A formal model of interpersonality

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Draft version 1.0, August 29, 2022. This paper has not been peer reviewed.

Author Note

Portions of these findings were presented as talk at the 53rd International Annual Meeting of the Society for Psychotherapy Research in 2022, Denver, Colorado, United States. We have no conflict of interest to disclose. The formal model is available as Mathematica notebook in Supplemental Material A. This study was not preregistered.

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Abstract

Capturing the complexity of interpersonal dynamics – emerging from the approach and avoidance motivation of two individuals in dyadic interplay that unfolds simultaneously on multiple time scales in order to satisfy their psychological needs – remains a scientific challenge. In line with calls for embracing complexity in psychological research using formal modeling, the purpose of this mathematical, non-data-driven (and thus not preregistered) study is to investigate the underlying mechanisms of the formation and maintenance of interpersonal relationships using evolutionary game theory.

After formalizing interpersonal situations based on the affiliative motives of their interactants, a relational state space is constructed that reflects the ways of relating available to the interactants in the momentary state of their interpersonal relationship. This allows for modeling the evolution of an interpersonal relationship as a trajectory – driven by positive and negative reinforcement – in the relational space.

Depending on the motives of both interactants, three qualitatively different interpersonal dynamics emerge: (1) global stability with only one relational attractor (e.g., an interpersonal relationship of pure friendliness in the long run), (2) bistability with two mutually exclusive relational attractors (e.g., either pure friendliness or pure distance), and (3) cyclicality with periodic orbits in the relational space (e.g., oscillation between friendliness and distance).

Grounded in empirically supported psychological constructs, the formal model generates the well-known pattern of interpersonal complementarity. Over and above, novel interpersonal patterns emerged that might point to some underlying mechanisms of the interpersonal maintenance of psychopathology. The numerous model limitations as well as avenues for empirical tests and further development are discussed.

Keywords: dyad; motive; approach; avoidance; evolutionary game theory

Word count: 12’585

A Formal Model of Interpersonality

The call for embracing complexity with formal modeling is pervasive in psychological research in general (e.g., Guest & Martin, 2021) and interpersonal theory in particular (Hopwood et al., 2021). However, the actual development of formal models of interpersonality is lagging behind. To contribute to fill this gap in interpersonal theory and to shed light on potential underlying mechanisms of the formation and maintenance of interpersonal relationships, an approach of pure formal modeling that allows for interpersonal complexity to emerge is adopted in this paper. To pave the way for subsequent formal modeling, the relevant psychological research drawn upon later is briefly outlined first.

Psychological Needs

Research on psychological needs and related constructs is distributed across basic and applied psychological disciplines, ranging from motivational science (Deci & Ryan, 2000; McClelland, 1987) and developmental and personality psychology (Dweck, 2017; S. Epstein, 2003) to clinical psychology and psychotherapy (Grawe, 1998; Westermann et al., 2019) as well as psychoanalysis (Blatt & Luyten, 2009; Lichtenberg, 1988), to name only a few.
A basic or fundamental need for affiliation is consistently proposed, although the specific term and theoretical background varies greatly (Baumeister & Leary, 1995: belonging; Deci & Ryan, 2000: relatedness; Dweck, 2017: acceptance; Grawe, 1998: attachment; Lichtenberg, 1988: attachment and affiliation; Luyten & Blatt, 2013: relatedness; McClelland, 1987: affiliation). Besides a need for affiliation, other psychological needs or basic human motivations have been defined and investigated, such as needs for control and self-esteem (S. Epstein, 2003; Grawe, 1998), autonomy and competence (Deci & Ryan, 2000), and self-definition (Luyten & Blatt, 2013). For the sake of simplicity, however, the formal model of interpersonality to be developed in this paper will be restricted to the need for affiliation.

Approach and Avoidance Motives

Addressing motivational processes in dynamic accounts of personality (Boag, 2018) and of interpersonal interaction (Hopwood et al., 2021; Horowitz et al., 2006) is deemed crucial. The psychological theory of motivation adopted here proposes that – in interaction with their environment – human beings strive to satisfy their psychological needs and to protect them from violation (Caspar, 1995; Carver, 2006). The intrapsychic processes that enable individuals to organize their experience and behavior in order to maintain their needs satisfied can be understood as motives (Caspar, 1995, 2011; Grawe, 1998). If a motive aims at eliciting or maintaining appetitive, need-satisfying experiences such as being accepted in the case of the need for affiliation, it is called an *approach motive* (Caspar, 1995; Elliot, 2006). In contrast, an *avoidance motive* aims at preventing or discontinuing aversive, need-violating experiences such as being ignored (Caspar, 1995; Elliot, 2006; Grosse Holtforth, 2008). Approach and avoidance motives are an integral part of the model to be developed.

Interpersonal Theory

An environment of particular importance for psychological needs are other human beings who constitute what can be called the *interpersonal environment* (e.g., Pincus & Ansell, 2003; Wiggins & Trobst, 1999). If they are motivated to do so, individuals in the interpersonal environment such as friends or colleagues can serve as means to satisfy psychological needs (Caspar, 1995; Deci & Ryan, 2014; Orehek et al., 2018). Because other individuals have their own, independent subjectivity which is assumed to be organized by motives, too (Fonagy et al., 2018; Frith & Frith, 2003; Winnicott, 1969), the interpersonal environment differs qualitatively from the inanimate environment as a means for need satisfaction. Specifically, the effect of a behavior in the interpersonal environment, such as asking another person to flip the light switch in order to light up the room, is assumed to depend on the momentarily relevant motives of that other individual. If the other is motivated to avoid being bossed around, the room is likely to remain dim (or the relationship quality diminishes, e.g., see Deci & Ryan, 2014). In dyads of two interacting individuals – where one individual is the interpersonal environment for the other and vice versa – the dependence of the effect of an interpersonal behavior on the motives of the other is bi-directional. This reciprocal influence in the interpersonal environment will be called *interpersonal interdependence*.

Over time, the interpersonal environment changes. On a slow timescale of weeks to years, the interpersonal environment is usually referred to as *interpersonal relationships* (e.g., a long-lasting friendship) or social network (Wrzus et al., 2013). On a faster timescale of minutes to hours, however, the interpersonal environment is commonly conceived of as interpersonal situations (e.g., jointly going to the movies; Pincus et al., 2020). With regard to the slowly changing interpersonal environment (i.e., interpersonal relationships), approach motives are assumed to aim at forming and maintaining satisfying interpersonal relationships such as a friendship, and avoidance motives at avoiding or ending need-threatening or -violating interpersonal relationships (e.g., Gable, 2006). With regard to the faster changing interpersonal situations, approach motives are assumed to aim at eliciting appetitive interpersonal experiences such as being greeted, and avoidance motives to aim at preventing or discontinuing aversive interpersonal experiences such as being ignored (Caspar, 1995; or see Nikitin & Freund, 2019). Interpersonal behaviors are often mapped onto the Interpersonal Circumplex (Kiesler, 1996; Pincus & Ansell, 2003), a circle spanned by the dimensions of affiliation and control in which an interpersonal complementarity is assumed (e.g., friendly behavior inviting for a friendly response). Both the slow and the fast timescale of interpersonal dynamics and their intertwining will be taken into account when modeling interpersonality later.

Formal Modeling

Interpersonal dynamics can be understood as emerging from interwoven dyadic processes concurrently unfolding within and between two individuals on multiple time scales, when drawing upon the body of research on psychological needs, approach and avoidance motives, and interpersonal theory that has been briefly outlined above. Embracing such complex interpersonal dynamics is a challenge for psychological research. A well-established way to handle interpersonal complexity in psychological research is to reduce it, for example by understanding interpersonal dynamics as (1) an expression of personality (Hopwood, 2018; Pincus & Ansell, 2003), (2) a window into the motives underlying instrumental behaviors (Caspar, 1995, 2011), (3) the core of personality pathology (Hopwood et al., 2013), (4) a means for the satisfaction of psychological needs (Orehek et al., 2018), and so on.

An alternative scientific approach to understand and explain complex phenomena that is common in many other disciplines is to *generate* them using formal modeling and empirically informed simulations. In computational social science (Hofman et al., 2021), this approach is often called generative explanation and follows the catchy phrase "If you didn't grow it [in a simulation], you didn't explain it.", coined by J. M. Epstein (2012, p. xxi). For example, Banisch and Olbrich (2019) addressed the polarization in opinion dynamics by 'growing' the phenomenon with an algorithmic model in a computer simulation using so-called agent-based modeling (Wilensky & Rand, 2015). Another approach to formal modeling, often adopted in theoretical biology, is the mathematical tool of differential equations. For example, population dynamics with competition, predation and mutualism between species are modelled using differential equations (e.g., Holland & Deangelis, 2010). A common feature of generative, both algorithmic and mathematical approaches is that they aim at producing, not at reducing complexity and that they allow for prediction that are falsifiable with empirical data.

In psychological research, (1) the call for embracing complexity using formal theories is increasingly pervasive (for an extensive but not exhaustive list, see Fried, 2020; Fried & Robinaugh, 2020; Guest & Martin, 2021; Haslbeck et al., 2021; Robinaugh et al., 2021; Roefs et al., 2022; Vallacher et al., 2015), (2) a growing number of theoretical papers aim to be understood as groundwork for subsequent formal modeling (e.g., Herzog et al., 2022), (3) and formal modeling is seen as a promising path out of the replication crisis in psychology (Oberauer & Lewandowsky, 2019). In interpersonal theory, formal models are also encouraged, for example by Hopwood and colleagues who invite to develop “computational models to examine interpersonal transaction cycles” (2021, p. 68). Thus, expectations are high regarding formal modeling. Actual formal models are, however, scarce (for two exceptions in the non-interpersonal domain, see Read et al., 2017, Robinaugh et al., preprint).

Purpose of the Present Study

To contribute to fill this gap and to shed light on potential underlying mechanisms of the formation and maintenance of interpersonal relationships, a theoretical approach of pure formal modeling that allows for interpersonal complexity to emerge will be adopted in this paper. The model is based on evolutionary game theory (Gaunersdorfer et al., 1991; Weibull, 2004), but prior knowledge of formal modeling or game theory is not necessary.

The first aim of this study is the development of a generative formal model of interpersonal dynamics that takes into account the intertwined dynamics at both relevant timescales, namely the slowly changing interpersonal relationships and the rapidly changing interpersonal situations. The ‘building blocks’ for modeling will be well-known, empirically supported and theoretically sound psychological concepts such as the need for affiliation, approach and avoidance motives, interpersonal behavior and instrumental conditioning. Because the few existing modeling approaches reduced the richness of subjective experiences drastically (e.g., to a single dimension of emotional valence; Baker et al., 2021), a special focus is placed on modeling interpersonal experience in a differentiated way. To sum up, the first aim of this study is a step-by-step construction of a formal model of interpersonality.

Due to modeling formally, (1) all assumptions are forced to be explicitly stated (which is often not the case in verbal theorizing), (2) simplifications are obvious and can become subject to subsequent criticism, and (3) model predictions are necessarily strong and thus easily falsifiable (e.g., Smaldino, 2017). As a useful side effect, formal modeling of interpersonality require researchers to develop a more differentiated mental model, too (Nowak et al., 2013).

The second aim of this paper is to investigate the long-term dynamics that the formal model of interpersonality exhibits. On the one hand, the emergence of well-known interpersonal patterns described in interpersonal theory such as reciprocal friendly or reciprocal distanced interpersonal behavior is expected (interpersonal complementarity; Kiesler, 1996) and would corroborate the validity of the model. On the other hand, the emergence of novel patterns of interpersonality that are psychologically plausible but not yet captured in empirical studies would underpin the utility of a formal modeling approach for interpersonal theory.

Method

The present paper is theoretical and employs a theory-driven approach of formal modeling. Step by step, the psychological assumptions and their mathematical formalization will be described in this section. The model per se is not new but well-known in theoretical biology (Gaunersdorfer et al., 1991). Nonetheless, we will re-develop it in the context of interpersonality. First, motivated interpersonal behavior in a dyad of two individuals in interaction with interpersonal interdependence is modeled. Next, a mathematical representation of an interpersonal relationship in a relational state space is constructed. Then, interpersonal dynamics are formally modelled as a change process of a relationship over time driven by instrumental conditioning. Finally, the emergent dynamics are outlined.

Formalization of Motivated Interpersonal Behavior in Dyadic Situations

An interpersonal relationship of two friends, two co-workers, or a therapist and a patient can be conceived of as a dyadic system of two interacting individuals A and B that evolves over time. Let the set of both interactants be $I = \{A, B\}$. Such a generic definition allows for a family of models with varying degrees of complexity. In this paper, merely one of the simplest dyadic models possible will be formulated and analyzed. First, based on a substantial body of psychological research that illustrate its centrality for interpersonal relationships (Deci & Ryan, 2000: relatedness; Dweck, 2017: acceptance; Luyten & Blatt, 2013: relatedness; McClelland, 1987: affiliation), let us assume that both individual in a dyad have a need for affiliation. To be explicit about what is omitted in modeling, we state as an auxiliary, unrealistic assumption that there are no other needs in the individuals.

