Interactions Between Latent Variables in Count Regression Models

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Abstract

In psychology and the social sciences, researchers often model count outcome variables accounting for latent covariates and their interaction effects. Even though neglecting measurement error in such count regression models (e.g., Poisson or negative binomial regression) can have unfavorable consequences like attenuation bias, such analyses are often carried out in the generalized linear model (GLM) framework using fallible covariates such as sum scores. An alternative are count regression models based on structural equation modeling, which allows to specify latent covariates and, thereby, account for measurement error. However, the issue of how and when to include interactions between latent covariates or between latent and manifest covariates is rarely discussed for count regression models. In this paper, we present a latent variable count regression modeling (LV-CRM) framework allowing for latent covariates as well as interactions among both latent and manifest covariates. We conducted three simulation studies, investigating the estimation accuracy of the LV-CRM framework and comparing it to GLM-based count regression models. Interestingly, we found that even in scenarios with high reliabilities, the regression coefficiencts from a GLM-based model can be severely biased. In contrast, even for moderate sample sizes the LV-CRM provided virtually unbiased regression coefficients. Additionally, statistical inferences yielded mixed results for the GLM-based models (i.e., low coverage rates, but acceptable empirical detection rates), but were generally acceptable using LV-CRM. We provide an applied example from clinical psychology illustrating how the LV-CRM framwork can be used to model count regressions with latent interactions.

Keywords: latent interactions, count outcomes, Poisson regression

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Introduction

In psychology and the social sciences, researchers often model and predict count outcomes accounting for latent covariates and their possible interaction effects. For example, Wilker et al. (2017) considered the regression of symptom severity (i.e., how often did symptoms occur) of posttraumatic stress disorder and dissociation on an interaction between traumatic load and mental defeat, two variables assessed using multiple items from psychometric questionaires. Further examples involve the interactive effect between psychological distress with gender on problematic drinking behavior (i.e., number of alcoholic drinks Rodriguez et al., 2020) as well as the interactive effect of callous-unemotional traits and gender on antisocial outcomes (e.g., number of arrests McMahon et al., 2010).

Such analyses are often carried out in the generalized linear model (GLM; McCullagh & Nelder, 1998) framework using fallible scores as covariates. Most prominent options with count outcomes are Poisson or negative binomial regression (Hilbe, 2011). These are GLMs with a logarithmic link function and a Poisson or negative binomial distributed random component, which take the discrete and non-negative nature of count outcomes into account. The predictors are assumed to be manifest and fixed by design. An advantage of this assumption is that interactions can be included as product terms of observed variables. A downside is that this implies measurement error-free observed values, which is often not plausible for psychological measurements such as test scores.

While it seems to be a wide-spread approach to neglect measurement error in such analyses (Cheung et al., 2021; Cortina et al., 2021), this procedure is well-known to have unfavorable consequences in GLMs: First, measurement error typically attenuates the estimated regression coefficients towards zero, but in settings with multiple fallible predictors this bias can actually also lead to both over- and underestimation (Carroll et al., 2006; Kiefer & Mayer, 2021a), making it difficult to identify relevant interaction effects.

Second, the reliability of the product term of two variables depends on their respective reliabilites and is typically lower than either of these (Bohrnstedt & Marwell, 1978; Busemeyer & Jones, 1983). Thus, products of fallible predictors can contribute to bias in parameter estimation. Consequently, there is a need for count regression models accounting for latent variables and their (latent) interactions.

While there is quite some literature on latent interaction models with continuous outcomes (e.g., Kelava et al., 2011; Klein & Moosbrugger, 2000) with possible extensions for non-normally distributed latent variable indicators (e.g., Jin et al., 2020), latent interactions are rarely discussed for regression models with count outcomes. One notable exception is the negative binomial multigroup structural equation model (NB-MG-SEM) framework, which was described by Kiefer and Mayer (2021a, 2021b). The framework comprises count regression models with latent continuous covariates and manifest categorical covariates. Interactions between the two types of covariates are included via the multigroup aspect of the framework. Recently, Rockwood (2021) proposed a generalized structural equation model (G-SEM) and illustrated how it can be used for estimation of count regression models with latent covariates. While the G-SEM framework is very versatile, the formulation and implementation of Rockwood (2021) does not allow for interaction terms between the latent variables.

In this paper, we want to contribute to the literature on latent interaction models for count outcomes in three ways: First, we present a general framework for latent variable count regression models allowing for interactions. We will derive this framework as an extension of the GLM framework by building on the G-SEM framework proposed by Rockwood (2021). Second, in three Monte Carlo simulation studies we compare the estimation accuracy of the proposed approach to GLM-based count regression models. Third, we provide an empirical example from clinical psychology to illustrate how the latent variable count regression model can be used to model count regressions with latent interactions in applied research.

Generalized Linear Models for Count Outcomes

In the following, we derive the LV-CRM as an extension of the GLM, because many applied researchers are familiar with the GLM notation as well as GLM-based count regression models like Poisson or negative binomial regression. Thus, we start with presenting the core elements of GLM-based Poisson regression model for count outcome variables, explain how interactions can be included within this framework, and discuss the impact of measurement error on the parameter estimation. In the next section, we extend this framework for latent covariates and latent interactions.

GLMs have been proposed by Nelder and Wedderburn (1972) and it can be shown that several well-known regression models, as for instance, the logistic regression model or the general linear model, are special cases of the GLM (McCullagh & Nelder, 1998). The key idea is that all these regression models can be decomposed into three main components: (a) a *random component* describing the conditional distribution of the outcome variable, (b) a weighted linear combination of the predictor variables (i.e., the *linear predictor*), and (c) a functional connection between the two, called the *link function*. In principle, each component can be modified independently from the other two, which results in a very flexible way to model regressive dependencies among manifest variables.

Poisson Regression Model

Generalized linear models for count outcomes are often referred to as the family of Poisson regression models (e.g., Coxe et al., 2009). This is because the standard Poisson regression model, while being parsimonious and comprehensible, is usually not the most suitable in many applied scenarios. Thus, alternatives like the negative binomial regression are also subsumed to the family of Poisson regressions.

Consider a vector of N i.i.d. sampled outcome variables $\boldsymbol{y} = (Y_1, \ldots, Y_N)'$, where the index $i = 1, \ldots, N$ indicates the individual observations, and for each individual the observation of m fixed covariate values $\boldsymbol{z}_i = (1, z_{1i}, \ldots, z_{mi})$ with index $j = 1, \ldots, m$, then, according to McCullagh and Nelder (1998), the GLM-formulation of a standard Poisson regression model involves the following three main components: (a) the random component is Poisson distributed, $Y_i \sim \mathcal{P}(\mu_{Y_i})$, that is, we consider each observation Y_i of our count outcome to follow a Poisson distribution with expectation

$$\mathrm{E}(Y_i) = \mu_{Y_i}$$

The Poisson distribution comes with the property of equidispersion, meaning that the variance and the expectation of Y_i are always the same. While equidispersion accounts for heteroscedasticity to some extent, researchers often encounter overdispersed count outcomes, that is, the variance of Y_i exceeds its mean. In this case, an additional variance component can be introduced to the model, leading to Poisson-mixture distributions such as the negative binomial distribution (i.e., a Poisson-Gamma mixture; Hilbe, 2011) or the Poisson-lognormal (PLN) distribution (Bulmer, 1974).

(b) The linear predictor π_i is defined as a weighted linear combination of the predictor variables $\boldsymbol{z}_i$, where the weights $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_m)'$ are the regression coefficients:

$$\begin{aligned} \pi_i &= \beta_0 + \sum_{j=1}^{m} \beta_j \cdot z_{ji} \\ &= \boldsymbol{z}_i \boldsymbol{\beta} \end{aligned}$$

Below, we show how interactions between two or more predictors can be included within this framework. It is important to note that the z_j are treated as fixed constants. As a consequence, they are treated as perfectly reliable measures of the predictors. However, this is not plausible if fallible scores of latent constructs (e.g., test scores from an intelligence test) are used as predictors and can lead to attenuation bias in the estimated regression coefficients, which we will explain in more detail below.

(c) For count outcomes, the mean of the outcome variable μ_{Y_i} and the linear predictor π_i are commonly connected via a logarithmic link function, that is,

$$\begin{aligned} \log(\mu_{Y_i}) &= \pi_i \\ \Leftrightarrow \mu_{Y_i} &= \exp(\pi_i) \end{aligned}$$

which naturally accounts for the lower bound of count outcomes at zero.

For an introduction to Poisson regression models including illustrative examples, see Coxe et al. (2009). Estimation of a Poisson regression model can be done via iteratively weighted least squares to find the maximum likelihood estimates and the corresponding standard errors. For more details, see McCullagh and Nelder (1998) and Nelder and Wedderburn (1972) Note that there are several extensions to the family of Poisson regression models, for example, accounting for inflation of observed zeros in the outcome (Lambert, 1992), or truncated Poisson distributions (Hilbe, 2011, Ch. 12).

Interactions in Poisson Regression Models

Whenever the effect of one predictor depends on the values of another, we can model this using interaction terms. In the GLM framework, we product terms of the observed variables can be added as a new variable to the linear predictor, e.g., $z_{3i} := z_{1i} \cdot z_{2i}$. In a simple example with only two covariates and their interaction, the equation for the linear predictor is:

$$\pi_i = \beta_0 + \beta_1 z_{1i} + \beta_2 z_{2i} + \beta_3 \cdot \underbrace{z_{3i}}_{= z_{1i} \cdot z_{2i}}$$

As in linear regressions, the substantive interpretation of the regression coefficients changes due to the interaction term and it is often recommended to compute conditional effects or simple slopes for an accessible substantive interpretation. That is, we compute the linear predictor containing only the predictor z_1 conditional on certain values of the other predictor z_2. If, for instance, z_2 is a binary trauma variable (e.g., $z_2 = 1$: trauma experienced vs. $z_2 = 0$ not experienced) and z_1 is age, we can compute the conditional regression of the count outcome on age given values of the trauma variable:

$$\pi_i = \beta_0 + \beta_1 z_{1i} \qquad \text{for } z_2 = 0$$

$$\pi_i = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) \cdot z_{1i} \qquad \text{for } z_2 = 1$$

The first equation represents the relationship between the outcome Y and the predictor *age* if a trauma was not experienced ($z_2 = 0$), and the second equation if a trauma was experienced, respectively.

For continuous variables, it is common to choose a reasonable set of values from their distribution to compute the simple slopes. For example, in Figure 9 of our empirical example, we compute the simple slope of dissociative symptoms on mental defeat at the mean of trauma load as well as one and two standard deviations above and below the mean. It is possible to obtain standard errors for the parameters of the simple slopes by using the Delta method (Raykov & Marcoulides, 2004). Note, however, that if the simple slopes are computed at values estimated from the sample (e.g., sample mean), their sampling variance has also to be taken into account for reliable inferences (Liu et al., 2017).

