Bayesian Uncertainty Estimation for Gaussian Graphical Models and Centrality Indices

J. Jongerling[1], S. Epskamp[2,3], and D.R. Williams[4]

[1]Department of Methodology and Statistics, Tilburg School of Social and Behavioral Sciences, Tilburg University.

[2]Department of Psychology, Faculty of Social and Behavioral Sciences, University of Amsterdam.

[3]Centre for Urban Mental Health, University of Amsterdam.

[4]Department of Psychology, University of California, Davis.

Abstract

In the network approach to psychopathology, psychological constructs are conceptualized as networks of interacting components (e.g., the symptoms of a disorder). In this network view, interest is on the degree to which symptoms influence each other, both directly and indirectly. These direct and indirect influences are often captured with centrality indices, however, the estimation method often used with these networks, the frequentist graphical LASSO (GLASSO), has difficulty estimating (uncertainty in) these measures. Bayesian estimation might provide a solution, as it is better suited to deal with bias in the sampling distribution of centrality indices. This study therefore compares estimation of symptom networks with Bayesian GLASSO- and Horseshoe priors to estimation using the frequentist GLASSO using extensive simulations. Results showed that the Bayesian GLASSO performed better than the Horseshoe, and that the Bayesian GLASSO outperformed the frequentist GLASSO with respect to bias in edge weights, centrality measures, correlation between estimated and true partial correlations, and specificity. Sensitivity was better for the frequentist GLASSO, but performance of the Bayesian GLASSO is usually close. With respect to uncertainty in the centrality measures, the Bayesian GLASSO shows good coverage for strength and closeness centrality, but uncertainty in betweenness centrality is estimated less well.

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Introduction

In recent years, psychological research is increasingly adopting a network perspective to psychological behavior (Borsboom, 2008; Borsboom, Cramer, Schmittmann, Epskamp, & Waldorp, 2011; Borsboom & Cramer, 2013; Cramer, Waldorp, van der Maas, & Borsboom, 2010; Schmittmann et al., 2013) in which psychological constructs are conceptualized as networks of interacting components referred to as nodes in network literature. Usually in psychological networks, these nodes represent observed variables (e.g., symptoms), while connections between nodes (edges) represent statistical relationships between the behaviors (Epskamp, Borsboom, & Fried, 2017a). Psychological networks are often estimated using Gaussian Graphical Models (GGM; Costantini et al., 2015; Lauritzen, 1996), which assume that the data are multivariate normally distributed, and in which edges can be interpreted as partial correlation coefficients, that is, correlations between two nodes after conditioning on all other nodes in the network, which are proportional to regression coefficients (Epskamp, Borsboom, & Fried, 2017a; Epskamp & Fried, 2018). Currently, a popular form of regularized GGM estimation is by using the graphical LASSO (GLASSO) (Friedman, Hastie, & Tibshirani, 2008; Epskamp & Fried, 2018), which limits the sum of the absolute edge weights, and therefore shrinks small estimates toward zero. As such, the GLASSO returns a sparse network model, in which only a relatively small number of edges are used to explain the covariation structure in the data (Epskamp, Borsboom, & Fried, 2017a). Because of this sparsity, the estimated models become more interpretable. Regularized estimation is not (always) necessary (especially with larger sample sizes), and unregularized estimation of GGMs is also possible (Liang, Song, & Qiu, 2015; Williams, Rhemtulla, Wysocki, & Rast, 2019). Regularized estimation is often used however, and is the estimation method on which we will focus in this study.

There are currently no real significance tests for edge weights in regularized networks (although they do exist for unregularized networks, for example in the *psychonetrics* R-package (Epskamp, 2020)). Instead, since regularization sets small edges equal to zero, the presence of an edge alone is taken as evidence that the two nodes are related to each (after conditioning on all other nodes in the network). The "importance" of nodes is often subsequently operationalized as how connected a node is to all others, either directly (*strength centrality*: the sum of the absolute values of all edges of a node (i.e., the partial correlations between the node and all other nodes)) or indirectly (*closeness centrality*: the inverse of the sum of largest indirect effects between nodes; *betweenness centrality*: the number of shortest paths between two other nodes that a node is part of) — although whether connectedness is indeed a good measure of importance, and whether it is always informative is under debate (see for example, Bringmann et al. (2019)). Yet, proper inference based on network models requires taking the accuracy of the estimates of edges and centrality measures into account, both to get an idea about the range of plausible value for these parameters, and to be able to determine which nodes can be considered as different from each other. Epskamp, Borsboom, and Fried (2017a) introduced a method, included in the *bootnet* R-package (Epskamp, Borsboom, & Fried, 2017b), to do this based on bootstrapping. However, while their method allows the estimation of intervals representing likely values of edges, called *bootstrapped confidence intervals*, these intervals are not the same as regular confidence intervals and can't be used to test the null hypothesis of no relation when regularization is used. In addition, simulations showed that it is difficult to get unbiased estimates and 95% confidence intervals for centrality indices. This is due to the instability in centrality indices caused by sampling variation and due to bias in their sampling distributions (i.e., strength centrality is calculated using the absolute value of edge weights which leads to skewed distributions, also see the supplementary material of Epskamp, Borsboom, and Fried (2017a)). In addition, their results showed that constructing bootstrapped CIs on very low significance levels is not feasible with a limited number of bootstrap samples (Epskamp, Borsboom, & Fried, 2017a).

In this paper, we therefore investigate Bayesian estimation of GGMs using a Bayesian version of the GLASSO (Wang, 2012) and using a Graphical Horseshoe prior (Li, Craig, & Bhadra, 2019) for the partial correlation matrix. We chose these two

Bayesian estimation methods because they are popular in the social science literature and therefore often used. In addition, we wanted to test alternative methods against the regularized estimation method that is quite common in the social sciences, which is the frequentist GLASSO. A Bayesian GLASSO is the most natural comparison method for this often used frequentist GLASSO in our opinion. Since the Bayesian estimation of GGMs provides posterior distributions for all sampled parameters and allows easy estimation of transformations and/or functions of these parameters, these methods could lead to measures of centrality that are less biased. For example, as mentioned above, strength centrality is calculated using the sum of the *absolute* values of the edge weights of a node (so positive and negative nodes don't cancel out), but this distorts the distribution of edges, and therefore the distribution of sums of (absolute values) of these edges as well (strength centrality) as well (see the left panel of Figure 1).

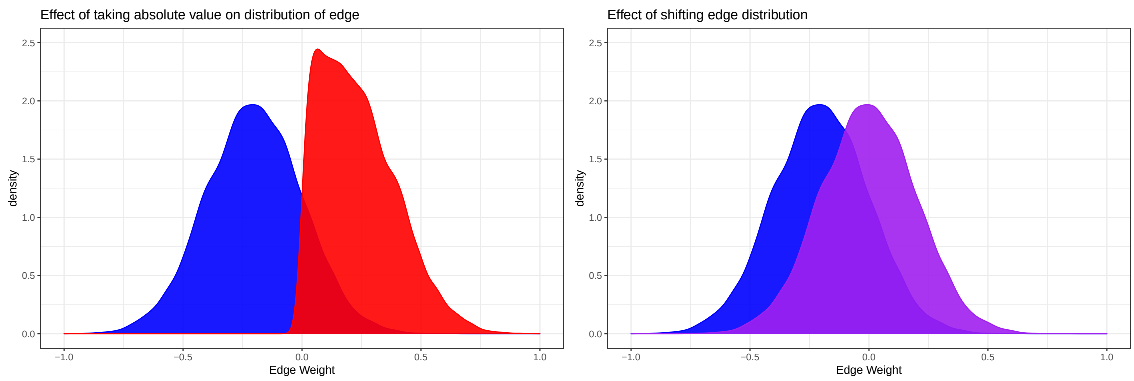


Figure 1. Effect of taking absolute values vs. shifting posteriors on the distribution of edges

With Bayesian analysis it's easy to use an alternative for absolute values. We could, for example, simply shift the posterior distributions of edge weights to be centered around positive values. If a posterior is centered around $-.20$ (the left distribution in both panels of Figure 1), we can shift the entire distribution .4 points to the right to center it around a *positive* value of .20 (right distribution in the right panel of Figure 1). As can be seen in Figure 1, shifting the entire posterior prevents distortion of the distributions of an edge weight (the right distribution in the right panel of Figure 1 is the exact same shape as the left distribution in that panel), unlike taking absolute values (the two distribution in the left panel of Figure 1 are clearly different). If we apply an appropriate shift to each individual edge (to make sure they are all centered around the positive 'version' of their individual point estimates), we can still prevent positive and negative (point estimates of) edges from canceling out when calculating strength centrality, but do so while distorting the distributions of the edges less. This could make the sum of these shifted posterior distributions a better behaved estimate of strength centrality than one based on absolute values. An extensive simulation study will therefore be undertaken to determine the ability of our Bayesian estimation methods to accurately estimate centrality measures and there uncertainty. In addition, to make sure that being able to estimate centralities and there uncertainty does not come at the cost of precision in estimation of individual edges, we also compare the Bayesian GLASSO and Horseshoe against the "standard" frequentist GLASSO (with tuning parameter selection based on the Extended Bayesian Information Criterion (EBIC) as implemented in the R-package *bootnet* (Epskamp, Borsboom, & Fried, 2017b) with respect to bias in edges, sensitivity, and specificity. This article is structured as follows. In the next section we will fist discuss the Bayesian GLASSO and Horseshoe estimation methods for GGMs. Then we will describe the different methods for estimating strength, closeness, and betweenness centrality that we will be testing. In the third section we will describe our simulation study, the results of which are presented in section four. We end with a discussion.

The Bayesian GLASSO and Horseshoe, and the estimation methods for the different centrality measures are incorporated into the R-package *BUEG* (Bayesian Uncertainty Estimation for GGMs) which is available on OSF (https://osf.io/9kjxv/).

Bayesian Estimation of GGMs

Assume we have observations from N individuals on P multivariate normally distributed variables $\mathbf{y_p}(p = 1, \ldots, P)$. If the means of these variables are equal to 0, the distribution of these data are given by,

$$\mathbf{y} \sim \mathrm{MVN}(\mathbf{0}, \mathbf{\Sigma}), \tag{1}$$

where $\mathbf{y}$ is a N by P matrix containing all item responses of the N individual on the P variables $\mathbf{y_p}$ (i.e., the vectors $\mathbf{y_p}$ make up the columns of matrix $\mathbf{y}$), and $\mathbf{\Sigma}$ is the variance-covariance matrix. The inverse of the variance-covariance matrix, $\mathbf{K}$, is called the precision matrix, which, after standardizing and inverting the signs of it's elements, contains the partial correlations between the P random variables on its off-diagonal elements. These partial correlations can be displayed as a weighted network, in which each variable $\mathbf{y_p}$ represents a node, and the partial correlations between variables show up as connections (edges) between the respective nodes. If a partial correlation between two variables is equal to 0, these two variables are conditionally independent given all other variables, and the nodes of these variables are unconnected in the graph of the weighted network. A challenge in precision matrix estimation is that the number of free parameters grows quadratically with the number of variables, which is why the precision matrix is often assumed to be sparse (i.e., some elements in $\mathbf{K}$ are expected to be zero even though every element of Σ may be non-zero).

Estimation of such a sparse model, termed a Gaussian Graphical Model (GGM; Epskamp, Rhemtulla, & Borsboom, 2017) when data are multivariate normally distributed, can be achieved by penalizing the likelihood. A popular penalized estimation method is the Graphical LASSO (GLASSO) proposed by Friedman et al. (2008), which uses the LASSO penalization (Tibshirani, 1996) and can be written as,

$$\log(\det \mathbf{K}) - \mathrm{tr}(\mathbf{SK}) - \sum_{j,k} \phi_\lambda(|\ \kappa_{jk}|), \tag{2}$$

where $\mathbf{S}$ is the covariance matrix, κ_{jk} is the entry on the jth row and kth column of $\mathbf{K}$ (proportional to the partial correlation between variables j and k), $\phi_\lambda(|\ \kappa_{jk}|) = \lambda|\kappa_{jk}|$ is the ℓ_1 penalty, and λ (with $\lambda > 0$) is a tuning parameter which controls the sparsity of $\mathbf{K}$ (with larger values of λ leading to more regularization and thus more sparsity) and that is typically chosen through cross-validation. The sum $\sum_{j,k} \phi_\lambda(|\ \kappa_{jk}|)$ in Equation 2 can be taken with or without a penalty on the diagonal terms (Rothman, Levina, &

Zhu, 2010; Meinshausen & Bühlmann, 2006; Yuan & Lin, 2007; Friedman et al., 2008).

Bayesian GLASSO

A Bayesian version of the GLASSO was introduced by Wang (2012). This Bayesian GLASSO is based on putting a double exponential prior on the off-diagonal entries of the precision matrix and an exponential prior on the diagonal entries,

$$p(\mathbf{K}|\lambda) \propto \prod_{j<k}(\mathrm{DE}(k_{jk}|\lambda)) \prod_{p=1}^{P}(\mathrm{EXP}(k_{jj}|\lambda/2))\mathbf{1}_{\mathbf{K}\in\mathbf{S_P}}, \tag{3}$$

where $\mathrm{DE}(x|\lambda))$ indicates the double exponential distribution with rate λ, $\mathrm{EXP}(x|\lambda/2)$ represents an exponential distribution with rate $\lambda/2$, and S_P is the space of $P \times P$ positive definite matrices (Li et al., 2019). The tuning parameter λ can be chosen by cross-validation as in a frequentist framework (Friedman et al., 2008; Rothman et al., 2010), or by specifying an appropriate hyperprior on this parameter (Li et al., 2019). The maximum a posteriori estimate of $\mathbf{K}$ under the prior in Equation 3 is equal to the regularized estimate of $\mathbf{K}$ you would obtain with the frequentist GLASSO (Li et al., 2019). Unlike the frequentist GLASSO, the Bayesian GLASSO does not set elements of the precision matrix to exactly 0. In fact, there is zero probability, according to the prior, that any of the partial correlations in $\mathbf{K}$ is exactly 0 (Wang, 2012). To make the Bayesian methods as comparable to the frequentist GLASSO as possible, we therefore have to apply either a discrete and continuous mixture prior distribution (Wang, 2012), such as the G-Wishart prior (Dawid & Lauritzen, 1993; Roverato, 2000), or use a heuristic decision rule to set elements of $\mathbf{K}$ to 0. In this study, we will use an additional decision rule as part of the Bayesian regularized estimation of GGMs, which sets edges whose 95% Credibility Intervals contain 0 to be equal to 0 (note that this approach is quite similar to frequentist hypothesis testing, and is only one possible decision rule). This second, decision rule (or pruning) step of our Bayesian regularization could be left out, in which case all regularization is done purely by the GLASSO or Horseshoe priors. This can be a useful alternative if having edges set to exactly to 0 is not required. However, as said above, we decided to add this second step as (an integral) part of our

Bayesian regularization to keep the similarity with the frequentist GLASSO as close as possible (and to have the calculation of sensitivity, specificity, etc. make sense).

Sampling from the Bayesian GLASSO was done using the data-augmented block Gibbs sampling scheme discussed in (Wang, 2012).

Bayesian Horseshoe

An alternative penalized Bayesian Estimation method is the Graphical Horseshoe introduced by Li et al. (2019). Instead of a double exponential prior, the graphical horseshoe puts a horseshoe prior on the off-diagonal elements of $\mathbf{K}$. The element-wise priors are specified for $(i, j = 1, \ldots, K)$ as (Li et al., 2019),

$$\begin{aligned} k_{ii} &\propto 1, \\ k_{ij:i\neq j} &\sim \text{Normal}(0, \lambda_{ij}^2 \tau^2), \\ \lambda_{ij:i\neq j} &\sim C^+(0, \mathbf{1}), \\ \tau &\sim C^+(0, \mathbf{1}), \end{aligned} \tag{4}$$

where C^+ denotes a half-Cauchy random variable with density $p(x) \sim (1 + x^2)^{-1}; x > 0$ (Carvalho, Polson, & Scott, 2010; Li et al., 2019). The horseshoe prior has two shrinkage parameters; the local shrinkage parameter λ_{ij} which is unique for each unique combination of variables, and the global shrinkage parameter τ which influences all partial correlation estimates. The global shrinkage parameter adapts to the sparsity of the entire matrix $\mathbf{K}$ and shrinks the estimates of the off-diagonal elements toward zero. In contrast, the local shrinkage parameters preserve the magnitude of non-zero off-diagonal elements, and ensure that the element-wise biases are not very large (Li et al., 2019). Based on the above the horseshoe prior for $\mathbf{K}$ can be written as (Li et al., 2019),

$$p(\mathbf{K}|\tau) \sim \prod_{i<j} \text{Normal}(k_{ij}|\lambda_{ij}^2, \tau^2) \prod_{i<j} C^2(\lambda_{ij}|0, \mathbf{1}) \mathbf{1}_{\mathbf{K}\in S_K} \tag{5}$$

where S_K again is the space of $K \times K$ positive definite matrices (Li et al., 2019). Like the Bayesian GLASSO, the Bayesian Horseshoe does not set elements of the precision matrix to exactly 0. For that it also requires an additional decision rule like setting elements whose 95% Credibility Intervals contain 0 to be equal to 0.

Sampling from the Bayesian Horseshoe was done using the data-augmented block Gibbs sampling scheme discussed in (Li et al., 2019).

Bayesian Methods for Estimating Centrality Indices

As mentioned in the introduction, Bayesian estimation of GGMs provides posterior distributions for all sampled parameters and allows easy estimation of transformations and/or functions of these parameters. In other words, with Bayesian estimation we can combine (posteriors of) model parameters in different ways to get (posteriors of) new, composite, parameters of interest. The different centrality estimates can be viewed as such composite parameters. We can combine (posteriors of) parameters any way we want, and as such can construct (the posterior of) a composite parameter of interest in different ways. As long as these different combinations make substantive sense, they can be viewed as representing (slightly) different operationalization of the same construct (i.e., composite parameter). As a simplistic example, if we want to construct a posterior for a composite parameter c from the posterior of model parameter a, with $c = 2 * a$, we could multiply the posterior of a by 2, or divide it by .5. Both would be different ways of getting to the posterior of c. In this study, we will test three different methods of combining (posteriors of) parameters to get estimates for the three centrality types (i.e., we will test three different Bayesian operationalizations for the centrality measures). All three methods are estimating the same constructs (the three centrality types), but do so in slightly different ways that might work more or less well in practice. Part of the aim of this study is determining which operationalization of the centrality estimates works best. The different estimation methods (operationalizations) for the three centrality types are discussed in the following three sub-sections.

Simple Gibbs-Sampler Estimation

The first method used to calculate the three different centrality measures for each node involved estimating each node's strength, closeness, and betweenness centrality in each iterations of the Gibbs-sampler. Specifically, this method consists of the following steps:

Algorithm 1 Simple Gibbs-Sampler Estimation

1: For each of the k ($k = 1, \ldots, K$) iteration of the Gibbs-sampler, use the current estimate for the inverse variance-covariance matrix as input for the calculations of strength, closeness, and betweenness centrality described in Opsahl, Agneessens, and Skvoretz (2010) and implemented in the *qgraph* R-package (Epskamp, Cramer, Waldorp, Schmittmann, & Borsboom, 2012).

2: For each node p ($p = 1, \ldots, P$), use the K estimates of strength, closeness, and betweenness centrality obtained in the previous step to obtain the posteriors for these three centrality types for each node.

3: For each node p ($p = 1, \ldots, P$), take the modes of the posterior distributions of the centrality measures as the point estimate for the centralities of the node.

4: For each node p ($p = 1, \ldots, P$), take the 2.5th and 97.5th percentile of the posterior distributions of the centrality measures to construct the 95% Credibility intervals for each node's strength, closeness, and betweenness centrality.

A problem with this method is that, as mentioned above, the Bayesian GLASSO and Horseshoe doesn't set edges to exactly 0, which is why the Bayesian regularization employed in this study also uses a decision rule that sets all edges whose 95% Credibility Interval contains 0, to 0. This decision rule can only be applied after all iterations of the Gibbs-sampler are done however, because it requires the complete posterior distribution of the edges. Since the Simple Gibbs-Sampler Estimation approach involves calculating centrality values in each iteration of the Gibbs-sampler (so before the Gibbs-sampler is completely done), the centrality estimations mentioned in step 1 above have to be done without the (heuristic) pruning step. This will lead to positive bias in the strength and closeness centralities estimates of this method, as edges that will eventually be set to exactly 0 (and should therefore not contribute to these centrality measures at all), are not set to 0 yet and will therefore have non-zero contributions to these centrality values. Betweenness centrality could also be biased, but since this measure depends on how many shortest paths between two nodes a third node is a part of, it is less directly influenced by the values of edges. After all, regularization does not necessarily change which path between nodes is shortest. As such, the extend and form of the bias introduced in this centrality measure by the absence of the pruning step is harder to predict. Note that the edge estimates used for the calculations of the centrality values in this method are still regularized to some degree by the prior, which pulls the values towards 0. They are not *completely* regularized however, because they can't be set to exactly zero without the pruning heuristic.

To solve for this issue, and to more fully correct our centrality estimates for regularization, we also devised the 2 methods described below.

Post-Processing Shift Estimation

The first method for calculating the centrality measures that takes complete Bayesian regularization (including pruning) into account is by using what we termed *Post-Processing Shift Estimation.* This approach consists of the following step:

Algorithm 2 Post-Processing Shift Estimation

1: Calculate the posteriors for centrality measures according to the Simple Gibbs-Sampler Estimation method discussed above.

2: Prune the final estimate of the precision matrix by setting edges whose 95% CI contains 0 to 0. Point estimates of the edges are based on the modes of the posterior (MAP estimates).

3: For each node p ($p = 1, \ldots, P$), estimate the three centrality values based on the pruned precision matrix from step 2 (using the calculations described in Opsahl et al. (2010) and implemented in the *qgraph* R-package (Epskamp et al., 2012)).

4: For each node p ($p = 1, \ldots, P$), calculate the difference between the point estimates of the three centrality measures obtained with Simple Gibbs-Sampler Estimation method (in step 3) and the point estimates obtained in the previous step. The difference between these two estimates can be interpreted as the bias introduced in the centrality estimates by not setting edges to exactly 0.

5: For each node p ($p = 1, \ldots, P$), use the difference between the two sets of point estimates calculated in the previous step to shift the posteriors for the three centrality measures obtained with the Simple Gibbs-Sampler Estimation method so that the modes of these posteriors become equal to the point estimates for the centralities obtained from the pruned precision matrix (step 3).

6: For each node p ($p = 1, \ldots, P$), take the 2.5th and 97.5th percentiles of the shifted posteriors from the previous step to construct the 95% Credibility intervals for each node's strength, closeness, and betweenness centrality.

Note that since we're merely shifting the posteriors obtained from the Simple Gibbs-Sampler Estimation method to construct our new posteriors with this method, the widths of the intervals will be the same across these two methods. However, in this method the posteriors are centered around less biased point estimates of strength, closeness, and betweenness centrality.

For the Simple Gibbs-Sampler Estimation method, the posterior distributions of the centrality measures were restricted to be larger than or equal to 0, as they should be, since the values of the centrality measures can't be smaller than 0. By shifting these posteriors, as we do in this second method, we could make some of the posteriors cover values smaller than 0 as well. In practice however, this will likely not cause any problems. First, in the tests analyses that we ran, the 95% Credibility Intervals for the centrality measures of the Post-Processing Shift method never went below 0. In addition, even if they did, this would just imply that we can't rule out that the corresponding node is unconnected to all others in the network (although the negative values should obviously not be interpreted).

Estimation based on Edge Weight Estimates

The second fully regularized estimation method for the centrality measures is not based on calculating the centrality values for each node in each iteration of the Gibbs-sampler. Instead it is based on the estimates of the edges themselves and the mathematical expressions for the variance of a(n) (inverse of a) sum.

For random variables $Y_1, \ldots, Y_k$, the variance of their sum is given by,

$$var\left[\sum_{k=1}^{K} Y_k\right] = \sum_{k=1}^{K} \sigma_k^2 + 2 \sum_{1 \leq k \leq l \leq K} \sigma_k \sigma_l, \tag{6}$$

where σ_k^2 is the variance of variable k. In addition, the variance of the inverse of a random variable Y is given by (Mood, Graybill, & Boes, 1985),

$$var\left[\frac{1}{Y}\right] \approx \left(\frac{1}{\mu_Y}\right)^2 \left(\frac{\sigma_Y^2}{\mu_Y^2}\right). \tag{7}$$

This second Equation can be obtained by a Taylor-series expansion and dropping all terms of order higher than 2 (Mood et al., 1985).

For strength centrality, the steps of this Estimation Based approach are as follows:

Algorithm 3 Estimation Based Strength Centrality

1: Prune the final estimate of the precision matrix by setting edges whose 95% CI contains 0 to 0. Point estimates of the edges are based on the modes of the posterior (MAP estimates).

2: For each node p ($p = 1, \ldots, P$), estimate the strength centrality based on the pruned precision matrix from the previous step (using the calculations described in Opsahl et al. (2010)).

3: For each edge k ($k = 1, \ldots, K$), that is, each element of the inverse covariance matrix, use it's posterior to determine its variance σ_k^2.

4: For each pair of edges k and l ($k = 1, \ldots, K$, $l = 1, \ldots, K$, $k \neq l$), use their values from each iteration of the Gibbs-Sampler to calculate the covariance between the two σ_{kl}.

5: For each node p ($p = 1, \ldots, P$), calculate the variance of the sum of it's edges (to all other nodes) using Equation 6: $\sigma^2_{Strength,p} = \sum_{k=1}^{S} \sigma_k^2 + 2\sum_{1 \leq k \leq l \leq S} \sigma_{kl}$ (for $k = 1, \ldots, S$, $l = 1, \ldots, S$, $k \neq l$), where σ_k^2 is the variance of edge k, σ_{kl} is the covariance between edge k and edge l, and S is the maximum number of edges of node p. Note that we also take the variance of nodes that are set to 0 after regularization into account when calculating the variance of the strength centrality of a node.

6: For each node p ($p = 1, \ldots, P$), calculate the 95% Credibility Intervals for each node's strength centrality by taking the point estimate for the strength centrality of that node (step 2) and adding and subtraction $1.96 * \sigma^2_{Strength,p}$ to the point estimate, where $\sigma^2_{Strength,p}$ is estimated in the previous step.

For closeness centrality, the steps of the Estimation Based approach are:

Algorithm 4 Estimation Based Closeness Centrality

1: Prune the final estimate of the precision matrix by setting edges whose 95% CI contains 0 to 0. Point estimates of the edges are based on the modes of the posterior (MAP estimates).

2: For each node p ($p = 1, \dots, P$), estimate the closeness centrality based on the pruned precision matrix from the previous step (using the calculations described in Opsahl et al. (2010) and implemented in the *qgraph* R-package (Epskamp et al., 2012)).

3: For each edge k ($k = 1, \dots, K$)(i.e., element of the inverse covariance matrix), use it's posterior to determine its variance σ_k^2.

4: For each pair of edges k and l ($k = 1, \dots, K$, $l = 1, \dots, K$, $k \neq l$), use their values from each iteration of the Gibbs-Sampler to calculate the covariance between the two σ_{kl}.

5: For each node p ($p = 1, \dots, P$), determine the edges on the largest/strongest shortest path from the node to all other nodes using Dijkstra's algorithm (Dijkstra, 1959) (implemented in the *qgraph* R-package (Epskamp et al., 2012))).

6: For each pair of nodes p and m ($p = 1, \dots, P$, $m = 1, \dots, P$, $p \neq m$), calculate the variance of the sum of the edges of the shortest path connecting the two nodes (determined in the previous step) using Equation 6: $\sigma^2_{\text{Shortest Path},pm} = \sum_{k=1}^{S} \sigma_k^2 + 2\sum_{1\leq k\leq l\leq S} \sigma_{kl}$, where σ_k^2 is the variance of edge k, σ_{kl} is the covariance between edges k and l, and S is the maximum number of edges on the shortest path between the two nodes.

