CITATION: Cheung, C-N., & Lourenco, S. F. (2019, in press). Does $1 + 1 = 2^{\text{nd}}$? The relations between children's understanding of ordinal position and their arithmetic performance. *Journal of Experimental Child Psychology*.

Does $1 + 1 = 2^{\text{nd}}$? The relations between children's understanding of ordinal position and their arithmetic performance

Chi-NGai Cheung and Stella F. Lourenco

Emory University

Author note

Chi-NGai Cheung, Department of Psychology, Emory University, 36 Eagle Row, Atlanta, GA 30322; Department of Psychology, University of South Florida, 4202 E Fowler Ave, Tampa, FL 33620; Jiann-Ping Hsu College of Public Health, Georgia Southern University, email: chingai.cheung@gmail.com. Stella F. Lourenco, Department of Psychology, Emory University, 36 Eagle Row, Atlanta, GA 30322, email: stella.lourenco@emory.edu

Direct correspondence to:

Chi-NGai Cheung (email: chingai.cheung@gmail.com)

Stella F. Lourenco (email: stella.lourenco@emory.edu)

Abstract

The current study examined the relations between 5- and 6-year-olds' understanding of ordinality and their mathematical competence. We focused specifically on 'positional operations', a property of ordinality not contingent on magnitude, in an effort to better understand the unique contributions of position-based ordinality to math development. Our findings revealed that two types of positional operations—the ability to execute representational movement along letter sequences, and the ability to update ordinal positions after item insertion or removal—predicted children's arithmetic performance. Nevertheless, these positional operations did not mediate the relation between magnitude processing (as measured by the acuity of the Approximate Number System) and arithmetic performance. Taken together, these findings suggest a unique role for positional ordinality in math development. We suggest that positional ordinality may aid children in their mental organization of number symbols, which may facilitate solving arithmetic computations and may support the development of novel numerical concepts.

Keywords: Ordinal ability; Approximate Number System (ANS); Mathematics

In increasingly technological societies, there is a growing demand for mathematical expertise. Greater math proficiency increases one's occupational opportunities and is associated with better quality of life (Parsons & Bynner, 2005). It is not especially controversial, then, to advocate for more research designed to better understand mathematical development. Significant strides must be made in our understanding of the cognitive mechanisms that underlie mathematical development if we are to inform theoretical debates about how children acquire different mathematical concepts and how they come to implement different types of computations.

A new and promising area of research focuses on the potential role of children's understanding of ordinality in their mathematical development, specifically arithmetic competence. In particular, studies have found that deficits in ordinal understanding may be linked to developmental dyscalculia, a learning disability that impairs arithmetic understanding (Attout, Salmon, & Majerus, 2015; Rubinsten & Sury, 2011). In typically-developing populations, performance on order judgment tasks, in which participants are required to distinguish between ascending and non-ascending numerical sequences, has been found to predict arithmetic performance in adults (Lyons & Beilock, 2011; Sasanguie, Lyons, De Smedt, & Reynvoet, 2017) and children (Lyons & Ansari, 2015; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Sasanguie & Vos, 2018). Nevertheless, it remains unclear why, and how, ordinal understanding may be related to mathematical development. In the current study, we investigated the potential contributions of ordinality to children's arithmetic understanding at an age when children are developing formal mathematics.

Ordinality as Position

Ordinality is typically conceived as encompassing relative magnitude (Brannon & Van de Walle, 2001). From this perspective, ordinality describes the magnitude relations between items such as numbers (e.g., $4 < 5$) or the magnitude relations among the arrangement of multiple items (e.g., ascending vs. descending numerical order). Magnitude-based ordinality contrasts with ordinality that describes positional relations (Nieder, 2005; Wiese, 2003). More specifically, position-based ordinality encompasses the specific position(s) of individual items within a sequence (e.g., an item in 1st or 2nd position; an item that is before or after another) or the overall arrangement of multiple positions within the sequence (e.g., alphabetical vs. reversed alphabetical order). The current study focuses on the positional property of ordinality, since its development and its potential links to mathematics are not well understood.

During the preschool years, children develop counting expertise (Wynn, 1990), which can be used in contexts that refer to magnitude or ordinal position. In the *magnitude* context, the last number recited in the counting sequence represents the set size, describing the totality (or ‘cardinality’) of the set, rather than a specific item (Gallistel & Gelman, 1978; Wynn, 1990). By contrast, in the *position* context, the last number recited refers specifically to the position of an item within the sequence. Noting the difference between the two types of counting, Colomé and Noël (2012) examined 3- to 5-year-olds’ counting ability in these two contexts. They found that although children’s ability to count for magnitude (e.g., reference to the quantity of the set) was related to their ability to count for position (e.g., reference to the item position), children acquired the ability to count for magnitude prior to counting for position.

In the school years, children are likely to acquire more complex ordinal knowledge in which positional information is manipulated. These “positional operations,” as we refer to them

here, are rule-governed and take different forms. Next, we describe the three types of positional operations examined in the current study.

- 1) Sequence traversal: This operation involves representational movement from one position to another within a sequence. For example, to determine which month comes two months after March, one can recite the month list from March, as if moving an attentional pointer forward in two steps (“April, May”) to the month of May. This type of recitation rests on two abilities: choosing the correct direction of movement (i.e., forward or backward) and tracking the accompanying movement.
- 2) Position reversal: This operation involves a systematic transformation of multiple item positions to implement the reversal of a sequence (e.g., the first item in a list becomes the last item and the last item becomes the first item). For example, if the original order of the sequence is “bunny, star, pencil”, then the reversed sequence is “pencil, star, bunny”. Position reversal requires the understanding that each item in the original sequence must occur in the reversed sequence once and once only. In the previous example, “bunny” would appear at the end of the sequence and not also at the beginning, as before. Position reversal differs from the backward direction of sequence traversal in that the items within the sequence are altered. There is no such manipulation in sequence traversal where the items themselves remain fixed. Moreover, the scope of position reversal always concerns the entire sequence; this is not the case for sequence traversal where traversal may only involve a segment of the sequence.
- 3) Position adjustment: Similar to position reversal, position adjustment involves structural changes to the sequence. However, in contrast to position reversal, this change involves the updating of one or more item positions in response to the insertion or removal of

item(s) in the sequence. That is, after an item is inserted, all items that follow are moved away from the beginning of the sequence by one step, whereas items before the location of insertion are unaffected. Analogous effects occur when an item is removed from the sequence, with the location of change determining the influence of the change. For example, when standing in a security line, if someone were to cut in front of you, your position would move back by one. This is not the case if a person joins the line behind you; your position would be unaffected.

We have described operations that apply to position-based ordinality because of their potential relevance to mathematical understanding. A large portion of mathematics (including arithmetic) involves the manipulation of symbols according to a set of rules. Thus, the ability to perform rule-governed manipulations on positional information may be particularly relevant to our understanding of the potential relations between ordinality and mathematical development. However, no study (to our knowledge) has provided a systematic investigation of these positional operations. A fuller understanding of the development of children's ordinal knowledge will require an examination of such ordinal concepts, as well as their links to arithmetic performance.

Potential Relations between Positional Ordinality and Mathematics

The extant studies on ordinal-math relations have largely utilized tasks involving judgments of numerical order (i.e., ascending vs. non-ascending judgment; e.g., Goffin & Ansari, 2016; Lyons et al., 2014; Rubinsten & Sury, 2011; Sasanguie et al., 2017), which can be solved by one of two strategies (Vos, Sasanguie, Gevers, & Reynvoet, 2017). One strategy is based on a comparison of magnitudes, such as judging the order of 2-3-4 by serially comparing the numerical values of 2 with 3 and 3 with 4. Since larger differences in magnitude give rise to

more efficient discrimination, this magnitude comparison strategy results in a canonical distance effect (e.g., better performance for 2-6-8 as compared to 2-3-4). Numerical order judgments can also be solved by a position strategy based on semantic memory, such as scanning one's memory of the number sequence to determine whether 2 comes before 3 and 3 comes before 4. Since it takes less time to scan or recite items that are closer together than those that are farther apart, this strategy results in a reverse distance effect (e.g., better performance for 2-3-4 as compared to 2-6-8). Both types of distance effects have been found in tasks involving judgments of numerical order (e.g., Lyons & Ansari, 2015; Sasanguie et al., 2017; Turconi, Campbell, & Seron, 2006; Vogel, Remark, & Ansari, 2015; Vos et al., 2017), depending on the stimulus type (e.g., number pair vs. triplet; ordered vs. non-ordered sequences), suggesting that both magnitude- and position-based ordinality may be invoked in these order judgments.