Measurement Error in the Covariates

As we stated before, the GLM framework assumes fixed predictors, which (a) are perfectly reliable and (b) do not vary from one sample to another. In psychological research, this is often not a realistic assumption, especially if predictors are randomly sampled, fallible measures of unobservable constructs (e.g., motivation, intelligence). If measurement error in predictors is ignored, the regression coefficients get attenuated towards zero. This phenomenon known as attenuation bias (Carroll et al., 2006) is illustrated in Figure 1. While the left panel shows a Poisson regression based on the true values of a covariate η, the right panel shows the attenuation of the regression line as an effect of measurement error ϵ added to the covariate. While attenuation bias is usually

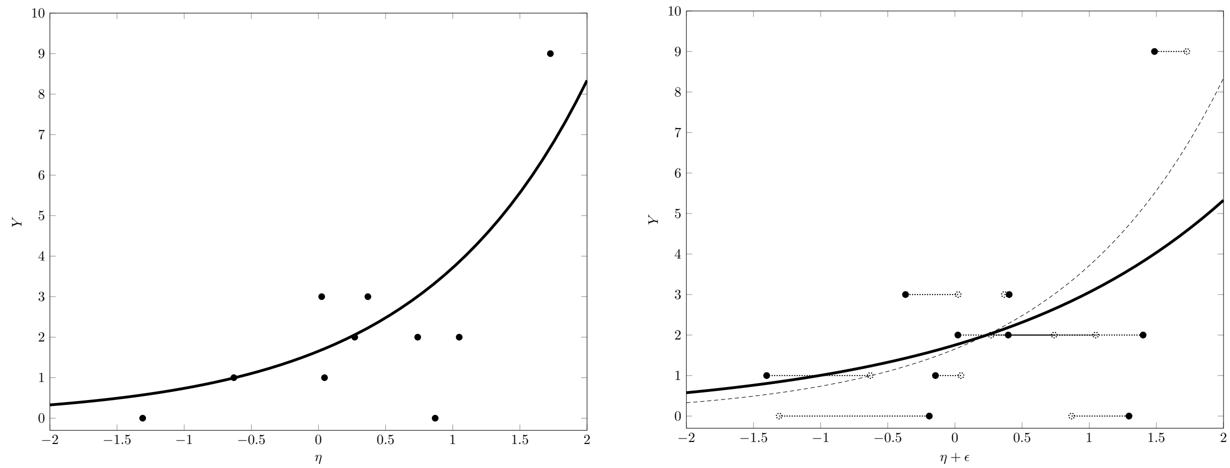

Figure 1

Left panel: Poisson regression (black line) of Y on the true scores of a predictor variable η (black dots). *Right panel:* Poisson regression (black line) of Y on fallible scores of the predictor variable (i.e., η plus a measurement error ϵ; black dots). Dashed regression lines reflect deviations from the Poisson regression with true scores.

associated with attenuation towards zero, biases in all directions can be observed with multiple fallible covariates (Carroll et al., 2006; Kiefer & Mayer, 2021a).

Attenuation bias affects fallible predictors in general, but is likely to be exacerbated when interaction terms from fallible predictors are involved. This is because the reliability of the product term is usually lower than that of either of the interacting variables (Busemeyer & Jones, 1983). Bohrnstedt and Marwell (1978) show that for two predictors with reliability of .8, the reliability of the product term can drop below .6 (depending on their scaling and correlation). Nevertheless, it still seems to be a wide-spread approach to neglect measurement error in regression analyses containing interactions (Cheung et al., 2021; Cortina et al., 2021).

Latent Variable Count Regression Model

In this section, we propose the latent variable count regression modeling (LV-CRM) framework for count regression models involving latent predictors, their interactions, manifest predictors, and possible latent-manifest interactions. For didactic reasons we show how the LV-CRM can be derived as an extension of the GLM and therefore also stick to

Table 1
Overview and model comparison

	GLM	**LV – CRM**
Mean model:	$\mathrm{E}(Y_i) = \mu_{Y_i}$	$\mathrm{E}(Y_i \vert \boldsymbol{\eta}_i) = \mu_{Y_i}$
Linear predictor:	$\pi_i = \boldsymbol{z}_i\boldsymbol{\beta}$	$\pi_i = \boldsymbol{z}_i\boldsymbol{\beta} + \boldsymbol{\Gamma}_1\boldsymbol{\eta}_i + \boldsymbol{\eta}_i'\boldsymbol{\Gamma}_2\boldsymbol{\eta}_i + \boldsymbol{\eta}_i'\boldsymbol{\Omega}_2\boldsymbol{z}_i$
Link function:	$\mu_{Y_i} = \exp(\pi_i)$	$\mu_{Y_i} = \exp(\pi_i)$
Measurement model:		$\boldsymbol{w}_i = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\epsilon}_i$

the common notation of the three main parts of the GLM. Note that there are several ways to arrive at this specific model, for example, from the broader modeling frameworks implemented in Mplus (Muthén & Muthén, 1998–2017) or in the GLLAMM approach (Skrondal & Rabe-Hesketh, 2004). The G-SEM framework (Rockwood, 2021) also overlaps with the LV-CRM, but does not allow for interaction terms involving latent variables.

We will present two steps to extend a GLM to a LV-CRM: (a) adding a measurement model as fourth model component and second, allowing latent variables, their interactions, and interactions between latent and manifest predictors as part of the linear predictor. Table 1 provides an overview and comparison of the GLM and the LV-CRM.

Measurement Model

In psychology and the social sciences, explicitly modeling measurement error and latent variables using a common factor technique (Bollen, 1989) is a popular approach . The key idea is that we have q measurements $\boldsymbol{w}_i = (W_{1i}, \ldots, W_q)'$ (e.g., items) intended to measure (multiple) latent variables (e.g., intelligence) and common factors $\boldsymbol{\eta}_i = (\eta_{1i}, \ldots, \eta_{pi})'$ with $p \leq q$ are introduced to model the correlations among the measurements:

$$\boldsymbol{w}_i = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\epsilon}_i$$

where $\boldsymbol{\nu}$ is a $q \times 1$ vector of intercepts; $\boldsymbol{\Lambda}$ is a $q \times p$ matrix of factor loadings; and $\boldsymbol{\epsilon}_i$ is a $q \times 1$ vector of measurement error variables. The observed indicators $\boldsymbol{w}_i$ are represented by a linear function of the latent variable plus measurement error. We assume that the measurement error variables $\boldsymbol{\epsilon}_i = (\epsilon_{1i}, \ldots, \epsilon_{qi})$ as well as the latent variables $\boldsymbol{\eta}_i$ are multivariate normally distributed with $\boldsymbol{\epsilon}_i \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta})$ and $\boldsymbol{\eta}_i \sim \mathcal{N}(\boldsymbol{\mu}_\eta, \boldsymbol{\Sigma}_\eta)$. Latent variables and measurement errors are independent from each other.

Latent Predictors and Interactions

Now, we can add the latent variables defined in the measurement model, and their possible interactions to the linear predictor component of the LV-CRM:

$$\pi_i = \underbrace{\beta_0 + \sum_{j=1}^{m} \beta_j \cdot z_{ji}}_{\text{GLM}} + \underbrace{\sum_{k=1}^{p} \gamma_k \cdot \eta_{ki}}_{\text{latent predictors}} + \underbrace{\sum_{k=1}^{p} \sum_{l=k+1}^{p} \gamma_{kl} \cdot \eta_{ki} \cdot \eta_{li}}_{\text{latent interactions}} + \underbrace{\sum_{j=1}^{m} \sum_{k=1}^{p} \omega_{jk} \cdot x_{ji} \cdot \eta_{ki}}_{\text{latent-manifest interactions}}$$

As denoted in the braces underneath the different sums, the first part is equivalent to the linear predictor of the GLM. By adding the latent variables with regression coefficients γ_k to the predictor, we obtain a special case of the G-SEM framework (Rockwood, 2021). The third part adds latent interactions with regression coefficients γ_{kl} to the linear predictor[1]. The fourth part allows for interactions between latent and observed predictors with regression coefficients ω_{jk}. Of course, some of the regression coefficients can be zero leading to more parsimonious models. The linear predictor in matrix notation is:

$$\pi_i = \boldsymbol{z}_i \boldsymbol{\beta} + \boldsymbol{\gamma}' \boldsymbol{\eta}_i + \boldsymbol{\eta}_i' \boldsymbol{\Gamma} \boldsymbol{\eta}_i + \boldsymbol{\eta}_i' \boldsymbol{\Omega} \boldsymbol{z}_i$$

where $\boldsymbol{\gamma}$ is a $p \times 1$ vector of regression coefficients; $\boldsymbol{\Gamma}$ is a $p \times p$ upper triagonal matrix of regression coefficients; $\boldsymbol{\Omega}$ is a $p \times m$ matrix of regression coefficients.

[1] It is possible to allow for quadratic terms of the latent variables (e.g., η_k^2) by changing the starting index of the second sum from $k + 1$ to k.

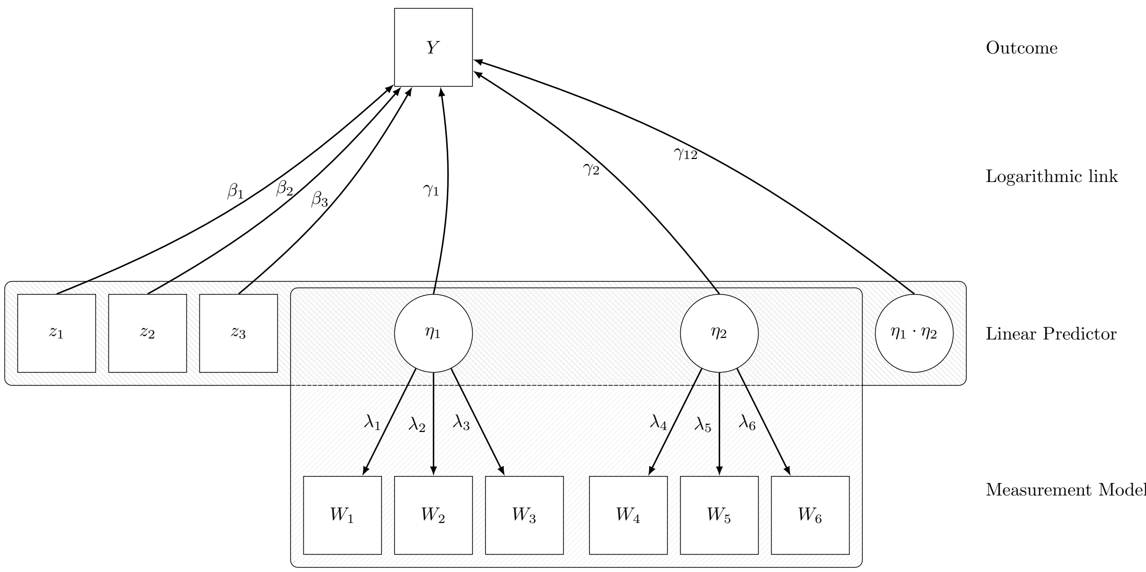

Figure 2

Path model depicting the four components of a LV-CRM. Example shows the Poisson regression of outcome variable Y on the manifest predictors z_1 to z_3, the latent predictors η_1 and η_2, and their interaction term $\eta_1 \cdot \eta_2$.

Parameter Estimation and Standard Errors

In this section, we provide a brief overview of maximum likelihood estimation of the proposed model. The overview is meant to give some intuition on the estimation technique and why no additional measurement models (e.g., as in a product-indicator approach; Kenny & Judd, 1984) or distributional assumptions (e.g., as in the LMS approach; Klein & Moosbrugger, 2000) are required. For a comprehensive illustration of the marginal likelihood technique and possible implementations, see Rockwood (2021) or Skrondal and Rabe-Hesketh (2004, Ch. 6). There exist alternative estimation methods for the LV-CRM, for instance, using Bayesian structural equation models (Asparouhov & Muthén, 2021).

For didactic reasons, we will restrict ourselves to describing the maximum likelihood estimation for the model depicted in Figure 2. This is actually the model used for simulation study 2 and very similar to the empirical example. The casewise log-likelihood function for this model can be written as:

$$\mathcal{L}_i(\boldsymbol{\theta}|y_i, \boldsymbol{z}_i, \boldsymbol{w}_i) = \int_{\eta_1}\int_{\eta_2} f(y_i|\boldsymbol{z}_i, \eta_1, \eta_2) \cdot f(\boldsymbol{w}_i|\eta_1, \eta_2) \cdot f(\eta_1, \eta_2)\, d(\eta_1, \eta_2)$$

where y_i, $\boldsymbol{z}_i = (z_{1i}, z_{2i}, z_{3i})$, and $\boldsymbol{w}_i = (W_{1i}, W_{2i}, W_{3i})$ are the individual values of the observed variables. Since the values of the latent variables η_1 and η_2 are not observed, they are integrated out.

