Language-specific numerical estimation in bilingual children

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Abstract

We tested 5- to 7-year-old bilingual learners of French and English ($N = 91$) to investigate how language-specific knowledge of verbal numerals affects numerical estimation. Participants made verbal estimates for rapidly presented random dot arrays in each of their two languages. Estimation accuracy differed across children's two languages, an effect which remained when controlling for children's familiarity with number words across their two languages. Also, children's estimates were equivalently well ordered in their two languages, suggesting that differences in accuracy were due to how children represented the relative distance between number words in each language. Overall, these results suggest that bilingual children have different mappings between their verbal and non-verbal counting systems across their two languages and that those differences in mappings are likely driven by an asymmetry in their knowledge of the structure of the count list across their languages. Implications for bilingual math education are discussed.

Keywords: Numerical cognition, number estimation, bilingualism, cognitive development

Introduction

The human ability to encode number in natural language has important consequences for our species, allowing us to perform exact numerical computations, construct models of the physical world, and to make precise predictions regarding distant places and hypothetical situations. Human cultures that lack labels for quantities greater than three or four are generally unable to represent larger sets exactly (e.g., Coppola, Spaepen, & Goldin-Meadow, 2013; Dixon, 2004; Epps, 2006; Frank, Everett, Fedorenko, & Gibson, 2008; Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004; Spaepen, Coppola, Spelke, Carey, & Goldin-Meadow, 2011; Spaepen, Coppola, Flaherty, Spelke, & Goldin-Meadow, 2013). Similarly, young children who have not yet learned to count are restricted to representing and comparing larger sets approximately (e.g., Feigenson, Dehaene, & Spelke, 2004; Wynn, 1990, 1992). Although body count systems (Ifrah, 2000; Menninger, 1969), ancient counting boards and abaci (Frank & Barner, 2012; Hatano, Miyake, & Binks, 1977) and non-verbal cognitive systems like the evolutionarily ancient “approximate number system” (Dehaene, 1997) all provide ways for humans to encode and reason about numerosity, language affords humans a distinctly flexible and productive format for describing and manipulating number, and, by some accounts, may be the conceptual basis for the creation of other symbolic representations of number (e.g., Chomsky, 2008; Di Sciullo, 2012; Spelke, 2017; Watanabe, 2017).

Like many non-human animals, humans are able to represent the approximate magnitudes of large sets independent of language using the approximate number system (ANS). For example, human infants, pigeons, rats, and even fish can discriminate different quantities of objects on the basis of their numerical ratio, consistent with Weber’s law (Agrillo, Dadda, Serena, & Bisazza, 2008; Platt & Johnson, 1971; Scarf, Hayne, & Colombo, 2011; Xu & Spelke, 2000). The acuity of

these representations improves quickly in infancy (Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005) and continues to change over the first 20 years of life, into adulthood (Barth, Kanwisher, & Spelke, 2003; Halberda & Feigenson, 2008). Still, even in adults, many small exact differences that we can represent in language, such as 87 vs. 88, cannot be discriminated by the ANS, suggesting that this system alone cannot explain the exact nature of linguistic numerical representations. Such findings raise the question of how language might allow humans to go beyond the limits of the ANS and to represent large exact magnitudes.

Several recent studies have argued that exact number representations (e.g., number word meanings, addition facts, or multiplication tables) are both language-dependent (meaning that they do not occur in absence of language), and language-specific (meaning that, in bilinguals, representations in one language are not automatically available to other languages), whereas approximate representations and computations are independent of natural language, such that training in one language generalizes automatically to a second language. For example, Spelke and Tsivkin (2001) trained Russian-English bilingual adults in one of their two languages on four types of mathematical problems, two of which required exact numerical responses (e.g., addition of multi-digit numbers), and two of which required approximate solutions (e.g., computing cube roots). After training, participants responded more slowly in their untrained language to questions that required precise answers (indicating language specificity), whereas there was no effect of language for questions that required approximate answers (for similar results, see Saalbach, Eckstein, Andri, Hobi, & Grabner, 2013). Similarly, in a neuroimaging study of bilingual adults, Dehaene et al. (1999) found that problems requiring exact computations, like multiplication tables, recruited areas of the brain involved in word-association processes, whereas approximate number tasks, like summing rapidly displayed arrays of dots, relied on areas of the brain associated with

visuo-spatial processing, rather than language, suggesting language-dependence (for similar results in monolinguals, see Dagenbach and McCloskey, 1992; Dehaene & Cohen, 1997; Dehaene, Molko, Cohen, & Wilson, 2004; Delazer & Benke, 1997; Domahs & Delazer, 2005; Ischebeck et al., 2006; Lampl, Eshel, Gilad, & Sarova-Pinhas, 1994; Lee, 2000; Lemer, Dehaene, Spelke, & Cohen, 2003; Pesenti, Seron, & Van Der Linden, 1994; van Harskamp & Cipolotti, 2001; van Harskamp, Rudge, & Cipolotti, 2002).