Although magnitude-based sequences may include information about position (e.g., 4 is both *less* than 5 and comes *before* 5 in the count sequence), accumulating evidence suggests that these properties of ordinality are dissociable. Position-based information can exist independent of magnitude, and there are reports of impaired magnitude-based ordinal abilities with relatively preserved positional ordinality (Dehaene & Cohen, 1997; Delazer & Butterworth, 1997) and vice versa (Dehaene & Cohen, 1997; Turconi & Seron, 2002). There are also different patterns of neural activation for the two types of ordinality (Turconi, Jemel, Rossion, & Seron, 2004; Zorzi, Di Bono, & Fias, 2011), and it has been found that they show differential associations with visuospatial processing in adulthood (Cheung & Lourenco, 2016). Importantly, dissociations have also been found in children. For example, preschoolers who understand the positional relations between number words (e.g., ten comes *after* six) may lack an understanding of the magnitude relations between the same numbers (i.e., ten is *greater* than six) (Davidson, Eng, &

Barner, 2012; Le Corre, 2014). Finally, although numerical sequences may include information related to both magnitude and positional properties (e.g., 4 is less than 5 and 4 comes before 5), other sequences such as letters of the alphabet are without the magnitude property, such that position-based ordinality can be specifically isolated. Two questions follow from the existing research. The first question concerns the role of positional ordinality in mathematical development. Given that magnitude- and position-based ordinality constitute different properties of ordinal sequences, it is an open question whether the understanding of positional ordinality is related to math competence, as has been shown for magnitude-based ordinality (e.g., Goffin & Ansari, 2016; Lyons et al., 2014; Rubinsten & Sury, 2011; Sasanguie et al., 2017). Tasks requiring a numerical order judgment may invoke both magnitude- and position-based ordinality, such that they are not especially well suited for testing the specific relation between positional ordinality and early mathematical competence. As discussed above, there are different types of ordinal operations (i.e., sequence traversal, position reversal, and position adjustment), and it is unknown how these ordinal operations develop in children. It is also unknown how position-based ordinality relates to mathematical competence in early development.

The second question concerns the link between the Approximate Number System (ANS), which has been implicated in representing numerical magnitude, and math performance. It has been suggested that the ANS serves as the foundation for formal mathematical abilities (e.g., Feigenson, Dehaene, & Spelke, 2004), though such a view is debated (Fuhs & McNeil, 2013; Halberda, Mazocco, & Feigenson, 2008; Lourenco & Bonny, 2016; Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013; for meta-analysis, see Schneider et al., 2017). In particular, there is debate about the reliability of this relation (Clayton, Gilmore, & Inglis, 2015; DeWind & Brannon, 2016; Smets, Sasanguie, Szűcs, & Reynvoet, 2015), and whether an ANS-math link is

better characterized by a mediating role of ordinality (Lyons & Beilock, 2011). Lyons and Beilock (2011) found that individual differences in ordinality understanding (as measured by numerical order judgments) mediated the influence of the ANS on symbolic arithmetic performance. It has been suggested that ordinality provides a bridge between the ANS and mathematical competence by representing the order of symbolic numbers (e.g., Lyons & Beilock, 2011; Odic & Starr, 2018; Szkudlarek & Brannon, 2017). However, it is unclear whether positional ordinality specifically serves this function, since the combination of magnitude- and position-based ordinality may be necessary. Here we tested whether positional ordinality serves a similar mediating role in the ANS-math link in children.

Current Study

Despite much interest in the cognitive foundations of formal mathematics, we know little about the individual differences that contribute to mathematical development. Few developmental studies have examined children's understanding of positional ordinality. Not only do we know little about the nature of children's understanding of this type of ordinality, but also there is no research (to our knowledge) on the relation between positional ordinality and formal mathematical concepts.

The goals of the current study were threefold. First, we sought to examine 5- and 6-year-olds' understanding of three types of positional ordinality (i.e., sequence traversal, position reversal, and position adjustment). Second, we addressed whether positional ordinality was related to children's arithmetic competence. Third, we tested the potential mediating role of positional ordinality in the link between ANS acuity and mathematical competence in children. To accomplish the three aforementioned goals, we devised three different tasks to assess 5- and 6-year-olds' understanding of positional ordinality. It should be noted that we focused

specifically on children of this age because they have already acquired a basic understanding of position-based ordinality (Colomé & Noël, 2012; Miller et al., 2000) and, thus, may also understand the more complex positional operations. In addition, children of this age have begun learning symbolic arithmetic but there is also development in the conceptual understanding and proficiency of arithmetic computation.

The position-based ordinality tasks administered to children were designed to tap the three types of positional operations described above. In the Sequence Traversal task, children were required to identify a target letter within a letter sequence by moving in a forward or backward direction along the sequence. In the Position Reversal task, we assessed children's ability to reverse an ordered sequence of items. In the Position Adjustment task, children reported the updated ordinal position of a target after a character either joined or left the queue before or after the target character.

To examine the relations between children's understanding of positional ordinality and mathematical competence, we included an arithmetic test that consisted of written addition and subtraction problems. We predicted that if an understanding of positional ordinality was related to children's arithmetic competence, then children who performed better on the ordinal tasks would have greater accuracy on the arithmetic task. We did not predict that the different ordinal tasks would vary in their relations to children's arithmetic competence, since all three positional operations involve the manipulation of information along a sequence, which we have suggested might be relevant to arithmetic computations. However, because these ordinal-math relations have not been previously examined in children of this age range, we considered the possibility that differential relations could exist. We discuss the implications of these differences in the General Discussion.

Importantly, we also assessed a collection of domain-general cognitive abilities (i.e., inhibition, working memory, processing speed, and vocabulary) that have been found to relate to mathematical competence (e.g., Clark et al., 2014; Cragg & Gilmore, 2014; Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013). By including these measures, we were able to evaluate the specificity of the relations between children's understanding of positional operations and arithmetic competence. More specifically, we were able to test the extent to which ordinal-math relations could be explained by other cognitive abilities.

Finally, in an effort to better understand the broader implications of the relations between position-based ordinality and mathematics, we also examined the potential mediating role of positional operations in the link between ANS acuity and arithmetic competence. As noted above, it has been suggested that ordinal understanding plays a mediating role in the ANS-math link. However, it is unclear whether this mediating role extends to position-based ordinality. By assessing children's understanding of positional operations, we shed light on the cognitive factors that contribute to math development at a formative period in childhood.

Method

Participants

Seventy-seven children (38 male; age range: 5.00 to 6.92 years; $M_{\text{age}} = 5.92$ years, $SD_{\text{age}} = 6.28$ months) participated in this study. One additional child was tested, but excluded from statistical analyses for failing to follow instructions. The majority of children were in kindergarten (68%), with the remaining children in preschool (12%), 1st (19%), and 2nd (1%) grade. Although demographic information was not collected systematically, children appeared to come from a range of socioeconomic and ethnic backgrounds, consistent with the demographics of the Atlanta metropolitan community. We recruited participants by contacting caregivers via

phone or email who had previously expressed interest in research studies at our university. All children received a small gift for participating. Informed consent was obtained on behalf of each child by a parent or legal guardian. Experimental procedures were approved by the local ethics committee.

Tasks and Procedure

Participants were tested individually in a university laboratory. All ten tasks were administered in a single experimental session (see Table 1), lasting approximately 90 minutes. Task order was randomized across participants with the constraint that tasks assessing the two main constructs of interest (i.e., positional operations and symbolic arithmetic) were administered prior to the other tasks (i.e., ANS and domain-general tasks; see Supplemental Materials for descriptions of the domain-general tasks).

*Table 1**Tasks Utilized in the Current Study*

Domain and Task	Description of Task Assessment
Positional Operations	
Sequence Traversal	Movement (forward or backward) along a sequence
Position Reversal	Conversion of ordinal positions to reverse the order of a sequence
Position Adjustment	Updating ordinal position after an item is added to or removed from a sequence
Numerical Cognition	
Arithmetic	Addition and subtraction of symbolic numbers
ANS	Precision of magnitude processing for non-symbolic numerosities
Domain General Cognition	
NEPSY-Inhibition	Inhibitory control
WJ-Decision Speed	Processing speed
WJ-Memory for Words	Verbal working memory span
WJ-Picture Recognition	Nonverbal working memory span
WJ-Picture Vocabulary	Vocabulary

Note. NEPSY stands for the Developmental NEuroPSYchological Assessment-2nd Edition (Korkman, Kirk, & Kemp, 2007); WJ is the acronym for the Woodcock-Johnson III Tests of Cognitive Abilities or Achievement (Woodcock, Mather, & McGrew, 2001; Woodcock, Mather, McGrew, & Schrank, 2001).

Ordinal Tasks

Sequence traversal. The Sequence Traversal task was designed to assess children's ability to move forward or backward within a sequence. We used letters, instead of numbers, in this task in order to isolate the effect of sequence traversal from arithmetic, which can be accomplished by counting up or down the numerical sequence. Children were required to perform sequence traversal on a subset of the alphabet sequence, which was presented as a row of cards (each card containing one letter). The distance to be traversed was depicted by a subset of cards in the row (see Figure 1). No number words were used in the task instructions. Only

children who successfully recognized printed letters and could successfully recite the alphabet were included. All children were screened for these abilities by requiring them to identify the letters A to K when printed on individual cards (order randomized) and to recite these letters in the correct order. Four children failed to pass the screening and, consequently, did not participate in this task.