There is no closed-form solution for the log-likelihood function and hence it has to be approximated through numerical techniques:

$$\mathcal{L}_i(\boldsymbol{\theta}|y_i, \boldsymbol{z}_i, \boldsymbol{w}_i) \approx \sum_{j=1}^{M} \omega_j \cdot f(y_i|\boldsymbol{z}_i, \eta^*_{1j}, \eta^*_{2j}) \cdot f(\boldsymbol{w}_i|\eta^*_{1j}, \eta^*_{2j})$$

where M is the number of integration points, ω_j is an integration weight, and η^*_{1j} and η^*_{2j} are the integration points, respectively. An advantage of this procedure is, that it provides a fixed set of values for the latent variables for each person in each iteration. Similar to the procedure in a GLM, we can use these latent variable values to compute product terms within the linear predictor. This is why the latent interaction term is presented as part of the linear predictor in Figure 2, but not as part of the measurement model. The product term is only part of the linear predictor and the linear predictor is only part of the density function of the outcome variable $f(y_i|\boldsymbol{z}_i, \eta_{1j}, \eta_{2j})$. Now, for each part sum of the approximated casewise likelihood, we can simply compute the linear predictor $\pi_i(j)$ as a function of the integration points:

$$\pi_i(j) = \beta_0 + \beta_1 \cdot z_{1i} + \beta_2 \cdot z_{2i} + \beta_3 \cdot z_{3i} + \gamma_1 \cdot \eta^*_{1j} + \gamma_2 \cdot \eta^*_{2j} + \gamma_{12} \cdot \eta^*_{1j} \cdot \eta^*_{2j}$$

These models can become computational demanding if the number of latent variables and, thus, the integration points, rises and we will discuss some approaches to reduce the computational burden. Standard errors can be derived using standard maximum likelihood theory, but this step is also computationally demanding as the second order derivatives of the log-likelihood have to be numerically approximated, too.

Simulation Studies

We conducted three Monte Carlo simulation studies to examine the performance of the LV-CRM framework under various empirical conditions and compared it to GLM-based Poisson or negative binomial regressions. From a substantive point of view, it is most interesting under which circumstances it is beneficial to go for the more complex LV-CRM framework and when the generalized linear model framework will provide acceptable results. The first simulation study focused on the extent of attenuation bias in a Poisson regression model with two latent variables and their interaction. It examines the question of how much bias one can expect given certain reliabilities of the sum scores, while still being computational feasible to replicate by the interested reader. The second simulation study focused on two questions, namely, (a) if and how much attenuation bias can spill over to regression coefficients of perfectly reliable measures, and (b) how the statistical inferences from the LV-CRM perform and how they compare to the GLM-based inferences. As this simulation study includes standard error estimation for the LV-CRM, it is computational more burdensome. The third simulation study focused on attenuation bias in more complex scenarios, where we considered three latent variables and their two-fold interactions. We examined attenuation bias for different combinations of interaction effects as well as correlational patterns among the latent variables. Due to the required three dimensional numerical integration, this simulation was computationally demanding, too. The corresponding R code as well as the final results for all three simulation studies are available from OSF[2].

To our knowledge, there is no previous study examining under which conditions the potential gains from a latent approach (e.g., reducing attenuation bias, increasing power) outweigh the costs (e.g., additional distributional assumptions, potential bias and numerical

[2] View-link for peer review: https://osf.io/q7knc/?view$_{o}nly = 363ff4f45c0d4e6785a66b64fc11a364$

instability for insufficient sample sizes). Thus, we aligned our simulation studies with the aim to provide guidelines for substantive researchers which model to prefer under which conditions. We examined the sample sizes required for an unbiased and reliable estimation, possible complexities of the model by varying the number of covariates, the number of interaction effects, interaction size, and the attenuation effect due to different reliabilities.

Simulation Study 1

The main focus of our first simulation study is to investigate the effect of different magnitudes of reliability of the score variables on attenuation bias and how well the LV-CRM can de-attenuate the estimated regression coefficients. We pursue this focus with a small-scale simulation that can be reproduced by the interested reader within reasonable time. In the simulation studies 2 and 3, we will investigate additional aspects of statistical inferences and higher-dimensional numerical integration which are computationally more demanding. Final results for all three simulation studies are included in the OSF repository.

Design

In this simulation study, we used a model with two latent variables, η_1 and η_2, and their interaction as predictors of the outcome variable Y in a Poisson regression model. The linear predictor was:

$$\pi_i = \beta_0 + \gamma_1 \cdot \eta_1 + \gamma_2 \cdot \eta_2 + \gamma_{12} \cdot \eta_1 \cdot \eta_2$$

where we are particularly interested in the estimation of the interaction parameter γ_{12}.

The latent variables were simulated as standard bivariate normally distributed with a correlation of $r = .3$ and measured with three indicators each. We also computed sum scores as fallible substitutes of the latent variables over the three indicators, respectively. The sum scores were z-standardized for comparability with the latent variables. The reliabilities of the sum scores were manipulated independently by altering the measurement error variances of the indicators. We investigated the six reliability combinations for the sum scores of both latent varibales, considering the reliablities of 0.7, 0.8, and 0.9 respectively. Additional design factors where the sample size ($N = 100,\ 200,\ 500,\ 1000$) and the size and direction of the interaction parameter ($\gamma_{12} = -0.3,\ 0,\ 0.3$)

We estimated the model with both a LV-CRM (where the means of the latent variables were fixed to 0 and the variances to 1) and a GLM (with z-standardized sum scores) and investigated and compared the bias and efficiency of the estimated interaction parameter $\hat{\gamma}_{12}$ (or $\hat{\beta}_4$ in the GLM, respectively).

Results

Convergence. We ran the simulation with $R = 1000$ replications including non-converged solutions first, in order to examine the convergence behavior of both approaches. Both approaches yielded convergence rates of virtually 100% in this simulation. Only in 6 out of 72 conditions, there was 1 out of 1000 replication where the LV-CRM did not converge. These 6 conditions had a positive interaction effect in common, but were unsystematic regarding the other design conditions (i.e., large and small sample sizes, high and low reliabilities) Thus, convergence rate seemed not to be an issue for the specified model in sample sizes from $N = 100$ upwards.

Bias. The following analyses are based on a second run of the simulation with $R = 1000$ replications *excluding* non-converged solutions, that is, if one of both models did not converge the replication was repeated until both models converged.

We investigated the bias of the estimated interaction coefficient $\hat{\gamma}_{12}$ in the LV-CRM and $\hat{\beta}_4$ the GLM, respectively. The results are presented (a) as bias for cases where the true parameter $\gamma_{12} = 0$ and (b) as relative bias for cases where the true parameter was non-zero, i.e., $\gamma_{12} \neq 0$. The results are illustrated in Figure 3 Both models yield very small biases in conditions where the true parameter γ_{12} was zero. For the GLM, the bias ranged between -0.006 and 0.002. The largest negative bias of about -0.015 was found for the LV-CRM in a condition with low reliabilites (both 0.7) and a small sample size of $N = 100$. The upper bound of the bias for the LV-CRM was 0.000. Overall, the LV-CRM tended to

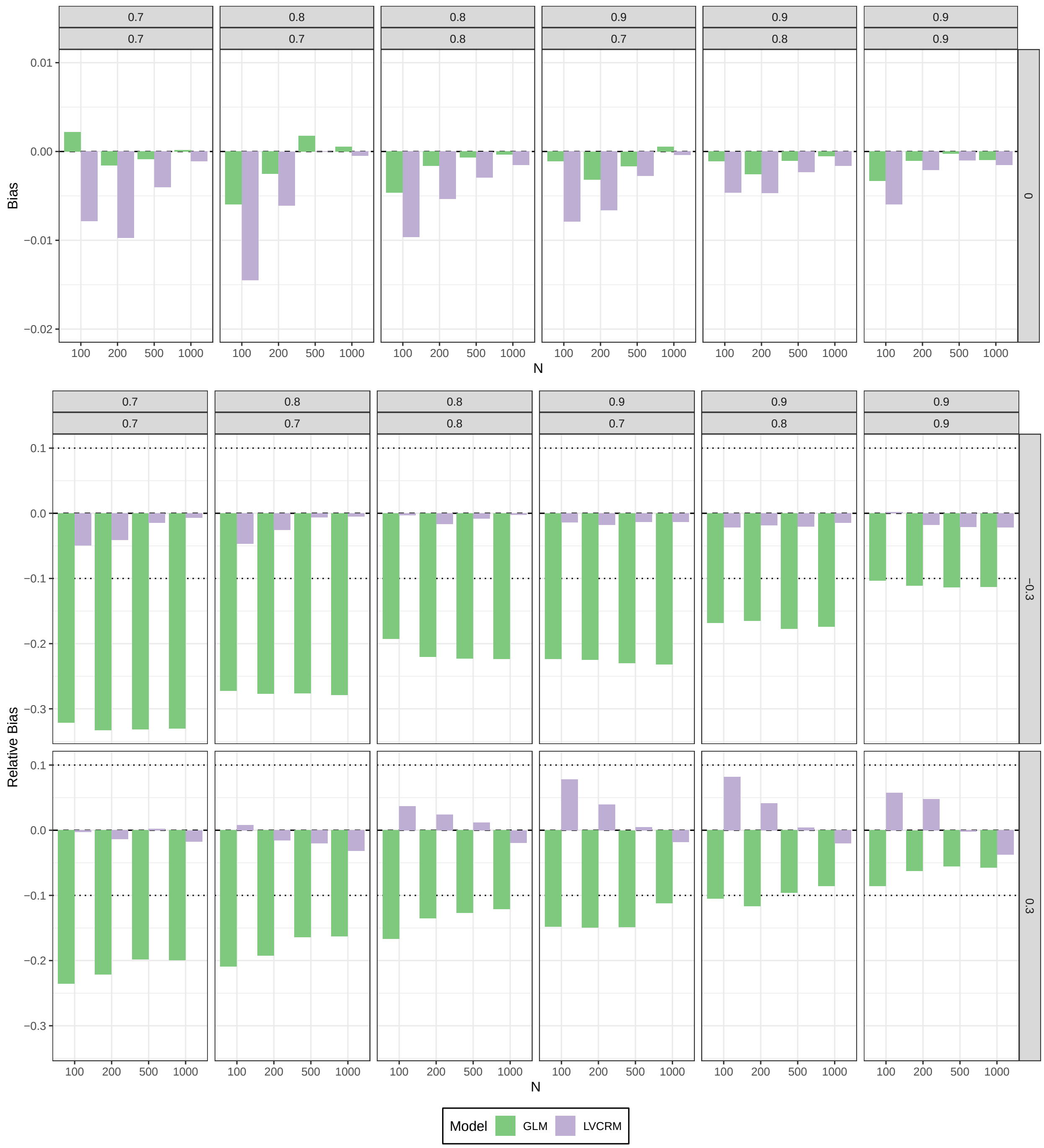

Figure 3

Simulation study 1: (Relative) bias of estimated interaction coefficients in the LV-CRM (purple) and the GLM (green). The upper panel shows bias for conditions with $\gamma_{12} = 0$, the lower panel shows relative bias for conditions with $\gamma_{12} \neq 0$. Columns reflect the six combinations of reliabilities of the sum scores, rows reflect the size and direction of the interaction effect, x-axis reflect sample size N.

underestimate the true parameter more than the GLM approach. This is not surprising, as measurement error is expected to attenuate the estimated regression coefficients of a GLM towards zero and in conditions with a true coefficient of zero, the attenuation is 'favorable' for the estimation of this zero.

The unfavorable effects of attenuation become clear, when looking at the relative bias of the estimated interaction coefficient in scenarios where the true parameter is not zero. The relative bias of the estimated interaction parameter in the GLM approach ranged between -5.6% and -33.2%. That is, even in scenarios with highly reliable score variables (both .9), we found at least 5 % underestimation. If at least one predictor had a reliability of .8 or lower, the underestimation was about 10% or more. On the other hand, the LV-CRM performed better and relative bias ranged between -4.9% and 8.2%. Interestingly, overestimation of the interaction parameter occurred in scenarios with positive interaction, highly reliable score variables, and small sample sizes. Overall, the LV-CRM provided more accurate estimates (i.e., less bias) for the interaction parameter than the GLM. For sample sizes of $N = 200$ or larger, the LV-CRM yielded relative bias below $\pm 5\%$ under all conditions.

