Performance of small group majorities in a single task

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Abstract

Aggregating decisions from larger groups typically results in outcomes with higher accuracy than decision outcomes from single individuals or smaller groups. Here I argue that it is important to consider not only overall proportion of correct decisions, but also individual competencies in terms of hits (h) and correct rejections (cr). I show that small groups can perform better than randomly selected individuals and larger groups in a single task when the average individual proportion correct is above .5, h and cr are asymmetric around .5, and $h + cr > 1$. If the average individual proportion correct is below .5 and $h + cr < 1$, small groups perform worse than individuals and larger groups. I also demonstrate that these two performance patterns can occur in empirical data from studies on violent recidivism, psychiatric morbidity, anxiety, and deception detection. I also show that the presence of correlations between decisions in a single task has both beneficial and detrimental effects when it comes to small group performance.

Keywords: Condorcet jury theorem, wisdom of crowds, hits, correct rejections

1 Introduction

It is typically assumed that aggregating decisions from larger groups improves accuracy compared to decisions of a single individual or from a smaller group. Can a smaller group ever outperform a larger group? Condorcet's Jury Theorem (CJT, Condorcet, 1785) provides one argument for the supremacy of larger groups, a phenomenon that is commonly referred to as the wisdom of crowds (Page, 2007; Sunstein, 2006; Surowiecki, 2004). The CJT states that if the individual competence, or accuracy, of independent group members is larger than .5, a majority vote in larger groups will be more accurate than in smaller groups and the probability that the majority vote is correct will approach 1.0 as the number of voters increases. This is the basis for the findings that accuracy increases with expert group size in a variety of settings (see e.g., Kurvers, de Zoete, Bachman, Algra, & Ostelo, 2018, for an example in the medical domain).

There are, however, situations where a small group can outperform individuals and larger groups. If the individuals are selected according to past performance, smaller groups can outperform larger groups (Budescu & Chen, 2015; Mannes, Soll, & Larrick, 2014; Olsson & Loveday, 2015). Even when individuals are randomly selected from a larger group of people, there are situations where smaller groups can outperform larger ones. If the group encounters tasks of different difficulties, Galesic, Barkoczi, and Katsikopoulos (2018) showed that smaller groups can achieve higher accuracy on average across tasks than either individ-

uals or larger groups.

I show that small randomly selected groups can outperform individuals and large groups even in a single task. A small-group advantage in terms of overall proportion correct P can be observed for a single task if the individual probability of hits (h , saying that the state of the world is 1 when it is indeed 1) and individual probability of correct rejections (cr , saying the state of the world is 0 when it is indeed 0; or equivalently 1 minus the probability of false alarms, fa) are not the same, as is assumed in the traditional formulation of the CJT. Specifically, a small-group advantage can be observed when the group proportion correct P is larger than .5, the individual competencies are asymmetric around .5 (i.e., h and cr are on the opposite sides of .5 and not equidistant from .5) and $h + cr > 1$ (lower case letters refer to individual performance, while uppercase letters refer to group performance). I will 1) describe four cases relating group size and accuracy and the conditions under which they occur, 2) show four empirical demonstrations from research on violent recidivism, psychiatric morbidity, anxiety, and deception detection, and 3) illustrate the effects of correlated decisions on the four cases relating group size and accuracy, as modeled with a leader-follower model (Boland, Proschan, & Tong, 1989; Spiekermann & Goodin, 2012).

2 Small-group advantage and disadvantage in a single task

In most discussions of CJT the definition of accuracy as proportion correct implicitly assumes that the accuracy when the event occurs is the same as the accuracy when the event does not occur. In the language of signal detection theory

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(Macmillan & Creelman, 2005; Sorkin, Hays, & West, 2001), this means that the individual probability of hits, h , and the individual probability of correct rejections, cr are the same. This assumption is problematic from policy and decision theoretic perspectives, where different outcomes can have different pay-offs or utilities. In a legal setting where a jury is to decide on a guilty or not guilty verdict, there might be a presumption that the legal system should be biased towards maximizing a jury's CR (or equivalently, minimizing FA , i.e., avoiding convicting innocent suspects). Juries might therefore be encouraged to achieve a group decision high CR , even if it comes at the expense of low H .