The task began with two practice trials. In the first practice trial, the experimenter lined up the letter cards A to K in a row and explained that the letters were arranged in “the way we say our letters” (i.e., alphabetical order). The experimenter then covered all the cards and removed a subset. Following this presentation, either the leftmost or rightmost card of the remaining cards was revealed as the anchor card. The experimenter then pointed to the covered target card, which was located at the other end of the row, and asked children to name the letter on the card. Children needed to make an inference based on the relative direction (i.e., before/after) and inter-item distance between the anchor and target cards (see Figure 1A). Since the number of covered cards between anchor and target cards corresponded to the inter-item distance between anchor and target letters, children could track the steps by making one-to-one correspondences between the covered cards and the recited letters (e.g., touching a card for each recited letter). Counting was not required. Corrective feedback was provided by uncovering the letters on the covered cards. The same procedure was used on the second practice trial and eight test trials, except that the test trials were computerized. Test trials were generated by crossing four levels of inter-item distance (i.e., 1 to 4; e.g., the inter-item distance of A and B is 1) and two levels of operation direction (i.e., forward vs. backward movement). Half of the test trials involved forward movement, and the other half involved backward movement. Operation direction was specific to the relative positions between anchor and target cards. For example, if

the anchor card was E, and the target card came after the anchor card, then the answer could be determined by reciting the letter sequence forward from the letter E. If the target card came before the anchor card, then one could recite the letters backwards to determine the answer. Performance was measured as children's accuracy on the test trials. Children were given unlimited time to respond.

Figure 1. Illustration of the ordinal tasks. Panel A is an example trial from the Sequence Traversal task. At the beginning of the trial, all letters were visible to illustrate the alphabetical order. The cards were then covered by an experimenter who also then removed a subset. In this example, all letters prior to H were removed. The leftmost card was then revealed (the anchor H in this example) and the experimenter pointed to the target card, asking children to name the letter. Panel B is an example trial from the Position Reversal task. At the beginning of the trial, the top and bottom rows of cards were identical. Then, the experimenter turned over all of the cards in the bottom row. Following this manipulation, the order of the cards in the bottom row was reversed. Finally, the last card in the bottom row was revealed, and the experimenter asked the child to name all of the remaining cards in the bottom row. Panel C is an example trial from the Position Adjustment task. (1) At the beginning of the trial, the experimenter told children the ordinal position of the target animal (indicated by an arrow on the computer screen). (2) Then, two occluders covered all the animals that came before and after the target (size and location of the occluders depended on the ordinal position of the target animal in that particular trial). (3) An animation showed that an animal either joined or left the queue either before or after the target. In this particular trial, an animal that came after the target (i.e., the penguin) left the queue. After the animation, children were asked to report the ordinal position of the target animal.

Position reversal. The Position Reversal task was designed to assess children's understanding of reversal in the context of an ordered sequence. Specifically, children needed to perform a systematic relocation of items to fully reverse the sequence. On each trial, children were presented with two identical rows of cards (see Figure 1B). Paper cards were used in the first practice trial to familiarize children with the task procedure. Subsequent trials were computerized. Because young children do not all understand terms such as "reverse order", we communicated this concept to them via the spatial arrangement of the cards, juxtaposing the original and reversed sequences. No number words were used in the instructions. The top row of cards showed the original order, and this row remained visible throughout the trial, preventing the need for memorizing the sequence order. The bottom row was used to demonstrate the reversal operation and to elicit a response. After explaining to children that the top and bottom rows were identical, the experimenter reversed the order of the bottom row and covered all but the rightmost (last) card of the row. The rightmost card remained uncovered to emphasize the reversal. Children were asked to name the content on the covered cards one by one from right to left. The opposite spatial directions across bottom and top rows were crucial for highlighting the reversal of the sequence while providing a potential scaffold for solving the task.

The task included two practice trials followed by six test trials. Test and practice trials were identical, except that in the test trials, the cards were covered before the reversal action. Trials included letter and picture sequences. We also included numbers as stimuli in this task to examine whether position reversal differed across different types of sequences, in an effort to test for domain-specific features of position reversal. All trials required that children respond on the basis of item position after a sequence was reversed. The six test trials were generated by crossing two levels of sequence length (i.e., 3 vs. 5 cards) and three levels of sequence type (i.e.,

letters, shapes, and numbers). Performance was measured as children's accuracy on the test trials. Children were given unlimited time to respond.

Position adjustment. The Position Adjustment task was designed to assess children's understanding of how to update ordinal position in response to the insertion or removal of an item within a sequence. Specifically, this task required knowing that inserting an item into a sequence changes the rank of all of the items following it, but not the items preceding it. All trials were computerized, and on each trial, children were shown a line of nine animals waiting in front of a cashier (see Figure 1C). The experimenter then indicated to the child the ordinal position of the target animal by counting from the first animal (next to the cashier) to the target animal. Counting was used to label the ordinal position (e.g., the animal "four" in line). Following this demonstration, two occluders appeared and covered all of the animals before and after the target animal, leaving only the target animal visible. The experimenter always confirmed that children knew the ordinal position of the target animal by asking for the number label. This information was repeated if children forgot it. Children were then shown an animation in which an animal either joined or left the line before or after the target animal. Children were asked to report the ordinal position of the target animal. Because all of the animals before or after the target were covered by occluders, children had to infer the answer based on the change shown in the animation.

The task began with two practice trials and was followed by eight test trials. Practice and test trials were identical except that there was corrective feedback during practice. Test trials were generated by crossing the type of operation (insertion or removal of an animal) and location (before or after the target animal). The ordinal position of the target animal (start of the trial)

ranged from three to seven. Performance was measured as children's accuracy on the test trials. Children were given unlimited time to respond.

Numerical Cognition

Arithmetic ability. Individual differences in children's mathematical ability were assessed with an arithmetic task. This task began with two practice questions (i.e., $2 + 1$ and $3 - 1$) in which the experimenter worked with the child to solve the problems using real objects. For example, to illustrate the practice problem of $2 + 1$, the experimenter showed children that there were two coins in a box; the box was then covered and one more coin was added; children were then asked to indicate the number of coins in the box (Jordan, Huttenlocher, & Levine, 1992). Following the practice questions, children completed 18 test trials presented in paper-and-pencil format, all as Arabic numerals. All test trials involved single-digit operands (half addition and half subtraction). No feedback was given on these trials, nor was there a time limit for responding. Performance was measured as children's accuracy on the test trials.

ANS acuity. To assess ANS acuity, children were required to judge which of two displays contained a larger number of items. This task was adapted from existing versions in the literature (Bonny & Lourenco, 2013; Libertus, Feigenson, & Halberda, 2011; Piazza et al., 2010). All trials were administered on computer, each of which was initiated by the experimenter who touched a virtual red star on the computer touchscreen. On each trial, children were presented with two arrays of rectangles that were shown side-by-side onscreen in front of two characters (i.e., Big Bird and Grover). Both arrays were made up of rectangles of different sizes and were presented for a brief period (2 s) to ensure that children's judgments reflected ANS acuity rather than explicit counting (Halberda & Feigenson, 2008). The arrays were then replaced by full body pictures of the characters Big Bird and Grover. Children were asked to

indicate which array was larger in numerosity (i.e., “Who had more boxes?”) by touching the corresponding characters onscreen (side of correct answer counterbalanced across trials).

Five different ratios were used across the 20 test trials (2:1, 4:3, 6:5, 15:13, and 22:20), with numerosities (in each array) ranging from 10 to 39. To ensure that children understood the task, they were given two practice trials with the easier ratios (2:1 and 4:3) and smaller numbers (2 to 4). Corrective feedback was provided on the practice trials, but not the test trials. The non-numerical properties in the arrays were controlled using the procedure of Libertus et al. (2011). In particular, on half the trials, cumulative area was matched across the two arrays and, on the other half, average element size was matched across the two arrays. Performance was measured as children’s accuracy on the test trials. Children were given unlimited time to respond.

Domain general cognition. We also utilized standardized tests to measure individual differences in domain-general cognitive abilities (see Table 1). Descriptions of these tasks are provided in the Supplemental Materials.

Results

Performance on Individual Measures

In this section, we report on children’s performance on each of the Ordinal tasks, followed by their performance on Arithmetic and ANS tasks (for domain-general cognitive tasks, see Supplemental Materials). For Ordinal, Arithmetic, and ANS tasks, we examined mean accuracy across different conditions using analyses of variance (ANOVAs). We include comparisons to chance for the ANS task. However, we do not include chance comparisons for Ordinal or Arithmetic tasks because it was not possible to define chance levels for the Position Adjustment task or the Arithmetic task, given that the possible responses were unlimited. For

Sequence Traversal and Position Reversal tasks, chance levels were either too small to provide meaningful comparisons or could not be straightforwardly determined¹.

