bayesian approach are person-specific and larger than the squared standard errors of the estimated factor scores based on the empirical Bayes estimator (Skrondal & Rabe-Hesketh, 2004).

Factor Score Estimation in The Presence of Missing Data

The existing literature on estimating factor scores in CFA models with missing data is rather sparse. Estabrook and Neale (2013) used a simulation study to investigate the properties of different factor score estimators when missing data were present in a one-factor model. Their simulation study indicates that the regression-FIML estimator, which the authors called *weighted full information maximum likelihood estimator*, performs better in terms of recovering the true rank-order of subjects than the Bartlett-FIML estimator, the traditional Bartlett method and simple sum scores. Estabrook and Neale (2013) focused on the recovery of the rank-order of subjects, which is particularly relevant for relative comparisons. However, in many settings (e.g., individual assessments, factor score regression), the actual values of the factor score are of interest. In these situations, the absolute deviations of the factor score estimates from the true scores is a more important criterion.

In another simulation study, Loncke et al. (2018) compared the performance of Bartlett-FIML and regression-FIML scores in a factor score regression model to a full SEM model and an ANOVA-based approach. Their results indicate that under the presence of

missing data the regression-FIML scores yield unbiased estimates that are comparable to the estimates from the full SEM model, whereas the Bartlett-FIML scores yield biased estimates.

To our knowledge, no study has yet investigated the accuracy of factor score estimates as well as the trustworthiness of their standard errors in MMM-PMD designs. Due to the unique missing data structure and its multi-factor structure, MMM-PMD designs impose very specific challenges to the estimation of factor scores. In particular, it is unclear whether valid factor scores with trustworthy standard errors can be estimated for subjects with planned missing data. Moreover, we are not aware of studies investigating the performance of fully Bayesian factor score estimation in CFA models with missing data.

Aim of the Study

The goal of the present study is to investigate whether accurate factor scores with trustworthy confidence or credible intervals can be estimated within MMM-PMD designs. Specifically, we use a comprehensive simulation study to compare the performance of the Bartlett-FIML, the regression-FIML and the fully Bayesian factor score estimation method in MMM-PMD designs. Moreover, we aim at identifying conditions that improve the accuracy of the estimated factor scores in MMM-PMD designs.

Methods

Simulation Overview

The simulation study consisted on the following four steps: In a first step, we simulated 500 true factor scores within nine different population models. In a second step, we used these true factor scores to generate manifest scores for different levels of reliability and varying amounts of planned missing data. In a third step, we estimated the factor scores and their corresponding confidence or credible intervals using the Bartlett-FIML, the Regression-FIML and the fully Bayesian estimator based on the generated manifest data. In a fourth step, we evaluated the quality of the factor score estimates using various criteria. In the following, we will illustrate the simulation design in detail.

Population Models

First, we experimentally varied the construct validity of the proxy methods. Specifically, we differentiated between highly valid and moderately valid proxy methods. Highly valid proxy methods were operationalized to correlate to .7 with the gold standard, whereas moderately valid proxy methods correlated to .5 with the gold standard. Using this approach, we defined three different conditions, where (a) either both proxy methods were highly valid (*high*), (b) one proxy method was highly valid and the other one moderately valid (*mixed*) or (c) both proxy methods were moderately valid (*moderate*).

Second, we varied the correlation of the bias factors ($\rho_{\text{Bias1}, \text{Bias2}}$) to examine how the amount of shared systematic bias variance of the proxy methods affects factor score estimation. In particular, we simulated the following three conditions: *common* bias ($\rho_{\text{Bias1}, \text{Bias2}} = .5$), *unique* bias ($\rho_{\text{Bias1}, \text{Bias2}} = .1$) and *inverse* bias ($\rho_{\text{Bias1}, \text{Bias2}} = -.5$). These correlations imply a specific correlation pattern for the model with correlated factors (see Table 1).

Fully crossing these conditions leads to 3 (*high, mixed, moderate*) x 3 (*common, unique, inverse*) = 9 population models. For each population model, we generated true factor scores for all three assessment methods (i.e., gold standard, proxy 1 and proxy 2) based on the correlated factors model (see Figure 1A) for 500 hypothetical subjects by drawing 500 values from the resulting multivariate normal distribution using R (R Core Team, 2017). The true factor scores were set to have a mean of zero and a variance of 1.

Manifest Data

Across all conditions, we assumed that three indicators (e.g., items or scales) per assessment method were available. We varied the amount and the quality of the available indicators to simulate different situations researchers might face.

Reliability of Indicators

The reliability of the indicators was either set to .5 (low reliability), .7 (moderate reliability) or .9 (high reliability). Within a given condition, the reliability was identical for all indicators, regardless of whether it was an indicator of the gold standard or a proxy method. Based on the true factor scores and the reliability in a given condition, we simulated the manifest data for all twelve indicators (i.e., three indicators per method). All factor loadings were set to 1. The manifest data were generated by adding a random error term to the true factor scores. All error terms were drawn from a normal distribution with a mean of zero and a variance $\sigma_{\varepsilon_k}^2$ using R (R Core Team, 2017). The value of $\sigma_{\varepsilon_k}^2$ is dependent on the assumed reliability in a given condition k . In conditions with high reliability ($Rel = .9$) $\sigma_{\varepsilon_k}^2$ is equal to 0.1, in conditions with moderate reliability ($Rel = .7$), $\sigma_{\varepsilon_k}^2$ is approximately equal to 0.43 and in conditions with low reliability ($Rel = .5$) $\sigma_{\varepsilon_k}^2$ is equal to 1. For each condition, we repeated this procedure 500 times to obtain 500 datasets (i.e. replications).

