1 TITLE

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3 **Analyzing the misperception of exponential growth in graphs**

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17 ABSTRACT

18 Exponential growth is frequently underestimated, an error that can have a heavy social cost in the
19 context of epidemics. To clarify its origins, we measured the human capacity ($N = 521$) to
20 extrapolate linear and exponential trends in scatterplots. Four factors were manipulated: the
21 function underlying the data (linear or exponential), the response modality (pointing or venturing
22 a number), the scale on the y axis (linear or logarithmic), and the amount of noise in the data.
23 While linear extrapolation was precise and largely unbiased, we observed a consistent
24 underestimation of noisy exponential growth, present for both pointing and numerical responses.
25 A biased ideal-observer model could explain these data as an occasional misperception of noisy
26 exponential graphs as quadratic curves. Importantly, this underestimation bias was mitigated by
27 participants' math knowledge, by using a logarithmic scale, and by presenting a noiseless
28 exponential curve rather than a noisy data plot, thus suggesting concrete avenues for
29 interventions.

30 STATEMENT OF RELEVANCE

31 Many natural and social phenomena, including the spread of epidemics, follow an exponential
32 growth. Although graphs depicting exponential trends are omnipresent in the media, people tend
33 to severely underestimate their fast progression, a misperception that can have a heavy social
34 cost. In order to offer evidence-based suggestions to reduce this underestimation bias, we
35 investigated its origins by asking hundreds of people to guess the evolution of graphical trends.
36 We found that the bias is reduced: 1) for people with a higher mathematical knowledge; 2) when
37 graphs are represented without noise; 3) when the vertical axis has a logarithmic scale; 4) when
38 people manually locate their prediction instead of making a numerical guess. These pieces of
39 evidence could inform and guide scientists, journalists and policy makers when they are asked to
40 graphically represent the evolution of exponential increases, such as, for example, the number of
41 infected people during a pandemic.

42 INTRODUCTION

43 Scatterplots are widely used to visually convey the evolution of a measure such as global
44 temperature, salary, or mortality over time (Friendly & Denis, 2005). They allow humans to
45 efficiently grasp a functional relationship at a glance, without serially processing the data in
46 numerical form, and to predict values beyond the available range (Ciccione & Dehaene, 2021).
47 Psychophysical research indicates that human adults can extract the trend underlying a graph and
48 that their extrapolation abilities partially extend to noisy representations of linear and non-linear
49 functions (Ciccione & Dehaene, 2021; Schulz et al., 2017).

50 The case of exponential functions is important because it captures many real-world
51 processes such as the increase in insect populations (Maino & Kearney, 2015), the compound
52 interest on a financial capital (Blackman, 1919), the area damaged in a fire (Ramachandran, 1986)
53 or the spread of epidemics and rumors (Bernoulli, 1760; Dietz, 1967; Ma, 2020). Exponentials are
54 mathematical functions of the form $y = a b^x$ where a is the intercept with the y -axis and b is a
55 positive real number. Exponential growth, which occurs when $b > 1$, may initially look slow, but it will
56 eventually overtake any linear or polynomial function.

57 Humans are known to consistently underestimate exponentials. This “exponential growth
58 bias” was originally described in behavioral studies in which participants extrapolated a new value
59 from numerical real-world data: their numerical extrapolation was consistently lower than the
60 correct one (Andreassen & Kraus, 1990; Eggleton, 1982; Wagenaar & Timmers, 1978), even when
61 the data was presented in a graphical format (Wagenaar & Sagaria, 1975) or as a computerized
62 representation of growing plants (Wagenaar & Timmers, 1979). The magnitude of this bias was
63 slightly reduced after a short lecture on it (Wagenaar & Sagaria, 1975), but did not correlate with
64 participants’ education level (Levy & Tasoff, 2016).

65 Misunderstanding exponential functions have dramatic consequences. During the 2019
66 coronavirus pandemic, people systematically misunderstood its exponential evolution (Lammers et
67 al., 2020): when asked to estimate the number of Covid-19 cases over the past days, they strongly
68 underestimated their acceleration, and this underestimation correlated with low personal
69 adherence to sanitary measures. Correcting their misperception significantly increased the support
70 for social distancing measures (Lammers et al., 2020; Schonger & Sele, 2020), thus confirming the
71 importance of a correct understanding of exponential functions to improve decision making and
72 social behavior.

73 In the present study, we investigated whether experimental conditions can be found in
74 which the exponential growth bias is reduced or even eliminated, in order to provide evidence-
75 based suggestions for a better graphic visualization of exponential trends. To this aim, we analyzed
76 in fine detail the factors that determine the misperception of exponential growths. First, we
77 investigated the role of the response modality: most previous studies asked participants to venture
78 a numerical guess, and thus their biases might have arisen from the numerical estimation stage
79 rather than from the analysis of the function itself. To separate those possibilities, we compared
80 two extrapolation conditions from visually presented scatterplots: one in which participants were
81 asked to point to the correct location of the extrapolated dot, and one in which they were asked to
82 provide a numerical answer.

83 Second, we analyzed the effect of noise in the dataset. Previous studies of linear scatterplots
84 (Ciccione & Dehaene, 2021) showed that, with increasing levels of noise, participants'
85 extrapolations depart more from the statistical ideal. In the case of exponentials, it is not known
86 whether the underestimation bias is already present for noiseless functions or if it only arises when
87 extracting an exponential trend from noise. To answer this question, we compared extrapolation
88 performance for noiseless versus noisy plots.

Third, we measured the costs and benefits of plotting the data on a log scale. When the y axis is logarithmic, exponential growth looks linear, and such linear trends can be perceived without much bias (Ciccione & Dehaene, 2021). Thus, we hypothesized that the advantage of extrapolating from a visually linear trend could eliminate the exponential growth bias altogether. In support of this hypothesis, framing exponential evolution in terms of doubling times (i.e., taking a logarithmic perspective on data) was shown to reduce the bias (Schonger & Sele, 2020). However, we also investigated whether this advantage was mitigated by the difficulty of interpolating numbers on a log scale.

