Testing the magnitude of correlations across experimental conditions

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Abstract

Comparisons between correlation coefficients are used to investigate data across multiple research fields, as they allow investigators to determine different degrees of correlation to independent variables. Such differences may be small, even with adequate sample size, but still scientifically relevant. To date, although much effort has gone into developing methods for estimating differences across correlation coefficients, adequate tools for variable sample sizes and correlational strengths have yet to be tested. The present study evaluated four different methods for detecting the difference between two correlations and tested the adequacy of each method using simulations with multiple data structures. These methods were Cohen's q , Fisher's method, linear mixed-effects models (LMEM), and an ad-hoc developed procedure that integrates bootstrap effect size estimation. Results showed that Fisher's method and the LMEM tended to reject the null hypothesis even in the presence of relevant differences between correlations and that Cohen's method was not sensitive to data structure. Bootstrap followed by effect size estimation resulted in a favorable, unbiased compromise for estimating quantitative differences between statistical associations and producing outputs that could be easily compared across studies.

1. Introduction

Comparing statistics is a frequent point of contention among researchers. The need to compare correlations is common and requires a specific assay to determine whether a continuous variable, often called covariate, has a different degree of correlation between two sets of data. Examples of fields in need of such an assay include cognitive psychology (e.g., correlation between the degree of task automation and behavioral performance in extroverted vs. introverted individuals), social psychology (e.g., correlation between social exclusion and job satisfaction in men vs. women), and cognitive neuroscience (e.g., correlation between brain activity and behavioral performance under negative vs. positive emotional stimulation). Calculated differences may be small, even with the recommended sample size, but can still be scientifically relevant (Ellis, 2010). Furthermore, modern science is gradually moving away from p -centric data interpretation towards effect-size-oriented approaches (Kelley & Preacher, 2012; Sullivan & Feinn, 2012). As such, reporting only r and p values and binarizing result interpretations as either significant or non-significant, depending on an (eventually corrected) threshold of $p < 0.05$, has become outdated (Nichols et al., 2017; Ioannidis, 2019).

The existence of a statistical association cannot be relied upon to evaluate whether the strength of the relation between two variables will always be the same. In a within-group design, one of the correlation coefficients between the outcome (DV) and the covariate (IV) may be more diminished in a specific experimental condition B than in another condition A. Alternatively, in a between-group design, the treatment group

may show a weaker correlation between the outcome and the covariate than in the control group, or vice versa. These context-dependent or group-dependent effects on the extent of the correlation between two variables should be investigated using the most suited methods so that scientists from different disciplines can assess the most fitting comparison between the two correlations. Of note, given that null hypotheses are always false when evaluated with large datasets (Cohen, 1990), and that both small samples and large samples can convey useful information (Lindquist et al., 2012; Friston, 2013), the effect of a correlational change should be investigated not only with various correlation values, but also various sample sizes. However, even though much effort has been invested into developing methodologies that can estimate differences across correlation coefficients, studies investigating this problem using a comprehensive approach and variable sample sizes have yet to be published. Several strategies have been developed to estimate differences between correlations. The simplest method was proposed by Jacob Cohen (Cohen, 1988), and estimated an effect size as the difference between two Fisher-transformed correlations. Fisher's method (Fisher, 1921) also accounts for sample size, and calculates the probability that two correlations will be different given their strength and the number of samples in the two groups. Whereas Cohen's and Fisher's methods rely exclusively on r values, ignoring the initial data structure, analysis of covariance (ANCOVA) and linear mixed-effects models (LMEM) retain this information. ANCOVA and LMEMs have been widely adopted for analyzing data in cognitive neuroscience experiments, wherein the parameters observed are affected by multiple factors (Garrett et al., 2010; Zilles & Amunts, 2013).

In a recent neuroimaging study, a bootstrap approach followed by calculation of the effect size was used to detect changes in correlations between neurophysiological parameters and behavioral performance across experimental conditions (Di Plinio et al., 2018). This assay allowed testing of the hypothesis that the association between functional connectivity across brain regions and behavioral performance (Hampson et al., 2010) was weakened by negative emotional stimulation. Finally, structural equation modelling has also been proposed for testing independent or dependent correlational hypotheses (Cheung & Chan, 2004). This method is grounded in confirmatory factor analysis and is useful when data includes both dependent and independent measures. However, this approach may be ill-suited to small sample sizes and is more appropriate for meta-analytic designs (Cheung & Chuan, 2005; Cheung, 2014).

