**The importance of gravity for the categorization of geometric objects in
humans**

*Sarah Placi

Center for Mind/Brain Sciences, University of Trento, 38068 Rovereto, Italy

Simon Stephan

Georg-August University of Göttingen, 37073 Göttingen, Germany

Michael R. Waldmann

Georg-August University of Göttingen, 37073 Göttingen, Germany

Giorgio Vallortigara

Center for Mind/Brain Sciences, University of Trento, 38068 Rovereto, Italy

Author Note

Sarah Placi <https://orcid.org/0000-0002-9652-8250>

Simon Stephan <https://orcid.org/0000-0002-6557-9637>

Michael R. Waldmann <https://orcid.org/0000-0002-8831-552X>

Giorgio Vallortigara <https://orcid.org/0000-0001-8192-9062>

*Correspondence concerning this article should be addressed to Sarah Placi
Center for Mind/Brain Sciences, University of Trento; Piazza Manifattura 1, 38068
Rovereto, Italy. E-mail: sarah.placi@unitn.it.

All experimental materials and data (including demo videos of the studies) are
provided in an OSF/ GitHub repository whose content can be accessed via a GitHub
page at <https://simonstephan31.github.io/>
[Geometric_categories_in_3D_and_2D_contexts/index.html](https://simonstephan31.github.io/Geometric_categories_in_3D_and_2D_contexts/index.html) or via
<https://osf.io/fnsyc/>.

Abstract

Several studies concluded that Euclidean principles underlie how people categorize objects differing in terms of their shape, size, sense, orientation, and position. We argue that Euclidean principles cannot satisfactorily describe how humans categorize geometric objects because Euclidean principles do not incorporate a fundamental feature of space: gravity. We present a new theory explaining the categorization of geometric objects in terms of the informativeness of objects' geometric features. We propose that in 3D contexts, gravity endows some geometric features (e.g., orientation) with more informativeness because they predict differences in objects' levels of energy (i.e., their gravitational potential energy). Consequently, some geometric objects (e.g., objects with different orientations) can be considered as being of different "physical" kinds, even if according to Euclidean geometry they are equal. We predict and show across several behavioural studies that humans are more likely to generalize information between objects of the same "physical kind" than between objects of the same "Euclidean kind". For 2D contexts, we predict and show that people treat differences in objects' geometric features more equally because objects in 2D contexts are deprived of physical properties. Our findings confirm the central role that gravity plays in how people categorize geometric objects in different contexts.

Keywords: Categorization of geometric objects, Euclidean geometry, Gravity, Invariant referents

Object categorization is a fundamental cognitive process underlying all forms of learning and reasoning associated with objects (Holyoak & Morrison, 2005; Mervis & Rosch, 1981; Murphy, 2002). Forming categories allows reasoners to generalize properties (e.g., function, causal capacity, etc.) of one object to all other objects within the same category (Holyoak & Morrison, 2005; Leyton, 1992; Mervis & Rosch, 1981; Murphy, 2002; Shepard, 2001), which in turn is useful to make predictions about hidden properties of new objects, and to efficiently store information (Corter & Gluck, 1992). Furthermore, categories play an essential role in causal learning and reasoning (Waldmann & Hagmayer, 2006), as both rely on cause and effect categories. The identification of the rules by which intelligent system form categories is, therefore, essential for the understanding of the organization of abstract knowledge (Tenenbaum, Kemp, Griffiths, & Goodman, 2011; Vallortigara, 2021).

One of the most basic differences that exist between objects is of geometric nature: all objects that animals encounter occupy different positions in space, and most also differ in terms of their orientation, their sense, their shape and/or their size (see Fig. 1 for an illustration of these differences). Do animals consider these differences when categorizing objects, and if they do, according to what abstract rule(s), if any, do they decide whether geometric objects belong to the same category or not?

Recently, the question was raised whether Euclidean principles underlie how minds, human minds in particular, conceptualize objects (Dehaene, Izard, Pica, & Spelke, 2006; Izard, Pica, Dehaene, Hinchey, & Spelke, 2011; Izard, Pica, & Spelke, 2021; Izard, Pica, Spelke, & Dehaene, 2011; Izard & Spelke, 2009; Spelke, Lee, & Izard, 2010). In fact, if Euclidean geometry was as natural as philosophers claim (Descartes, 1637/2001; Kant, 1781/2003; Plato, ca. 380 B.C./1949), then it would not be surprising that evolution shaped minds so that they incorporate Euclidean principles in their perception and categorization of objects (Gallistel, 1990; Shepard, 2001; Spelke et al., 2010).

Categorizing objects based on Euclidean principles would entail considering some geometric features of objects while ignoring other features. In Euclidean geometry, be it

Figure 1
Euclidean transformations

Note: From top to bottom are illustrated four types of Euclidean transformations: translations, rotations, reflections, and homothecies. Translations are characterised by the linear movement of an object along a line and vary an object's position. Rotations are characterised by the rotation of an object around a point (2D) or a line (3D) and vary an object's orientation. Reflections are characterised by a reflection of an object over a line (2D) or by a rotation of an object around a line (3D) and vary an object's sense. Homothecies are characterised by the proportional increase or decrease in the size of an object and vary an object's size (or global size).

74 plane geometry or the geometry of solid objects, the orientation and the position of
 75 objects do never matter: they are not defining properties of objects. As a result,
 76 rotations and translations (see Fig. 1 for an illustration of Euclidean transformations)
 77 are considered invariant Euclidean transformations. On the other hand, the angle and
 78 length proportions of objects (i.e., their shape) are defining properties of objects, so
 79 that objects varying in terms of these features might be categorized differently (e.g.,
 80 consider a rectangle and a parallelogram). The status of sense (i.e., whether or not two
 81 objects are the result of a reflection over a line, i.e., they are mirror images of each
 82 other) and of size depends on the variant of Euclidean geometry. For example, in plane

geometry (i.e., 2D geometry), both sense and size are irrelevant features and can be ignored when categorizing objects because they do not affect objects' angle and length proportions (see ig. 1). In the geometry of solid objects on the other hand, both sense and size are relevant and reflect intrinsic differences between objects (Izard, Pica, Dehaene, et al., 2011).