7: For each node p ($p = 1, \dots, P$), calculate the variance of the sum of all of its shortest paths to all other nodes using Equation 6: $\sigma^2_{\text{Sum of Shortest Paths of node p}} = \sum_{m=1}^{P} \sigma^2_{\text{Shortest Path},pm} + 2\sum_{1\leq m\leq n\leq P} \sigma_{pm,pn}$ (for $m \neq p$), where $\sigma^2_{\text{Shortest Path},pm}$ is the variance of the shortest path between nodes p and m (determined in the previous step), and $\sigma_{pm,pn}$ is the covariance between the shortest path from node p to node m, and the shortest path from node p to node n.

8: For each node p ($p = 1, \dots, P$), calculate the mean of the sum of all of its shortest paths to all other nodes. This is done by calculating the sum of the shortest paths between the node and all other nodes in each iteration of the Gibbs-sampler, and then taking averaging across iterations.

9: For each node p ($p = 1, \dots, P$), calculate the variance of the closeness centrality using Equation 7: $\sigma^2_{Clsnss,p} = \frac{\sigma^2_{\text{Sum of Shortest Paths of node p}}}{\mu^4_{\text{Sum of Shortest Paths of node p}}}$, where $\mu_{\text{Sum of Shortest Paths of node p}}$ is the mean of the sum of all of shortest paths from node p to all other nodes (calculated in previous step).

10: For each node p ($p = 1, \dots, P$), calculate the 95% Credibility Intervals for each node's closeness centrality by taking the point estimate for the closeness centrality of that node (step 2) and adding and subtraction $1.96 * \sigma^2_{Clsnss,p}$ to the point estimate, where $\sigma^2_{Clsnss,p}$ is estimated in the previous step.

Note that in the steps above we have two "types" of paths when combining variances and covariances into total variance. The first type are the successive edges on the same shortest path (e.g., on the path A - B - C, we have the edges/paths A - B and B - C), and the second are the separate shortest paths emanating from the same node (e.g., the paths A - B - C and A - D - E). We aren't sure whether it makes sense for these two types of paths to be correlated amongst each other (i.e., for successive edges on the same shortest path to be correlated, or for separate shortest paths emanating from the same node to be correlated). As a result we used four different variations of our edge weight based calculation of closeness centrality. We calculated the total closeness centrality assuming that 1) both successive edges on the same shortest path and different shortest paths emanating from the same node could be correlated, 2) only separate shortest paths emanating from the same node could be correlated, 3) only successive edges on the same shortest paths could be correlated, and 4) assuming that both shortest and successive paths were uncorrelated.

A downside of this second, edge weight based, method is that it can't be used for a new estimate of betweenness centrality, as it isn't a direct function of individual edges weights. Instead, betweenness centrality is about how many shortest paths between two other nodes a node is part of. In addition, it assumes normally distributed random variables. This second downside, isn't necessarily a large problem however. In our data, the posterior distributions of most edges look pretty normal. Furthermore, the width of the CIs for strength centrality of this method also appear quite similar to those obtained with the other two methods (that don't assume normality). Sometimes they are a little wider, sometimes a little narrower but the difference is always in the second decimal place or even smaller.The width of CIs for closeness centrality are also quite similar. They tend to be a little narrower of wider depending on whether one assumes successive edges on shortest paths to be correlated (if successive edges on a shortest path are assumed to be correlated the intervals tend to be a little wider than those of the other methods, while assuming independent successive edges on a shortest path leads to slightly narrower intervals than the other methods), but these differences tend to be in the 4 decimal place or even smaller (typically the fifth decimal place).

Simulation Study

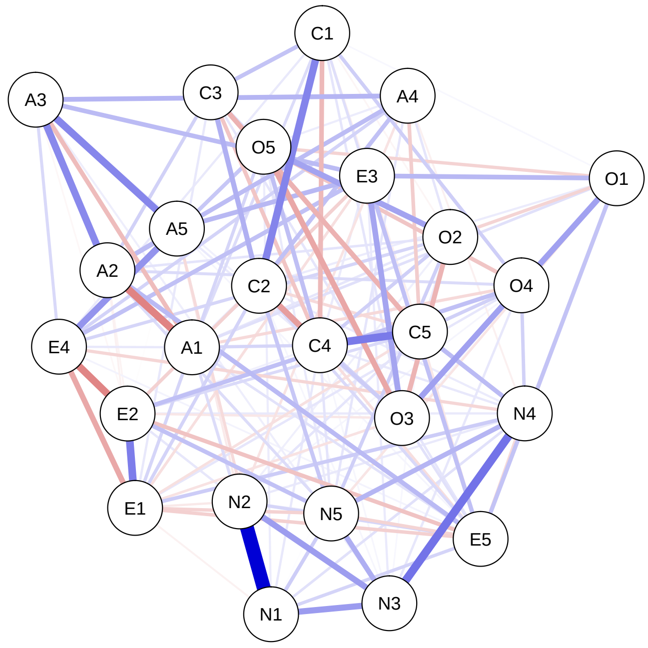
To test the performance of the two Bayesian estimation methods mentioned above and the different centrality estimation methods we ran an extensive simulation study. Our focus for this paper is on the social sciences (specifically psychology), which is why we generated data that is representative for data encountered in this field (while also looking at the effect of network structure and sparsity on performance). Specifically, we generated data for two different network structures: 1) a network based on the partial correlation from the bfi-data in the *Psych* R-package (Revelle, 2019), which contains information on 25 personality self-report items taken from the International Personality Item Pool (ipip.ori.org), and 2) a random network structure for 25 nodes generated with the *genGGM* function from the *bootnet* package (Epskamp, Borsboom, & Fried, 2017b)(Figure 2). We generated data for sample sizes of $N = 900, 5,000$ and $10,000$ (which gives us N (sample size) to p (number of estimated parameters) ratios of 3, 16.67, and 33.33 respectively). In addition, we also varied the density of the random network so that it either closely matched that of the bfi-data (.537 vs .517) or was equal to about half that of the bfi-data (.263 vs .517). The partial correlations in the bfi-data ranged from $-.256$ to .528 (mean=.023, sd=.080), the partial correlations in the random network of equal density ranged from $-.216$ to .492 (mean=.013, sd=.093), and the partial correlations in the random network of half-density ranged from $-.319$ to .510 (mean=$-.001$, sd=.089).For each scenario (i.e., combination of N and network-structure) we ran 1000 replications.

For the Bayesian estimation of the generated data we used the Bayesian GLASSO prior (with $\lambda \sim gamma(1, .01)$, i.e. we used a gamma distribution with rate parameter of 1 and scale parameter of .01 as hyper-prior for the tuning parameter) and Bayesian Horseshoe prior (with $C^{+}(0, \mathbf{1})$ hyperpriors for both the local (λ_{ij}) and global (τ) shrinkage parameters) discussed above. For both methods we used $10,000$ burn-in iterations and $10,000$ subsequent iterations. Traceplots showed good convergence with these number of iterations. We also analyzed the generated data using a frequentist graphical LASSO for comparison. For this frequentist estimation we used the estimation method used in the popular *bootnet* R-package (Epskamp, Borsboom, & Fried, 2017b), in which the optimal value for the regularization parameter was selected using the Extended Bayesian Information Criterion (EBIC). Specifically, this frequentist method runs the Glasso estimation 100 times (with 100 different values for the tuning parameter). The values of the tuning parameter in these runs are logarithmically spaced between the maximal value of the tuning parameter at which all edges are zero

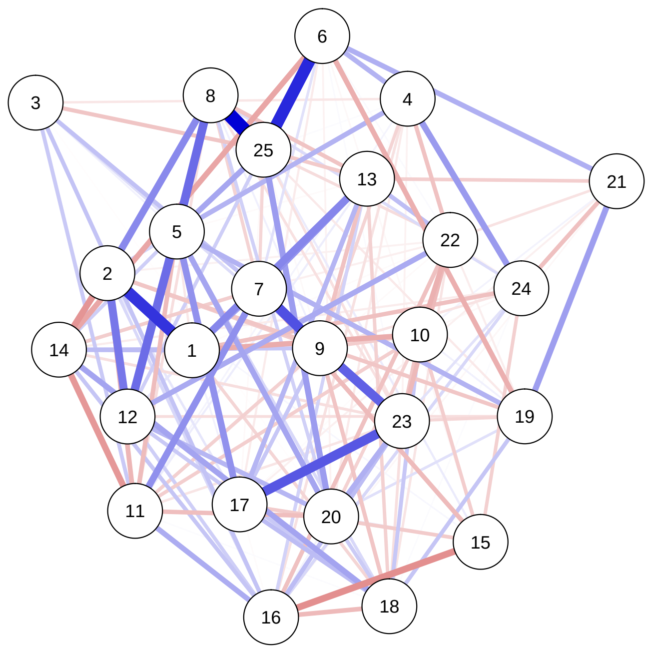
(λ_{max}) and $\lambda_{max}/100$. For each of the resulting graphs the EBIC is computed and the graph with the best EBIC is selected (Epskamp et al., 2012).

To assess performance, we first compared the performance of the Bayesian GLASSO and Horseshoe to frequentist GLASSO estimation on KL-loss, the Frobenius norm, the correlation between the estimated and true precision-matrix elements, F1 (the harmonic mean of the positive predictive value and sensitivity), sensitivity, and specificity. The KL-loss and Frobenius norm are measures of the "distance" between the true and estimated precision matrix and can therefore be viewed as measures of bias in edge estimates. The correlation between the estimated and true precision-matrix elements give an indication of how well the rank-order of edges are maintained. The sensitivity and specificity give an indication of how well edges are classified as present or absent respectively, while F1 gives and indication of how well edges that are present in the population are identified as such.

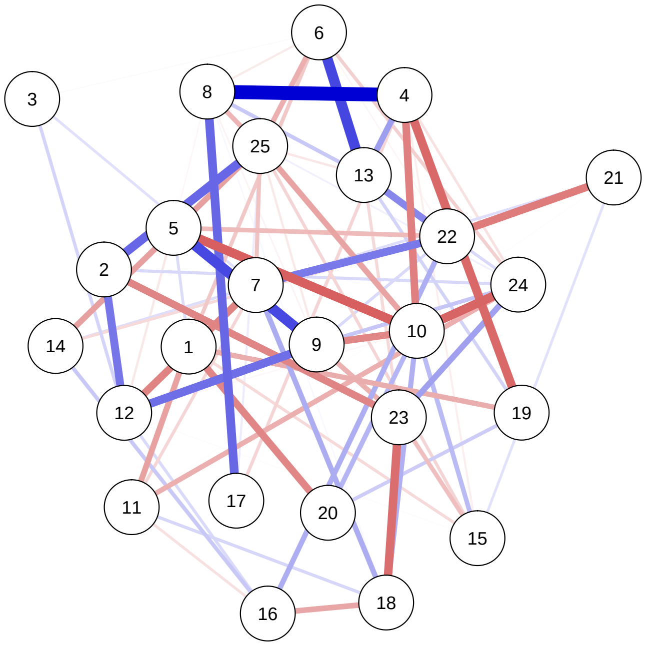
Next, we assessed performance of the Bayesian methods with regard to the different centrality measures by looking at the mean square error of the centrality estimates, the correlation between the true and estimated centrality measures, and the coverage of the 95% Credibility Intervals (both per node and on average). Coverages between .92 and .98 were deemed acceptable (Bradley, 1978), and per node coverage was deemed sufficient if at least 75% of nodes had acceptable coverage.



a Psych Network: Network Based on the bfi-data



b RED Network: Random Network with density equal to the Psych Network



c RHD Network: Random Network with density half that of the Psych Network

Figure 2. The Three Network-Structures Under Which Data Were Generated

Results

Comparison to Frequentist GLASSO

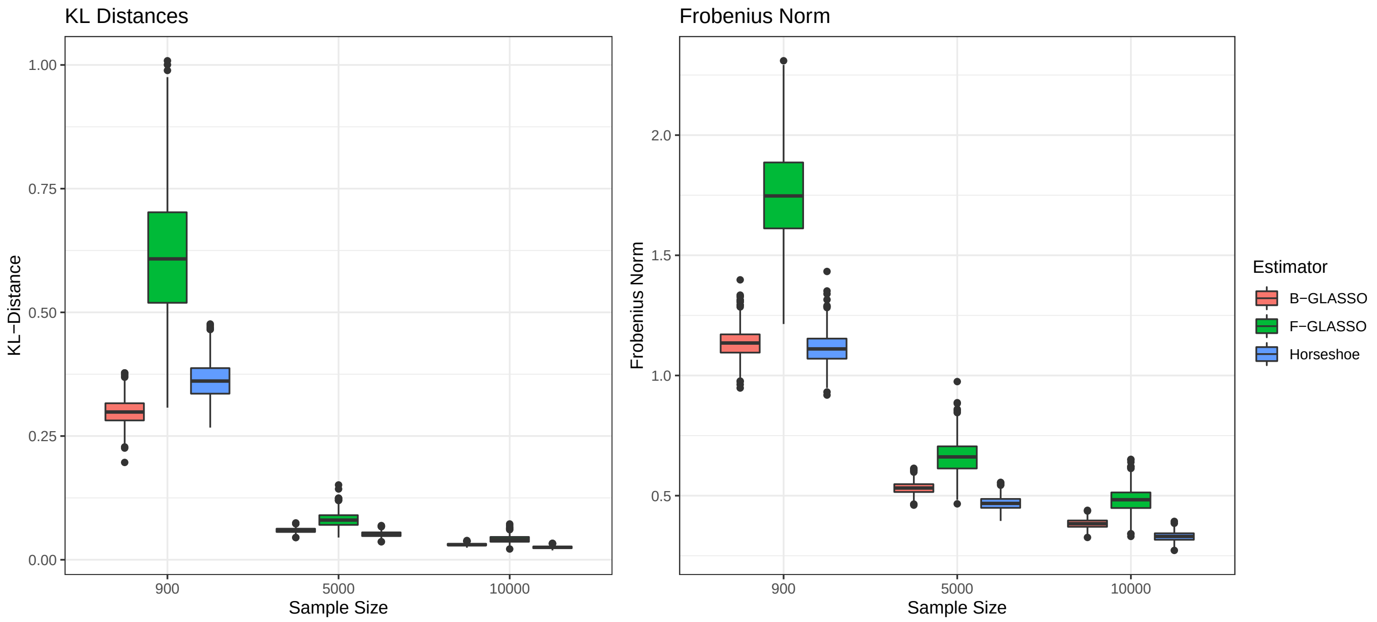
Figure 3 shows that the Bayesian estimation methods compare well to the frequentist GLASSO in terms of total bias in the (regularized) partial correlation matrix. For both the network based on the bfi-data from the *Psych* R-package (the Psych Network) (Revelle, 2019) and the Random Network with half the density of the Psych Network (the Random Half Density or *RHD* Network) the two Bayesian methods show lower KL-distance and lower Frobenius norms than the frequentist GLASSO for all three sample sizes. For the Random Network with equal density (the Random Equal Density or *RED* Network), the frequentist GLASSO performs better that the Horseshoe, and slightly better than the Bayesian GLASSO, with respect to KL-distance at N=900. For sample size of 5,000 and 10,000, the two Bayesian methods perform better however. The Bayesian GLASSO and Horseshoe also perform better in terms of the Frobenius Norm for the RED Network at all sample sizes.

To improve readability, figures for the the correlation between the estimated and true precision-matrix elements, sensitivity, and specificity are given in Appendix A. Figure A.1 shows that all three estimation methods show high correlations between true and estimated edge weights at all sample sizes (with correlations becoming larger as N increases). Again the Bayesian method appear to have a slight edge over the frequentist GLASSO. For all networks, the Bayesian GLASSO always has higher correlations than its frequentist counterpart, while the Horseshoe has higher correlations for $N > 900$.

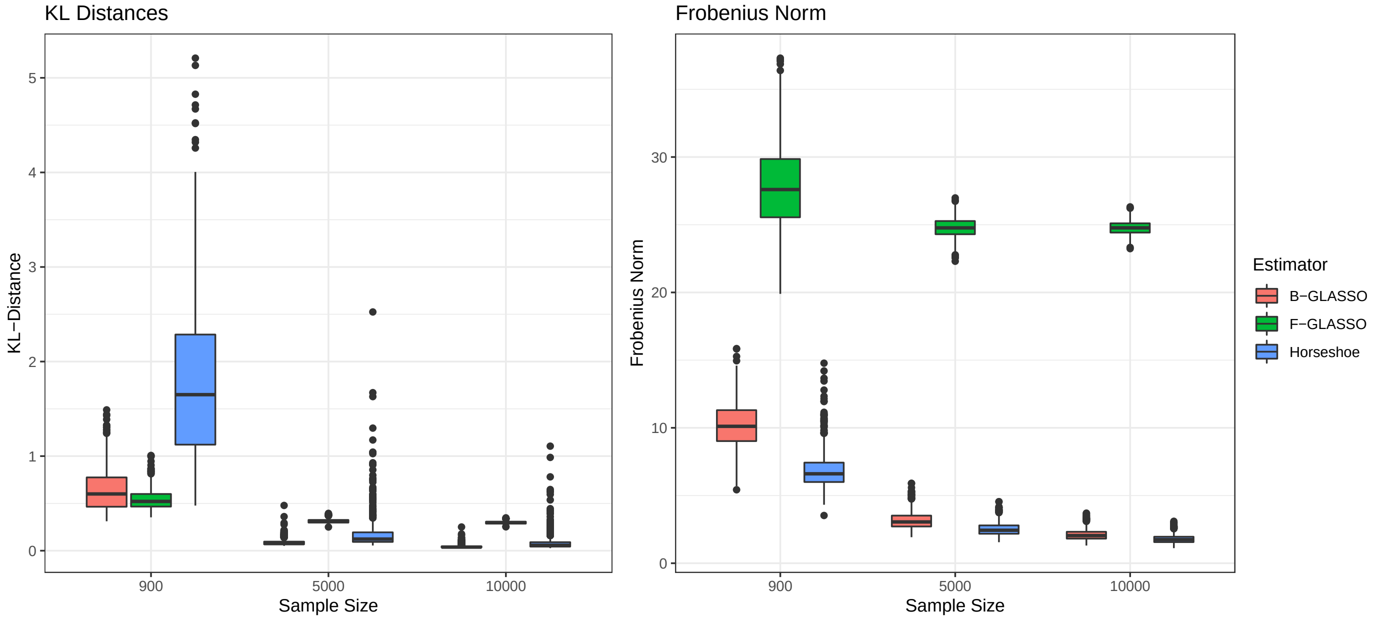
Figures A.2 shows that for the Psych Network, the frequentist GLASSO has better sensitivity that the Bayesian methods at all sample sizes. For the RED Network (Figure A.3) the frequentist GLASSO works better for $N < 10,000$. At $N = 10,000$ the Bayesian GLASSO performs best (although the difference with the frequentist GLASSO is not large). For the RHD Network (Figure A.4), the frequentist GLASSO always has better sensitivity than the Graphical Horseshoe. At $N > 900$, however, the sensitivity of the Bayesian GLASSO is equal or slightly better. The specificity is always better for the Bayesian methods, while F1 is better for all networks for sample size larger than

900. For the two random networks the F1 for the Bayesian GLASSO is also larger than that of the frequentist GLASSO at $N = 900$.

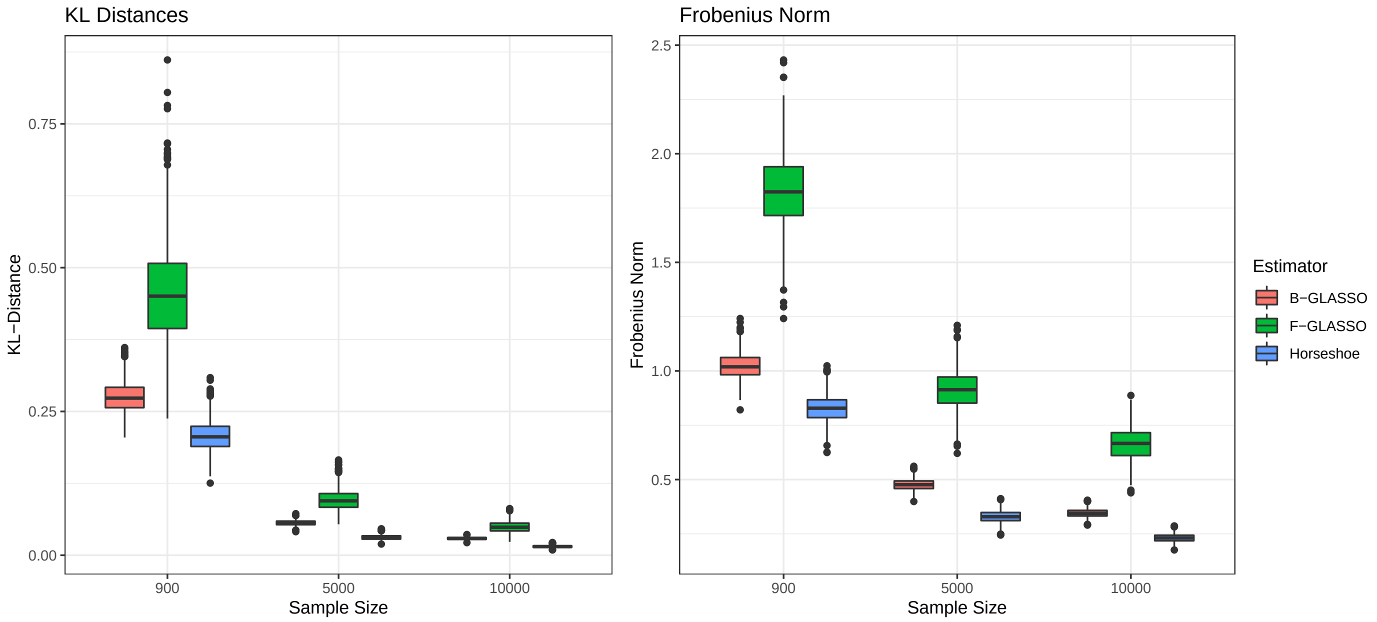
Figures A.5, A.6, and A.7 show the correlation between the true and estimated centrality measures. For the Psych network (Figure A.5), the correlation between true and estimated strength and closeness centrality is always higher for the Bayesian methods (although the frequentist GLASSO also has high correlations). The correlation between true and estimated betweenness centrality is about the same for all estimation methods. For the RED network (Figure A.6), the Bayesian methods have higher correlations between true and estimated closeness and betweenness centrality. The correlation between true and estimated strength centrality is better for this network if $N > 900$. All three measures are always really close in performance. Finally, for the RHD network (Figure A.7), the Bayesian GLASSO always has higher correlation between true and estimated strength and closeness centrality, and has the highest correlations for betweenness centrality if $N > 900$ (otherwise the variability in correlation is slightly larger than for the other methods). The Horseshoe has higher correlations than the frequentist GLASSO for strength and betweenness centrality, but the correlation for closeness centrality in this network is really unstable for this estimation method.



a Bias for Psych Network



b Bias for Random Network



c Bias for Random Network (Half Density)

Figure 3. Bias in Estimated Edge Weights for the Different Estimation Methods at N=900, 5,000, and 10,000

Table 1

Average Coverage Rates for the Bayesian GLASSO

Centrality	Estimation Method	Psych Network Sample Size 900		5,000		10,000		Random Network (Eq. Density) Sample Size 900		5,000		10,000		Random Network (Half Density) Sample Size 900		5,000		10,000	
		Average	%	Average	%	Average	%	Average	%	Average	%	Average	%	Average	%	Average	%	Average	%
Strength	Post-Processing Shift	.661	.120	.949	.760	.969	.920	.775	.160	.941	.760	.951	.800	.923	.400	.978	.440	.980	.480
	Simple Gibbs-Sampler	.152	.000	.135	.000	.124	.000	.139	.000	.141	.000	.148	.000	.008	.000	.005	.000	.004	.000
	Based on Edge Weights	.583	.040	.910	.480	.939	.840	.776	.160	.935	.800	.943	.840	.967	.200	.993	.080	.993	.000
Closeness	Post-Processing Shift	.932	.720	.938	.880	.940	.880	.939	.840	.948	.960	.950	1.000	.687	.000	.888	.240	.920	.600
	Simple Gibbs-Sampler	.564	.120	.899	.600	.923	.720	.908	.320	.952	.920	.956	.960	.926	.440	.912	.680	.932	.720
	Based on Edge Weights:																		
	Correlated Successive and Shortest Paths	1.000	.000	.979	.520	.964	1.000	1.000	.000	.970	.920	.959	1.000	.955	.680	.935	.720	.930	.720
	Correlated Successive Paths	1.000	.000	.913	.560	.871	.120	1.000	.000	.884	.320	.854	.160	.949	.720	.769	.000	.749	.000
	Correlated Shortest Paths	1.000	.000	.979	.480	.964	1.000	1.000	.000	.970	.840	.959	1.000	.955	.680	.935	.720	.930	.720
	Uncorrelated Successive and Shortest Paths	1.000	.000	.914	.600	.871	.120	1.000	.000	.885	.680	.854	.160	.949	.720	.769	.000	.749	.000
Betweenness	Post-Processing Shift	.806	.000	.891	.600	.905	.720	.845	.240	.903	.640	.917	.640	.857	.400	.953	.640	.955	.600
	Simple Gibbs-Sampler	.956	.440	.973	.520	.979	.400	.963	.320	.973	.320	.980	.200	.932	.240	.991	.200	.990	.200