Assumption (1): Need for affiliation – Each individual has a need for affiliation and no other needs

Subjective experiences can satisfy or violate psychological needs (Elliot, 2006; Grosse Holtforth, 2008; Westermann et al., 2019). Whereas *appetitive experiences* of belonging, closeness or inclusion satisfy the need for affiliation (e.g., a joint visit to the cinema), *aversive experiences* involving rejection or exclusion threaten or violate the need for affiliation (e.g., a decline of an invitation). In the dyadic model to be formulated here, individuals are not considered as responding to appetitive and aversive experiences that they passively face in their environment in stimulus-response fashion. Rather, let us assume that individuals proactively impact on their environment in order to generate or maintain appetitive experiences and to avoid or discontinue aversive experiences. This is in line with

Perceptual Control Theory (Powers, 1973) and related models in motivation science (e.g., Carver & Scheier, 1998) as well as clinical psychology (Caspar, 1995, 2011; Grawe, 1998).

The psychological processes that organize the experience and behavior of an individual to generate appetitive experiences and discontinue aversive experiences are called *motives*. Thus, in the context of this model, motives are motivational processes and not dispositions. Let us assume that each individual has two motives: an affiliative approach motive and an affiliative avoidance motive.

Assumption (2) – Motives: Each individual has two motives, an approach motive that strives to satisfy the need for affiliation, and an avoidance motives that strives to prevent the need for affiliation from being threatened or violated.

Next, drawing upon interpersonal theory (Kiesler, 1996; Pincus & Ansell, 2003) and related work (Caspar, 1995; Horowitz et al., 2006), let us assume that interpersonal behaviors serve as means that motives use to generate or avoid subjective experiences. Specifically and for the sake of simplicity, only the interpersonal behaviors located on the horizontal affiliation dimension of the Interpersonal Circumplex (Kiesler, 1996) will be employed to formally model the behavioral means of the motives. Affiliative approach motives use friendly interpersonal behaviors (F) from the right-hand side of the affiliative dimension of the IPC in order to generate appetitive subjective experience, such as smiling or arranging to meet for sports. Affiliative avoidance motives use distancing or hostile behaviors (D) – such as avoiding eye contact or ignoring an invitation – from the left-hand side of the affiliative dimension in order to avoid aversive experience of rejection.

Assumption (3) – Interpersonal affiliative behaviors: Affiliative approach motives use friendly interpersonal behaviors (F) to control appetitive subjective experience and affiliative avoidance motives use distancing interpersonal behaviors (D) to control aversive subjective experience of rejection.

Thus, let the set of available classes of motivated behaviors that is shared by both interactants be M_i = {F, D} = {‘friendly interpersonal behaviors’, ‘distancing interpersonal behaviors'}. In a specific interpersonal situation, individual A is either approach-motivated and behaves friendly, a_F, or avoidance-motivated and behaves distancing, a_D, and also B is motivated to behave either friendly, b_F, or distancing, b_D. So far, we have constructed two elements of our formal dyadic model of interpersonality: a set of individuals in interaction, $I = \{A, B\}$, and a set of motivated behaviors, $M_i = \{F, D\}$, with $i \in I$, that serve only a single need, namely affiliation.

Let us assume that each individual is either approach- or avoidance-motivated *before* the actual start of the interaction in each encounter. Suppose, for example, that two acquaintances, called Mr. A and Ms. B, unexpectedly meet on the street. Immediately in that moment and before they know what the other will do, both simultaneously choose to greet or to ignore the other. Of course, Mr. A substantially contributes with his own behavior – for example, by greeting (F) – to his subsequent subjective interpersonal experience. However, his experience does also critically depend on the behavior of Ms. B. She might also greet (F), or she might instead ignore Mr. A (D), which would be an outcome of unrequited greeting for him that he likely experiences as aversive. Conversely, Ms. B's experience also depends on her behavior and the behavior of Mr. A. This kind of reciprocal dependence, which is assumed to be essential to many interpersonal interactions, is defined as *interdependency*[1] here. In other words, interdependency states that the effect of one's own behavior depends on the other individual, and vice versa. The formal model of interpersonality to be developed here is an attempt to contribute to a better understanding of the dynamics stemming from dyadic interdependency.

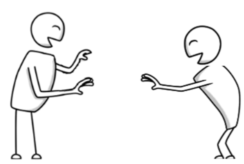
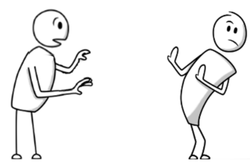
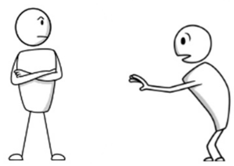
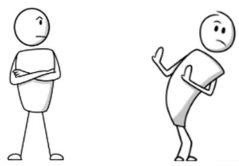
In an interpersonal interaction with simultaneous choices as in our example, four outcomes are possible: both A and B greet, both A and B ignore, A greets and B ignores, and

[1] Please note that this game-theoretical definition of interdependence differs from definitions in interpersonal models such as the structural analysis of social behavior (SASB; Benjamin, 1974).

A ignores and B greets. This corresponds to four possible dyadic combinations of motives in an interpersonal situation. For example, A might be avoidant and behaving distanced, a_D, and B might be approach-motivated and friendly, b_F. The four potential situations are displayed in a tabular form in Figure 1, where the two rows correspond to the motives of A and the two columns correspond to the motives of B. Formally, each of the four situations can be expressed as a vector of the motive of A and the motive of B as components, for example $\mathbf{s} = (a_D, b_F)$. Such a vector with a combination of motives will be called *situative motive profile* (or just *situation*), because it represents the specific motives of both interactants in a situation. The dyadic *motive space*, then, consists of all dyadic pairs of motives that are possible, $S = \{(a_F, b_F), (a_D, b_F), (a_F, b_D), (a_D, b_D)\}$, which correspond to the four cells of the table in Figure 1. Mapped onto the Interpersonal Circumplex, the motive space consists of the four possible arrangements of two individuals on the affiliative axis (A left or right, and B left or right). To sum up, we have constructed a dyadic motive space S from the interactants $I = \{A, B\}$ and their motivated behaviors $M_i = \{F, D\}$, with each point in the space corresponding to a motive profile $\mathbf{s}$.

Figure 1

The Dyadic Motive Space S Spanned by the Approach and Avoidance Motivation of Individual A (Rows) and Individual B (Columns)

		Individual B	
		Approach motive b_F (affiliative approach motivation) 	*Avoidance motive b_D* (affiliative avoidance motivation) 
Individual A	*Approach motive a_F* (affiliative approach motivation) 	**s** = (a_F, b_F)  $\pi_A = +3$	**s** = (a_F, b_D)  $\pi_A = -1$
	Avoidance motive a_D (affiliative avoidance motivation) 	**s** = (a_D, b_F)  $\pi_A = 0$	**s** = (a_D, b_D)  $\pi_A = +1$

Note. Tabular representation of the motive space S = {(a_F, b_F), (a_D, b_F), (a_F, b_D), (a_D, b_D)}. Each motive profile (i.e., situation), **s** ∈ S, 1) corresponds to a cell in the table, 2) refers to the motives of both interactants in the situation, and 3) is accompanied by a subjective experience π_A of individual A varying in valence. The experience π_B of individual B is not displayed here.

A shared interpersonal situation – such as going to the movies with approach motivation – can be and usually will be subjectively experienced (or anticipated to be experienced) in different ways by two individuals. For example, one interactant might like the joint visit to the cinema more than the other. Let the function π_i represent the subjective experience that interactant i ∈ {A, B} attaches to a specific, shared situation **s** = (a_v, b_w), with v ∈ M_A and w ∈ M_B. We assume that subjective experiences can be expressed on a single dimension of valence that ranges from aversiveness to appetitiveness. Thus, the function π_i maps interactant i's subjective experience of a situation **s** onto the real numbers, π_i: B → ℝ, and is a utility function in a game-theoretical sense. Then, extreme aversiveness is represented by large negative numbers, extreme appetitiveness by large positive numbers, and neutral indifference by numbers around zero. Within an individual, the subjective experiences in different situations can be compared with each other. For example, if A likes situations where both interactants are approach-motivated and friendly, $\mathbf{s_{FF}}$ = (a_F, b_F), more than situations where both are avoidance-motivated and distanced, $\mathbf{s_{DD}}$ = (a_D, b_D), then this can be expressed as $\pi_A(\mathbf{s_{FF}}) > \pi_A(\mathbf{s_{DD}})$. However, the comparison of subjective experience between individuals, even of the same shared situation, is not well defined.

> *Assumption (4) – Subjective experience. The subjective experience that an interactant attaches to an upcoming or momentary interpersonal situation can be represented as a single number on a dimension ranging from aversiveness to appetitiveness, and can be compared with other subjective experiences of the same interactant.*

Drawing upon the theory of instrumental conditioning (Skinner, 1953), let us define a positive reinforcement as a positive difference of *appetitive* or neutral experiences, $\pi_i' \geq 0$, $\pi_i > 0$, so that $\pi_i - \pi_i' > 0$ (e.g., $\pi_i - \pi_i' = 5 - 1 = 4 > 0$). In other words, the transition from one situation, **s'** ∈ S – such as oneself being avoidant and the other approach-motivated, with $\pi_i' = +1$, – to another situation, **s** ∈ S with **s** ≠ **s'**, – such as oneself being approach-motivated and the other being approach-motivated, too, with $\pi_i = +5$ – acts as a rewarding positive reinforcement. The change of the situation can be due to oneself changing the motivated behavior or the other. Next, we define a positive difference of *aversive* or neutral experiences, $\pi_i' < 0$, $\pi_i \leq 0$, $\pi_i - \pi_i' > 0$, as a negative reinforcement (e.g., $\pi_i - \pi_i' = -2 - (-3) = +1 > 0$). For example, the change from being approach-motivated and the other being avoidant (e.g., $\pi_i' = -3$) to being avoidant and the other being avoidant, too (e.g., $\pi_i = -2$), acts as a rewarding negative reinforcement. All other positive differences are defined as mixed reinforcements (e.g., from negative to positive experience). Analogous to this, 1) a negative difference of appetitive experiences is defined as a positive punishment (i.e., omission), 2) a negative difference of aversive experiences is defined as a negative punishment, and 3) all other negative differences are defined as mixed punishments. All differences that result in zero are defined as indifferences.

The assumption that individuals differ in their subjective experiences of an objectively shared situation results in a dyadic asymmetry of subjectivity. For our formal model this implies that two tables (or matrices) are necessary – one for each individual – to formalize the interdependency of the consequences of the motivation of one interactant and the motivation of the other interactant. If, for example, B is approach-motivated (left-hand column in the table in Figure 1), the consequence for A being also approach-motivated would be $\pi_A(a_F, b_F) = +3$, and the consequence for A being instead avoidance-motivated would be $\pi_A(a_D, b_F) = 0$. In this case, switching from avoidance (0) to approach motivation (+3) is a positive reinforcement (+3 – 0 = +3), and switching the other way around would be a negative punishment (0 - 3 = -3). However, given B is avoidant with regard to affiliation, being also avoidant would be more attractive for A ($\pi_A = +1$) than being approach-motivated ($\pi_A = -1$). The tabular display of consequences for A (and also B) can be written as a 2x2 matrix and will be called *motive matrix*:

$$A = \begin{pmatrix} +3 & -1 \\ 0 & +1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \text{ and } B = \begin{pmatrix} +2 & -2 \\ -1 & +1 \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

In game theory such matrices are called payoff matrices and two of them define an asymmetric game. The concrete values chosen for A and B are psychologically plausible but have only an illustrative purpose here.

Up to now, we have constructed a dyadic motive space S from the interactants $I = \{A, B\}$ and their motivated behaviors $M_i = \{F, D\}$. Each point in the space corresponds to a situative motive profile (i.e., a situation) and can be mapped to two subjective experiences,

π_A and π_B. Transitions from one motive profile to another motive profile (from one situation to another) imply reinforcements or punishments. Although the formal dyadic model is not yet dynamic, a preliminary static analysis within the constraints of the model reveals that interpersonal situations are possible in which neither interactant has an incentive to individually change his or her motivation. For example, if the interactants A and B described in Figure 1 are avoidant, $\mathbf{s} = (a_D, b_D)$, switching to approach motivation would be a punishment for A (-1 - (+1) = -2) and also for B (-2 - (+1) = -3). Generally, such a state is a Nash equilibrium – a state in which neither of two interactants has an advantage of changing his or her behavior individually – and in the context of interpersonality such states in the motive space S will be called interpersonal Nash equilibria. Another result of a preliminary analysis is that despite interpersonal interdependence, an individual can prefer to consistently choose the same motivation irrespectively of the motivation of the other interactant. For example, suppose that the subjective experience of an interactant A^* is given by

$$A^* = \begin{pmatrix} +3 & -1 \\ 0 & -2 \end{pmatrix}$$

In this case, A^* would prefer approach motivation if the other is approach-motivated (+3 > 0) and also if the other is avoidance-motivated (-1 > -2). That is, A^* would be approach-motivated no matter what B intends. In line with terminology of game theory, this is called a strictly dominant motive.[2] Beyond such static analyses of interpersonal behavior, the aim of this paper is to formally model the interpersonal *relationships*, not only interpersonal situations. Thus, building upon our formal groundwork derived from four assumptions, we now turn our attention away from single interpersonal situations (motive profiles) to interpersonal relationships and their dynamics over time.

