Explaining forgetting at different timescales requires a time-variant forgetting function

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Abstract

Employing memory models in practical applications provides an opportunity to test and refine theories on a large scale. The better models work in real-world scenarios, the more they can benefit educational practice. Using recall data spanning timescales from minutes to weeks, we find that a single forgetting curve cannot simultaneously capture short- and long-term performance, and show that an accurate model therefore requires a forgetting function that changes with time.

1 Main text

The growing availability of large data sets collected in naturalistic environments makes it easier than ever to evaluate and improve models of cognition outside the confines of laboratory studies. These data sets enable real-world testing of psychological theories in larger and more diverse samples, providing new insights that would be difficult or impossible to derive from traditional experiments [1, 2, 3, 4]. Such efforts are especially important for models of human memory, which can inform and sometimes directly shape educational practice [5, 6]. Adaptive learning systems built on a model of students' internal memory state have been demonstrated to support learning more effectively than less adaptive systems [7, 8, 9, 10, 11, 12]. For these systems to function optimally, a good memory model is critical.

Memory models employed in educational settings should ideally be able to capture learners' performance both on the short timescale of a single learning session (seconds to minutes) as well as on longer timescales spanning multiple learning sessions (hours to days and weeks). That way, the adaptive scheduling of study material can be optimised both within and across learning sessions. However, this is typically not the case: models are generally good at predicting either short-term or long-term retention, but not both using the same parameters. Here, we illustrate this problem in a commonly used memory model, and demonstrate a solution using time-variant forgetting parameters.

We use a naturalistic data set consisting of 25,843 fact learning sequences spanning a wide range of retention intervals. The data were collected from 218 learners through an adaptive learning system used in the context of an undergraduate cognitive psychology course (see [6] for details). Learners repeatedly studied course-related facts across multiple retrieval practice sessions. They decided by themselves when to use the system, resulting in between-session