Finally, we checked whether education in science and mathematics can help overcome the exponential growth bias. To this end, we asked whether the exponential bias varied with participants' knowledge of mathematics as well as their type and level of education.

METHODS

Experimental design

We used a factorial design with factors of response modality, noise, axis scale, and function type (figure 1A). On each trial, participants saw a scatterplot and extrapolated a new point outside the original data range. In two distinct blocks, they either placed a dot (pointing task) or ventured a numerical value (number task). Within each block, the stimuli consisted in a mixture of noisy or noiseless scatterplots, whose y-axis scale could either be linear or logarithmic. Importantly, the same five equidistant ticks were shown, and only their labels were changed (0-250-500-750-1000 in the linear scale condition; 1-10-100-1000-10000 in the log scale condition). Finally, the type of function could either be linear or exponential. In the "same-graph" condition, the function was *visually* linear in both the linear and the log scale conditions (across subjects, the same exact displays were used). In the "same-function" condition, the function was *numerically* exponential in both the

113 linear and the log scale conditions (across subjects, the same exact numerical values were used),
 114 thus resulting in an exponential-looking graph when plotted on a linear scale and in a linear-looking
 115 graph when plotted on a log scale (figure 1A). Each trial consisted in a scatterplot comprising 33
 116 horizontally equally spaced white points on a black background inside a square graph.

117

118 **Figure 1. Experimental paradigm to study the human extrapolation of linear and exponential graphs.** A: the four experimental
 119 conditions, with a sample noisy stimulus for each. Right, distribution of the stimulus datapoints, with some sample noiseless curves
 120 in red. B: distribution of the participants' extrapolation responses (in purple) compared with the distribution of correct responses (in
 121 green). Numerical (left) and pointing (right) responses are separated for sake of clarity. At the right of each subplot, the average error
 122 (i.e., the difference between the participants' response and the correct responses) is indicated, for each location of the extrapolation
 123 area, separately for noisy and noiseless conditions.

124 Stimuli generation and characteristics

125 To avoid possible biases due to a reluctance to use the extremes of the available scale, we
126 matched the linear and exponential stimuli in two different ways, as explained below.

127 In the “same-graph” condition, the slopes and intercepts were randomly varied in order to
128 generate an approximately uniform distribution of desired *pointing* responses in the range 150-850.
129 Concretely, stimuli were generated by uniformly sampling a value Y in $[150, 850]$ and then
130 repeatedly uniformly sampling a slope A greater than 50 or smaller than -50 until the line of
131 equation $y = Ax + (Y - A)$ would take a value between 150 and 850 for $x=-1$ (the leftmost value).
132 Then, 33 equally spaced points were sampled along the line of equation $y = Ax + (Y - A) + \varepsilon_\sigma$
133 for x in $[-1, 0.5]$, where ε_σ was centered noise of variance σ , independent for each point. Finally,
134 either a linear or a log scale was displayed on the left, without changing the screen locations of the
135 dots.

136 In the “same-function” condition, the two parameters of the function were randomly chosen
137 in order to generate an approximately flat distribution of desired *numerical* responses in the range
138 150-850. Concretely, we first sampled a value Y' in $[150, 850]$ and then computed $Y =$
139 $\frac{1000}{4} \log_{10}(Y')$. Then we sampled parameters with the same constraints as the “same-graph”
140 condition (plus the requirement that the slope would be positive), to define a line of equation $y =$
141 $Ax + (Y - A) + \varepsilon_\sigma$. Then we sampled along this line on similarly to the “same-graph” condition.
142 Then, we either presented the points with a log scale on the left (log scale condition), or computed,
143 for each point, the numerical value when presented on the log scale, and then presented these
144 values on a linear scale, thereby displaying a function of equation $y = 10^{\frac{4}{10} * (Ax + (Y - A) + \varepsilon_\sigma)}$. By doing
145 so we ensured that the correct response ranged from 150 to 850 in the linear scale condition.
146 The noise level σ was set to either zero (noiseless condition) or 10. This value was chosen to add a
147 moderate amount of noise without hiding the underlying linear or exponential trend.

148 Participants

149 625 participants took part in this on-line experiment. The experiment was advertised on
150 Twitter and Facebook, starting from the 18th of February, 2021. People interested to participate
151 could simply click on the provided link, choose their preferred language among six (English, French,
152 Italian, Spanish, Portuguese and Chinese) and read and accept a written consent, in which they
153 declared not to be legally minor. Participants were informed they could withdraw from the
154 experiment at any moment by simply quitting the webpage. The procedure and the consent were
155 approved by the local ethical committee (reference: CER-Paris-Saclay-2019-063). Data collection for
156 the purpose of the study was stopped two months later, the 18th of April, 2021. 521 participants
157 met our criteria for data analysis, i.e. answering to all questions proposed and not giving the default
158 answer to the four questions on self-evaluation. Note that 33 participants declared passing the
159 experiment more than once, but the results were unchanged when their data was excluded. Their
160 data was merely discarded from correlational analyses.

161 Experimental procedure

162 The experimental procedure started with a series of questions on demographic aspects
163 (country of origin, gender, age, highest degree obtained; average grade in mathematics and
164 academic field) and on subjective self-evaluations assessments, with answers on a Likert scale from
165 1 to 10: current skills in mathematics; current skills in first language; familiarity with graphs;
166 knowledge of statistics. Then, participants performed one experimental task (the pointing task or
167 the number task), preceded by short instructions. At the end of their first task, participants were
168 invited to continue with the second task, which was again preceded by the instructions. The order
169 of the tasks was randomly assigned for each participant. Both instruction texts were each
170 accompanied with a simple explanatory figure. The instructions to the pointing task were the
171 following: “In this part of the experiment, you will see scatterplots such as in the figure below. Place

172 a point in the grey zone where you think that the curve will go. When you are happy with the
173 location you chose, just click. Try to be as precise as possible but do not use rulers and do not make
174 calculations: we want your intuitive response!”. The instructions to the number task were the
175 following: “In this part of the experiment, you will see scatterplots such as in the figure below.
176 Consider the grey zone and estimate the value that the curve would reach if it continued till that
177 zone. Try to be as precise as possible but do not use rulers and do not make calculations: we want
178 your intuitive response!”.