Ordinal measures.

Sequence Traversal task. Children's accuracy ($M = 65.41\%$, $SD = 19.15\%$) on this task was analyzed using a mixed-factor ANOVA, with direction (forward or backward) as the within-subjects factor and age (5- or 6-year-olds) as the between-subjects factor. There were significant main effects of direction, $F(1, 71) = 84.65$, $MSE = 0.66$, $p < .001$, $\eta_p^2 = .54$, and age, $F(1, 71) = 6.77$, $MSE = 1.09$, $p = .011$, $\eta_p^2 = .09$, but no significant interaction between these variables ($p > .9$). Children performed better on forward ($M = 80.82\%$, $SD = 23.76\%$) than backward trials ($M = 50\%$, $SD = 23.94\%$), and 6-year-olds ($M = 70.95\%$, $SD = 15.61\%$) performed better than 5-year-olds ($M = 59.72\%$, $SD = 20.94\%$). These findings are consistent with previous work showing that reciting numbers backwards is more difficult than reciting numbers in a forward direction (Fuson, 1988). Thus, as has been shown for numerical sequences, children have greater difficulty traversing sequences of letters in backward than forward direction, an effect observed in both 5- and 6-year-olds, though there was improvement in this ability across this age range.

One might ask whether the performance difference for the forward and backward direction arose from a conceptual failure in children's understanding of the correct direction for a given problem, or whether it was an execution problem in implementing the correct operation. To answer this question, we analyzed the direction of children's responses. For example, if both the correct answer (e.g., F) and the child's response (e.g., H) came after the anchor letter (e.g.,

¹ The Sequence Traversal task was not a forced-choice task; children could respond with any letter (i.e., A to Z). Chance in this case was 0.038 (assuming an equal likelihood of responding with each letter), a value too small to be meaningful when compared to children's performance. Determination of chance in the Position Reversal task was not straightforward. Within a trial, children had to indicate the content on all of the covered cards. The choice of each card was not independent, since a given choice could affect a subsequent choice, making it difficult to estimate chance performance.

B), then the response was considered correct in this analysis, even though it was not an exact match. Among the forward trials, more than half of 5- and 6-year-olds responded correctly on all trials (5-year-olds: 100%; 6-year-olds: 94.59%; binomial test: $ps < .004$). For backward trials, more than half of 6-year-olds (75.00%; binomial test: $p < .001$), but not 5-year-olds (55.56%, binomial test: $p > .6$), responded in the correct direction. Together, these findings suggest that 5-year-olds performed worse in backward than forward trials because they failed to choose the correct representational direction in the backward trials. That is, they had difficulty with a key component of this positional understanding: direction. Nevertheless, children gradually acquired knowledge about backward representational movement, such that, by 6 years of age, children choose the correct direction for solving sequence traversal problems.

Position Reversal task. Children's accuracy ($M = 67.97\%$, $SD = 26.98\%$) on this task was analyzed using a mixed-factor ANOVA, with sequence type (letter, shape, or number sequences) and sequence length (3- or 5-items) as the within-subjects factors, and age (5- or 6-year-olds) as the between-subjects factor. There were significant main effects of sequence length, $F(1, 75) = 24.75$, $MSE = 0.19$, $p < .001$, $\eta_p^2 = .25$, and age, $F(1, 75) = 6.15$, $MSE = 0.41$, $p = .015$, $\eta_p^2 = .08$. Children performed better with 3-item ($M = 78.14\%$, $SD = 28.00\%$) than 5-item sequences ($M = 57.79\%$, $SD = 36.36\%$), and 6-year-olds ($M = 75.44\%$, $SD = 25.39\%$) performed better than 5-year-olds ($M = 60.68\%$, $SD = 26.80\%$). No other effects reached statistical significance ($ps > .05$).

To better understand how children performed on this task, we asked whether children understood a critical rule—namely, that every item in the original sequence could occur only once in the reversed sequence. An analysis of the number of children who consistently followed this rule on all trials revealed that more than half of the 5-year-olds were consistent for 3-item

sequences (69.23%, binomial test: $p = .024$), but not for 5-item sequences (46.15%, binomial test: $p > .7$). By contrast, more than half of the 6-year-olds consistently followed this rule for both sequences (3-item: 81.58%, binomial test: $p < .001$; 5-item: 73.68%, binomial test: $p = .005$). Thus, whereas 5-year-olds were only capable of consistently following the “every item can occur only once” rule in the shorter sequences, 6-year-olds generalized this principle of position reversal to longer sequences.

Position Adjustment task. Children’s accuracy ($M = 57.79\%$, $SD = 22.12\%$) on this task was analyzed using a mixed-factor ANOVA, with operation (insertion or removal) and location (before or after the target) as the within-subjects factors, and age (5- or 6-year-olds) as the between-subjects factor. This analysis revealed significant main effects of location, $F(1, 75) = 5.61$, $MSE = 1.99$, $p = .020$, $\eta_p^2 = .07$, and age, $F(1, 75) = 7.42$, $MSE = 0.72$, $p = .008$, $\eta_p^2 = .09$. Children’s performance was better when changes occurred before the target ($M = 67.21\%$, $SD = 40.00\%$) than after the target ($M = 48.38\%$, $SD = 44.12\%$), and 6-year-olds ($M = 64.47\%$, $SD = 22.80\%$) performed better than 5-year-olds ($M = 51.28\%$, $SD = 19.62\%$). No other effects reached statistical significance ($ps > .06$). These results suggest that children understood how changes before the target affected the target’s ordinal position prior to understanding how to handle changes occurring after the target, with a better understanding of this difference by older than younger children.

Non-ordinal measures.

Arithmetic task. Children’s accuracy on this task ($M = 69.70\%$, $SD = 29.21\%$) was analyzed using a mixed-factor ANOVA, with question type (addition or subtraction) as the within-subjects factor, and age (5- or 6-year-olds) as the between-subjects factor. There was a significant main effect of age, $F(1, 75) = 21.71$, $MSE = 10.86$, $p < .001$, $\eta_p^2 = .22$, such that 6-year-

olds ($M = 83.63\%$, $SD = 21.34\%$) performed better than 5-year-olds ($M = 56.13\%$, $SD = 29.67\%$), as would be expected given known developmental improvement in arithmetic competence over this age range. Neither the main effect of question type, nor its interaction with age was significant ($ps > .5$). Children of both ages performed comparably on addition and subtraction problems. Although other research has found that children tend to perform better on addition than subtraction problems (e.g., Das, LeFevre, & Penner-Wilger, 2010), this effect may depend on the size of the numerical values implicated in the problems. Here, the more difficult addition problems involved double digit answers, whereas subtraction always involved single digit answers, which may have made the two types of problems comparable in difficulty.

ANS task. Children's accuracy on this task ($M = 75.78\%$, $SD = 11.62\%$) was analyzed using a mixed-factor ANOVA, with ratio (2:1, 4:3, 6:5, 15:13, and 22:20) and control type (matched cumulative area vs. matched average element size) as the within-subjects factors, and age (5- or 6-year-olds) as the between-subjects factor. As expected, we found a significant ratio effect, $F(3.287, 246.49) = 39.91$ (Greenhouse-Geisser corrected), $MSE = 0.43$, $p < .001$, $\eta_p^2 = .35$ (linear trend analysis: $F[1, 75] = 162.04$, $p < .001$, $\eta_p^2 = .68$), consistent with extant findings showing that magnitude precision follows Weber's law (e.g., Dehaene, 2003; Halberda & Feigenson, 2008). Performance was significantly above the chance level of 50% at all ratios ($ps < .001$), except for the most difficult ratio (22:20, $ps > .07$), for children of both age groups. There was also a significant effect of control type, $F(1, 75) = 16.03$, $MSE = 0.26$, $p < .001$, $\eta_p^2 = .18$, such that children's performance was better for trials with matched element size ($M = 79.48\%$, $SD = 12.7\%$) than cumulative area ($M = 72.1\%$, $SD = 15.5\%$), consistent with other research showing that the latter trial type is more challenging (Gilmore et al., 2013). No other effects reached statistical significance ($ps > .09$).

Relations between Children's Ordinal Abilities and their Arithmetic Performance

We used regression analyses to examine the extent to which children's understanding of the different positional operations related to their arithmetic performance. We also conducted mediation analyses to assess the potential role that children's ordinal abilities might have on the link between ANS acuity and arithmetic competence. Adjusted scores for each task, calculated without the lowest scoring item, were used in these analyses to ensure sufficient reliability (see Table 2 for reliabilities)². Children who failed to complete all of the tasks were excluded from these analyses. To ensure that these analyses were not unduly influenced by outliers, cases in which any score deviated from the task mean by 3 SDs were also excluded ($N = 68$).