While it is clear that approximate and exact linguistic representations of number arise from distinct learning processes and rely on different cortical regions of the brain, previous studies leave open the mechanisms by which the systems become related, and to what degree mappings between systems might differ across a bilingual learner's languages. In order to estimate the number of dots presented on a screen - an ability which emerges in development sometime after the age of 4 - children must relate symbolic representations of number - i.e., words in their verbal count list - to non-symbolic ANS values (Barth, Starr, & Sullivan, 2009; Gunderson et al., 2015; Le Corre & Carey, 2007; Lipton & Spelke, 2005; Mejias & Schiltz, 2013; Siegler & Booth, 2004; Sullivan & Barner, 2014; Wagner & Johnson, 2011). While much is known about changes in children's estimation abilities over development, less is known about the mechanisms that drive change. Changes in estimation ability could be driven by maturation of the ANS and/or acquired experience with the count list. Here, we ask how experience with a count list influences estimation performance, controlling for ANS ability. We approach this question by studying bilingual children who are acquiring two count lists (French and English). One possibility is that when children acquire two count lists (e.g., one in French and another in English), knowledge of these lists fails to transfer across languages, because the words - which differ phonologically across languages - are mapped on an item-by-item basis to individual ANS values, such that knowledge

cannot generalize to another language. Another possibility, not incompatible with the first, is that estimation doesn't depend solely on item-based associations between words and ANS values, but also draws on more general knowledge that children may glean from experience estimating, or from the structure of counting itself (Izard & Dehaene, 2008; Sullivan & Barner, 2014). This approach allows us to investigate the independent effect of language (e.g., familiarity with number words, the structure of the count list, etc) on estimation.

Although no previous study has tested these questions in the context of estimation, evidence from young bilingual children indicates that knowledge of the first number words (i.e., meanings for 'one', 'two', and 'three') does not transfer across languages, probably because these words are represented as item-based associations between words and cardinal values (Wagner, Kimura, Cheung, & Barner, 2015). Critically, however, the same study finds that the knowledge of counting procedures that allow children to count and accurately give large sets (sometimes called "cardinal principle" knowledge) does transfer across a bilingual child's two languages. This raises the possibility that, if early estimation abilities are governed chiefly by item-based associations between words and ANS values, transfer may not occur between languages. However, if estimation is governed in part by more general principles - not specific to particular words - transfer might occur. For example, a child who has extensive experience making estimates in English might notice a "later-greater" principle - that numbers later in the count list denote greater quantities (Davidson, Eng, & Barner, 2012; Le Corre, 2014), and that the relative distance between two magnitudes corresponds to the distance between their verbal labels in the count list. This type of general principle might be learned in one language and extended to a second language, thereby facilitating learning in the L2.

Understanding how estimation abilities are mediated by language in bilingual learners is not only theoretically important, but also has practical implications. First, given the global prevalence of exposure to bilingual education, it's important to know whether numerical knowledge acquired in one language will readily transfer to a child's second language, or whether educators should dedicate time to conducting training in both languages separately. For example, in the context of a bilingual immersion curriculum, children who live in a culture that requires math fluency in one language (e.g., English) but who receive their primary training in a different language (e.g., French), may benefit from learning certain foundational math skills in both of their languages. Second, previous studies have found relationships between estimation abilities and symbolic arithmetic (Booth & Siegler, 2008; Desoete, Ceulemans, De Weerd, & Pieters, 2012; De Smedt, Noël, Gilmore, & Ansari, 2013; Fazio, Bailey, Thompson, & Siegler, 2014; Holloway & Ansari, 2009; Sullivan, Frank, & Barner, 2016). Bilingual learners provide an interesting test case, because individual learners make estimates in multiple languages and yet draw on a single non-symbolic approximate number system to do so. Consequently, studying bilingual learners permits the isolation of effects of language experience on estimation, separate from, e.g., the acuity of ANS representations, and which of these components might better explain later mathematics achievement.

To explore the role of language in the development of estimation, we presented 5- to 7-year-old French-English bilinguals with a dot-array estimation task in their two languages. We also tested how high children could count in each language as a proxy of relative exposure to each language's counting system. We then asked whether children generate different estimates in their two languages in terms of (1) accuracy and (2) degree of variability (i.e., uncertainty). We also asked whether any such differences might be explained by children's knowledge of counting

structure in each language, including factors like number words access in each language, and their knowledge of the later-greater principle - i.e., whether on consecutive trials an increase or decrease in the size of a presented dot array was reflected by a verbal estimate that was earlier or later in the count list.