179

180 RESULTS

181 Evaluating bias versus variance in participants’ responses

182 Using a terminology borrowed from machine learning literature (Geman et al., 1992), participants
183 could be inaccurate in two ways: bias versus variance (Ciccione & Dehaene, 2021). Bias refers to the
184 average error: a positive bias means that participants overestimate their extrapolation, whereas a
185 negative bias means that they underestimate it. Variance, or rather variability, here measured by
186 standard deviation, describes how much participants’ errors are spread around their average value,
187 thus indicating erratic responding.

188 On each trial, we measured the signed difference between subject’s answer and the correct answer.
189 Pointing responses were evaluated to the nearest pixels and linearly rescaled to range from 0 to
190 1000, thus coinciding with the linear axis values. Numerical responses were similarly transformed
191 to screen units (see supplemental material for discussion and alternative measures). We could then
192 compute each subjects’ mean error (bias) and standard deviation (variability) in each condition.
193 Figure 1B shows the distributions of participants’ responses and their average bias for each desired
194 response, and figure 2 shows the average bias and standard deviation in the sixteen conditions of
195 the design. Bias and variability were primarily analyzed with repeated-measures ANOVAs,

separately for the “same-graph” (tables 1A and 1B) and the “same-function” condition (tables 2A and 2B). All t-tests were performed under the conservative assumption of samples’ unequal variances (Welch, 1947).

Figure 2. Mean bias and mean standard deviation of extrapolation responses, separately for each response modality, y axis scale and noise level. On the left: results from the same-graph (= linear) conditions. On the right, results from the same-function (= exponential) conditions. The graph makes it clear that responses to exponential curves on a linear axis differ considerably from the other conditions, both in bias and in variability across trials.

Pointing leads to better extrapolations than venturing a number

We first describe participants’ extrapolation performance for linear functions on a linear scale (top line in figure 1, and leftmost points in Figure 2). Pointing responses were both less biased and less variable than numerical guesses. Indeed, participants were very precise at pointing towards the correct extrapolation value: no bias was found (mean bias=.79; $t(520)=.91$, $p=.3$). When asked

210 to type in a numerical answer, the bias remained small but became significant (mean bias=-14.6,
211 $t(520)=-5.55$, $p<.001$).

212 As concerns variability, pointing answers were also less variable than numerical ones
213 (significant effect of response modality on variability, table 1B). Combined, these results reveal that
214 participants were accurate at extrapolating linear trends, and significantly more so when pointing
215 than when venturing a numerical guess.

216 Numerical answers are anchored to round numbers

217 Examination of the distribution of responses indicated that numerical answers displayed a
218 strong anchoring effect: responses spiked for round numbers (purple peaks in figure 1B) while
219 pointing responses followed the flat distribution of desired responses. Reference tick values (0-250-
220 500-750-1000) were used significantly more commonly as numerical guesses ($433/3126=13.9\%$)
221 than as pointing responses ($31/3126=.01\%$; $\chi^2=374.34$, $df=1$, $p<.0001$). Furthermore, responses
222 corresponding to tens and hundreds were significantly more frequent than the expected frequency
223 under the null hypothesis that each number had the same probability to be chosen (binomial test,
224 empirical proportion $2597/3126=83\%$, expected proportion $313/3126=10\%$, $p<.0001$). Those
225 findings fit with the known human preference for round numbers, particularly powers of ten
226 (Dehaene & Mehler, 1992; Sigurd, 1988).

227 Extrapolations from noisy linear trends are more variable but still accurate

228 We next tested the effect of noise on linear extrapolation. Participants' bias was small when
229 extrapolating both noiseless and noisy scatterplots. Their average errors were virtually identical in
230 both cases (figure 2, top), with no significant bias in the pointing condition (t-tests against zero, both
231 $p>.05$), and only a modest, non-significantly larger bias for noisy stimuli in the numerical condition
232 ($bias_{noisy}=-17.71$, $bias_{noiseless}=-13.27$, $t(1038)=-1.12$, $p=.26$). Their pointing responses were more

233 variable for noisy scatterplots ($sd_{noisy}=27.92$, $sd_{noiseless}=13.20$; $t(1015.8)=8.64$, $p<.0001$), in
234 agreement with previous research (Ciccione & Dehaene, 2021).

235 For numerical answers, surprisingly, additional noise did not increase participants' response
236 variability (two-sample t-test, $p=.64$), probably because the data was dominated by variability due
237 to the anchoring effect.

238 Participants are poor at reading numbers from a log scale, except at tick values

239 We next examined the pure effect of asking participants to read out the numbers from a log
240 scale rather than from a linear one, using identical linear graphs ("same-graph" condition; second
241 line in figure 1, and second data point in figure 2). As expected, the scale had no effect when
242 participants responded by pointing (bias: $bias_{log}=.51$, $bias_{linear}=.79$; $t(1039.4)=-.23$, $p=.82$; variability:
243 $sd_{log}=24.7$, $sd_{linear}=23.68$; $t(1014.6)=-.59$, $p=.56$). However, there was a large influence on numerical
244 responses. While response variability was only slightly larger on a log scale ($sd_{log}=51$, $sd_{linear}=44.91$;
245 $t(1019.7)=2.52$, $p=.01$), response bias became systematically positive ($bias_{log}=15.58$) and was
246 significantly different from both zero ($t(520)=7.5$, $p<.0001$) and from the linear axis condition where
247 bias was negative ($bias_{log}=15.58$, $bias_{linear}=-14.64$; $t(992.45)=9.1$, $p<.0001$).