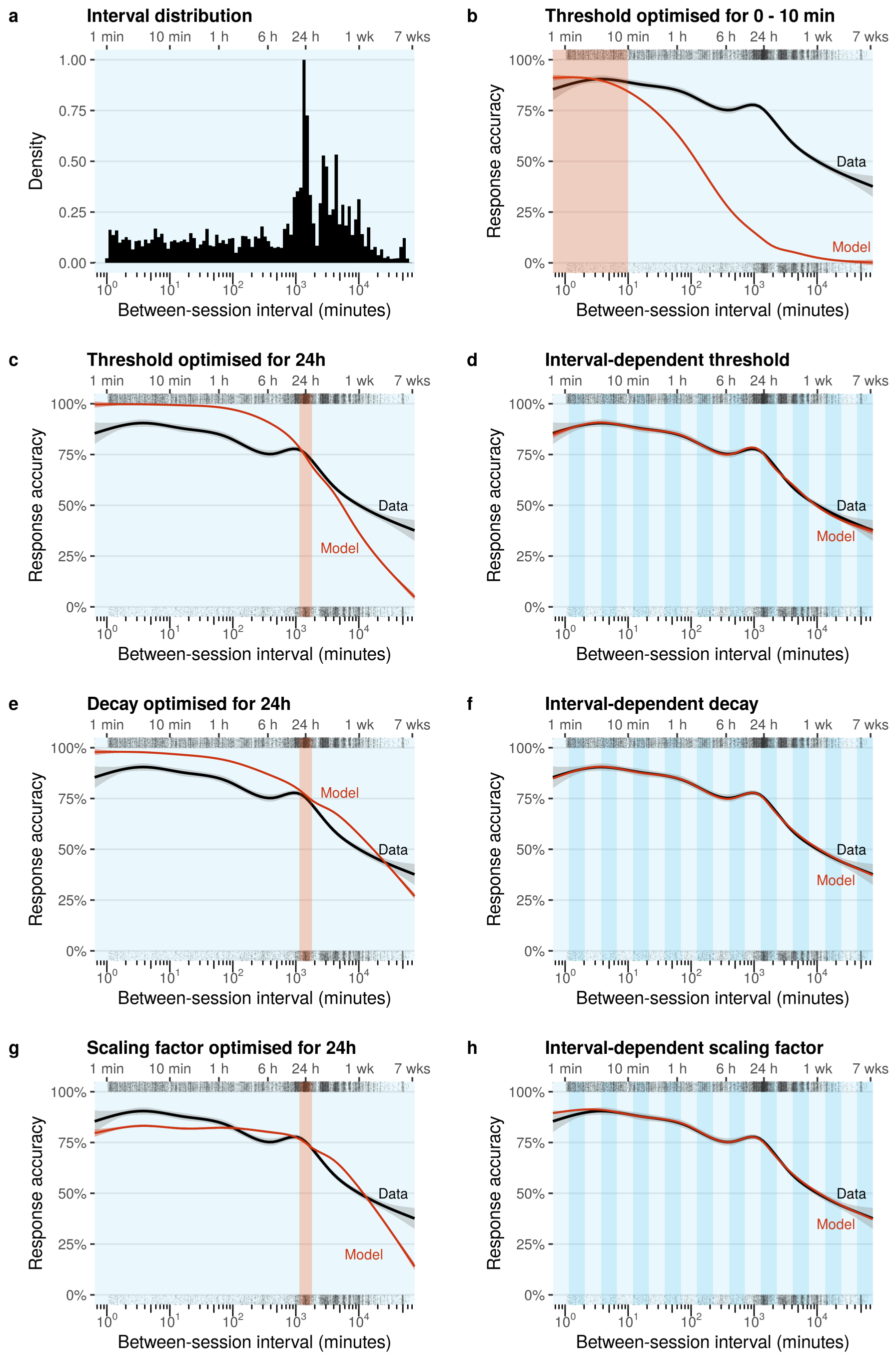

Figure 1: Data and fitted ACT-R memory models. Curves are fitted GAMs smooths (shaded areas show the 95% CI). **a**: Distribution of between-session intervals. **b**: Standard model with threshold fitted to short intervals up to 10 minutes (vertical orange bar). **c**, **e**, and **g**: Standard models with a single parameter fitted to intervals around 24 hours (vertical orange bar). **d**, **f**, and **h**: Models with a single parameter fitted separately to the data in each time bin (vertical blue stripes).

intervals that varied naturally from several seconds (when a learner did back-to-back learning sessions) to multiple weeks, with a median interval of about 20 hours (see Figure 1a). As is typical, response accuracy gradually decreased with longer intervals (see the black curve in Figure 1b).

To model these data, we fit a standard ACT-R model of declarative memory, which is an instantiation of trace decay theory [13]. Like others (e.g., [14, 10]), ACT-R assumes that the forgetting function is the same across short and long intervals. Previous work has shown that the predictions of an ACT-R model can be harnessed for more efficiently spaced practice—both within [7, 8] and between sessions [15]. However, there is a trade-off: when configured to reproduce performance on intervals up to 10 minutes, the model becomes far too pessimistic in its predictions as intervals get longer (Figure 1b), while the same model fitted to longer intervals overestimates accuracy on short intervals (Figure 1c).

Similar to many other memory models (e.g., [16, 17, 18, 19, 20]), ACT-R assumes that forgetting over time is described by a power law. Recall of an item is contingent on the combined activation of its associated traces being above a certain threshold. By changing the rate of decay (the exponent d) and/or the threshold (τ), optimal fits can be obtained at different intervals. Figures 1c and 1e show the result of optimising τ or d, respectively, for intervals around 24h. In these cases, while recall is now captured correctly around 24h, projected retention at other intervals deviates from the data.

Others have shown how scaling between-session intervals by some factor h (between 0 and 1) can improve long-term predictions, the underlying assumption being that memories suffer less interference—and are therefore forgotten slower—in off-task periods [21, 22]. If we apply a scaling factor such that recall at 24h intervals is captured correctly, the model does perform better, but still mispredicts retention on shorter and longer timescales (Figure 1g).

These attempts at one-size-fits-all fitting show that the ACT-R model cannot reproduce the whole recall curve using a single set of parameters. This issue likely affects other models that assume a time-invariant forgetting process, whether represented by a power function, exponential function, or in some other form [23, 18]. The practical implication is that a well-calibrated model may tell us the optimal moment to repeat an item within a single session, but after a delay of an hour or more, that same model will already systematically underestimate retention, hampering its ability to make useful scheduling recommendations in a subsequent learning session. This is unfortunate, because estimated individual differences in memory strength between items tend to persist over time, and could therefore be exploited to improve learning outcomes if correctly modelled [24, 25].

We can solve this problem by factoring in the duration of the interval when setting model parameters. One way to achieve this, demonstrated here, is to bin the learning sequences based on the between-session interval, and then determining model parameters separately for each bin. We use a 20-bin split, with bins of equal width on a logarithmic scale. As Figures 1d, 1f, and 1h show, this procedure produces a strong correspondence between data and model, irrespective of which parameter we vary.

Furthermore, Figure 2 confirms that there is a strong relationship between the geometric mean interval of each bin and the fitted parameter value, indicating systematic change over time. This regularity makes it possible to find appropriate model settings for any interval within the observed range, and potentially for longer intervals through extrapolation.