2. Method

Participants

In total, 93 French-English bilingual 5- to 7-year-old children participated in the study ($M = 6.8$ years, $SD = 0.8$, range = 4.9 - 8.0). This sample size represents the maximum number of children that could be tested in a 3-week field trip to Comox, British Columbia ($n = 64$), plus a smaller group of children tested at a French immersion school in San Diego, California ($n = 26$) or at a lab at UCSD ($n=3$). Two children were excluded due to failure to complete more than half of the trials, leaving a final N of 91.

The field site in British Columbia was chosen because of the availability of both L1 English-speaking and L1 French-speaking children who lived in an English-dominant environment but attended French immersion schools, resulting in high levels of balanced bilingual number knowledge. At one school, attendance was restricted to children whose caregivers spoke French as a first language ($n=34$), while the other school targeted children from English-speaking households ($n=30$). The school in San Diego ($n= 26$) served children from a variety of linguistic and cultural backgrounds (e.g., French citizens, French Canadians, and monolingual English families interested in French education). In all three schools, formal instruction was given almost exclusively in French, including math curriculum. As part of the consent process, parents received a short questionnaire which asked whether their children were bilingual. Children who were not

identified as bilingual by caregivers did not participate in the study. Participants were, in general, from medium to high SES families. The study received approval by the ethics committee of UCSD.

Procedure

Each participant completed a dot-array estimation task and a counting assessment in both English and in French. Each child received two blocks of tasks - an English block and a French block - the order of which varied between subjects. Within each block, children were always presented with the Dot Array Estimation task before the Highest Count task. In the English block, children were tested by an English-speaking researcher. In the French block, children were tested by a French-English bilingual researcher. The majority of the participants ($n=75$) were tested in both languages by the same French-English bilingual researcher and the remainder ($n=16$) were tested by two different researchers to maximize data collection in the available time. Testing lasted on average 30 minutes. All participants were tested at their respective schools, with the exception of three children who were tested at the UC San Diego Language and Development Lab.

Dot Array Estimation task

Participants were presented with dot-arrays on a computer screen. Arrays ranged in magnitude from 4 to 98. Some items were presented 8 times each (4, 16, 32, 60, 80), and to avoid boredom these trials were interspersed with other items that were presented twice each (8, 12, 24, 44, 70, 98). Stimuli were presented to each participant in two blocks of trials, where Block A and Block B contained identical items, but in different randomized orders. Each participant received both blocks in both languages, and the order of the blocks was counterbalanced. Hence, participants were assigned to one of four conditions in a 2 [English-first vs. French-first] x 2 [Block A First vs. Block B First] design.

Participants were first introduced to an Experimenter who explained that they would be tested in two languages. This sentence was presented in French for children tested with the French block first, and in English for children tested with the English block first. Next, children were introduced to the task in that block's language (e.g., in the French block, instructions were in French). Children were instructed to look at the dots on the screen and guess how many there were. All participants' responses were elicited verbally in the language of test (French or English depending on language block) and recorded by the experimenter. The experimenter provided no information to children about the range of numbers in the task. Non-informative verbal encouragement was given to the participants in order to keep them motivated (e.g., "You are very attentive"; "You already did half of the game!"). Within each language block, participants were tested for ten minutes or until the completion of all 52 trials, whichever came first.

Highest Count Task

Participants were asked to count as high as they could in English and in French. They were stopped at 100, or when they made their first error.

3. Results

Data Management

Highest Count. We recorded each child's Highest Count in each of their two languages, which was defined as the highest number counted to without making an error (with a maximum value of 100), in order to describe the counting abilities of our sample.

Estimation. Before conducting our primary analyses, we excluded all estimates that were not provided in the format of a unique verbal numeral ($n = 64/9464$), such as expressions of addition or multiplication (e.g., "*There are six plus three!*") or other non-numbers (e.g., "*Eleventy-billions*"). We also removed outlier estimates that were 10 times larger or smaller than the

presented numerosities ($n = 247/9464$) or that were at least 5 *SD* from the mean estimation for each numerosity presented ($n = 35/9153$). Except for our Ordinality analyses, all analyses reported below were performed on the remaining 9118 responses. Ordinality analyses cannot be impacted by outliers (since they consider only order and not absolute distance between estimates), and therefore outliers were not removed for this analysis.

We computed four main measures of estimation performance: Raw Estimates, Percent Absolute Error (PAE), Coefficient of Variation (COV), and Ordinality. Raw Estimates were children's numerical estimates. Consistent with previous work (Barth et al., 2009; Lipton & Spelke, 2005; Sullivan & Barner, 2014) our first set of analyses tested the relation between children's verbal estimates and the number of dots presented. To determine whether language affected estimates, analyses reported probed whether language predicted a difference in the slope of estimates (i.e., whether there was an interaction between Language x Numerosity).