248 What could be the reason for this effect? It appears that, on a log scale, participants have
249 trouble finding which number corresponds to a given location. Indeed, the organization of a
250 logarithmic scale can be counterintuitive. For instance, the value that falls in the middle of 10 and
251 100 is not 55, as on a linear scale, but 31.6 (the geometric mean of 10 and 100). Participants' failure
252 to understand this property could explain their numerical overestimations on a log scale: they would
253 know the correct location (as indicated by their accurate pointing), but fail to turn it into a correct
254 number, instead choosing either the closest tick mark, or using a linear interpolation of the two
255 nearest tick marks, two strategies that would lead to overestimation.

256 Indeed, both strategies were attested in our data. First, for log scales, response distributions
257 showed prominent spikes at tick values 10, 100 and 1000, which were absent at corresponding
258 points 250, 500 and 750 on the linear scale (Figure 1b). Indeed, the anchoring effect was much
259 stronger on a log scale: participants were more likely to make numerical guesses that fell at or close
260 to the reference ticks' values (± 25 screen units) on a log scale (1268/3126=41%) than on a linear
261 scale (814/3126=26%; $\chi^2=147.77$, $df=1$, $p<.0001$).

262 However, there was no significant difference in the frequency of answers corresponding
263 *exactly* to the ticks (log, 452/3126=14.5%; linear, 433/3126=13.9%; $\chi^2=.43$, $df=1$, $p=.51$). Thus,
264 participants were still trying to venture an accurate guess, not just responding with the tick value.
265 We next tested the idea that they performed a linear rather than logarithmic interpolation between
266 ticks. We extracted all log-scale trials whose desired response location fell between the second and
267 third tick marks (i.e. between 10-100 on a log scale) and submitted subjects' numerical answers to
268 a multiple regression analysis with two regressors: the correct numerical answer and the answer
269 that would have been correct under linear interpolation. We found that the latter regressor entered
270 as a significant predictor ($\beta=.97$, $p=.001$), whereas the correct numerical response did not ($\beta=.14$,
271 $p=.65$). Similar results were found in the range 100-1000 (but both predictors were significant:
272 $\beta_{\text{linear_interpolation}}=4.94$, $p<.001$; $\beta_{\text{correct_response}}=.5$, $p<.001$).

273 Only noisy scatterplots lead to the exponential underestimation bias

274 We next turned to the perception of exponential functions, first characterizing the
275 conditions under which the underestimation bias is present when data is plotted on a linear scale,
276 then investigating whether this bias can be mitigated by using a log scale.

277 On a linear scale (third data point in figure 2), we observed the classical exponential
278 underestimation bias. Importantly, this negative bias was only found for noisy functions, for both
279 pointing responses (bias=-13.23, $t(520)=-2.37$, $p=.02$) and numerical guesses (bias=-50.44, $t(520)=-$

280 8.85, $p < .0001$). On the contrary, participants' answers often overshoot the correct prediction in the
281 noiseless condition, for both numerical (bias=56.67, $t(520)=13.93$, $p < .0001$) and pointing answers
282 (bias=19.12, $t(520)=3.37$, $p < .001$).

283 These two opposite biases are clearly visible in the heatmaps in figure 3, which show the
284 entire set of participants' responses for exponential functions on a linear scale. In the noisy
285 condition, the green line, which represents the median response given by participants as a function
286 of the correct location, is consistently lower than the red line, which represents the desired
287 response. For noiseless functions, however, the green line is consistently above the red one,
288 meaning that participants overestimate their extrapolations. Note also that both biases become
289 noticeably larger as the exponential function becomes steeper, and thus the desired response
290 increases. This trend was confirmed by a linear regression between the amount of average bias and
291 the correct location ($R^2=.07$, $F[1,6250]=435.5$, $p < .0001$). Thus, participants are not far from correct
292 for shallow exponential functions, but they misperceive steep ones.

293 In passing, the marginal histograms in figure 3 (top) exhibit spikes at ticks' and round
294 numbers' locations, confirming the anchoring effect found for linear functions (see supplemental
295 material).

296 Participants' responses on a linear scale were dramatically more variable for exponential
297 functions than for linear ones (figure 2; $t(965.43)=-29.47$, $p < .0001$). This was true for both noise
298 levels and response modalities (all $p < .0001$), confirming that participants always experience
299 considerable difficulties in correctly estimating the behavior of exponential functions.
300 Extrapolations of exponentials on a linear scale were also more variable for noisy data than for
301 noiseless functions (figure 2; as confirmed by a significant main effect of noise on variability, table
302 2B): this was true for both pointing ($sd_{noise}=125.38$, average $sd_{noiseless}=74.77$, $t(897.31)=11.4$,
303 $p < .0001$) and numerical responses ($sd_{noise}=122.33$, $sd_{noiseless}=78.91$, $t(908.9)=9.68$, $p < .0001$).

Figure 3. Distribution of the participants' responses to exponential functions on a linear scale, as a function of the expected correct locations (binsize = 25). Plots are shown separately for each response modality and noise level. On the right of each plot, a histogram of the frequency of responses is shown in blue. The diagonal red line indicates optimal performance. The green line shows the participants' median response for each bin of correct location.

The underestimation bias disappears when exponential functions are plotted on a log scale

Finally, we examined whether the perception of exponential trends could be rescued by plotting the same data on a log scale. Figure 2B shows that, when participants were asked to extrapolate exponentials plotted on a log scale, their answers became much less biased and more precise for both response modalities and noise levels. Pointing answers were significantly less biased on a log axis than on a linear axis, for both noisy ($t(580.4)=2.03$, $p<.05$) and noiseless scatterplots ($t(579.45)=-14.71$, $p<.0001$). The same was true for numerical guesses, which were significantly less biased on a log axis for both noisy ($t(754.55)=8.65$, $p<.0001$) and noiseless scatterplots ($t(724.27)=-2.881$, $p<.01$). Analogously, participants' extrapolations of exponentials were considerably less variable on a log scale than on a linear scale ($t(770.06)=-33.89$, $p<.0001$).