Amount of Missing Data

The total sample size within each condition was 500. Following the rationale of MMM-PMD designs, all subjects were assessed on both proxy methods. The number of subjects with data on the gold standard method was experimentally varied according to the following five conditions: 125, 250, 375, 475 and 500. We chose a lower limit of 125 complete cases because prior research indicated that at least 125 complete cases are needed for MMM-PMD designs to converge (Lawes et al., 2020; Rhemtulla & Little, 2012). To introduce planned missingness to the data, we deleted the observations of all indicators of the gold standard for a random subsample of subjects according to the different missing data proportions. In all conditions with the same amount of missing data, the expensive indicators were deleted for the same subjects in order to compare the results across replications and

conditions. The data generating scripts as well as the raw data files can be found at <https://osf.io/a85j7/>.

Analysis Models

We analyzed the generated manifest data using the 2MM-PMD design (Graham et al., 2006), and the 3MM-PMD design (Lawes et al., 2020). The 2MM-PMD design incorporates the (partially observed) indicators of the gold standard as well as the indicators of the first proxy method. Therefore, the population model in the *mixed* and *valid* conditions is identical in 2MM-PMD designs. The 3MM-PMD design incorporates data from the indicators of both proxy methods as well as the gold standard method. We set the factor loadings of the first indicators of each method to 1 to identify the measurement models. To obtain the bias factors, we re-parametrized the 2MM and 3MM model so that the factor(s) of the proxy method(s) are regressed onto the factor of the gold standard method (Figure 1B). This allows to analyze the data within the tradition of MMM-PMD designs (e.g., examining the bias factors) in a model that is data equivalent to the data generating model (i.e., correlated factors model).

Factor Score Estimation

Bartlett-FIML and Regression-FIML Estimator

We fit the analysis models to the manifest data using the implemented FIML algorithm in lavaan (Rosseel, 2012). To estimate the Bartlett-FIML and regression-FIML factor scores, we used the `lavPredict()`-function. As described earlier, the Bartlett-FIML method always yields factor score estimates of zero for the gold standard method in MMM-PMD designs for subjects with planned missing data. Thus, we manually replaced these with missing values. The analysis scripts of the simulation are available at <https://osf.io/a85j7/>.

Fully Bayesian Estimator

We used Mplus 8.1 (L. K. Muthén & Muthén, 2017) to estimate the fully Bayesian models and to extract the factor scores.¹ We used the default uninformative priors and two MCMC chains to estimate the posterior distributions of all parameters. The first half of the MCMC iterations were discarded as warmup iterations. To control for high autocorrelations, we used only every 20th iteration (i.e., thinning). At least 5000 MCMC iterations were run after the warmup iterations to ensure convergence of the MCMC chains, if needed we continued sampling from the posterior until the potential scale reduction value (PSR) was below 1.05 for all parameters or a total of 50,000 iterations was reached. Following previous work by Asparouhov & Muthen (2010) we obtained the posterior distributions of the factor scores by drawing 500 plausible values for each factor score. The means of the posterior distributions were used as point estimates for the different parameters. Mplus outputs were imported into R using the software package MplusAutomation (Hallquist & Wiley, 2018) for further analyses.

Evaluation Criteria

In this study, we only focus on the quality of the estimated factor scores of the gold standard method (and not the factor scores of the bias factors). We computed the following evaluation criteria separately for complete cases and planned missingness cases.

¹Besides Mplus, numerous open-source programs like stan (Carpenter et al., 2017) or jags (Denwood, 2016), which are both implemented in R, are available to conduct the fully Bayesian analysis. The R-package blavaan (Merkle & Rosseel, 2018) provides a user-friendly interface based on the lavaan syntax to specify Bayesian structural equation models, which can be run using stan or jags. Unfortunately, the runtime of the simulated models in blavaan is much higher compared to the runtime of Mplus. Therefore, we used Mplus.

1 ***Mean Absolute Error***

2 As an accuracy measure, we calculated the mean absolute error (MAE) of the
 3 estimated factor scores by $\frac{1}{N_j} \sum_{j=1}^{N_j} |\widehat{FS}_{j,r} - FS_j|$, where N_j is equal to the number of subjects
 4 (i.e. 500) and $\widehat{FS}_{j,r}$ is the estimated factor score of subject j in replication r and FS_j is the
 5 true factor score of subject j . To obtain summary statistics for each condition, we averaged the
 6 MAEs across all proper replications. Low MAE values are desirable – especially, when the
 7 levels of the factor scores are interpreted in respect to a cut-off value.