Table 2

Average Coverage Rates for the Bayesian Horseshoe

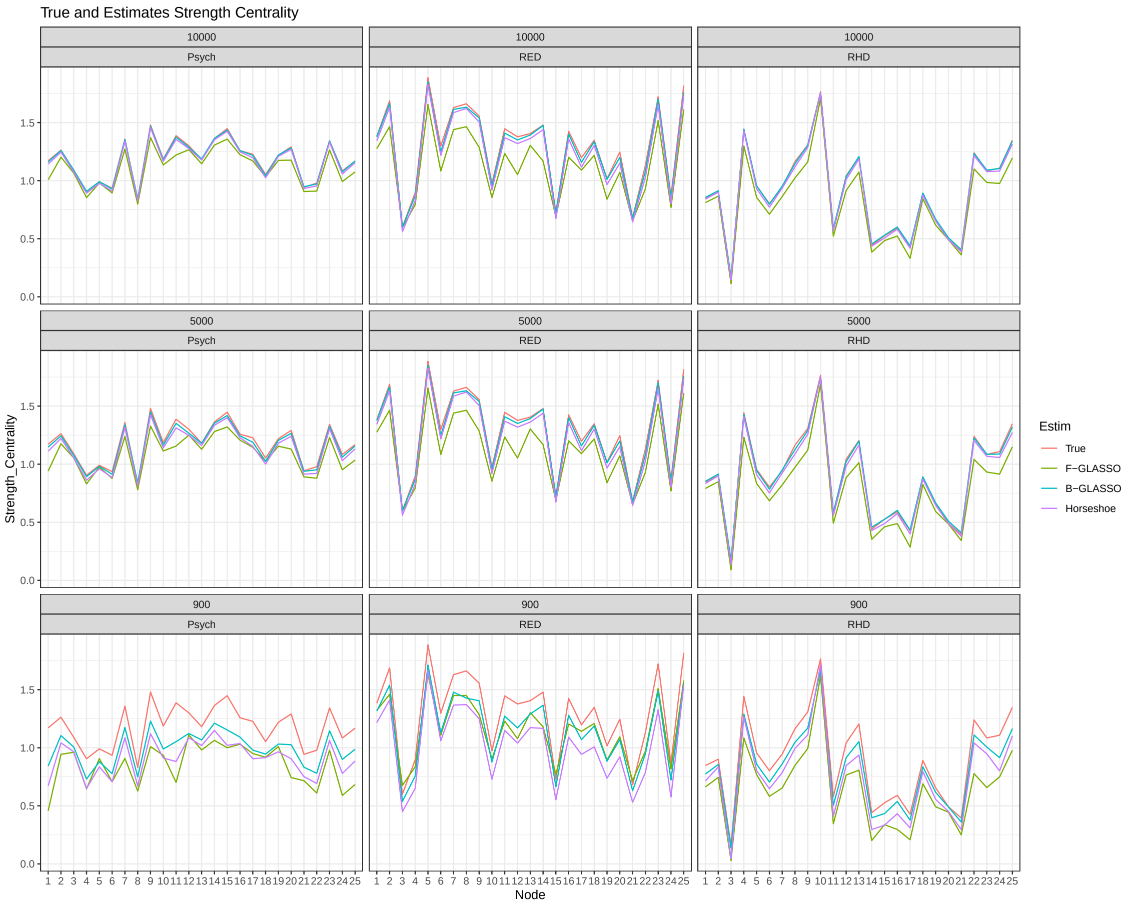
Centrality	Estimation Method	Psych Network Sample Size 900		5,000		10,000		Random Network (Eq. Density) Sample Size 900		5,000		10,000		Random Network (Half Density) Sample Size 900		5,000		10,000	
		Average	%	Average	%	Average	%	Average	%	Average	%	Average	%	Average	%	Average	%	Average	%
Strength	Post-Processing Shift	.389	.000	.876	.280	.950	.600	.373	.000	.813	.160	.874	.440	.703	.160	.933	.160	.966	.320
	Simple Gibbs-Sampler	.646	.200	.432	.000	.360	.000	.773	.440	.610	.080	.148	.040	.482	.040	.260	.000	.198	.000
	Based on Edge Weights	.336	.000	.818	.160	.911	.440	.350	.000	.783	.120	.849	.320	.742	.080	.946	.020	.975	.240
Closeness	Post-Processing Shift	.921	.680	.937	.800	.941	.800	.882	.240	.943	.840	.950	.960	.344	.000	.892	.280	.931	.720
	Simple Gibbs-Sampler	.752	.240	.898	.560	.924	.760	.952	.680	.962	.960	.965	.960	.848	.360	.917	.640	.943	.880
	Based on Edge Weights:																		
	Correlated Successive and Shortest Paths	1.000	.000	.978	.400	.964	.920	1.000	.000	.968	.800	.960	.840	.851	.320	.958	.840	.941	.880
	Correlated Successive Paths	1.000	.000	.912	.440	.869	.120	1.000	.000	.878	.320	.856	.200	.839	.240	.825	.160	.780	.000
	Correlated Shortest Paths	1.000	.000	.978	.400	.964	.920	1.000	.000	.968	.800	.960	.840	.851	.320	.958	.840	.941	.880
	Uncorrelated Successive and Shortest Paths	1.000	.000	.912	.480	.870	.120	1.000	.000	.878	.320	.856	.200	.839	.240	.826	.160	.780	.000
Betweenness	Post-Processing Shift	.777	.000	.895	.600	.907	.760	.819	.320	.897	.640	.918	.600	.893	.480	.952	.600	.958	.560
	Simple Gibbs-Sampler	.967	.360	.975	.560	.982	.360	.960	.280	.973	.360	.981	.200	.978	.120	.991	.160	.991	.160

Centrality Measure Coverage

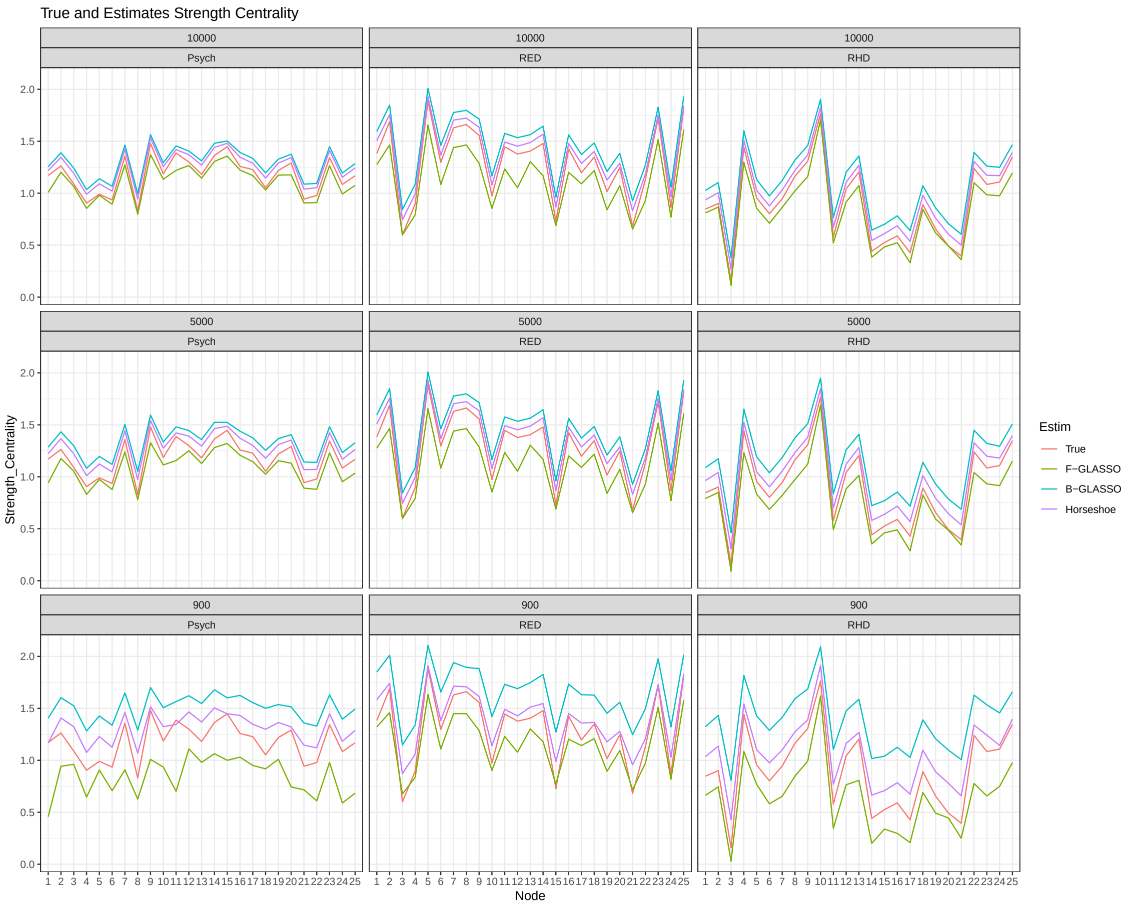
For the Psych, RED, and RHD networks, Figures 4a and 4b, 6a and 6b, and 8a and 8b show the true Strength, Closeness, and Betweenness centrality of each of the 25 nodes respectively (in red). In addition, they show the estimated values of these centralities for the Frequentist GLASSO (in green), the Bayesian GLASSO (in blue), and the Graphical Horseshoe (in purple), for each of the three sample sizes under which data was generated from the networks. For the Bayesian methods, estimated obtained with the Post-Processing Shift estimation method are given in Figures 4a, 6a, and 8a, while the estimated obtained with the Simple Gibbs-Sampler estimation method are given in Figures 4b, 6b, and 8b.

For the Bayesian estimation methods, Figures 5, 7, and 9 give information on the average coverage rates and the variability in these rates across the 25 nodes (for Strength, Closeness, and Betweenness centrality respectively), for each combination of the three networks and the three generated sample sizes.

For readability, tables with the detailed centrality information for each of the 25 nodes in each of the three networks are given in Appendix A.



a True Strength Centrality for Post-Processing Shift Estimation



b True and Estimated Strength Centrality for Simple Gibbs-Sampler

Figure 4. True and Estimated Strength Centrality

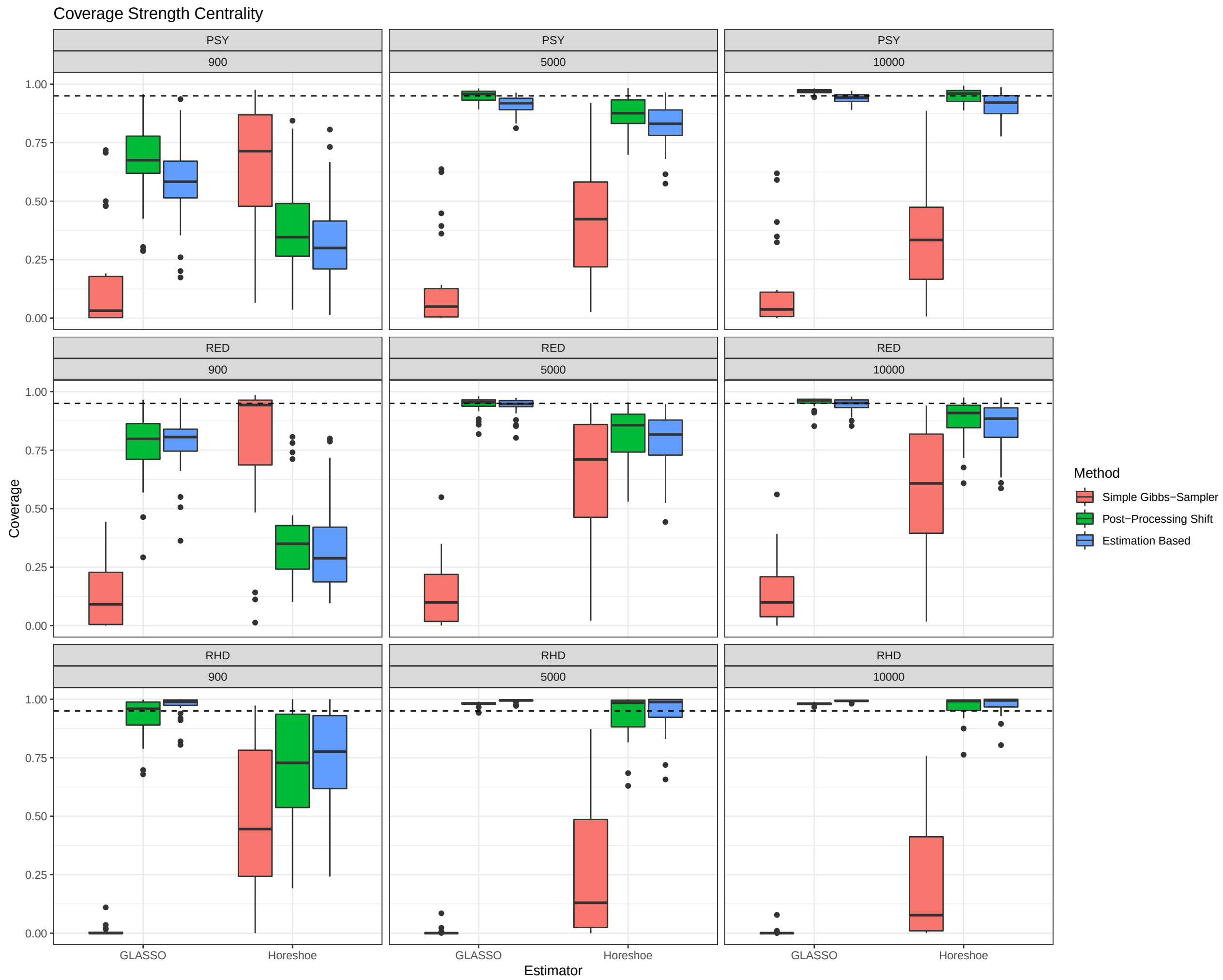


Figure 5. Coverage for Strength Centrality

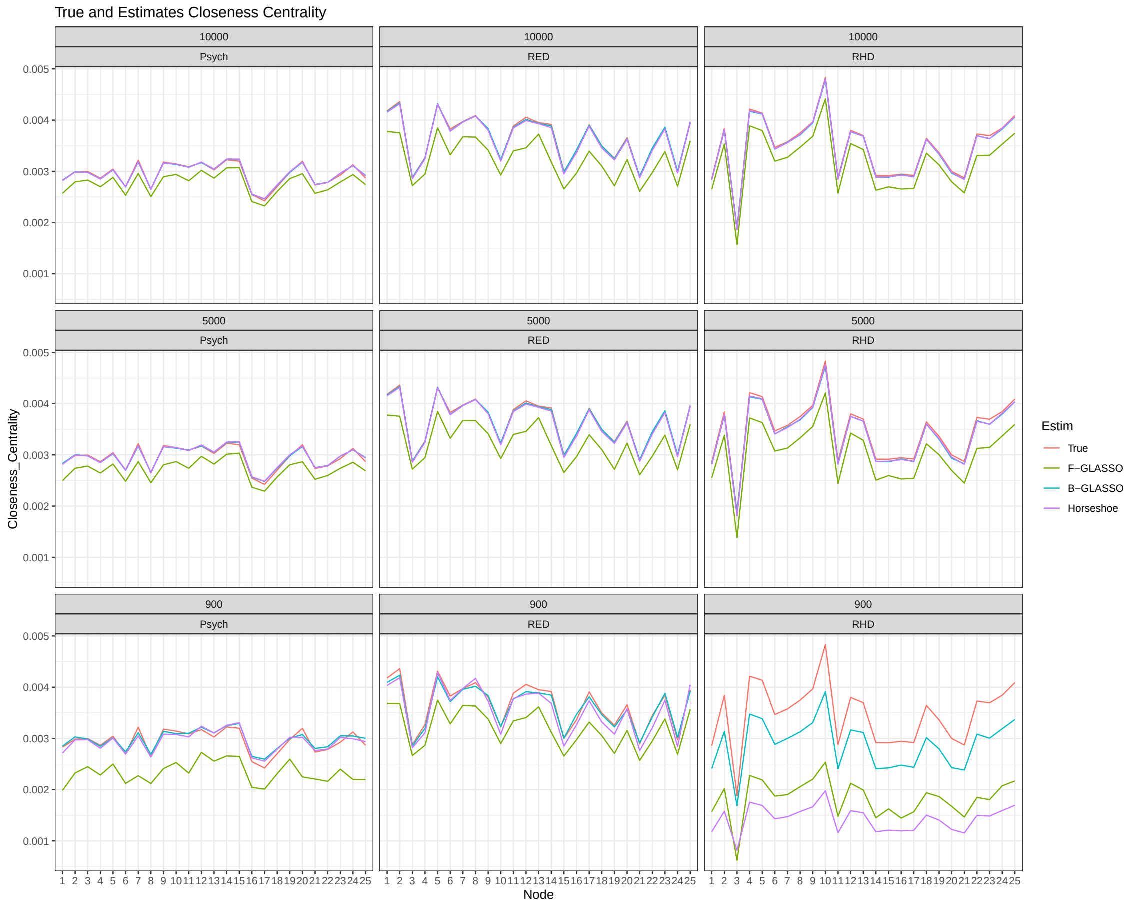
Strength. When using the Bayesian GLASSO, strength centrality of all networks is estimated best by the Post-Processing Shift estimation method. This method has the lowest MSE (tied with the estimation method based on edge weights which uses the same point estimate)(see Figures 4a - 4b and Tables A.1 - A.3).

Importantly, for $N > 900$, this method also has good coverage for the two denser networks (the Psych- and RED Network)(Figure 5 and Table 1). For the Psych Network, the average coverage rate and proportion of nodes with sufficient coverage is .949 and .760 for $N = 5,000$, and .969 and .920 for $N = 10,000$. For the RED network the average coverage and proportion of nodes with sufficient coverage is .941 and .760 for $N = 5,000$ and .951 and .800 for $N = 10,000$. For both these denser networks coverage at $N = 900$ is too low due to negative bias in the point estimates for each nodes strength centrality. For the less dense RHD network, the Post-Processing Shift estimation method has good average coverage at all sample sizes, but the proportion of nodes with sufficient coverage is too low, although, at $N > 900$, this is due to the method being too conservative (i.e., to many coverages above .98 (Table A.3)).

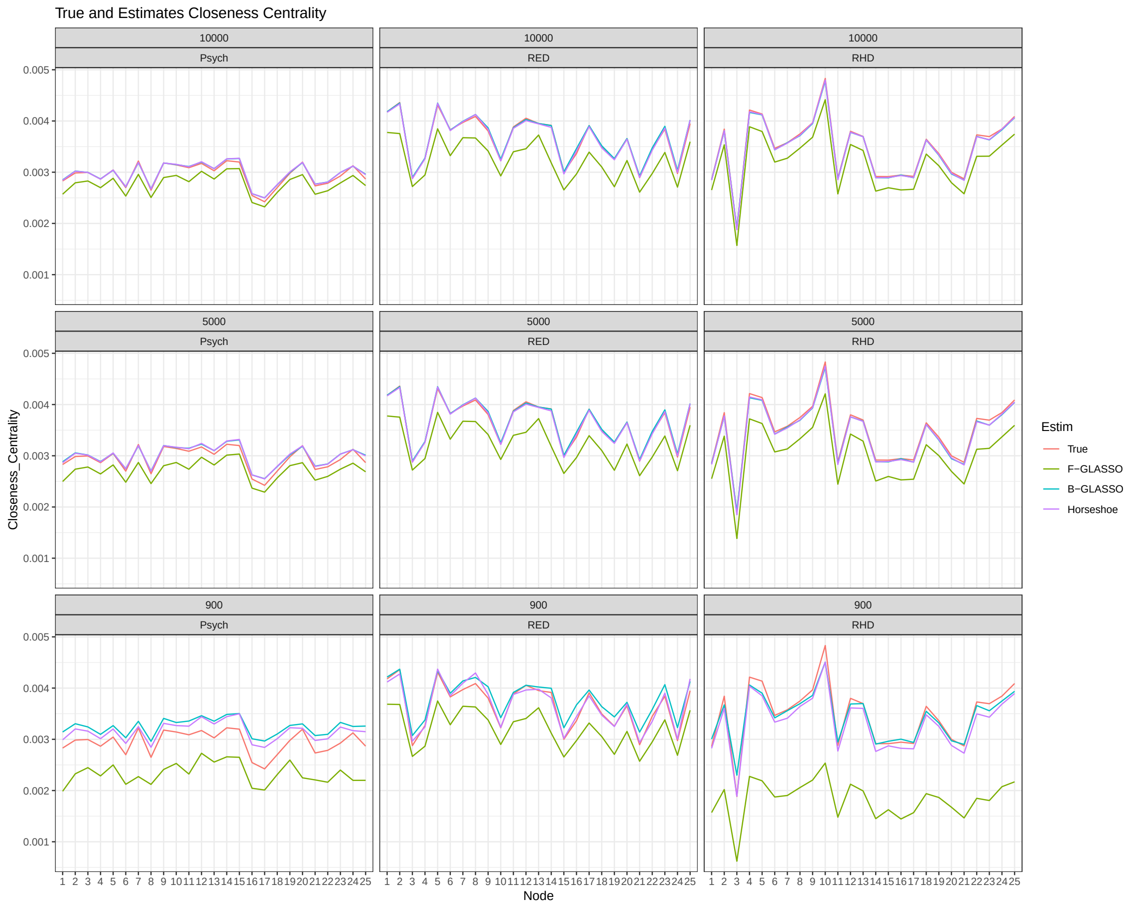
Estimation using the Simple Gibbs-sampler estimation method resulted in positive bias in the strength estimates as expected, and as a result average and per node coverage where too low for all networks and sample sizes.

Estimation based on edge weights performed well in terms of MSE, but performed worse than the Post-Processing Shift approach in terms of coverage. This method only showed good coverage for $N = 10,000$ in the Psych network, and $N > 900$ in the RED network.

When using the graphical Horseshoe, strength centrality is estimated less well than when the Bayesian GLASSO is used (see Table 2, Tables A.4 - A.6, and Figures 4a and 4b). The MSE's of both the Post-Processing Shift estimation method and the estimation based on edge weights are slightly larger (although the difference is very small) while the average and per node coverage of these methods is insufficient for all networks and sample sizes. Simple Gibbs-sampler estimation method does have smaller MSE's when the Horseshoe is used, but as mentioned above, there is substantial positive bias, and therefore, low coverage when using this method.



a True and Estimated Closeness Centrality for Post-Processing Shift Estimation



b True and Estimated Closeness Centrality for Simple Gibbs-Sampler

Figure 6. True and Estimated Closeness Centrality

Figure 7. Coverage for Closeness Centrality

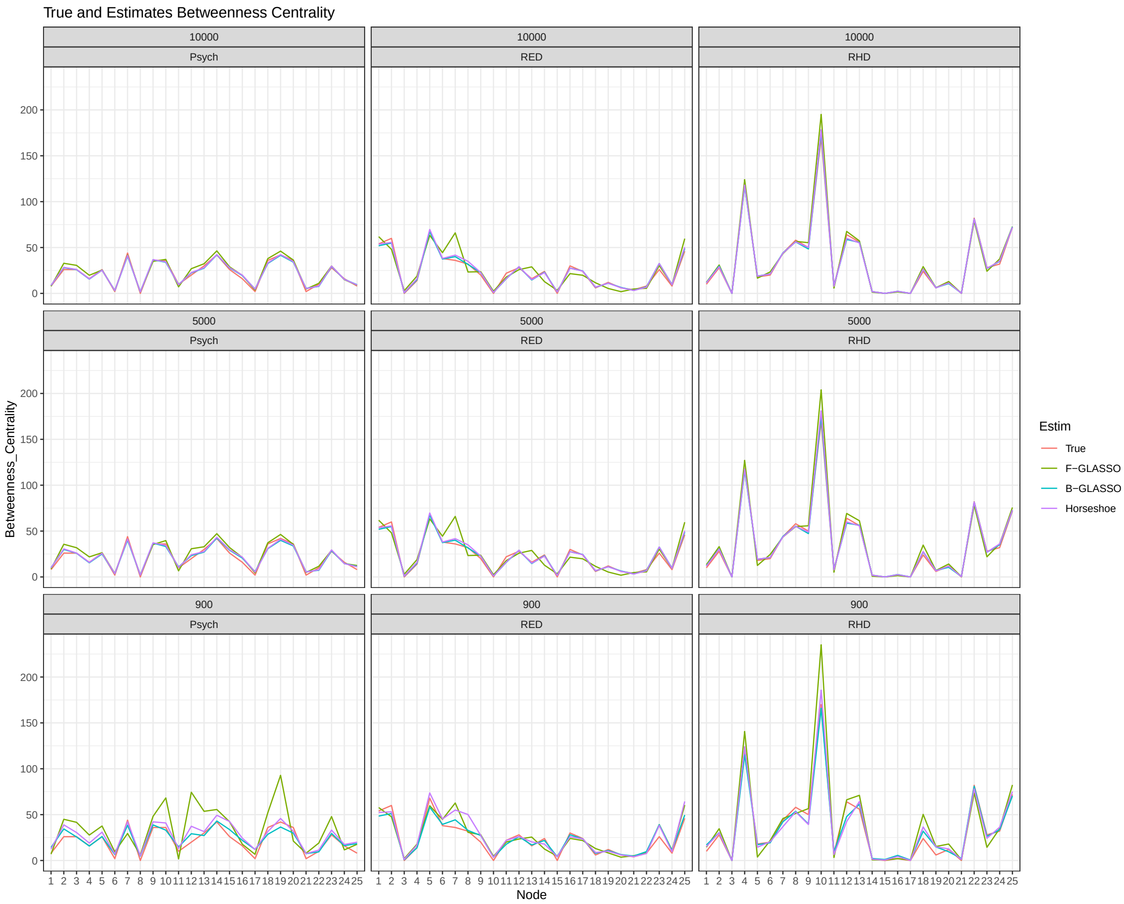
Closeness. When using the Bayesian GLASSO, closeness centrality of the denser Psych- and RED networks is again estimated best by the Post-Processing Shift Estimation method. This method had low MSE and good average coverage for all N (see Table 1, Tables A.1 - A.3 and Figures 6a and 6b)).

The proportion of nodes with sufficient coverage is also good for this method in these denser networks (Table 1), except for $N = 900$ in the Psych network where it is a little too low (.720). For the sparser RHD network, the coverage of closeness centrality of this method is too low.

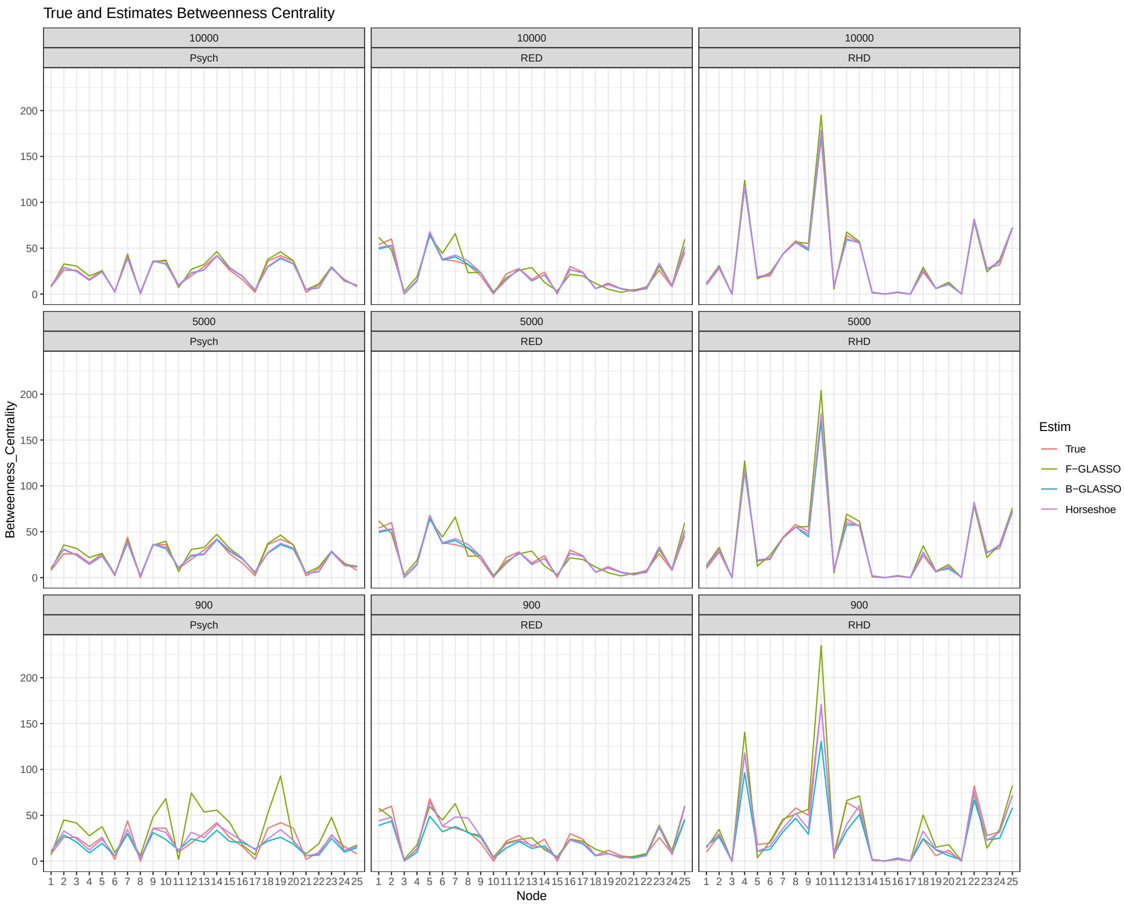
Estimation using the Simple Gibbs-Sampler estimation method resulted in positive bias in the closeness estimates, as expected, for the denser Psych and RED networks. For the RHD network, there was a small negative bias (except at $N = 900$). This is probably due to the fact that there a lot more edges that were close to 0 (in each iteration of the Gibbs-sampler) in this network, which could have limited the effect of the Bayesian GLASSO not setting edges explicitly to 0 when estimating a network. Average and per node coverage where too low for this method however, except for N of $5,000$ or $10,000$ in the RED network.

Of the 4 estimation methods based on edge weights, the methods with correlated successive and shortest paths and the method with correlated shortest paths worked best. In fact, performance of these two methods was exactly the same, as was that of the two methods assuming either uncorrelated shortest paths or uncorrelated successive and shortest paths. This indicates that successive paths on a shortest route between nodes are (effectively) uncorrelated. In the rest of this paper we will therefore only distinguish between methods with correlated shortest paths and the methods with uncorrelated shortest paths. The method with correlated successive paths showed good coverage for $N = 10,000$ in the Psych Network, and for $N > 900$ in the RED network. For lower sample sizes, this method was too conservative in these denser networks. In the sparser RHD network, this method did not have good coverage. This could be due to shrinkage being "stronger" in this network (due to it's sparser nature), which could pose a problem for the assumption of normality that this method is based on.