In this paper, four different approaches are examined to evaluate their power to detect an effect. The term *effect* in this study refers to “a change in the correlation between two conditions or two groups.” Although *p*-values are undergoing a theoretical revision by the scientific community, they still provide a universally recognized statistic (Greenland, 2019; Goodman, 2019). As such, both *p*-values and effect estimates are reported for each method. The methods examined in this study are Cohen's' *q*, Fisher's method, LMEM, and bootstrap with effect size estimation. A series of simulations were implemented so that the four methods could be tested in an environment wherein correlational strength and sample size vary from one cycle to the next. Method performances are then discussed.

2 Materials and Methods

2.1. Simulation parameters

In the simulations used in the present study, values of the first correlation coefficient, r_1 , occurred in the interval $[-0.5 \ 0.5]$ in steps of 0.01. For each value of r_1 , the second correlation value r_2 occurred in the interval $[(r_1 - 0.5) \ (r_1 + 0.5)]$ in steps of 0.01. For each cycle, N samples were simulated for each condition. As such, N will be the number of samples considered. N varied from 10 to 180, in steps of 2. As our focus was on the comparison of repeated measures (within-subject design without missing data), the number of samples for each condition was set to be equal; that is, $N = n_1 = n_2$.

Correlation strengths and sample sizes were chosen in accordance with cognitive psychology- and neuroscience-like scenarios, but results can be extended to other research fields such as medicine and social psychology. Data were processed using MATLAB 9.2 (The Math Works Inc., Natick, MA). Statistics obtained for each method were averaged across values of r_1 and zero-centered. Each plot shows the average statistics (e.g., p -value, effect size) with respect to increasing values of r_2 and N , and across values of r_1 .

For LMEM and bootstrap models, sample data and covariate values were randomly generated and normally distributed, using the MatLab function *randn*. This led to the creation of two distributions for the dependent variable (DV), namely $\mathbf{X}_1$ and $\mathbf{X}_2$, with a mean $\mu_1 = \mu_2 = 0$ and a standard deviation $\sigma_1 = \sigma_2 = 1$. Independent variable (IV) and N covariate values were simulated for each condition and computed to yield the desired

value of correlation with the DV. The covariates γ_1 and γ_2 were obtained as follows:

$$\gamma_i = \sigma_\gamma \left(r_i * \mathbf{x}_i + \sqrt{1 - r_i^2} * \Psi_i \right) + \mu_\gamma \quad (1)$$

where μ_γ and σ_γ are the desired mean and standard deviation of the covariate (with $\mu_\gamma = 0$ and $\sigma_\gamma = 1$), respectively, while Ψ is a pseudo-random set of N normally distributed values. This procedure was based on the Cholesky decomposition, which is commonly used in Monte-Carlo simulations of multiple correlated variables (Press et al., 1992).

2.2. Methods

2.1.1. Cohen's q

Cohen proposed a simple method for interpreting the difference between two correlations (Cohen, 1988). Initially, to reduce skewness (asymmetry derived from the definition of r_i in the interval $[-1, 1]$), r values were transformed to z values via the Fisher procedure:

$$z_i = (0.5) \log \left(\frac{1+r_i}{1-r_i} \right) \quad (2)$$

Then, the absolute value of the difference between the two z values was computed, such that $q = |z_1 - z_2|$. The value q is the estimate of the effect size. The following intervals were proposed by Cohen to interpret these values: $q < 0.1$, *no* effect; $0.1 \leq q < 0.3$, *small* effect; $0.3 \leq q < 0.5$, *medium* effect; $q \geq 0.5$, *large* effect. Since no p -value is associated with Cohen's method, the only statistic reported is the effect size q .

2.1.2. Fisher's method

Fisher's method (Fisher, 1921) is used to calculate the probability of two correlations being different, given the differences between r values and the size of the two samples.

The null hypothesis is that the correlation between $\mathbf{X}_1$ and $\mathbf{y}_1$ will be the same as the correlation between $\mathbf{X}_2$ and $\mathbf{y}_2$ for sample sizes n_1 and n_2 . Correlation values r_1 and r_2 are converted to z values as described in Equation 2. The test statistic t is then calculated:

$$t = \frac{(z_1 - z_2)}{\sqrt{1/(n_1 - 3) + 1/(n_2 - 3)}} \quad (3)$$

Finally, using the cumulative distribution function of t in a standard distribution with mean $\mu_t = 0$ and standard deviation $\sigma_t = 1$, the p -value is calculated to assess whether the null hypothesis can be trusted or not. Statistics reported for Fisher's method are the p -value and t -statistic.