[2] Please note that the use of the adjective 'dominant' here in a game-theoretical sense differs from its use in interpersonal models that refer to interpersonal dominance. Being friendly all the time would be a strictly dominant motive in the present model but is not a dominant interpersonal behavior in Interpersonal Theory.

The Relational State Space

What are the available options for two interactants A and B to relate with each other in general, not in a specific situation? What are their potential ways of relating? Within the constraints of the current state of formal modeling in this paper, there are only four possible ways of dyadic relating in interpersonal interactions. Each way of relating will be called a *relation* and corresponds to a behavioral type with two roles in evolutionary game theory (Gaunersdorfer et al., 1991). A relation has two positions, namely the position of interactant A and the position of interactant B. For example, the two positions of one of the four relations can be described as 'A is friendly and B is distancing', $a_F b_D$. If A realizes this way of dyadic relating, he or she behaves friendly and expects the other to behave distancing. If, on the other hand, B realizes the same way of relating ($a_F b_D$), he or she behaves distancing and expects the other to be approach-motivated and behaving friendly. Importantly, relations are not to be confused with situative motive profiles, which refer to concrete interpersonal situations.

> *Assumption (5) – Dyadic relations: Patterns of dyadic relating have two positions that specify a motive for each of the two interactants and can be realized in concrete interpersonal situations.*

Formally, a relation consists of two motivated behaviors, a_v and b_w, with $v \in M_A$ and $w \in M_B$. The set of relations R contains four elements: 1) both A and B are approach-motivated, $R_1=a_F b_F$, 2) A is avoidance- and B is approach-motivated, $R_2=a_D b_F$, 3) A is approach- and B avoidance-motivated, $R_3=a_F b_D$, and 4) both A and B are avoidance-motivated, $R_4=a_D b_D$. Thus, let the set of dyadic relations be $R = \{R_1, R_2, R_3, R_4\}$.

In general, a single observation of the interpersonal behaviors of two interactants in a specific situation (i.e., a motive profile) is not sufficient to characterize the interpersonal relationship of those interactants. For example, if two individuals are friendly to each other once, that does not imply they are friends. Consequently, let us assume that interpersonal behaviors are not a sufficient basis for a formal model of interpersonality. Instead, building upon the formal definition of relations, we describe an interpersonal relationship with the momentary availability of relations – that is, different ways of relating – in situations within that relationship. For example, an interpersonal relationship in which a relation of reciprocal friendliness is much likelier than the other relations results in many friendly interpersonal situations, justifying its characterization as friendship. Formally, let us define a relational state space as a probability distribution of the availability of the relations in an interpersonal relationship. Thus, the availability of ways of relating in a relationship is modelled as a probability. This allows for defining the momentary state of a relationship as a vector **x**:

$$\mathbf{x} = (\text{prob}(R_1), \text{prob}(R_2), \text{prob}(R_3), \text{prob}(R_4))$$
$$= (x_1, x_2, x_3, x_4), \text{ with } x_1 + x_2 + x_3 + x_4 = 1.$$

For example, $\mathbf{x} = (.70, .20, .05, .05)$ tells us that an interactant in a specific situation realizes the relation $R_1=a_Fb_F$ (A is approach-motivated and B is approach-motivated) with a probability of 70%, the relation $R_2=a_Db_F$ (A avoidance-motivated, B approach-motivated) with 20%, and so on. This representation allows us to formalize the momentary state of an interpersonal relationship as a point in a relational state space.

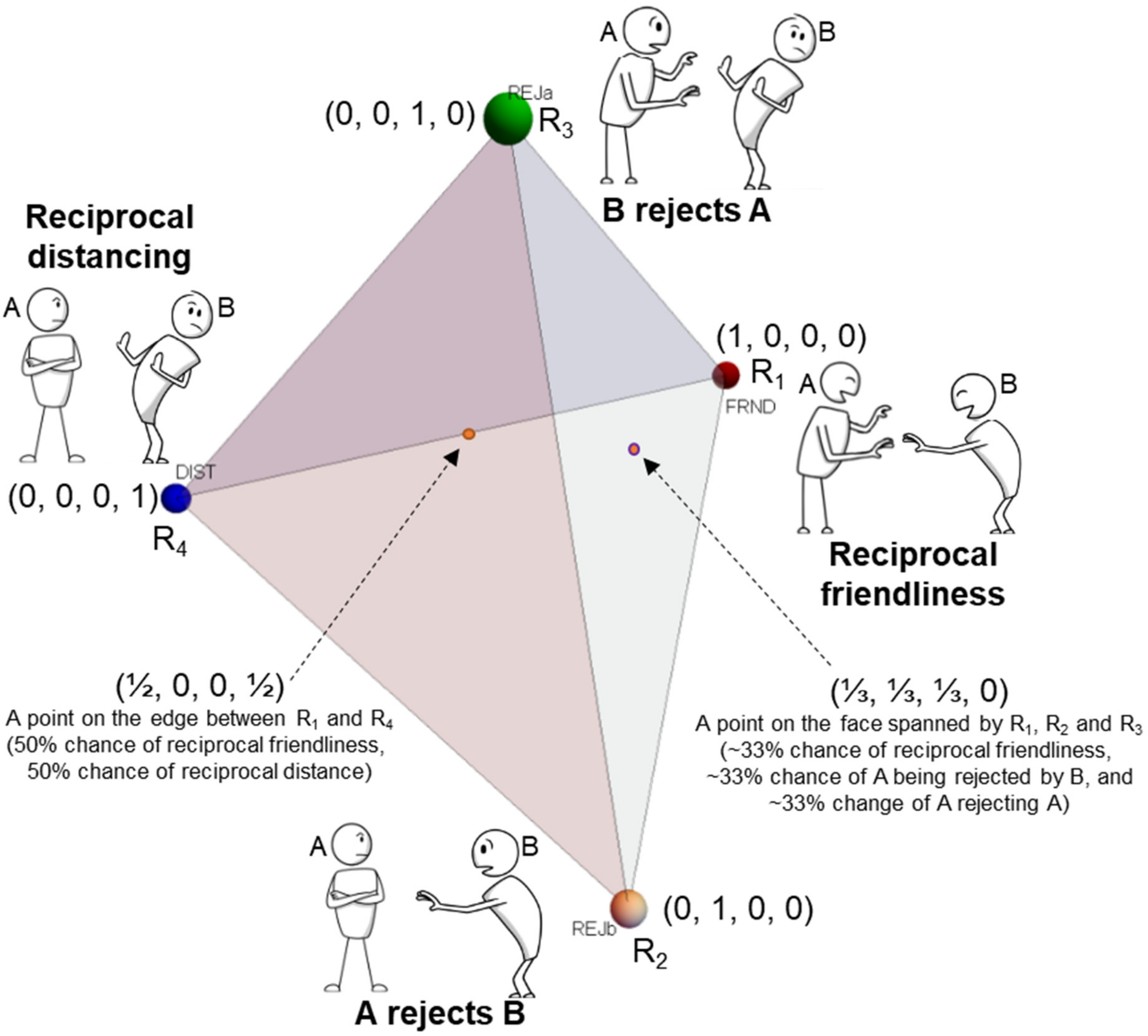
Assumption (6) – Relational space. The momentary state of an interpersonal relationship can be captured by the probability of its dyadic relations.

The mathematical representation of an interpersonal relationship as a numerical vector in a relational state space paves the way for formally modeling the development of an interpersonal relationship over time as a dynamic system. However, before we start modeling interpersonal dynamics in the next section, a visual representation of the relational space and trajectories within that space is constructed.

The relational space represents all possible states that an interpersonal relationship can assume. In the current model, it is 4-dimensional due to the four ways of relating defined above. Fortunately, such a non-imaginable 4-dimensional probability distribution can be projected onto a simple, imaginable 3-dimensional geometric form called simplex. Visualized in that way, the relational space appears as a pyramid with a triangular base (technically, a so-called tetrahedron). Figure 2 shows the tetrahedral representation of the relational space.

Figure 2

Graphical Representation of the Relational Space Containing All Potential States of an Interpersonal Relationship



Note. The relational space in its tetrahedral representation consists of 1) four corners (or vertices; i.e., relationships with pure relations) that are illustrated by pictograms, 2) six edges (relationships with mixes of two relations), 3) four triangular faces (relationships with mixes of three relations), and 4) one interior (relationships with mixes of all four possible relations). Each relationship state is a point the relational space and refers to a vector of probabilities.

The four corners (technically, vertices or nodes) of the relational space in Figure 2 represent the four possible interpersonal relationships which completely rely on a single relation (i.e., a single way of relating). For instance, **x** = (1, 0, 0, 0) is the rightmost corner in Figure 2 and represents relationships where only approach-motivated friendly interactions are available. Relationships with a mix of only two relations are expressed by the six edges (i.e., lines between the corners). For example, **x** = (¾, ¼, 0, 0) refers to a relationship which consists of both interactants either being approach-motivated or A being avoidant and B approaching. The four triangular faces of the relational space (the triangles between each three corners) represent relationships with mixes of three relations. Finally, the interior of the relational space reflects relationships where each of the four relations is possible, $x_i > 0$.

An interpersonal relationship that changes over time is represented as a movement of the relationship within the relational space. Technically, such a movement is called a trajectory. Points in the relational space to that the trajectories of the neighborhood evolve to over time are called (relational) attractors. Points in the relational space from that the trajectories of the neighborhood evolve *away* are called (relational) repellers. Now, with a formal model based on six psychologically plausible assumptions as well as a relational state space and its graphical representation, we are finally in a position to model and subsequently analyze interpersonal dynamics: the change of interpersonal relationships over time.

Formally Modeling Interpersonal Dynamics

Drawing upon evolutionary game theory, particularly and heavily on the work by Gaunersdorfer, Hofbauer, and Sigmund (1991), let us assume that relations (i.e., ways of relating) that are fitter than average become more available and thus occur more often in the interpersonal relationship. In other words, relations whose realization result in more appetitive subjective experiences (and thus are positively reinforced) or in less aversive subjective experience (and thus are negatively reinforced) than the average of relations increase in relative availability. In this way, we are specifying a dynamic system on the relational space that is driven by an equation that describes the change of the state $\mathbf{x}$ of the interpersonal relationship over time, namely $\frac{dx}{dt}$.

> *Assumption (7) – Positive and negative reinforcement. Relations that result in more appetitive or less aversive experience than the average relation increase in availability.*

Specifically, let us assume that prior to each interpersonal situation in a relationship, two relations are randomly drawn from the probability distribution that is defined by the momentary state of the relationship in the relational space. In the example of two acquaintances meeting accidentally on the street, this refers to a random sampling of two relations in the moment they meet and before they actually start to interact. With a probability of p = ½, the first relation is assigned to individual A and the other relation is assigned to individual B, and with the same probability 1 – p = ½, the first relation is assigned to individual B and the other relation is assigned to individual A. In other words, one of the randomly drawn relations motivates the experience and behavior of a specific interactant such as individual A with a probability of 50%. Let us assume that each individual activates the motive of the position that refers to him or her in the assigned relation and expects the other individual to activate the motive of the position that refers to the other. For example, if individual A realizes the relation $R_3=a_Fb_D$, he or she is approach-motivated with regard to affiliation and behaves friendly and expects the other interactant to be avoidant and behaving accordingly. If, instead, individual B realizes the same relation R_3, she or he would be avoidant and expect the other to demonstrate approach-motivated behavior.