When using the graphical Horseshoe, closeness centrality is estimated with a little more bias than with the Bayesian GLASSO, however coverage is better for all methods except the Post-Processing Shift method (see Table 2, Tables A.4 - A.6, and Figures 6a and 4b). When using the Graphical Horseshoe, coverage of the Post-Processing Shift method is only sufficient for $N > 900$ in the Psych and RED network, and never sufficient for the RHD network. The coverage for the estimation based method with correlated successive paths shows lower average and per node coverage than when the Bayesian GLASSO is used, but this method does have good average and per node coverage in the RHD network now for $N > 900$. The Simple Gibbs-Sampler estimation method again works a little better with the Horseshoe prior and now has good average and per node coverage for $N = 10,000$ in all three networks, and for $N > 900$ in the RED network.



a True and Estimated Betweenness Centrality for Post-Processing Shift Estimation



b True and Estimated Betweenness Centrality for Simple Gibbs-Sampler

Figure 8. True and Estimated Betweenness Centrality

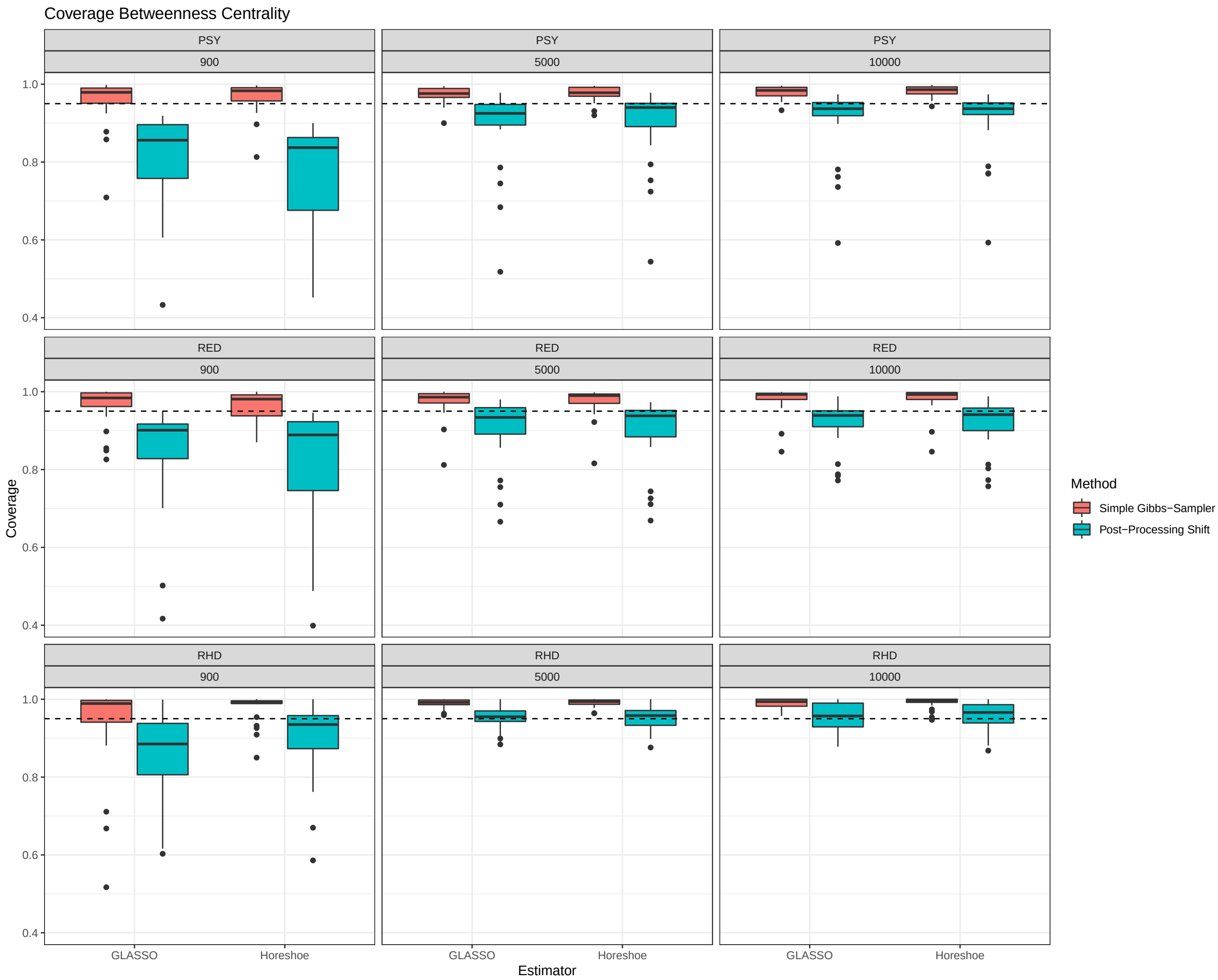


Figure 9. Coverage for Betweenness Centrality

Betweenness. Out of the 3 centrality estimates, betweenness centrality was estimated the least well by all methods. When using the Bayesian GLASSO, the Post-Processing Shift Estimation method again tends to have the lowest MSE (except for $N = 10,000$ in the RHD network)(see Tables A.1 - A.3 and Figures 8a and 8b)).

Unlike for strength and closeness centrality, coverage for betweenness centrality is not sufficient with any method. Also, coverage for the Post-Processing Shift estimation method is worse than that of the Simple Gibbs-Sampler (Table 1). This last method has better average coverage rates, and only has insufficient average coverage for $N > 900$ in the RHD network, where average coverage is too conservative. The per node coverage of the Simple Gibbs-Sampler is insufficient however for all networks and sample sizes, although at $N > 900$ the method tend to be too conservative.

When using the Graphical Horseshoe, the MSE's for the betweenness centrality estimates again tend to be larger than when the Bayesian GLASSO is used (see Tables A.4 - A.6, and Figures 8a and 8b). With this prior the MSE of the Post-Processing Shift method tend to be smaller than that of the Simple Gibbs-Sampler method for the denser Psych and RED networks (except when $N = 900$). For the RHD network, the Simple Gibbs-Sampler always has the lowest MSE. In terms of coverage the overall picture is similar to that seen with the Bayesian GLASSO (although coverage for Post-Processing Shift method appears somewhat lower, and that of the Simple-Gibbs Sampler method somewhat higher)(see Table 2) . The average and per node coverage for the Post-Processing Shift method is too low, while the Simple Gibbs-Sampler method has good average coverage, with average coverage only being insufficient for $N > 900$ in the RHD network (where average coverage is too conservative). In addition, as was the case when the Bayesian GLASSO was used, the per node coverage of the Simple Gibbs-Sampler is insufficient for all networks and sample sizes, although at $N > 900$ the method tend to be too conservative.

Conclusion

Taken together, results of our simulation study shows that the Bayesian methods (particularly the Bayesian GLASSO) are strong alternatives for the frequentist GLASSO. The Bayesian GLASSO outperforms the frequentist GLASSO with respect to a) bias in edge weights, b) bias in the centrality measures, and c) the correlation between the estimated and true partial-correlations. The Bayesian Horseshoe also typically outperforms the frequentist GLASSO on these three measures (except at $N = 900$ where the frequentist GLASSO shows less bias in edge weights and higher correlations between estimated and true values).

In terms of sensitivity and specificity results are more mixed. The frequentist GLASSO has better sensitivity overall (although the Bayesian GLASSO is very close or slightly better in the RED and RHD networks for $N \geq 5,000$), while the Bayesian methods have better specificity. A choice between methods here therefore appears to come down to a preference for erring on the side of caution (deciding an edge is absent while it isn't) versus erring on the sides of discovery (deciding an edge is there while it isn't). However, the Bayesian GLASSO does perform better than the frequentist GLASSO (except at $N = 900$ in the Psych network) on F1 which gives a harmonic mean of precision (the proportion of truly positive edges out of all the edges identified as positive by a method) and recall (the proportion of all true positive edges that gets identified as positive), while the Graphical Horseshoe performs better on this measure for $N \geq 900$.

In terms of (coverage for) the three different centrality measures, the Bayesian GLASSO outperform the Graphical Horseshoe, and shows good coverage for strength and closeness centrality at sample sizes of $N = 5,000$ or higher (in which case there are 16.66 observations for each partial correlation with 25 nodes). For smaller sample sizes the Bayesian GLASSO has insufficient coverage for strength centrality in the denser Psych- and RED networks, and insufficient coverage for closeness centrality in the less dense RHD network. This is likely the case because there is less shrinkage at $N > 900$. As mentioned, Bayesian regularization is a two-step approach; 1) estimates are pulled towards zero (but *not* set to zero) by the GLASSO or Horseshoe priors, and 2) estimates whose 95% Credibility Intervals contain zero are set to 0. At $N > 900$, the shrinkage in the first of these steps is less “severe” than at $N = 900$, likely leading to less distorted posteriors. For strength and closeness centrality the Post-Processing Shift estimation method provided the best performance out of all methods tested with the Bayesian GLASSO in both coverage en MSE. Performance with regard to betweenness centrality was worse than for the other two centrality measures, and coverage is insufficient for both the Bayesian GLASSO and Graphical Horseshoe at all N for all networks. For this centrality measure the Simple-Gibbs estimation appear to be the best choice. It has a larger MSE than the Post-Processing Shift method, but shows better coverage rates and tends to have intervals that are too wide for $N \geq 900$, implying that it will err on the side of being too conservative.

Finally, performance of the Bayesian GLASSO (and the Bayesian methods in general) appears to be better when networks are not to sparse, but this should not pose a problem in social sciences. With regard to network-structure, the methods worked well for both the structure of the bfi-data from the *psych-package*, and for a random network.

Limitations

In this study we evaluated the performance of different estimation methods by looking at their ability to recover an underlying true- or generating model. This is only one way of evaluating estimation methods, and some argue not the most important one (Cudeck & Henly, 2003), as in practice there are no true models, and not being able to recover a generating model in a simulation study does not mean that a method does not have other merits (like simplicity or interpretability) or cannot capture relevant aspects of underlying processes in practical research (Cudeck & Henly, 2003; Hand & Vinciotti, 2003). Future research should also look at accuracy of model-based prediction and/or cross-validation as performance measures.

In addition, as is always the case with simulation studies, results only generalize within the scope of the settings and scenarios used in this study. That is, we only looked at a subset of possible data generating- and prior models. We did not consider an exhaustive set of different generating network structures because our focus was on the use of GGMs in the social-, and specifically, psychological sciences. We therefore wanted to compare methods on network structures that researchers in these field are likely to encounter (such as the bfi-data). Also, when determining the effect of network structure on the performance of the different methods, by also generating data based on networks with a different structure and/or density as that of the bfi-data, we wanted to keep these variations in structure within the limits that they might be encountered by, and therefore relevant to, applied social scientists (e.g., the size of the edge weights and the number of variables). Getting a more fine grained picture of the effect of different network characteristics (i.e., size of edge weights, general structure, sparsity, etcetera) on the performance of different estimation methods is important however. Future research should therefore look at more diverse sets of data, including but not limited to i) networks whose precision matrices includes edge weights that are (mostly) very close to zero, ii) networks much denser than the ones used in this study, or even iii) from networks who's characteristics are less common (or perhaps even unlikely) in psychological practice (to determine boundary performance). Regarding priors, we chose to focus on the Bayesian GLASSO and Horseshoe priors, again because of our focus on the use of GGMs in the social sciences. These two priors are currently quite popular in that field and therefore often used. However, many alternatives to these two choices exist. One interesting alternative is the already mentioned G-Wishart prior (Dawid & Lauritzen, 1993; Roverato, 2000), a continuous and discrete mixture prior that can set edges to 0 without the need for a heuristic like the one we used in this article. In addition, future research might also want to consider priors with Ridge-type penalization as in the Eigenvalue decomposition based Wishart prior introduced by Kuismin and Sillanpää (2016). Ridge penalization tends to shrink to 0 less strongly than LASSO regularization, and could therefore lead to better results with respect to sensitivity, albeit likely at the cost of specificity. Studying alternative priors will give valuable information about how they compare to each other and what analytic goals are best served by what prior. As suggested above, the GLASSO could be preferred when specificity is more important for example, while Ridge-type priors could be the better choice if sensitivity is more important. Lastly, future research might also want to study variations on the priors used in this study in more detail. For example, by setting different hyper-priors on the tuning parameters of the GLASSO and Horseshoe priors, or by using different heuristics for setting edges to 0, to further investigate the impact of choices in those aspects of the priors in different contexts.

In addition, we only looked at forms of regularization in which edges are set to exactly 0. As mentioned, Bayesian regularization does not set elements of the precision matrix to exactly 0 by default. It pulls estimates towards zero, but does not make them exactly zero. This less strict regularization might actually be a better option in some circumstances, depending on how realistic the assumption of sparsity is for the data (or population) at hand. Another, interesting difference between Bayesian regularization and frequentist regularization, is that, as Bayesian regularization is achieved by means of priors, the amount of regularization diminishes as the sample size goes up. This is actually really useful behavior. The more signal there is in the data, the less estimates are pulled to 0. Frequentist regularization also prunes less edges as data increases, as the EBIC will select denser graphs, but how the amount of regularization depends on the amount of data, and how this dependence is different than for the Bayesian methods is not very clear. Looking into Bayesian regularization (and how it compares to it's Frequentist cousin) in more detail, both from an applied and more theoretical point of view, is therefore needed.

Next to looking more closely into the differences between frequentist and Bayesian regularization mentioned above, future research could also look into non-regularized estimation of GGMs in more detail (Liang et al., 2015; Williams et al., 2019). Regularization makes inference more difficult as the meaning of p-values becomes less clear. What is the exact null-hypothesis being tested, for example? And how can one account for the fact that the data is used not only for model estimation, but also for the process used to determine the penalty term (and therefore the sparsity of the final model)? As mentioned in the introduction, usually the presence of an edge is taken as proof for the presence of a relation between nodes, while the absence is taken as proof for conditional independence. This might seem to side-step the mentioned issues with p-values, but doesn't really. For every method (not just regularized methods) 1 - sensitivity gives the false negative rate, which indicates we cannot simply use the absence of an edge/connection as evidence the a relationship is null. Similarly, 1 - specificity gives the false positive rate (and the frequentist GLASSO has been show to have a quite a few false positives, although this is less the case for the Bayesian GLASSO, see above), which indicates we cannot simply use the presence of an edge/connection as evidence the a relationship is exists. Non-regularized estimation could circumvent some of the inference problems mentioned above (although issues with post-selection inference apply to non-regularized estimation as well (Berk, Brown, Buja, Zhang, & Zhao, 2013)), but it's performance (compared to regularized estimation) has not been extensively studied yet.

Finally, there is some debate on whether the different centrality indices should be used in the context of psychological networks (Bringmann et al., 2019). In their paper Bringmann et al. (2019), observe that centrality indices display wide confidence intervals (Bringmann et al., 2013), low stability in cross-sectional data (Epskamp, Borsboom, & Fried, 2017a), inconsistency (regarding which node is most central) across data sets (Bringmann, Pe, & Vissers, 2016; Forbes, Wright, Markon, & Krueger, 2017; however also see Borsboom et al., 2017), and are not always linked to external measure of interest (Rodebaugh et al., 2018). In addition, Bringmann et al. (2019) also discuss more conceptual difficulties with the actual meaning of the different centrality estimates in the context of psychological networks. Particularly betweenness and closeness centrality appear to be unsuitable as measures of node importance in psychological networks context. As mentioned in the introduction however, proper inference based on network models requires taking the accuracy of the estimates, including centrality measures, into account. Until now it wasn't possible to do this properly, which is why we looked into the Bayesian estimation of centrality indices in this paper. Proper uncertainty estimation for the centrality measures might make them more useful in practice, at least with regard to (consistently) ranking nodes based on centrality, and/or with regard to relating node centrality to external measures of interest or outcomes. For example, it could be that only when a node is clearly more central than another (based on it's 95% Credibility Interval and those of others), that it can be viewed as more important, but that so far identifying truly more central nodes was not possible due tot the limitations in estimating the uncertainty of node centralities. In addition, when truly more central nodes can be identified from among all other nodes, centrality might also become more usefully related to external outcomes. The problem raised by Bringmann et al. (2019) on how the indices should be interpreted in the context of psychological networks, and on how they are unsuited to capture certain processes in a network won't be solved by proper uncertainty estimation however. In addition, our Bayesian methods also don't provide proper uncertainty estimates for betweenness centrality, so this measure will remain problematic regardless.

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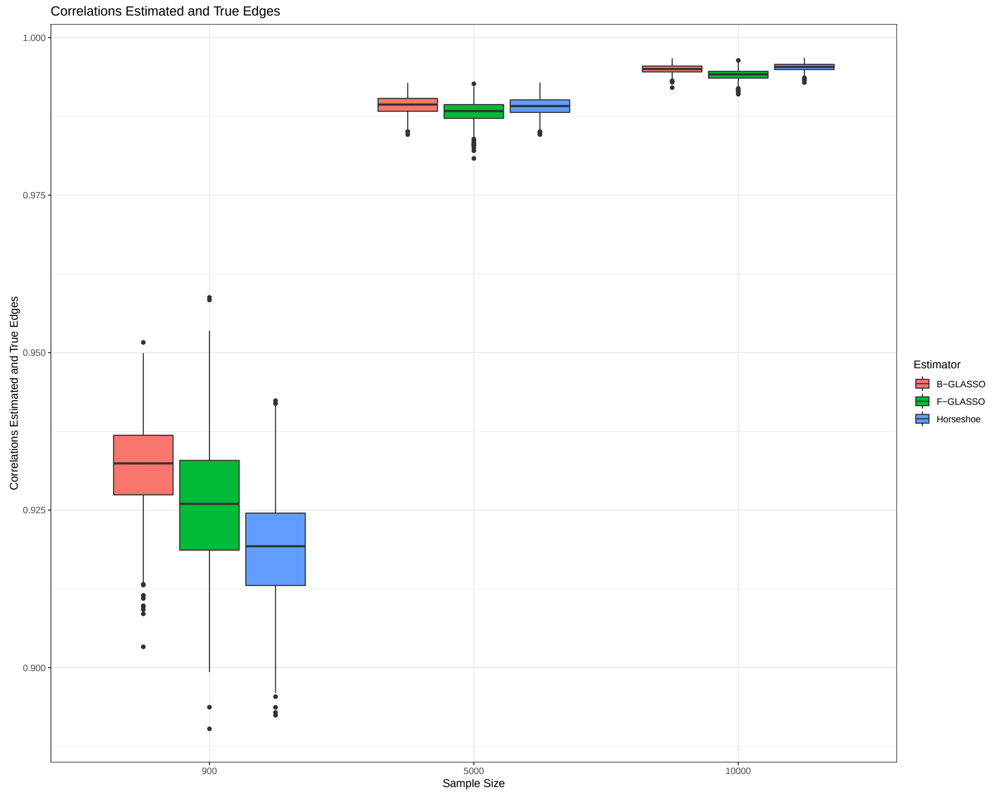
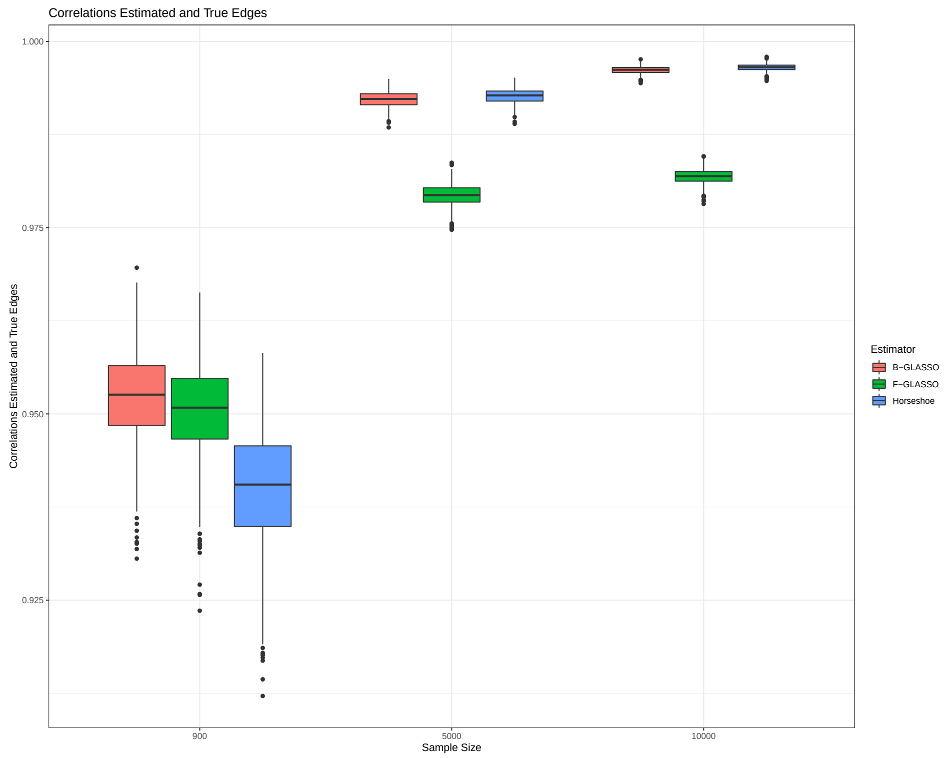
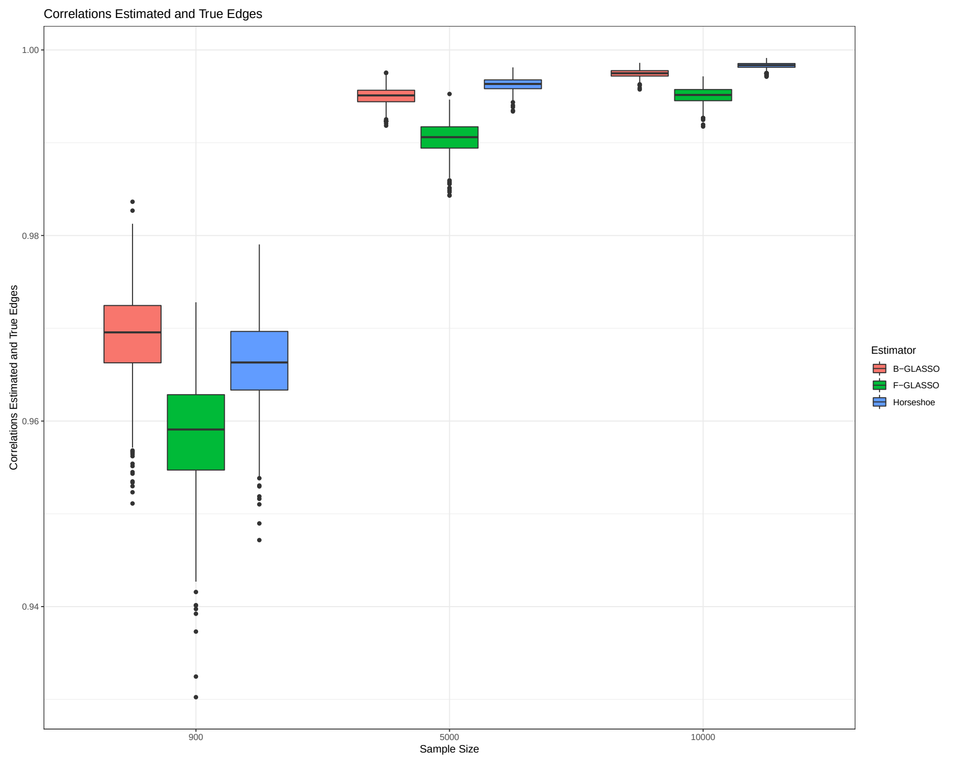
Appendix A



a Correlation Between Estimated and True Edges for Psych Network



b Correlation Between Estimated and True Edges for Random Network



c Correlation Between Estimated and True Edges for Random Network (Half Density)