Empirical asymmetries between average individual levels of hits, h , and correct rejections cr , have indeed been observed in many real-world situations. In any decision situation where the decision criterion can be varied to obtain different h and cr , situations can occur where h is above .5 and the cr is below .5. In other situations, h can be found to be below .5 and cr above .5. For example, for the case of predicting violent recidivism Rice and Harris (1995) found that for low base rates (.15) of violent recidivism h (.35) was much lower than cr (.88, or equivalently a fa of .12).

Assuming that H and CR are based on independent Bernoulli trials as in the CJT, the relation between H , CR and P for a certain group size n , can be expressed as:

$$P_n = brH_n + (1 - br)CR_n \quad (1)$$

where P_n is the average accuracy for a group of size n , br is the base rate of the event, H_n is the hit rate for the majority decision made by a group of size n , and CR_n is the correct rejection rate for the majority decision made by a group of size n . H_n and CR_n are determined by the binomial cumulative distribution function with parameters n , h and cr . The values of h and cr are assumed to be the averages of the individual values.

The convergence properties for P expressed as a function of H and CR are dependent upon the values of the individual competencies, h and cr (throughout the paper I assume homogeneous individual competencies, hence the omission of the bar over h and cr). As n grows large P converges to:

$$P = \begin{cases} 1 & \text{if } h > .5 \text{ and } cr > .5 \\ 0 & \text{if } h < .5 \text{ and } cr < .5 \\ br & \text{if } h > .5 \text{ and } cr < .5, \text{ as } n \rightarrow \infty \\ 1 - br & \text{if } h < .5 \text{ and } cr > .5 \\ .5 & \text{if } h = .5 \text{ and } cr = .5 \end{cases} \quad (2)$$

When both h and cr are on the same side of .5, the results are analogues to CJT results: P converges to 1 when $h > .5$ and $cr > .5$ (analogous to the case in CJT where the average of individual competencies, $\bar{p}$, is above .5) and converges to 0 when $h < .5$ and $cr < .5$ (analogous to the case in CJT where the average of the individual competencies, $\bar{p}$,

is below .5). When h and cr are on the opposite sides of .5, however, P converges to either the base rate, br , or to $1 - br$. The reason for the convergence to the br is that when n becomes large and $h > .5$, H converges to 1, and when $cr < .5$, CR converges to 0, leaving the base rate of the event: $br \cdot 1 + (1 - br) \cdot 0 = br$. When $h < .5$, on the other hand, H converges to 0, and when $cr > .5$, CR converges to 1, leaving one minus the base rate of the event: $br \cdot 0 + (1 - br) \cdot 1 = 1 - br$. Figure 1 shows simulation results for H , CR , and P as a function of group size for the first four cases in Equation 2. The black circles in Figure 1 indicate the asymptotic value for P when $n = 10,001$ and $br = .6$.

Next, I discuss four cases relating group size and accuracy: group advantage, individual advantage, small-group advantage, and small-group disadvantage. I define large-group advantage as $P_{n+2} > P_n$ for all n . An individual advantage is defined as $P_n > P_{n+2}$ for all n . A small-group advantage occurs when there exists a group size s for which $P_s > P_{n \leq s-2}$ and $P_s > P_{n \geq s+2}$, for all $n \neq s$. A small-group disadvantage is defined as having a global proportion correct minimum for a group of size n , with $P_s < P_1$ (where $P_1 = \bar{p}$), and $P_s < P_n$.