The expected (average) subjective experience for relation $R_3=a_Fb_D$ – when realized by individual A as friendly behavior – depends on the interaction with the four relations that can be realized by the other individual B. For example, when A realizes the relation $R_3=a_Fb_D$ and faces the relation $R_1=a_Fb_F$ in individual B, individual B behaves friendly. In effect, both behave friendly. This corresponds to the subjective experience a_{11} in matrix A (or the upper-left cell in the table in Figure 1). Across many situations, the probability of the occurrence of that situative motive profile is x_1 (which is the probability of the relation R_1 in the first component of vector **x**). Therefore, the expected consequence for R_3 when realized by A who faces relation R_1 realized by B is $x_1 \cdot a_{11}$. In the same way, the other three expected consequences are determined and aggregated:

$$\pi_A(R_3 \mid \text{individual A}) = x_1 \cdot a_{11} + x_2 \cdot a_{11} + x_3 \cdot a_{12} + x_4 \cdot a_{12}$$

However, the relation in our example, $R_3{=}a_F b_D$, can also be realized by individual B. Then, he or she is avoidant and demonstrates distancing behavior. The expected (average) subjective experience for the relation, now realized by B, across many situations is then:

$$\pi_B(R_3 \mid \text{individual B}) = x_1 \cdot b_{21} + x_2 \cdot b_{22} + x_3 \cdot b_{21} + x_4 \cdot b_{22}$$

The expected consequence for the relation R_3, irrespective of which individual realizes it, with $p = ½$ is then:[3]

$$\pi(R_3) = p \cdot \pi_A(R_3 \mid \text{individual A}) + (1 - p) \cdot \pi_B(R_3 \mid \text{individual B})$$

$$= ½ \cdot \pi_A(R_3 \mid \text{individual A}) + ½ \cdot \pi_B(R_3 \mid \text{individual B})$$

$$= ½ \cdot (x_1 \cdot a_{11} + x_2 \cdot a_{11} + x_3 \cdot a_{12} + x_4 \cdot a_{12} + x_1 \cdot b_{21} + x_2 \cdot b_{22} + x_3 \cdot b_{21} + x_4 \cdot b_{22})$$

$$= ½ \cdot (x_1 (a_{11} + b_{21}) + x_2 (a_{11} + b_{22}) + x_3 (a_{12} + b_{21}) + x_4 (a_{12} + b_{22}))$$

Let us neglect the factor ½, which does not change the long-term dynamics of the system we are interested in here, and define a 4x4 matrix M with the parameters derived above for R_3, namely $a_{11} + b_{21}$, $a_{11} + b_{22}$, and so on, as entries in the third row, and the

[3] Here, the subjective experiences of two individuals are added up which implies the psychologically implausible assumption that subjective experience of two individuals is on the same scale. To explicitly formalize that problematic assumption, one could define a function Ω that maps the subjective experience of individuals to an intersubjectively shared scale, $\Omega: \pi_i \longrightarrow \mathbb{R}$. Here, the function is simply the identity function, $\Omega(\pi_i) = \pi_i$.

corresponding parameters for R_1, R_2 and R_4 as entries in the first, second and fourth rows, respectively:

$$M = \begin{pmatrix} a_{11}+b_{11} & a_{11}+b_{12} & a_{12}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{11} & a_{21}+b_{12} & a_{22}+b_{11} & a_{22}+b_{12} \\ a_{11}+b_{21} & a_{11}+b_{22} & a_{12}+b_{21} & a_{12}+b_{22} \\ a_{21}+b_{21} & a_{21}+b_{22} & a_{22}+b_{21} & a_{22}+b_{22} \end{pmatrix}$$

This allows to express the expected consequence for a relation R_i as $(Mx)_i$. For example, for relation R_3, this is:

$$(Mx)_3 = \Sigma\, x_i \cdot M_{3i} = x_1\,(a_{11} + b_{21}) + x_2\,(a_{11} + b_{22}) + x_3\,(a_{12} + b_{21}) + x_4\,(a_{12} + b_{22}),$$

which is exactly what we derived above (except for the neglected factor ½). The matrix M will be called *relational matrix* and that single matrix is sufficient to characterize the interpersonal dynamics of the dyadic system of two individuals in interaction. According with assumption 7) and in line with evolutionary game theory (Gaunersdorfer et al., 1991; Weibull, 2004), we assume that relations with consequences that are accompanied by a higher (positive or negative) reinforcement – that is, numerically higher consequences – than the average reinforcement of the relations increase:

$$\frac{dx_i}{dx} = x_i\,[(M\boldsymbol{x})_i - \bar{M}], \text{ with } \bar{M} = \sum x_i\,(M\boldsymbol{x})_i \qquad \text{(Eq. 1)}$$

Equation (1) is the so-called replicator equation which frames relations – that is, dyadic ways of relating – as relational replicators in the context of the present formal model. As can be seen in the Equation (1), interpersonal dynamics depend on the current state of the relationship **x** *and* the motives M of the interactants[4].

For the analysis of the dynamic system just defined (i.e., the bifurcation analysis), a simplification of the parameter space is convenient. Specifically, the eight parameters of the matrices A and B (that are combined to 16 parameters in the relational matrix M) can be

[4] The formal model is available as Mathematica notebook in Supplemental Material A.

reduced to four free parameters without loss of generality (Gaunersdorfer, Hofbauer, & Sigmund, 1993). The four parameters describe the interdependency of individual motives:

- *Withdrawal of A:* $W = a_{21} - a_{11}$, where positive values indicate that avoidance motivation is more attractive for A given B is approach-motivated, and
- *Withdrawal of B:* $w = b_{21} - b_{11}$, where positive values indicate that avoidance motivation is more attractive for B given A is approach-motivated, and
- *Persistence of A*: $P = a_{12} - a_{22}$, where positive values indicate that approach motivation is more attractive for A given B is avoidant (i.e., clinginess of A), and
- *Persistence of B:* $p = b_{12} - b_{22}$, where positive values indicate that approach motivation is more attractive for B given A is avoidant (i.e., clinginess of B).

For example, if A prefers to be approach-motivated when B is approach-motivated, and to be avoidant when B is avoidant, and B has similar preferences, all parameters are negative. That is, both interactants would be non-withdrawing and non-persisting.

Bifurcation Analysis

Decades ago, Gaunersdorfer, Hofbauer and Sigmund (1993) have conducted an in-depth mathematical analysis of the dynamic system that has been re-formulated in the context of interpersonality in this section. To avoid redundancy, in the following only a short, non-mathematical, narrative summary of their bifurcation analysis as well as implications for interpersonal dynamics are presented as an overview. Then, in the results section, the psychological prerequisites for the emergence of the different interpersonal dynamics in the relational space will be focused upon in detail.

The dynamics of the system in the interior of the relational state space can be differentiated into three qualitatively different classes, if the motives in interactants A and B are independent (Gaunersdorfer, Hofbauer and Sigmund, 1993) as we assume here. Applied to our formal model of interpersonality, the first class of dynamics is one of a global relational stability (see Figure 3A and Supplemental Material B for an animated illustration). Here, only a single relation remains available to the individuals in the relationship on the long run, irrespective of the initial state of the relationship.

The second class of the dyadic system is one of bistability (see Figure 3B and Supplemental Material C for an animated illustration). In that case there are two relational attractors in opposing corners of the relational space and the relational field is divided into two basins of attraction. One the long run, the relationship evolves to one of the two relational attractors. As an example, one attractor might be reciprocal friendliness and the other reciprocal distancing.

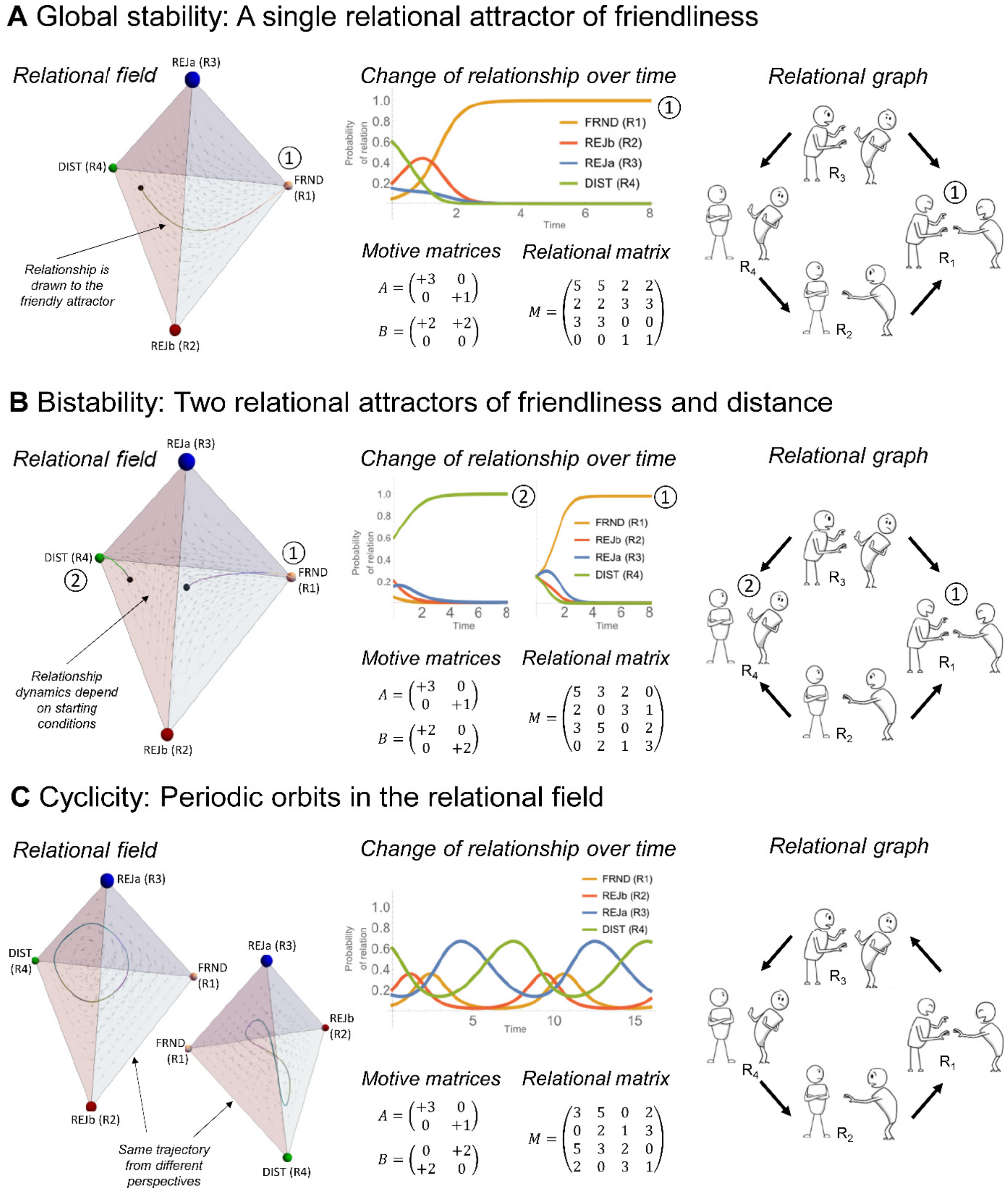
The third class of dynamics refers to the dyadic system being cyclic (see Figure 3C and Supplemental Material D for an animated illustration). In this case, the state of the relationship in the relational space cycles on period orbits on the long run and never settles. For example, 1) after a phase of very high availability of reciprocal friendliness in the relationship, 2) the relation of A rejecting B becomes predominant, 3) followed by a phase of reciprocal distancing being the most easily available relation, 4) which is then replaced by a phase of B rejecting B, and 5) a predominance of reciprocal friendliness, restarting the cycle.

Results

In the following, the approach and avoidance motives of both interactants that give rise to the qualitatively differing dynamics are investigated from a psychological point of view. In each case, we assume that the affiliative approach and avoidance motivations of individuals A and B are independent.

Figure 3

Three Classes of Interpersonal Dynamics in the Relational State Space with Examples



Note. The relational flow that is induced in the relational space by the replicator equation is illustrated with small grey arrows, and the totality of the flow is called relational field. The initial conditions of the exemplary trajectories were (.05, .20, .15, .60) in subfigure A, (.05, .20, .15, .60) and (.25, .25, .25, .25) in subfigure B, and (.05, .20, .15, .60) in subfigure C. Each trajectory is also plotted as a line chart with time as x-axis and the availabilities of the relations in the interpersonal relationship on the y-axis. For convenience, the parameters that give rise to the dynamics are displayed as matrices with a total of 24 parameters. However, effectively there are only four free parameters in the model. A simplified illustration of the interpersonal dynamics in a relational graph is displayed in each subfigure, with arrows pointing to the more reinforcing relation. In Supplemental Materials B to D, video animations of the three subfigures are available.

Global Stability: A Single Attractor in the Relational Space

In each of the relational fields included in the class of globally stable interpersonal dynamics, only one of the four edges of the relational space in Figure 2 – that correspond to the four pure relations of reciprocal friendliness, $R_1=a_Fb_F$, reciprocal distance, $R_4=a_Db_D$, etc. – is a relational attractor with the entire relational space as its global attractor basin. In total, there are twelve different relational fields in this class (see Table 1), with each of the four edges being a relational attractor in three similar relational fields (e.g., reciprocal friendliness, $R_1=a_Fb_F$, in rows #2, #9, and #10 in Table 1).

The motivational, intrapsychic conditions that give rise to the interpersonal emergence of such dynamics within the constraints of our formal model involve the presence of strictly dominant motives. In other words, if the situative motivation of an interactant does not depend on the motivation of the other interactant (e.g., A is motivated to be friendly irrespectively of B being friendly or distanced), the dyadic interdependence which usually characterizes interpersonal interactions does not occur. For example, if A has a strictly dominant approach motive and behaves friendly all the time, and B is responsive and conditions his or her motivation on the dominant approach motivation of A, then a single dyadic attractor emerges that corresponds to that dominating motive ($R_1=a_Fb_F$, see Figure 3A and row #2 in Table 1). Another case is A *and* B both having a strictly dominant motive: If, for example, A is strictly avoidant and B strictly approach-motivated, then a single relational attractor emerges that corresponds to the dominating motives of both interactants (e.g., $R_3=a_Fb_D$, row #6 in Table 1). Non-interdependence gives rise to globally stable dynamics…

- … if B is indifferent to the motives of A, but A is responsive to the motives of B (A depends on B but B does not depend on A; see rows #2 and #3 in Table 1),
- … if A is indifferent to the motives of B, but B is responsive to the motives of A (A does not depend on B but B depends on A; see rows #5 and #9 in Table 1),

- … if both interactants are indifferent to the other's motive (A and B do not depend on each other; see rows #6, #7, #10 and #11 in Table 1), or
- … if one of the interactants has a strictly dominant motive and the other interactant is antagonistic, meaning the he or she is avoidant when the other is approach-motivated and approach-motivated if the other is avoidant (see rows #8, #12, #14 and #15 in Table 1).