8 ***Correlations of True Scores and Factor Score Estimates***

9 Within each replication, we calculated the Pearson product moment correlation of the
 10 estimated factor scores and the true factor scores by $cor(\widehat{FS}_r, FS)$, where $\widehat{FS}_r$ corresponds to
 11 a matrix of estimated factor scores in replication r and FS is a matrix of true factor scores.
 12 Sometimes this index is interpreted as the factor score reliability (Estabrook & Neale, 2013).
 13 To obtain summary statistics, we averaged these correlations across all proper replications.
 14 High correlations indicate that the rank-order of the true scores is preserved by the factor
 15 score estimates (Hardt et al., 2019). Preserving the correct rank-order is essential, when
 16 researchers want to compare subjects among each other.

17 ***Standard Errors***

18 For the Bartlett-FIML and regression-FIML method, we used the standard errors
 19 reported by lavaan (Rosseel, 2012). In the fully Bayesian approach, we used the standard
 20 deviations of the posterior distributions of the factor score estimates as standard errors.
 21 Smaller standard errors are desirable as they indicate that the factor scores have been
 22 estimated with higher precision. However, it is important to evaluate whether the standard
 23 error estimates are trustworthy and reflect the true uncertainty associated with the factor score
 24 estimates.

Coverage

To investigate whether the confidence or credible intervals are trustworthy, we determined the 95%-coverage rates by computing the relative proportion of replications in which the 95%- confidence or credible interval contained the true factor scores. Ideally, the coverage rates should be close to .95. For the Bartlett-FIML scores and regression-FIML scores, we calculated the 95%-confidence or credible intervals by $\widehat{FS}_{j,r} \pm 1.96 \cdot \widehat{SE}_{j,r}$, where $\widehat{FS}_{j,r}$ corresponds to the estimated factor score of subject j in replication r and $\widehat{SE}_{j,r}$ is the corresponding standard error. For the fully Bayesian method, Mplus returns the 2.5% and 97.5% percentiles of the posterior distribution of the factor scores, which we used to define the 95% - credible interval.

Results

Convergence

All 67,500 replications converged in the maximum-likelihood-based model estimation (i.e., for the Bartlett-FIML and regression-FIML factor score estimation). However, 19 replications yielded improper solutions (e.g., negative variances). All improper solutions resulted from 3MM-PMD models in the *high-inverse* condition with low reliability. These improper replications were dropped from all further analyses. We investigated the convergence of the fully Bayesian models by inspecting the PSR values first. Three models did not reach the convergence criterion of a PSR value below 1.05 for all parameters after the specified maximum number of 50,000 iterations. All three of these models were 3MM-PMD models in the *high-unique* condition with low reliability and 125 complete cases. We excluded these replications from all further analyses. After the exclusion, the highest PSR value was 1.044 and the mean PSR value across all replications was 1.02. Moreover, we investigated the proper mixing of the MCMC chains as well as the size of the autocorrelations for a number of randomly drawn conditions using the available plots in Mplus. All inspected

plots looked satisfactory so that we concluded that all but the three excluded Bayesian models converged.

Correlations Among the Different Factor Score Estimators

Table S2 in the supplementary materials depicts the average Pearson correlation of the three factor score estimation methods across all proper replications for the different population models. Averaged across all conditions, the regression-FIML scores correlated to .9994 with the fully Bayesian scores and to .9859 with the Bartlett-FIML scores for complete cases. Further, the Bartlett-FIML scores correlated to .9849 with the fully Bayesian scores for subjects with complete data across all conditions. For subjects with planned missing data, the estimated factor scores of the fully Bayesian approach correlated to .9941 with the regression-FIML scores across all conditions. As described before, the Bartlett-FIML method is not suited to estimate factor scores of the gold standard for subjects with planned missing data. Thus, no correlations with the Bartlett-FIML scores for planned missingness cases could be computed.

MAE and Correlations of Factor Score Estimates with True Scores

Figure 2 depicts the MAE values and the correlations of the estimated factor scores with the true factor scores. For complete cases the MAE values varied between 0.14 and 0.48 ($M = 0.28$) and the correlations with the true scores varied between .83 and .99 ($M = .93$). Highly reliable indicators improved the quality of the estimated factor scores in terms of MAE and the correlations with the true scores. Moreover, the narrow boxplots indicate that the quality of the estimated factor scores of complete cases was highly stable across the different population models, analysis designs (i.e., 3MM vs. 2MM) and number of complete cases in the research design. Compared to the complete cases, the quality of the estimated factor scores of planned missingness cases was poorer and more variable across the simulated conditions. Specifically, the MAE varied between 0.37 and 0.79 ($M = 0.61$) and the correlations with the true scores varied between .39 and .89 ($M = .64$). The number of

complete cases present in the design did not have a strong effect on the quality of the estimated factor scores (see Figure S2 in the online supplement).

Properties and Availability of Proxy Methods

Figure 3 depicts the MAE of the regression-FIML and fully Bayesian scores for subjects with planned missing data. The MAE was smaller, if reliable indicators are available and the proxy methods were highly correlated with the gold standard (i.e., high validity). Moreover, the availability of a second proxy method (i.e., using a 3MM-PMD design) yielded more accurate factor score estimates compared to using just one proxy method (i.e., using a 2MM-PMD design). 3MM-PMD designs yielded especially accurate factor scores, if the proxy methods were highly correlated with the gold standard (i.e., high validity) and not highly positively correlated with each other (i.e., low or negative correlation of the bias factors). Importantly, conditions that minimized the MAE also maximized the correlation of the estimated factor scores with the true scores and minimized the estimated standard error (see Figures S3 and S4 in the online supplement).