The conditions for when the small-group advantage occurs follow the same logic as their occurrence when there is a mix of easy and difficult tasks (Galesic et al., 2018). When $h > .5$ and $cr < .5$ or when $h < .5$ and $cr > .5$, a small-group advantage is observed when $h + cr > 1$ and P_1 is larger than .5 (as calculated by eq. 1). Conversely, a small-group disadvantage is observed when $h + cr < 1$ and P_1 is smaller than .5. I should note that there are cases when the small-group effects do not occur in these conditions. Specifically, when P_1 is close to .5 the expected group size for the small-group advantage will be in the region of $1 \leq n \leq 3$ and therefore sometimes undetectable.

3 Empirical demonstrations

In wisdom of crowd studies it is typically assumed, and empirically confirmed, that increasing group size will lead to higher accuracy, at least for non-interacting individuals with less dependent decisions. For example, studies on the accuracy of physician's detection performance of various conditions such as breast or skin cancer (e.g., Kurvers et al., 2016) almost invariably show a monotonically increasing accuracy with group size (e.g., Kurvers et al., 2018). These studies, however, rely mostly on accepted tests and criteria that have average individual hits and correct rejections well above .5. There are, however, other predictive and diagnostic situations where the average individual hits and correct rejections are on the opposite sides of .5. Figure 2 shows four examples: violent recidivism ($h = .37$, $cr = .88$, $br = .15$; Rice & Harris, 1995), physician's detection of psychiatric morbid-

FIGURE 1: Simulation results for H , CR , and P as a function of group size n for examples of the first four cases in Equation 2. White circles are the minimum values of P_n for $n \leq 101$. Gray circles are the maximum values of P_n for $n \leq 101$. Black circles are the asymptotic values of P_n for $n = 10,001$.

FIGURE 2: Simulation results based on empirical H , CR , and P as a function of group size n . White circles are the minimum values of P_n for $n \leq 101$. Gray circles are the maximum values of P_n for $n \leq 101$. Black circles are the asymptotic values of P_n for $n = 10,001$.

ity among cancer patients ($h = .29$, $cr = .85$, $br = .36$; Fallowfield, Ratcliffe, Jenkins, & Saul, 2001), physician's detection of anxiety among dermatology patients ($h = .36$, $cr = .86$, $br = .17$; Dalgard et al., 2018), and lie detection in a laboratory setting ($h = .35$, $cr = .54$, $br = .50$; Reinhard, Greifeneder, & Scharmach, 2013). The first three examples have $h + cr > 1$ and therefore a small-group advantage is observed for these cases. The lie detection example, however, has $h + cr < 1$ and a small-group disadvantage is observed.

4 Leader-follower convergences

So far, I have assumed that individual decisions are independent from each other. However, in many real-world situations this does not hold. Therefore, it is essential to also understand the effects of correlations on the accuracy of group decisions. I implemented dependencies among individual decisions with a leader-follower model (Boland et al., 1989; Galesic et al., 2018; Kao & Couzin, 2014; Spiekermann & Goodin, 2012). The leader-follower model has a parameter r that determines the proportion of group members that follow the leader, effectively controlling the correlations between group members. I implemented the leader-follower model as an extension of Equation 1 with expressions that allow for the combined effects of H_n (here depending on h and r), h , CR_n (here depending on cr and r), cr , leader hits h_l , leader correct rejections cr_l , and br . First, define the probabilities in the cumulative binomial distribution as $h' = h(1 - r) + r$, $h'' = h(1 - r)$, $cr' = cr(1 - r) + r$, and $cr'' = cr(1 - r)$. Then, define $H_{n,h'}$, $H_{n,h''}$, $CR_{n,cr'}$, and $CR_{n,cr''}$ as the corresponding values of the group hit rates and correct rejections for n and r . The proportion correct for a certain group size is then:

$$P_n = br[h_l H_{n,h'} + (1 - h_l) H_{n,h''}] + (1 - br)[cr_l CR_{n,cr'} + (1 - cr_l) CR_{n,cr''}] \quad (3)$$

Extensive computer simulations showed that across the whole range of r , the superiority of moderate group sizes still holds (not shown).