Performance on Position Adjustment, NEPSY-Inhibition, WJ-Decision Speed, WJ-Memory of Words, WJ-Picture Recognition, and WJ-Picture Vocabulary tasks had acceptable skewness and kurtosis (i.e., within 2 SEs; Tabachnick & Fidell, 1996). However, this was not the case for the Sequence Traversal, Position Reversal, Arithmetic, and ANS tasks. We corrected less than acceptable skewness and/or kurtosis by applying a Rank-based Inverse Normal (RIN) transformation (Bishara & Hittner, 2012) to these tasks (skewness and kurtosis were acceptable for all tasks post-transformation).

² The worst performing item was removed from Ordinal, Arithmetic, and ANS tasks to reduce noise associated with random responding. No items were removed from the general cognitive tasks, which were all standardized measures with acceptable reliabilities. Note that the pattern of results remains the same whether the analyses involve adjusted or unadjusted scores.

*Table 2**Task Reliabilities*

Task Name	Reliability
Sequence Traversal ^a	.64
Position Reversal ^a	.60
Position Adjustment ^a	.73
Arithmetic ^a	.87
ANS ^a	.61
NEPSY-Inhibition ^b	.82
WJ-Decision Speed ^b	.87
WJ-Memory for Words ^b	.78
WJ-Picture Recognition ^b	.72
WJ Picture Vocabulary ^b	.81

^aSplit-half correlations (Spearman-brown formula) computed on adjusted scores.

^bValues from respective manuals of standardized tests.

Children's accuracy on the Arithmetic task was regressed on the three ordinal tasks (i.e., Sequence Traversal, Position Reversal and Position Adjustment). To ensure that the effects were specific to children's understanding of positional ordinality, we also entered into the model age and their performance on the general cognitive tasks (i.e., NEPSY-Inhibition, WJ-Decision Speed, WJ-Memory for Words, WJ-Picture Recognition, and WJ-Picture Vocabulary) and ANS task. Children's performance on both the Sequence Traversal and Position Adjustment tasks were significant predictors of their arithmetic scores, but their performance on the Position Reversal task was not (see Table 3). Moreover, hierarchical regression analyses revealed that Sequence Traversal and Position Adjustment performance explained unique variance when entered as the last step of the analysis (see Table 4). To summarize, children's understanding of sequence traversal and position adjustment were predictive of their performance on the Arithmetic task, even when accounting for other cognitive abilities and ANS acuity, but their understanding of positional reversal was not.

Table 3

Results of the Multiple Regression Analysis Predicting Children's Performance on the Arithmetic Test

Predictor	β	t	p	r_{partial}
Age	.419	4.58	< .001	.518
Sequence Traversal	.206	2.37	.021	.299
Position Reversal	.048	.515	.609	.068
Position Adjustment	.243	2.56	.013	.321
ANS	.197	2.19	.03	.278
NEPSY Inhibition	-.001	-.006	.995	-.001
WJ-Decision Speed	> -.001	-.001	.999	> -.001
WJ-Memory for Words	.154	1.79	.079	.231
WJ-Picture Recognition	-.196	-2.34	.023	-.296
WJ-Picture Vocabulary	.132	1.43	.158	.186

Note. Partial correlations reported in the right-most column were calculated by controlling for all other predictors in the model.

Table 4

Results of Hierarchical Regression Analyses Predicting Children's Performance on the Arithmetic Test

		ΔR^2	F	p
Step 1	Age	.31	30.21	<.001
Step 2	General Cognitive tests	.18	4.34	.002
Step 3	ANS	.05	5.96	.018
Step 4	<i>Non-target ordinal tasks</i>			
	Position Reversal & Position Adjustment	.05	3.74	.030
	Sequence Traversal & Position Adjustment	.09	6.79	.002
	Sequence Traversal & Position Reversal	.05	3.25	.046
Step 5	<i>Target ordinal tasks</i>			
	Sequence Traversal	.04	5.60	.021
	Position Reversal	<.01	.27	.609
	Position Adjustment	.04	6.54	.013

Note. Full model: $R^2 = .63$; $F(10, 57) = 9.65$, $p < .001$.

In Step 4, the different rows represent the order in which the ordinal tasks were entered into the analysis (excluding the target ordinal task in the corresponding row in Step 5). In Step 5, the corresponding rows in the next step of the analysis are reported (excluding the non-target ordinal tasks in the corresponding row in Step 4). For example, the first row of Steps 4 and 5 together represent entering Position Reversal and Position Adjustment tasks in Step 4 and entering the Sequence Traversal task in Step 5.

We did not find significant correlations among the three types of positional operations (see Table 5 for zero-order and age-controlled correlations), which suggests that these abilities represent dissociable ordinal processes. Thus, even within the domain of positional ordinality, there may be distinct abilities, as measured by the three different tasks. Moreover, and consistent with possible dissociations, there were differences in the cognitive abilities that correlated with each of the positional operations (see Table 5). Whereas children's performance on the Sequence Traversal task was significantly correlated with their performance on the WJ-Decision Speed task, their performance on Position Reversal was significantly correlated with NEPSY-Inhibition, and their performance on Position Adjustment was significantly correlated with WJ-Picture Vocabulary and WJ-Decision Speed tasks. These findings suggest that different mechanisms may support performance on different Ordinal tasks (see General Discussion for further discussion) and, together with the regression analyses above, suggest both commonalities and differences in the relations between children's understanding of positional operations and their arithmetic competence.

Does positional ordinality mediate the relation between the ANS and symbolic arithmetic? Children's ANS acuity was a significant predictor of arithmetic performance, as revealed by a multiple regression analysis (see Table 3). In subsequent analyses, we probed whether this link was mediated by children's ordinal understanding, as has been found in adults (Lyons & Beilock, 2011). We conducted a mediation analysis to test whether the three types of positional operations mediated the influence of ANS acuity on arithmetic ability (see Figure 2). Although not all ordinal tasks were significantly correlated with ANS or arithmetic tasks (see Table 5), the presence or absence of a mediation effect should be verified with a direct statistical test, since correlations provide only indirect evidence for a potential mediation effect. Thus,

following recent recommendations (Hayes, 2013), and an a priori mediation-by-design analysis plan (Fairchild & McDaniel, 2017), all three tasks of positional operations were included in the mediation analyses. These analyses were conducted using the bootstrapping procedure implemented by Hayes (2013) in SPSS. Confidence intervals (CIs) were calculated for each indirect effect using bootstrapping (10,000 iterations). As in the analyses above, age (in months) and children's performance on the general cognitive tasks were included as control variables. These analyses revealed a significant direct effect of ANS acuity on arithmetic ability, but no evidence of significant mediation for any type of positional operation (see Figure 2). More specifically, the value of zero was included in the 95% CIs of all three indirect effects. Thus, these analyses failed to provide support for mediating effects of positional operations on the relation between children's ANS acuity and arithmetic performance. Taken together, our findings suggest that an understanding of specific positional operations (i.e., sequence traversal and position adjustment) and ANS acuity may make unique contributions to children's symbolic arithmetic performance.

Table 5

Zero Order and Age-controlled Pearson Correlations Between Tasks (N = 68)

	1	2	3	4	5	6	7	8	9	10
1. Sequence Traversal	—	.124	.150	.369**	.191	.132	.274*	.142	.045	.100
2. Position Reversal	.161	—	.187	.234	.100	.378**	.207	.096	-.071	.237
3. Position Adjustment	.202	.243*	—	.425**	.228	.099	.245*	.140	.162	.368**
4. Arithmetic task	.409**	.310**	.514**	—	.387**	.192	.243**	.334*	-.107	.368**
5. ANS	.193	.104	.225	.336**	—	.252*	.276*	.211	.169	.220
6. NEPSY-Inhibition	.109	.346**	.061	.102	.247*	—	.192	.121	-.035	.133
7. WJ-Decision Speed	.250*	.181	.201	.149	.272*	.199	—	.043	.059	.233
8. WJ-Word Memory	.126	.079	.110	.238	.208	.127	.049	—	.086	.257*
9. WJ-Picture Recognition	.035	-.080	.137	-.117	.168	-.030	.064	.089	—	.130
10. WJ-Picture Vocabulary	.108	.242*	.364**	.334**	.221	.127	.227	.253*	.127	—

Note. The values above the diagonal are age-controlled correlations between tasks. The values below the diagonal are the zero order correlations.

* $p < .05$, ** $p < .01$

Figure 2. A summary of the mediation model tested. Control variables are listed in the dotted box. The analysis found a significant direct effect of ANS acuity on arithmetic ability (c') but there was no evidence of indirect effects (ab), suggesting no mediation by any of the positional ordinality variables.