2.1.3. Linear mixed-effects model

Linear mixed-effect models (LMEM) were applied in the form [DV ~ IV*condition +(1|subject)] and thus included a fixed effect (the experimental condition), a continuous effect (the covariate IV), and a random intercept at the subject level (1|subject) to account for inter-individual variability. This type of model is applied frequently in psychology and neuroscience. At each cycle, a model was fitted using the MatLab function *fitlme* and the p -value and β statistics for the interaction between the experimental condition and the covariate were extracted.

Among applicable generalized linear models, the choice of the LMEM over, say, ANCOVA is due to the former's increased flexibility and sensitivity (Schneider et al., 2015; Brysbaert & Stevens, 2018). Of note, as the aim of the present work is to investigate the power to predict an *effect*, corrections for multiple comparisons were unnecessary. Furthermore, since within-subject variability at the interaction level (subject : covariate) was not simulated, the introduction of random slopes was not necessary for the purposes of the study.

2.1.4. Bootstrap method and effect size estimation

The bootstrap method is a resampling technique often used to estimate confidence intervals and allows one to approximate the sampling distribution of a statistic (Efron & Tibshirani, 1986). In this study, we used a univariate, bias-corrected, accelerated bootstrap with replacement (Efron, 1987) to sample the correlation value r_i . A sampling distribution was obtained by resampling the original data k times and obtaining k samples with sizes equal to the starting sample ($N_k = N$); that is, k is the number of bootstrap cycles. A similar sampling approach has been described for correlations (see Olkin & Finn, 1990, 1995); however, the method presented here implements a bias-corrected bootstrap procedure that can accommodate small sample sizes and outliers and includes an effect size estimation statistic.

Each cycle was bootstrapped by estimating the correlation between the DV and covariate for each condition. Individually bootstrapping each correlation allows the estimation of the bootstrapped effect size for each condition, which may be useful for descriptive purposes. After k bootstrap cycles, two distributions of correlations were

obtained for each condition and transformed into z-values. Each distribution possessed an associated mean and standard deviation (μ_{z1} , σ_{z1} ; μ_{z2} , σ_{z2}). These distributions were then used for analyses. For each cycle (i.e., for each pair of r_2 and N values), the difference between the two z-distributions was represented as indicated in Equations 4 and 5, with the effect size estimated using Cohen's d (Lipsey & Wilson, 2001; Ellis, 2010):

$$ES_{12} = d = \frac{\mu_{z1} - \mu_{z2}}{\sigma_{\text{pooled}}} \quad (4)$$

where

$$\sigma_{\text{pooled}} = \sqrt{\frac{(n_1 - 1) \sigma_{z1}^2 + (n_2 - 1) \sigma_{z2}^2}{n_1 + n_2 - 2}} \quad (5)$$

A Z-test was performed comparing the distribution obtained by subtracting the two bootstrapped correlation distributions against a zero-centered distribution, to prevent biases caused by large sample sizes in estimating a p -value. Effect sizes (ES) can also be interpreted in terms of the percentage of nonoverlap of the first group's scores with those of the second group (Cohen, 1988). For example, ESs of 0.0, 0.8, and 1.7 indicate that the distribution of scores for the first group overlapped with the distribution of scores for the second group with 0, 47.45, and 75.4% of nonoverlap, respectively.

3. Results

Results of the simulations are reported in Figures 1-4. In these figures, the x- and y-axes represent sample size and correlation difference, respectively, between r_1 and r_2 .

Values are averaged across different levels of r_1 tested. Total time to run the simulation

was approximately 75 hours on a personal computer with eight 2.40GHz core processors.

Figure 1 shows the results of Cohen's q method, which returns an estimate of the effect (q) independent from the sample size N , reflecting only the difference between correlations. Empirically, small, medium, and large effects are defined based upon the magnitude of q . The interpretation of the result needs to be contextualized, however; a *large* difference between two correlations in a small sample ($N < 10$) is not systematically trustworthy. Conversely, a *small* effect might still be an important one (Rosnow & Rosenthal, 1989).