Figure A.1. Correlations Between Estimated and True Edge Weights

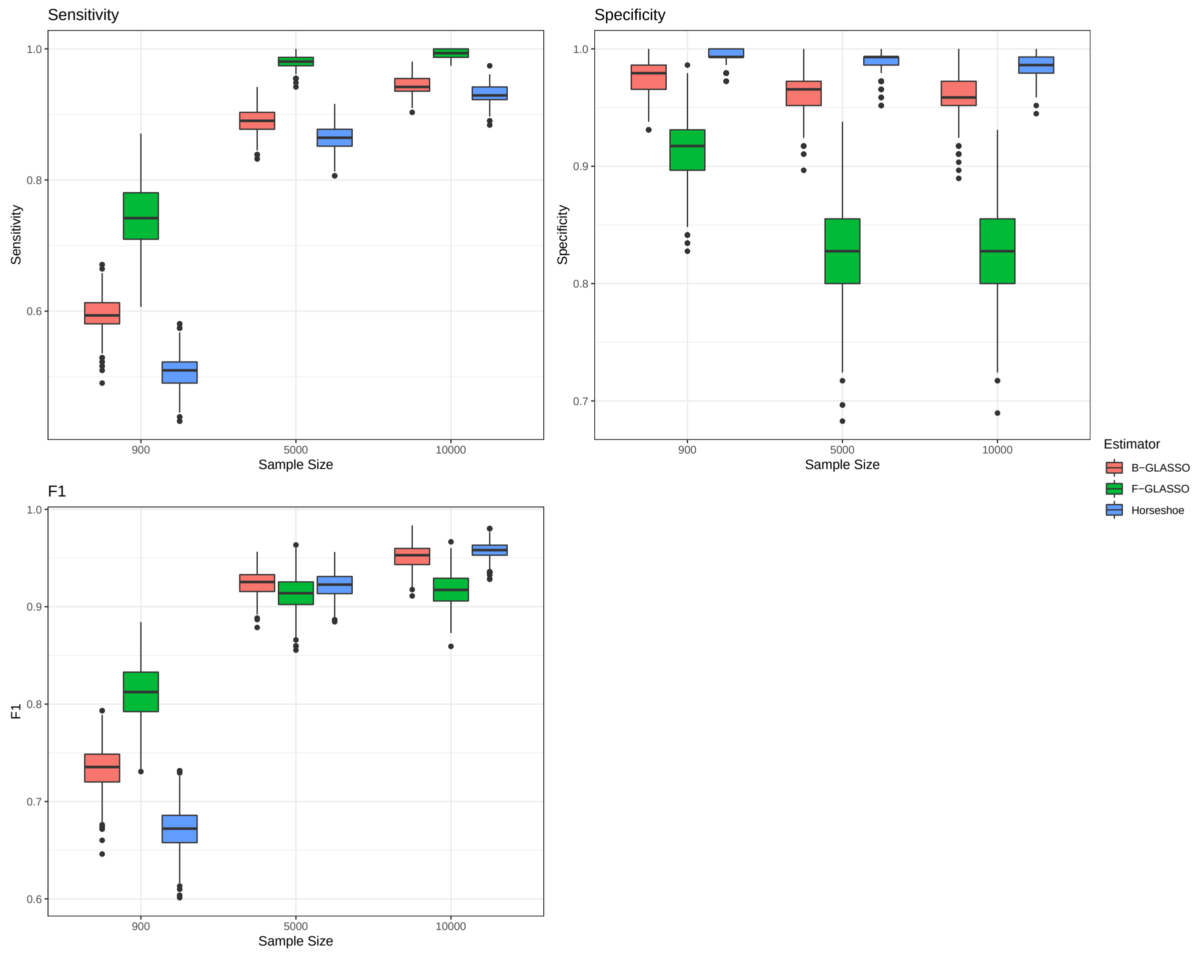


Figure A.2. Sensitivity, Specificity, and F1 for the Psych Network

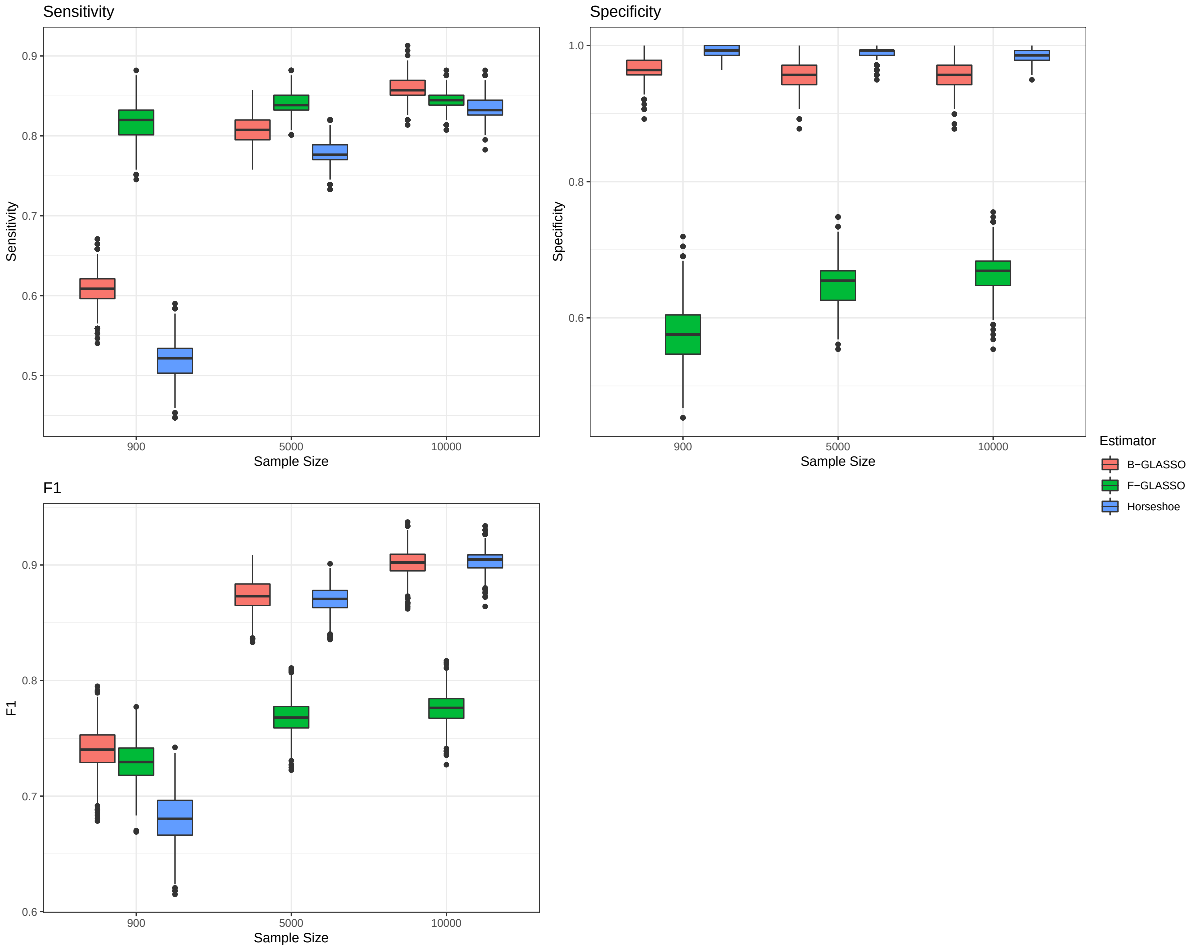


Figure A.3. Sensitivity, Specificity, and F1 for the Random Network

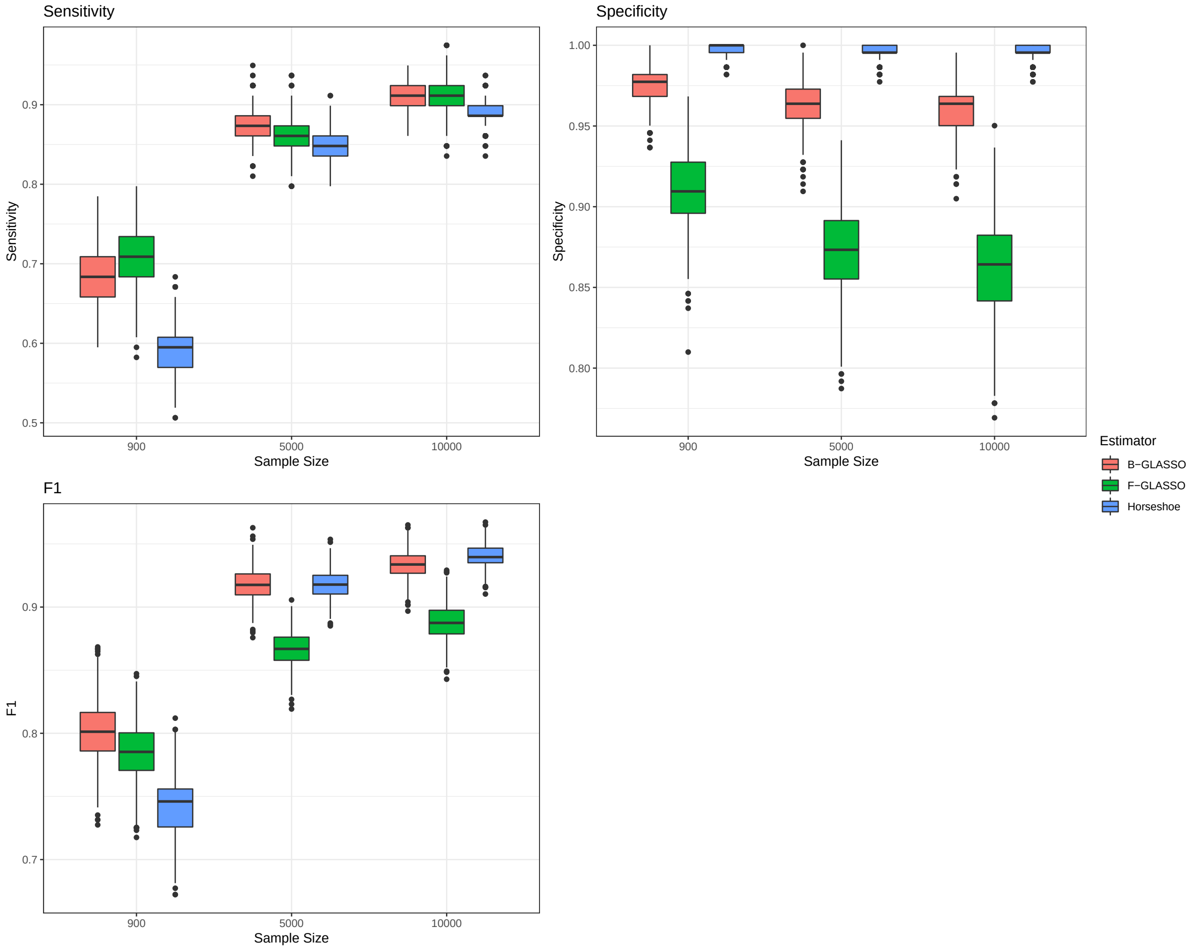


Figure A.4. Sensitivity, Specificity, and F1 for the Random Network (Half Density)

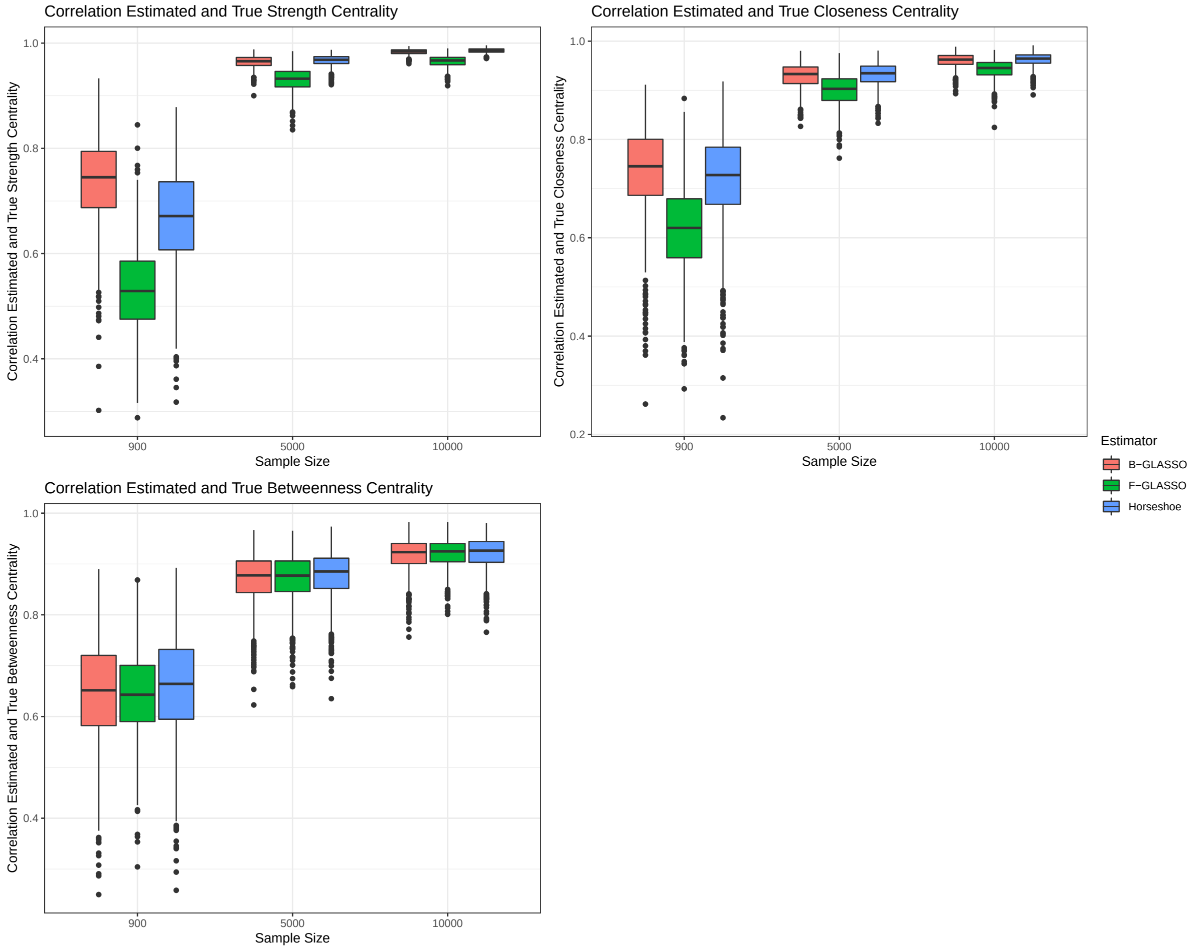


Figure A.5. Correlation between the Estimated and True Centrality Values for the Psych Network

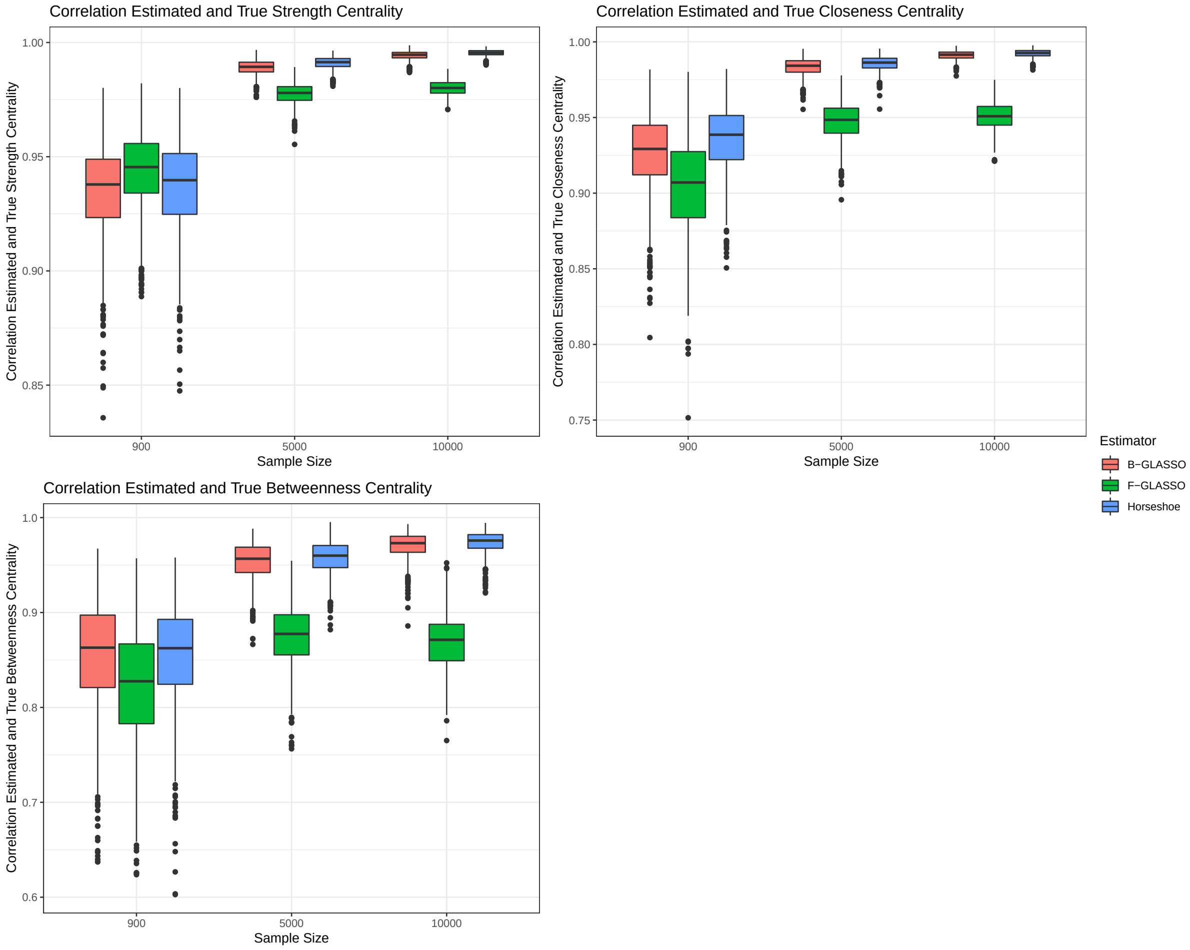


Figure A.6. Correlation between the Estimated and True Centrality Values for the Random Network

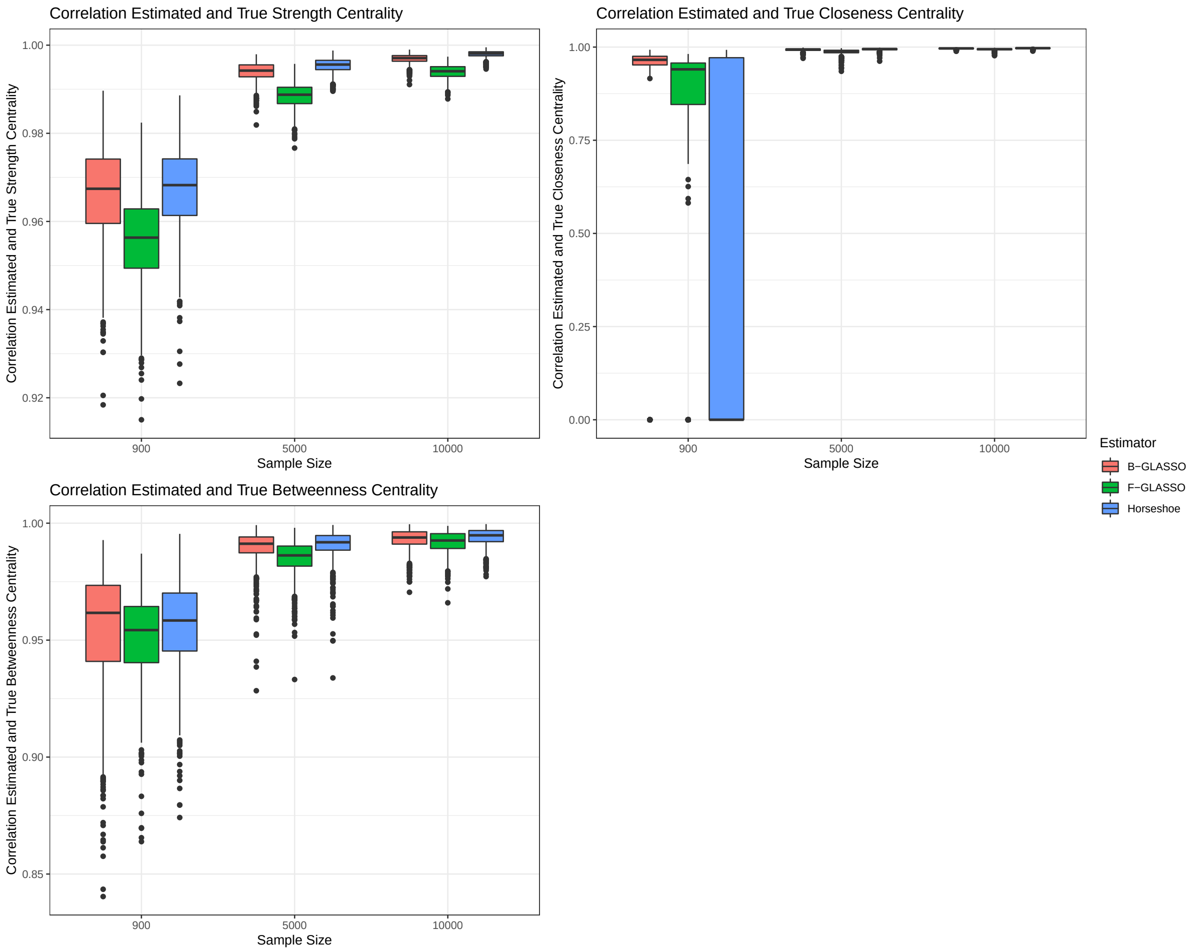


Figure A.7. Correlation between the Estimated and True Centrality Values for the Random Network (Half Density)

Table A.1

Per Node True- and Estimated Scores, MSE, and Coverage Rates for the Bayesian GLASSO (Psych Network)

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
900	Strngth	True	1.171	1.263	1.090	.905	.991	.935	1.358	.830	1.480	1.186	1.387	1.300	1.181	1.364	1.448	1.258	1.227	1.053	1.219	1.291	.943	.979	1.343	1.084	1.169
		Post-Processing Shift Estimation																									
		Estimate	.843	1.105	1.006	.731	.880	.777	1.174	.749	1.230	.989	1.053	1.123	1.067	1.211	1.153	1.092	.980	.945	1.032	1.026	.833	.781	1.146	.899	.987
		MSE	.108	.025	.007	.030	.012	.025	.034	.006	.063	.039	.112	.031	.013	.023	.087	.028	.061	.012	.035	.070	.012	.039	.039	.034	.033
		Coverage	.304	.778	.957	.687	.885	.738	.675	.932	.455	.620	.287	.673	.870	.729	.289	.706	.425	.934	.619	.456	.867	.662	.622	.669	.694
		Simple Gibbs Sampler																									
		Estimate	.405	1.602	1.525	1.283	1.427	1.338	1.649	1.292	1.700	1.506	1.565	1.622	1.547	1.679	1.601	1.625	1.556	1.501	1.537	1.515	1.359	1.330	1.630	1.396	1.493
		MSE	.055	.115	.190	.143	.191	.162	.084	.213	.049	.103	.032	.103	.134	.100	.023	.135	.108	.201	.101	.050	.173	.123	.083	.097	.106
		Coverage	.479	.051	.000	.001	.000	.000	.191	.000	.482	.059	.707	.022	.010	.061	.718	.004	.027	.000	.032	.500	.002	.026	.178	.122	.121
		Estimation Based Estimate																									
		Estimate	.843	1.105	1.006	.731	.880	.777	1.174	.749	1.230	.989	1.053	1.123	1.067	1.211	1.153	1.092	.980	.945	1.032	1.026	.833	.781	1.146	.899	.987
		MSE	.108	.025	.007	.030	.012	.025	.034	.006	.063	.039	.112	.031	.013	.023	.087	.028	.061	.012	.035	.070	.012	.039	.039	.034	.033
		Coverage	.201	.671	.936	.597	.845	.661	.578	.869	.354	.514	.174	.583	.828	.668	.260	.632	.358	.889	.575	.374	.789	.537	.521	.583	.584
	Clsnss	True	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		Post-Processing Shift Estimation																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.935	.940	.977	.962	.960	.954	.898	.964	.958	.957	.936	.952	.936	.953	.937	.901	.828	.903	.966	.887	.929	.941	.917	.920	.882
		Simple Gibbs Sampler																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.004	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.666	.485	.628	.819	.768	.416	.945	.512	.812	.890	.722	.526	.370	.657	.669	.067	.005	.075	.604	.960	.361	.606	.196	.952	.397
		Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		Estimation Based Estimate: Correlated Successive Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000
		Estimation Based Estimate: Correlated Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000
	Btwns	True	8	26	26	16	26	2	44	0	36	36	10	20	30	42	26	16	2	36	42	36	2	10	28	16	8

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Table A.1 – continued from previous page

N	Centr.		Psych Network Node 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
		Post-Processing Shift Estimation																									
		Estimate	14.218	34.438	25.674	15.774	26.118	6.522	38.498	4.756	38.830	33.074	14.840	29.194	27.192	43.040	33.700	21.982	12.072	28.556	36.476	29.850	7.478	9.460	29.530	16.244	18.436
		MSE	38.664	71.200	.106	.051	.014	2.448	3.272	22.620	8.009	8.561	23.426	84.530	7.885	1.082	59.290	35.784	101.445	55.413	3.515	37.823	3.008	.292	2.341	.060	108.910
		Coverage	.731	.779	.913	.861	.888	.688	.877	.433	.891	.904	.815	.758	.914	.899	.770	.841	.634	.896	.886	.856	.606	.899	.919	.855	.635
		Simple Gibbs Sampler																									
		Estimate	1.973	28.621	2.851	9.074	19.461	5.831	3.233	2.905	31.314	23.623	12.231	24.333	2.950	33.725	21.487	2.409	13.350	21.960	26.167	18.621	6.013	6.658	24.846	9.951	14.717
		MSE	8.841	6.867	26.512	47.967	42.754	14.676	189.536	8.440	21.962	153.179	4.978	18.778	81.901	68.481	2.364	19.438	128.827	197.112	25.698	302.033	16.103	11.170	9.945	36.592	45.115
		Coverage	.980	.989	.993	.979	.988	.955	.925	.972	.994	.951	.991	.981	.963	.976	.997	.990	.709	.943	.878	.858	.950	.998	.996	.987	.958
5,000	Strngth	True	1.171	1.263	1.090	.905	.991	.935	1.358	.830	1.480	1.186	1.387	1.300	1.181	1.364	1.448	1.258	1.227	1.053	1.219	1.291	.943	.979	1.343	1.084	1.169
		Post-Processing Shift Estimation																									
		Estimate	1.147	1.244	1.077	.893	.982	.913	1.338	.837	1.451	1.164	1.352	1.267	1.180	1.353	1.419	1.245	1.189	1.025	1.205	1.265	.939	.951	1.326	1.061	1.157
		MSE	.001	.000	.000	.000	.000	.001	.000	.000	.001	.000	.001	.001	.000	.000	.001	.000	.001	.001	.000	.001	.000	.001	.000	.001	.000
		Coverage	.944	.964	.978	.970	.974	.963	.952	.982	.914	.954	.902	.907	.963	.978	.904	.967	.892	.942	.944	.932	.980	.927	.971	.958	.960
		Simple Gibbs Sampler																									
		Estimate	1.285	1.433	1.298	1.081	1.196	1.119	1.504	1.052	1.594	1.334	1.481	1.445	1.358	1.524	1.523	1.439	1.375	1.255	1.367	1.405	1.142	1.138	1.482	1.233	1.327
		MSE	.013	.029	.043	.031	.042	.034	.021	.049	.013	.022	.009	.021	.031	.026	.006	.033	.022	.041	.022	.013	.039	.025	.019	.022	.025
		Coverage	.448	.028	.000	.016	.000	.005	.126	.000	.361	.086	.624	.043	.005	.035	.637	.006	.109	.000	.049	.394	.001	.062	.142	.088	.108
		Estimation Based Estimate																									
		Estimate	1.147	1.244	1.077	.893	.982	.913	1.338	.837	1.451	1.164	1.352	1.267	1.180	1.353	1.419	1.245	1.189	1.025	1.205	1.265	.939	.951	1.326	1.061	1.157
		MSE	.001	.000	.000	.000	.000	.001	.000	.000	.001	.000	.001	.001	.000	.000	.001	.000	.001	.001	.000	.001	.000	.001	.000	.001	.000
		Coverage	.865	.918	.956	.927	.964	.920	.915	.958	.854	.906	.812	.861	.950	.943	.895	.940	.832	.919	.935	.891	.955	.873	.932	.920	.908
	Clsnss	True	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		Post-Processing Shift Estimation																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.952	.938	.939	.947	.939	.959	.926	.957	.940	.945	.938	.956	.946	.951	.913	.951	.912	.926	.949	.934	.945	.950	.921	.946	.858
		Simple Gibbs Sampler																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.946	.892	.965	.969	.974	.942	.967	.931	.964	.969	.929	.928	.884	.919	.797	.902	.658	.789	.944	.965	.883	.937	.776	.973	.678
		Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.990	.983	.980	.985	.984	.989	.974	.978	.977	.987	.979	.986	.982	.980	.967	.987	.975	.974	.976	.981	.976	.983	.957	.984	.960
		Estimation Based Estimate: Correlated Successive Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.966	.953	.928	.939	.934	.914	.836	.868	.922	.919	.924	.954	.914	.936	.929	.931	.918	.931	.869	.901	.920	.936	.857	.868	.863
		Estimation Based Estimate: Correlated Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.990	.983	.980	.985	.984	.989	.975	.978	.977	.987	.979	.986	.982	.981	.967	.987	.975	.974	.976	.981	.976	.983	.957	.984	.960

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N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Psych Network Node																								
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.966	.953	.928	.939	.934	.917	.838	.869	.922	.920	.924	.955	.914	.938	.929	.931	.919	.932	.870	.901	.920	.936	.858	.868	.863
	Btwns	True	8	26	26	16	26	2	44	0	36	36	10	20	30	42	26	16	2	36	42	36	2	10	28	16	8
			Post-Processing Shift Estimation																								
		Estimate	1.124	29.994	25.840	15.468	25.084	3.638	4.214	2.182	36.862	33.250	1.852	23.290	27.104	42.448	29.196	2.604	5.742	3.840	39.924	33.800	5.718	7.292	28.696	14.680	11.510
		MSE	4.511	15.952	.026	.283	.839	2.683	14.334	4.761	.743	7.563	.726	1.824	8.387	.201	1.214	21.197	14.003	26.626	4.310	4.840	13.824	7.333	.484	1.742	12.320
		Coverage	.907	.920	.978	.954	.944	.745	.928	.518	.946	.949	.948	.885	.925	.936	.918	.895	.786	.900	.939	.957	.684	.922	.965	.951	.884
			Simple Gibbs Sampler																								
		Estimate	1.365	3.613	24.234	14.526	23.502	3.562	37.906	1.608	36.201	31.715	1.944	23.539	25.364	41.343	28.764	2.195	6.022	26.599	35.950	31.093	4.684	6.212	27.904	13.305	11.917
		MSE	5.592	21.283	3.120	2.174	6.241	2.439	37.143	2.585	.041	18.360	.892	12.528	21.491	.431	7.641	17.596	16.175	88.377	36.602	24.077	7.206	14.347	.009	7.263	15.340
		Coverage	.982	.970	.995	.988	.992	.982	.956	.974	.991	.977	.989	.963	.966	.994	.983	.969	.940	.900	.951	.976	.966	.967	.992	.992	.965
10,000	Strngth	True	1.171	1.263	1.090	.905	.991	.935	1.358	.830	1.480	1.186	1.387	1.300	1.181	1.364	1.448	1.258	1.227	1.053	1.219	1.291	.943	.979	1.343	1.084	1.169
			Post-Processing Shift Estimation																								
		Estimate	1.161	1.259	1.090	.908	.991	.928	1.353	.842	1.469	1.179	1.375	1.288	1.187	1.364	1.436	1.258	1.218	1.043	1.221	1.282	.947	.972	1.342	1.075	1.169
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.952	.976	.981	.970	.980	.979	.971	.973	.956	.973	.964	.944	.974	.980	.957	.982	.968	.956	.980	.970	.970	.966	.974	.963	.969
			Simple Gibbs Sampler																								
		Estimate	1.255	1.389	1.242	1.035	1.139	1.067	1.464	.994	1.564	1.293	1.455	1.406	1.311	1.481	1.502	1.392	1.335	1.196	1.328	1.376	1.087	1.095	1.449	1.195	1.285
		MSE	.007	.016	.023	.017	.022	.017	.011	.027	.007	.012	.005	.011	.017	.014	.003	.018	.012	.021	.012	.007	.021	.013	.011	.012	.013
		Coverage	.411	.026	.000	.012	.000	.004	.121	.000	.324	.073	.591	.037	.007	.029	.619	.011	.111	.001	.035	.349	.000	.058	.106	.073	.102
			Estimation Based Estimate																								
		Estimate	1.161	1.259	1.090	.908	.991	.928	1.353	.842	1.469	1.179	1.375	1.288	1.187	1.364	1.436	1.258	1.218	1.043	1.221	1.282	.947	.972	1.342	1.075	1.169
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.895	.943	.961	.945	.972	.948	.943	.946	.917	.938	.890	.905	.963	.955	.948	.956	.921	.941	.972	.929	.955	.924	.947	.926	.932
	Clsnss	True	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
			Post-Processing Shift Estimation																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.957	.951	.942	.934	.931	.955	.914	.966	.946	.955	.954	.951	.957	.958	.934	.948	.921	.938	.955	.932	.934	.943	.904	.942	.879
			Simple Gibbs Sampler																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.966	.943	.973	.953	.961	.963	.938	.946	.977	.968	.960	.952	.915	.938	.842	.940	.792	.852	.962	.955	.916	.958	.808	.966	.740
			Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.978	.973	.972	.960	.949	.975	.944	.974	.969	.973	.973	.966	.969	.974	.963	.975	.950	.969	.969	.958	.959	.978	.925	.966	.945