When categorizing objects, do humans spontaneously consider objects' shape (and maybe their size and sense) while ignoring their orientation and position? To investigate human sensitivity to objects' geometric features, Izard et al. (Izard, Pica, Dehaene, et al., 2011; Izard, Pica, & Spelke, 2019; Izard et al., 2021; Izard & Spelke, 2009; Spelke et al., 2010), used a deviant detection paradigm (after Dehaene et al., 2006). Participants of different age categories (children and adults) and cultures (US participants, and Mundurucu participants who could not read and did not receive education in formal geometry) were presented with panels displaying six 2D forms (see Fig. 2). Five of these forms shared a geometric property (e.g., their shape, their size, or their sense). A sixth form, the deviant, did not share that property (see panel (d) of Fig. 2). Participants were asked to identify the form that appears to be "different". In *pure* trials, all forms were rotated to different degrees. In *interference* trials, all forms were rotated to different degrees and either their shape, their size, or their sense was varied. For example, in *interference* trials in which size was the deviant, forms also had different shapes or senses, in addition to different orientations. Results in the *pure* trials were that across age and culture, shape and size deviants were more often detected than sense deviants. While American adults were still able to detect sense deviants, the performance of children and Mundurucu participants was at chance. Results in the *interference* trials were that participants had more difficulties finding the deviants when size or shape interfered in addition to orientation, than when sense interfered in addition to orientation. Based on these findings, Izard and colleagues concluded that human intuitions are coherent with the non-oriented Euclidean geometry of solid objects, in which shape and size are defining features of objects, but not sense, orientation, and position, which are easily overlooked (Izard, Pica, Dehaene, et al.,

2011; Izard et al., 2019, 2021; Izard & Spelke, 2009; Spelke et al., 2010).

We argue that this conclusion and the used methodology bear several weaknesses. First, the conclusion that humans tend to overlook differences in orientation and position when classifying shapes is not supported by any empirical data. Izard et al. only tested humans' sensitivity to shape, size, and sense. Also, it is possible that the different orientations at which shapes were presented interfered with participants' detection of sense and that participants did, therefore, not overlook orientation. In fact, when forms are not simultaneously rotated, children and Mundurucu participants can easily detect sense deviants (Dehaene et al., 2006; Izard & Spelke, 2009). Likewise, it is possible that in the interference trials the only feature that was actually interfering with the detection of the sense deviants was orientation, and not size or shape.

Another problem with Izard and colleagues' conclusion and, more generally, with the argument that humans categorize geometric objects based on Euclidean principles, is that several studies with children and adults show that orientation matters in how humans categorize objects (Arguin & Leek, 2003; Clements, Swaminathan, Hannibal, & Sarama, 1999; Dillon, Izard, & Spelke, 2020; Schwartz, Day, & Cohen, 1979; Shepard & Metzler, 1971; Tarr & Pinker, 1989). For example, both adults and infants seem to consider squares and diamonds (i.e., squares that are rotated at 45°) as different objects (Schwartz et al., 1979). Furthermore, children and adults alike seem to make a distinction between vertical sense (e.g., objects reflected over a horizontal line, i.e., vertical mirror images) and horizontal sense (e.g., objects reflected over a vertical line, i.e., horizontal mirror images) (Cairns & Steward, 1970; Sekuler & Houlihan, 2007). More particularly, both adults and children find it more difficult to distinguish between horizontal mirror images than between vertical mirror images. In Euclidean geometry, however, there is no difference between vertical and horizontal lines, as each can be converted into the other one by a 90° rotation. As a consequence, in people categorized objects based on Euclidean principles, they should not differentiate between vertical and horizontal sense.

If not according to Euclidean principles, according to which other abstract rules

Figure 2*Example of a deviant detection task.*

Note: Participants are presented with six forms, five of which share a geometrical features, e.g., sense, that the sixth figure does not share. The orientation of the six forms vary, covering 360°. Participants are asked to identify the form that looks different, which in this example is the form in panel (d).

could humans categorize objects that vary in terms of their geometric features, such as position, orientation, sense, shape and size? A novel theory, that we explore and test in this article, is that humans, and maybe other animals as well, spontaneously categorize objects based on their hidden physical properties that are determined by the relationship between their geometric features and gravity. However, humans are special insofar as their culture also creates artificial 2D environments in which objects are abstract shapes deprived of physical properties. In such artificial environments, we argue that education plays a role in how humans categorize objects.

149 The new theory

150 Object concepts are building blocks of our understanding of the world (Murphy,
151 2002). Concepts capture commonalities between objects that are *informative* about
152 their hidden properties (Corter & Gluck, 1992), such as their function and causal
153 capacities (Izard & Spelke, 2009; Mervis & Rosch, 1981; Murphy, 2002). At the same
154 time, they overlook differences between objects that are irrelevant. This directly
155 influences generalization and categorization, as properties will be generalized between
156 objects whose differences are overlooked, while objects whose differences are considered
157 as important will be classified into different categories.

158 The rationale of our new theory is as follows: (i) differences in objects' geometric
159 features can only be informative if minds can detect them. Detection requires referents
160 that are independent from the viewer's perspective and stable over time (Weiss, 1993).
161 (ii) Even if differences in objects' geometric features can objectively be detected, not all
162 of them are necessarily informative in all contexts.

163 We propose that this is where gravity comes into play. We propose that gravity
164 plays at least two important roles in the categorization of geometric features. First, as a
165 vertical force vector — which is the way animals experience gravity — gravity is an
166 essential invariant referent for minds to recognize several geometric features. Shape and
167 size are absolute values and might not require referents when movement can occur. Yet,
168 the presence of gravity is necessary to assess differences in the orientation of objects
169 around the horizontal axis X (which affects what we call *vertical sense* and can also be
170 obtained in a 2D plane by a reflection over the X axis) and Z (which affects what we
171 call *orientation* and can also be obtained in the 2D plane by a rotation around a point
172 of the plane), and to assess differences in vertical position (see Fig. 3 for a
173 representation of the orthogonal axes we refer to and of the differences in geometric
174 features that can be referred to them). The only geometric differences that cannot be
175 related to the gravity vector are differences in objects' orientation around the Y-axis
176 (which affects what we call *horizontal sense* and can also be obtained in the 2D plane
177 by a reflection over the Y axis), and differences in objects' horizontal position (see

Fig. 3). This does not mean that horizontal sense and position can not be perceived; however, the perception of these features depends on the presence of further arbitrary cues, such as landmarks or cultural or statistical norms that are not always present. As an example, consider a field covered with sunflowers that are all, except one, turned in the same direction. In this case the statistical norm established by the majority of sunflowers can help detect the deviant flower.