320 Those results (figure 2), hold because we compare performance on both axes using the
321 common currency of screen location units. When considering the actual numbers given, the
322 variability for answers on a log scale increases relative to the linear scale but, crucially, the
323 underestimation bias still vanishes (see supplementary materials and figure 1). Taken together,
324 these results suggest that, for the very same exponential trend, plotting the data on a log rather
325 than a linear scale eliminates the underestimation bias.

326 Mathematical education mitigates the exponential growth bias

327 We next examined inter-individual variability in the underestimation bias. For each
328 participant, we computed the magnitude of their exponential growth bias for noisy scatterplots.
329 The distribution of individual biases, shown in figure 4A, covered a broad range from ~0 (no bias) to
330 -100 or lower (strong bias). Was this variability just noise, or did it vary systematically with
331 knowledge and education? Before the test, participants self-evaluated their skills in mathematics
332 on a scale from 1 to 10. For the following analysis, we excluded participants that answered 1 or 10,
333 since they were considerably fewer than the other ones (only two and nine respectively).

334 Participants' self-evaluated mathematical knowledge predicted the size of their
335 underestimation bias for noisy exponentials (figure 4B), for both pointing ($R=.12$, $df=476$, $p<.01$) and
336 numerical answers ($R=.19$, $df=476$, $p<.0001$). This correlation was specific to mathematics and was
337 not found between participants' bias and their self-evaluation of first-language skills (regardless of
338 noise and response modality, all $p>.05$). For noiseless exponentials, no such correlation was found
339 for pointing responses ($R=.05$, $df=476$, $p=.26$) and only a moderate one for numerical answers
340 ($R=.09$, $df=476$, $p<.05$). To further analyze how the exponential growth bias for noisy data varied
341 with education, we analyzed it as a function of the academic field in which participants obtained
342 their university degree (figure 4C). We observed that graduates in science, engineering and health

343 (n=221; average bias=-8) performed considerably better in extrapolating exponentials from noisy
 344 scatterplots than graduates in the other fields (n=196; average bias=-49; $t(410.68)=-4.5$, $p<.0001$).

345 Other correlation analyses are available as supplementary materials. In particular, we found
 346 that the precision of linear extrapolation increased with mathematical knowledge, both for
 347 numerical and for pointing responses.

348
 349
 350 **Figure 4. Inter-individual variability in the underestimation bias for noisy exponentials.** **A:** distribution of the bias across
 351 participants. **B:** participants' bias as a function of their self-evaluated mathematical knowledge. **C:** participants' bias as a function of
 352 their academic field of study.

353 Putative origins of the exponential bias in graph perception

354 Why do human adults underestimate the growth of noisy exponentials? Participants did not
355 simply prolong the curve linearly: their extrapolation responses were significantly higher than the
356 values predicted by the tangent at the last point (paired t-test: $t(2969)=17.64$, $p<.0001$). This
357 remained true even when including solely the participants whose mathematical education was
358 below the median (paired t-test: $t(1529)=9.22$, $p<.0001$). Thus, participants clearly understood that
359 exponential data are positively accelerated.

360 Another possibility is that participants mistook exponential growth for another simpler and
361 shallower relationship such as a quadratic one. We could reject this model because its predictions
362 were significantly lower than human data (paired t-test: $t(2969)=24.19$, $p<.0001$), and this was true
363 even for the least educated participants (paired t-test: $t(1529)=14.13$, $p<.0001$). Furthermore, this
364 theory can only account for the underestimation in the noisy condition, and falls short of explaining
365 success or overestimation in the noiseless condition.

366 The latter point suggests that humans possess an intuition of exponential growth, but do not
367 systematically apply it. The underestimation bias for exponential functions could arise from a subset
368 of trials in which noise would make it hard for participants to distinguish exponentials from other
369 functions. The high variability of responses to noisy exponentials (figures 2, 3) would arise from a
370 mixture of trials in which participants correctly detect the exponential trend and trials in which they
371 do not. In fact, this might be rational: noise could prevent the detection of exponentials even for an
372 optimal observer, who then reverts to the simpler hypothesis of linear or quadratic growth.

373 To test this idea, we modeled a biased Bayesian ideal observer that attempted to select the
374 best-fitting curve from three alternatives: linear, quadratic and exponential, with a special penalty
375 against exponentials (figure 5A). The model, adapted from Ciccione and Dehaene (2021), selects
376 the best-fitting function according to a penalized Bayesian Information Criterion (BIC).

377 When penalty=0 the model is unbiased and almost always correctly detected exponentials
378 (1000 trials, accuracy=91.2%). This finding validates our choice of noise level (it suffices to prevent
379 the confusion of exponentials with other functions) but shows that the unbiased optimal observer
380 does not fit human data. However, participants may not consider linear and exponential
381 relationships as equally likely or equally complex. It seems likely that, with less and less
382 mathematical education, participants would increasingly consider the exponential as a more exotic
383 relationship than the linear one. By biasing the model with a single free parameter (the penalty K
384 associated to choosing exponentials) we get a good fit to the data (figure 5). For a penalty $K=3$ (which
385 cancels the difference between quadratic and exponentials, the former having one extra free
386 parameter), exponentials were frequently misclassified either as quadratic (22.6% of trials) or as
387 linear (8.5%), both of which led to an underestimation of extrapolated responses.

388 We systematically searched for the penalty that provided the best fit to the human data,
389 obtained by minimizing the Jensen–Shannon Distance between the 2D distributions of the model
390 and the data in 3000 randomly selected trials. The best fit was obtained for a value of $K=12.62$, in
391 which case the biased ideal observer model accounted for several features of the data. First, while
392 it underestimated noisy exponentials, its average response always fell in between quadratic and
393 exponential extrapolation, similar to humans. This finding arose because when faced with a noisy
394 exponential graph, the model sometimes opted for a quadratic and sometimes for an exponential,
395 but rarely for a linear function. The model also provided a plausible account of the effect of
396 mathematical education, by assuming that less educated participants exhibit a larger prior against
397 exponentials (figure 5C). Indeed, when we separately fitted the data from participants in the first
398 and last quartiles of mathematical education, we found that a lower exponential penalty was
399 required for the former than for the latter (respectively 7.1 vs 15.8; figure 5D). Importantly, these
400 observations held only for noisy exponentials. For noiseless trials, even the heavily biased Bayesian

401 observer (corresponding to a participant with low mathematical education) systematically opted for
402 an exponential function when an exponential was actually present, explaining the absence of a
403 negative bias for noiseless graphs.