General Discussion

Accumulating evidence suggests an important relation between an individual's understanding of ordinality and their mathematical ability (Goffin & Ansari, 2016; Lyons et al., 2014; Rubinsten & Sury, 2011). Despite this evidence, there are open questions about the nature of this relation and the developmental origins. Extant studies on this topic have tended to overlook important nuances that characterize different types of ordinality, namely magnitude- versus position-based ordinality, and the types of positional operations. Here, we asked about the relations between children's understanding of different positional operations and early arithmetic competence. We also asked about the potential mediating role of ordinal understanding in the relation between ANS precision and math performance in children.

Our findings revealed that two types of positional ordinality (sequence traversal and position adjustment) predicted arithmetic understanding in 5- and 6-year-olds, providing evidence for ordinal-math associations that are not likely explained by magnitude processes common to ordinal and mathematical competencies. Our findings also revealed that, contrary to previous findings obtained with numerical order judgments in adults, positional ordinality may not serve as a mediator between ANS acuity and arithmetic competence in childhood. Taken together with previous findings, the current research suggests that positional ordinality, like magnitude-based ordinality, is predictive of arithmetic competence, but that these different properties of ordinality may play distinct roles in mathematical development.

The Development of Positional Ordinality

One type of positional operation examined here was sequence traversal, which we described as the ability to execute representational movement along a sequence. We found that children first acquire sequence traversal for the forward direction, with an understanding of the

distinction between forward and backward movement emerging later. Another type of positional operation was position reversal, described as the conversion of ordinal positions into a reversed sequence and premised on the rule that each item can occur only once in the reversed sequence. Our data suggest that knowledge of this rule first emerges for short sequences and then generalizes to longer sequences, with children able to consistently apply this rule to sequences as long as five items. The third positional operation was position adjustment, which we have described as the updating of ordinal position after an item is inserted, or removed from, a sequence. A key component of understanding this positional operation is that only items that come after the inserted/removed item should be updated. We found that children understood how to update changed items prior to understanding when items should remain in the same position. Taken together, the current study provides insight into 5- and 6-year-olds' ordinal understanding, specifically positional operations.

Perhaps a surprising result of the current study was that children's performance on the different positional operations tasks were not correlated with one another. We described these measures as subtypes of position-based ordinality that differ from magnitude-based ordinality. It is possible, however, that these types of positional operations are also dissociable. The correlations to different cognitive tasks would seem consistent with this possibility. Inhibitory control was correlated with children's performance on the Position Reversal task. It is possible that position reversal requires inhibition because memory for the original order may cause interference during the reversal process. The association between vocabulary and Position Adjustment suggests that language, perhaps especially words for cardinal versus ordinal number (e.g., "one" vs. "first"), may play a role in differentiating the impact of magnitude- versus positional-based ordinality of insertion or removal on a sequence. Speed of processing abstract

concepts was associated with position adjustment and sequence traversal. Such abstract thinking might aid in distinguishing forward versus backward direction in sequence traversal, as well as changes before versus after in position adjustment. Although we have speculated about the possible relations between children's understanding of positional ordinality and their general cognitive abilities, we urge caution in over-interpreting these correlations, especially since only age was controlled for in these analyses. More research will be needed to understand the potential dissociations between the different types of positional operations.

The Role of Positional Operations in Symbolic Arithmetic

Consistent with our hypotheses, we found that sequence traversal and position adjustment were both predictive of children's arithmetic performance, even when controlling for children's age and their performance on other cognitive tasks. However, we found no evidence to support the proposed relation between position reversal and arithmetic performance. We discuss these findings below.

Previous studies have demonstrated that the computation of addition and subtraction may involve a traversal of the count sequence—that is, reciting number words forward or backward in steps corresponding to the operands (e.g., Fuson, 1988). Our study extends this literature by demonstrating that children's accuracy in traversing the letter sequence is predictive of their success in solving arithmetic problems and, thus, suggests a more general relation between the understanding of ordinality and arithmetic.

The current study also found a significant association between children's ability to engage in position adjustment and their arithmetic understanding. Both position adjustment and arithmetic describe how a sequence changes after items are added or removed. However, position adjustment, as measured by our Position Adjustment task, corresponds to changes in

item position, whereas arithmetic involves changes in magnitude. The difference between the two types of changes lies in the role of location. Although the location of insertion or removal defines how item position should be altered, it has no effect on magnitude. Magnitude always increases after insertion and decreases after removal regardless of the location of insertion or removal. To succeed in the Position Adjustment task, children must understand the difference between changes in position and magnitude, and apply the appropriate set of rules to the task. The ability to appreciate positional relations, independent of magnitude, is crucial for a mature understanding of the symbolic number system. For example, a concept such as parity (i.e., odd vs. even) captures an important distinction between position and magnitude. In the sequences of odd/even numbers, the positions of the constituent numbers are different from that of the count list (e.g., 3 is the third word in the count list but it comes second in the list of odd numbers), though their numerical values remain the same. In other words, the odd/even number sequences break the direct correspondence between quantity and position (i.e., the Nth word represents N quantity) — a critical feature of the count list. The ability to differentiate position and magnitude information enables one to appreciate the differences and similarities between different types of numerical sequences. Such ability may also enable children to deploy position and magnitude concepts with flexibility. Alternatively, children who have better conceptual understanding of arithmetic may be in a better position to appreciate how manipulations of magnitude in arithmetic differ from position adjustment, and, thus, may perform better on both tasks.

The current study does not provide support for a relation between position reversal and arithmetic. Children's understanding of position reversal, as measured by the Position Reversal task, did not predict their arithmetic ability, unlike the other positional operations assessed in this study. What might account for the lack of an effect? One possibility is that an understanding of

position reversal has no bearing on the development of simple arithmetic. The reason for this could be that when addition or subtraction is solved by counting, children tend to rely on representational movement along the count list. Information about the distance to be traversed is critical for the operation and it is specified by the problem (i.e., addend or subtrahend). However, position reversal does not involve a tracking of movement. Information about distance (i.e., how far each item should be moved from the original position) is not necessary for the operation (e.g., the first item becomes the last item regardless of distance). Yet it remains an open question whether position reversal may be relevant to other mathematical concepts, such as the understanding of negative numbers, which can be considered as a complete reversal of the positive numbers. Future research would do well to examine the potential relevance of different positional operations on different mathematical concepts.

The relations among ANS, arithmetic, and positional ordinality. The current study also provides insight into the much studied relation between the ANS and mathematics (Schneider et al., 2017). Although previous work has suggested a mediating role for ordinality in this relation (Lyons & Beilock, 2011), precisely how ordinality might mediate such a relation remains unclear. A potential explanation for this mediation is that the magnitude property of ordinality is linked with the ANS on the one hand, and its positional property is linked with symbolic math on the other hand, such that ordinality forms a bridge that enables the two systems to work together to support mathematical thinking. According to this view, positional ordinality alone may not be sufficient to mediate such a relation, since magnitude-based processing would also be necessary. Indeed, the current study found that our tasks, which we have argued targeted children's understanding of positional ordinality, did not mediate the relation between ANS acuity and arithmetic competence in children, even though both ANS

acuity and the understanding of positional ordinality were predictive of children's performance on an arithmetic test.

It is important to note that Lyons et al. (2014) found that although judgments of numerical order were a weak predictor of math ability in 1st- and 2nd- graders, this relation increased with age. Similarly, Sasanguie and Vos (2018) found that numerical order judgments mediated the relation between symbolic number comparisons and arithmetic in older children but not younger children. Given these findings, it is possible that positional ordinality may mediate the link between ANS acuity and mathematical understanding in older children. If the effects are dependent on age, then mediation of positional ordinality could emerge later in development. However, another possibility is that the mediation effect is specific to ordinality that incorporates magnitude information (e.g., judgments of numerical order), in which case there may be differences between measures that capture the different properties of ordinality.

Yet another possibility is that a mediation effect of ordinality may be specific to arithmetic fluency. Lyons and Beilock (2011) assessed arithmetic ability with a timed arithmetic task, in which fluency is beneficial because the quick retrieval of arithmetic facts is generally faster than computation (Siegler, 1987). If speed turns out to be an important element for the mediation effect, then our tasks, in which children were given unlimited time to respond, would not capture such a relation.