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Table A.1 – continued from previous page

N	Centr.		Psych Network Node 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Estimation Based Estimate: Correlated Successive Paths																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.921	.933	.902	.873	.864	.876	.752	.853	.899	.898	.882	.906	.893	.919	.938	.856	.839	.895	.850	.808	.887	.881	.787	.820	.837
			Estimation Based Estimate: Correlated Shortest Paths																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.978	.974	.972	.960	.949	.975	.944	.974	.970	.973	.973	.966	.969	.974	.963	.975	.950	.970	.969	.958	.959	.978	.925	.966	.946
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.921	.932	.903	.874	.865	.876	.754	.854	.899	.898	.882	.906	.894	.919	.939	.856	.839	.895	.850	.808	.888	.881	.788	.820	.838
	Btwns	True	8	26	26	16	26	2	44	0	36	36	10	20	30	42	26	16	2	36	42	36	2	10	28	16	8
			Post-Processing Shift Estimation																								
		Estimate	8.684	28.040	25.894	15.566	25.310	3.086	4.708	1.730	36.676	33.918	9.594	22.360	27.526	42.060	27.690	19.534	4.760	32.646	41.560	34.114	5.530	7.658	29.294	15.360	9.472
		MSE	.468	4.162	.011	.188	.476	1.179	1.837	2.993	.457	4.335	.165	5.570	6.121	.004	2.856	12.489	7.618	11.249	.194	3.557	12.461	5.485	1.674	.410	2.167
		Coverage	.956	.933	.974	.955	.935	.762	.928	.592	.940	.947	.965	.898	.937	.943	.948	.902	.781	.926	.965	.958	.736	.919	.953	.946	.928
			Simple Gibbs Sampler																								
		Estimate	9.098	28.953	24.761	15.097	24.140	3.028	38.951	1.425	36.080	32.689	9.647	22.830	26.237	41.595	28.021	19.414	4.647	29.505	38.795	32.965	4.773	6.636	28.928	14.194	9.775
		MSE	1.205	8.718	1.535	.816	3.461	1.057	25.493	2.030	.006	1.959	.125	8.010	14.163	.164	4.083	11.659	7.008	42.184	1.270	9.214	7.690	11.315	.861	3.261	3.151
		Coverage	.989	.981	.992	.993	.990	.991	.954	.980	.995	.971	.995	.970	.963	.996	.985	.969	.964	.933	.994	.984	.978	.954	.993	.986	.984

Table A.2

Per Node True- and Estimated Scores, MSE, and Coverage Rates for the Bayesian GLASSO (Random Network)

			Random Network Node																								
N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
900	Strngth	True	1.383	1.688	.599	.897	1.886	1.297	1.629	1.662	1.556	.974	1.447	1.377	1.405	1.478	.728	1.425	1.197	1.348	1.017	1.246	.682	1.131	1.722	.863	1.818
			Post-Processing Shift Estimation																								
		Estimate	1.316	1.539	.538	.757	1.712	1.129	1.478	1.427	1.404	.877	1.272	1.171	1.291	1.365	.664	1.280	1.070	1.187	.884	1.071	.630	.948	1.487	.723	1.552
		MSE	.005	.022	.004	.020	.030	.028	.023	.055	.023	.009	.030	.042	.013	.013	.004	.021	.016	.026	.017	.031	.003	.034	.055	.020	.071
		Coverage	.965	.793	.964	.845	.695	.766	.767	.464	.798	.892	.706	.668	.832	.871	.955	.823	.864	.747	.852	.711	.963	.750	.569	.830	.292
			Simple Gibbs Sampler																								
		Estimate	1.850	2.011	1.147	1.340	2.105	1.658	1.941	1.895	1.883	1.421	1.732	1.690	1.748	1.826	1.272	1.734	1.632	1.627	1.453	1.559	1.248	1.496	1.977	1.322	2.017
		MSE	.217	.104	.300	.197	.048	.130	.097	.054	.107	.200	.081	.098	.117	.121	.297	.095	.189	.078	.190	.098	.320	.134	.065	.210	.040
		Coverage	.001	.116	.000	.000	.374	.031	.091	.291	.220	.005	.201	.174	.089	.228	.000	.303	.002	.308	.018	.050	.000	.112	.444	.005	.423
			Estimation Based Estimate																								
		Estimate	1.316	1.539	.538	.757	1.712	1.129	1.478	1.427	1.404	.877	1.272	1.171	1.291	1.365	.664	1.280	1.070	1.187	.884	1.071	.630	.948	1.487	.723	1.552
		MSE	.005	.022	.004	.020	.030	.028	.023	.055	.023	.009	.030	.042	.013	.013	.004	.021	.016	.026	.017	.031	.003	.034	.055	.020	.071
		Coverage	.961	.810	.973	.836	.806	.752	.826	.506	.774	.857	.703	.661	.840	.817	.938	.786	.889	.746	.824	.763	.957	.675	.550	.797	.363
	Clsnss	True	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
			Post-Processing Shift Estimation																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.932	.894	.947	.933	.916	.925	.963	.956	.963	.951	.927	.871	.950	.940	.963	.160	.944	.952	.950	.920	.942	.935	.968	.942	.969
			Simple Gibbs Sampler																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.004	.004	.004	.003	.004	.003	.004	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.975	.988	.902	.948	.986	.969	.902	.947	.797	.850	.988	.982	.976	.963	.827	.709	.982	.917	.902	.974	.800	.933	.787	.818	.865
			Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			Estimation Based Estimate: Correlated Successive Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.999	.999	.999	.998	1.000	1.000	1.000	1.000	1.000	.999	1.000	.999	1.000	.998	.998	1.000	1.000	1.000	.999	1.000	.998	1.000	1.000	1.000	1.000
			Estimation Based Estimate: Correlated Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.999	.999	.999	.998	1.000	1.000	1.000	1.000	1.000	.999	1.000	.999	1.000	.998	.998	1.000	1.000	1.000	.999	1.000	.998	1.000	1.000	1.000	1.000
	Btwns	True	54	60	0	14	68	38	36	32	20	0	22	28	16	24	0	30	24	6	12	6	4	8	26	8	46

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Table A.2 – continued from previous page

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Random Network Node																								
			Post-Processing Shift Estimation																								
		Estimate	48.420	51.220	.868	13.946	58.154	39.378	44.332	32.792	27.224	3.622	17.724	25.992	16.930	21.876	4.112	28.096	23.552	7.862	1.870	6.442	4.848	9.714	39.002	1.974	49.662
		MSE	31.136	77.088	.753	.003	96.944	1.899	69.422	.627	52.186	13.119	18.284	4.032	.865	4.511	16.909	3.625	.201	3.467	1.277	.195	.719	2.938	169.052	8.845	13.410
		Coverage	.908	.885	.743	.913	.866	.921	.841	.950	.814	.502	.917	.940	.917	.927	.417	.914	.923	.828	.937	.901	.880	.871	.701	.792	.908
			Simple Gibbs Sampler																								
		Estimate	38.953	43.889	.195	9.868	49.074	32.084	37.856	31.212	25.837	3.054	14.426	21.503	13.969	16.722	2.921	22.994	18.962	5.897	8.266	4.849	3.755	7.467	36.661	7.538	45.055
		MSE	226.425	259.577	.038	17.074	358.198	35.003	3.445	.621	34.071	9.330	57.363	42.217	4.127	52.972	8.534	49.089	25.380	.011	13.944	1.326	.060	.284	113.667	.213	.894
		Coverage	.849	.855	1.000	.989	.826	.978	.996	1.000	.968	.938	.969	.980	.998	.962	.936	.967	.992	.998	.984	1.000	.997	.990	.898	.994	1.000
5,000	Strngth	True	1.383	1.688	.599	.897	1.886	1.297	1.629	1.662	1.556	.974	1.447	1.377	1.405	1.478	.728	1.425	1.197	1.348	1.017	1.246	.682	1.131	1.722	.863	1.818
			Post-Processing Shift Estimation																								
		Estimate	1.376	1.664	.598	.869	1.851	1.246	1.614	1.632	1.540	.955	1.409	1.351	1.393	1.474	.716	1.401	1.157	1.336	1.011	1.200	.676	1.081	1.701	.846	1.760
		MSE	.000	.001	.000	.001	.001	.003	.000	.001	.000	.000	.001	.001	.000	.000	.000	.001	.002	.000	.000	.002	.000	.002	.000	.000	.003
		Coverage	.979	.952	.981	.950	.934	.883	.955	.947	.971	.961	.917	.945	.965	.972	.961	.956	.938	.978	.958	.859	.970	.872	.938	.955	.819
			Simple Gibbs Sampler																								
		Estimate	1.594	1.849	.843	1.088	2.009	1.461	1.777	1.798	1.714	1.166	1.576	1.535	1.564	1.646	.961	1.562	1.372	1.484	1.209	1.384	.927	1.271	1.827	1.058	1.932
		MSE	.044	.026	.059	.036	.015	.027	.022	.019	.025	.037	.017	.025	.025	.028	.055	.019	.031	.019	.037	.019	.060	.020	.011	.038	.013
		Coverage	.003	.098	.000	.016	.219	.082	.099	.208	.172	.018	.259	.139	.078	.159	.001	.350	.015	.255	.029	.172	.000	.306	.549	.019	.282
			Estimation Based Estimate																								
		Estimate	1.376	1.664	.598	.869	1.851	1.246	1.614	1.632	1.540	.955	1.409	1.351	1.393	1.474	.716	1.401	1.157	1.336	1.011	1.200	.676	1.081	1.701	.846	1.760
		MSE	.000	.001	.000	.001	.001	.003	.000	.001	.000	.000	.001	.001	.000	.000	.000	.001	.002	.000	.000	.002	.000	.002	.000	.000	.003
		Coverage	.973	.954	.974	.943	.962	.853	.965	.955	.959	.936	.907	.940	.973	.950	.943	.938	.948	.971	.948	.879	.971	.803	.930	.937	.858
	Clsnss	True	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
			Post-Processing Shift Estimation																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.960	.951	.956	.959	.967	.930	.958	.968	.934	.953	.939	.917	.944	.946	.955	.920	.953	.950	.952	.934	.951	.953	.937	.950	.969
			Simple Gibbs Sampler																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.004	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.968	.973	.960	.969	.966	.971	.973	.958	.920	.955	.962	.945	.966	.968	.968	.858	.964	.963	.966	.958	.951	.960	.927	.922	.915
			Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.974	.963	.978	.989	.980	.960	.978	.979	.964	.965	.956	.938	.972	.960	.975	.964	.968	.976	.973	.956	.969	.981	.970	.972	.985
			Estimation Based Estimate: Correlated Successive Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.942	.914	.794	.871	.948	.890	.960	.942	.931	.864	.851	.867	.935	.847	.837	.835	.911	.911	.807	.855	.818	.867	.939	.813	.961
			Estimation Based Estimate: Correlated Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.974	.963	.978	.989	.981	.961	.978	.979	.964	.965	.956	.938	.972	.960	.975	.964	.968	.976	.973	.956	.969	.981	.970	.972	.985

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Table A.2 – continued from previous page

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Random Network Node																								
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.942	.915	.794	.871	.948	.890	.960	.942	.931	.864	.851	.868	.935	.848	.837	.835	.912	.911	.807	.855	.816	.867	.938	.814	.961
	Btwns	True	54	60	0	14	68	38	36	32	20	0	22	28	16	24	0	30	24	6	12	6	4	8	26	8	46
			Post-Processing Shift Estimation																								
		Estimate	51.920	54.930	.744	14.976	67.438	37.430	4.074	31.636	22.630	1.106	15.888	29.048	14.620	23.152	1.686	27.574	24.506	6.718	11.100	6.548	3.230	7.156	32.614	9.576	48.962
		MSE	4.326	25.705	.554	.953	.316	.325	16.597	.132	6.917	1.223	37.357	1.098	1.904	.719	2.843	5.885	.256	.516	.810	.300	.593	.712	43.745	2.484	8.773
		Coverage	.923	.907	.772	.959	.954	.964	.912	.980	.891	.710	.856	.969	.951	.959	.666	.944	.934	.943	.933	.961	.966	.945	.755	.887	.933
			Simple Gibbs Sampler																								
		Estimate	49.258	52.520	.383	14.267	65.208	37.032	4.578	32.649	23.265	.975	15.370	27.617	14.249	21.058	1.575	26.561	23.291	6.014	1.389	5.935	3.120	6.694	33.552	9.110	5.556
		MSE	22.490	55.950	.146	.071	7.798	.937	2.962	.422	1.658	.950	43.953	.147	3.065	8.658	2.482	11.829	.502	.000	2.595	.004	.774	1.706	57.035	1.231	2.761
		Coverage	.955	.946	1.000	.995	.981	.986	.969	.995	.972	.996	.903	.998	.981	.977	.971	.977	.989	.994	.989	.998	.996	.994	.812	.991	.969
10,000	Strngth	True	1.383	1.688	.599	.897	1.886	1.297	1.629	1.662	1.556	.974	1.447	1.377	1.405	1.478	.728	1.425	1.197	1.348	1.017	1.246	.682	1.131	1.722	.863	1.818
			Post-Processing Shift Estimation																								
		Estimate	1.384	1.680	.604	.884	1.870	1.265	1.631	1.653	1.549	.966	1.428	1.368	1.402	1.485	.724	1.412	1.173	1.340	1.021	1.223	.683	1.104	1.714	.854	1.782
		MSE	.000	.000	.000	.000	.000	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.000	.000	.001	.000	.001	.000	.000	.001
		Coverage	.968	.968	.967	.963	.955	.912	.969	.965	.954	.963	.938	.964	.971	.967	.971	.951	.919	.970	.966	.911	.970	.913	.955	.963	.853
			Simple Gibbs Sampler																								
		Estimate	1.533	1.807	.773	1.031	1.978	1.413	1.739	1.768	1.670	1.110	1.541	1.492	1.518	1.603	.891	1.522	1.316	1.445	1.152	1.343	.855	1.225	1.796	.997	1.900
		MSE	.022	.014	.030	.018	.008	.013	.012	.011	.013	.018	.009	.013	.013	.016	.027	.009	.014	.009	.018	.009	.030	.009	.006	.018	.007
		Coverage	.006	.089	.000	.024	.209	.099	.079	.147	.181	.022	.265	.122	.091	.128	.000	.368	.052	.275	.041	.191	.000	.392	.561	.038	.315
			Estimation Based Estimate																								
		Estimate	1.384	1.680	.604	.884	1.870	1.265	1.631	1.653	1.549	.966	1.428	1.368	1.402	1.485	.724	1.412	1.173	1.340	1.021	1.223	.683	1.104	1.714	.854	1.782
		MSE	.000	.000	.000	.000	.000	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.000	.000	.001	.000	.001	.000	.000	.001
		Coverage	.966	.965	.970	.959	.973	.876	.979	.971	.932	.932	.920	.953	.978	.947	.952	.919	.937	.962	.953	.920	.965	.854	.949	.943	.890
	Clsnss	True	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
			Post-Processing Shift Estimation																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.954	.947	.959	.950	.940	.937	.958	.946	.955	.963	.946	.933	.947	.956	.953	.921	.945	.946	.972	.954	.955	.958	.950	.947	.957
			Simple Gibbs Sampler																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.004	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.970	.972	.963	.959	.936	.976	.977	.967	.945	.958	.950	.954	.961	.972	.968	.872	.956	.953	.976	.966	.964	.960	.953	.931	.939
			Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.964	.949	.968	.960	.934	.952	.971	.959	.970	.968	.952	.942	.958	.958	.964	.941	.946	.958	.974	.960	.965	.969	.958	.956	.970

Continued on next page

Table A.2 – continued from previous page

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Random Network Node																								
			Estimation Based Estimate: Correlated Successive Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.927	.903	.714	.830	.896	.875	.944	.910	.949	.839	.825	.854	.914	.812	.783	.810	.856	.864	.770	.846	.776	.831	.930	.781	.910
			Estimation Based Estimate: Correlated Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.964	.949	.968	.960	.934	.952	.971	.959	.970	.968	.952	.942	.958	.958	.964	.941	.947	.957	.974	.960	.965	.969	.958	.956	.969
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.927	.904	.714	.830	.896	.876	.944	.910	.949	.839	.825	.855	.914	.812	.783	.811	.856	.862	.770	.848	.776	.829	.931	.781	.911
	Btwns	True	54	6	0	14	68	38	36	32	2	0	22	28	16	24		3	24	6	12	6	4	8	26	8	46
			Post-Processing Shift Estimation																								
		Estimate	53.434	56.834	.716	14.928	68.138	37.122	38.678	3.600	21.072	.834	16.248	29.500	14.640	24.520	1.292	28.010	24.490	6.754	11.332	6.584	3.076	7.150	3.762	8.936	46.874
		MSE	.320	1.024	.513	.861	.019	.771	7.172	1.960	1.149	.696	33.086	2.250	1.850	.270	1.669	3.960	.240	.569	.446	.341	.854	.723	22.677	.876	.764
		Coverage	.941	.928	.881	.923	.946	.968	.910	.988	.939	.784	.814	.939	.950	.948	.788	.951	.938	.927	.928	.953	.986	.967	.772	.904	.962
			Simple Gibbs Sampler																								
		Estimate	51.393	54.721	.517	14.482	67.445	36.822	39.301	31.441	21.852	.683	15.615	28.633	14.172	22.811	1.205	27.234	23.870	6.304	1.913	6.246	3.047	6.722	31.733	8.680	48.417
		MSE	6.794	27.864	.268	.232	.308	1.388	1.900	.313	3.429	.467	4.766	.401	3.341	1.414	1.452	7.651	.017	.092	1.181	.060	.907	1.634	32.868	.462	5.843
		Coverage	.981	.958	.998	.994	.985	.995	.977	.998	.993	.995	.892	.996	.980	.998	.981	.974	.998	.998	.991	.999	.993	.996	.846	.992	.980

Table A.3

Per Node True- and Estimated Scores, MSE, and Coverage Rates for the Bayesian GLASSO (Random Network Half the Density of Psych Network)

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			(Random Network Half the Density of Psych Network) Node																								
900	Strngth	True	.847	.901	.156	1.442	.955	.802	.945	1.164	1.308	1.765	.579	1.042	1.204	.441	.526	.590	.427	.891	.655	.491	.395	1.240	1.085	1.108	1.348
			Post-Processing Shift Estimation																								
		Estimate	.774	.856	.136	1.289	.859	.704	.861	1.047	1.169	1.669	.505	.914	1.053	.397	.433	.537	.376	.836	.617	.490	.360	1.110	1.006	.914	1.165
		MSE	.005	.002	.000	.023	.009	.010	.007	.014	.019	.009	.005	.016	.023	.002	.009	.003	.003	.003	.001	.000	.001	.017	.006	.037	.034
		Coverage	.953	.996	.994	.789	.959	.948	.961	.949	.866	.959	.953	.890	.788	.981	.926	.973	.998	.988	.991	.997	.993	.877	.961	.679	.697
			Simple Gibbs Sampler																								
		Gibbs	1.324	1.432	.809	1.817	1.428	1.288	1.414	1.593	1.688	2.094	1.105	1.476	1.588	1.017	1.040	1.126	1.028	1.390	1.207	1.098	1.007	1.627	1.534	1.456	1.659
		Estimate	.227	.282	.425	.140	.223	.236	.220	.185	.144	.108	.276	.188	.147	.332	.264	.287	.361	.249	.304	.368	.374	.150	.202	.121	.097
		Coverage	.001	.000	.000	.004	.000	.000	.000	.000	.004	.017	.000	.000	.005	.000	.000	.000	.000	.000	.000	.000	.000	.020	.001	.035	.110
			Estimation Based Estimate																								
		Estimate	.774	.856	.136	1.289	.859	.704	.861	1.047	1.169	1.669	.505	.914	1.053	.397	.433	.537	.376	.836	.617	.490	.360	1.110	1.006	.914	1.165
		MSE	.005	.002	.000	.023	.009	.010	.007	.014	.019	.009	.005	.016	.023	.002	.009	.003	.003	.003	.001	.000	.001	.017	.006	.037	.034
		Coverage	.984	1.000	.997	.919	.995	.979	.990	.985	.974	.991	.979	.962	.910	.994	.986	.990	1.000	1.000	1.000	.998	.999	.939	.989	.805	.820
	Clsnss	True	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
			Post-Processing Shift Estimation																								
		Estimate	.002	.003	.002	.003	.003	.003	.003	.003	.003	.004	.002	.003	.003	.002	.002	.002	.002	.003	.003	.002	.002	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.819	.506	.854	.573	.510	.722	.768	.678	.662	.373	.773	.637	.760	.750	.775	.812	.793	.689	.741	.713	.767	.649	.559	.656	.637
			Simple Gibbs Sampler																								
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.935	.876	.676	.908	.734	.984	.991	.981	.944	.580	.973	.952	.991	.977	.977	.976	.984	.954	.989	.985	.990	.969	.936	.962	.918
			Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																								
		Estimate	.002	.003	.002	.003	.003	.003	.003	.003	.003	.004	.002	.003	.003	.002	.002	.002	.002	.003	.003	.002	.002	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.989	.937	NA	.910	.938	.961	.964	.921	.932	.919	.990	.940	.962	.979	.980	.981	.930	.956	.931	.955	.986	.983	.969	.948	.954
			Estimation Based Estimate: Correlated Successive Paths																								
		Estimate	.002	.003	.002	.003	.003	.003	.003	.003	.003	.004	.002	.003	.003	.002	.002	.002	.002	.003	.003	.002	.002	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.986	.933	NA	.897	.935	.955	.960	.917	.927	.914	.980	.935	.960	.974	.974	.978	.921	.952	.918	.941	.985	.980	.963	.942	.951
			Estimation Based Estimate: Correlated Shortest Paths																								
		Estimate	.002	.003	.002	.003	.003	.003	.003	.003	.003	.004	.002	.003	.003	.002	.002	.002	.002	.003	.003	.002	.002	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.989	.937	NA	.910	.938	.961	.964	.921	.932	.919	.990	.940	.962	.979	.980	.981	.931	.956	.931	.955	.986	.983	.969	.948	.954
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.002	.003	.002	.003	.003	.003	.003	.003	.003	.004	.002	.003	.003	.002	.002	.002	.002	.003	.003	.002	.002	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.986	.933	NA	.897	.935	.955	.960	.917	.927	.914	.980	.935	.960	.974	.974	.978	.921	.952	.919	.941	.985	.980	.963	.942	.951
	Btwns	True	10	28	0	118	18	20	44	58	50	170	8	64	56	2	0	2	0	24	6	12	0	82	28	32	72

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Table A.3 – continued from previous page

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Random Network Half the Density of Psych Network, Node																								
		Post-Processing Shift Estimation																									
		Estimate	16.940	3.356	.002	115.210	17.194	19.358	42.210	53.608	39.830	165.928	9.344	47.836	61.382	2.192	1.102	5.578	.524	31.954	14.796	9.540	2.104	8.886	26.376	33.526	7.374
		MSE	48.164	5.551	.000	7.784	.650	.412	3.204	19.290	103.429	16.581	1.806	261.275	28.966	.037	1.214	12.802	.275	63.266	77.370	6.052	4.427	1.241	2.637	2.329	2.644
		Coverage	.777	.884	.999	.941	.920	.881	.875	.974	.844	.939	.855	.667	.916	.885	.759	.616	.928	.806	.603	.926	.641	.952	.938	.934	.964
		Simple Gibbs Sampler																									
		Estimate	16.634	26.434	.000	96.197	1.724	13.010	31.805	46.882	29.395	13.431	6.187	33.322	5.923	1.465	.272	3.395	.190	24.788	12.170	6.069	1.169	66.512	23.371	24.959	58.251
		MSE	44.013	2.451	.000	475.362	52.947	48.859	148.726	123.612	424.570	1565.745	3.287	941.156	25.780	.286	.074	1.947	.036	.621	38.074	35.179	1.366	239.866	21.427	49.581	189.037
		Coverage	.930	1.000	1.000	.885	.998	.978	.957	.959	.711	.668	.996	.517	.997	1.000	.999	.988	1.000	.995	.946	.989	.995	.881	.992	.989	.941
5,000	Strngth	True	.847	.901	.156	1.442	.955	.802	.945	1.164	1.308	1.765	.579	1.042	1.204	.441	.526	.590	.427	.891	.655	.491	.395	1.240	1.085	1.108	1.348
		Post-Processing Shift Estimation																									
		Estimate	.853	.915	.170	1.428	.944	.785	.950	1.125	1.290	1.749	.585	1.021	1.199	.457	.525	.602	.433	.890	.667	.509	.411	1.225	1.085	1.087	1.319
		MSE	.000	.000	.000	.000	.000	.000	.000	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001
		Coverage	.988	.990	.979	.982	.981	.983	.988	.942	.979	.988	.987	.980	.978	.974	.979	.984	.986	.985	.981	.976	.979	.983	.985	.966	.945
		Simple Gibbs Sampler																									
		Estimate	1.089	1.173	.462	1.652	1.192	1.039	1.185	1.375	1.511	1.951	.836	1.258	1.409	.722	.770	.853	.717	1.139	.929	.785	.688	1.448	1.321	1.294	1.509
		MSE	.058	.074	.094	.044	.056	.056	.057	.045	.041	.034	.066	.047	.042	.079	.060	.069	.084	.062	.075	.086	.086	.043	.056	.035	.026
		Coverage	.000	.000	.000	.000	.000	.000	.000	.001	.004	.000	.000	.000	.002	.000	.000	.000	.000	.000	.000	.000	.000	.007	.000	.023	.085
		Estimation Based Estimate																									
		Estimate	.853	.915	.170	1.428	.944	.785	.950	1.125	1.290	1.749	.585	1.021	1.199	.457	.525	.602	.433	.890	.667	.509	.411	1.225	1.085	1.087	1.319
		MSE	.000	.000	.000	.000	.000	.000	.000	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001
		Coverage	.997	.998	.995	.996	.993	.997	.994	.976	.994	1.000	.998	.990	.996	.991	.991	.996	.995	.996	.996	.997	.995	.994	.995	.988	.972
	Clsnss	True	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		Post-Processing Shift Estimation																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.944	.853	.928	.818	.904	.888	.944	.862	.916	.811	.915	.881	.919	.925	.914	.942	.852	.920	.860	.901	.889	.866	.748	.911	.910
		Simple Gibbs Sampler																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.975	.856	.949	.835	.914	.929	.965	.915	.947	.789	.950	.932	.957	.945	.942	.966	.898	.943	.877	.921	.920	.925	.766	.942	.931
		Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.963	.919	.976	.890	.954	.931	.974	.882	.954	.913	.955	.942	.956	.960	.976	.971	.864	.950	.886	.950	.963	.949	.799	.943	.953
		Estimation Based Estimate: Correlated Successive Paths																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.837	.778	.481	.696	.858	.751	.919	.710	.834	.831	.781	.865	.873	.750	.799	.834	.593	.781	.656	.509	.891	.844	.629	.875	.847
		Estimation Based Estimate: Correlated Shortest Paths																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