Standard Errors and Coverage

The estimated standard errors obtained in a prototypical replication for the 500 subjects is depicted in Figure S5 in the supplementary materials. The figure illustrates the previously described differences among the estimators: The Bartlett-FIML method yields the same standard error for all subjects with complete data and is not suited to estimate factor scores or standard errors for subjects with planned missing data. Contrarily, the regression-FIML estimator yields the same standard error within each missingness pattern. The fully Bayesian estimator yields person-specific standard errors. Standard errors of factor scores of subjects with planned missing data are higher compared to those of complete cases. Figure 4 depicts the coverage rates of the 95% confidence or credible intervals. For complete cases the coverage rates ranged from .87 to .96 ($M = .94$). The coverage rates for complete cases were closer to the nominal level of .95 when at least 250 complete cases were present compared to

conditions in which only 125 complete cases were present. Particularly, the coverage rates of the fully Bayesian estimator varied considerably when only 125 complete cases were present. The coverage rates of subjects with planned missing data ranged from .85 to 1 ($M = .94$). In conditions, where 250 or 375 complete cases were present the coverage rates for subjects with planned missing data were more homogenous across conditions and closer to the nominal level of .95 (range: .9-.98) compared to conditions with 125 or 475 complete cases (range: .85-1). When 3MM designs with highly valid proxy methods (i.e., *valid* conditions) were used and the bias factors had low or negative correlations (i.e., *unique* or *inverse* conditions) coverage rates for subjects with planned missing data were very close to the nominal level and ranged from .94 to .96.

Discussion

Multi-method measurement designs with planned missing data (MMM-PMD) are elegant and cost-efficient research designs for group level-analyses and combine the advantages of expensive gold standard methods with those of cheaper proxy methods. The present simulation study extended the work on MMM-PMD designs by investigating whether these designs can be used to obtain accurate factor scores with trustworthy confidence or credible intervals for the gold standard method. The major challenge for estimating factor scores in MMM-PMD designs is that only some subjects have complete data whereas many other subjects have data on the proxy methods but not on the gold standard method. We showed that the regression-FIML and fully Bayesian estimator are in principle suited for estimating factor scores in MMM-PMD designs as they appropriately deal with the unique missing data structure, whereas the Bartlett-FIML method is not.

The results of the present simulation study indicate that the quality of the estimated factor scores of the gold standard method heavily depends on whether or not the gold standard method was administered. For subjects with data on the gold standard method (i.e., complete cases), the MAE of the estimated factor scores was on average 0.15 of a standard deviation,

when highly reliable indicators ($Rel = .9$) were used. This is a small to medium effect according to Cohen's (1992) classification of the effect size d for differences in the means. The simulation underlined that factor scores can be estimated with higher accuracy, if highly reliable indicators are used. This relationship is well known and has been discussed intensively by other authors (e.g., Guttman, 1955). The specific factor score estimator, the underlying population model, and the number of complete cases in the design did not have a large impact on the accuracy of the estimated factor scores of subjects with complete data.

As expected, the estimated factor scores of subjects with planned missing data were less accurate than the factor scores estimates of subjects with complete data. Even in conditions with highly reliable indicators ($Rel = .9$), the MAE of the estimated factor scores of subjects with planned missing data was on average 0.58 of a standard deviation, which is a medium to large effect according to Cohen's (1992) classification. Thus, the factor scores of subjects with planned missing data were generally estimated with considerable bias.

Administering two valid proxy methods instead of one increased the accuracy of the estimated factor scores for subjects with planned missing data. Moreover, a low or negative correlation of the bias factors was beneficial for accurately estimating factors scores for subjects with planned missing data. This result is consistent with existing research on group-level properties of 3MM-PMD designs (Lawes et al., 2020) and can be explained by the fact that smaller correlations of the bias factors result in smaller correlations of the proxy methods (see Table 1). Just like in traditional regression analysis, it is particularly valuable for predicting a dependent variable (here: factor scores of the gold standard method) if the independent variables (here: proxy measures) are highly correlated with the dependent variable but uncorrelated among each other. In order for the proxy methods to be uncorrelated with each other but highly correlated with the gold standard method, the correlations of the corresponding bias factors (which are partial correlations with respect to the gold standard method) have to be negative. Still, even in the best performing condition, the MAE of the

1 estimated factor scores of subjects with planned missing data was 0.37 indicating substantial
2 bias in the estimation of the factor scores for individuals with planned missing data. Thus,
3 individual scores obtained from MMM-PMD designs are generally not suited for highly
4 consequential decisions (e.g., high stakes testing, forensic diagnostics).

5 Besides the accuracy of the estimated factor scores, we also investigated the
6 trustworthiness of the corresponding confidence or credible intervals. Our simulation showed
7 that the confidence or credible intervals of the estimated factor scores of complete cases were
8 generally trustworthy as the coverage rates were only slightly below the nominal level. For
9 subjects with planned missing data, the coverage rates were on average also close to the
10 nominal level but varied considerably across conditions -- especially when few complete
11 cases or few subjects with planned missing data were present. We recommend to always
12 report the confidence or credible intervals of the factor score estimates from MMM-PMD
13 designs in order to communicate the different levels of uncertainty associated with the
14 estimated factor scores of subjects with and without planned missing data. Importantly, those
15 conditions that maximized the accuracy of the estimated factor scores (i.e., using multiple
16 reliable and valid proxy methods that have low inter-correlations) also minimized the
17 estimated standard errors and yielded trustworthy confidence and credible intervals when
18 enough complete cases and subjects with planned missing data were present.