Positional ordinality and working memory. In the current study, we were also able to disentangle the effect of children's understanding of ordinality from that of working memory (WM) on their arithmetic competence. Previous studies have shown that higher WM capacity is related to higher mathematical achievement (e.g., Friso-van den Bos et al., 2013; Geary, 2011; Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout, 2013). Moreover, recent studies

have suggested that it is WM for order information (i.e., order WM, e.g., memorizing the order of a random letter sequence over a short period of time), not WM for the identity of items (e.g., memorizing which letters constitute the sequence), that predicts mathematical performance (Attout & Majerus, 2015; Attout, Noël, & Majerus, 2014; De Visscher, Szmalec, Van Der Linden, & Noël, 2015). These findings raise the question of whether the association between ordinality and math is the result of individual differences in order WM. We would argue that although ordinal reasoning requires WM resources, ordinality and order WM are not interchangeable. An understanding of ordinality involves more than memorizing order; it encompasses a collection of concepts and rules that govern how ordinal information should be manipulated. By contrast, order WM is specific to the maintenance of order information over a short period of time (Attout et al., 2014). When designing the tasks in the current study, we made our best efforts to ensure that the tasks assessed children's conceptual understanding and their ability to execute the positional operations, not simply their short-term retention of order information. We deliberately minimized the demands on order WM by drawing on the long-term memory of alphabetical order in the Sequence Traversal task and by keeping the original order in full view throughout the trials in the Positional Reversal task. In the Position Adjustment task, the demand on short-term retention was minimal since children only needed to remember the rank of one target item. More importantly, our analyses indicated that children's performance on Sequence Traversal and Positional Adjustment tasks were significant predictors of their arithmetic competence even after controlling for individual differences in WM. These results suggest that the association between children's understanding of positional ordinality and mathematical competence is not likely reducible to the effect of WM.

Limitations

We have argued that position-based ordinality, particularly sequence traversal and position adjustment, is related to arithmetic competence in children. We have also suggested that position- and magnitude-based ordinality may play unique roles in mathematical development. However, one might question whether the tasks in the present study were truly independent of magnitude processing. Although we noted that the Sequence Traversal task did not involve number words and that numerical processing was not required, one could still ask whether children solved this task by counting the inter-item distance between the target and anchor letters. We believe this is unlikely, at least for Sequence Traversal task, because it would be difficult to recite letters and perform counting simultaneously, and we would emphasize that counting can be used for determining a quantity (i.e., magnitude) or an ordinal position, depending on the counting principle adopted for the task (Colomé & Noël, 2012). However, we acknowledge that the potential involvement of magnitude processing cannot be ruled out definitively, and thus, more research with these, and other, tasks will be needed. Relatedly, we did not compare children's performance on position- and magnitude-based ordinal tasks (e.g., numerical order judgments). Future research designed to test how these different tasks relate to children's arithmetic performance will be crucial in isolating how these properties of ordinal understanding contribute to mathematical competence.

Another limitation of the current research was the relatively narrow age range of participants. Given other research showing developmental change (e.g., Lyons et al., 2014; Sasanguie & Vos, 2018), an important consideration for further research will be whether the relations between the different positional operations and arithmetic competence change over time as children's ordinal knowledge continues to develop.

Conclusion

The current study sheds light on our understanding of ordinal-math associations in children. This was achieved by highlighting the distinction between position- and magnitude-based ordinality; revealing 5- and 6-year-olds' knowledge and limitations of positional operations; and pinpointing the specific types of positional ordinality that are associated with an early understanding of arithmetic. These findings are particularly relevant to children who have just begun learning arithmetic, as they may provide potentially useful information for designing effective teaching materials for children learning mathematics.

Acknowledgments

The authors thank Lijia Wang and Jenna Sung for assistance with data collection. This study was supported by Laney Graduate School Professional Development Support Funds from Emory University.

References

- Attout, L., & Majerus, S. (2015). Working memory deficits in developmental dyscalculia: The importance of serial order. *Child Neuropsychology*, 21(4), 432-450.
doi:10.1080/09297049.2014.922170
- Attout, L., Noël, M.-P., & Majerus, S. (2014). The relationship between working memory for serial order and numerical development: A longitudinal study. *Developmental Psychology*, 50(6), 1667-1679. doi:10.1037/a0036496
- Attout, L., Salmon, E., & Majerus, S. (2015). Working Memory for Serial Order Is Dysfunctional in Adults With a History of Developmental Dyscalculia: Evidence From Behavioral and Neuroimaging Data. *Developmental Neuropsychology*, 40(4), 230-247.
doi:10.1080/87565641.2015.1036993
- Bonny, J. W., & Lourenco, S. F. (2013). The approximate number system and its relation to early math achievement: Evidence from the preschool years. *Journal of Experimental Child Psychology*, 114(3), 375-388. doi:10.1016/j.jecp.2012.09.015
- Brannon, E. M., & Van de Walle, G. A. (2001). The Development of Ordinal Numerical Competence in Young Children. *Cognitive Psychology*, 43(1), 53-81.
doi:10.1006/cogp.2001.0756
- Cheung, C.-N., & Lourenco, S. F. (2016). The associations between space and order in numerical and non-numerical sequences. *Consciousness and Cognition*, 45, 124-134.
doi:<http://dx.doi.org/10.1016/j.concog.2016.08.013>
- Clark, C. A., Nelson, J. M., Garza, J., Sheffield, T. D., Wiebe, S. A., & Espy, K. A. (2014). Gaining control: changing relations between executive control and processing speed and

their relevance for mathematics achievement over course of the preschool period.

Frontiers in Psychology, 5, 107. doi:10.3389/fpsyg.2014.00107

Clayton, S., Gilmore, C., & Inglis, M. (2015). Dot comparison stimuli are not all alike: the effect of different visual controls on ANS measurement. *Acta Psychol (Amst)*, 161, 177-184. doi:10.1016/j.actpsy.2015.09.007

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education*, 3(2), 63-68. doi:10.1016/j.tine.2013.12.001

Das, R., LeFevre, J. A., & Penner-Wilger, M. (2010). Negative numbers in simple arithmetic. *Quarterly Journal of Experimental Psychology*, 63, 1943-1952. doi:10.1080/17470210903564359

Davidson, K., Eng, K., & Barner, D. (2012). Does learning to count involve a semantic induction? *Cognition*, 123(1), 162-173. doi:10.1016/j.cognition.2011.12.013

De Visscher, A., Szmalec, A., Van Der Linden, L., & Noël, M.-P. (2015). Serial-order learning impairment and hypersensitivity-to-interference in dyscalculia. *Cognition*, 144, 38-48.

Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex: A Journal Devoted to the Study of the Nervous System and Behavior*, 33(2), 219-250. doi:10.1016/s0010-9452(08)70002-9

Delazer, M., & Butterworth, B. (1997). A Dissociation of Number Meanings. *Cognitive Neuropsychology*, 14(4), 613. doi:10.1080/026432997381501

- DeWind, N. K., & Brannon, E. M. (2016). Significant Inter-Test Reliability across Approximate Number System Assessments. *Frontiers in Psychology*, 7, 310.
doi:10.3389/fpsyg.2016.00310
- Fairchild, A. J., & McDaniel, H. L. (2017). Best (but oft-forgotten) practices: mediation analysis. *The American Journal of Clinical Nutrition*, 105(6), 1259-1271.
doi:10.3945/ajcn.117.152546 %J The American Journal of Clinical Nutrition
- Feigenson, L., Dehaene, S., & Spelke, E. S. (2004). Core systems of number. *Trends in Cognitive Sciences*, 8(7), 307-314. doi:10.1016/j.tics.2004.05.002
- Friso-van den Bos, I., van der Ven, S. H. G., Kroesbergen, E. H., & van Luit, J. E. H. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review*, 10, 29-44. doi:10.1016/j.edurev.2013.05.003
- Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.
- Gallistel, C. R., & Gelman, R. (1978). The child's understanding of number. In: Cambridge: Harvard University Press.
- Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology*, 47(6), 1539-1552. doi:10.1037/a0025510
- Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., . . . Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PLoS ONE*, 8(6), e67374. doi:10.1371/journal.pone.0067374
- Goffin, C., & Ansari, D. (2016). Beyond magnitude: Judging ordinality of symbolic number is unrelated to magnitude comparison and independently relates to individual differences in arithmetic. *Cognition*, 150, 68-76. doi:10.1016/j.cognition.2016.01.018

- Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, 44(5), 1457-1465. doi:10.1037/a0012682
- Hayes, A. F. (2013). *Introduction to Mediation, Moderation, and Conditional Process Analysis: A Regression-Based Approach*: Guilford Publications.
- Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1992). Differential calculation abilities in young children from middle- and low-income families. *Developmental Psychology*, 28(4), 644-653. doi:10.1037/0012-1649.28.4.644
- Le Corre, M. (2014). Children acquire the later-greater principle after the cardinal principle. *British Journal of Developmental Psychology*, 32(2), 163-177. doi:10.1111/bjdp.12029
- Libertus, M. E., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. *Developmental Science*, 14(6), 1292-1300. doi:10.1111/j.1467-7687.2011.01080.x
- Lyons, I. M., & Ansari, D. (2015). Numerical Order Processing in Children: From Reversing the Distance-Effect to Predicting Arithmetic. *Mind, Brain, and Education*, 9(4), 207-221. doi:10.1111/mbe.12094
- Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition*, 121(2), 256-261. doi:10.1016/j.cognition.2011.07.009
- Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science*, 17(5), 714-726. doi:10.1111/desc.12152