Table 1

Complete Listing of Possible Interpersonal Dynamics Depending on The Four Model Parameters

#	With-drawal		Persis-tence		Intrapsychic conditions		Interpersonal dynamics	
	A	*B*	*A*	*B*	*A*	*B*	*Relational field*	*Relational attractor(s)*
1	No	No	No	No	Responsive	Responsive	Bistable	Friendly (R_1), Distanced (R_4)
2	No	No	No	Yes	Responsive	Persisting	Globally stable	Friendly (R_1)
3	No	Yes	No	No	Responsive	Withdrawing	Globally stable	Distanced (R_4)
4	No	Yes	No	Yes	Responsive	Antagonistic	Cyclical	Period orbits ($R_1 \rightarrow R_3 \rightarrow R_4 \rightarrow R_2 \rightarrow R_1$)
5	Yes	No	No	No	Withdrawing	Responsive	Globally stable	Distanced (R_4)
6	Yes	No	No	Yes	Withdrawing	Persisting	Globally stable	B rejected (R_2)
7	Yes	Yes	No	No	Withdrawing	Withdrawing	Globally stable	Distanced (R_4)
8	Yes	Yes	No	Yes	Withdrawing	Antagonistic	Globally stable	B rejected (R_2)
9	No	No	Yes	No	Persisting	Responsive	Globally stable	Friendly (R_1)
10	No	No	Yes	Yes	Persisting	Persisting	Globally stable	Friendly (R_1)
11	No	Yes	Yes	No	Persisting	Withdrawing	Globally stable	A rejected (R_3)
12	No	Yes	Yes	Yes	Persisting	Antagonistic	Globally stable	A rejected (R_3)
13	Yes	No	Yes	No	Antagonistic	Responsive	Cyclical	Period orbits ($R_1 \rightarrow R_2 \rightarrow R_4 \rightarrow R_3 \rightarrow R_1$)
13	Yes	No	Yes	Yes	Antagonistic	Persisting	Globally stable	B rejected (R_2)
15	Yes	Yes	Yes	No	Antagonistic	Withdrawing	Globally stable	A rejected (R_3)
16	Yes	Yes	Yes	Yes	Antagonistic	Antagonistic	Bistable	A rejected (R_3), B rejected (R_2)

Note. Intrapsychic conditions stem from the withdrawal and persistence parameters. Responsive interactants condition their motivation on the other interactant's motivation, withdrawing interactants are strictly avoidant, persisting interactants are strictly approach-motivated, and Antagonistic interactants are avoidant if the other is approach-motivated and approach-motivated if the other is avoidant. The relations are $R_1=a_F b_F$ (reciprocal friendliness), $R_2=a_D b_F$ (A is avoidant, B is approach-motivated), $R_3=a_F b_D$ (A is approach-motivated, B is avoidant), and $R_4=a_D b_D$ (reciprocal distancing).

In Figure 3A an example of an interpersonal relationship in the globally stable class of interpersonal dynamics is displayed. Irrespectively of where within the relational space the interactants start, the relationship will be drawn to the friendly relational attractor ($R_1=a_F b_F$). This is due to interactant B preferring approach motivation irrespective of the other's motivation (see motive matrix of B in Figure 3A). Thus, even if A is avoidant, it is more reinforcing for B to be affiliatively approach-motivated than to be avoidant (e.g., to remain friendly despite being rejected). Such a pattern can also be expressed as a relational graph. In such a graph, the relations that share a motive in one position are connected with a directed edge (see Figure 3A, right-hand side). The directed edges point to the relation that is more reinforcing. If the globally stable dynamics were kind of a rock-paper-scissors-like game, one of the moves would the optimal move to play all the time for both interactants (e.g., being motivated to be friendly always wins).

With regard to the four interdependency parameters defined above *[see page 24]*, individual A in Figure 3A is neither withdrawing, $W = a_{21} - a_{11} = 0 - (+3) = -3 < 0$, nor persisting, $P = a_{12} - a_{22} = 0 - (+2) = -2 < 0$. Individual B is also not withdrawing, $w = b_{21} - b_{11} = 0 - (+2) = -2 < 0$, however, he or she is persisting, $p = b_{12} - b_{22} = +2 - 0 = +2 > 0$. That is, individual B persists being approach-motivated and behaving friendly even if individual A withdraws. This gives rise to the globally stable, rigid interpersonal dynamics.

Taken together, global relational stability stems from a lack of responsiveness in at least one interactant that results in broken dyadic interdependence. In these cases, global stability of relations with *different* motives (e.g., A is approach-motivated and B is avoidant) is only possible when both interactants have strictly dominant but differing motives. If one of the interactants is responsive in the sense of not-persisting and not-withdrawing, relations are globally stable that have pure motives (e.g., A and B are avoidant), because then the rigid interactant teaches the more flexible interactant how to be motivated. Lastly, in such intrapsychic circumstances of strictly dominant motives, an interpersonal relationship with *mixed relations* – that is, multiple available ways of relating – is not possible on the long run.

Bistability: Two Dyadic Attractors in the Relational Space

In this class of bistable interpersonal dynamics, two types of relational fields are possible that correspond to the two pairs of opposite relations: First, reciprocal friendliness, $R_1=a_F b_F=(1, 0, 0, 0)$, versus reciprocal distancing, $R_4=a_D b_D=(0, 0, 0, 1)$, and second, A rejecting B, $R_2=a_D b_F=(0, 1, 0, 0)$, versus B rejecting A, $R_1=a_F b_D=(0, 0, 1, 0)$. Psychologically speaking, within the constraints of our formal model, such a dynamic emerges when both interactants take the motivation of the other into account, which induces symmetric dyadic interdependency. Thus, if both interactants have intrapsychic conditions that allow for responsiveness, their relationship will either evolve to reciprocal friendliness or to reciprocal distancing on the long run, depending on the initial state of the relationship (see Figure 3B and row #1 in Table 1). Here, both interactants prefer to be approach-motivated when the other is approach-motivated and both prefer to be avoidance-motivated when the other is avoidance-motivated. This corresponds to the principle of complementarity of the affiliative axis of the Interpersonal Circumplex.

In the case of bistability, the according relational graph has two relations which have only incoming and no outgoing directed edges (i.e., 'sinks'; see Figure 3B, right-hand side). Again, if this was a rock-paper-scissors-like game, it would be optimal for both players to coordinate to consistently use the same of two advantageous moves. With regard to the four interdependency parameters defined above, the individual A in Figure 3B is neither withdrawing, $W = a_{21} – a_{11} = 0 – (+3) = -3$, nor persisting, $P = a_{12} – a_{22} = 0 – (+2) = -2$, and also individual B is neither withdrawing, $w = b_{21} – b_{11} = 0 – (+2) = -2$, nor persisting, $p = b_{12} – b_{22} = 0 – (+2) = -2$. That is, both individuals are responsive.

However, when both prefer to be approach-motivated if the other is avoidance-motivated and vice versa (see row #16 in Table 1), bistability also emerges. Then, the relational attractors in the bistable relational field are $R_2=a_Db_F$ (A is avoidant, B approaching) and $R_3=a_Fb_D$ (A is approaching, B is avoidant). In this case, both interactants are withdrawing and persisting and thus antagonistic in their motivation.

Cyclicality: Periodic Orbits in the Relational Space

Finally, if both interactants have no strictly dominant strategies (so that there actually is dyadic interdependence), but one is responsive and the other is antagonistic, cyclical relational patterns emerge. Specifically,

- if the interactant A prefers to be approach-motivated if the other is approach-motivated and to be avoidance-motivated if the other is avoidant (responsiveness),
- but interactant B prefers to be avoidance-motivated if the other is approach-motivated and to be approach-motivated if the other is avoidant (inconsistency),

one of two possible cyclical relational fields emerges (see Figure 3C and row #4 in Table 1). Here, reciprocal friendliness, $R_1=a_Fb_F$, is more reinforcing than B being rejected by A, $R_2=a_Db_F$, which in turn is more reinforcing than reciprocal distancing, $R_4=a_Db_D$, which in turn is more reinforcing than A being rejected by A, $R_3=a_Fb_H$, which in turn is more reinforcing than reciprocal friendliness, $R_1=a_Fb_F$, so that the loop is closed ($R_1\rightarrow R_3\rightarrow R_4\rightarrow R_2\rightarrow R_1$).

Expressed as a psychological narrative this means that if both are avoidantly distancing (R_4), interactant B’s approach motivation and friendly behavior increases and interactant A’s expectation that B is approach-motivated also increases (R_2; $dx_2/dt > 0$). Shortly afterwards, this raises the reinforcement value for interactant A to be approach-motivated and behave friendly and for interactant B to expect the other to be approach-motivated (R_1; $dx_1/dt > 0$). Soon, the attractivity for interactant B to be avoidant and behave distancing and for interactant A to expect the other to be avoidant increases (R_3; $dx_3/dt > 0$). Next, this evokes a stronger reinforcement for A to avoidantly withdraw and for B to expect A to do so (R_4; $dx_4/dt > 0$). This gives rise to the next cycle. Of course, a relational field with a counterclockwise rotational component is also possible (see row #13 in Table 1 for details).

In the relational graph in Figure 3C (right-hand side), the cyclical interpersonal pattern is easily recognizable. If this was a rock-paper-scissors-like game, no move would be optimal, which is of course the case in the actual rock-paper-scissors game. Playing the move 'reciprocal distancing' very often makes it more attractive to play the move 'B seeks affiliation although B is distanced', but then the move 'reciprocal affiliation' is motivationally more attractive, and so on. Such cyclical patterns are not predicted by interpersonal models such as the interpersonal circumplex (Kiesler, 1996) or the structural analysis of social behavior (Benjamin, 1974). Cyclical interpersonal dynamics resemble an interpersonal relationship where a stable dyadic pattern is not possible, resulting in a 'stable' pattern of an unstable relationship.

Discussion

The light that the present formal modeling approach of evolutionary game theory (Gaunersdorfer et al., 1991) sheds on interpersonality is of theoretical nature and should not be mistaken for empirical findings. Having that in mind, the formal model of interpersonality allows for (1) predicting well-known and also novel interpersonal dynamics, (2) verifying that the assumed underlying psychological mechanisms are actually able to generate interpersonal dynamics (i.e., generative sufficiency), and (3) providing a generic mathematical framework for formal modeling in interpersonal theory and related disciplines.

Regarding the interpersonal dynamics, three qualitatively different patterns of interpersonal relationships emerged. First, bistable relational fields within which, for example, both interactants are either approach-motivated and friendly or both are avoidance-motivated and distanced on the long run. This dynamic resembles the interpersonal complementarity at the affiliation axis of the interpersonal circumplex (Kiesler, 1996). Second, a rigid and globally stable relational field within that only a single available way of relating is possible. Such dynamics stem from not taking into account the motives of the other and might capture processes relevant to the interpersonal formation and maintenance of personality pathology (Hopwood et al., 2013). Third, a novel, cyclic relational field that implies an endless oscillation between closeness and distance can emerge (Hopwood et al., 2021; Wachtel, 1994) when one interactant is antagonistic and prefers distance when closeness is easily available and vice versa.

As a second key finding, the formal model developed here demonstrates that mechanisms that potentially underly interpersonal dynamics can be explicitly modelled (Guest & Martin, 2021; Oberauer & Lewandowsky, 2019; Robinaugh et al., 2021) as an interplay of empirically supported psychological constructs such as approach and avoidance motives, relational patterns and instrumental conditioning. Particularly, interpersonal interdependence (and the lack of it in one or both interactants) turned out to be crucial for the model dynamics (which is, for example, in line with the theory of mentalization that posits detrimental effects of not taking the motivation of others into account; Fonagy et al., 2018). With its clear separation of the formally specified mechanisms on the one hand and the emerging interpersonal dynamics on the other hand (J. M. Epstein, 2012), the formal model offers a generative explanation of interpersonality that helps to avoid logical fallacies which verbal theories are prone to (Boag, 2018). For example, using the term ‘attractor’ in verbal theories is often a reification accompanied by circular explanations (e.g., ‘Interpersonal complementarity means that there are two attractors, and those two attractors explain interpersonal complementarity’), whereas using the term in the context of the formal model of interpersonality developed here is appropriate (Gelfand et al., 2018).

The third, more general key finding is that a useful, not overly simplistic formal foundation of interpersonal constructs with mathematical definitions is indeed possible. Central concepts such as situative motive profile (i.e., interpersonal situation), motive space, motive matrix, relational matrix, relation (i.e., way of dyadic relating), and relational space (i..e, relationship representation) are now available to be used, refined, extended, and criticized by other researchers in interpersonal theory and related disciplines such as clinical psychology and psychotherapy research (Fried & Robinaugh, 2020). In addition, the dynamic mathematical structures that arose from the definitions and equations – such as relational attractor, relational field and relational replicator – (1) have a clear, well-defined, non-metaphorical meaning (Gelfand et al., 2018; Gelfand & Engelhart, 2012), (2) allow for nomothetic and also idiographic empirical studies (e.g., by adding a statistical layer to the deterministic model, Bürkner, 2017; Molenaar, 2004), (3) have a subjectively meaningful, experiential quality expressible in pictograms and probably accessible by lay persons (see Figure 2), potentially of use for psychological interventions (e.g., Babl et al., 2022), and (4) hopefully contribute to trans-disciplinary definitions of interpersonal and related psychological constructs (Baldwin, 1992). Auxiliary concepts such as the relational graph (see Figure 3, right-hand side) might be useful for other applications, too, such as Markov chain analyses (Benjamin, 1979) or participatory interpersonal paradigms (Westermann & Sibilis, 2022).