Figure 2 show p -values and t -statistics obtained using Fisher's method. The test did not return significant values (Figure 1b, red) for relatively small sample sizes, even those with large differences between r_1 and r_2 ($\Delta r = 0.4$). For larger samples ($N = 40$), however, this method accepted the null hypothesis as indicated by $\Delta r = 0.15$. In the formula used for the t -statistic, sample size increases cause the test statistics to increase logarithmically, with the p -values showing a logarithmic decrease.

Results of the mixed methods analysis (LMEM) are reported in Figure 3. Like ANCOVA, mixed models are likely to accept the null hypothesis even in the presence of a large difference between r_1 and r_2 , as it interprets this difference as not being significant. This happens even with adequate sample sizes, such as an N of 40, by neuroscience standards. An advantage of LMEMs, however, is that the means and standard deviations of the original data are retained. Results thereby depend upon how the data points are distributed, and graphs tend to be more scattered than those described in previous paragraphs.

The p -value and effect size d obtained with bootstrapping simulations are reported in Figure 4. Like in previous methods, N and r_2 vary while r_1 is fixed. The number (k) of random samplings was set to 200 ($k = 200$). Equivalent results were obtained for pilot simulations with $k = 500$ and $k = 1,000$; however, as these simulations included a reduced number of cycles for computational purposes, their results are not included here. As for Cohen's method, the bootstrap approach provides an effect size estimation but not a p -value. Moreover, the procedure is sensitive to variability in the data, as indicated by the smoothness of the colors.

A post-hoc comparison among p -values gathered using Fisher, LMEM, and bootstrap methods is reported in Figure 5. The bootstrap method was quite unaffected by the sample size, accepting more frequently the null hypothesis only with very small samples (e.g., $N < 20$). Conversely, both Fisher and LMEM accepted the null hypothesis even with large differences between correlations, whereas the inverse bias was observed with large samples.

Since it may be pointed out that the results described until now may be based on averages across values of r_1 , an example with a specific value of r_1 ($r_1 = 0.30$) is illustrated in Figure 6. On the one hand, these results confirm the observations made until this point: Cohen's method ignores sample size; Cohen's and Fisher's methods do not account for variability in the data structure; Fisher's method and LMEM tend to accept the null hypothesis even with large differences in the correlational strength. However, the presence of overestimated effect sizes (see for example some points with $N < 20$ in Figure 6f for LMEM) shows that the variability in the data structure may endanger the estimation of fixed-effects statistics in mixed-models (Faraway, 2006)

probably falling on Simpson's paradoxes (Good & Mittal, 1987). The bootstrap procedure still accounted for variability in the data, but these paradoxical cases were not observed. This probably happened because the bootstrap procedure limits the pitfalls of classical inferences methods (Killeen, 2005).

4. Discussion

The present study compared vintage and modern statistical methods used to evaluate differences across correlations in the fields of psychology, medicine, and related disciplines.

A direct comparison between correlation coefficients as provided by Cohen's method can be useful, given that the calculation only requires the two correlation values (Cohen, 1988). However, Cohen's method estimates the effect magnitude irrespectively of sample size. Conversely, the effect estimated by Fisher's method increases with increasing sample size. Although these two methods can be useful when the original data structure is unavailable, neither Fisher's nor Cohen's method consider potential variability in the data. As such, the effects estimated with these methods should be carefully contextualized and interpreted.

It has been suggested that LMEMs may generate p -values that are too small, possibly overestimating the importance of a given effect (Faraway, 2006). The primary purpose of the LMEM is to use data from a continuous variable to estimate differences between levels of a fixed factor, including repeated-measures or longitudinal scenarios (Robinson, 1991). Given the results presented here, LMEM (and other generalized

linear models, like the ANCOVA) may be useful for explorative purposes but may not be the best choice if the aim is to test the difference between the two correlations. In fact, the presence of overestimated effect sizes for LMEM suggested possible miscalculation on fixed-effects coefficients in mixed-models (Faraway, 2006; Good & Mittal, 1987).

The bootstrap procedure followed by a test to determine effect size accurately estimated the difference between the strength of the two correlations. Key features of this method are that it considers the sample size and the variability of the initial data, returns a descriptive measure of the difference between the two correlations, and provides a p -value not biased by too small or too large sample, and not affected by the pitfalls of classical inferences methods (Killeen, 2005).