404 Finally, we examined whether the humans and the model failed on similar trials. Using the
405 best-fitting penalty value of 12.62, we split noisy exponential trials according to the model's
406 classifications (Figure 5E). When the model correctly classified the function as exponential (25.5%
407 of trials), participants' bias was not significantly different from 0 (mean=-13.1, $p>.05$) and smaller
408 than when the model misclassified it as either quadratic (mean=-34.5; 66.2% of trials) or linear
409 (mean=-49.5; 8.3% of trials). The bias for exponentials was significantly different from the two
410 others (pairwise differences in a generalized mixed effect model; linear-exponential: $t(2960)=-2.62$,
411 $p=.024$; quadratic-exponential: $t(2955)=-2.42$, $p=.041$). Those results suggest that human
412 participants, like the model, are not blind to exponential growth, but that bias and noise lead them
413 to often misinterpret the curve as a linear or quadratic growth.

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Figure 5. Ideal observer model of graph perception and extrapolation. **A:** architecture of the model: for a given trial, the model extrapolates the best-fitting candidate function that minimizes the Bayesian information Criterion (BIC) plus an adjustable penalty for exponentials. **B:** comparison of the distribution of answers to noisy exponentials for humans (left) and for the model with three different values of the exponential penalty (right; numbers indicate the Euclidean distance between the data and model distributions). **C:** average exponential underestimation bias of the model as a function of penalty K. **D:** modelling of participants with high and low mathematical knowledge: Jensen-Shannon Distance as a function of the penalty for the first and last quartile of math level, plus indication of the penalty K minimizing the distance. **E:** splitting of the distribution of responses according to the choice of the ideal observer, both for the model and for the behavioral data.

423 DISCUSSION

424 In a large-scale online study, we studied the factors that modulate the misperception of
425 exponential growth in graphs. Our systematic extrapolation study replicated the exponential growth
426 underestimation bias (Andreassen & Kraus, 1990; Wagenaar & Sagaria, 1975) and extended it to
427 pointing responses. Indeed, although the bias was higher for numerical extrapolations, it was still
428 significantly present when subjects were asked to extrapolate a curve by pointing. This suggests that
429 the bias arises, at least in part, at the stage of perceiving and extrapolating the scattered dots, and
430 not just when the plot is converted into numerical values. This finding converges with prior studies
431 showing that human adults have difficulties in interpolating or extrapolating accelerating functions
432 (Schulz et al., 2017), including quadratics (Ciccione & Dehaene, 2021). However, we discovered that
433 the exponential growth bias is present when the data are noisy, but not when a noiseless
434 exponential curve is displayed. This suggests that participants do not lack an intuitive understanding
435 of accelerating functions, but fail to deploy it properly in the presence of noisy data trends.

436 We confirmed this suggestion by exploring the properties of an ideal-observer model of
437 graph perception and extrapolation. The model is based on the idea that participants have to
438 mentally chose between several candidate functions and that the presence of noise in the data may
439 prevent the selection of the proper curve, thereby leading to a quadratic or even linear
440 extrapolation of exponentials. We could reject an ideal observer model that does not penalize
441 exponentials. However, we found that a biased model, with an additional penalty against
442 exponentials, could fit the data. This analysis suggests that participants had intuitions of exponential
443 growth but were biased against this interpretation and failed to correctly chose an exponential
444 trend in the presence of noise.

445 We also discovered an independent source of bias in human graph extrapolation, which
446 occurred solely for numerical guesses: participants anchored their answers to round integer values

447 and to tens and hundreds. This “rounding” behavior is intuitive and fits with the higher cross-
448 linguistic frequency of number words corresponding to tens (Sigurd, 1988). It may partially arise
449 from an anchoring heuristic (Jacowitz & Kahneman, 1995) elicited by the values displayed on the y-
450 axis.

451 One last finding merits discussion: the overestimation bias for noiseless functions. This
452 aspect contradicts Hutzler et al. (2021), who found an underestimation. However, their stimuli did
453 not depict genuine exponential functions, but exponentials with a temporally decreasing rate; also,
454 their graphs were either so curved that the correct extrapolation fell way above the depicted y axis
455 or too shallow to be perceived as exponential. In our stimuli, we carefully chose the parameters a
456 and b to avoid those problems. Further research will be required to understand why our noiseless
457 exponentials were slightly overestimated. One possibility is that participants visually extended such
458 curves using inappropriate but intuitive geometric operations (Dehaene et al., 2006; Sablé-Meyer
459 et al., 2021). Another possibility is that participants knew that humans underestimate exponentials
460 and overcorrected their responses when they were confident they saw an exponential. Indeed, the
461 experiment was conducted one year after the beginning of the Covid-19 pandemic and most
462 participants could have been aware of the general underestimation in the number of new
463 contaminated people during Covid-19 outbreak.

464 Evidence-based suggestions to improve graph perception

465 Many scientists recently called for an improvement in data visualization and graphical
466 representations as a fundamental step towards a stronger public appropriation of societal,
467 environmental, and health phenomena (Concilio et al., 2021; Dixon et al., 2021; Harold et al., 2016;
468 Murray et al., 2020). The present results lead to the following suggestions to improve the perception
469 of exponentials in scatterplots.