The remaining three analysis types were planned in order to probe the nature of any effects of language on estimation. First, because it is possible for estimates to exhibit a slope approaching 1 despite being inaccurate (e.g., in cases where the intercept is not 0; in cases where estimates for a particular numerosity were highly variable but averaged out to the "correct" response), our second set of analyses measured accuracy as the Percent Absolute Error (PAE) of estimates, defined as the absolute distance between an estimate and its target numerosity on a particular trial:

$$PAE = \left| \frac{Estimate - Numerosity}{Numerosity} \right| \times 100$$

PAE values were log transformed to normalize a skewed distribution of residuals (to satisfy the assumptions of our linear regression analyses, below). Second, we also asked how variable estimates were, by calculating the Coefficient of Variation – i.e., the standard deviation of estimates relative to the mean estimate for a particular target:

$$COV_N = \frac{\text{Standard Deviation of Estimate}_N}{\text{Mean Estimate}_N}$$

For each subject, this generated one COV per numerosity (N) per language.

Finally, we calculated the Ordinality of participants' responses (as a measure of "later-greater" knowledge). Participants' raw estimates, COV, and PAE are all sensitive to the relative distance between their estimates. For example, if a participant sees a set of 40 dots and estimates "forty", their PAE, and raw estimate will be different than if they had estimated "forty one" (and, in most contexts, so will their COV). However, as noted by Sullivan and Barner (2013, 2014) children often make estimates that are wildly inaccurate despite nevertheless being well ordered: If a child estimated "four" after seeing 4 dots, and then saw 44 dots on the next trial, their response on this second trial would be considered ordinal whether they responded "five", "five million", or any other number greater than 4. Testing for ordinality is important because the presence of ordinal estimates would indicate that poor estimation accuracy cannot be attributed to poor knowledge of how number words are ordered, or that numbers later in the count list denote greater quantities. Instead, such evidence would implicate other factors, such as the problem of relating *distance* between numbers in the count list to corresponding differences between ANS values. We return to the significance of this distinction in the General Discussion.

To calculate the ordinality of each numerical estimate, n , we coded whether it differed from the previous trial's estimate in the correct direction. For example, if a child saw 4 dots on trial $n-1$ and then saw 44 dots on trial n , their estimate was coded as ordinal only if they provided a larger estimate on trial n than trial $n-1$.

Preliminary Analyses

For these and all analyses, models were generalized linear mixed effects models constructed in R using the 'lme4' package (Bates, Maechler, Bolker, & Walker, 2015; R Core

Team, 2019). All model outputs are available in our OSF repository (https://osf.io/f8v7b/?view_only=01910a3ad63c4fa39071617e24c10ab3).

Highest Count. Highest Count was used to measure children's familiarity with the count lists in French and in English. As shown in Figure 1, participants generally counted higher in English than in French ($t(90) = 5.59, p < .0001$). On average, children counted up to 50.5 in French ($SD = 31$; range = 3-100) and to 71 in English ($SD = 37$; range = 8-100). Fifty children counted to 100 in English while fourteen counted to 100 in French. Forty-nine children counted higher in English than in French, and 17 counted higher in French than in English. Twenty-five children were "balanced counters", in that their highest count differed by less than 10% across their two languages.

Figure 1. Highest count performance in English (black/solid/closed circle) and French (white/dotted/open circle) by age.

Estimation. We conducted preliminary analyses in order to ensure that there were no effects of Language Order (i.e., which language the child was tested in first) on estimates. To test this, we predicted each measure of estimation performance (Raw Estimates, Ordinality, PAE, and CoV) from Language Order and Numerosity; for all models, Subject and Numerosity were treated as random factors;¹ we generated p -values by comparing our full model to one that excluded Language Order. We found no effect of Language Order for any of our measures of estimation (all $p > .10$).

Because there are typically age effects on estimation performance, we also conducted preliminary analyses predicting each estimation measure from Age (z -scored), Numerosity, and their interaction. For raw estimates, there was a significant interaction of Age and Numerosity ($B = .12$, $SE = .01$, $t = 14.28$, $p < .0001$). There were significant effects of age for PAE ($B = -.25$, $SE = .03$, $t = -8.99$, $p < .0001$) and Ordinality ($B = .35$, $SE = .05$, $z = 7.04$, $p < .0001$). For COV, there were no effects of Age ($B = -.02$, $SE = .01$, $t = -1.31$, $p > 0.1$) or Numerosity ($B = .001$, $SE = .002$, $t = .93$, $p > 0.1$).

Because Language Order had no effect in preliminary analyses, we did not consider it in subsequent analyses. However, we included Age, Numerosity, and the interaction of Age and Numerosity in our Raw Estimate models. We included Age and Numerosity in our PAE models and Ordinality models. We did not include Age or Numerosity in our COV models because it had no effect in preliminary analyses.

Main Analyses

¹ Numerosity was also added as a random factor in our models in order to account for the fact that some estimation items were presented eight times while others only four times (see methods).