Relative Efficiency. The results for the relative efficiency of the LV-CRM compared to the GLM (i.e., ratio of the respective RMSE) are presented in Figure 4. In conditions with a true interaction parameter of $\gamma_{12} = 0$, the relative efficiency of the LV-CRM approach compared to the GLM approach ranged between 1.048 and 1.270. That is, the RMSE of the LV-CRM approach is about 4.8% to 27% higher than that of the GLM. Again, that is not surprising given the 'favorable' effect of the attenuation bias in these conditions.

In conditions with a true interaction parameter of $\gamma_{12} \neq 0$, the relative efficiency ranges from 0.304 to 1.622. As can be seen in Figure 4, the LV-CRM typically more efficient in scenarios with a negative interaction parameter, especially with larger sample sizes. With a positive interaction parameter, the LV-CRM is typically more efficient if at

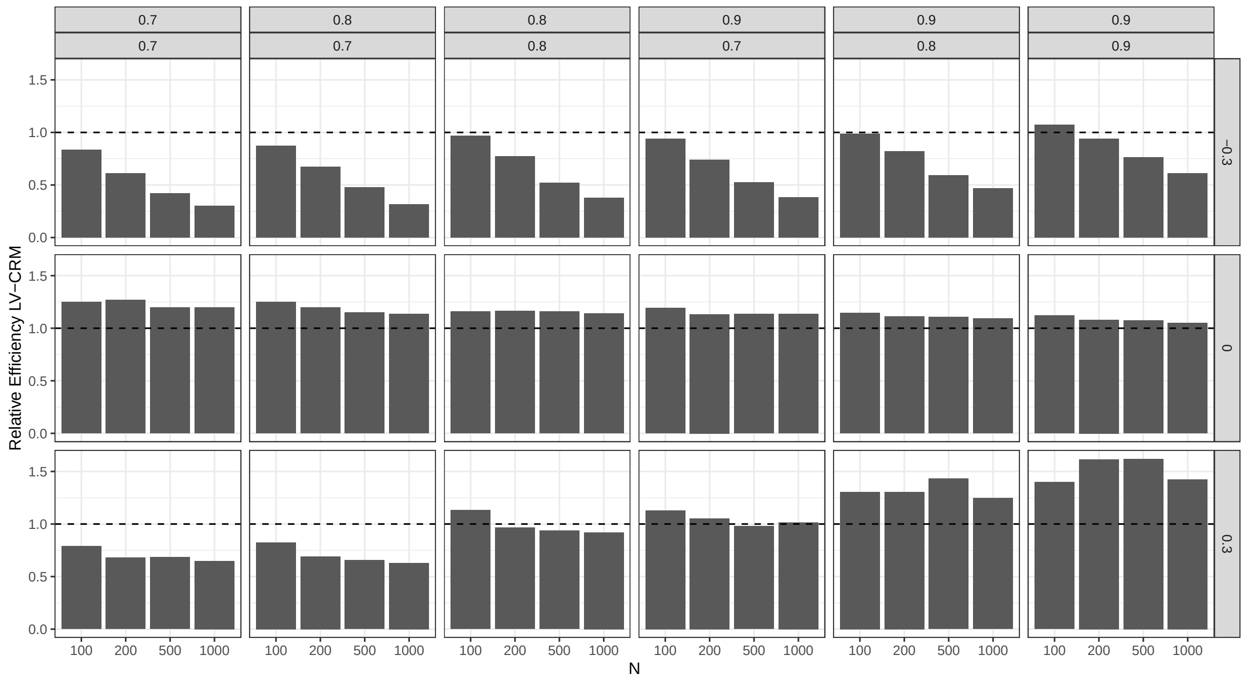

Figure 4

Simulation study 1: Relative Efficiency of estimated interaction coefficient in the LV-CRM compared to GLM (i.e., RMSE of LV-CRM divided by the RMSE of the GLM). Columns reflect the six combinations of reliabilities of the sum scores, rows reflect the size and direction of the interaction effect, x-axis reflect sample size N.

least one predictor has a reliability of 0.7 or both reliabilites were .8 (with few expections in sample sizes of $N = 100$). However, with increasing reliability of the fallible score variable the LV-CRM was less efficient than the GLM.

Simulation Study 2

In the second simulation study, we extended the design of our first study in two regards: First, we investigated whether attenuation bias can have a spill-over effect on other regression coefficients, for instance, those of perfectly reliable predictors. Second, we examined whether the bias reduction in the LV-CRM approach also comes with improved statistical inferences, for example, an increase of power to detect interaction effects. Thus, we computed 95 % confidence intervals (CIs) and the empirical detection rate for each condition.

Design

The simulation design followed our first simulation study with a few additions: First, three additional manifest and perfectly reliable predictors were added to the regression model. These predictors were generated as independent from each other and from the latent variables. This was done to investigate potential spill-over effects of the fallible score variables. The linear predictor was

$$\pi_i = \beta_0 + \beta_1 \cdot x_{1i} + \beta_2 \cdot x_{2i} + \beta_3 \cdot x_{3i} + \gamma_1 \cdot \eta_1 + \gamma_2 \cdot \eta_2 + \gamma_{12} \cdot \eta_1 \cdot \eta_2$$

Second, standard errors, confidence intervals, and the empirical detection rate for the interaction parameter γ_{12} were computed. Third, the random component was choosen as negative binomial instead of a Poisson distribution. This is a more realistic scenario as it incorporates additional variance in the outcome not explained for by the predictors (which is usually the case in applied settings), but the estimation is slightly more demanding. It is also closely related to our empirical example below, where we use a negative binomial regression model.

Results

Spill-over effect. We used the Euclidean norm of the biases of the three regression coefficients of the observed covariates (i.e., $B(\hat{\beta}_1)$, $B(\hat{\beta}_2)$, $B(\hat{\beta}_3)$) to get an overall evaluation of possible spill-over effects, that is,

$$S(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3) = \sqrt{B(\hat{\beta}_1)^2 + B(\hat{\beta}_2)^2 + B(\hat{\beta}_3)^2}$$

The results are summarized in Figure 5. We also computed bias and relative efficiency of the remaining coefficients (i.e., of the latent variables and the interaction term), but do not illustrate the results here as they closely resemble our findings from the first simulation study. The complete results can be found in the OSF repository.

Figure 5

Simulation study 2: Spill-over effect $S(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$ of estimated regression coefficients in the LV-CRM and in the GLM. Columns reflect the six combinations of reliabilities of the sum scores, rows reflect the size and direction of the interaction effect, x-axis reflect sample size N.

Overall, the results indicate no spill-over effect of the fallible score variables on the estimated regression coefficients of the perflectly reliable covariates. The spill-over effect $S(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$ ranged from 0.0004 to 0.0149 for the GLM and from 0.0007 to 0.0149 for the LV-CRM. The highest values were obtained in scenarios with mixed reliabilities (i.e., one high, one low) on both fallible scores and rather low sample sizes. However, the corresponding regression coefficients appeared virtually unbiased under all conditions.

Coverage and Empirical Detection Rate. We examined the coverage rate (i.e., the proportion of CIs including the true parameter value), and the empirical detection rate (i.e., the proportion of CIs not including zero) for the 95 % confidence intervals (CI) of the interaction parameter γ_{12} estimated with both approaches (i.e., $\hat{\gamma}_{12}$ in the LV-CRM and $\hat{\beta}_6$ in the GLM). The results are summarized in Figures 6.

In scenarios, where the true interaction parameter was $\gamma_{12} = 0$, both the coverage rate and the empirical detection rate (i.e., the Type I error rate in these scenarios) were acceptable for both approaches. For the GLM, the actual coverage rate of the CIs ranged between $[0.918,\ 0.948]$ and the empirical detection rate between $[0.052,\ 0.085]$, respectively. For the LV-CRM, the actual coverage rate of the CIs ranged between $[0.923,\ 0.957]$ and the empirical detection rate between $[0.043,\ 0.077]$, respectively.

In contrast, in scenarios where the true interaction effect was not zero (i.e., $\gamma_{12} \neq 0$), coverage and empirical detection rate yielded diverging results. For the GLM, the actual coverage rate of the CIs ranged between $[0.199,\ 0.951]$ and the empirical detection rate (i.e., the power in these scenarios) between $[0.127,\ 1.000]$, respectively. Notably the coverage rate was more accurate for small interaction sizes (i.e., $\gamma_{12} = |0.1|$), but barely acceptable for larger interaction sizes (i.e., $\gamma_{12} = |0.3|$). For the LV-CRM, the actual coverage rate of the CIs ranged between $[0.904,\ 0.966]$ and the empirical detection rate between $[0.121,\ 1.000]$, respectively. Overall, the power was similar for both approaches. On average, the power was 0.7% higher for the LV-CRM, where the differences in power between the two approaches ranged from -3.9% (i.e., higher power in the GLM) to 8.2% (i.e., higher power in the LV-CRM).

When it comes to statistical inferences, these findings indicate that attenuation bias in the GLM is somewhat compensated for by an overconfident (i.e., too small) standard error. As a result, hypothesis testing seemed to work reasonably well, but the confidence intervals were too narrow and did often (i.e., up to 90.1%) not include the true parameter value. In contrast, the LV-CRM yielded unbiased point estimates and accurately accounted for multiple sources of uncertainty (e.g., measurement error, regression residual) resulting in wider CIs (compared to the GLM). Thus, null hypothesis testing would be expected to work with both approaches, but substantive interpretation of the CI is more reliable with

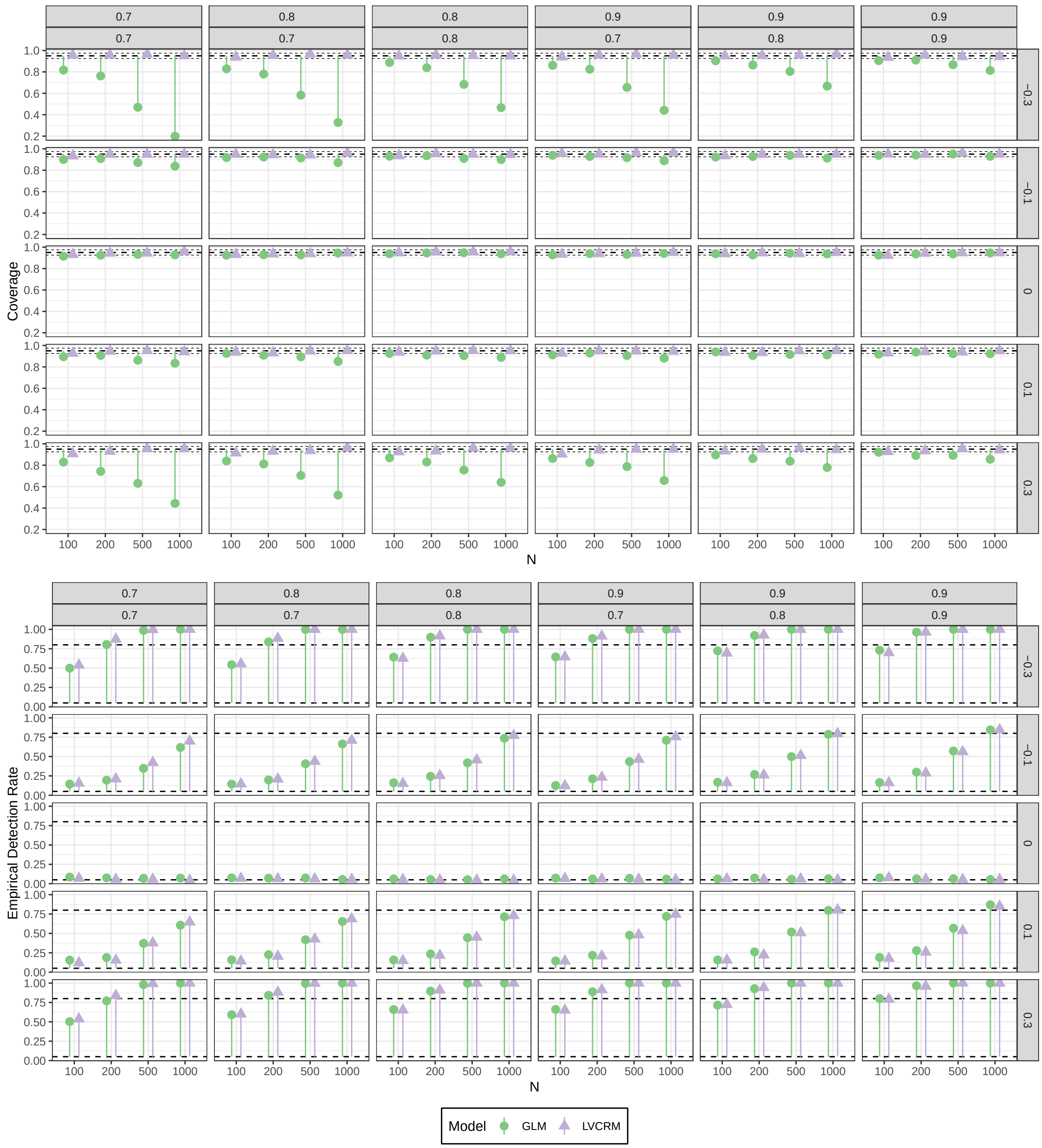

Figure 6

Simulation study 2: Coverage rates (upper panel) and empirical detection rates (lower panel) of estimated interaction coefficient $\hat{\gamma}_{12}$ in the LV-CRM and in the GLM. Columns reflect the six combinations of reliabilities of the sum scores, rows reflect the size and direction of the interaction effect, x-axis reflect sample size N.

the LV-CRM.