A second important role of gravity is that it endows massive objects with gravitational potential energy. Gravitational potential energy is a function of an object's mass and of the elevation of its center of mass. From this it follows that objects with different shapes and sizes, with different orientations around the plane axes, and with different vertical positions, can have different levels of gravitational potential energy (see Fig. 4). Crucially, this is not true for objects with different orientations around the Y axis (i.e., objects with different horizontal senses), and with different horizontal positions (see Fig. 4).

Why does an object's level of gravitational potential energy matter at all for categorization? Gravitational potential energy can be converted into other types of energy, for example kinetic energy (i.e., the energy of motion). The kinetic energy of objects can be transferred to other objects, creating a causal chain of events. Objects with different levels of gravitational potential energy can, therefore, be said to be of a different physical "kind", as they have different potentials to bring about changes in the world (i.e., events).

Importantly, if an object's geometric features change due to an external intervention, for example because an animal or a person manipulates an object, then differences in vertical position, vertical sense, and orientation should no longer be informative about differences in objects' intrinsic levels of gravitational potential energy. This is because the energy that creates these differences does not come from within the objects but from the entity manipulating it. One could imagine two pens, one lying on a table, and the other either (i) being carried one meter above the table by a person, or (ii) levitating on its own. In the first case, one would probably not infer that the two

Figure 3

Axes used in 3D and 2D environments and how geometrical features can be referred to them.

Note: (A) The upper and lower picture show the axes systems referred to in 3D and 2D contexts, respectively. (B) From left to right and top to bottom are displayed different geometrical differences between objects. Only differences in vertical position, vertical sense and orientation can be referred to the Y-axis. Differences in horizontal sense and horizontal position have to be referred to one of the plane axes (e.g. the X-axis).

pens are (fundamentally) different whereas in the second case one would.

In sum, gravity helps minds detect directional (e.g., orientation and vertical sense) and positional features (e.g., vertical position) and it endows these features with more informativeness about the object's physical kind than they would have in an environment without gravity (e.g., Euclidean spaces). The only geometric features not affected by gravity, neither in their likelihood to be detected nor in their importance in predicting differences in objects' physical properties, are horizontal sense and horizontal position. We argue that this *explains* why humans tend to overlook horizontal sense more than vertical sense (Cairns & Steward, 1970; Sekuler & Houlihan, 2007).

216 *The special case of 2D environments*

217 Human culture created a new type of environment: artificial 2D spaces. 2D
 218 spaces are special in so far as they are devoid of gravity, but in contrast to 3D spaces
 219 come with a 2D frame that has a beginning (an origin) and an end. 2D frames are also
 220 often associated with up, down, left and right referents (consider books or computer
 221 screens for example). From this it follows that any object within such a frame has an
 222 absolute vertical and horizontal position, orientation, vertical and horizontal sense, size,
 223 and shape. All geometric features can, therefore, be recognized and used for
 224 categorization. However, whether some geometric features are more informative than
 225 others depends on human decisions and norms; to maximize the informativeness of 2D
 226 objects, all geometric features should be treated as different. If we take the example of
 227 letters written on a screen, “b” is different from its vertical mirror image “p”, but also
 228 from its horizontal mirror image “d”; all are also different from “a”, which has a
 229 different shape, “C” is different from the smaller “c”, and this first “c” is different from
 230 this second translated “c”, because reading has a left/right (or right/left) and up/down
 231 direction. Crucially, as 2D spaces are an invention of human culture, we assume that
 232 the new referents of 2D spaces (i.e., the left/right referents) have to be learnt.

233 The results of Izard and colleagues can also be explained in light of our new
 234 theory. American adults were found to have more difficulties finding the sense deviants
 235 than finding the size or shape deviants because 2D sense is relative to the left/right or
 236 up/down referents of 2D frames. In the experiments in which the shapes were rotated
 237 at different degrees (Izard, Pica, Dehaene, et al., 2011; Izard et al., 2019, 2021; Izard &
 238 Spelke, 2009; Spelke et al., 2010), the relationship between the shape and its sense
 239 referents was lost and could only be retrieved if participants had the idea to mentally
 240 rotate the shapes. American adults who learnt the informativeness of horizontal sense
 241 at school were still more likely to find the sense deviants than American children and
 242 Mundurucu adults who did not learn the informativeness of sense, and did, therefore,
 243 not think about mental rotation. In the experiments in which the shapes were not
 244 simultaneously rotated (Dehaene et al., 2006; Izard & Spelke, 2009), children and

Figure 4

Geometrically transformed objects with their center of masses.

Note: From left to right are displayed the original L-shaped object used in our studies, followed by objects that vary in terms of their horizontal sense (Y-Refl.), their size (Size), their shape (Shape), their orientation (90°-Rot. and 180°-Rot.), their vertical sense (X-Refl.), their vertical position (Y-Tran), and their horizontal position (X-Tran). The red dots depict the center of mass of each object and the dotted line depicts the height of the center of mass of the original object (First object). Only the Y-Refl. and X-Tran. objects have center of masses at the same height than the first object, which means that only these three objects have the same gravitational potential energy.

Mundurucu adults had no problem finding the sense deviants, because they could use the statistical norm of the five shapes turned towards the same direction to detect the deviant turned in the opposite direction.

Results

Testing the new theory

As a test of our theory, we presented participants with a fictitious scenario in which scientists who were exploring a new planet came across and photographed an unfamiliar object. Subjects were then shown a picture of that object (see “1st object” in Fig. 5A). It was then mentioned that the scientists soon learned that the object is emitting “alpha rays”. We tested three different contexts. In a 3D test context without a manipulation of objects (Exp. 1a, $N = 854$), the presented object stood on its own on the ground. In a 3D test context with manipulation of objects (Exp. 2, $N = 868$), the object was first picked up by a robot. In a 2D test context (Exp. 3, $N = 854$), the object was a shape displayed on a wall centered in a rectangular frame. After the initial

object, participants were shown another picture taken by the scientists in which they saw two new objects (see Fig. 5B). Each of the two new objects differed from the first object by one geometric feature (e.g., size, shape, vertical position, horizontal position, 90 degrees rotation, 180 degrees rotation, vertical sense, horizontal sense). One object always had a different horizontal sense (see Y-Reflection in Fig. 5B), while the other object was either an object with a different size, a different shape, a different orientation, a different vertical sense, a different vertical position, or a different horizontal position (see examples in Fig. 5). Different combinations of objects were tested between subjects, which means that each experiment had seven conditions: Size, Shape, 90-Rotation and 180-Rotation (for orientation), X-Reflection (for vertical sense), Y-Translation (for vertical position), and X-Translation (for horizontal position; see Supplementary materials for illustrations of all conditions). Participants were told that one of the new objects is emitting the same intensity of alpha rays as the first object, while the other object is emitting a *different* intensity of rays. Participants were asked to choose the object they believed is emitting the different intensity of rays.