All analyses reported below were conducted using mixed-effects models, such that all estimates were entered into a single regression, with slopes fitted for each participant.

Our primary question was whether Language of Test impacted estimation performance. To test this, we first constructed a model predicting Raw Estimates from Age (in months), Numerosity, Language of Test, and their interactions. If estimates differ across languages, then we should find an interaction of Numerosity and Language of Test: This would indicate that there is a difference in the slope of estimates for English, relative to for French. In our main analyses, we planned to interpret the highest order interaction containing Language of Test. We found a significant three-way interaction of Age, Numerosity, and Language of Test (see Figure 2; $B = .11$, $SE = .01$, $t = 11.02$, $p < .0001$), which reflected a difference in estimation behavior between French and English language, that was greatest among younger children.

Figure 2. Relationship between numerosity and estimates in English (blue/solid/open circle) and French (red/dashed/closed circle). Panels are grouped by age in years (5YOs $n = 20$, 6YOs $n = 30$, and 7YOs $n = 41$). Data points represent binned means; bars denote bootstrapped 95% confidence intervals.

This first finding suggests that fluent bilingual children, who are immersed through schooling and their community in a French- and English-speaking environment, exhibited

significantly different estimation abilities across their two languages. However, this first analysis leaves open the precise nature of this difference. A strong interpretation of our finding is that children's estimates differ across their languages because they have made different mappings between number words and the ANS in each case. However, while our analysis found a difference in estimates across languages, it leaves open the possibility that differences were due to the fact that children accessed and used different number words in each of their languages. For example, a child who is familiar with numbers up to ~80 in one language and ~150 in the other might make similar use of the numbers up to 80 in each language - perhaps with equal accuracy or noise - but perform differently for larger numbers that they can only access in one language (e.g., whether more or less accurately).

To identify whether children made different use of the numbers that overlapped across their two languages, we first identified the child's "highest common estimate", which was defined as the highest number that the child produced *in both languages* during the estimation task (e.g., if the child's highest verbal estimate in English was 80 and in French was 150, that child's highest common estimate was 80). We reasoned that if access to number words limited performance and caused cross-linguistic differences in estimation, it should have done so only outside of this productive estimation range. Differences within the highest common estimate, however, would suggest that children's estimates differ not purely because of their differing abilities to access number words, but also because of how the words they *can* access are mapped to ANS values. In a first *post hoc* analysis² that included only verbal estimates that were within the child's highest

² All 91 participants were represented in these *post hoc* analyses, which contained 76.34% - 92.94% of the original dataset (depending on the analysis). None of the analyses on Raw Estimates, PAE and Ordinality were conducted on datasets containing fewer than 7400 trials. The dataset used for the main analysis of COV contained 1992 trials, since a unique COV was calculated for each Numerosity, in each language per child. The COV post hoc analyses were performed on datasets containing 1937 and 1634 trials.

common estimate range we once again found a three-way interaction of Numerosity, Language of Test, and Age ($B = .03$, $SE = 0.01$, $t = 3.39$, $p < .0001$). Our second *post hoc* analysis considered only trials where the presented numerosity had a value that was within the child's highest common estimate (e.g., considering only those trials on which the target numerosity was 80 or smaller). Once more, we found a three-way interaction of Numerosity, Language of Test, and Age ($B = .09$, $SE = .01$, $t = 7.56$, $p < .0001$). Thus, even when only considering trials that were within the child's estimation range for both languages, estimates differed as a function of language suggesting that bilingual children appear to have different mappings between ANS values and familiar number words across their two languages. Our next analyses address the nature of this difference.

As a first step to understanding the nature of children's different estimates across French and English, we considered estimation accuracy, as measured by the percent absolute error of estimates (i.e., log PAE). If language impacts the accuracy of estimates, then we should expect PAE to differ for English vs. French. Evidence for this would include either a main effect of Language of Test, or an interaction of Numerosity and Language of Test, such that participants are more accurate at estimating in one numerical range for one language, but in a different numerical range for the other language. To test this, we predicted PAE from Age, Numerosity, Language of Test, and the interaction of Numerosity and Language of Test. This analysis found an effect of Age ($B = -.25$, $SE = .03$, $t = -9.00$, $p < .0001$), reflecting the fact that accuracy was greater among older children (see Figure 3). Importantly, there was also a significant interaction of Numerosity and Language of Test ($B = .005$, $SE = .001$, $t = 7.29$, $p < .0001$), suggesting that, at least in some numerical ranges, there was a difference in accuracy across the child's two languages. Following the method outlined above, we next asked whether the effect of language persisted when

examining only numbers within their highest common estimate. We found that the interaction between Numerosity and Language of Test remained even when considering only verbal estimates or magnitudes within the child's highest common estimate (Trimmed Estimation Range: $B = .004$, $SE = .001$, $t = 5.04$, $p < .0001$; Trimmed Numerosity Range: $B = -.004$, $SE = .001$, $t = 4.56$, $p < .0001$; see Figure 3). As shown in Figure 3, this result was likely driven by the underestimation of larger numbers (60-100 range) in French for 5- to 7-year-olds (see Figures 2 & 3). Thus, children's estimates in French and English differed in accuracy, and this difference remained when considering only numbers used in both languages.