19 Thus, estimated factor scores from MMM-PMD designs might be promising when
20 used in secondary analyses that rely on group-level properties (i.e. rank-order of individuals)
21 rather than on the absolute levels of the individual factor scores. For example, estimated
22 factor scores from MMM-PMD designs could be used in factor score regression (Devlieger &
23 Rosseel, 2017) or for computing propensity scores based on latent covariates (Raykov, 2012).
24 Even though we did not directly assess the performance of factor scores from MMM-PMD
25 designs in these contexts, our simulation provides important insights that might help to
26 effectively design MMM-PMD studies for these purposes. First, multiple reliable proxy

methods that are highly valid (i.e., high correlation with the gold standard method) but uncorrelated with each (i.e., contain independent construct-relevant information) should be used in order to increase the accuracy of the estimated factor scores. Our simulation indicates that under these conditions the estimated factor scores correlate to well above .8 with the true scores for individuals with planned missing data. Second, at least 250 complete cases and 125 subjects with planned missing data should be present in the design in order to obtain trustworthy confidence or credible intervals for the factor score estimates. Third, the uncertainty of the estimated factor scores should be incorporated in the analysis model. In this regard, we advise to use a fully Bayesian approach, which has the additional advantage of accounting for the uncertainty associated with the structural parameters when estimating the factor scores. Unfortunately, the fully Bayesian approach comes with the cost of higher computational demands. In our simulation, a fully Bayesian model took up to two minutes, whereas the regression-FIML models converged in less than one second.

Conclusions

Taken together, the present study illustrates that MMM-PMD designs are not a panacea for obtaining individual scores when assessment methods are expensive. While MMM-PMD designs yielded accurate factor scores for subjects with complete data, factor scores of subjects with planned missing data were considerably biased. Therefore, MMM-PMD designs are generally not suited for situations in which it is critical to obtain highly accurate individual scores (i.e., high stakes testing, forensic decisions). Additional studies are needed to examine the usefulness of factor scores obtained from MMM-PMD studies for secondary analyses such as factor score regression or the computation of propensity scores.

Limitations and Future Research

An important direction for future research is to identify real-life applications for MMM-PMD designs that yield the desired correlational pattern, where the cheap proxy methods are highly correlated with the gold standard method but not with each other.

Unfortunately, we are unaware of an applications that produces this specific pattern. A potential way to achieve this constellation could be to employ research methods that use different response modes (e.g. interview vs. questionnaire) or data sources (e.g., self-report and informant rating).

Further, more research is needed to evaluate the feasibility and cost-effectiveness of MMM-PMD designs when the estimated factor scores are used in secondary analyses such as factor score regression (Devlieger & Rosseel, 2017), integrative data analyses (Curran & Hussong, 2009) or propensity scores estimation (Raykov, 2012). Comprehensive simulation studies for each of these approaches would be helpful to determine the usefulness of MMM-PMD designs in these contexts. In addition, studies investigating the quality of factor score estimates in other PMD designs in which subjects are randomly assigned to subsets of items (Graham et al., 2001, 2006) or measurement waves (Brandmaier et al., 2020; Graham et al., 2001; McArdle & Woodcock, 1997; Wu et al., 2016) are needed.

Moreover, in some research domains it might not be possible to administer highly reliable measures. In such situations, it can be beneficial to increase the number of indicators (e.g., present more items in a self-report questionnaire). Thus, it seems worthwhile to investigate the interplay between the number of indicators and the number of proxy methods for estimating factor scores. In this context, it would be helpful to develop tools that support applied researchers in designing optimal research designs for a given research budget. Finding the optimal assessment designs can either be done using simulation studies (e.g., Lawes et al., 2020; Wu et al., 2016) or by using analytical approaches that rely on the asymptotic properties of the research designs (e.g., Brandmaier et al., 2020; Rhemtulla et al., 2016). Optimally, an easy-to-use software package should be developed in which researchers can simply plug in the possible design choices, the available budget as well as the criterion to optimize (e.g., accuracy of factor scores).