While the three different time-variant solutions shown here all achieve a good fit across timescales, each solution has distinct implications for modelling

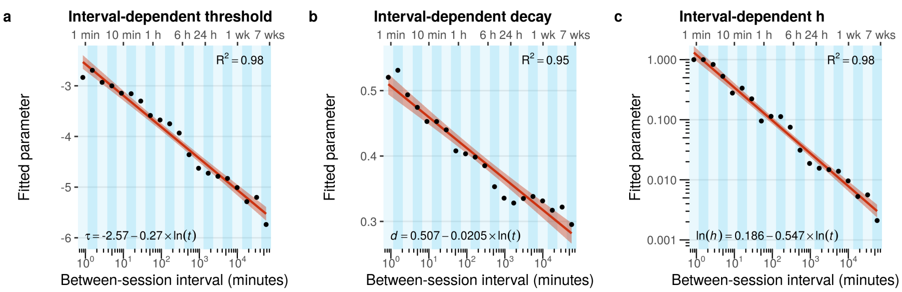

Figure 2: Optimal model parameters per time bin. Points mark the geometric mean of each bin and were used to fit the red regression lines (red shaded areas show the 95% CI). The regression equation is shown in the bottom left of each sub-figure; the fit is shown in the top right.

memory, which we will outline in turn. Firstly, a declining threshold over time suggests that retrieval effort increases as intervals get longer: while the rate of decay of the memory trace may be unaffected by the interval, learners could (temporarily) expend more effort on retrieval. This fits with accounts of short-term and long-term variability in threshold as a type of speed-accuracy trade-off [26, 27].

Secondly, a time-variant decay exponent is consistent with the notion of ever stronger consolidation of memories as they age [28, 16, 29]. This explanation has similarities to the Multiscale Context Model (which represents items using a cascade of leaky accumulators that also have sequentially slower decay rates; [18]), as well as the sum-of-exponentials model [30]. While an apparent slowing of decay can have multiple causes, one explanation is that a longer interval between sessions increases the chance of a memory being reactivated in the meantime. In an educational setting, where materials share a common theme, specific memories could easily be reactivated during class or in another study session. Memory reactivation may also happen involuntarily at random moments [31], and during sleep [32].

Finally, scaling intervals between sessions by h operationalises the idea that memory traces experience less interference outside learning sessions [21, 22, 33]. A decreasing h may reflect that the rate at which interfering information is encountered continues to decline over time. Importantly, our finding that the optimal value of h is time-variant indicates that h should not be thought of as a stable factor. While a single value may work well enough in settings with a limited range of intervals (e.g., [22, 33]), it is suboptimal for more general (educational) applications.

In summary, these analyses show that where a single forgetting curve falls short, a time-variant model of forgetting does capture memory retention across timescales (Figure 1). Using naturalistic data with intervals ranging from seconds to weeks, we have found that optimal model parameters change in a predictable manner (Figure 2). This makes it possible to tailor the predictions of a single model to a wide range of intervals. In educational applications, a model of memory that is accurate both within and between learning sessions can be a key element in improving learning outcomes.

2 Methods

2.1 Data

We use data of undergraduate students using an adaptive fact learning system in a university course, described in more detail in the original study [6]. Participants gave informed consent, and the Ethical Committee Psychology at the University of Groningen approved the use of their data (ID: 18072-O). Students could use the system to study glossary terms associated with the course through retrieval practice. Learning sessions were self-initiated, but the scheduling of practice trials within sessions was determined by the adaptive learning system. The resulting data consisted of learning sequences, each of which was associated with a particular learner studying a particular fact over multiple repetitions. Facts were studied through open-answer retrieval practice trials: a cue appeared on screen and the learner typed the associated answer into a text box. On the first presentation of a fact, the answer was provided along with the cue. In all trials, response accuracy (a binary outcome) was determined by comparing the typed answer to the correct answer, with some allowance for typographic errors. More extensive details about the scheduling and presentation of items are given in Section 3.1 of [6].

From the full set of learning data, we selected learning sequences of three or more practice trials in one learning session, followed by (at least) one practice trial in the next learning session. Each sequence therefore includes exactly one between-session interval: the time from the end of the first session to the start of the second session. We only predict response accuracy for the first trial in the second session; subsequent repetitions are discarded. The subset of the data used in the current analysis contained a total of 171,219 trials, making up 25,843 learning sequences from 218 learners.

2.2 Fitting the memory model

The memory model we use consists of two functions: a function that computes an item's activation given its history, and a function that maps an item's activation onto a retrieval probability. We use functions from the ACT-R cognitive architecture to fulfil these roles.