Here I go further and provide convergence properties for overall proportion correct $\bar{P}$, assuming equal proportion of hits and correct rejections for the leader and the group members. Convergence results can be derived by noting that for a single task in the leader-follower model, P converges to p_l if $r > (p - 0.5)/p$ (Goodin & Spiekermann, 2012). First, simplify the right side of the inequality to $1 - (0.5/p)$. Then, substitute p with the minimum or maximum value of h and cr or their complements $h' = 1 - h$ and $cr' = 1 - cr$ (contrary to p in the previous inequality, h and cr range between zero and one). The convergence properties are summarized in Table 1.

Thus, P will converge to p_l , 0, 1, br , or $1 - br$ depending on the values of h , cr , and r . If the values of r are between the boundaries specified in Table 1, the convergence of P can be calculated by using asymptotic values in Equation 3. For example, if the term $H_{n,h'}$ is larger than .5 it will converge to 1 and if the term is below .5 it will converge to 0.

Figures 3 and 4 illustrate the effects of r for the same four cases shown in Figure 1. In all panels of Figure 3 the value of r was set to be lower than .167, which is the lower boundary of r for all combinations of h and r in the different panels. In all panels of Figure 4, the value of r was set to be larger than .375, which is the upper boundary of r for all combinations of h and r in the different panels. The average proportions correct $p_l = \bar{p}$ were .72, .28, .64, and .36 (calculated with Equation 2) for the four different combinations of br , h and cr .

Figure 3 shows that a value of r of .15 has little effect on proportion correct P compared to the case with independent decisions illustrated in Figure 1: The advantages and disadvantages show the same patterns, although the convergence rates are lower. The one difference is that in Figure 3D, there are both an individual advantage and a large group advantage as there are two maxima at $n = 1$ and when n goes to infinity (i.e., P_1 and the leader's proportion correct, p_l , are the same). When $r = .5$, the patterns change. In Figure 4A we observe a small group advantage, while Figures 1A and 3A show a large group advantage. Figure 4B shows a small group disadvantage, while Figures 1B and 3B show an individual advantage. Figure 4C shows a small group advantage, as do Figures 1C and 3C. Figure 4D shows a small group disadvantage, the same as in Figures 1D and 3D.

5 Discussion

In this paper I showed that a small-group advantage can be achieved for a single task, given that the average proportion correct for a single individual is above .5, the individual competencies are asymmetric around .5 (i.e., h and cr are on the opposite sides of .5 and not equidistant from .5) and $h + cr > 1$. If $h + cr < 1$ a small-group disadvantage is observed. The empirical demonstrations showed that these patterns can occur in real-world decision situations. Finally, I showed that introducing correlations between decisions can have both detrimental and beneficial effects when it comes to a small-group advantage.

The results on this paper show that When planning for the size of committees for real-world decisions, it is important to consider not only overall proportion correct, but also hits, correct rejections, and the degree of correlation between decisions. For example experts deciding if a convicted criminal is likely to reoffend can have low hit rates and high correct rejection rates (Rice & Harris, 1995). In situations such as these, a small group of experts might outperform a larger one

TABLE 1: Leader-Follower convergences as $n \rightarrow \infty$. $h' = 1 - h$ and $cr' = 1 - cr$.

$if\ h > .5\ and\ cr > .5:$	$if\ r < 1 - \frac{0.5}{\min(h, cr)}$	$then\ P = 1,$	$if\ r > 1 - \frac{0.5}{\max(h, cr)}$	$then\ P = p_l$
$if\ h < .5\ and\ cr < .5:$	$if\ r < 1 - \frac{0.5}{\min(h', cr')}$	$then\ P = 0,$	$if\ r > 1 - \frac{0.5}{\max(h', cr')}$	$then\ P = p_l$
$if\ h > .5\ and\ cr < .5:$	$if\ r < 1 - \frac{0.5}{\min(h, cr')}$	$then\ P = br,$	$if\ r > 1 - \frac{0.5}{\max(h, cr')}$	$then\ P = p_l$
$if\ h < .5\ and\ cr > .5:$	$if\ r < 1 - \frac{0.5}{\min(h', cr)}$	$then\ P = 1 - br,$	$if\ r > 1 - \frac{0.5}{\max(h', cr)}$	$then\ P = p_l$