As noted by an anonymous reviewer, though PAE is a standard measure used to capture estimation accuracy, it is also sensitive to variability around the mean of a participant's estimates. Though this is not a problem from the perspective of testing whether children make different estimates across their languages, it may make it more difficult to isolate accuracy as distinct from variability. To address this, we reproduced our PAE analyses using a modified measure of accuracy, signed error rate, defined as $(\text{estimate} - \text{numerosity})/\text{numerosity}$. Using this measure, we again found that the accuracy of children's estimates differed reliably across their two languages. Specifically, we found an interaction between Age and Numerosity ($B = .001$, $SE = .0002$, $t = 5.80$, $p < .0001$), and a significant interaction of Numerosity and Language of Test ($B = -.001$, $SE = .001$, $t = -2.81$, $p < .001$). These effects remained significant when considering only trimmed data (Trimmed Estimation Range: $B = -.001$, $SE = .0004$, $t = -2.25$, $p < .05$. When considering trimmed Numerosity Range, the main effect of Language of Test remained significant ($B = -.72$, $SE = .008$, $t = 5.46$, $p < .0001$), though the interaction between Language of Test and Numerosity did not ($p = .08$). Thus, this series of *post hoc* analyses suggests that when accuracy is

isolated from the variability of estimates (i.e., using signed error rate as a measure), our results resemble those reported for PAE, though in one case (trimmed Numerosity Range), the effect of Language of Test does not differ as a function of numerosity.

Figure 3. Relationship between numerosity and Log Percent Absolute Error in English (blue/solid/closed circle) and French (red/dashed/open circle) across age (5YOs $n = 20$, 6YOs $n =$

30, and 7YOs $n = 41$). Data points represent binned means; error bars are bootstrapped 95% confidence intervals.

We next asked whether children's estimates differed in how variable they were across their two languages - e.g., whether, on average, the standard deviation of estimates for a particular numerosity was greater in one language than in the other. We measured this by computing the coefficient of variation (COV) for each target value, for each of the child's two languages, as described above. Next, we constructed a model predicting COV from Language of Test (recall that neither numerosity nor age were related to COV in preliminary models and were therefore excluded from this model). Children's COV in English ($M = 0.35$) was modestly greater than in French ($M = 0.32$), a potentially surprising result given that children were overall more accurate in English than in French (see Figure 4). However, this effect did not reach the alpha threshold of $p < .05$ ($B = -.023$, $SE = .0118$, $t = -1.96$, $p = .0502$). When analyses were restricted to only estimates within the child's productive estimation range, or when analyses were restricted to numerosities within the child's productive estimation range, Language of Test did not predict COV (all $p > .10$). This suggests that, to the extent that there was a difference in COV in the first analysis, this was likely due to greater variability for numbers that children only produced in English.

Figure 4. Average Coefficient of Variation (COV) in English (blue) and French (red). Error bars represent 95% confidence intervals.

Our last set of analyses considered ordinality. Recall that the accuracy and variability of estimates may each be affected by (a) a child's knowledge that words later in the count list denote larger magnitudes (i.e., ordinality / later-greater), (b) their knowledge of how relative distance between verbal estimates in the count list corresponds to distance between ANS values (numerical distance), or (c) both. To assess whether ordinality differed across languages, we constructed a model predicting Ordinality from Language of Test, Numerosity, and Age. We found a main effect of Age ($B = .35$, $SE = .05$, $z = 7.05$, $p < .0001$), such that older children gave more ordinal estimates than younger children, and a main effect of Language of Test ($B = -.14$, $SE = .06$, $z = -2.45$, $p = .014$), such that Ordinality rates were higher in English (82.8%) than in French (80.9%), though this effect was very small (2% difference); see Figure 5. There was no reliable effect of Numerosity ($B = -.02$, $SE = .01$, $z = -1.76$, $p = .079$). When we considered only estimates that were within the child's highest common estimate, the difference between languages became smaller (English = 83.8%; French = 82.2%) and the effect of Language of Test was no longer significant ($B = -.107$, $SE = .062$, $z = -1.74$, $p = .083$). Likewise, when we considered only numerosities that were within the child's productive estimation range, we also found no significant effect of language on Ordinality (English = 83.8%; French = 82.4%; $B = -.107$, $SE = .064$, $z = -1.67$, $p = .095$). These data indicate that although children exhibit significant differences in the ordinality across all their estimates in French and English, these differences are very small and cannot likely explain the much larger differences in accuracy reported above. Also, when analyses are restricted to numbers that are familiar across both of a child's languages, these already small effects become non-significant.