Bridges to Interpersonal Theory, Personality Psychology, Psychopathology and Psychotherapy

Interpersonal theory. Although minimalistic with only one need and two types of motives in both interactants (Nowak, 2004; Smaldino, 2017), the formal model of interpersonality is surprisingly successful in generating the interpersonal complementarity of the affiliation axis of the interpersonal circumplex (Kiesler, 1996) with psychological mechanisms that are not part of the original circumplex model (Horowitz et al., 2006). If the emergent relational field is understood as a self-organized, implicit process (Lewin, 1943) that is driven by basic instrumental learning processes, then deliberate, self-regulatory, planned interpersonal behaviors are perturbations of the interpersonal relationship in the relational field (Carver & Scheier, 2002). For instance, if a dyad in the bistable relational field is near the distanced attractor, a series of deliberate friendly behaviors could 'push' the dyad into the basin of the friendly attractor. Of note, such a combination of self-organization and self-regulation (Carver & Scheier, 2002) could underly the predictive principle of antithesis in the structural analysis of social behavior model (Benjamin, 1974).

The idea of cyclic dynamics is not new to interpersonal theory and also psychodynamic approaches (Mitchell, 1988; Pincus et al., 2020; Wachtel, 1994; Wagner et al., 1995), but modeling cyclic interpersonal dynamics remained challenging up to now (Hopwood et al., 2021). The formal model of interpersonality was not designed to exhibit periodic orbits, but is able to let circular dynamics emerge under specific conditions, based on a particular interplay of approach and avoidance motives. Thereby, it paves the way for a more precise understanding of the maintenance of 'stably unstable' interpersonal relationships as a special case of interpersonality. Although the circular relational field stemming from an oscillation between approach and avoidance is very likely to be dysfunctional, a circularity in a field of two or more *approach* motives of different psychological needs is expected to be adaptive in order to satisfy needs one by one over time (Sadler et al., 2009).

Motivation and personality psychology. The interplay of approach and avoidance motives within a single individual has already been studied using monadic (in contrast to dyadic) formal models (Ballard et al., 2017; Gernigon et al., 2015; Read et al., 2017). The present model adds the idea of dyadic, reciprocal interpersonal environments to motive interactions, thereby realizing the principle of reciprocal interactionism more closely, which has been proposed in theoretical frameworks of personality (e.g., "[...] people develop in interaction with environments that are partly shaped by their own actions", Cervone, 2004, p. 184; Back et al., 2011; Bandura, 1978). Lastly, a crucial aspect of the formal model, namely the responsiveness that is reflected in the interpersonal interdependence, seems to be empirically associated with relationship quality (Nikitin & Freund, 2019).

Psychopathology. Two interpersonal dynamics emerging in the model differ from the expected principle of complementarity of interpersonal theory, namely (1) the globally stable relational field where interpersonality collapses into a single dimension and (2) the cyclical relational field of a constantly changing interpersonal relationship. A rigidity of interpersonal behavior (and the underlying motives) is assumed in individuals with personality disorders (Caspar, 1995, 2011; Hopwood et al., 2013) and an irresponsiveness to the motives and interpersonal behaviors of others is proposed to be a mechanism that underlies chronic depression (McCullough, 2003). The concept of strictly dominant motives introduced in this paper, which results in the lack of interpersonal interdependence, might help to explain these psychopathological processes and to pave the way for advancing psychological interventions.

When the expectation of affiliative approach motivation evokes avoidance motivation and the expectation of affiliative avoidance motivation evokes approach motivation, periodic orbits in the relational space emerge that are likely to be relevant for forms of psychopathology, too. The hypothesis that conflicts between motivational goals give rise to cyclical, mutually interfering behaviors is not new, of course (for a perspective of perceptual control theory on this topic, see Mansell, 2010). However, the cyclical dynamics in the formal model of interpersonality might contribute to a better understanding of psychopathology with oscillations over time by taking into account multiple timescales. The interplay of (1) interpersonal relationships as context for short-term interpersonal situations and (2) interpersonal situations as driver of long-term changes of interpersonal relationships is likely to shed a more differentiated light one the underlying mechanisms of, for example, demand/withdraw dynamics in depression (e.g., Knobloch-Fedders et al., 2014), or interpersonal patters in borderline personality disorder (e.g., Lazarus et al., 2020). The cyclical dynamics of closeness and distance are not due to an external trigger in the formal model, so that interpersonal problems of individuals with borderline personality disorder in terms of switching between idealization and devaluation of others could be understood as an intrinsic process. Importantly, a bistable relational field might also become maladaptive when the attractor basin of the avoidantly distanced attractor is larger than that of the friendly attractor (e.g., Gottman et al., 2002).

Psychological therapies. Formal models of interpersonality might be a neutral and sufficiently distant point of view to serve as a novel starting point for the ongoing efforts to integrate the disparate models of the formation, maintenance and treatment of mental disorders that different approaches to psychotherapy have developed (Castonguay et al., 2015; Grawe, 1998). From the perspective of cognitive behavioral therapies, the interpersonal maintenance of problems outlined in the formal model might appear as a dyadic, reciprocal, time-dependent reinforcement schedule (that can be impacted on not by changing the other, but one’s own behavior as proposed in CBASP and realized with interpersonal discrimination exercises and situational analyses; McCullough, 2003). From the neo-humanistic, experiential perspective of emotion-focused therapy (Goldman & Greenberg, 2007; Greenberg, 2017), the cyclical patterns emergent under certain conditions in the interpersonal model can be understood (1) as self-splits in the monadic, intrapsychic domain of individual psychotherapy (e.g., self-critical splits that are allowed two unfold and to change within a therapeutic two-chair dialogue) or (2) as self-reinforcing negative interactive cycles in the dyadic, interpersonal domain of couple therapy. From the perspective of psychodynamic therapies,

(1) the interaction of dyadic relations in the formal model could be interpreted as competing self-object configurations in line with modern object relations theory[5] (e.g., Kernberg, 2019), (2) the field in the relational state space of the formal model could resemble aspects of the relational field outlined in relational psychoanalysis (Mitchell, 1988), with attractors and also repellers in the relational field granting or restricting relational freedom (for both interactants, also in a dyadic therapeutic interaction; Stern, 2013), and (3) the intrinsic cyclical interpersonal patterns could be an expression of the self-perpetuating processes proposed in cyclical psychodynamics (Wachtel, 2014) or of the oscillation between a 'good' and a 'bad' internal part object in the paranoid-schizoid position (Klein, 1946).

The discussion of the many bridges between the formal model of interpersonality and interpersonal theory, motivation and personality psychology, clinical psychology as well as psychotherapy in the previous paragraphs has been extensive. This might reflect the utility of formal modeling and simulations as a fruitful approach in psychological research that has great potential to become a third methodological pillar next to to verbal theories and empirical approaches.

Formal Modeling and Open Theory

Formal models can be shared between research labs as codebooks or algorithmic programs and are likely to contribute to open science by allowing for 'Open Theory' (for example, see Waade et al., 2022). Importantly, they "[..] are explicit in the assumptions they make about how the parts of a system work and interact, and moreover are explicit in the aspects of reality they omit" (Smaldino, 2017, p. 328). For example, a researcher who is interested in a more fine-grained resolution of the fast changing component of the interpersonal environment, namely interpersonal situations (Pincus et al., 2020), would

[5] However, in a formal, generative way beyond verbal theories with a metaphorical use of the concept of an attractor (e.g., Connolly, 2019).

recognize that this process is modelled in a very abstract way in the current formal model, omitting important aspects of interpersonal reality. He or she could then extend or revise the previously shared, open formal model to address that aspect in a more sophisticated way (e.g., by understanding interpersonal situations as sequential games; Yoshida et al., 2008).

Another advantage of formal models is that they are able to incorporate complex, simultaneous, interactive processes taking place within and between individuals (i.e., intrapsychic and interpersonal, respectively) at multiple time scales, which is necessary for meeting the needs of modern psychological theorizing (Hofmann & Hayes, 2019; Pincus et al., 2020). For instance, the assumption of stationarity of interpersonal experiential and behavioral processes is often questionable, as the short-term dynamics can and often do qualitatively change depending on the interaction partner (and his or her motives). Such a non-stationarity has to be addressed as it is a violation the assumptions of many statistical models (Boker et al., 2016), but that might result in more explained variance if taken into account.

Limitations and Strengths

Due to the simplistic nature of this first formal model of interpersonality, the list of substantial limitations is long. The model does not allow for stable, mixed relational attractors in the interior of the relational space (Gaunersdorfer et al., 1991; Selten, 1988), that is, interpersonal relationships with multiple available ways of relating are not possible on the long run in the constraints of the model. Furthermore, limitations of the model are due to the unrealistic, psychologically implausible modeling assumptions of (1) only one need, namely affiliation, (2) a shared relational space (instead of a relational representation in each individual, potentially subject to parataxic distortion, Sullivan, 1953), implying perfect communication, (3) no satisfaction or frustration of needs, (4) a one-to-one relationship of motivation and behavior (Caspar, 2011; Horowitz et al., 2006), (5) no perceptive and interpretative processes (Pincus et al., 2020), and (6) no relevance of affective processes (Hopwood et al., 2021). However, each of these aspects could be addressed in future versions of the model. For example, the level of need satisfaction could be introduced as a novel, dynamic variable (dN/dt) that impacts on the motive matrix of the individual. Then, approach and avoidance motives would not only depend on the expectation of the other's motives and the accompanying consequences, but also the momentary frustration of needs.

Another obvious and important shortcoming is that the validity of a model does not necessarily follow from generative sufficiency (J. M. Epstein, 2006). Just as wetting the road with a bucket of water has the same effect as rainfall but tells very little about the underlying mechanisms of rain, the formal model of interpersonality developed in this paper might only imitate interpersonal dynamics with artificial, inappropriate mechanisms. However, the model specifies what data is necessary for its falsification, paving the way for empirical tests with behavioral observations (e.g., as in Knobloch-Fedders et al., 2014), experience sampling (e.g., Himmelstein et al., 2019), participatory interpersonal paradigms (Westermann & Sibilis, 2022), or a combination thereof, as well as recent statistical approaches that allow recovering bistable dynamics from timeseries (Haslbeck & Ryan, 2021). Up to now, however, the formal model of interpersonality remains empirically untested and speculative despite its foundation in empirically supported psychological constructs.

Beyond what has already been outlined as advantages for the model up to now, the strengths of the interpersonal model of interpersonality include (1) the very sparse parameter space that can be fully described with four bits (see Table 1) and thus provides room for more complex extensions of the model, (2) the relational state space that takes both experiential and the behavioral aspects into account, because a relation consists of one's own motivated behavior and the expected motivated behavior of the other, (3) a relational state space that is richer than in other models (e.g., Butler et al., 2017; Liebovitch et al., 2011), preventing the complexity of interpersonal meanings to collapse into a single dimension of valence due to restrictive modeling instead of restrictive dynamics, and (4) the idiographic dyad as genuine basic unit of interpersonal analysis.

Outlook

By allowing for empirically informed simulations, formal models of interpersonal dynamics have the potential to address numerous timely and relevant open research question: What underlying mechanisms account for some interpersonal relationships being stable over time and others being unstable? Under what circumstances are maladaptive interpersonal patterns maintained in dyadic interaction despite being distressing? How are ways of dyadic relating transmitted from one generation to the next over a time span of years or even decades? In which way do psychological therapies change the interpersonal patterns patients use to relate to others? Empirical investigations addressing such questions face multiple challenges that pose no problem to formal models and simulations. For example, ethical considerations forbid to induce psychopathology in real – but not in simulated – individuals, and the feasibility of empirical research project with a timeframe of several decades is very limited – but the simulation of billions of interactions taking place over a hundred years of simulated time in a couple of hours of computing time is not a problem. Hopefully, the formal model of interpersonality contributes to formulating and investigating research questions that have been difficult to answer in interpersonal theory and related fields such as clinical psychology without formal modeling and simulation.

More specifically, there are many opportunities for future research to advance the present formal model. For example, (1) by exchanging the need for affiliation with the need for control and its according behavioral dimension of the interpersonal circumplex, (2) by extending the model to two needs for each interactants, allowing for cross-motive interactions (e.g., with a relational field in that affiliation is conditioned on submission), (3) by modeling individual, non-shared relationship representations that can differ from each other, (4) by introducing imperfect, noisy communication to assess the stability of interpersonal relationships despite misunderstandings, (5) by modeling deliberate self-regulation as external perturbation of the dyadic system (Carver & Scheier, 2002; Henry et al., 2022) with a psychological metric of the distance between two relational attractors (e.g., how much self-regulated behavior is necessary to change the relationship on the long run?), (6) by modeling learning in the sense of a dynamic on the motive matrices of both interactants (dA/dt and dB/dt) or only one interactant such as a patient in psychotherapy (dP/dt), and (7) by translating the mathematical model into an algorithmic pendant and vice versa (Banisch, 2016; Scalco et al., 2018) to allow for simulation of complex processes such as dyadic dynamics on a social graphs in which relations spread from one interactant to the next, or transmission of relational patterns over multiple generations.