References

- Asparouhov, T., & Muthén, B. O. (2010). Plausible values for latent variables using Mplus. *Mplus Technical Report*. <http://www.statmodel.com/download/Plausible.pdf>
- Bartlett, M. S. (1937). The statistical conception of mental factors. *British Journal of Psychology*, 28(1), 97–104. <https://doi.org/doi:10.1111/j.2044-8295.1937.tb00863.x>
- Bentler, P. M., & Yuan, K. H. (1997). Optimal conditionally unbiased equivariant factor score estimators,. In M. Berkane (Ed.), *Latent Variable Modeling and Applications to Causality* (pp. 259–281). Springer.
- Brandmaier, A. M., Ghisletta, P., & von Oertzen, T. (2020). Optimal planned missing data design for linear latent growth curve models. *Behavior Research Methods*, 52, 1445–1458. <https://doi.org/10.3758/s13428-019-01325-y>
- Carpenter, B., Gelman, A., Hoffman, M. D., Lee, D., Goodrich, B., Betancourt, M., Brubaker, M. A., Guo, J., Li, P., & Riddell, A. (2017). Stan: A probabilistic programming language. *Journal of Statistical Software*, 76(1), 1–32. <https://doi.org/10.18637/jss.v076.i01>
- Cohen, J. (1992). A Power Primer. *Journal of Organometallic Chemistry*, 112(1), 155–159. <https://doi.org/https://doi.org/10.1037/0033-2909.112.1.155>
- Curran, P. J., & Hussong, A. M. (2009). Integrative Data Analysis: The Simultaneous Analysis of Multiple Data Sets. *Psychological Methods*, 14(2), 81–100. <https://doi.org/10.1037/a0015914>
- Curran, P. J., McGinley, J. S., Bauer, D. J., Hussong, A. M., Burns, A., Chassin, L., Sher, K., & Zucker, R. (2014). A Moderated Nonlinear Factor Model for the Development of Commensurate Measures in Integrative Data Analysis. *Multivariate Behavioral Research*, 49(3), 214–231. <https://doi.org/10.1080/00273171.2014.889594>
- Denwood, M. J. (2016). runjags: An R package providing interface utilities, model templates, parallel computing methods and additional distributions for MCMC models in JAGS.

- 1 *Journal of Statistical Software*, 71(2016), 1–25. <https://doi.org/10.18637/jss.v071.i09>
- 2 Devlieger, I., & Rosseel, Y. (2017). Factor score path analysis: An alternative for SEM?
- 3 *Methodology*, 13, 31–38. <https://doi.org/10.1027/1614-2241/a000130>
- 4 Eid, M. (2020). Multi-Faceted Constructs in Abnormal Psychology: Implications of the
- 5 Bifactor S - 1 Model for Individual Clinical Assessment. *Journal of Abnormal Child*
- 6 *Psychology*, 48(7), 895–900. <https://doi.org/10.1007/s10802-020-00624-9>
- 7 Eid, M., Geiser, C., & Koch, T. (2016). Measuring method effects: From traditional to design-
- 8 oriented approaches. *Current Directions in Psychological Science*, 25(4), 275–280.
- 9 <https://doi.org/10.1177/0963721416649624>
- 10 Eid, M., Geiser, C., Koch, T., & Heene, M. (2017). Anomalous results in G-factor models:
- 11 Explanations and alternatives. *Psychological Methods*, 22(3), 541–562.
- 12 Eid, M., Krumm, S., Koch, T., & Schulze, J. (2018). Bifactor models for predicting criteria by
- 13 general and specific factors: Problems of nonidentifiability and alternative solutions.
- 14 *Journal of Intelligence*, 6(3), 42. <https://doi.org/10.3390/jintelligence6030042>
- 15 Eid, M., Lischetzke, T., Nussbeck, F. W., & Trierweiler, L. I. (2003). Separating trait effects
- 16 from trait-specific method effects in multitrait-multimethod models: A multiple-indicator
- 17 CT-C(M-1) model. *Psychological Methods*, 8(1), 38–60. [https://doi.org/10.1037/1082-](https://doi.org/10.1037/1082-989X.8.1.38)
- 18 989X.8.1.38
- 19 Estabrook, R., & Neale, M. (2013). A comparison of factor score estimation methods in the
- 20 presence of missing data: reliability and an application to nicotine dependence.
- 21 *Multivariate Behavioral Research*, 48(1), 1–27.
- 22 <https://doi.org/10.1080/00273171.2012.730072.A>
- 23 Geiser, C., Eid, M., West, S. G., Lischetzke, T., & Nussbeck, F. W. (2012). A Comparison of
- 24 Method Effects in Two Confirmatory Factor Models for Structurally Different Methods.
- 25 *Structural Equation Modeling*, 19(3), 409–436.
- 26 <https://doi.org/10.1080/10705511.2012.687658>

- 1 Graham, J. W. (2012). *Missing Data Analysis and Design*. Springer Science & Business
2 Media. <https://doi.org/10.1007/978-1-4614-4018-5>
- 3 Graham, J. W., Taylor, B., & Cumsille, P. (2001). Planned missing-data designs in analysis of
4 change. In L. M. Collins & A. G. Sayer (Eds.), *New methods for the analysis of change*
5 (pp. 335–353). American Psychological Association.
6 <https://doi.org/10.1146/annurev.psych.58.110405.085530>
- 7 Graham, J. W., Taylor, B. J., Olchowski, A. E., & Cumsille, P. E. (2006). Planned missing
8 data designs in psychological research. *Psychological Methods*, 11(4), 323–343.
9 <https://doi.org/10.1037/1082-989X.11.4.323>
- 10 Grice, J. W. (2001). Computing and evaluating factor scores. In *Psychological Methods* (Vol.
11 6, Issue 4, pp. 430–450). <https://doi.org/10.1037/1082-989X.6.4.430>
- 12 Guttman, L. (1955). The determinacy of factor score matrices with implications for five other
13 basic problems of common-factor theory. *British Journal of Statistical Psychology*, 8,
14 65–81. <https://doi.org/10.1111/j.2044-8317.1955.tb00321.x>
- 15 Hallquist, M. N., & Wiley, J. F. (2018). MplusAutomation: An R Package for Facilitating
16 Large-Scale Latent Variable Analyses in Mplus. *Structural Equation Modeling*, 25(4),
17 621–638. <https://doi.org/10.1080/10705511.2017.1402334>
- 18 Hardt, K., Hecht, M., Oud, J. H. L., & Voelkle, M. C. (2019). Where Have the Persons
19 Gone?—An Illustration of Individual Score Methods in Autoregressive Panel Models.
20 *Structural Equation Modeling*, 26(2), 310–323.
21 <https://doi.org/10.1080/10705511.2018.1517355>
- 22 Hardt, K., Hecht, M., & Voelkle, M. C. (2020). Robustness of individual score methods
23 against model misspecification in autoregressive panel models. *Structural Equation*
24 *Modeling*, 27(2), 240–254. <https://doi.org/10.1080/10705511.2019.1642755>
- 25 Hektner, J. M., Schmidt, J. A., & Csikszentmihalyi, M. (2007). *Experience sampling method:*
26 *measuring the quality of everyday life*. Sage Publications.