- 470 1. **Improve people's mathematical education.** We found that, the higher the participants' self-
471 evaluated math knowledge, the smaller their exponential underestimation bias (as well as
472 their variability in linear extrapolation). Thus, although correlation is not causation, it seems
473 likely that explicitly educating people to the fast-growing mathematical properties of
474 exponentials could mitigate their bias. Indeed, a short lecture on the exponential growth
475 bias was shown to reduce the size of the underestimation (W. Wagenaar & Sagaria, 1975).
476 Math education correlates with participants' numerosity perception (Ciccione & Dehaene,
477 2020; Halberda et al., 2008, 2012; Piazza et al., 2013), and intervention studies suggest a
478 causal effect on both number acuity (Jordan & Dyson, 2016; Wilson et al., 2009) and intuitive
479 mathematics (Dillon et al., 2017). Recent evidence also suggests that mathematical skills
480 correlate with complex graph understanding (Ludewig et al., 2020). Future studies should
481 confirm our findings with psychophysical experiments in a laboratory context, and test
482 causality through a randomized control trial with an education intervention on graph
483 perception.
- 484 2. **Use pointing rather than numerical responses.** Participants were more precise and less
485 variable when pointing to the extrapolation location, rather than venturing a numerical
486 response. Thus, inviting the reader to carefully locate the extrapolation point, for instance
487 by clicking before making a numerical estimation, may help. The latter suggestion is
488 particularly relevant for interactive visualizations: readers could click on the extrapolated
489 location and the software provides the corresponding number.
- 490 3. **Use a logarithmic scale, but with a high density of labels.** A logarithmic scale makes
491 exponential data look linear, and therefore easier to extrapolate without bias. Our data
492 indicate that log scales have pros and cons: they remove the exponential bias, but lead to an
493 overestimation of numerical responses. The latter effect arises from the difficulty of

understanding which numbers fall in between tick marks on a log scale. As a result, subjects either select the nearest tick, or interpolate linearly between ticks. Those issues could be mitigated by providing a higher density of numerical labels (e.g., not just 10-100-1000 but also the intermediate locations for decades and hundreds). Our study also replicates and extends previous research showing that exponential extrapolations from a log scale are more variable than those from a linear scale (Menge et al., 2018; Romano et al., 2020). However, this is true only if we consider the actual numerical distance from the correct answer (supplementary figure 1) but not if we consider the distance in the space of the graph.

4. **Reduce the noise in the data plot.** One of the most interesting results in our study is that the exponential underestimation bias vanished (and indeed became an overestimation) when we presented a noiseless exponential function rather than noisy data. According to our model, reducing the noise facilitates the choice of the adequate family of functions, and therefore leads to better extrapolations. This suggests that a more accurate human perception of the exponential growth in real data could be obtained by presenting a smoother graph, e.g. the best-fitting exponential curve or a moving average, instead of noisy datapoints.

We end by noting that the last two suggestions not only eliminate the underestimation bias, but lead to overestimation. One should therefore consider the concrete implications of the conveyed message and the cost associated to either type of error. If, for example, the purpose is to alert the readers to the fast-growing progression of a pandemic, overestimation may be less problematic than underestimation. However, overestimating the future number of contaminated people may have adverse effects as well, such as excessive sanitary measures. In sum, the preference for a data visualization over another should be based on a thorough analysis of the represented data and of the message that the graph is supposed to convey. While such decisions

518 are beyond the scope of cognitive psychology, the present study highlights the crucial importance
519 of informing them by psychophysical research on graph perception.

520

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535 Conceptualization: LC, MSM, SD

536 Methodology: LC, MSM, SD

537 Analyses: LC, MSM, SD

538 Visualization: LC, MSM, SD

539 Supervision: SD

540 Bayesian model: MSM, SD

541 Writing – original draft: LC

542 Writing – review & editing: LC, MSM, SD

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544 [Competing interests](#)

545 Authors declare that they have no competing interests.

546

547 [Data availability](#)

548 All data and scripts for the analyses are available on the Open Science Framework at:

549 <https://osf.io/sh8n5/files/>.

550 TABLES

551

Effect	df	F	p	Partial η^2
Response modality	1, 520	.09	.77	.0002
Noise	1, 520	9.32	<.01	.02
Axis scale	1, 520	152.24	<.0001	.23
Response modality:Noise	1, 520	1.53	.22	.003
Response modality:Axis scale	1, 520	167.41	<.0001	.24
Noise:Axis scale	1, 520	.07	.79	.0001
Response modality:Axis scale:Noise	1, 520	.24	0.63	.0005

552

553 **Table 1A.** Repeated measures omnibus ANOVA on extrapolation bias (for “same-graph” conditions).

554

Effect	df	F	p	Partial η^2
Response modality	1, 520	241.11	<.0001	.32
Noise	1, 520	72.12	<.0001	.12
Axis scale	1, 520	9.44	<.01	.02
Response modality:Noise	1, 520	51.80	<.0001	.09
Response modality:Axis scale	1, 520	6.52	.01	.01
Noise:Axis scale	1, 520	.12	.73	.0002
Response modality:Axis scale:Noise	1, 520	<.0001	<.0001	.97

555

556 **Table 1B.** Repeated measures omnibus ANOVA on extrapolation variability (for “same-graph”
557 conditions).

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Effect	df	F	p	Partial η^2
Response modality	1, 520	18.82	<.0001	.03
Noise	1, 520	229.54	<.0001	.31
Axis scale	1, 520	1.02	.31	.002
Response modality:Noise	1, 520	<.0001	.97	<.0001
Response modality:Axis scale	1, 520	105.4	<.0001	.17
Noise:Axis scale	1, 520	285.95	<.0001	.35
Response modality:Axis scale:Noise	1, 520	<.0001	0.96	<.0001

560

561 **Table 2A.** Repeated measures omnibus ANOVA on extrapolation bias (for “same-function”
562 conditions).

563

Effect	df	F	p	Partial η^2
Response modality	1, 520	20.93	<.0001	.04
Noise	1, 520	315.26	<.0001	.38
Axis scale	1, 520	1615.38	<.0001	.76
Response modality:Noise	1, 520	7.06	<.01	.01
Response modality:Axis scale	1, 520	20.06	<.0001	.04
Noise:Axis scale	1, 520	126.55	<.0001	.20
Response modality:Axis scale:Noise	1, 520	.04	<.0001	.83

564

565 **Table 2B.** Repeated measures omnibus ANOVA on extrapolation variability (for “same-function”
566 conditions).