Simulation Study 3

In the third simulation study, we focused on scenarios with three latent variables and their two-fold interactions. Our goal was to investigate the extent of attenuation bias in this complex settings given different combinations of reliability, correlations, and interactional patterns among the latent variables. Similar to the first simulation study, we restricted ourselves to consider bias and relative efficiency of the estimated interaction parameters alone and did not investigate statistical inferences in order to keep the simulation computationally feasible.

Design

The design of the simulation study is similar to the first simulation study, but with three latent variables and their three two-fold interactions. The linear predictor was:

$$\pi_i = \beta_0 + \gamma_1 \cdot \eta_1 + \gamma_2 \cdot \eta_2 + \gamma_3 \cdot \eta_3 + \gamma_{12} \cdot \eta_1 \cdot \eta_2 + \gamma_{13} \cdot \eta_1 \cdot \eta_3 + \gamma_{23} \cdot \eta_2 \cdot \eta_3$$

We manipulated the following three factors: First, reliability of the sum scores of each latent variable could take the values 0.7 or 0.9, resulting in eight reliability combinations. Second, we investigated four different correlational patterns among the latent variables. These four patterns where (a) small negative correlations ($r = -.3$) among all LVs, (b) small positive correlations ($r = .3$) among all LVs, (c) no correlations ($r = 0$) among all LVs, and (d) mixed correlations (negative, positive, null) among the LVs. Third, we investigated four different interactional patterns. These where (similar to the correlations) (a) small negative interaction coefficients ($\gamma = -.3$) for all LVs, (b) small positive interaction coefficients ($\gamma = .3$) for all LVs, (c) no interaction effect ($\gamma = 0$) for all LVs, and (d) mixed interaction coefficients (negative, positive, null) for the LVs.

We did not investigate different sample sizes in this simulation, but choose a single sample size of $N = 500$ throughout all conditions.

Results

Bias. The results on attenuation bias are presented (a) as bias for scenarios where all interaction coefficients were zero and (b) as relative bias for non-zero interaction coefficients. An overview of the results is given in Figure 7.

In conditions where all three interaction coefficients were zero, the bias ranged between -0.002 and 0.002 for the GLM and between -0.004 and 0.003 for the LV-CRM. As in the first simulation study, this result is not surprising given the 'favorable' effect of attenuation bias in these conditions.

In conditions where interaction coefficients could differ from zero, we found an common pattern of relative bias in most conditions, but with a notable exception (i.e., all correlations and interactions positive) which we will discuss separately. The common pattern shows a substantial relative bias for all three estimated interaction coefficients in the GLM (between -0.451 and 0.016), but comparably low relative bias for the estimated interaction coefficiens for the LV-CRM (between -0.112 and 0.020).

However, in conditions with positive correlations among the LVs and three positive interaction effects, a rather unsystematic pattern of relative bias occured – as displayed in Figure 7. Here, relative biases in both directions were observed, that is, between -0.225 and 0.261 for the GLM and between -0.169 and 0.190 for the LV-CRM. In order to examine, if this pattern was caused by the medium sample size and possible estimation issues, we re-run these conditions with a larger sample size of $N = 2000$. However, we found the same pattern again. We are not sure, why the relative bias behaves so differently under these conditions, but suspect an unfavorable combination of multicollinearity of the latent variables and their interaction terms, measurement error, and rather steep conditional effects (i.e., simple slopes) leading to highly dispersed outcome values. However, it shows that attenuation bias can have rather unexpected effects in complex scenarios involving multiple latent variables.

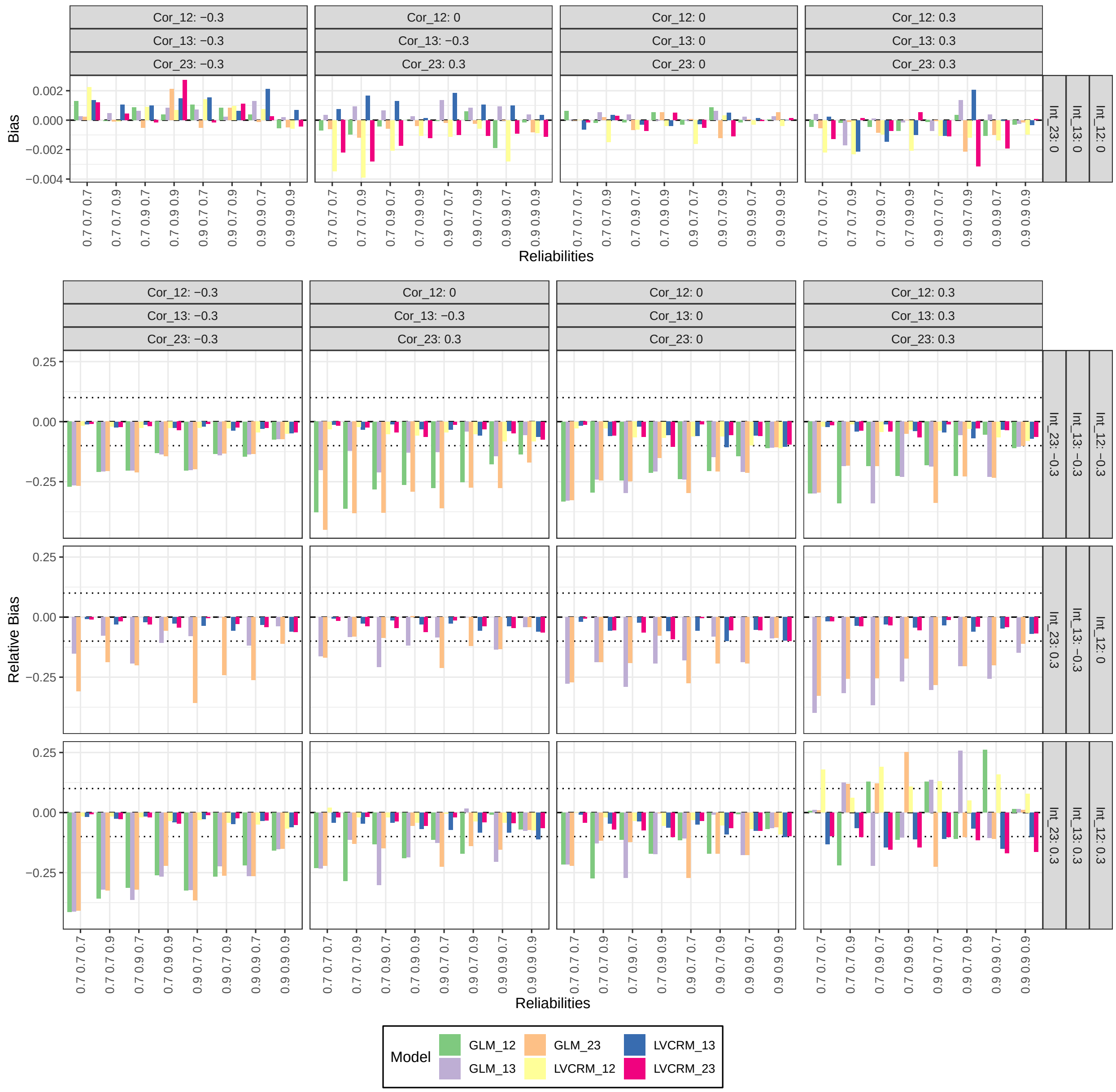

Figure 7

Simulation study 3: (Relative) bias of estimated interaction coefficients in the LV-CRM (yellow, blue, magenta) and the GLM (green, purple, orange). The upper panel shows bias for conditions where all interaction coefficients were zero, the lower panel shows relative bias for the remaining conditions. In conditions with mixed coefficients, (relative) bias for $\gamma_{12} = 0$ is not shown. Columns reflect the correlational patterns among the LVs, rows reflect the size and direction of the interaction effects, x-axis reflects different combinations of reliabilities of the sum scores.

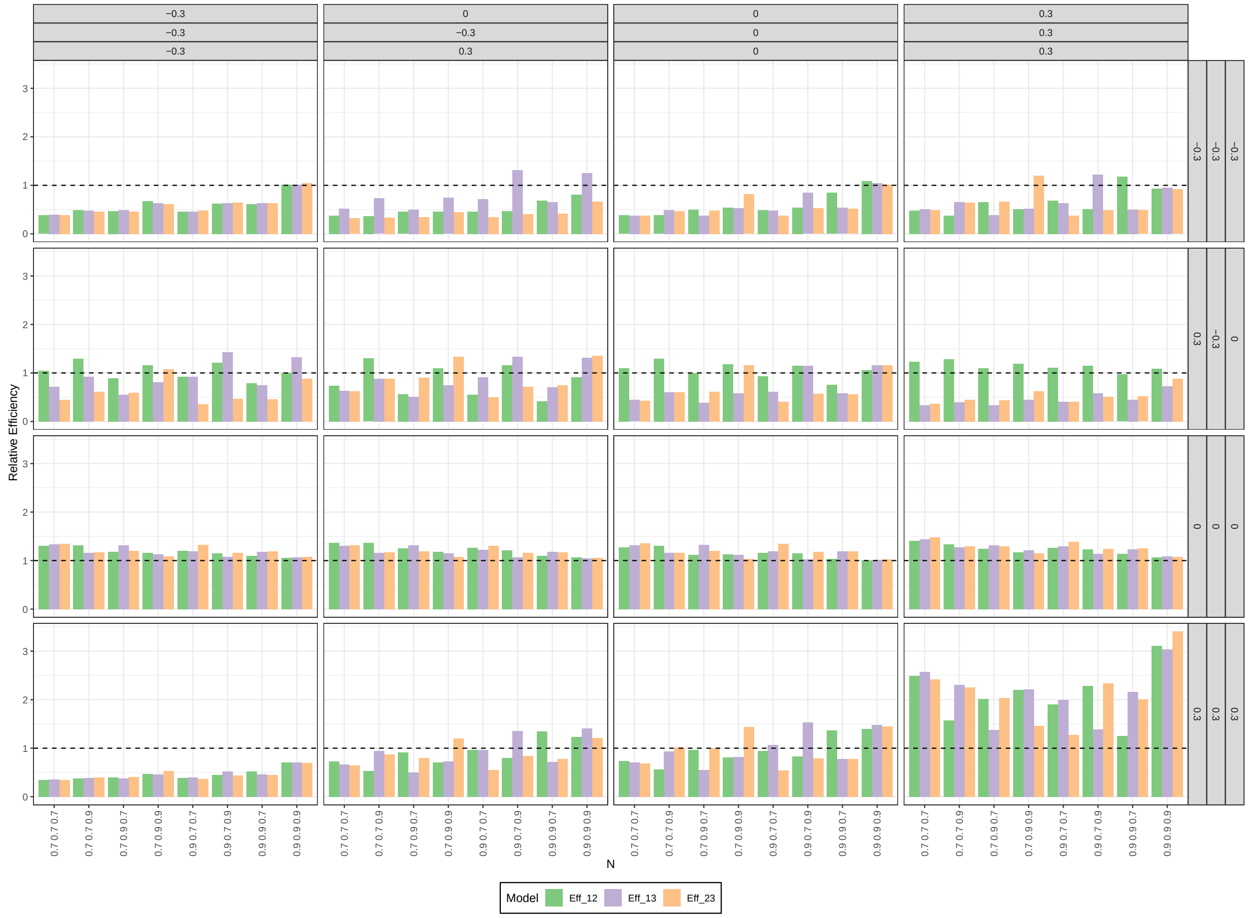

Figure 8

Simulation study 3: Relative Efficiency of estimated interaction coefficients in the LV-CRM compared to GLM (i.e., RMSE of LV-CRM divided by the RMSE of the GLM). Columns reflect the correlational patterns among the LVs, rows reflect the size and direction of the interaction effects, x-axis reflects different combinations of reliabilities of the sum scores.