- 1 Hoshino, T., & Bentler, P. M. (2013). Bias in factor score regression and a simple solution. In
- 2 A. R. De Leon & K. C. Chough (Eds.), *Analysis of mixed data: Methods & applications*
- 3 (pp. 43–61). CRC Press, Taylor & Francis Group.
- 4 [https://doi.org/https://doi.org/10.1201/b14571](https://doi.org/10.1201/b14571)
- 5 Kahneman, D., Krueger, A. B., Schkade, D. A., Schwarz, N., & Stone, A. A. (2004). A survey
- 6 method for characterizing daily life experience: the day reconstruction method. *Science*
- 7 (*New York, N.Y.*), 306(5702), 1776–1780. <https://doi.org/10.1126/science.1103572>
- 8 Lawes, M., Schultze, M., & Eid, M. (2020). Making the Most of Your Research Budget:
- 9 Efficiency of a Three-Method Measurement Design With Planned Missing Data.
- 10 *Assessment*, 27(5), 903–920. <https://doi.org/10.1177/1073191118798050>
- 11 Lawley, D. N., & Maxwell, A. E. (1971). *Factor analysis as a statistical method* (2nd Ed.).
- 12 American Elsevier Pub. Co.
- 13 Little, T. D., & Rhemtulla, M. (2013). Planned missing data designs for developmental
- 14 researchers. *Child Development Perspectives*, 7(4), 199–204.
- 15 <https://doi.org/10.1111/cdep.12043>
- 16 Loncke, J., Eichelsheim, V. I., Branje, S. J. T. T., Buysse, A., Meeus, W. H. J. J., & Loeys, T.
- 17 (2018). Factor Score Regression With Social Relations Model Components: A Case
- 18 Study Exploring Antecedents and Consequences of Perceived Support in Families.
- 19 *Frontiers in Psychology*, 9(1699), 1–19. <https://doi.org/10.3389/fpsyg.2018.01699>
- 20 Lucas, R. E., Wallsworth, C., Anusic, I., & Donnellan, M. B. (2021). A direct comparison of
- 21 the day reconstruction method (DRM) and the experience sampling method (ESM).
- 22 *Journal of Personality and Social Psychology*, 120(3), 816–835.
- 23 <https://doi.org/10.1037/pspp0000289>
- 24 McArdle, J. J., & Woodcock, R. W. (1997). Expanding test-retest designs to include
- 25 developmental time-lag components. *Psychological Methods*, 2(4), 403–435.
- 26 <https://doi.org/10.1037/1082-989X.2.4.403>

- 1 McElreath, R. (2015). *Statistical Rethinking: A Bayesian Course With Examples in R and*
2 *Stan*. CRC Press. <https://doi.org/10.1201/9781315372495>
- 3 Merkle, E. C., & Rosseel, Y. (2018). blavaan: Bayesian structural equation models via
4 parameter expansion. *Journal of Statistical Software*, 85(4), 1–30.
5 <https://doi.org/10.18637/jss.v085.i04>
- 6 Muthén, B. O., & Muthén, L. K. (2004). *Mplus Technical Appendices*. Muthén & Muthén.
- 7 Muthén, L. K., & Muthén, B. O. (2017). *Mplus user's guide*. (8th ed.). CA: Muthén &
8 Muthén.
- 9 Palmer, R. F., Dwyer, J. H., & Semmer, N. (1994). A measurement model of adolescent
10 smoking. *Addictive Behaviors*, 19(5), 477–489. <https://doi.org/10.1016/0306->
11 [4603\(94\)90003-5](https://doi.org/10.1016/0306-4603(94)90003-5)
- 12 R Core Team. (2017). *R: a language and environment for statistical computing*.
13 <https://www.r-project.org/>
- 14 Raykov, T. (2012). Propensity score analysis with fallible covariates: A note on a latent
15 variable modeling approach. *Educational and Psychological Measurement*, 72(5), 715–
16 733. <https://doi.org/10.1177/0013164412440999>
- 17 Reynolds, C. R., & Suzuki, L. A. (2013). Bias in Psychological Assessment: An Empirical
18 Review and Recommendations. In J. R. Grahams, J. A. Naglieri, & I. B. Weiner (Eds.),
19 *Handbook of psychology: Assessment psychology* (pp. 82–113). John Wiley & Sons,
20 Inc.. <https://doi.org/10.1002/0471264385.wei1004>
- 21 Rhemtulla, M., & Little, T. D. (2012). Planned missing data designs for research in cognitive
22 development. *Journal of Cognition and Development*, 13(4), 425–438.
23 <https://doi.org/10.1080/15248372.2012.717340>
- 24 Rhemtulla, M., Savalei, V., & Little, T. D. (2016). On the asymptotic relative efficiency of
25 planned missingness designs. *Psychometrika*, 81(1), 60–89.
26 <https://doi.org/10.1007/s11336-014-9422-0>