567

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SUPPLEMENTAL MATERIAL

686

Choice of error measure

688 We used the difference in screen units (normalized to the range 0-1000) as an error measure since
689 it offers a common currency for comparing conditions with different axes. Concretely, answers for
690 stimuli presented on a log scale were translated into their corresponding values in screen units.
691 Consider a participant making a numerical guess of 100 for an extrapolation from a log scale. If the
692 correct answer was 1000, the numerical error would be 900. However, this corresponds, in terms
693 of screen units, to the difference between two tick marks, i.e. to the same error that a participant
694 would make if they answered 500 instead of a correct answer of 750 on the linear scale. Thus, in
695 both cases, this example corresponds to an error of 250 screen units. Using screen units is natural
696 since it assigns the same weight to the errors resulting (on a log scale) from answering 1000 instead
697 of 100 and answering 100 instead of 10. For a log scale, this is analogous to calculating the ratio
698 between the given answer and the correct one; however, using error ratios for extrapolations on a
699 linear scale would be problematic since it assigns a different weight to the error resulting from a
700 given answer of 750 instead of 500 and a given answer of 500 instead of 250: in the first case the
701 error ratio is of 1,5 and in the second case it is of 2, although the overestimation bias is, for both
702 cases, of 250 screen units.

703 An alternative error measure would be using the numerical difference between the given
704 and the correct answer. In figure S1, results are plotted in such units. This measure is informative
705 about absolute numerical distance from the correct value, but it makes impossible to quantitatively
706 compare errors for stimuli on different scales. When considering the actual numbers given, answers'
707 variability on a log scale increases relative to the linear scale: this is unsurprising given that, with a
708 log scale, a very small change on screen can result in a very large change in number. However, the

709 exponential underestimation bias vanishes for pointing responses ($t(520)=-0.16$, $p=.87$), and turns
710 into an overestimation for number responses ($t(520)=8.54$, $p<.0001$; figure S1), which is reasonably
711 low given the wide numerical extension of the log scale.

712 Anchoring effects for exponential extrapolations

713 For exponential functions on a linear scale, numerical responses were more likely than pointing
714 responses to correspond to tick values ($672/3126=21\%$ versus $24/3126=1\%$; $\chi^2=676.79$, $df=1$,
715 $p<.0001$), and numerical responses corresponding to tens and hundreds were significantly more
716 frequent than the expected frequency under the null hypothesis that each number had the same
717 probability to be chosen by participants (binomial test, empirical proportion $2645/3126=85\%$,
718 expected proportion $313/3126=10\%$, $p<.0001$). These findings replicate the anchoring effects found
719 for linear functions. Combined, they suggest that for both linear and exponential functions,
720 participants' numerical answers were biased towards round numbers.

721 Influence of mathematical knowledge on linear extrapolations

722 We also tested whether mathematical knowledge modulated the precision of linear extrapolations.
723 Self-evaluated math knowledge did not correlate with participants' bias, for any experimental
724 conditions (all $p>.05$), thus indicating that for linear functions, the bias remained null regardless of
725 mathematical education. However, math knowledge negatively correlated with participants'
726 response variability, confirming that participants with a higher mathematical education are more
727 precise; this was true for both linear ($R=-0.17$, $df=476$, $p<.001$) and log scale ($R=-0.13$, $df=476$,
728 $p<.01$), for both pointing ($R=-0.09$, $df=476$, $p<.05$) and numerical responses ($R=-0.17$, $df=476$,
729 $p<.001$) and for both noiseless ($R=-0.12$, $df=476$, $p<.01$) and noisy functions ($R=-0.2$, $df=476$,
730 $p<.0001$). Crucially, no correlation was found with self-evaluation of participants' first language skills
731 (all related $p>.05$). These findings show that, even for simple linear scatterplots and even when just

732 pointing to an extrapolated data point, participants' precision in responding is modulated by their
733 math knowledge.

734 Influence of other factors on the exponential growth bias

735 The underestimation bias of noisy exponentials did not correlate with the highest degree obtained
736 (in agreement with previous research: Levy & Tasoff, 2016), nor with age or sex. Other than with
737 mathematical knowledge, the bias also correlated with familiarity with graphs and statistical
738 knowledge and with participants' (reported) average math grade at university (all $p < .05$). Again,
739 no correlation was found with the self-evaluated knowledge of first language.

740 Effect of task order

741 As explained in the methods' section, participants either started with the pointing task or the
742 number task. To test for an effect of order on extrapolation performance, we conducted two
743 ANOVAs (one on bias, the other on variability) with the task order as a between-subject factor. For
744 the "same-graph" condition, no main effect of order was found for bias ($F[1,519]=1.04$, partial
745 $\eta^2=.002$, $p=.31$), nor for variability ($F[1,519]=.12$, partial $\eta^2=.0002$, $p=.72$); no interaction effects of
746 order with the other factors were found either (all $p > .05$). For the "same-function" condition, only
747 a small interaction of order with axis scale was found for the bias ($F[1,519]=5.24$, partial $\eta^2=.01$,
748 $p < .05$). In terms of variability, an interaction effect of order with response modality was found
749 ($F[1,519]=13.13$, partial $\eta^2=.02$, $p < .001$): indeed, when participants started with the pointing task,
750 their average variability (61.3) remained relatively stable for the number task (63.1); however,
751 when they started with the number task, the average variability (69.7) shrunk to 56.5 in the
752 pointing task. Plausibly, performing numerical guesses requires a higher level of graphical analysis
753 and might have pushed participants to be more precise in the subsequent pointing task.

Supplementary figure S1. Mean bias and mean standard deviation of extrapolation responses, separately for experimental condition. Same format as figure 2, but the unit of measurement here is the difference between participants' **numerical** answers and the correct responses (in figure 2, we used screen units as measurement). For conditions with a linear scale, the plot therefore corresponds to figure 2. For conditions with a logarithmic scale, the higher bias and standard deviation are due to the much longer numerical interval covered by the y axis scale for log (range: 1-10000) than for linear scales (1-1000).