Relative efficiency. The results for the relative efficiency of the estimated interaction coefficients in the LV-CRM compared to the GLM were similar to our findings from simulation studies 1 and 2 and are therefore not discussed in detail again. An overview is given in Figure 8 and the complete results are available from the OSF repository.

Empirical Example

We use an empirical example from clinical psychology to illustrate how the LV-CRM framework can be applied to model count regressions with latent interactions. Wilker et al. (2017) examined the effects of trauma load, mental defeat, and their interaction on symptom severity (i.e., incidence of symptoms) of Posttraumatic Stress Disorder (PTSD) and dissociation in Ugandan rebel war survivors.

Theoretical Background

The experience of traumatic events, such as war, torture, sexual violence, accidents or natural disasters can lead to the development of PTSD. The disorder is characterized by intrusive re-experiencing of the traumatic events, avoidance of trauma reminders, persistent alterations of mood and cognition and a state of elevated arousal (American Psychiatric Association, 2013). In addition to these symptoms, survivors of multiple and interpersonal trauma are at elevated risk to show dissociative symptoms, which include feelings of derealization (e.g. feeling as if the own experience is not real), depersonalization (e.g., feeling as if being outside the own body), dizziness, and an incapability to move (Schauer & Elbert, 2010; Vermetten & Spiegel, 2014).

Importantly, after a single or few traumatic events, the majority of individuals do not develop trauma-associated psychopathology (Kessler et al., 2005). Whether an individual will develop mental health symptoms after a traumatic event largely depends on individual risk and resilience factors as well as on trauma-related predictors (Kessler et al., 2021). However, research from post conflict settings indicates that, with an increasing number of different types of traumatic events (termed traumatic load) almost every individual will develop mental health symptoms, and individual risk and resilience factors only play a subordinate role (Neuner et al., 2004; Wilker et al., 2015).

Peritraumatic cognitive processes, referring to thoughts which occur at the time of the trauma, have been identified to influence both the memory and the appraisal of the traumatic event. Therefore, they represent risk factors for trauma-associated psychopathology and important targets for trauma-focused interventions which aim at the modification of trauma memories and associated negative cognitions. One important peritraumatic cognitive process is termed mental defeat (Kleim et al., 2012) and refers to a loss of mental resistance and human dignity during the trauma (Dunmore et al., 1999, 2001). The experience of mental defeat during a trauma is associated with the development of permanent negative cognitions about the self (e.g. “I am weak” or “I am destroyed”) and the world (e.g. “I can trust nobody”), which are known to be important symptoms of PTSD. At the same time, they lead to increased avoidance of trauma-associated memories and thereby lead to the maintenance and chronification of psychopathology (Dunmore et al., 2001; Ehlers et al., 1998).

While there is a lot of evidence indicating that the peritraumatic cognitive process of mental defeat is a central risk factor for PTSD in individuals from relatively peaceful, industrialized countries, research from post-conflict settings on mental defeat was completely lacking. Since the burden of PTSD is much higher in post conflict settings compared to industrialized countries (Charlson et al., 2019), research from this context is urgently needed to better understand factors central to trauma-associated psychopathology and its treatment in this context. Therefore, Wilker et al. (2017) conducted a study to investigate whether mental load would be an important predictor of PTSD and dissociative symptoms in a post-conflict population from Northern Uganda. In more detail, they investigated the interplay of trauma load and mental defeat on PTSD risk, PTSD symptoms and dissociative symptoms. Because previous research showed that individual predictors become less important at higher levels of trauma load, potential trauma load × mental defeat interaction effects were of particular interest to the study.

Method

The description of the methods is taken from Wilker et al. (2017, pp. 3–5). For the complete methods, the reader is referred to the original article.

Sample. Data collection took place in villages of Nwoya district in Northern Uganda. This area was severely affected by the war between the Lord’s Resistance Army (LRA) rebel group and the Ugandan governmental forces, which lasted almost two decades. The atrocities committed during this war included forced recruitment and abductions of children and young adults, killings, mutilations, and sexual offenses. Data collection took place in 2013, eight years after the cease-fire agreement between the LRA and the governmental troops in 2005. The final sample of $N = 227$ was 54% female, with a mean age of 33.29 (SD = 10.56, range = 18—62).

Measures. Trauma exposure was assessed by means of a 62-item event list. This event list comprised general traumatic experiences (e.g., natural disasters, accidents), war-related traumatic events (e.g., being close to combat), as well as events specific for the LRA conflict (e.g., being forced to kill somebody by the LRA). We calculated the number of different traumatic event types experienced to assess the amount of trauma exposure (traumatic load). As previously shown in the same sample, the retest reliability of this variable was $r = .82$ (Wilker et al., 2015) and, thus, was treated as a latent predictor using a single indicator approach in our analysis.

The extent of mental defeat was assessed for the worst traumatic event using the Mental Defeat Questionnaire (MDQ) in the form of an interview (Dunmore et al., 1999, 2001). The MDQ comprises 11 unipolar items (e.g., "I lost any will-power", "I felt destroyed as a person") and requires responses on a 5-point Likert-type scale ranging from not at all to very strong. The MDQ showed a good internal consistency in the present sample (Cronbach's $\alpha = .89$) and was modeled as a latent predictor using a multiple indicator approach in our analysis.

The main outcome of our analysis were dissociative symptoms assessed by means of the Shutdown Dissociation Scale (Shut-D; Schalinski et al., 2015). The Shut-D includes 13 unipolar items (e.g., "Have you fainted?" "Have you felt like you were outside of your body?" "Have you felt suddenly weak and warm?" "Have you felt nauseous? Have you felt as though you were about to throw up? Have you felt yourself break out in a cold sweat?") investigating current bodily dissociative symptoms for the past 6 months. Participants were requested to answer on a 4-point scale ranging from 0 (*never*) to 3 (*several times a week*). Thus, the scale score acts as a proxy of the incidence of dissociative symptoms and behaves similarly as a count variable, that is, the lower bound represents zero symptom occurences, the variable only takes non-negative integer values, and a certain amount of heteroscedasticity is present. Thus, Wilker et al. (2017) handled the outcome as a count variable. The Shut-D showed a high internal reliability in the present study (Cronbach's $\alpha = .91$).

Statistical Analysis

Wilker et al. (2017) compared models of varying complexity (i.e. with and without including the covariates age, sex, and age at worst event and with and without considering potential trauma load - mental defeat interaction effects). In this study, in order to investigate the differences between the GLM and the LV-CRM, we calculated the full model. Accordingly, our model included main effects of sex, age, and age at worst event. Further, trauma load, mental defeat as well their interaction were included as predictors in the regression models.

Negative Binomial Regression

As in the original study by Wilker et al. (2017), we estimated a negative binomial regression. That is, the outcome variable Y_i (i.e., the Shut-D score) is linked to the linear predictor with a logarithmic link function and is assumed to follow a negative binomial distribution.

The linear predictor in this model was

$$\begin{aligned}\pi_i = &\beta_0 + \beta_1 \cdot \text{Sex}_i + \beta_2 \cdot \text{Age}_i + \beta_3 \cdot (\text{Age at Worst Event})_i \\ &+ \beta_4 \cdot \underbrace{\text{TLS}_i}_{\substack{\text{Standardized Score}\\ \text{Trauma Load}}} + \beta_5 \cdot \underbrace{\text{MDQ}_i}_{\substack{\text{Standardized Score}\\ \text{Mental Defeat}}} + \beta_6 \cdot \text{TLS}_i \cdot \text{MDQ}_i\end{aligned}$$

LV-CRM

In addition, we used the LV-CRM framework to carry out the same analysis, but including a measurement model for the latent trauma load ($\eta_{\text{TL};i}$) and mental defeat ($\eta_{\text{MD};i}$) variables in order to account for measurement error:

$$
\begin{pmatrix} \mathrm{MDQ}_{1;i} \\ \mathrm{MDQ}_{2;i} \\ \mathrm{MDQ}_{3;i} \\ \mathrm{MDQ}_{4;i} \\ \mathrm{MDQ}_{5;i} \\ \mathrm{MDQ}_{6;i} \\ \mathrm{MDQ}_{7;i} \\ \mathrm{MDQ}_{8;i} \\ \mathrm{MDQ}_{9;i} \\ \mathrm{MDQ}_{10;i} \\ \mathrm{MDQ}_{11;i} \\ \mathrm{TLS}_{i} \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \\ \nu_6 \\ \nu_7 \\ \nu_8 \\ \nu_9 \\ \nu_{10} \\ \nu_{11} \\ \nu_{12} \end{pmatrix} + \begin{pmatrix} \lambda_1 & 0 \\ \lambda_2 & 0 \\ \lambda_3 & 0 \\ \lambda_4 & 0 \\ \lambda_5 & 0 \\ \lambda_6 & 0 \\ \lambda_7 & 0 \\ \lambda_8 & 0 \\ \lambda_9 & 0 \\ \lambda_{10} & 0 \\ \lambda_{11} & 0 \\ 0 & \lambda_{12} \end{pmatrix} \cdot \begin{pmatrix} \eta_{\mathrm{MD};i} \\ \eta_{\mathrm{TL};i} \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \\ \epsilon_9 \\ \epsilon_{10} \\ \epsilon_{11} \\ \epsilon_{12} \end{pmatrix}
$$

Both scales were fixed to a mean of zero and a variance of one and, consequently, all intercepts and loadings as well as the latent covariance were estimated freely.

Note that we modeled trauma load $\eta_{\mathrm{TL};i}$ using a fixed-reliability single indicator approach as proposed by (Savalei, 2019) for two reasons: First, trauma load is not a traditional psychometric variable, but the items reflect different traumatic event types. The items are not meant to measure a common factor and are likely to be uncorrelated to some extent (i.e., experiencing a natural disaster is not necessarily correlated to having an accident). Thus, a multiple indicator approach would not have been appropriate. Second, the retest reliability of 0.82 (Wilker et al., 2015) indicates that trauma load cannot be assessed exactly, meaning there is some kind of measurement error involved. According to our simulation studies, we would expect a substantial attenuation bias on the interaction effect given this level of reliability and, thus, explicitly controlling for this measurement seems warranted. In order to fix the reliability of trauma load to 0.82, we constrained its measurement error variance to:

$$\mathrm{Var}(\epsilon_{12}) = \lambda_{12}^2 \cdot \left(\frac{1}{\mathrm{Rel}_{\mathrm{TLS}}} - 1\right)$$
$$\text{where } \mathrm{Rel}_{\mathrm{TLS}} = 0.82$$

Then, the linear predictor for the LV-CRAM was

$$\begin{aligned}\pi_i =& \beta_0 + \beta_1 \cdot \mathrm{Sex}_i + \beta_2 \cdot \mathrm{Age}_i + \beta_3 \cdot (\text{Age of Worst Event})_i \\ &+ \gamma_1 \cdot \underbrace{\eta_{\mathrm{TL};i}}_{\substack{\text{Latent Variable}\\ \text{Trauma Load}}} + \gamma_2 \cdot \underbrace{\eta_{\mathrm{MD};i}}_{\substack{\text{Latent Variable}\\ \text{Mental Defeat}}} + \gamma_{12} \cdot \eta_{\mathrm{TL};i} \cdot \eta_{\mathrm{MD};i}\end{aligned}$$

where the standardized test scores for mental defeat and trauma load are replaced with the corresponding latent variables.