Implications

The implications of a formal working model of interpersonality are potentially manifold. Interpersonal theory as well as personality, social and clinical psychology might adopt formal modeling that is grounded in psychological theories and allows to test complex theories in a simulation prior to expensive empirical tests. Thereby formal modeling might evolve to an important counterweight against a too strong focus on data-driven, atheoretical approaches in psychological research. More specifically, personality psychology might embrace this opportunity to overcome the separation of personality and environment by instead dovetailing of person- and environment processes on different timescales in formal models and simulation. Researchers in social psychology and computational social science might be interested in bringing the interpersonal processes (Kelley, 1991) to a larger scale, generalizing the model dynamics from the dyad to the triad, to small groups, or even large social networks. Research in clinical psychology and psychotherapy is likely to benefit from more complex, dynamic models of psychopathology (e.g., of borderline personality disorder), particularly with regard to the formation, maintenance and treatment of psychopathology that is not an isolated, intrapsychic process within an individual, but relies on interpersonal processes for its maintenance. Importantly, if formal modeling were used in all these areas of research, it would not be a replacement for but an enrichment of verbal theorizing and empirical studies.

Conclusion

The dyadic interplay of two individuals forming, maintaining and ending their interpersonal relationship to satisfy and protect their psychological needs in numerous interpersonal situations is complex and a challenging research topic. However, a simplistic formal model of dyadic interaction with very constrained but plausible and empirically supported psychological assumptions was sufficient to generate a substantial part of that interpersonal complexity including well-known but also novel regularities such as cyclic interpersonal dynamics. As generative sufficiency of a formal model is not at all a guarantee for validity, only future empirical tests can corroborate or falsify the formal model. Yet, the model is one of the few models of interpersonality that are indeed formal and hopefully paves the way for other researchers to use formal modeling and empirically informed simulations in research on interpersonal theory and other psychological disciplines.

Acknowledgements

I thank LY for the illustrations of the dyadic pictograms used throughout the figures of this paper, TB and TL for the helpful comments on the initial draft of this paper, and MLW for giving me the impetus to write the manuscript. This work was supported by the Swiss National Science Foundation (grant number 160412; international short research visit of SW).

References

Babl, A., Berger, T., Eubanks, C. F., Gómez Penedo, J. M., Caspar, F., Sachse, R., & Kramer, U. (2022). Addressing interpersonal patterns in patients with personality disorders partially explains psychotherapy outcome via changes in interaction patterns: A mediation analysis. *Psychotherapy Research*, 1–11. https://doi.org/10.1080/10503307.2022.2036383

Back, M. D., Baumert, A., Denissen, J. J. A., Hartung, F., Penke, L., Schmukle, S. C., Schönbrodt, F. D., Schröder–Abé, M., Vollmann, M., Wagner, J., & Wrzus, C. (2011). PERSOC: A Unified Framework for Understanding the Dynamic Interplay of Personality and Social Relationships. *European Journal of Personality*, *25*(2), 90–107. https://doi.org/10.1002/per.811

Baker, A. Z., Peluso, P. R., Freund, R., Diaz, P., & Ghaness, A. (2021). Using dynamical systems mathematical modeling to examine the impact emotional expression on the therapeutic relationship: A demonstration across three psychotherapeutic theoretical approaches. *Psychotherapy Research: Journal of the Society for Psychotherapy Research*, 1–15. https://doi.org/10.1080/10503307.2021.1921303

Baldwin, M. W. (1992). Relational schemas and the processing of social information. *Psychological Bulletin*, *112*(3), 461–484. https://doi.org/10/cx7t9b

Ballard, T., Yeo, G., B. Vancouver, J., & Neal, A. (2017). The dynamics of avoidance goal regulation. *Motivation and Emotion*, *41*(6), 698–707. https://doi.org/10.1007/s11031-017-9640-8

Bandura, A. (1978). The self system in reciprocal determinism. *American Psychologist*, *33*(4), 344–358. https://doi.org/10.1037/0003-066X.33.4.344

Banisch, S. (2016). *Markov Chain Aggregation for Agent-Based Models*. Springer International Publishing. https://doi.org/10.1007/978-3-319-24877-6

Banisch, S., & Olbrich, E. (2019). Opinion polarization by learning from social feedback. *The Journal of Mathematical Sociology*, *43*(2), 76–103. https://doi.org/10.1080/0022250X.2018.1517761

Baumeister, R. F., & Leary, M. R. (1995). The need to belong: Desire for interpersonal attachments as a fundamental human motivation. *Psychological Bulletin*, *117*(3), 497–529. https://doi.org/10.1037/0033-2909.117.3.497

Benjamin, L. S. (1974). Structural analysis of social behavior. *Psychological Review*, *81*(5), 392–425. https://doi.org/10/cksftd

Benjamin, L. S. (1979). Use of structural analysis of social behavior (SASB) and Markov chains to study dyadic interactions. *Journal of Abnormal Psychology*, *88*(3), 303–319. https://doi.org/10/dngm4d

Blatt, S. J., & Luyten, P. (2009). A structural–developmental psychodynamic approach to psychopathology: Two polarities of experience across the life span. *Development and Psychopathology*, *21*(03), 793–814. https://doi.org/10/fvdzq2

Boag, S. (2018). Personality Dynamics, Motivation, and the Logic of Explanation. *Review of General Psychology*, *22*(4), 427–436. https://doi.org/10/gfsrwm

Boker, S. M., Staples, A. D., & Hu, Y. (2016). Dynamics of Change and Change in Dynamics. *Journal for Person-Oriented Research*, *2*(1–2), 34–55. https://doi.org/10/gfpn88

Bürkner, P.-C. (2017). **brms**: An *R* Package for Bayesian Multilevel Models Using *Stan*. *Journal of Statistical Software*, *80*(1). https://doi.org/10.18637/jss.v080.i01

Butler, E. A., Guan, J., Predoehl, A., Brau, E., Simek, K., & Barnard, K. (2017). Computational Temporal Interpersonal Emotion Systems. In R. R. Vallacher, S. J. Read, & A. Nowak (Eds.), *Computational Social Psychology* (1st ed., pp. 127–143). Routledge. https://doi.org/10.4324/9781315173726-7

Carver, C. S. (2006). Approach, Avoidance, and the Self-Regulation of Affect and Action. *Motivation and Emotion*, *30*(2), 105–110. https://doi.org/10/dg4w27

Carver, C. S., & Scheier, M. F. (1998). *On the self-regulation of behavior.* Cambridge University Press.

Carver, C. S., & Scheier, M. F. (2002). Control Processes and Self-Organization as Complementary Principles Underlying Behavior. *Personality and Social Psychology Review*, *6*(4), 304–315. https://doi.org/10/c4sr97

Caspar, F. (1995). *Plan Analysis: Toward optimizing psychotherapy*. Hogrefe.

Caspar, F. (2011). Plan analysis. In T. D. Eells (Ed.), *Handbook of psychotherapy case formulation* (2nd ed., pp. 251–289). Guilford Press.

Castonguay, L. G., Eubanks, C. F., Goldfried, M. R., Muran, J. C., & Lutz, W. (2015). Research on psychotherapy integration: Building on the past, looking to the future. *Psychotherapy Research*, *25*(3), 365–382. https://doi.org/10.1080/10503307.2015.1014010

Cervone, D. (2004). The Architecture of Personality. *Psychological Review*, *111*(1), 183–204. https://doi.org/10.1037/0033-295X.111.1.183

Connolly, P. (2019). The Gravity of Objects: How Affectively Organized Generative Models Influence Perception and Social Behavior. *Frontiers in Psychology*, *10*, 2599. https://doi.org/10/ggffpr

Deci, E. L., & Ryan, R. M. (2000). The “what” and “why” of goal pursuits: Human needs and the self-determination of behavior. *Psychological Inquiry*, *11*(4), 227–268. https://doi.org/10/bfn2hn

Deci, E. L., & Ryan, R. M. (2014). Autonomy and Need Satisfaction in Close Relationships: Relationships Motivation Theory. In N. Weinstein (Ed.), *Human Motivation and*

Interpersonal Relationships (pp. 53–73). Springer Netherlands. https://doi.org/10.1007/978-94-017-8542-6_3

Dweck, C. S. (2017). From needs to goals and representations: Foundations for a unified theory of motivation, personality, and development. *Psychological Review*, *124*(6), 689–719. https://doi.org/10/gc93cx

Elliot, A. J. (2006). The Hierarchical Model of Approach-Avoidance Motivation. *Motivation and Emotion*, *30*(2), 111–116. https://doi.org/10/bmstk9

Epstein, J. M. (2006). Remarks on the Foundations of Agent-Based Generative Social Science. In L. Tesfatsion & K. Judd (Eds.), *Handbook of Computational Economics* (Vol. 2, pp. 1585–1604). North Holland. https://doi.org/10.1016/S1574-0021(05)02034-4

Epstein, J. M. (2012). *Generative Social Science: Studies in Agent-Based Computational Modeling:* Princeton University Press. https://doi.org/10.1515/9781400842872

Epstein, S. (2003). Cognitive-experiential self-theory of personality. In T. Millon & M. J. Lerner (Eds.), *Handbook of psychology* (pp. 159–184). Wiley & Sons.

Fonagy, P., Gergely, G., Jurist, E. L., & Target, M. (2018). *Affect Regulation, Mentalization, and the Development of the Self* (P. Fonagy, G. Gergely, & E. L. Jurist, Eds.; 1st ed.). Routledge. https://doi.org/10.4324/9780429471643

Fried, E. I. (2020). Theories and Models: What They Are, What They Are for, and What They Are About. *Psychological Inquiry*, *31*(4), 336–344. https://doi.org/10.1080/1047840X.2020.1854011

Fried, E. I., & Robinaugh, D. J. (2020). Systems all the way down: Embracing complexity in mental health research. *BMC Medicine*, *18*(1), 205, s12916-020-01668-w. https://doi.org/10.1186/s12916-020-01668-w

Frith, U., & Frith, C. D. (2003). Development and neurophysiology of mentalizing. *Philosophical Transactions of the Royal Society of London. Series B: Biological Sciences*, *358*(1431), 459–473. https://doi.org/10.1098/rstb.2002.1218

Gable, S. L. (2006). Approach and Avoidance Social Motives and Goals. *Journal of Personality*, *74*(1), 175–222. https://doi.org/10/bm9ztj

Gaunersdorfer, A., Hofbauer, J., & Sigmund, K. (1991). On the dynamics of asymmetric games. *Theoretical Population Biology*, *39*(3), 345–357. https://doi.org/10.1016/0040-5809(91)90028-E

Gelfand, L. A., & Engelhart, S. (2012). Dynamical Systems Theory in Psychology: Assistance for the Lay Reader is Required. *Frontiers in Psychology*, *3*(October), 382. https://doi.org/10.3389/fpsyg.2012.00382

Gelfand, L. A., Ervin, M. C., & Germ, S. R. (2018). Commentary: Pattern destabilization and emotional processing in cognitive therapy for personality disorders. *Frontiers in Psychology*, *9*(FEB), 1845. https://doi.org/10/gf7kdg

Gernigon, C., Vallacher, R. R., Nowak, A., & Conroy, D. E. (2015). Rethinking Approach and Avoidance in Achievement Contexts: The Perspective of Dynamical Systems. *Review of General Psychology*, *19*(4), 443–457. https://doi.org/10/gf7j9b

Goldman, R. N., & Greenberg, L. S. (2007). Integrating Love and Power in Emotion-Focused Couple Therapy. *European Psychotherapy*, *7*(1), 117–135.

Gottman, J., Swanson, C., & Swanson, K. (2002). A General Systems Theory of Marriage: Nonlinear Difference Equation Modeling of Marital Interaction. *Personality and Social Psychology Review*, *6*(4), 326–340. https://doi.org/10/b9pnfm

Grawe, K. (1998). *Psychologische Therape [Psychological Therapy]*. Bern: Hogrefe.