FIGURE 3: Effects of $r = .15$ on P for the same values of h , cr , and br as in Figure 1. p_l is the leader's proportion correct. White circles are the minimum values of P_n for $n \leq 101$. Gray circles are the maximum values of P_n for $n \leq 101$. Black circles are the asymptotic values of P_n for $n = 10,001$.

(see Figure 2A, 2B, 2C). However, if $h + cr < 1$ a small group of experts is less accurate than a randomly chosen expert and a larger group of experts (See Figure 2D). More generally, past information about the pattern of hits and correct rejections can give clues as to what is to be expected: large-group advantage, individual advantage, small-group advantage, or small-group disadvantage.

My results also show the importance of dependencies among individual decisions. A high correlation can produce a small-group advantage even when hits and correct rejections are both above .5, but this same correlation diminishes the small-group advantage found when hits and correct rejections are asymmetric around .5. In some real-world decision situations, such as jury deliberations, it might not be the case that the hits and correct rejections which are asymmetric around .5 will produce a small-group advantage. The nature of the decision situation would nevertheless be conducive to such an effect as the information sharing among individuals would result in dependencies among individuals

which can produce a small-group advantage even when the hits and correct rejections are above .5. As with decision situations without dependencies among individuals, knowing the hits, correct rejections, and the level of dependencies can give clues as to what is to be expected in terms of accuracy when group size increases and how to best determine the size of a committee. For example, the boosting of a small-group advantage when there are high correlations among individual decisions might provide an additional mechanism that can help explain contradictory findings of small or negative effects of group size on accuracy in interacting groups.

In this paper I focused on proportion correct, hits, and false alarms. There are many other measures of accuracy such as positive predictive value, negative predictive value, positive likelihood ratio, diagnostic odds ratio and so on (for an overview, see Singh, 2013). It would be worthwhile to investigate the prevalence of large-group advantage, individual advantage, small-group advantage, or small-group disadvantage for these measures too. However, if the mea-

FIGURE 4: Effects of $r = .5$ on P for the same values of h , cr , and br as in Figure 1. p_l is the leader's proportion correct. White circles are the minimum values of P_n for $n \leq 101$. Gray circles are the maximum values of P_n for $n \leq 101$. Black circles are the asymptotic values of P_n for $n = 10,001$.

asures include components that are monotonically related to hits and false alarms, for example, the diagnostic odds ratio one would expect that such measures also would exhibit similar patterns as proportion correct with asymmetric hits and correct rejections.

The demonstrations in this paper used the binomial distribution function, assumed equal competencies across individuals, and implemented dependencies between individuals in a simple leader-follower model. All these assumptions and model choices in the simulations can be challenged. One crucial assumption that needs to be further investigated is different assumptions made about the dependencies among individuals. For example, in the leader-follower model used here, the parameter r can never cause the correlations among individuals to be negative and the competencies for the leader were the same as for the individuals in the group.

The results in this paper are only applicable to categorical decisions. If continuous judgments are aggregated and the resulting group error is measured with mean squared error (or any convex loss function), a larger group will always have smaller error than the groups average individual. This follows from what has been called the “averaging principle” (Larrick & Soll, 2006), the “diversity prediction theorem” (Page, 2007) or the “ambiguity decomposition” (Krogh & Vedelsby, 1995): the error of an aggregate estimate is equal to the difference between average individual error and the variability of the individual estimates around the aggregate

estimate, or their diversity. In other words, for continuous judgments the group will be better than the average individual as long as there is some diversity in the individual estimates. As I show here, this is not always the case for categorical decisions.

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