Results

Table 2 shows the regression coefficients, their standard errors, and p-values of the estimated GLM and the LV-CRM, respectively. As it can be seen, the standardized coefficients of the relevant parameter trauma load, mental defeat, and their interaction are larger if measurement errors are considered in the LV-CRM. These results of the real-data example are in line with our simulation results on attenuation bias and illustrate that the GLM is likely to underestimate the true parameter effects even if the reliability of the latent variables is relatively high.

Most importantly, in the clinical psychology example, the interaction effect of trauma load and mental defeat was not significant. Therefore, Wilker et al. (2017) identified a main effect model as the most parsimonious model with the best data fit and reported their results from this model. By contrast, the LV-CRM was able to identify a significant trauma load × mental defeat interaction effect. This allowed for the calculation of simple slopes, illustrated in Figure 9. The color indicates the extend of trauma load (green to red); the latent variable η (MDQ) has a standardized scale, that is, mean of 0

Table 2

Covariate	GLM	Estimate	SE	p	LV-CRM	Estimate	SE	p
Intercept	$\hat{\beta}_0$	1.812	0.389	< 0.001	$\hat{\beta}_0$	1.811	0.407	< 0.001
Sex	$\hat{\beta}_1$	-0.349	0.231	0.130	$\hat{\beta}_1$	-0.321	0.241	0.182
Age	$\hat{\beta}_2$	-0.023	0.017	0.180	$\hat{\beta}_2$	-0.027	0.018	0.150
Age at Worst Event	$\hat{\beta}_3$	0.010	0.020	0.613	$\hat{\beta}_3$	0.015	0.021	0.467
MDQ	$\hat{\beta}_4$	0.675	0.131	< 0.001	$\hat{\gamma}_1$	0.713	0.174	< 0.001
Trauma Load	$\hat{\beta}_5$	0.543	0.136	< 0.001	$\hat{\gamma}_2$	0.628	0.194	0.001
MDQ : Trauma Load	$\hat{\beta}_6$	-0.253	0.141	0.073	$\hat{\gamma}_{12}$	-0.379	0.192	0.049

and standard deviation of 1. If trauma load was below the average (green lines; 1 and 2 SD below average, respectively), there was a strong positive association between mental defeat and dissociative symptoms (dark green slope = 1.470, $p < .001$; light green slope = 1.091, $p < .001$. The association was smaller at an average trauma load (yellow line), but remained significant (slope = 0.713, $p < .001$). However, at higher levels of trauma load (red lines; 1 and 2 SD above average, respectively), mental defeat was no longer a significant predictor of dissociative symptoms (orange slope = 0.334, $p = .075$; red slope = -0.045, $p = .546$).

Discussion of Empirical Example

Previous research showed that at higher levels of trauma load, the interindividual variability in trauma-associated symptoms decreases and individual risk factors may only play a subordinate role (Kolassa et al., 2010; Mollica et al., 1998; Neuner et al., 2004; Wilker et al., 2015). This should be reflected by significant interactions between risk factors, such as peritraumatic mental defeat, and trauma load on the outcome variable. While this effect was only present at a trend level when employing classical negative binomial regression models, the novel method introduced in this paper allowed us to discover such interaction effects.

At the same time, the strong importance of both trauma load and mental defeat as predictors of Shutdown dissociation were replicated by the novel analyses. Due to the

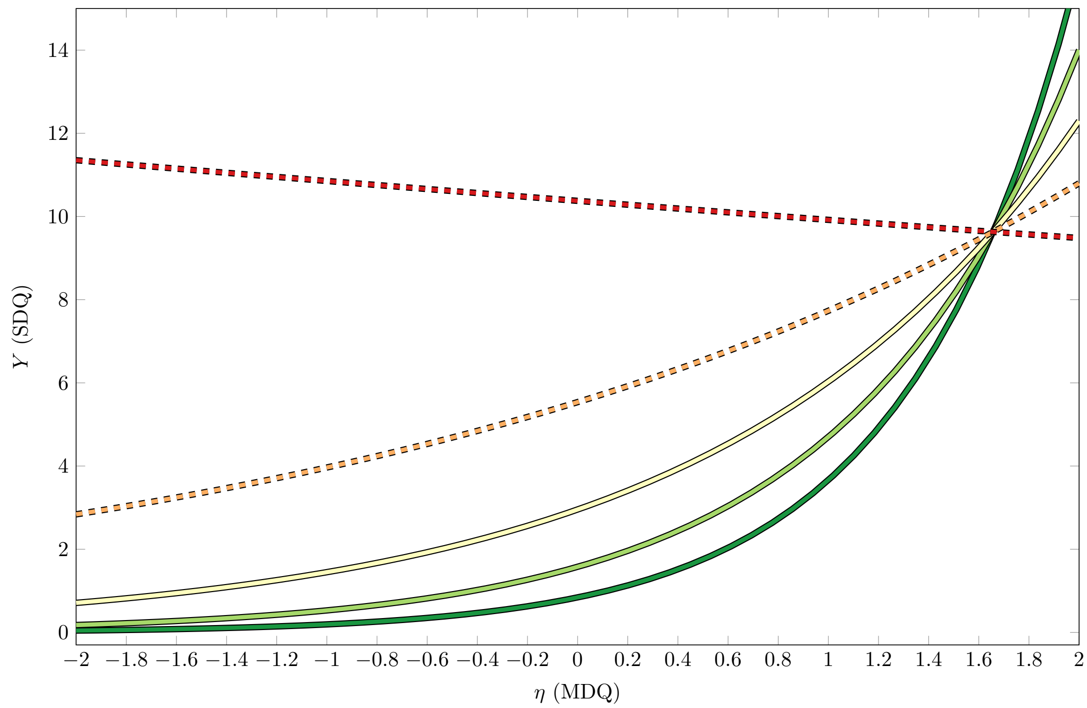

Figure 9

Simple slopes for the relation between latent mental defeat (MDQ) and dissociative symptoms (SDQ) given several values of trauma load (at 2 SD below mean in dark green; at 1 SD below mean in light green; at mean in yellow; at 1 SD above mean in orange; at 2 SD above mean in red)

increased power, the effects were even stronger than reported in the original analyses.

Discussion

In psychology and the social sciences, interactions between latent predictors in count regressons are often of interest. While it is well-known, that using fallible scores and not accounting for measurement error generally leads to attenuation bias in the estimated regression coefficients (Carroll et al., 2006), the extent of these effects has not been previously studied for count regression models. In this paper, we introduced the latent variable count regression model (LV-CRM) framework. We examined its performance regarding point estimation as well as statistical inferences using simulation studies and illustrated its use in an empirical example from clinical psychology.

In our simulation studies, we could show that severe attenuation bias (i.e., relative bias below -10%) for the interaction terms can occur even if the fallible score variables have considerably high reliabilites (i.e., both 0.9). For non-zero interaction effects, the estimated interaction parameters from the GLM were attenuated up to -33%. Attenuation bias this high was observed both in scenarios with two and three latent variables and their respective interactions. In contrast, the LV-CRM yielded virtually unbiased under most conditions and seemed to be a suitable alternative to de-attenuate the point estimates. In our third simulation study, we found a notable exception from this rule: In scenarios with three positively correlated latent variables and three positive interaction effects, the relative bias fluctuated rather unsystematically for both the GLM and the LV-CRM with larger biases for the newly proposed approach. For interaction effects of zero, however, both approaches worked equally well, with a slight advantage for the GLM due to the attenuation bias.

With regard to the relative efficiency of the point estimates, we found similar results. That is, for scenarios with an interaction effect of zero the GLM was slightly more efficient, as the attenuation bias has a 'favorable' effect in this case. In scenarios with non-zero interaction effect, however, the LV-CRM is often considerably more efficient than the GLM, especially if reliabilities of 0.8 or below are involved.

Our second simulation study also investigated statistical inferences for the GLM and the LV-CRM. Interestingly, we found that the empirical detection rate (i.e., Type I error rates and power) were acceptable and on the same level for both approaches. In the GLM, the biased point estimates are compensated by overconfident standard error estimates, that is, even though the point estimates are systematically closer to zero, the confidence intervals are too narrow and, therefore, do not necessarily include the zero too often. In contrast, the coverage rates were often poor for the GLM, especially in scenarios with larger interaction effects. That is, the confidence intervals were often too narrow to include the true interaction parameter, leading to coverage rates up to 19.9%. The LV-CRM, however, yielded accurate coverage rates under all conditions.

Limitations and Extensions

A well-known limitation of the marginal maximum likelihood approach that we chose for estimating the LV-CRM is the computational burden, that can quickly become unfeasible if multiple latent variables are involved. However, there exist different techniques to alleviate the computational burden (see Skrondal & Rabe-Hesketh, 2004, Ch. 6, for an accessible overview): First, numerically efficient techniques from the family of Gauss-Hermite quadratures can be used for normally distributed latent variables. Here the integration points and weights are derived through rule-based computations. Exponential growth of the number of integration points can be drastically reduced with techniques like adaptive Gauss-Hermite quadrature or Laplace approximation. An alternative can be the use of sparse grids (Heiss & Winschel, 2008), where integration are removed if their weight falls below a certain cutoff, resulting in a considerably smaller grid. Second, Monte Carlo integration offers an alternative for high-dimensional integration problems as well as in situations with non-normal latent variables. Here the integration points and weights are randomly drawn from the target distribution. In contrast to Gauss-Hermite techniques, the number of integration points does not necessarily grow exponentially and the weights are always equal to 1. Especially for non-normally distributed latent variables, this technique can be facilitated with a Gauss-Hermite rule-based importance sampling approach (Elvira et al., 2021). Third, in some cases it is possible to reduce the dimension of numerical integration below the number of latent variables. Rockwood (2021) illustrates this in an example with five latent dimensions, where one dimension of integration suffices after a re-parameterization of the model. While this reduction technique does not generalize directly to a model with interaction terms, it can be useful in situations where only few of the latent variables are actually involved in interactions.

While the LV-CRM is an extension of both the G-SEM framework (Rockwood, 2021) and the NB-MG-SEM framework (Kiefer & Mayer, 2021a, 2021b), it is more restrictive as these frameworks in some regards. The LV-CRM can in principle be extended to be a full generalization of the G-SEM framework, when also considering multiple outcome variables, a structural model among the latent variables, and allowing manifest covariates in the measurement model. An important advantage of such a generalization would be the inclusion of multiple outcomes, allowing for zero-inflated count regression models. The two main differences (besides the latent interactions) between the LV-CRM and the NB-MG-SEM are: First, the NB-MG-SEM is based on a multigroup framework which allows for more modeling flexibility when it comes to group-specific effects. For example, it is possible to estimate group-specific overdispersion parameters or measurement error variances. Thus, it offers an alternative to model heterogeneity in parameters. Second, in the LV-CRM we distinguished between stochastic and fixed observed variables. That is, manifest predictors in the LV-CRM are considered as fixed by design. The NB-MG-SEM models all observed variables as being stochastic (i.e., randomly sampled). While this distinction should have no effect on the estimation of the model parameters (Kiefer & Mayer, 2019), the stochastic approach additionally estimates various moments (i.e., expectation, variance, covariance) of the manifest predictors, which can be used for further analyses.

Data Availablity

The datasets generated during and/or analysed during the simulation studies of the current study are available in the OSF repository,
https://osf.io/q7knc/?view$_{o}nly = 363ff4f45c0d4e6785a66b64fc11a364$

The dataset analysed for the empirical example of the current study are not publicly available due to data privacy laws. Because of this limitation, analysis code is illustrated with a synthetic dataset, which is also available from the OSF repository.