This paradigm allowed us to test which geometric features are relevant for categorization, and in particular, which features participants consider or ignore when generalizing hidden properties of objects, (in this case, the intensity at which objects emit alpha rays). For example, if participants tend to consider horizontal sense as irrelevant, then they should infer that the first object and its horizontal mirror image (i.e., the Y-Reflection object) emit the same intensity of alpha rays. If they tend to consider vertical position as informative, then they should infer that the object with a different vertical position (i.e., the Y-Translation object) emits a different intensity of alpha rays than the first object. If they think that all geometric features are equally informative, they should choose at chance. The only geometric feature that was hard to test in the 3D context was horizontal position. Given that in our scenario the two new objects are encountered by the scientists in another location, they all have different horizontal positions, which makes this feature irrelevant. This problem does not occur in the 2D context, however, as all objects are presented within a defined frame in which

Figure 5
Stimuli used in the experiments.

Note: (A) First pictures shown to participants in the 3D context without manipulation (i-ii), in the 3D context with manipulation (iii) and in the 2D context (iv). In Exp. 1a (i), Exp. 2 (iii) and Exp. 3 (iv) participants learned that the object was emitting alpha rays and in Exp. 1b (ii) that both objects were emitting the same intensity of alpha rays. (B) Example of the second pictures shown to participants. In Exp. 1a (i), Exp. 2 (ii) and Exp. 3 (iii), participants learnt that one of these two new objects was emitting the same intensity of alpha rays as the first object, whereas the other object was emitting a different intensity of alpha ray. In Exp. 1b (i), participants were told that a change happened and while one object was still emitting the same intensity of alpha rays as before, the other object was emitting a different intensity of alpha rays. In all four experiments, participants were asked to guess which object was emitting the different intensity of alpha rays.

their position is fixed.

To test more reliably how participants treat differences in horizontal position in a 3D context, we conducted a slightly modified experiment, Exp. 1b ($N = 854$; see Fig. 5). In this experiment, the scientists in the scenario first discovered two objects, i.e., two identical retinal instances of the first object, which were then both assumed to undergo a change. Participants were shown a new picture of both transformed objects. This second picture was identical to the test stimulus used in Exp. 1a, but the first picture was different, as it showed two objects instead of one (see Fig. 5A for an example). In this experiment, all objects kept their horizontal position except in the X-Translation condition. One might object that Exp. 1b is not comparable to the other

experiments because the geometric differences are observed within objects rather than between. We still believe that these experiments are comparable given that in all of them, except for the geometric differences between objects or between objects' states, all else is kept equal. In Exp. 1a, Exp. 2, and Exp. 3, all objects are new so that this newness becomes irrelevant. In Exp. 1b, both objects undergo a geometric transformation so that the mere fact that they undergo a change becomes irrelevant. Only the geometric differences between objects in Exp. 1a, Exp. 2, and Exp. 3, and the geometric differences between objects' states in Exp. 1b can be informative about the objects' hidden properties (i.e, their emission of alpha rays). In all four experiments, these geometric differences are kept equal between conditions.

In Exp. 1a and Exp. 1b, we predicted that participants would generalize the intensity of alpha rays emission of the first object (or of the first state of the object) to the object in the second picture that had the same gravitational potential energy as the first object. We thus predicted that participants, to explain the difference in intensity of alpha rays, would select the object whose gravitational potential energy changed. Participants were, therefore, expected to choose the object that was not the Y-Reflection object, except in X-Translation condition.

In Exp. 2 we predicted that subjects choices would be closer to 50% in those conditions in which the geometric differences were directional and positional, but not in those conditions in which the geometric differences between the objects were size and shape, the features that remained unaffected by the external intervention. The reason is that an external causal intervention that changes an object's directional and positional features renders these features undiagnostic with respect to the object's "natural" properties. Overall, compared to Exp. 1a and 2b the proportion of subjects selecting the non-Y-Reflection object is expected to become smaller and get closer to 50%.

In Exp. 3, we predicted that participants' preferences would be closer to 50% in all conditions (corresponding to random selection; and thus indifference), given that objects were deprived of gravitational potential energy, and given that all participants in our experiments learnt to relate 2D objects to the left/right and up/down referents of

Figure 6
Subjects' selections in the Exp. 1-4.

Note: (A-D) Subjects' selections of the object that they thought emitted the different intensity of alpha rays in the seven tested conditions ($n = 122$ per condition in Exp. 1a and 1b, and in Exp. 3, $n = 124$ per condition in Exp. 2). Blue represents selections of the test object that differed from the first object in terms of the geometric transformations indicated by the X-axes labels. Red indicates selections of the test object that had the same horizontal sense as the first object. Error bars represent the 95% CIs of the proportion estimations. P-values (two-sided) indicate results of binomial tests against chance (50%). (E) Aggregated subjects' selections for all four experiments. P-values (two-sided) indicate results of proportion difference tests.

human culture, and also learnt that all these differences were informative (i.e., they could all read and write and all received formal education).

A summary of the results is shown in Fig. 6A. As can be seen there, as predicted by our theory we found that most participants in the 3D context without manipulation (Exp. 1a and Exp. 1b) chose the non-Y-Reflection object in all except the X-Translation condition (see Fig. 6A and B). Participants, therefore, made a distinction between differences in horizontal and vertical sense, but also between differences in horizontal and vertical position. All in all, the geometric features they perceived as crucial or irrelevant were those predicted by our theory.

Also, in line with our theory, the way participants categorized the same geometric features distinctly differed between a 3D context with manipulation and a 3D context without manipulation. Overall, 61% of our subjects this time selected the non-Y-Reflection objects, whereas this was the case for only 71% of the subjects in Exp. 1a, and for only 76% in Exp. 1b. Proportion difference tests confirmed that the proportion of subjects selecting the non-Y-Reflection objects in the 3D contexts with non-manipulated objects (Exp. 1a and 1b) was significantly higher than in the present 3D context with manipulated objects (Exp. 1a vs. Exp. 2: $\chi^2(1) = 19.24$, $p < .001$, 95% CI of difference [0.06, 0.15]; Exp. 1b vs. Exp. 2: $\chi^2(1) = 42.96$, $p < .001$, 95% CI of difference [0.10, 0.19]). Also, as predicted by our theory the preference for the non-Y-Reflection object in the size condition remained high. However, contrary to what we predicted, we found a decreased preference for the non-Y-Reflection object in the shape condition. We return to this unexpected finding in the General Discussion.