Figure 5. Average proportion of ordinal responses in English (blue) and French (red) across age (5YOs $n = 20$, 6YOs $n = 30$, and 7YOs $n = 41$). Error bars represent 95% confidence intervals.

General Discussion

We tested whether, when children acquire multiple languages, their estimation abilities differ across their two languages. In particular, we asked whether children's mappings between

number words and magnitudes are language-specific (e.g., in French vs. English), or whether certain components of these mappings might transfer across languages in bilingual learners. To do this, we asked French-English bilingual children to make dot-array estimates in each of their two languages. By eliciting estimates in both languages, we examined the effect of language, independent of the acuity of non-linguistic (ANS-based) numerical representations, which remained constant (since language was manipulated within-subjects). We found that estimates differed across children's two languages, and in particular that their estimates were less accurate in French than in English, especially among younger participants. Interestingly, this difference in estimation performance remained even when analyses were restricted to the range of numbers that children produced in their weaker language (i.e., their "highest common estimate"), as well as when analyses were restricted to dot-array numerosities that fell within this same range. These data suggest that the accuracy of mappings between number words and numerical magnitudes differs across a single child's two languages, and that this difference cannot be explained by differences in the precision of the ANS (since children use the same ANS system in both languages). These data are most consistent with the view that the structures that support mature estimation are, at least in part, specific to particular languages, and therefore do not readily transfer from a highly trained language to a less trained language.

How might a child's linguistic representations of number differ, such that estimates differ across languages? We addressed this question by asking which types of knowledge differ across languages, and are therefore language-specific and not readily transferred from one language to the other. As noted above, we found that accuracy differed significantly across children's French and English estimates. However, other aspects of their estimates did not differ across languages. For example, although there was a significant difference in ordinality across French and English

this difference was very small (2%) and disappeared when analyses were restricted to numbers known in both French and English. This suggests that once a child exhibits later-greater knowledge in one language, they show roughly equal competence in deploying it in their second language for familiar numbers - a pattern that is compatible with the use of a language-general principle, rather than language- and item-specific learning. Also, we found no difference in the variability of estimates for familiar numbers across languages; although children mapped their number words to magnitudes less accurately in French than in English, their mis-mappings were nevertheless every bit as consistent.

One hypothesis that may explain this pattern of findings is Structure Mapping, a mechanism discussed in previous studies of number word learning (Carey, 2004, 2009; Gentner, 2010; Wynn, 1992), dot-array estimation (Izard & Dehaene, 2008; Lyons, Ansari, & Beilock, 2012; Sullivan & Barner, 2013; 2014), and number line reasoning (Cohen & Sarnecka, 2014; Siegler, Thompson, & Schneider, 2011; Thompson & Opfer, 2010; for review, see Marchand & Barner, 2017). On this view, mapping numerical symbols to magnitudes is not merely a process of forming item-based associations between individual words and ANS values, but instead involves a process of noticing - and exploiting - the analogous structures of the count list and ANS values. For example, to make an accurate estimate for a dot array containing 20 items, a child who has a robust item-based association between 10 and the word *ten* might first use the later-greater principle to infer that since the 20-dot array is greater in number, it merits a label that comes later in the count list - thereby exploiting knowledge of the structure of counting (and a principle that might be transferred across languages, compatible with our finding that ordinality does not differ across children's languages). For this estimate to be accurate, however, the child must further exploit the structure of counting in a way that is language-specific. Using a process similar to

anchoring and adjustment (Tversky & Kahneman, 1974), a child who knows (or guesses) that a particular dot array contains *ten* items and subsequently sees an array twice as large could make an estimate of *twenty*, provided that they know that *twenty* is twice as far into the count list than *ten*. Thus, they must know not only the ordering of number words in their language, but also the relative distance between number words in their count list - i.e., knowledge which depends on language-specific mastery of the count list.

On this Structure Mapping hypothesis, there are multiple ways in which bilinguals might become better estimators in one language than another. First, they might acquire better knowledge of the relative distance between numbers in a particular language. Although the order of the count list makes the distance between numbers implicitly available to children, this knowledge is likely difficult to deploy, much as it's difficult for even adults to estimate the relative distance between letters of the alphabet (Klahr, Chase & Lovelace, 1983). Knowledge that, e.g., 40 is twice 20, likely requires experience beyond reciting the count list, and may benefit from practice counting with decades or reciting basic multiplication facts. Second, differences in estimation across languages may be explained, in part, by differences in calibration. Although it's unlikely that all large number words have item-specific associative mappings to ANS values, a small number of anchor points for especially frequent numbers may exist (Sullivan & Barner, 2013, 2014). A child who lacks any anchors might know the later-greater principle and that 40 is twice 20, but might still systematically under- or over-estimate due to mis-calibration (i.e., not having associative mappings to ANS values that act as anchors for larger estimates that rely on structural inference). Finally, also compatible with Structure Mapping, if bilingual children have limited access to number words in their L2, they might stretch their response grid (Izard & Dehaene, 2008), thereby changing their estimates for all numbers that do not exhibit a strong associative mapping, thereby causing them

to be less accurate for larger numbers even if they don't differ in ordinality or COV. Future studies should explore this question by directly probing children's structural knowledge of the count list, and by testing the effects of calibration events on estimation accuracy across a bilingual child's two languages.