Greenberg, L. S. (2017). Emotion-focused therapy of depression. *Person-Centered and Experiential Psychotherapies*, *16*(2), 106–117. https://doi.org/10/gf7j9d

Grosse Holtforth, M. (2008). Avoidance motivation in psychological problems and psychotherapy. *Psychotherapy Research*, *18*(2), 147–159. https://doi.org/10/dbff47

Guest, O., & Martin, A. E. (2021). How Computational Modeling Can Force Theory Building in Psychological Science. *Perspectives on Psychological Science*, 174569162097058. https://doi.org/10.1177/1745691620970585

Haslbeck, J. M. B., & Ryan, O. (2021). Recovering Within-Person Dynamics from Psychological Time Series. *Multivariate Behavioral Research.* Advance online publication. https://doi.org/10.1080/00273171.2021.1896353

Haslbeck, J. M. B., Ryan, O., Robinaugh, D., Waldorp, L., & Borsboom, D. (2021). Modeling Psychopathology: From Data Models to Formal Theories. *Psychological Methods.* Advance online publication. https://doi.org/10.31234/osf.io/jgm7f

Henry, T. R., Robinaugh, D. J., & Fried, E. I. (2022). On the Control of Psychological Networks. *Psychometrika*, *87*(1), 188–213. https://doi.org/10.1007/s11336-021-09796-9

Herzog, P., Kube, T., & Fassbinder, E. (2022). How childhood maltreatment alters perception and cognition – the predictive processing account of borderline personality disorder. *Psychological Medicine*, 1–18. https://doi.org/10.1017/S0033291722002458

Himmelstein, P. H., Woods, W. C., & Wright, A. G. C. (2019). A comparison of signal- and event-contingent ambulatory assessment of interpersonal behavior and affect in social situations. *Psychological Assessment*, *31*(7), 952–960. https://doi.org/10.1037/pas0000718

Hofman, J. M., Watts, D. J., Athey, S., Garip, F., Griffiths, T. L., Kleinberg, J., Margetts, H., Mullainathan, S., Salganik, M. J., Vazire, S., Vespignani, A., & Yarkoni, T. (2021). Integrating explanation and prediction in computational social science. *Nature*, *595*(7866), 181–188. https://doi.org/10.1038/s41586-021-03659-0

Hofmann, S. G., & Hayes, S. C. (2019). The Future of Intervention Science: Process-Based Therapy. *Clinical Psychological Science*, *7*(1), 37–50. https://doi.org/10.1177/2167702618772296

Holland, J. N., & Deangelis, D. L. (2010). A consumer-resource approach to the density-dependent population dynamics of mutualism. *Ecology*, *91*(5), 1286–1295. https://doi.org/10/b4sqw8

Hopwood, C. J. (2018). Interpersonal Dynamics in Personality and Personality Disorders. *European Journal of Personality*, *32*(5), 499–524. https://doi.org/10/gfh38v

Hopwood, C. J., Pincus, A. L., & Wright, A. G. C. (2021). Six assumptions of contemporary integrative interpersonal theory of personality and psychopathology. *Current Opinion in Psychology*, *41*, 65–70. https://doi.org/10.1016/j.copsyc.2021.03.007

Hopwood, C. J., Wright, A. G. C., Ansell, E. B., & Pincus, A. L. (2013). The Interpersonal Core of Personality Pathology. *Journal of Personality Disorders*, *27*(3), 270–295. https://doi.org/10.1521/pedi.2013.27.3.270

Horowitz, L. M., Wilson, K. R., Turan, B., Zolotsev, P., Constantino, M. J., & Henderson, L. (2006). How Interpersonal Motives Clarify the Meaning of Interpersonal Behavior: A Revised Circumplex Model. *Personality and Social Psychology Review*, *10*(1), 67–86. https://doi.org/10/cwbbz5

Kelley, H. H. (1991). Lewin, Situations, and Interdependence. *Journal of Social Issues*, *47*(2), 211–233. https://doi.org/10.1111/j.1540-4560.1991.tb00297.x

Kernberg, O. F. (2019). Therapeutic Implications of Transference Structures in Various Personality Pathologies. *Journal of the American Psychoanalytic Association*, *67*(6), 951–986. https://doi.org/10.1177/0003065119898190

Kiesler, D. J. (1996). *Contemporary interpersonal theory and research: Personality, psychopathology, and psychotherapy.* John Wiley & Sons.

Klein, M. (1946). Notes on some schizoid mechanisms. *The International Journal of Psycho-Analysis*, *27*(Pt 3-4), 99–110.

Knobloch-Fedders, L. M., Critchfield, K. L., Boisson, T., Woods, N., Bitman, R., & Durbin, C. E. (2014). Depression, relationship quality, and couples' demand/withdraw and demand/submit sequential interactions. *Journal of Counseling Psychology*, *61*(2), 264–279. https://doi.org/10.1037/a0035241

Lazarus, S. A., Beeney, J. E., Howard, K. P., Strunk, D. R., Pilkonis, Paul. A., & Cheavens, J. S. (2020). Characterization of relationship instability in women with borderline personality disorder: A social network analysis. *Personality Disorders: Theory, Research, and Treatment*, *11*(5), 312–320. https://doi.org/10.1037/per0000380

Lewin, K. (1943). Defining the'field at a given time.'. *Psychological Review*, *50*(3), 292–310. https://doi.org/10/ch4jgm

Lichtenberg, J. D. (1988). A theory of motivational-functional systems as psychic structures. *Journal of the American Psychoanalytic Association*, *36*, 57–72.

Liebovitch, L. S., Peluso, P. R., Norman, M. D., Su, J., & Gottman, J. M. (2011). Mathematical model of the dynamics of psychotherapy. *Cognitive Neurodynamics*, *5*(3), 265–275. https://doi.org/10/c6q8st

Luyten, P., & Blatt, S. J. (2013). Interpersonal relatedness and self-definition in normal and disrupted personality development: Retrospect and prospect. *American Psychologist*, *68*(3), 172–183. https://doi.org/10/f4t8zb

Mansell, W. (2010). Control theory and psychopathology: An integrative approach. *Psychology and Psychotherapy: Theory, Research and Practice*. https://doi.org/10.1348/147608304X21400

McClelland, D. C. (1987). *Human motivation*. CUP Archive.

McCullough, J. P. (2003). Treatment for chronic depression using Cognitive Behavioral Analysis System of Psychotherapy (CBASP). *Journal of Clinical Psychology, 59*(8). https://doi.org/10.1002/jclp.10176

Mitchell, S. A. (1988). *Relational Concepts in Psychoanalysis: An Integration*. Harvard University Press.

Molenaar, P. C. M. (2004). A Manifesto on Psychology as Idiographic Science: Bringing the Person Back Into Scientific Psychology, This Time Forever. *Measurement: Interdisciplinary Research & Perspective*, *2*(4), 201–218. https://doi.org/10/fqvbnc

Nikitin, J., & Freund, A. M. (2019). Who Cares? Effects of Social Approach and Avoidance Motivation on Responsiveness to Others. *Personality and Social Psychology Bulletin*, *45*(2), 182–195. https://doi.org/10.1177/0146167218781335

Nowak, A. (2004). Dynamical Minimalism: Why Less is More in Psychology. *Personality and Social Psychology Review*, *8*(2), 183–192. https://doi.org/10.1207/s15327957pspr0802_12

Nowak, A., Rychwalska, A., & Borkowski, W. (2013). Why Simulate? To Develop a Mental Model. *Journal of Artificial Societies and Social Simulation*, *16*(3), 12. https://doi.org/10/gf7j9g

Oberauer, K., & Lewandowsky, S. (2019). Addressing the theory crisis in psychology. *Psychonomic Bulletin & Review*, *26*(5), 1596–1618. https://doi.org/10.3758/s13423-019-01645-2

Orehek, E., Forest, A. L., & Wingrove, S. (2018). People as Means to Multiple Goals: Implications for Interpersonal Relationships. *Personality and Social Psychology Bulletin.*, 46. https://doi.org/10.1177/0146167218769869

Pincus, A. L., & Ansell, E. B. (2003). Interpersonal Theory of Personality. In T. Millon & M. Lerner (Eds.), *Handbook of Psychology: Vol. Personality and social psychology* (pp. 209–229). John Wiley. https://doi.org/10.1002/0471264385.wei0509

Pincus, A. L., Hopwood, C. J., & Wright, A. G. C. (2020). The Interpersonal Situation: An Integrative Framework for the Study of Personality, Psychopathology, and Psychotherapy. In J. F. Rauthmann, R. A. Sherman, & D. C. Funder (Eds.), *The Oxford Handbook of Psychological Situations* (pp. 123–142). Oxford University Press. https://doi.org/10.1093/oxfordhb/9780190263348.013.4

Powers, W. T. (1973). *Behavior: The Control of Perception*. De Gruyter, Chicago.

Read, S. J., Smith, B. J., Droutman, V., & Miller, L. C. (2017). Virtual personalities: Using computational modeling to understand within-person variability. *Journal of Research in Personality*, *69*, 237–249. https://doi.org/10/gdngft

Robinaugh, D., Haslbeck, J. M. B., Waldorp, L., Kossakowski, J. J., Fried, E. I., Millner, A., McNally, R. J., van Nes, E. H., Scheffer, M., Kendler, K. S., & Borsboom, D. (preprint). *Advancing the Network Theory of Mental Disorders: A Computational Model of Panic Disorder*. https://doi.org/10.31234/osf.io/km37w

Robinaugh, D., Haslbeck, J., Ryan, O., Fried, E. I., & Waldorp, L. (2021). Invisible Hands and Fine Calipers: A Call to Use Formal Theory as a Toolkit for Theory Construction. *Perspectives on Psychological Science, 16*(4), 725 –743. https://doi.org/10.1177/1745691620974697

Roefs, A., Fried, E. I., Kindt, M., Martijn, C., Elzinga, B., Evers, A. W. M., Wiers, R. W., Borsboom, D., & Jansen, A. (2022). A new science of mental disorders: Using personalised, transdiagnostic, dynamical systems to understand, model, diagnose and treat psychopathology. *Behaviour Research and Therapy*, *153*, 104096. https://doi.org/10.1016/j.brat.2022.104096

Sadler, P., Ethier, N., Gunn, G. R., Duong, D., & Woody, E. (2009). Are we on the same wavelength? Interpersonal complementarity as shared cyclical patterns during interactions. *Journal of Personality and Social Psychology*, *97*(6), 1005–1020. https://doi.org/10.1037/a0016232

Scalco, A., Ceschi, A., & Sartori, R. (2018). Application of Psychological Theories in Agent-Based Modeling: The Case of the Theory of Planned Behavior. *Nonlinear Dynamics, Psychology, and Life Sciences*, *22*(1), 15–33.

Selten, R. (1988). A Note on Evolutionarily Stable Strategies in Asymmetric Animal Conflicts. In W. Leinfellner, G. Eberlein, & S. H. Tijs (Eds.), *Models of Strategic Rationality* (Vol. 2, pp. 67–75). Springer Netherlands. https://doi.org/10.1007/978-94-015-7774-8_3

Skinner, B. F. (1953). *Science and Human Behavior*. MacMillan.

Smaldino, P. E. (2017). Models Are Stupid, and We Need More of Them. In R. R. Vallacher, S. J. Read, & A. Nowak (Eds.), *Computational Social Psychology* (1st ed., pp. 311–331). Routledge. https://doi.org/10.4324/9781315173726-14

Stern, D. B. (2013). Relational Freedom and Therapeutic Action. *Journal of the American Psychoanalytic Association*, *61*(2), 227–256. https://doi.org/10.1177/0003065113484060

Sullivan, H. S. (1953). *The interpersonal theory of psychiatry*. Norton.

Vallacher, R. R., Van Geert, P., & Nowak, A. (2015). The Intrinsic Dynamics of Psychological Process. *Current Directions in Psychological Science*, *24*(1), 58–64. https://doi.org/10.1177/0963721414551571

Waade, P. T., Enevoldsen, K. C., Vermillet, A.-Q., Simonsen, A., & Fusaroli, R. (2022). Introducing tomsup: Theory of mind simulations using Python. *Behavior Research Methods*. https://doi.org/10.3758/s13428-022-01827-2

Wachtel, P. L. (1994). Cyclical processes in personality and psychopathology. *Journal of Abnormal Psychology*, *103*(1), 51–54. https://doi.org/10.1037/0021-843X.103.1.51

Wachtel, P. L. (2014). An integrative relational point of view. *Psychotherapy*, *51*(3), 342–349. https://doi.org/10.1037/a0037219

Wagner, C. C., Kiesler, D. J., & Schmidt, J. A. (1995). Assessing the interpersonal transaction cycle: Convergence of action and reaction interpersonal circumplex measures. *Journal of Personality and Social Psychology*, *69*(5), 938–949. https://doi.org/10/cfv8jr

Weibull, J. W. (2004). *Evolutionary game theory*. MIT Press.

Westermann, S., grosse Holtforth, M., & Michalak, J. (2019). Motivation in Psychotherapy. In R. M. Ryan (Ed.), *The Oxford Handbook of Human Motivation* (pp. 415–442). Oxford University Press. https://doi.org/10.1093/oxfordhb/9780190666453.013.23

Westermann, S., & Sibilis, A. (2022). *Emergence and assessment of interpersonal experience and behavior in a nonverbal, generative, game-like paradigm*. https://doi.org/10.1016/j.actpsy.2022.103689

Wiggins, J. S., & Trobst, K. K. (1999). The fields of interpersonal behavior. In L. A. Pervin & O. P. John (Eds.), *Handbook of personality: Theory and research* (pp. 653–670). Guilford Press.

Wilensky, U., & Rand, W. (2015). *An introduction to agent-based modeling: Modeling natural, social, and engineered complex systems with NetLogo*.

Winnicott, D. W. (1969). The use of an object and relating through identification. In *Playing and reality* (pp. 86–94). Basic Books.

Wrzus, C., Hänel, M., Wagner, J., & Neyer, F. J. (2013). Social network changes and life events across the life span: A meta-analysis. *Psychological Bulletin*, *139*(1), 53–80. https://doi.org/10.1037/a0028601

Yoshida, W., Dolan, R. J., & Friston, K. J. (2008). Game Theory of Mind. *PLoS Computational Biology*, *4*(12), e1000254. https://doi.org/10.1371/journal.pcbi.1000254