Finally, as predicted by our theory, participants' selection differed between the 3D context without manipulation and the 2D context. Subjects' preferences in the 2D context were much closer to indifference (see Fig. 6C, D and E). Overall, 57% of our subjects this time selected the non-Y-Reflection objects, whereas this was the case for only 71% of the subjects in Exp. 1a, and for only 76% in Exp. 1b. Proportion difference tests confirmed that the proportion of subjects selecting the non-Y-Reflection objects in the 3D contexts with non-manipulated objects (Exp. 1a and 1b) was significantly higher than in the present 2D context (Exp. 1a vs. Exp. 3: $\chi^2(1) = 37.18$, $p < .001$, 95% CI of difference [0.10, 0.19]; Exp. 1b vs. Exp. 3: $\chi^2(1) = 67.90$, $p < .001$, 95% CI of difference [0.14, 0.23]). The proportions between the 3D context with manipulated objects and the 2D context was not significant, $\chi^2(1) = 2.90$, $p < .001$, 95% CI of difference [-0.09, 0.01].

Deviant detection experiment

In addition to these four experiments, we also ran a deviant detection experiment to test whether what drove the results of Izard and colleagues (Izard, Pica, Dehaene, et

al., 2011; Izard et al., 2019, 2021; Izard & Spelke, 2009; Spelke et al., 2010) was actually the interference of orientation with sense rather than a general failure to detect sense. In addition, as this has never been tested before, we also tested whether participants could easily find position and orientation deviants. In *pure* trials, participants saw six forms, five of which shared the same geometric feature, i.e., their size, shape, vertical position, horizontal position, vertical sense, horizontal sense, or orientation, while the sixth form did not. Importantly, none of the forms was additionally rotated, as this would not constitute a pure trial. In *interference* trials, five out of six forms shared either their size, their shape, their orientation, or their sense, while at the same time one of the other features was interfering. For example, in trials where the deviant had a different sense, all forms could also either have a different size, a different shape or a different orientation (see Supplementary materials for illustrations).

We predicted that in *pure* trials, participants would detect all the deviants without difficulty. In the *interference* trials, we predicted that sense would be harder to detect only when orientation would interfere with it as this interference would blur out the relationship between the deviant and its sense referents.

Results were (see Fig. 7) that in the *pure* trials, participants detected without problems all the deviants, except the horizontal position deviants that seemed a bit harder to detect. In the *interference* trials, as predicted, the detection of sense only differed from the detection of the other deviants when orientation interfered with it.

General Discussion

We proposed a new theory for the categorization of geometric objects that puts physics at the center. In particular, we argued that in 3D contexts, gravity plays an essential role for the categorization of geometric objects. First, it allows minds to detect differences in objects' vertical sense (but not horizontal sense), orientation, and vertical position (but not horizontal position). Second, gravity endows differences in orientation, vertical sense, and vertical position (alongside shape and size) with more informativeness than differences in horizontal sense and horizontal position as they

Figure 7
Results of the deviant detection experiment.

Note: In pure trials, participants detected easily all deviants, although they had more difficulty identifying the horizontal position deviant. In the interference trials, size, shape, and sense did not interfere with the detection of one deviant more than with another. Orientation, on the other hand, interfered with the detection of sense deviants more than with the detection of size or shape deviants. Error bars represent the 95% CIs of the proportion estimations. P-values (two-sided) indicate results of difference of proportion difference tests.

predict differences in objects' level of gravitational potential energy.

In line with the novel theory, our findings indicate that rather than categorizing geometric objects according to Euclidean principles, people categorize geometric objects according to their informativeness in different contexts. In 3D context with gravity, our findings show that people spontaneously generalize information between objects whose geometric features predict similar levels of gravitational potential energy. Instead of grouping geometric objects together based on their “Euclidean kind”, people seem, therefore, to group geometric objects together based on their “physical kind”. This is well illustrated by the fact that in the 3D context without manipulation (Exp. 1; see

Fig. 6A and B), subjects were more likely to generalize information between objects that had a different horizontal sense or a different horizontal position than between objects that had a different vertical sense or a different vertical position. This was predicted by our theory because in such a context, differences in vertical sense and vertical position are informative about differences in the gravitational potential energy of objects, but not differences in horizontal sense and horizontal position. If people categorized objects based on Euclidean principles, they should be as likely to generalize information between objects that have different vertical and horizontal senses, or between objects that have different vertical and horizontal positions because Euclidean geometry does not differentiate between vertical and horizontal axis. Furthermore, if people categorized objects based on Euclidean principles, they should be either more likely or equally likely to generalize information between objects that have different orientations than between objects that have different senses. This is because orientation is never considered a relevant feature in Euclidean geometry, whereas sense can be, depending on the variant of Euclidean geometry. Our findings show the opposite trend: Participants were more likely to generalize information between objects that were horizontal mirror images of each other than between objects that had different orientations. Again, this supports our theory that people spontaneously categorize objects based on their physical kind.

Our theory is further supported by the fact that when differences between objects' level of gravitational potential energy are not intrinsic to the objects, e.g., when they are determined by external entities manipulating the objects, people seem to find these differences less relevant for categorization. This is particularly well illustrated by the Y-Translation condition, in which participants' preference for the object with a different vertical position dramatically dropped from 77% (Exp. 1a) and 90% (Exp. 1b) in the context without manipulation to 43% in the context with manipulation (Exp. 2). One could wonder why participants' preference for the object with a different shape also dropped in the context with manipulation. This was indeed not predicted by our theory. A possible explanation for this deviation from the predictions is that subjects might

have thought that the intervention of the robot, in addition to position and direction, may also have altered the angle of the object's two arms, rendering this geometric difference between the objects undiagnostic for its natural propensity to produce alpha rays of a certain intensity. Indeed, the modular shape of the objects (they could be consisting of two arms of different lengths connected by a round ball joint) – that we chose because it facilitated transformations – might have invited the idea that the shape of the objects can easily be altered. This explanation also fits with the predicted high preference for the non-Y-Reflection object in the size condition, as it is implausible to assume that the robot's picking up the object caused a change in the object's size.

Also in line with our theory, people show less of a categorization preference in a 2D context. This can be explained by the fact that in such a context, objects cannot be differentiated in terms of their physical kind anymore given that they are deprived of physical properties, e.g., gravitational potential energy. In a 2D context, differences in objects' geometric features are, therefore, more equivalent in terms of their informativeness.