Although Fisher, LMEM, and bootstrap methods returned a p -value, which is traditionally used to assess the presence or absence of an effect, the debate about how and even if these statistics should be used persists (Goodman, 2019). Certain seminal cognitive neuroimaging studies dealt with relatively small samples (Lindquist et al., 2012; Friston, 2013) due to logistical constraints, and such samples may not be large enough to detect significant differences using linear regression (LMEM). Recently, p -values have been shown to be misleading measures of the strength of the evidence against the null hypothesis (Berger & Selke, 1987; Hupé, 2017); also, they do not directly provide an index of effect magnitude (Sullivan & Feinn, 2012). The fact that the bootstrap approach provided a descriptive effect size which was close to the effect estimated by mixed-models underlines its effectiveness and is in line with the trend towards p -independent assays in psychology (Ellis, 2010; Kelley & Preacher, 2012; Nichols et al., 2017). Furthermore, the implementation of a z -test provided an

unbiased, universally accepted null-hypothesis statistic that may still be useful for p -generation researchers (Ioannidis, 2019; Greenland, 2019). Bootstrapping correlations have been discussed extensively in the literature, and researchers have noted that monotonic, transformation invariant procedures like the bootstrap are ill-suited to estimating confidence intervals or testing a null hypothesis (Lunneborg, 1985; Efron & Tibshirani, 1986; Rasmussen, 1987; Strube, 1988; Efron, 1988; Olkin & Finn, 1995). The procedure presented here and compared with other methods, however, represents a slightly new approach that combines bootstrap with effect size estimation and null-hypothesis testing.

5. Conclusion

The present study evaluated the efficacy of four different methods for investigating differences in correlations across experimental conditions. Bootstrapping followed by effect size estimation was the most successful, providing a statistic that accounted for both inter-individual and sample size variability in comparing correlation coefficients between experimental conditions. This method is easily implementable in MatLab through the *bootes* function made available online by the author at MathWorks. Although these findings have implications for researchers interested in comparing the magnitude of correlations between different experimental conditions, this study has two important limitations that must be acknowledged. First, the series of simulations implemented here only considered normally-distributed sample data with zero mean or unitary variance. Second, the bootstrap procedure presented here works well for

within-subject analyses and may be applied without complications to those between subjects, but is not yet applicable to experimental designs including both between- and within-subject factors. Future studies should evaluate these and other methods in such alternative situations, to uncover other easily-implemented, bias-free tools for researchers in the fields of psychology, neuroscience, and medicine.

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FIGURES AND CAPTIONS

Figure 1. Results of the simulations using Cohen's q . Effect sizes are averaged across values of r_1 . Black horizontal lines are used to separate the different levels of effect: *small* (blue), *medium* (cyan), *large* (green). Sample size is represented in the horizontal axis; values of r_2 with respect to r_1 are represented in the vertical axis.

Figure 2. Results of the simulations using Fisher's method. Statistics are averaged across values of r_1 . T -statistics are reported on the left panel, p -values on the right panel. Sample size is represented in the horizontal axis; values of r_2 with respect to r_1 are represented in the vertical axis. Dashed and dotted lines represent thresholds of $p < 0.05$ and $p < 0.01$, respectively.

Figure 3. Results of the simulations using mixed-models (LMEM). Statistics are averaged across values of r_1 . β values for the interaction *condition : covariate* divided by their standard errors (SE) are reported on the left panel, corresponding p -values are reported on the right panel. Sample size is represented in the horizontal axis; values of r_2 with respect to r_1 are represented in the vertical axis.

Figure 4. Results of the simulations using bootstrap (Boot+ES). Statistics are averaged across values of r_1 . Effect sizes (d) are reported on the left panel, corresponding p -values are reported on the right panel. Sample size is represented in the horizontal axis; values of r_2 with respect to r_1 are represented in the vertical axis.

Figure 5. Comparison among Fisher (in blue), LMEM (in red), and bootstrap (in green) p -values. Thresholds of $p=0.05$ (dashed lines) and $p=0.01$ (dotted lines) are represented for each method. For graphical clarity, only the lower half of the values are reported here.

Figure 6. Results of the simulations using the four methods, with $r_1=0.30$. Dashed and dotted lines represent thresholds of $p < 0.05$ and $p < 0.01$, respectively. Sample size is represented in the horizontal axis; values of r_2 with respect to r_1 are represented in the vertical axis. **(a)** Cohen's method. **(b-c)** Fisher's method. **(d-e)** LMEM. **(f-g)** Bootstrap followed by effect size estimation.