- 1 Rosseel, Y. (2012). {lavaan}: an {R} package for structural equation modeling. *Journal of*
2 *Statistical Software*, 48(2), 1–36. <http://www.jstatsoft.org/v48/i02/>
- 3 Skrondal, A., & Rabe-Hesketh, S. (2004). *Generalized latent variable modeling. Multilevel,*
4 *longitudinal, and structural equation models*. Chapman & Hall.
5 <https://doi.org/10.1201/9780203489437>
- 6 Thomson, G. H. (1938). Methods of estimating mental factors. *Nature*, 141(3562), 246.
7 <https://doi.org/doi:10.1038/141246a0>
- 8 Thurstone, L. L. (1934). The vectors of the mind. *Psychological Review*, 41(1), 1–32.
9 <https://doi.org/doi:10.1037/h0075959>
- 10 Wu, W., Jia, F., Rhemtulla, M., & Little, T. D. (2016). Search for efficient complete and
11 planned missing data designs for analysis of change. *Behavior Research Methods*, 48(3),
12 1047–1061. <https://doi.org/10.3758/s13428-015-0629-5>
- 13 Zawadzki, M. J., Graham, J. W., & Gerin, W. (2012). Increasing the validity and efficiency of
14 blood pressure estimates using ambulatory and clinic measurements and modern missing
15 data methods. *American Journal of Hypertension*, 25(7), 764–769.
16 <https://doi.org/10.1038/ajh.2012.40>
- 17

1

Table 1*Population Models*

	Correlated Factors Model			Restricted CTC(M-1) Model
	Gold Standard and Proxy 1	Gold Standard and Proxy 2	Proxy 1 and Proxy 2	Bias Factors
Model	$\rho_{GS, Proxy1}$	$\rho_{GS, Proxy2}$	$\rho_{Proxy1, Proxy2}$	$\rho_{Bias1, Bias2}$
High-Common	.7	.7	.74	.5
High-Unique	.7	.7	.54	.1
High-Inverse	.7	.7	.23	-.5
Mixed-Common	.7	.5	.66	.5
Mixed-Unique	.7	.5	.41	.1
Mixed-Inverse	.7	.5	.04	-.5
Moderate-Common	.5	.5	.62	.5
Moderate-Unique	.5	.5	.32	.1
Moderate-Inverse	.5	.5	-.12	-.5

Note. The correlation structure in the correlated factors model implies a specific correlation of the bias factors in the data-equivalent restricted CTC(M-1) model

2

Figure 1*Data-Equivalent Parametrizations of a 3MM Design with Three Indicators Per Assessment**Method*

Note. Panel A shows how the 3MM can be parametrized as a correlated factors model based on the factors corresponding to the gold standard method (GS) and the proxy methods (Proxy1, Proxy2). Panel B depicts how the 3MM model can be parametrized by regressing the factors of the proxy methods (Proxy1, Proxy2) onto the factor of the gold standard (GS), which allows to specify the bias factors (Bias1, Bias2). Panel C illustrates how the 3MM design can be parametrized as a restricted CTC(M-1) model. Models A, B and C are data-equivalent re-parametrizations of each other (see Geiser et al., 2012).

Figure 2*Accuracy of Estimated Factor Scores*

Note. Panel A depicts boxplots of the mean absolute error (MAE) for the estimated factor scores of the gold standard method across the different conditions. The x-axis corresponds to the reliability of the indicators in the conditions. Panel B analogously depicts the Pearson correlation of the estimated factor scores with the true factor scores. The filled boxplots represent complete cases (cc), the lightly grey filled boxplots represent planned missingness cases (pmf). The Bartlett-FIML estimator is not suited for estimating factor scores for subjects with planned missing data and is thus not displayed for planned missingness cases.

Figure 3

The Impact of Design Features on the MAE of Factor Score Estimates from Planned Missingness Cases

Note. The x-axis corresponds to the reliability of the indicators in the conditions. The columns reference the validity of the proxy method, the rows indicate the correlation of the bias factors (see Table 1). 2MM refers to conditions with one proxy method and 3MM refers to conditions with two proxy methods. The Bartlett-FIML estimator is not suited for estimating factor scores for subjects with planned missing data and is thus not displayed.

1

2

Figure 4*Coverage Rates Depending On the Number of Available Complete Cases*

Note. The figure depicts boxplots for the coverage rates of the 95%- confidence or credible intervals of the estimated factor scores of the gold standard method across the different conditions. The filled boxplots represent complete cases (cc), the lightly grey filled boxplots represent planned missingness cases (pmf). The Bartlett-FIML estimator is not suited for estimating factor scores for subjects with planned missing data and is thus not displayed for planned missingness cases.