One could wonder why participants' preferences did not completely vanish in the 2D context. One reason could be that some participants still considered objects as if they were massive objects affected by gravity. Another reason could be that some differences in objects' geometric features were perceived as changing the object to a higher level than other differences. For instance, a 90°-rotation does not only change the orientation of the object, but also the orientation of the object's body axis. A 180°-rotation, by contrast, does not affect the body axis.

Are there obvious alternative explanations of our findings? One possibility that readily comes to mind is that lower level features, e.g., purely perceptual differences such as pixel level changes, rather than cues that are informative for differences in gravitational potential energy might have shaped subjects decisions in our tasks. Importantly, though, we can rule out such alternative explanations. Our findings cannot be explained in terms of purely perceptual differences between objects because we kept perceptual differences constant between experiments. Only the context in which objects

were presented varied between experiments. If participants had generalized information only in terms of pixel level changes between objects, then their preferences should not have varied between experiments. Yet, we found systematic differences between the experiments.

Our findings also show that the reason why subjects in Izard and colleagues' deviant detection experiment had more difficulties detecting the sense deviants (Izard, Pica, Dehaene, et al., 2011; Izard et al., 2019, 2021; Izard & Spelke, 2009; Spelke et al., 2010) is the interference of orientation. We show that in trials in which orientation does not interfere with sense (i.e., in our *pure* trials), people find sense deviants as easily as all other deviants, except for horizontal position deviants (see Fig. 7A). The reason why participants had a harder time finding horizontal position deviants could be due to the fact that people considered objects as a group of forms presented in 3×2 arrays. Deviants that had a different vertical position could easily be compared to the two other forms presented in the same line, whereas the deviant that had a different horizontal position could only be compared to one form in the same column; one form, however, does not create a statistical norm. Our findings also show that in *interference* trials, sense is only harder to detect when orientation is interfering with it, but not when size or shape are interfering. All in all, these findings show that (i) sense is not harder to detect than other geometric features, and neither are orientation and at least vertical position; (ii) orientation is not easy to ignore, at least not when it interferes with sense. These findings do not support Izard and colleagues' conclusion that people have Euclidean intuitions because they tend to consider size and shape more than sense, orientation and position Izard, Pica, Dehaene, et al. (2011). However, these findings support our theory that in a 2D context, or when statistical norms are available, all geometric features should be easily detectable. We only tested WEIRD adults and not children or Mundurucu participants, but previous results Dehaene et al. (2006); Izard and Spelke (2009) also show that when forms are not simultaneously rotated at different intervals, Mundurucu participants and children have no trouble detecting the sense deviants.

Ideas and speculations

Although our article focused on humans it has to be mentioned that similar categorization behavior has been documented in other species. For example, pigeons (Cerella, 1990) are more likely to generalize information between geometric objects that are horizontal mirror images of each other or have a different horizontal position, than between objects that vary in terms of their orientation, shape or size. Rats (Kinsbourne, 1971) seem to consider horizontal mirror images as more similar than vertical mirror images or than geometric objects that have different orientations. Like humans, goldfish (Bowman & Sutherland, 1970; Sutherland & Bowman, 1969) seem to consider square and diamonds as different objects. Several species, including octopuses (Sutherland, 1960), cichlids (Gierszewski, Bleckmann, & Schluessel, 2013), cats (Warren, 1969), and rhesus monkeys (Riopelle, Rahm, Itoigawa, & Draper, 1964) seem to consider horizontal mirror images as more similar than vertical mirror images. It is possible that other animals too spontaneously categorize geometric objects based on their physical kind in absence of any additional information. This categorization tendency could be evolutionarily ancient and thus shared amongst many species. Intuitions about objects' physical properties may be part of animals' core knowledge for physics (Kubricht, Holyoak, & Lu, 2017; Wu, Yildirim, Lim, Freeman, & Tenenbaum, 2015) that helps them structure their abstract knowledge (Tenenbaum et al., 2011; Vallortigara, 2021). In fact, our findings are also a compelling case of causal reasoning in the absence of any statistical data in which people's intuitions help them build the categories necessary for causal learning and reasoning (see also Waldmann & Hagmayer, 2006, for the relevance of categorization in causality). We showed that in the absence of statistical data or of any dynamic scenes, people can intuitively retrieve physical information from static objects based solely on their geometric differences. These intuitions about objects' physical properties could be one of the ways in which evolution created minds that "get so much from so little" (Tenenbaum et al., 2011, p. 1).

Future research could try to investigate these questions in more depth in other animals, especially in newborn animals that have not yet any experience with the world.

Similarly, future research could investigate the same questions in young children who have not yet learnt the cultural referents of 2D spaces nor the cultural informativeness of 2D geometric features. This would give us a better idea of the universality the categorization of objects based on their physical properties, and of whether the intuitions underlying it are present already at birth.

Methods

Ethical statement

This research was conducted in accordance with the Declaration of Helsinki and the Ethical Principles of the German Psychological Society (DGPs), the Association of German Professional Psychologists (BDP), and the American Psychological Association (APA). It involved no invasive or otherwise ethically problematic techniques and no deception (and therefore, according to National jurisdiction, did not require a separate vote by a local Institutional Review Board; see the regulations on freedom of research in the German Constitution (§ 5 (3)), and the German University Law (§ 22)).

Testing the new theory

Participants ($N = 2,562$ in total) were recruited via the online platform Prolific (www.prolific.co). Inclusion criteria were: age range between 18 and 99 years old, a subject's approval rate concerning participation in previous studies of at least 90%. To ensure that subjects could read and understand the materials, we excluded subjects who received no formal education. In Exp. 1a, subjects' mean age was 34 years ($SD = 13$ years, Median = 31 years). 346 were male, 493 were female, 11 were non-binary, and 4 preferred not to report their gender. In Exp. 1b, subjects mean age was 35 years ($SD = 13$ years, Median = 32 years). 368 were male, 478 were female, 5 were non-binary, and 3 preferred not to report their gender. In Exp. 2, subjects mean age was 32 years ($SD = 11.5$ years, Median = 29 years). 256 were male, 594 were female, 3 were non-binary. In Exp. 3, subjects mean age was 35 years ($SD = 12$ years, Median = 33 years). 405 were male, 437 were female, 7 were non-binary, and 5 preferred not to

reveal their gender. In all studies, participants received a link that led them directly to the online questionnaire, which was created with the software Unipark (<https://ww2.unipark.de/>). Subjects received a monetary reward for their participation.