Continued on next page

Table A.3 – continued from previous page

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Random Network Half the Density of Psych Network – Node																								
		Coverage	.963	.919	.976	.890	.954	.931	.974	.882	.954	.913	.955	.941	.956	.960	.976	.971	.864	.950	.886	.950	.963	.949	.799	.943	.953
		Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.837	.776	.481	.694	.858	.752	.919	.711	.836	.831	.781	.864	.872	.753	.799	.834	.593	.781	.659	.509	.892	.844	.628	.876	.847
	Btwns	True	10	28	0	118	18	20	44	58	50	170	8	64	56	2	0	2	0	24	6	12	0	82	28	32	72
		Post-Processing Shift Estimation																									
		Estimate	12.204	3.598	.000	115.974	19.292	21.150	44.154	55.312	47.250	175.138	8.250	58.650	56.332	1.894	.032	2.656	.012	27.336	7.026	1.556	.482	81.600	26.976	35.092	72.800
		MSE	4.858	6.750	.000	4.105	1.669	1.323	.024	7.225	7.563	26.399	.063	28.623	.110	.011	.001	.430	.000	11.129	1.053	2.085	.232	.160	1.049	9.560	.640
		Coverage	.906	.952	1.000	.978	.946	.966	.957	.982	.947	.947	.960	.930	.964	.994	.992	.884	.997	.899	.909	.955	.936	.943	.970	.949	.962
		Simple Gibbs Sampler																									
		Estimate	12.374	29.997	.000	115.270	18.845	2.963	42.888	55.342	44.565	171.417	8.099	57.493	57.218	1.857	.026	2.281	.002	27.101	7.021	9.762	.468	81.413	26.633	34.947	71.864
		MSE	5.635	3.990	.000	7.451	.713	.927	1.236	7.062	29.536	2.007	.010	42.335	1.483	.020	.001	.079	.000	9.619	1.042	5.008	.219	.345	1.870	8.685	.019
		Coverage	.968	.992	1.000	.996	.992	.981	.993	.998	.959	.987	.996	.963	.992	1.000	1.000	1.000	1.000	.976	.999	.978	.996	.989	.993	.986	.989
10,000	Strngth	True	.847	.901	.156	1.442	.955	.802	.945	1.164	1.308	1.765	.579	1.042	1.204	.441	.526	.590	.427	.891	.655	.491	.395	1.240	1.085	1.108	1.348
		Post-Processing Shift Estimation																									
		Estimate	.856	.913	.168	1.443	.955	.796	.953	1.147	1.305	1.757	.587	1.033	1.207	.455	.530	.601	.439	.893	.668	.508	.405	1.234	1.090	1.104	1.341
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.977	.980	.982	.989	.983	.981	.982	.967	.978	.988	.981	.984	.979	.973	.977	.982	.976	.985	.977	.973	.985	.982	.978	.983	.969
		Simple Gibbs Sampler																									
		Estimate	1.027	1.103	.378	1.602	1.132	.974	1.123	1.319	1.460	1.905	.767	1.202	1.358	.644	.702	.782	.641	1.072	.857	.707	.605	1.393	1.261	1.249	1.468
		MSE	.032	.041	.049	.026	.031	.030	.032	.024	.023	.019	.035	.026	.024	.041	.031	.037	.045	.033	.041	.046	.044	.024	.031	.020	.014
		Coverage	.000	.000	.000	.000	.000	.000	.000	.001	.001	.000	.000	.000	.003	.000	.000	.000	.000	.000	.000	.000	.000	.005	.000	.010	.078
		Estimation Based Estimate																									
		Estimate	.856	.913	.168	1.443	.955	.796	.953	1.147	1.305	1.757	.587	1.033	1.207	.455	.530	.601	.439	.893	.668	.508	.405	1.234	1.090	1.104	1.341
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.992	.992	.998	.994	.997	.997	.993	.992	.993	.999	.986	.995	.992	.993	.992	.996	.995	.995	.992	.989	.998	.990	.990	.990	.981
	Clsnss	True	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		Post-Processing Shift Estimation																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.948	.907	.951	.883	.945	.919	.957	.881	.945	.894	.925	.918	.942	.920	.940	.943	.885	.942	.907	.927	.922	.912	.806	.923	.947
		Simple Gibbs Sampler																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.958	.903	.956	.876	.951	.937	.958	.909	.960	.884	.948	.940	.962	.936	.950	.960	.906	.961	.904	.934	.938	.935	.816	.968	.952
		Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

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Table A.3 – continued from previous page

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Random Network Half the Density of Psych Network — Node																								
		Coverage	.949	.923	.972	.902	.950	.929	.955	.876	.946	.913	.949	.930	.960	.943	.957	.955	.879	.954	.915	.947	.949	.936	.794	.917	.954
			Estimation Based Estimate: Correlated Successive Paths																								
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.778	.781	.395	.749	.831	.731	.834	.726	.810	.841	.709	.833	.862	.727	.730	.787	.632	.802	.694	.481	.873	.820	.612	.847	.841
			Estimation Based Estimate: Correlated Shortest Paths																								
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.948	.923	.972	.902	.950	.929	.956	.876	.946	.913	.949	.930	.960	.943	.957	.955	.879	.954	.916	.947	.949	.936	.794	.917	.954
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.778	.781	.396	.749	.831	.731	.835	.726	.810	.841	.709	.832	.862	.728	.730	.787	.632	.802	.695	.480	.872	.820	.612	.848	.841
	Btwns	True	10	28	0	118	18	20	44	58	50	170	8	64	56	2	0	2	0	24	6	12	0	82	28	32	72
			Post-Processing Shift Estimation																								
		Estimate	11.692	29.836	.000	117.010	18.898	21.226	44.154	56.116	48.434	175.274	8.232	58.778	55.542	1.948	.004	2.322	.000	26.338	6.230	1.756	.252	81.120	27.196	34.882	72.526
		MSE	2.863	3.371	.000	.980	.806	1.503	.024	3.549	2.452	27.815	.054	27.269	.210	.003	.000	.104	.000	5.466	.053	1.548	.064	.774	.646	8.306	.277
		Coverage	.878	.971	1.000	.973	.925	.926	.957	.992	.937	.929	.953	.909	.974	.994	1.000	.946	1.000	.921	.991	.931	.990	.946	.969	.900	.970
			Simple Gibbs Sampler																								
		Estimate	11.732	29.786	.000	116.916	18.903	21.551	43.729	56.095	47.781	174.712	8.205	59.166	56.273	1.944	.004	2.153	.000	26.072	6.231	1.527	.256	81.375	27.173	35.158	72.212
		MSE	3.001	3.191	.000	1.174	.816	2.405	.074	3.628	4.924	22.203	.042	23.364	.075	.003	.000	.023	.000	4.293	.053	2.169	.066	.391	.684	9.974	.045
		Coverage	.957	.993	1.000	.993	.993	.971	.996	1.000	.981	.976	.995	.975	.995	1.000	1.000	1.000	1.000	.984	1.000	.982	1.000	.994	.997	.971	.992

Table A.4

Per Node True- and Estimated Scores, MSE, and Coverage Rates for the Bayesian Horseshoe

N	Centr.		Psych Network Node 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
900	Strngth	True	1.171	1.263	1.090	.905	.991	.935	1.358	.830	1.480	1.186	1.387	1.300	1.181	1.364	1.448	1.258	1.227	1.053	1.219	1.291	.943	.979	1.343	1.084	1.169
		Post-Processing Shift Estimation																									
		Estimate	.672	1.042	.966	.649	.836	.707	1.085	.670	1.121	.912	.881	1.086	1.019	1.149	1.021	1.036	.905	.915	.965	.905	.751	.692	1.063	.779	.886
		MSE	.249	.049	.015	.065	.024	.052	.074	.025	.129	.075	.256	.046	.026	.046	.183	.049	.104	.019	.065	.149	.037	.082	.078	.093	.080
		Coverage	.049	.490	.844	.346	.711	.431	.343	.744	.136	.307	.036	.439	.673	.465	.067	.429	.134	.810	.313	.155	.591	.288	.291	.265	.362
		Simple Gibbs Sampler																									
		Estimate	1.166	1.407	1.324	1.076	1.229	1.126	1.460	1.069	1.516	1.324	1.344	1.466	1.369	1.505	1.448	1.432	1.347	1.299	1.364	1.321	1.146	1.120	1.448	1.182	1.288
		MSE	.000	.021	.055	.029	.057	.036	.010	.057	.001	.019	.002	.027	.035	.020	.000	.030	.014	.060	.021	.001	.041	.020	.011	.010	.014
		Coverage	.973	.714	.131	.561	.066	.478	.862	.203	.963	.738	.973	.518	.454	.703	.977	.552	.823	.080	.641	.958	.417	.769	.869	.884	.839
		Estimation Based Estimate																									
		Estimate	.672	1.042	.966	.649	.836	.707	1.085	.670	1.121	.912	.881	1.086	1.019	1.149	1.021	1.036	.905	.915	.965	.905	.751	.692	1.063	.779	.886
		MSE	.249	.049	.015	.065	.024	.052	.074	.025	.129	.075	.256	.046	.026	.046	.183	.049	.104	.019	.065	.149	.037	.082	.078	.093	.080
		Coverage	.022	.393	.806	.254	.662	.360	.300	.635	.100	.238	.014	.399	.668	.415	.061	.380	.106	.732	.321	.111	.504	.210	.232	.217	.265
	Clsnss	True	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		Post-Processing Shift Estimation																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.886	.939	.965	.951	.949	.949	.837	.960	.928	.943	.906	.926	.927	.935	.916	.926	.874	.899	.954	.838	.955	.944	.931	.871	.920
		Simple Gibbs Sampler																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.917	.789	.844	.928	.885	.782	.975	.843	.947	.940	.897	.622	.581	.762	.712	.450	.161	.342	.736	.978	.686	.839	.528	.976	.673
		Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	.999
		Estimation Based Estimate: Correlated Successive Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	1.000	1.000	1.000	1.000	.999
		Estimation Based Estimate: Correlated Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	.999
		Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	1.000	1.000	1.000	1.000	.999
	Btwns	True	8	26	26	16	26	2	44	0	36	36	10	20	30	42	26	16	2	36	42	36	2	10	28	16	8

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Table A.4 – continued from previous page

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Psych Network Node																								
		Post-Processing Shift Estimation																									
		Estimate	12.526	38.864	3.486	19.004	3.922	7.166	42.148	5.168	42.126	4.930	13.138	37.326	31.522	49.322	42.500	24.366	12.010	31.556	45.800	31.794	7.512	11.390	33.158	17.474	19.992
		MSE	2.485	165.482	2.124	9.024	24.226	26.688	3.430	26.708	37.528	24.305	9.847	30.190	2.316	53.612	272.250	69.990	10.200	19.749	14.440	17.690	3.382	1.932	26.605	2.173	143.808
		Coverage	.790	.709	.863	.831	.839	.633	.869	.452	.863	.880	.847	.594	.897	.834	.676	.805	.652	.900	.851	.862	.598	.849	.875	.837	.627
		Simple Gibbs Sampler																									
		Estimate	1.406	32.897	24.458	12.259	24.014	5.833	34.660	2.744	35.663	31.409	1.877	31.537	25.698	4.175	3.821	22.365	12.402	24.311	34.477	22.656	5.964	8.210	28.979	11.585	16.189
		MSE	5.789	47.566	2.377	13.995	3.944	14.691	87.227	7.529	.114	21.079	.770	133.092	18.509	3.332	23.238	4.517	108.201	136.630	56.592	178.075	15.709	3.205	.958	19.490	67.059
		Coverage	.983	.963	.992	.993	.997	.951	.957	.978	.994	.994	.991	.897	.991	.991	.986	.968	.813	.964	.982	.933	.948	.994	.987	.990	.926
5,000	Strngth	True	1.171	1.263	1.090	.905	.991	.935	1.358	.830	1.480	1.186	1.387	1.300	1.181	1.364	1.448	1.258	1.227	1.053	1.219	1.291	.943	.979	1.343	1.084	1.169
		Post-Processing Shift Estimation																									
		Estimate	1.112	1.222	1.054	.862	.961	.885	1.316	.814	1.427	1.140	1.313	1.249	1.161	1.335	1.401	1.228	1.153	1.001	1.180	1.241	.914	.920	1.306	1.031	1.132
		MSE	.003	.002	.001	.002	.001	.003	.002	.000	.003	.002	.006	.003	.000	.001	.002	.001	.006	.003	.002	.002	.001	.003	.001	.003	.001
		Coverage	.816	.923	.933	.891	.945	.876	.880	.983	.829	.861	.721	.817	.951	.943	.832	.941	.698	.869	.876	.857	.960	.813	.916	.864	.902
		Simple Gibbs Sampler																									
		Estimate	1.222	1.366	1.222	1.011	1.123	1.047	1.444	.973	1.542	1.275	1.422	1.392	1.295	1.464	1.486	1.371	1.302	1.179	1.310	1.353	1.068	1.071	1.426	1.168	1.263
		MSE	.003	.011	.017	.011	.017	.012	.007	.021	.004	.008	.001	.008	.013	.010	.001	.013	.006	.016	.008	.004	.016	.009	.007	.007	.009
		Coverage	.857	.361	.034	.301	.026	.219	.549	.046	.770	.489	.919	.339	.198	.343	.886	.260	.652	.067	.423	.782	.125	.495	.582	.580	.508
		Estimation Based Estimate																									
		Estimate	1.112	1.222	1.054	.862	.961	.885	1.316	.814	1.427	1.140	1.313	1.249	1.161	1.335	1.401	1.228	1.153	1.001	1.180	1.241	.914	.920	1.306	1.031	1.132
		MSE	.003	.002	.001	.002	.001	.003	.002	.000	.003	.002	.006	.003	.000	.001	.002	.001	.006	.003	.002	.002	.001	.003	.001	.003	.001
		Coverage	.680	.863	.890	.817	.926	.810	.831	.965	.744	.802	.575	.761	.939	.907	.831	.904	.615	.797	.850	.781	.926	.728	.870	.792	.845
	Clsnss	True	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		Post-Processing Shift Estimation																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.956	.950	.942	.945	.947	.960	.939	.963	.947	.948	.945	.940	.945	.939	.898	.951	.903	.902	.928	.943	.946	.952	.908	.949	.876
		Simple Gibbs Sampler																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.961	.909	.971	.971	.977	.946	.968	.946	.971	.971	.946	.898	.882	.901	.754	.896	.669	.748	.920	.969	.901	.936	.762	.972	.709
		Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.991	.990	.979	.983	.988	.987	.981	.981	.981	.982	.982	.977	.979	.982	.968	.979	.967	.955	.961	.986	.982	.988	.952	.985	.965
		Estimation Based Estimate: Correlated Successive Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.974	.958	.923	.934	.938	.920	.856	.888	.935	.928	.937	.927	.910	.919	.919	.918	.908	.904	.853	.899	.916	.941	.830	.879	.877
		Estimation Based Estimate: Correlated Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

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Table A.4 – continued from previous page

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Psych Network Node																								
		Coverage	.991	.989	.979	.983	.988	.987	.981	.981	.982	.982	.982	.977	.980	.982	.968	.979	.968	.955	.962	.986	.982	.989	.952	.985	.965
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.974	.958	.923	.935	.938	.920	.857	.890	.935	.928	.936	.927	.910	.920	.919	.918	.909	.904	.854	.900	.917	.941	.832	.880	.877
	Btwns	True	8	26	26	16	26	2	44	0	36	36	10	20	30	42	26	16	2	36	42	36	2	10	28	16	8
			Post-Processing Shift Estimation																								
		Estimate	9.950	3.620	26.072	15.884	25.776	3.620	41.328	2.026	37.184	34.382	1.360	24.240	27.710	43.234	29.702	21.540	5.304	31.196	41.046	34.694	5.416	7.734	29.476	14.550	11.144
		MSE	3.803	21.344	.005	.013	.050	2.624	7.140	4.105	1.402	2.618	.130	17.978	5.244	1.523	13.705	3.692	1.916	23.078	.910	1.706	11.669	5.135	2.179	2.103	9.885
		Coverage	.912	.898	.978	.958	.941	.753	.948	.544	.945	.957	.950	.843	.944	.933	.923	.871	.794	.908	.951	.961	.724	.940	.957	.956	.891
			Simple Gibbs Sampler																								
		Estimate	1.179	31.242	24.570	14.908	24.168	3.556	39.031	1.494	36.467	33.028	1.427	24.517	25.986	42.283	29.734	21.065	5.431	27.253	37.428	32.332	4.574	6.566	28.685	13.354	11.460
		MSE	4.747	27.482	2.045	1.192	3.357	2.420	24.687	2.233	.218	8.836	.182	2.399	16.109	.080	13.941	25.652	11.774	76.514	2.904	13.455	6.628	11.790	.470	7.001	11.968
		Coverage	.982	.960	.996	.992	.996	.980	.969	.974	.992	.984	.990	.931	.972	.996	.978	.951	.954	.920	.972	.982	.971	.972	.992	.992	.968
10,000	Strngth	True	1.171	1.263	1.090	.905	.991	.935	1.358	.830	1.480	1.186	1.387	1.300	1.181	1.364	1.448	1.258	1.227	1.053	1.219	1.291	.943	.979	1.343	1.084	1.169
			Post-Processing Shift Estimation																								
		Estimate	1.143	1.245	1.075	.892	.977	.911	1.339	.828	1.456	1.166	1.355	1.276	1.177	1.353	1.425	1.247	1.198	1.024	1.210	1.269	.932	.955	1.332	1.057	1.156
		MSE	.001	.000	.000	.000	.000	.001	.000	.000	.001	.000	.001	.001	.000	.000	.001	.000	.001	.001	.000	.000	.000	.001	.000	.001	.000
		Coverage	.914	.964	.973	.969	.987	.945	.960	.994	.931	.955	.893	.904	.986	.980	.919	.973	.888	.916	.969	.950	.987	.932	.973	.926	.972
			Simple Gibbs Sampler																								
		Estimate	1.218	1.345	1.191	.990	1.091	1.020	1.425	.943	1.531	1.255	1.420	1.371	1.270	1.442	1.480	1.346	1.289	1.146	1.291	1.343	1.039	1.053	1.412	1.153	1.244
		MSE	.002	.007	.010	.007	.010	.007	.004	.013	.003	.005	.001	.005	.008	.006	.001	.008	.004	.009	.005	.003	.009	.006	.005	.005	.006
		Coverage	.786	.238	.022	.194	.007	.166	.474	.014	.698	.423	.886	.268	.135	.257	.851	.166	.566	.054	.334	.722	.084	.371	.435	.438	.414
			Estimation Based Estimate																								
		Estimate	1.143	1.245	1.075	.892	.977	.911	1.339	.828	1.456	1.166	1.355	1.276	1.177	1.353	1.425	1.247	1.198	1.024	1.210	1.269	.932	.955	1.332	1.057	1.156
		MSE	.001	.000	.000	.000	.000	.001	.000	.000	.001	.000	.001	.001	.000	.000	.001	.000	.001	.001	.000	.000	.000	.001	.000	.001	.000
		Coverage	.813	.917	.946	.939	.978	.905	.923	.987	.872	.912	.777	.874	.985	.957	.921	.950	.816	.852	.958	.890	.970	.881	.951	.867	.930
	Clsnss	True	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
			Post-Processing Shift Estimation																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.968	.952	.940	.936	.939	.959	.916	.970	.950	.959	.954	.945	.953	.955	.928	.946	.907	.916	.955	.933	.937	.947	.905	.945	.897
			Simple Gibbs Sampler																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.967	.947	.975	.956	.966	.962	.946	.953	.972	.973	.966	.940	.921	.939	.824	.932	.793	.824	.957	.955	.919	.962	.803	.966	.775
			Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																								
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003

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Table A.4 – continued from previous page

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Psych Network Node																								
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.983	.979	.969	.967	.956	.972	.947	.981	.964	.978	.974	.961	.972	.971	.957	.964	.945	.952	.967	.955	.961	.977	.922	.966	.948
		Estimation Based Estimate: Correlated Successive Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.920	.938	.902	.884	.874	.875	.760	.846	.897	.901	.879	.896	.908	.906	.924	.844	.820	.859	.845	.823	.885	.883	.788	.828	.843
		Estimation Based Estimate: Correlated Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.983	.979	.968	.967	.956	.972	.947	.981	.964	.978	.973	.961	.972	.972	.958	.964	.945	.952	.967	.955	.961	.977	.922	.966	.948
		Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																									
		Estimate	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.003	.003	.003	.003	.003	.003	.003	.003
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.921	.939	.902	.885	.874	.875	.763	.847	.898	.901	.879	.896	.908	.907	.924	.844	.821	.859	.845	.823	.885	.883	.789	.828	.844
	Btwns	True	8	26	26	16	26	2	44	0	36	36	10	20	30	42	26	16	2	36	42	36	2	10	28	16	8
		Post-Processing Shift Estimation																									
		Estimate	8.678	28.378	26.112	15.824	25.262	3.040	41.364	1.652	36.780	34.422	9.370	22.726	27.962	42.296	27.988	19.964	4.624	33.208	42.282	34.656	5.256	7.980	29.738	15.274	9.236
		MSE	.460	5.655	.013	.031	.545	1.082	6.948	2.729	.608	2.490	.397	7.431	4.153	.088	3.952	15.713	6.885	7.795	.080	1.806	1.602	4.080	3.021	.527	1.528
		Coverage	.953	.922	.966	.952	.936	.770	.935	.593	.938	.957	.974	.882	.945	.935	.946	.899	.771	.930	.963	.962	.789	.933	.937	.943	.947
		Simple Gibbs Sampler																									
		Estimate	9.057	29.220	24.949	15.269	24.349	3.008	39.589	1.363	36.173	33.296	9.375	23.242	26.651	42.052	28.354	19.807	4.351	3.094	39.705	33.502	4.691	6.934	29.320	14.261	9.460
		MSE	1.118	1.369	1.104	.534	2.727	1.017	19.459	1.858	.030	7.311	.390	1.514	11.216	.003	5.543	14.495	5.526	34.881	5.266	6.241	7.240	9.403	1.742	3.024	2.133
		Coverage	.992	.978	.989	.995	.991	.993	.964	.981	.997	.979	.996	.962	.973	.998	.982	.957	.975	.943	.996	.991	.981	.967	.994	.986	.989

Table A.5

Per Node True- and Estimated Scores, MSE, and Coverage Rates for the Bayesian Horseshoe (Random Network)

N	Centr.		Random Network Node 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
900	Strngth	True	1.383	1.688	.599	.897	1.886	1.297	1.629	1.662	1.556	.974	1.447	1.377	1.405	1.478	.728	1.425	1.197	1.348	1.017	1.246	.682	1.131	1.722	.863	1.818
			Post-Processing Shift Estimation																								
		Estimate	1.217	1.411	.449	.650	1.662	1.060	1.367	1.372	1.251	.729	1.149	1.041	1.173	1.166	.553	1.091	.943	1.007	.738	.922	.530	.788	1.327	.577	1.540
		MSE	.028	.077	.022	.061	.050	.056	.069	.084	.093	.060	.089	.113	.054	.097	.030	.111	.065	.116	.078	.105	.023	.118	.156	.082	.077
		Coverage	.741	.305	.781	.428	.368	.394	.285	.193	.350	.459	.257	.242	.471	.378	.712	.303	.380	.188	.363	.138	.807	.211	.101	.329	.143
			Simple Gibbs Sampler																								
		Estimate	1.583	1.741	.871	1.059	1.912	1.383	1.714	1.708	1.614	1.140	1.491	1.427	1.512	1.547	.989	1.451	1.359	1.365	1.178	1.279	.957	1.208	1.730	1.030	1.835
		MSE	.040	.003	.074	.026	.001	.007	.007	.002	.003	.028	.002	.002	.011	.005	.068	.001	.026	.000	.026	.001	.076	.006	.000	.028	.000
		Coverage	.484	.955	.013	.664	.976	.892	.905	.952	.946	.687	.964	.946	.841	.943	.142	.971	.659	.968	.714	.972	.112	.946	.984	.693	.985
			Estimation Based Estimate																								
		Estimate	1.217	1.411	.449	.650	1.662	1.060	1.367	1.372	1.251	.729	1.149	1.041	1.173	1.166	.553	1.091	.943	1.007	.738	.922	.530	.788	1.327	.577	1.540
		MSE	.028	.077	.022	.061	.050	.056	.069	.084	.093	.060	.089	.113	.054	.097	.030	.111	.065	.116	.078	.105	.023	.118	.156	.082	.077
		Coverage	.718	.288	.800	.377	.456	.310	.350	.177	.317	.421	.236	.181	.452	.279	.639	.273	.397	.187	.277	.158	.787	.136	.096	.288	.148
	Clsnss	True	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
			Post-Processing Shift Estimation																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.837	.819	.940	.872	.944	.908	.960	.918	.933	.865	.910	.780	.942	.767	.888	.915	.820	.858	.859	.911	.877	.786	.938	.891	.911
			Simple Gibbs Sampler																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.973	.945	.975	.978	.970	.976	.945	.767	.976	.982	.989	.943	.990	.942	.983	.967	.977	.984	.981	.979	.977	.944	.980	.965	.716
			Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	1.000	.999	.999	1.000	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999
			Estimation Based Estimate: Correlated Successive Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	.999	.999	.999	1.000	1.000	1.000	1.000	1.000	1.000	.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999
			Estimation Based Estimate: Correlated Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	1.000	.999	.999	1.000	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	1.000	1.000	.999	.999	.999	1.000	1.000	1.000	1.000	1.000	1.000	.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999
	Btwns	True	54	60	0	14	68	38	36	32	20	0	22	28	16	24	0	30	24	6	12	6	4	8	26	8	46