Note that the ability to use the structure of counting to guide estimates may depend on how accessible this structure is in a particular language, which may be relevant to understanding differences between French and English estimation. In English, the words *seventy*, *eighty*, and *ninety* are relatively regular and are composed of the morphemes -ty and “seven”, “eight”, and “nine” respectively. In contrast, in French the word *soixante-dix* (70) translates as “sixty-ten”, *quatre-vingts* (80) as “four-twenty”, and *quatre-vingt-dix* (90) as “four-twenty-ten”. These structural irregularities may make these numbers harder to learn in French than in English, predicting poorer estimation. Conversely, they may make estimation easier by making the multiplicative relations between numbers more transparent. Unfortunately, in our dataset this question is difficult to test, since we didn't assess whether children had decomposed these words according to rules, and also because the irregularities in French happen to arise specifically with larger numbers. Consequently, although estimates for these numbers could be poorer due to irregularities, less accurate estimates could also result from the fact that, in a weaker language, larger numbers are simply not familiar as they are in a child's primary language. Although no previous studies have examined the impact of counting regularity on estimation, past work has shown that children who are exposed to highly regular counting systems like Cantonese learn to count earlier than children learning less regular systems or Hindi, and may also be quicker to learn other numerical skills, though such effects are variable across studies, and some attribute them to educational practices rather than language (Dowker, Bala, & Lloyd, 2008; Dowker & Roberts,

2015; Lefevre, Clarke, & Stringer, 2002; Mark & Dowker, 2015; Miller, Kelly, & Zhou, 2005; Miller, Smith, Zhu, & Zhang, 2005; Miller & Stigler, 1987; Miura et al., 1998; Miura & Okamoto, 1989, 2013; Schneider et al., 2020).³ However, these findings notwithstanding, there is reason to believe that irregularities in how languages represent decade terms like *sixty* and *seventy* don't likely contribute to these differences. For example, although English and Cantonese speaking children readily learn rules to combine decade labels (10-90) with unit labels (1-9), children who speak these languages don't appear to know that decade labels are generated by rules, and instead treat words like "sixty" as unanalyzed morphemes (as shown by the fact that, when asked to count, children frequently count up to decade transitions like 29, 39, 49, but can't use a rule to generate decade labels; Schneider et al, 2020). Thus, evidence from other languages suggests that French-speaking children likely represent words like *soixante-dix* and *quatre-vingts* as unanalyzed words, such that their relative complexity therefore doesn't impact estimation, either positively or negatively. Future studies should explore this question.

In summary, this study shows that estimation abilities differ across a bilingual child's two languages and that these differences are not explained by count list knowledge alone, since they persist even within numbers used in both languages. These results have potentially important consequences for mathematics education, since although previous studies show associations between ANS acuity and mathematics achievement (Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011; Starr, Libertus, & Brannon, 2013), an equal number find a link between math achievement and estimation abilities, with some studies arguing that this second relationship is stronger (Booth & Siegler, 2006; 2008; Gunderson, Ramirez, Beilock, &

³ Note that Lefevre et al. (2002) find that French Canadian children don't count as fluidly as English-speaking children, but also that they also don't receive as much counting input, making it unclear whether linguistic differences in the count list play a role.

Levine, 2012; Kolkman, Kroesbergen, & Leseman, 2013; Moore & Ashcraft, 2015; Siegler & Booth, 2004; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Sullivan, Frank, & Barner, 2016). These factors, combined with our findings, suggest that if caregivers, preschool teachers, or other educators wish to train estimation abilities as a mechanism for promoting later mathematics ability, it may be important that this training take place in the expected language of later mathematics instruction. Likewise, children who are likely to transition midway through their mathematics training from one modality of instruction (e.g., French) to another (e.g., English), may benefit from estimation training in both languages. In particular, bilingual children may benefit from being explicitly taught about language-specific aspects of estimation in their L2, such as the distance between numerals (e.g., counting by 10s, learning that $20 = \text{double } 10$, etc). Developing bilinguals' understanding of the structure of their counting systems in both languages may improve their estimation abilities and more generally, their mappings between the linguistic and non-linguistic representations of magnitudes.

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