Sample size rationale

For every of the seven between-subjects conditions in each of our experiments, we wanted to be able to detect differences from chance (chance level = .50) of .15 with more than 90% probability. Although our theory makes specific predictions that warrant directed (one-sided) testing, we calculated the required sample size based on the test power for two-sided binomial tests (one sample case; difference from constant). To calculate the sample size, we used the software G*Power30 Faul, Erdfelder, Buchner, and Lang (2009) with “Sensitivity” as the type of power analyses, which resulted in a sample size of $n = 122$ per condition ($N = 854$ for all seven conditions in Exp. 1a, Exp. 1b, and Exp. 3). In Exp. 2 we tested $N = 868$ participants ($n = 124$ per condition). This larger sample size resulted from an additional counterbalancing factor that we included in this study (the side from which the displayed robot arm held the initial object).

Randomization

In each of the four experiments, subjects were alternately assigned to one of the seven between-subjects conditions.

Blinding

In all experiments, subjects were not informed about the hypotheses that we tested.

Analyses

For each experiment, we first conducted two-sided binomial tests (see Tables 1 to 4 of the Supplementary materials document) to test whether subjects’ selections in each

of the seven conditions ($N = 122$ per condition for Exp. 1a, Exp. 1b, and Exp. 3, and $N = 124$ for Exp. 2) can plausibly be assumed to be explained by chance (50%). In Exp. 1a, these tests revealed that this is not the case for all except the X-Translation condition (see Fig. 6A). Except in the X-Translation condition, subjects tended to select the non-Y-Reflection object over the Y-Reflection object. In Exp. 1b, the tests revealed significant differences from chance in all conditions (see Fig. 6B). Unlike in Exp. 1a, subjects' preference for the non-Y-Reflection object in the X-Translation condition this time significantly differed from chance, but in the other direction than in the other conditions. In Exp. 2, subjects' selections were not significantly different from chance in the Shape, the Y-Translation, and the X-Translation conditions (see Fig. 6C). In Exp. 3, subjects' selections were not significantly different from chance in the Size, the 180°-Rotation, the X-Reflection, and the X-Translation condition (see Fig. 6D). In the Y-Translation condition, they were found to have a significant preference for the Y-Reflection object.

In a second step, we conducted five equality-of proportion tests (2-sample test for equality of proportions with continuity correction; two-sided) to test whether subjects' preferences for the non-Y-Reflection objects that we observed in our two studies of 3D contexts with non-manipulated objects (Exp. 1a and b) were significantly higher than subjects' preference for the non-Y-Reflection objects in the 3D context with manipulated objects (Exp. 2) and in 2D context (Exp. 3). The first two tests confirmed that the proportion of subjects selecting the non-Y-Reflection object in the 3D contexts with non-manipulated objects (Exp. 1a and 1b) was significantly higher than in 3D contexts with manipulated objects (Exp. 2), Exp. 1a vs. Exp. 2: $\chi^2(1) = 19.24$, $p < .001$, 95% CI of difference [0.06, 0.15]; Exp. 1b vs. Exp. 2: $\chi^2(1) = 42.96$, $p < .001$, 95% CI of difference [0.10, 0.19].

The third and fourth test confirmed that the proportion of subjects selecting the non-Y-Reflection object was significantly higher in the 3D contexts with non-manipulated objects (Exp. 1a and 1b) than in the 2D context (Exp. 3), Exp. 1a vs. Exp. 3: $\chi^2(1) = 37.18$, $p < .001$, 95% CI of difference [0.10, 0.19]; Exp. 1b vs. Exp. 3:

$\chi^2(1) = 67.90$, $p < .001$, 95% CI of difference [0.14, 0.23]. A fifth test comparing these proportions between the 3D context with manipulated objects and the 2D context was not significant, $\chi^2(1) = 2.90$, $p < .001$, 95% CI of difference [-0.09, 0.01].

Deviant detection paradigm

Participants ($N = 29$) were recruited via the online platform Prolific (<https://www.prolific.co/>). Inclusion criteria were: age range between 18 and 99 years old, a subject's approval rate concerning participation in previous studies of at least 90%. To ensure that subjects could read and understand the materials, we excluded subjects who received no formal education. Subjects' mean age was 33 years ($SD = 11$ years, Median = 34 years). 19 were female, 10 were male.

Blinding

Subjects were not informed about the hypotheses that we tested.

Analyses

For the interference trials we compared the proportions of correct selections for each triplet in the four different interference conditions (see Fig. 7B) using tests for equality of proportions with continuity correction. The only significant difference was obtained in the condition in which orientation was the interference factor. Here, the sense transformation, $P_{correct} = 0.38$, was significantly harder to detect than the shape, $P_{correct} = 0.93$, and the size transformation, $P_{correct} = 0.93$. Shape vs. sense: $\chi^2(1) = 17.17$, $p < .001$, 95% CI of difference [0.32, 0.79]; size vs. sense: $\chi^2(1) = 17.17$, $p < .001$, 95% CI of difference [0.32, 0.79].

Data and materials availability

All experimental (and supplementary) materials (including demo videos of the studies), the data (including descriptions), and the analyses scripts (including comments) are available in an online repository, which can be accessed via the GitHub page <https://simonstephan31.github.io/>

Geometric_categories_in_3D_and_2D_contexts/ or via the link
<https://osf.io/fnsyc/> that leads to an OSF repository. There are no restrictions on
materials.

Acknowledgments

We thank the members of the Animal Brain & Cognition research group of the
Animal Cognition and Neuroscience lab (Center for Mind/Brain Sciences, University of
Trento) for their insights and comments on this project.

This project has been funded with an Early Postdoc.Mobility fellowship received
from the Swiss National Science Foundation with grant No P2SKP1_191311.

This project has also received funding from the European Research Council
(ERC) under the European Union's Horizon 2020 research and innovation program with
grant agreement No 833504.

Author contributions

Author contributions: S.P. conceptualized the theory and the studies, S.P. and
S.S. created and ran the experiments and wrote the manuscript, S.S. analysed the data,
M.R.W and G.V. reviewed the manuscript.

Declaration of interests

All authors declare that they have no competing interests.