ACT-R proposes that each encounter of an item generates a trace that decays over time following a power function. The rate of decay is captured in the exponent d. The activation is the logarithm of the summed traces, representing the log-odds of needing the item at the current time [34]:

$$A = \ln\left(\sum_j t_j^{-d}\right) \tag{1}$$

Although activation values are commonly observed to fall within a certain range, an item's activation is theoretically unbounded. We therefore need a function to link activation to a predicted probability of recall, which is constrained to the interval $[0, 1]$. A logistic function serves this purpose:

$$p = \frac{1}{1 + e^{-(A-\tau)/s}} \tag{2}$$

The midpoint τ, which ACT-R refers to as the retrieval threshold, represents the point at which retrieval is equally likely to succeed or fail. The steepness of the logistic curve is controlled through the activation noise parameter s. We hold the noise constant at $s = 0.5$, in the middle of the recommended range $[0.2, 0.8]$. It follows that any change in predicted probability of retrieving an item is therefore the result of a change in activation, a change in the threshold, or both. Given a set of learning sequences, we can find the optimal values for these parameters using a logistic regression.

2.2.1 Finding τ

Equation 2 can be rearranged as follows:

$$p = \frac{1}{1 + e^{-(\frac{-\tau}{s} + \frac{1}{s}A)}} \tag{3}$$

Using the default decay $(d = 0.5)$ to calculate activation, we can fit a logistic regression model with coefficients $\beta_0 = \frac{-\tau}{s}$ and $\beta_1 = \frac{1}{s}$. Since s is a known parameter in our memory model, β_1 can be held constant using an offset term.

This enables us to derive the threshold τ from the estimated intercept: $\tau = -\beta_0 * s$.

2.2.2 Finding A

Similarly, Equation 2 can be rewritten as follows:

$$p = \frac{1}{1 + e^{-(\frac{A}{s} - \frac{1}{s}\tau)}} \tag{4}$$

We can again solve this as a logistic regression with coefficients $\beta_0 = \frac{A}{s}$ and $\beta_1 = -\frac{1}{s}$. To find A, we hold the threshold τ at a constant value. Fitting the regression model to the data then yields an estimate for the activation: $A = \beta_0 * s$.

2.2.3 Finding d

Since activation is itself a calculated value (see Equation 1), we can work out which value of the decay parameter d produces a certain activation, given an item's encounter history. We use a binary search to identify the value of d that minimises the mismatch between the target activation and the calculated activation.

2.2.4 Finding h

Following earlier work [21, 22], we can extend Equation 1 to incorporate a "psychological time" component, which shrinks the between-session interval t_b by some scaling factor h while leaving within-session intervals t_w as they are:

$$A = \ln\left(\sum_j (t_{wj} + h * t_b)^{-d}\right) \tag{5}$$

The scaling factor h is constrained to the interval $[0, 1]$ to achieve a compression of clock time. When $h = 1$ the calculation simplifies to Equation 1. We can again use a binary search to find the value of h that produces a given target activation, while holding d constant at its default value.

2.3 Modelling changes over time

The methods described above can be used to find optimal model parameters for a whole data set, or for specific parts of the data, as we did in Figures 1b, 1c, 1e, and 1g. However, to identify changes in optimal parameters over time, we divided the data over bins based on the between-session interval, and fitted the memory model separately to each bin. Bins were of constant width on a logarithmic scale. The smaller the bins, the more localised the estimates get, but at the cost of having fewer observations to constrain the model fit. A division into 20 bins was found to work well for the current data set.

All regression models were fitted using the lme4 package (version 1.1-21) [35] for R (version 3.6.3) [36]. We visualised the (continuous) predicted recall probability of the fitted ACT-R models alongside the (binary) observed recall performance using generalised additive models (GAMs; [37]).

3 Data Availability

The data are available from OSF at `https://doi.org/10.17605/osf.io/mzd6s`.

4 Code Availability

The analysis code is available from OSF at `https://doi.org/10.17605/osf.io/mzd6s`.

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6 Author contributions

M.v.d.V., F.S., and H.v.R. collected the data. M.v.d.V. developed the method and conducted the analysis in consultation with F.S., J.P.B., and H.v.R. M.v.d.V. wrote the manuscript with substantial input from F.S., J.P.B., and H.v.R.

7 Competing interests

An earlier version of the adaptive learning system used to collect the data discussed in this manuscript is licensed to SlimStampen B.V., a University of Groningen supported spin-off directed by H.v.R. No commercial or financial interests have influenced the setup, analysis, or reporting of this study. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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