References

Asparouhov, T., & Muthén, B. (2021). Expanding the Bayesian structural equation, multilevel and mixture models to logit, negative-binomial, and nominal variables. *Structural Equation Modeling: A Multidisciplinary Journal*, *28*(4), 622–637. https://doi.org/10.1080/10705511.2021.1878896

Bohrnstedt, G. W., & Marwell, G. (1978). The Reliability of Products of Two Random Variables. *Sociological Methodology*, *9*, 254. https://doi.org/10.2307/270812

Bollen, K. A. (1989). *Structural Equations with Latent Variables*. John Wiley & Sons, Inc. https://doi.org/10.1002/9781118619179

Bulmer, M. G. (1974). On Fitting the Poisson Lognormal Distribution to Species-Abundance Data. *Biometrics*, *30*(1), 101. https://doi.org/10.2307/2529621

Busemeyer, J. R., & Jones, L. E. (1983). Analysis of multiplicative combination rules when the causal variables are measured with error. *Psychological Bulletin*, *93*(3), 549–562. https://doi.org/10.1037/0033-2909.93.3.549

Carroll, R. J., Ruppert, D., Stefanski, L. A., & Crainiceanu, C. M. (Eds.). (2006). *Measurement error in nonlinear models: A modern perspective* (2nd ed). Chapman & Hall/CRC
OCLC: ocm67405697.

Charlson, F., Van Ommeren, M., Flaxman, A., Cornett, J., Whiteford, H., & Saxena, S. (2019). New WHO prevalence estimates of mental disorders in conflict settings: A systematic review and meta-analysis. *The Lancet*, *394*(10194), 240–248. https://doi.org/10.1016/S0140-6736(19)30934-1

Cheung, G. W., Cooper-Thomas, H. D., Lau, R. S., & Wang, L. C. (2021). Testing Moderation in Business and Psychological Studies with Latent Moderated Structural Equations. *Journal of Business and Psychology*, *36*(6), 1009–1033. https://doi.org/10.1007/s10869-020-09717-0

Cortina, J. M., Markell-Goldstein, H. M., Green, J. P., & Chang, Y. (2021). How Are We Testing Interactions in Latent Variable Models? Surging Forward or Fighting Shy? *Organizational Research Methods*, *24*(1), 26–54. https://doi.org/10.1177/1094428119872531

Coxe, S., West, S. G., & Aiken, L. S. (2009). The Analysis of Count Data: A Gentle Introduction to Poisson Regression and Its Alternatives. *Journal of Personality Assessment*, *91*(2), 121–136. https://doi.org/10.1080/00223890802634175

Dunmore, E., Clark, D. M., & Ehlers, A. (1999). Cognitive factors involved in the onset and maintenance of posttraumatic stress disorder (PTSD) after physical or sexual assault. *Behaviour Research and Therapy*, *37*(9), 809–829. https://doi.org/10.1016/S0005-7967(98)00181-8

Dunmore, E., Clark, D. M., & Ehlers, A. (2001). A prospective investigation of the role of cognitive factors in persistent Posttraumatic Stress Disorder (PTSD) after physical or sexual assault. *Behaviour Research and Therapy*, *39*(9), 1063–1084. https://doi.org/10.1016/S0005-7967(00)00088-7

Ehlers, A., Clark, D. M., Dunmore, E., Jaycox, L., Meadows, E., & Foa, E. B. (1998). Predicting response to exposure treatment in PTSD: The role of mental defeat and alienation. *Journal of Traumatic Stress*, *11*(3), 457–471. https://doi.org/10.1023/A:1024448511504

Elvira, V., Martino, L., & Closas, P. (2021). Importance Gaussian Quadrature. *IEEE Transactions on Signal Processing*, *69*, 474–488. https://doi.org/10.1109/TSP.2020.3045526

Heiss, F., & Winschel, V. (2008). Likelihood approximation by numerical integration on sparse grids. *Journal of Econometrics*, *144*(1), 62–80. https://doi.org/10.1016/j.jeconom.2007.12.004

Hilbe, J. M. (2011). *Negative binomial regression* (2nd ed). Cambridge University Press OCLC: ocn694679188.

Jin, S., Vegelius, J., & Yang-Wallentin, F. (2020). A Marginal Maximum Likelihood Approach for Extended Quadratic Structural Equation Modeling with Ordinal Data. *Structural Equation Modeling: A Multidisciplinary Journal*, *27*(6), 864–873. https://doi.org/10.1080/10705511.2020.1712552

Kelava, A., Werner, C. S., Schermelleh-Engel, K., Moosbrugger, H., Zapf, D., Ma, Y., Cham, H., Aiken, L. S., & West, S. G. (2011). Advanced Nonlinear Latent Variable Modeling: Distribution Analytic LMS and QML Estimators of Interaction and Quadratic Effects. *Structural Equation Modeling: A Multidisciplinary Journal*, *18*(3), 465–491. https://doi.org/10.1080/10705511.2011.582408

Kenny, D. A., & Judd, C. M. (1984). Estimating the Nonlinear and Interactive Effects of Latent Variables. https://doi.org/10.1037/0033-2909.96.1.201

Kessler, R. C., Berglund, P., Demler, O., Jin, R., Merikangas, K. R., & Walters, E. E. (2005). Lifetime Prevalence and Age-of-Onset Distributions of DSM-IV Disorders in the National Comorbidity Survey Replication. *Archives of General Psychiatry*, *62*(6), 593. https://doi.org/10.1001/archpsyc.62.6.593

Kessler, R. C., Ressler, K. J., House, S. L., Beaudoin, F. L., An, X., Stevens, J. S., Zeng, D., Neylan, T. C., Linnstaedt, S. D., Germine, L. T., Musey, P. I., Hendry, P. L., Sheikh, S., Storrow, A. B., Jones, C. W., Punches, B. E., Datner, E. M., Mohiuddin, K., Gentile, N. T., … McLean, S. A. (2021). Socio-demographic and trauma-related predictors of PTSD within 8 weeks of a motor vehicle collision in the AURORA study. *Molecular Psychiatry*, *26*(7), 3108–3121. https://doi.org/10.1038/s41380-020-00911-3

Kiefer, C., & Mayer, A. (2019). Average effects based on regressions with a logarithmic link function: A new approach with stochastic covariates. *Psychometrika*, *84*(2), 422–446. https://doi.org/10.1007/s11336-018-09654-1

Kiefer, C., & Mayer, A. (2021a). Accounting for latent covariates in average effects from count regressions. *Multivariate Behavioral Research*, *56*(4), 579–594. https://doi.org/10.1080/00273171.2020.1751027

Kiefer, C., & Mayer, A. (2021b). Treatment effects on count outcomes with non-normal covariates. *British Journal of Mathematical and Statistical Psychology*, *74*(3), 513–540. https://doi.org/10.1111/bmsp.12237

Kleim, B., Ehlers, A., & Glucksman, E. (2012). Investigating cognitive pathways to psychopathology: Predicting depression and posttraumatic stress disorder from early responses after assault. *Psychological Trauma: Theory, Research, Practice, and Policy*, *4*(5), 527–537. https://doi.org/10.1037/a0027006

Klein, A., & Moosbrugger, H. (2000). Maximum likelihood estimation of latent interaction effects with the LMS method. *Psychometrika*, *65*(4), 457–474. https://doi.org/10.1007/BF02296338

Kolassa, I.-T., Ertl, V., Eckart, C., Kolassa, S., Onyut, L. P., & Elbert, T. (2010). Spontaneous remission from PTSD depends on the number of traumatic event types experienced. *Psychological Trauma: Theory, Research, Practice, and Policy*, *2*(3), 169–174. https://doi.org/10.1037/a0019362

Lambert, D. (1992). Zero-Inflated Poisson Regression, with an Application to Defects in Manufacturing. *Technometrics*, *34*(1), 1. https://doi.org/10.2307/1269547

Liu, Y., West, S. G., Levy, R., & Aiken, L. S. (2017). Tests of simple slopes in multiple regression models with an interaction: Comparison of four approaches. *Multivariate Behavioral Research*, *52*(4), 445–464. https://doi.org/10.1080/00273171.2017.1309261

McCullagh, P., & Nelder, J. A. (1998). *Generalized linear models* (2nd ed). Chapman & Hall/CRC.

McMahon, R. J., Witkiewitz, K., Kotler, J. S., & The Conduct Problems Prevention Research Group. (2010). Predictive validity of callous–unemotional traits measured in early adolescence with respect to multiple antisocial outcomes. *Journal of Abnormal Psychology*, *119*(4), 752–763. https://doi.org/10.1037/a0020796

Mollica, R. F., McInnes, K., Pool, C., & Tor, S. (1998). Dose-effect relationships of trauma to symptoms of depression and post-traumatic stress disorder among Cambodian survivors of mass violence. *British Journal of Psychiatry*, *173*(6), 482–488. https://doi.org/10.1192/bjp.173.6.482

Muthén, L. K., & Muthén, B. O. (1998–2017). *Mplus User's Guide. Eighth Edition*. Los Angeles, CA: Muthén & Muthén.

Nelder, J. A., & Wedderburn, R. W. M. (1972). Generalized Linear Models. *Journal of the Royal Statistical Society. Series A (General)*, *135*(3), 370. https://doi.org/10.2307/2344614

Neuner, F., Schauer, M., Karunakara, U., Klaschik, C., Robert, C., & Elbert, T. (2004). Psychological trauma and evidence for enhanced vulnerability for posttraumatic stress disorder through previous trauma among West Nile refugees. *BMC Psychiatry*, *4*(1), 34. https://doi.org/10.1186/1471-244X-4-34

Raykov, T., & Marcoulides, G. A. (2004). Using the Delta method for approximate interval estimation of parameter functions in SEM. *Structural Equation Modeling: A Multidisciplinary Journal*, *11*(4), 621–637. https://doi.org/10.1207/s15328007sem1104_7

Rockwood, N. J. (2021). Efficient Likelihood Estimation of Generalized Structural Equation Models with a Mix of Normal and Nonnormal Responses. *Psychometrika*, *86*(2), 642–667. https://doi.org/10.1007/s11336-021-09770-5

Rodriguez, L. M., Litt, D. M., & Stewart, S. H. (2020). Drinking to cope with the pandemic: The unique associations of COVID-19-related perceived threat and psychological distress to drinking behaviors in American men and women. *Addictive Behaviors*, *110*, 106532. https://doi.org/10.1016/j.addbeh.2020.106532

Savalei, V. (2019). A comparison of several approaches for controlling measurement error in small samples. *Psychological Methods*, *24*(3), 352–370. https://doi.org/10.1037/met0000181

Schalinski, I., Schauer, M., & Elbert, T. (2015). The Shutdown Dissociation Scale (Shut-D). *European Journal of Psychotraumatology*, *6*(1), 25652. https://doi.org/10.3402/ejpt.v6.25652

Schauer, M., & Elbert, T. (2010). Dissociation Following Traumatic Stress: Etiology and Treatment. *Zeitschrift für Psychologie / Journal of Psychology*, *218*(2), 109–127. https://doi.org/10.1027/0044-3409/a000018

Skrondal, A., & Rabe-Hesketh, S. (2004). *Generalized latent variable modeling: Multilevel, longitudinal, and structural equation models*. Chapman & Hall/CRC.

Vermetten, E., & Spiegel, D. (2014). Trauma and Dissociation: Implications for Borderline Personality Disorder. *Current Psychiatry Reports*, *16*(2), 434. https://doi.org/10.1007/s11920-013-0434-8

Wilker, S., Kleim, B., Geiling, A., Pfeiffer, A., Elbert, T., & Kolassa, I.-T. (2017). Mental Defeat and Cumulative Trauma Experiences Predict Trauma-Related Psychopathology: Evidence From a Postconflict Population in Northern Uganda. *Clinical Psychological Science*, *5*(6), 974–984. https://doi.org/10.1177/2167702617719946

Wilker, S., Pfeiffer, A., Kolassa, S., Koslowski, D., Elbert, T., & Kolassa, I.-T. (2015). How to quantify exposure to traumatic stress? Reliability and predictive validity of measures for cumulative trauma exposure in a post-conflict population. *European Journal of Psychotraumatology*, *6*(1), 28306. https://doi.org/10.3402/ejpt.v6.28306