References

- Arguin, M., & Leek, E. C. (2003). Orientation invariance in visual object priming depends on prime—target asynchrony. *Perception & Psychophysics*, 65(3), 469–477.
- Bowman, R. S., & Sutherland, N. (1970). Shape discrimination by goldfish: Coding of irregularities. *Journal of Comparative and Physiological Psychology*, 72(1), 90..97.
- Cairns, N. U., & Steward, M. S. (1970). Young children’s orientation of letters as a function of axis of symmetry and stimulus alignment. *Child Development*, 41, 993–1002.
- Cerella, J. (1990). Pigeon pattern perception: Limits on perspective invariance. *Perception*, 19(2), 141–159.
- Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children’s concepts of shape. *Journal for Research in Mathematics Education*, 192–212.
- Corter, J. E., & Gluck, M. A. (1992). Explaining basic categories: Feature predictability and information. *Psychological Bulletin*, 111(2), 291–303.
- Dehaene, S., Izard, V., Pica, P., & Spelke, E. (2006). Core knowledge of geometry in an amazonian indigene group. *Science*, 311(5759), 381–384.
- Descartes, R. (1637/2001). *Discourse on method, optics, geometry, and meteorology*. P. J. Olscamp, Ed. and Trans. Hackett Publishing.
- Dillon, M. R., Izard, V., & Spelke, E. S. (2020). Infants’ sensitivity to shape changes in 2d visual forms. *Infancy*, 25(5), 618–639.
- Faul, F., Erdfelder, E., Buchner, A., & Lang, A.-G. (2009). Statistical power analyses using g* power 3.1: Tests for correlation and regression analyses. *Behavior Research Methods*, 41(4), 1149–1160.
- Gallistel, C. R. (1990). *The organization of learning*. The MIT Press.
- Gierszewski, S., Bleckmann, H., & Schluessel, V. (2013). Cognitive abilities in malawi cichlids (*pseudotropheus* sp.): matching-to-sample and image/mirror-image discriminations. *PLoS One*, 8(2), e57363.

- Holyoak, K. J., & Morrison, R. G. (2005). *The cambridge handbook of thinking and reasoning* (Vol. 137). Cambridge University Press Cambridge.
- Izard, V., Pica, P., Dehaene, S., Hinchey, D., & Spelke, E. (2011). Geometry as a universal mental construction. In *Space, time and number in the brain* (pp. 319–332). Elsevier.
- Izard, V., Pica, P., & Spelke, E. (2019). Perceptual foundations of euclidean geometry. In *Meeting of the european society for cognitive psychology*.
- Izard, V., Pica, P., & Spelke, E. (2021). Visual foundations of euclidean geometry. *PsyArXiv*. (<https://doi.org/10.31234/osf.io/rmdeh>) doi: <https://doi.org/10.31234/osf.io/rmdeh>
- Izard, V., Pica, P., Spelke, E. S., & Dehaene, S. (2011). Flexible intuitions of euclidean geometry in an amazonian indigene group. *Proceedings of the National Academy of Sciences*, 108(24), 9782–9787.
- Izard, V., & Spelke, E. S. (2009). Development of sensitivity to geometry in visual forms. *Human Evolution*, 23(3), 213–248.
- Kant, I. (1781/2003). *The critique of pure reason*. JMD Meiklejohn, Trans. Mineola, NY: Dover.
- Kinsbourne, M. (1971). Discrimination of orientation by rats. *Psychonomic Science*, 22(1), 50.
- Kubricht, J. R., Holyoak, K. J., & Lu, H. (2017). Intuitive physics: Current research and controversies. *Trends in Cognitive Sciences*, 21(10), 749–759.
- Leyton, M. (1992). *Symmetry, causality, mind*. MIT press.
- Mervis, C. B., & Rosch, E. (1981). Categorization of natural objects. *Annual Review of Psychology*, 32(1), 89–115.
- Murphy, G. L. (2002). *The big book of concepts*. MIT press.
- Plato. (ca. 380 B.C./1949). *Meno* (B. Jowett, Trans.). Indianapolis, Bobbs-Merrill.
- Riopelle, A., Rahm, U., Itoigawa, N., & Draper, W. (1964). Discrimination of mirror-image patterns by rhesus monkeys. *Perceptual and Motor Skills*, 19(2), 383–389.

- 701 Schwartz, M., Day, R. H., & Cohen, L. B. (1979). Visual shape perception in early
 702 infancy. *Monographs of the Society for Research in Child Development*, 1–63.
- 703 Sekuler, R. W., & Houlihan, K. (2007). Discrimination of mirror-images: Choice time
 704 analysis of human adult performance. *The Quarterly Journal of Experimental*
 705 *Psychology*, 20(2), 204–207.
- 706 Shepard, R. N. (2001). Perceptual-cognitive universals as reflections of the world.
 707 *Behavioral and Brain Sciences*, 24(4), 581–601.
- 708 Shepard, R. N., & Metzler, J. (1971). Mental rotation of three-dimensional objects.
 709 *Science*, 171(3972), 701–703.
- 710 Spelke, E., Lee, S. A., & Izard, V. (2010). Beyond core knowledge: Natural geometry.
 711 *Cognitive Science*, 34(5), 863–884.
- 712 Sutherland, N. (1960). Visual discrimination of orientation by octopus: Mirror images.
 713 *British Journal of Psychology*, 51(1), 9–18.
- 714 Sutherland, N., & Bowman, R. (1969). Discrimination of circles and squares with and
 715 without knobs by goldfish. *The Quarterly Journal of Experimental Psychology*,
 716 21(4), 330–338.
- 717 Tarr, M. J., & Pinker, S. (1989). Mental rotation and orientation-dependence in shape
 718 recognition. *Cognitive Psychology*, 21(2), 233–282.
- 719 Tenenbaum, J. B., Kemp, C., Griffiths, T. L., & Goodman, N. D. (2011). How to grow
 720 a mind: Statistics, structure, and abstraction. *Science*, 331(6022), 1279–1285.
- 721 Vallortigara, G. (2021). *Born knowing: Imprinting and the origins of knowledge*. MIT
 722 press.
- 723 Waldmann, M. R., & Hagmayer, Y. (2006). Categories and causality: The neglected
 724 direction. *Cognitive Psychology*, 53(1), 27–58.
- 725 Warren, J. (1969). Discrimination of mirror images by cats. *Journal of Comparative*
 726 *and Physiological Psychology*, 69(1), 9–11.
- 727 Weiss, I. (1993). Geometric invariants and object recognition. *International Journal of*
 728 *Computer 11263on*, 10(3), 207–231.
- 729 Wu, J., Yildirim, I., Lim, J. J., Freeman, B., & Tenenbaum, J. (2015). Galileo:

730 Perceiving physical object properties by integrating a physics engine with deep
731 learning. *Advances in Neural Information Processing Systems*, 28, 127–135.