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N	Centr.		Random Network Node 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
		Post-Processing Shift Estimation																									
		Estimate	52.362	53.106	1.218	16.882	73.668	45.284	55.046	5.150	27.334	3.460	21.924	26.144	2.260	17.904	4.178	25.916	23.546	8.680	1.110	6.120	4.214	7.624	38.236	9.552	64.366
		MSE	2.683	47.527	1.484	8.306	32.126	53.057	362.750	329.423	53.788	11.972	.006	3.445	18.148	37.161	17.456	16.679	.206	7.182	3.572	.014	.046	.141	149.720	2.409	337.310
		Coverage	.946	.911	.684	.889	.898	.857	.626	.708	.832	.488	.921	.937	.876	.914	.399	.929	.932	.824	.942	.923	.898	.924	.746	.862	.606
		Simple Gibbs Sampler																									
		Estimate	43.985	47.875	.301	12.914	64.459	38.405	48.261	47.192	27.115	2.770	18.746	22.578	17.408	14.540	2.810	23.150	2.038	6.599	8.154	4.685	3.185	5.934	37.680	7.104	59.272
		MSE	10.299	147.013	.090	1.179	12.538	.164	15.320	23.799	5.626	7.673	1.585	29.401	1.982	89.497	7.897	46.922	15.697	.359	14.795	1.729	.664	4.269	136.426	.802	176.142
		Coverage	.941	.936	1.000	.992	.984	.981	.907	.870	.949	.949	.994	.980	.991	.915	.938	.967	.991	.995	.982	1.000	.999	.989	.870	.993	.878
5,000	Strngth	True	1.383	1.688	.599	.897	1.886	1.297	1.629	1.662	1.556	.974	1.447	1.377	1.405	1.478	.728	1.425	1.197	1.348	1.017	1.246	.682	1.131	1.722	.863	1.818
		Post-Processing Shift Estimation																									
		Estimate	1.343	1.632	.560	.828	1.826	1.218	1.586	1.621	1.505	.921	1.370	1.319	1.363	1.439	.674	1.359	1.116	1.303	.967	1.150	.643	1.028	1.652	.804	1.733
		MSE	.002	.003	.002	.005	.004	.006	.002	.002	.003	.003	.006	.003	.002	.002	.003	.004	.007	.002	.002	.009	.001	.011	.005	.004	.007
		Coverage	.934	.849	.955	.780	.798	.696	.879	.910	.904	.874	.709	.857	.901	.933	.884	.831	.663	.911	.872	.531	.944	.584	.742	.845	.530
		Simple Gibbs Sampler																									
		Estimate	1.506	1.757	.742	.995	1.927	1.365	1.704	1.723	1.635	1.079	1.492	1.453	1.488	1.569	.866	1.478	1.287	1.402	1.127	1.287	.831	1.184	1.754	.966	1.842
		MSE	.015	.005	.020	.010	.002	.005	.006	.004	.006	.011	.002	.006	.007	.008	.019	.003	.008	.003	.012	.002	.022	.003	.001	.011	.001
		Coverage	.197	.724	.021	.463	.894	.752	.638	.769	.710	.404	.880	.720	.573	.643	.079	.860	.516	.828	.438	.879	.036	.882	.929	.463	.951
		Estimation Based Estimate																									
		Estimate	1.343	1.632	.560	.828	1.826	1.218	1.586	1.621	1.505	.921	1.370	1.319	1.363	1.439	.674	1.359	1.116	1.303	.967	1.150	.643	1.028	1.652	.804	1.733
		MSE	.002	.003	.002	.005	.004	.006	.002	.002	.003	.003	.006	.003	.002	.002	.003	.004	.007	.002	.002	.009	.001	.011	.005	.004	.007
		Coverage	.922	.815	.947	.742	.829	.617	.906	.879	.871	.832	.654	.812	.896	.869	.826	.783	.682	.900	.817	.541	.934	.443	.729	.800	.524
	Clsnss	True	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		Post-Processing Shift Estimation																									
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.942	.939	.966	.955	.969	.915	.963	.975	.961	.951	.938	.881	.951	.911	.918	.954	.944	.939	.944	.920	.945	.938	.947	.952	.967
		Simple Gibbs Sampler																									
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.964	.971	.977	.968	.964	.982	.979	.963	.960	.967	.955	.926	.974	.955	.960	.946	.961	.962	.969	.951	.962	.963	.971	.961	.929
		Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																									
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.967	.962	.987	.985	.979	.958	.988	.981	.978	.972	.956	.929	.979	.935	.963	.974	.955	.965	.973	.938	.974	.974	.975	.973	.985
		Estimation Based Estimate: Correlated Successive Paths																									
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.930	.883	.817	.876	.956	.874	.962	.949	.958	.851	.851	.820	.950	.775	.779	.895	.900	.886	.788	.852	.835	.833	.946	.843	.944
		Estimation Based Estimate: Correlated Shortest Paths																									
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.968	.962	.987	.985	.979	.958	.988	.981	.978	.972	.956	.929	.979	.935	.963	.974	.955	.965	.972	.938	.974	.974	.973	.973	.985

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N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Random Network Node																								
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.930	.885	.817	.875	.957	.873	.961	.948	.958	.851	.850	.820	.949	.774	.779	.895	.900	.886	.787	.852	.835	.833	.946	.843	.944
	Btwns	True	54	60	0	14	68	38	36	32	20	0	22	28	16	24	0	30	24	6	12	6	4	8	26	8	46
			Post-Processing Shift Estimation																								
		Estimate	53.642	55.640	.864	15.866	69.720	38.116	41.738	35.016	22.722	1.122	16.190	29.144	15.088	22.664	1.656	27.464	24.332	6.872	11.126	6.162	3.214	6.718	32.574	9.406	5.384
		MSE	.128	19.010	.746	3.482	2.958	.013	32.925	9.096	7.409	1.259	33.756	1.309	.832	1.785	2.742	6.431	.110	.760	.764	.026	.618	1.644	43.217	1.977	19.219
		Coverage	.952	.934	.744	.925	.948	.969	.858	.966	.884	.711	.864	.969	.952	.941	.669	.942	.950	.931	.938	.962	.973	.941	.726	.893	.893
			Simple Gibbs Sampler																								
		Estimate	5.690	53.257	.445	15.139	67.660	38.031	42.468	36.149	23.504	.943	15.757	27.796	14.738	2.753	1.546	26.365	23.323	6.134	1.395	5.571	3.072	6.256	33.443	8.905	52.038
		MSE	1.959	45.463	.198	1.298	.116	.001	41.829	17.217	12.280	.889	38.981	.042	1.593	1.541	2.389	13.216	.459	.018	2.574	.184	.862	3.042	55.396	.818	36.463
		Coverage	.977	.957	.998	.994	.994	.992	.942	.990	.970	.996	.922	.998	.983	.967	.972	.973	.992	.996	.988	.998	.996	.992	.816	.990	.942
10,000	Strngth	True	1.383	1.688	.599	.897	1.886	1.297	1.629	1.662	1.556	.974	1.447	1.377	1.405	1.478	.728	1.425	1.197	1.348	1.017	1.246	.682	1.131	1.722	.863	1.818
			Post-Processing Shift Estimation																								
		Estimate	1.364	1.659	.577	.859	1.849	1.242	1.616	1.642	1.525	.942	1.402	1.346	1.383	1.464	.694	1.384	1.148	1.319	.997	1.189	.662	1.072	1.691	.826	1.759
		MSE	.000	.001	.000	.001	.001	.003	.000	.000	.001	.001	.002	.001	.000	.000	.001	.002	.002	.001	.000	.003	.000	.003	.001	.001	.003
		Coverage	.965	.909	.975	.875	.846	.717	.966	.942	.923	.900	.809	.926	.934	.967	.915	.849	.776	.933	.949	.676	.966	.735	.902	.890	.609
			Simple Gibbs Sampler																								
		Estimate	1.478	1.748	.708	.973	1.923	1.350	1.693	1.716	1.619	1.054	1.486	1.440	1.470	1.556	.830	1.469	1.262	1.392	1.103	1.281	.792	1.172	1.755	.939	1.838
		MSE	.009	.004	.012	.006	.001	.003	.004	.003	.004	.006	.002	.004	.004	.006	.010	.002	.004	.002	.008	.001	.012	.002	.001	.006	.000
		Coverage	.134	.608	.017	.395	.845	.729	.503	.645	.659	.359	.819	.615	.505	.497	.086	.820	.505	.782	.311	.868	.030	.851	.896	.457	.941
			Estimation Based Estimate																								
		Estimate	1.364	1.659	.577	.859	1.849	1.242	1.616	1.642	1.525	.942	1.402	1.346	1.383	1.464	.694	1.384	1.148	1.319	.997	1.189	.662	1.072	1.691	.826	1.759
		MSE	.000	.001	.000	.001	.001	.003	.000	.000	.001	.001	.002	.001	.000	.000	.001	.002	.002	.001	.000	.003	.000	.003	.001	.001	.003
		Coverage	.956	.895	.973	.844	.867	.634	.975	.918	.885	.869	.767	.902	.940	.935	.863	.805	.783	.922	.931	.680	.954	.610	.888	.845	.587
	Clsnss	True	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
			Post-Processing Shift Estimation																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.962	.939	.972	.955	.956	.942	.965	.956	.967	.951	.946	.899	.953	.938	.930	.955	.960	.936	.954	.961	.947	.946	.957	.953	.970
			Simple Gibbs Sampler																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.971	.975	.976	.964	.953	.980	.985	.978	.971	.967	.954	.936	.970	.964	.948	.933	.965	.959	.974	.977	.971	.962	.980	.960	.952
			Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																								
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.965	.943	.983	.972	.953	.955	.981	.958	.982	.960	.952	.909	.962	.934	.956	.966	.960	.954	.971	.961	.964	.966	.962	.968	.969

Continued on next page

Table A.5 – continued from previous page

N	Centr.		Random Network Node 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
		Estimation Based Estimate: Correlated Successive Paths																									
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.932	.869	.762	.848	.922	.863	.947	.905	.955	.833	.830	.797	.910	.790	.750	.856	.875	.878	.775	.856	.778	.815	.933	.801	.917
		Estimation Based Estimate: Correlated Shortest Paths																									
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.965	.942	.984	.972	.953	.955	.981	.958	.982	.960	.952	.909	.962	.933	.956	.966	.960	.954	.970	.961	.964	.966	.961	.968	.969
		Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																									
		Estimate	.004	.004	.003	.003	.004	.004	.004	.004	.004	.003	.004	.004	.004	.004	.003	.003	.004	.003	.003	.004	.003	.003	.004	.003	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.932	.870	.762	.846	.924	.862	.947	.905	.955	.833	.830	.797	.910	.790	.751	.856	.876	.878	.776	.858	.778	.813	.932	.801	.917
	Btwns	True	54	6	0	14	68	38	36	32	2	0	22	28	16	24		3	24	6	12	6	4	8	26	8	46
		Post-Processing Shift Estimation																									
		Estimate	54.606	57.504	.804	15.420	69.056	37.536	39.636	32.916	21.388	.830	16.216	29.652	14.862	24.312	1.272	27.918	24.278	6.754	11.372	6.208	3.058	6.934	3.966	8.862	47.722
		MSE	.367	6.230	.646	2.016	1.115	.215	13.220	.839	1.927	.689	33.455	2.729	1.295	.097	1.618	4.335	.077	.569	.394	.043	.887	1.136	24.661	.743	2.965
		Coverage	.941	.937	.877	.890	.957	.971	.900	.988	.936	.803	.813	.933	.959	.958	.773	.946	.957	.931	.947	.973	.987	.958	.757	.908	.941
		Simple Gibbs Sampler																									
		Estimate	52.320	55.310	.592	14.965	68.485	37.282	4.228	33.888	22.064	.678	15.537	28.785	14.432	22.704	1.190	27.216	23.727	6.361	1.973	5.964	3.035	6.520	31.847	8.565	49.078
		MSE	2.823	21.997	.350	.932	.235	.516	17.873	3.565	4.261	.460	41.770	.617	2.459	1.680	1.417	7.751	.074	.130	1.056	.001	.932	2.190	34.184	.320	9.472
		Coverage	.995	.965	.999	.992	.989	.998	.970	.998	.992	.993	.897	.994	.988	.999	.980	.975	.999	.998	.993	.999	.995	.993	.846	.995	.970

Table A.6

Per Node True- and Estimated Scores, MSE, and Coverage Rates for the Bayesian Horseshoe (Random Network Half the Density of Psych Network)

			(Random Network Half the Density of Psych Network) Node																								
N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
900	Strngth	True	.847	.901	.156	1.442	.955	.802	.945	1.164	1.308	1.765	.579	1.042	1.204	.441	.526	.590	.427	.891	.655	.491	.395	1.240	1.085	1.108	1.348
			Post-Processing Shift Estimation																								
		Estimate	.715	.834	.045	1.264	.803	.647	.786	.994	1.111	1.718	.411	.846	.936	.296	.333	.432	.310	.796	.556	.451	.295	1.041	.948	.800	1.102
		MSE	.017	.004	.012	.032	.023	.024	.025	.029	.039	.002	.028	.038	.072	.021	.037	.025	.014	.009	.010	.002	.010	.039	.019	.094	.060
		Coverage	.772	.980	1.000	.513	.714	.728	.733	.621	.447	.978	.662	.501	.250	.767	.549	.720	.972	.936	.899	.990	.986	.537	.802	.192	.319
			Simple Gibbs Sampler																								
		Estimate	1.035	1.137	.430	1.543	1.102	.976	1.099	1.275	1.391	1.914	.770	1.163	1.268	.665	.709	.785	.674	1.098	.888	.779	.658	1.340	1.242	1.144	1.396
		MSE	.035	.056	.075	.010	.022	.030	.024	.012	.007	.022	.036	.015	.004	.050	.033	.038	.061	.043	.054	.083	.069	.010	.025	.001	.002
		Coverage	.362	.055	.000	.850	.550	.407	.623	.768	.883	.475	.391	.782	.941	.243	.405	.445	.078	.243	.102	.006	.012	.863	.619	.973	.965
			Estimation Based Estimate																								
		Estimate	.715	.834	.045	1.264	.803	.647	.786	.994	1.111	1.718	.411	.846	.936	.296	.333	.432	.310	.796	.556	.451	.295	1.041	.948	.800	1.102
		MSE	.017	.004	.012	.032	.023	.024	.025	.029	.039	.002	.028	.038	.072	.021	.037	.025	.014	.009	.010	.002	.010	.039	.019	.094	.060
		Coverage	.826	.991	1.000	.540	.798	.785	.776	.669	.618	.993	.706	.518	.290	.775	.715	.746	.930	.981	.922	.999	.989	.542	.804	.242	.386
	Clsnss	True	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
			Post-Processing Shift Estimation																								
		Estimate	.001	.002	.001	.002	.002	.001	.001	.002	.002	.002	.001	.002	.002	.001	.001	.001	.001	.002	.001	.001	.001	.001	.001	.002	.002
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.374	.294	.433	.353	.285	.354	.344	.383	.370	.265	.335	.363	.385	.348	.385	.354	.364	.351	.390	.378	.294	.259	.231	.339	.362
			Simple Gibbs Sampler																								
		Estimate	.035	.056	.075	.010	.022	.030	.024	.012	.007	.022	.036	.015	.004	.050	.033	.038	.061	.043	.054	.083	.069	.010	.025	.001	.002
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.989	.628	.993	.844	.577	.920	.882	.933	.837	.536	.941	.752	.945	.888	.975	.938	.923	.870	.934	.944	.881	.755	.605	.875	.842
			Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																								
		Estimate	.001	.002	.001	.002	.002	.001	.001	.002	.002	.002	.001	.002	.002	.001	.001	.001	.001	.002	.001	.001	.001	.001	.001	.002	.002
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.955	.823	NA	.650	.865	.932	.907	.703	.788	.738	.946	.799	.862	.947	.933	.952	.690	.839	.699	.873	.958	.960	.875	.825	.906
			Estimation Based Estimate: Correlated Successive Paths																								
		Estimate	.001	.002	.001	.002	.002	.001	.001	.002	.002	.002	.001	.002	.002	.001	.001	.001	.001	.002	.001	.001	.001	.001	.001	.002	.002
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.939	.814	NA	.644	.850	.916	.902	.682	.782	.725	.930	.798	.851	.934	.906	.940	.673	.825	.688	.857	.950	.957	.863	.814	.897
			Estimation Based Estimate: Correlated Shortest Paths																								
		Estimate	.001	.002	.001	.002	.002	.001	.001	.002	.002	.002	.001	.002	.002	.001	.001	.001	.001	.002	.001	.001	.001	.001	.001	.002	.002
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.955	.823	NA	.650	.865	.932	.907	.703	.788	.738	.946	.799	.862	.947	.933	.952	.690	.839	.699	.873	.958	.960	.875	.825	.906
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.001	.002	.001	.002	.002	.001	.001	.002	.002	.002	.001	.002	.002	.001	.001	.001	.001	.002	.001	.001	.001	.001	.001	.002	.002
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.939	.814	NA	.644	.850	.916	.902	.682	.782	.725	.930	.798	.851	.934	.906	.940	.673	.825	.688	.857	.950	.957	.863	.814	.897
	Btwns	True	10	28	0	118	18	20	44	58	50	170	8	64	56	2	0	2	0	24	6	12	0	82	28	32	72

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Table A.6 – continued from previous page

N	Centr.		Random Network Half the Density of Psych Network – Node 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
		Post-Processing Shift Estimation																									
		Estimate	15.280	3.264	.000	124.038	14.444	2.490	36.892	53.046	39.228	185.710	6.540	42.500	64.274	1.432	.496	4.386	.102	36.538	14.924	12.660	1.046	78.118	24.212	36.316	75.140
		MSE	27.878	5.126	.000	36.457	12.645	.240	5.524	24.542	116.036	246.804	2.132	462.250	68.459	.323	.246	5.693	.010	157.201	79.638	.436	1.094	15.070	14.349	18.628	9.860
		Coverage	.837	.908	1.000	.941	.961	.935	.942	.988	.929	.900	.953	.762	.886	.958	.873	.670	.993	.775	.586	.920	.772	.979	.967	.953	.947
		Simple Gibbs Sampler																									
		Estimate	15.801	29.861	.000	117.594	1.715	16.344	35.651	52.273	35.103	17.823	5.711	4.177	6.731	.977	.173	2.818	.025	32.589	12.817	8.910	.706	75.821	22.640	31.763	71.582
		MSE	33.656	3.464	.000	.165	53.077	13.366	69.707	32.802	221.919	.677	5.238	567.526	22.380	1.047	.030	.669	.001	73.773	46.471	9.550	.499	38.185	28.734	.056	.175
		Coverage	.926	.996	1.000	.994	.994	.990	.991	.995	.954	.990	.995	.850	.990	.999	.999	.989	1.000	.932	.909	.992	.997	.987	.989	.998	.991
5,000	Strngth	True	.847	.901	.156	1.442	.955	.802	.945	1.164	1.308	1.765	.579	1.042	1.204	.441	.526	.590	.427	.891	.655	.491	.395	1.240	1.085	1.108	1.348
		Post-Processing Shift Estimation																									
		Estimate	.834	.899	.136	1.408	.909	.753	.924	1.088	1.258	1.750	.558	.984	1.165	.429	.489	.575	.400	.871	.647	.488	.380	1.204	1.068	1.058	1.273
		MSE	.000	.000	.000	.001	.002	.002	.000	.006	.003	.000	.000	.003	.002	.000	.001	.000	.001	.000	.000	.000	.000	.001	.000	.002	.006
		Coverage	.996	1.000	.993	.947	.882	.880	.986	.630	.839	.997	.986	.816	.938	1.000	.946	.996	.984	.996	.997	.999	.993	.961	.995	.874	.684
		Simple Gibbs Sampler																									
		Estimate	.964	1.041	.307	1.525	1.048	.903	1.055	1.232	1.381	1.850	.700	1.121	1.282	.580	.639	.717	.569	1.011	.792	.644	.538	1.328	1.198	1.180	1.395
		MSE	.014	.020	.023	.007	.009	.010	.012	.005	.005	.007	.015	.006	.006	.019	.013	.016	.020	.014	.019	.023	.020	.008	.013	.005	.002
		Coverage	.065	.000	.004	.451	.326	.268	.172	.641	.613	.254	.090	.562	.588	.023	.123	.083	.024	.056	.007	.000	.005	.486	.130	.666	.872
		Estimation Based Estimate																									
		Estimate	.834	.899	.136	1.408	.909	.753	.924	1.088	1.258	1.750	.558	.984	1.165	.429	.489	.575	.400	.871	.647	.488	.380	1.204	1.068	1.058	1.273
		MSE	.000	.000	.000	.001	.002	.002	.000	.006	.003	.000	.000	.003	.002	.000	.001	.000	.001	.000	.000	.000	.000	.001	.000	.002	.006
		Coverage	1.000	1.000	.994	.957	.905	.899	.988	.657	.903	1.000	.994	.830	.962	1.000	.980	.998	.987	.999	.999	1.000	.995	.963	.995	.923	.719
	Clsnss	True	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		Post-Processing Shift Estimation																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.941	.859	.903	.858	.914	.889	.919	.876	.930	.864	.892	.899	.919	.914	.927	.930	.873	.927	.883	.940	.844	.860	.727	.924	.899
		Simple Gibbs Sampler																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.972	.860	.942	.869	.921	.923	.953	.906	.951	.848	.940	.936	.956	.942	.959	.961	.894	.949	.888	.945	.886	.899	.738	.946	.930
		Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.963	.950	.990	.944	.979	.947	.978	.934	.972	.966	.962	.971	.965	.974	.982	.975	.919	.969	.941	.973	.963	.953	.832	.972	.969
		Estimation Based Estimate: Correlated Successive Paths																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.863	.861	.449	.844	.927	.798	.929	.836	.877	.932	.793	.902	.897	.796	.852	.861	.678	.860	.794	.635	.889	.871	.684	.922	.886
		Estimation Based Estimate: Correlated Shortest Paths																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

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Table A.6 – continued from previous page

Random Network Half the Density of Psych Network — Node

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
		Coverage	.963	.950	.990	.944	.979	.947	.978	.934	.972	.966	.962	.971	.965	.974	.982	.975	.919	.969	.942	.973	.963	.953	.831	.972	.968
		Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.863	.861	.451	.843	.927	.800	.928	.836	.878	.932	.793	.902	.897	.796	.852	.861	.678	.860	.794	.635	.889	.871	.683	.922	.886
	Btwns	True	10	28	0	118	18	20	44	58	50	170	8	64	56	2	0	2	0	24	6	12	0	82	28	32	72
		Post-Processing Shift Estimation																									
		Estimate	11.774	3.360	.000	117.734	19.388	21.184	43.616	55.164	48.946	181.022	7.748	59.946	55.770	1.792	.018	2.500	.002	27.710	6.656	11.666	.354	81.542	27.010	36.054	73.030
		MSE	3.147	5.570	.000	.071	1.927	1.402	.147	8.043	1.111	121.484	.064	16.435	.053	.043	.000	.250	.000	13.764	.430	.112	.125	.210	.980	16.435	1.061
		Coverage	.905	.955	1.000	.971	.933	.958	.956	.986	.958	.876	.972	.923	.971	.996	.995	.898	.999	.905	.950	.968	.941	.950	.968	.908	.961
		Simple Gibbs Sampler																									
		Estimate	11.817	3.228	.000	117.383	19.234	21.249	42.842	55.160	47.272	178.587	7.673	6.285	56.484	1.766	.012	2.107	.000	27.839	6.665	11.179	.337	81.501	26.658	35.982	72.682
		MSE	3.300	4.963	.000	.380	1.523	1.560	1.340	8.065	7.441	73.730	.107	13.802	.235	.055	.000	.011	.000	14.738	.443	.675	.113	.249	1.800	15.856	.465
		Coverage	.981	.994	1.000	.994	.992	.979	.994	.998	.987	.964	.996	.983	.994	1.000	1.000	1.000	1.000	.978	1.000	.993	.998	.993	.994	.964	.993
10,000	Strngth	True	.847	.901	.156	1.442	.955	.802	.945	1.164	1.308	1.765	.579	1.042	1.204	.441	.526	.590	.427	.891	.655	.491	.395	1.240	1.085	1.108	1.348
		Post-Processing Shift Estimation																									
		Estimate	.841	.900	.145	1.429	.931	.772	.936	1.119	1.286	1.754	.567	1.009	1.187	.435	.506	.582	.418	.874	.653	.491	.387	1.215	1.077	1.083	1.315
		MSE	.000	.000	.000	.000	.001	.001	.000	.002	.001	.000	.000	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.000	.001	.001
		Coverage	.995	.998	.999	.979	.952	.936	.998	.763	.944	.993	.992	.919	.969	.998	.969	.997	.983	.993	.996	.998	1.000	.962	.995	.947	.875
		Simple Gibbs Sampler																									
		Estimate	.936	1.005	.270	1.511	1.029	.878	1.031	1.220	1.371	1.829	.672	1.106	1.270	.545	.611	.686	.537	.979	.759	.605	.500	1.306	1.173	1.169	1.391
		MSE	.008	.011	.013	.005	.005	.006	.007	.003	.004	.004	.009	.004	.004	.011	.007	.009	.012	.008	.011	.013	.011	.004	.008	.004	.002
		Coverage	.029	.001	.001	.287	.241	.230	.108	.519	.460	.201	.057	.427	.412	.010	.077	.041	.010	.049	.002	.000	.003	.439	.069	.519	.759
		Estimation Based Estimate																									
		Estimate	.841	.900	.145	1.429	.931	.772	.936	1.119	1.286	1.754	.567	1.009	1.187	.435	.506	.582	.418	.874	.653	.491	.387	1.215	1.077	1.083	1.315
		MSE	.000	.000	.000	.000	.001	.001	.000	.002	.001	.000	.000	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.000	.001	.001
		Coverage	1.000	1.000	.999	.983	.966	.940	1.000	.804	.974	.999	.995	.928	.985	.998	.992	1.000	.986	.999	1.000	1.000	1.000	.965	.998	.967	.895
	Clsnss	True	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		Post-Processing Shift Estimation																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.954	.931	.953	.916	.962	.930	.950	.906	.961	.935	.925	.918	.954	.933	.957	.939	.907	.953	.926	.957	.893	.900	.813	.954	.949
		Simple Gibbs Sampler																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.972	.920	.964	.909	.961	.947	.965	.928	.970	.935	.937	.951	.971	.945	.962	.964	.920	.970	.929	.960	.913	.926	.823	.975	.956
		Estimation Based Estimate: Correlated Successive Paths & Shortest Paths																									
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

Continued on next page

Table A.6 – continued from previous page

N	Centr.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
			Random Network Half the Density of Psych Network Node																								
		Coverage	.952	.951	.974	.932	.965	.939	.952	.905	.961	.964	.934	.938	.961	.952	.969	.947	.906	.960	.940	.967	.932	.926	.787	.935	.965
			Estimation Based Estimate: Correlated Successive Paths																								
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.782	.831	.383	.809	.881	.772	.848	.786	.839	.915	.690	.868	.886	.755	.766	.811	.670	.843	.760	.563	.865	.820	.615	.877	.862
			Estimation Based Estimate: Correlated Shortest Paths																								
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.952	.950	.974	.932	.965	.939	.952	.905	.961	.964	.934	.938	.961	.952	.969	.947	.906	.960	.940	.967	.932	.926	.786	.935	.966
			Estimation Based Estimate: Uncorrelated Successive Paths & Shortest Paths																								
		Estimate	.003	.004	.002	.004	.004	.003	.004	.004	.004	.005	.003	.004	.004	.003	.003	.003	.003	.004	.003	.003	.003	.004	.004	.004	.004
		MSE	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
		Coverage	.782	.831	.383	.809	.881	.772	.851	.787	.840	.915	.690	.868	.886	.756	.766	.811	.670	.843	.762	.563	.866	.820	.615	.878	.862
	Btwns	True	10	28	0	118	18	20	44	58	50	170	8	64	56	2	0	2	0	24	6	12	0	82	28	32	72
			Post-Processing Shift Estimation																								
		Estimate	11.476	29.128	.000	118.134	18.800	21.148	43.794	56.272	49.972	178.404	7.856	6.016	55.014	1.930	.000	2.334	.000	26.492	6.090	11.398	.168	8.958	27.284	35.292	72.174
		MSE	2.179	1.272	.000	.018	.640	1.318	.042	2.986	.001	7.627	.021	15.872	.972	.005	.000	.112	.000	6.210	.008	.362	.028	1.086	.513	1.837	.030
		Coverage	.881	.975	1.000	.966	.910	.927	.966	.998	.966	.881	.976	.939	.968	.999	1.000	.935	1.000	.939	.997	.947	.986	.965	.974	.868	.974
			Simple Gibbs Sampler																								
		Estimate	11.524	29.259	.000	117.998	18.834	21.594	43.601	56.247	49.376	178.155	7.927	6.248	55.637	1.928	.000	2.122	.000	26.327	6.086	11.176	.170	81.180	27.264	35.577	72.148
		MSE	2.322	1.586	.000	.000	.695	2.540	.159	3.072	.389	66.497	.005	14.075	.132	.005	.000	.015	.000	5.416	.007	.679	.029	.673	.541	12.792	.022
		Coverage	.968	.997	1.000	.995	.996	.974	.996	1.000	.994	.947	.996	.984	.997	1.000	1.000	1.000	1.000	.991	1.000	.994	1.000	.992	.998	.954	.995