- Nieder, A. (2005). Counting on neurons: the neurobiology of numerical competence. *Nature Reviews Neuroscience*, 6(3), 177-190. doi:10.1038/nrn1626
- Odic, D., & Starr, A. (2018). An Introduction to the Approximate Number System. *Child Development Perspectives*. doi:10.1111/cdep.12288
- Parsons, S., & Bynner, J. (2005). *Does numeracy matter more?* London: National Research and Development Centre for Adult Literacy and Numeracy.
- Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., . . . Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, 116(1), 33-41.
- Rubinsten, O., & Sury, D. (2011). Processing Ordinality and Quantity: The Case of Developmental Dyscalculia. *PLoS ONE*, 6(9), e24079. doi:10.1371/journal.pone.0024079
- Sasanguie, D., Lyons, I. M., De Smedt, B., & Reynvoet, B. (2017). Unpacking symbolic number comparison and its relation with arithmetic in adults. *Cognition*, 165, 26-38. doi:10.1016/j.cognition.2017.04.007
- Sasanguie, D., & Vos, H. (2018). About why there is a shift from cardinal to ordinal processing in the association with arithmetic between first and second grade. 21(5), e12653. doi:doi:10.1111/desc.12653
- Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De Smedt, B. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: a meta-analysis. *Developmental Science*, 20(3), e12372. doi:10.1111/desc.12372

- Siegler, R. S. (1987). The perils of averaging data over strategies: An example from children's addition. *Journal of Experimental Psychology: General*, 116(3), 250-264.
doi:10.1037/0096-3445.116.3.250
- Siegler, R. S. (2016). Magnitude knowledge: the common core of numerical development. *Developmental Science*, 19(3), 341-361. doi:10.1111/desc.12395
- Smets, K., Sasanguie, D., Szűcs, D., & Reynvoet, B. (2015). The effect of different methods to construct non-symbolic stimuli in numerosity estimation and comparison. *Journal of Cognitive Psychology*, 27(3), 310-325. doi:10.1080/20445911.2014.996568
- Szkudlarek, E., & Brannon, E. M. (2017). Does the Approximate Number System Serve as a Foundation for Symbolic Mathematics? *Language Learning and Development*, 13(2), 171-190. doi:10.1080/15475441.2016.1263573
- Turconi, E., Campbell, J. I. D., & Seron, X. (2006). Numerical order and quantity processing in number comparison. *Cognition*, 98(3), 273-285. doi:10.1016/j.cognition.2004.12.002
- Turconi, E., Jemel, B., Rossion, B., & Seron, X. (2004). Electrophysiological evidence for differential processing of numerical quantity and order in humans. *Cognitive Brain Research*, 21(1), 22-38. doi:10.1016/j.cogbrainres.2004.05.003
- Turconi, E., & Seron, X. (2002). Dissociation Between Order and Quantity Meanings in a Patient with Gerstmann Syndrome. *Cortex*, 38(5), 911-914. doi:10.1016/s0010-9452(08)70069-8
- Vogel, S. E., Remark, A., & Ansari, D. (2015). Differential processing of symbolic numerical magnitude and order in first-grade children. *Journal of Experimental Child Psychology*, 129(0), 26-39. doi:10.1016/j.jecp.2014.07.010

- Vos, H., Sasanguie, D., Gevers, W., & Reynvoet, B. (2017). The role of general and number-specific order processing in adults' arithmetic performance. *Journal of Cognitive Psychology*, 29(4), 469-482. doi:10.1080/20445911.2017.1282490
- Wiese, H. (2003). *Numbers, language, and the human mind*. Cambridge, UK: Cambridge University Press.
- Wynn, K. (1990). Children's understanding of counting. *Cognition*, 36(2), 155-193. doi:10.1016/0010-0277(90)90003-3
- Xenidou-Dervou, I., De Smedt, B., van der Schoot, M., & van Lieshout, E. C. D. M. (2013). Individual differences in kindergarten math achievement: The integrative roles of approximation skills and working memory. *Learning and Individual Differences*, 28, 119-129. doi:10.1016/j.lindif.2013.09.012
- Zorzi, M., Di Bono, M. G., & Fias, W. (2011). Distinct representations of numerical and non-numerical order in the human intraparietal sulcus revealed by multivariate pattern recognition. *Neuroimage*, 56(2), 674-680. doi:10.1016/j.neuroimage.2010.06.035

Supplemental Online Materials for “Does $1 + 1 = 2^{\text{nd}}$? The relations between children’s understanding of ordinal position and their arithmetic performance”

Chi-Ngai Cheung and Stella F. Lourenco

Emory University

Task Descriptions

Domain-general Cognitive Abilities

Inhibitory control. Individual differences in inhibitory control were assessed with the Inhibition subtest of the Developmental NEuroPSYchological Assessment-2nd Edition (NEPSY; Korkman, Kirk, & Kemp, 2007). On this task, children labeled shapes (circle vs. square) or the direction of arrows (up vs. down) as quickly as possible such that responses were either congruent (e.g., say “circle” if the stimulus was a circle; Naming condition) or incongruent (e.g., say “circle” if the stimulus was a square; Inhibition condition) with the presented stimulus. Scores for Naming and Inhibition conditions were scaled based on completion time and number of errors in the respective conditions. Children’s overall performance was measured by a contrast score scaled based on both Naming and Inhibition scores, which measures children’s inhibitory control while taking into account their baseline naming speed.

Decision speed. Individual differences in processing speed were assessed with the Decision Speed subtest of the Woodcock-Johnson III Tests of Cognitive Abilities (WJ; Woodcock, Mather, McGrew, & Schrank, 2001). Children were given a worksheet that contained 40 test items. Each item consisted of seven different pictures, two of which belonged to the same category (e.g., sun and moon). Children were instructed to indicate which pictures belonged to the same category. Children completed as many trials as possible within three minutes. Performance was measured using standard scores for this task (scaled based on $M = 100$ and $SD = 15$).

Verbal working memory. Individual differences in children’s verbal working memory (WM) capacity were assessed by the Memory for Words subtest of the WJ Tests of Cognitive Abilities (Woodcock, Mather, McGrew, et al., 2001). On each trial of this test, children were

tasked with memorizing a list of words presented with an audio recording. Children were required to produce the words in the order in which they were presented. The length of word lists varied from 1 to 7 words. This task incorporated an adaptive procedure, such that trials were presented in increasing difficulty and termination depended on the child's performance. The test ended when children failed all three lists in a group. Performance was measured using standard scores (scaled based on $M = 100$ and $SD = 15$).

Nonverbal working memory. Individual differences in nonverbal working memory (WM) span were assessed by the Picture Recognition subtest of the WJ Tests of Cognitive Abilities (Woodcock, Mather, McGrew, et al., 2001). On each trial, children were tasked with memorizing a list of target pictures (list length: 1 to 4) presented for five seconds. Then, children were asked to identify the targets from an array that contained both target and non-target pictures. Performance was measured using standard scores for this task (scaled based on $M = 100$ and $SD = 15$).

Vocabulary. Individual differences in children's vocabulary were assessed by the Picture Vocabulary subtest of the WJ Tests of Achievement (Woodcock, Mather, & McGrew, 2001). Children were asked to name pictures of objects in a test booklet with multiple pictures on each page. On each trial, the experimenter pointed to a picture on the page and children were required to provide the name of the referenced object. This task used an adaptive procedure, such that trials were presented in order of increasing difficulty, and termination of the task depended on children's performance. More specifically, the task ended when children made six consecutive errors on a page, or when they reached the last item of the test. Performance was measured using standard scores (scaled based on $M = 100$ and $SD = 15$).

Results

Standardized scores for domain-general cognitive tests are presented in Supplemental Table 1. The results show that, for all of these measures, children's mean scores were within the normal range.

Supplemental Table 1

Mean scores (SD in parentheses) for each age group of the domain-general cognitive tests.

Tasks	Age	
	5-year-olds	6-year-olds
NEPSY-Inhibition	10.69 (2.68)	10.16 (2.69)
WJ-Decision Speed	107.34 (14.28)	107.55 (14.07)
WJ-Memory for Words	110.69 (14.94)	111.71 (16.80)
WJ-Picture Recognition	113.23 (10.35)	111.55 (11.68)
WJ-Picture Vocabulary	106.15 (11.14)	108.79 (10.54)

References

Korkman, M., Kirk, U., & Kemp, S. (2007). *NEPSY-II: A developmental neuropsychological assessment*. San Antonio, TX: The Psychological Corporation.

Woodcock, R. W., Mather, N., & McGrew, K. S. (2001). *Woodcock-Johnson III tests of achievement*. Itasca, Ill.: Riverside Publishing Company.

Woodcock, R. W., Mather, N., McGrew, K. S., & Schrank, F. A. (2001). *Woodcock-Johnson III tests of cognitive abilities*. Itasca, IL: Riverside Publishing Company.