Latin Square Tasks: A multi-study evaluation

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Abstract

The Latin Square Task (LST) was proposed as a theoretically well-grounded paradigm for measuring fluid intelligence. In four studies (total $N = 3,439$) we systematically investigated the psychometric properties of LSTs. Results provided evidence that (a) the construct is unidimensional, (b) the administered stimulus types and the rotation of the item matrix only played a minor role, and (c) the relations with other measures of reasoning ability were in the expected range (about $r = .50$), confirming the validity of LSTs. For unsupervised automated test construction, item difficulty was only insufficiently accounted for by relational complexity and the number of steps that need to be memorized for solving an item. We discuss possible remedies for improving the paradigm including its generalizability. Furthermore, we propose that the binding-hypothesis of working memory is theoretically suited to account for item difficulties in LSTs.

Keywords: latin square task, working memory, relational complexity, reasoning, processing capacity.

Latin Square Tasks: A multi-study evaluation

LSTs have been developed to measure relational reasoning in line with *Relational Complexity* (RC) Theory (Birney, 2002; Halford et al., 1998). This theory assumes that the processing load in reasoning tasks can be expressed in terms of the complexity of the mental models a person uses to solve a task (Halford, 1993). Typically, a Latin square is a matrix consisting of n rows and n columns, that are partly filled with stimuli (e.g., numbers). The defining rule of a Latin square is that each stimulus can only appear once in each row and each column of the matrix, and, thus, no more than n times. As the typical LST matrix is of size $n \times n$, the rows and columns can be considered interchangeable. Hence, we will refer to both, rows and columns, with the more general term *vectors*. In psychological research (e.g., Birney, 2006), incomplete Latin squares are presented with a highlighted empty target cell (see “?” in Figure 1). Test-takers have to infer the stimulus that needs to be inserted into this specific target cell to solve the task correctly.

Psychometric Properties of LST

Measurement quality is essential for replicable and cumulative research (Flake & Fried, 2020). To be of practical value in applied settings, several psychometric criteria must be met. Accordingly, LSTs as indicators of fluid intelligence (gf) have to offer a satisfying reliability, good discriminative power, and construct validity. Furthermore, the task should also be easy to instruct, come with efficient speed-accuracy trade-offs, possess incremental validity, and be invariant across relevant moderators. Finally, it would be beneficial, if items could be rationally constructed based on theory-based features, that influence item difficulties (Irvine & Kyllonen, 2002).

Previous reliability estimates of LSTs for university students ranged from low to satisfactory for figural stimuli ($\alpha = .56$ to $.84$; Bertling, 2012; Birney, 2002; Birney et al., 2006; Birney & Bowman, 2009). Unfortunately, the majority of studies did not report reliability estimates (Andrews et al., 2006, 2013; Bateman et al., 2017, 2019; Bateman &

Birney, 2019; Daniel, 2016; Double & Birney, 2018; Perret et al., 2011; Zeuch, 2010; Zeuch et al., 2011; Zhang et al., 2009). Moreover, the reliability estimates of previous studies could be considered biased, since Cronbach's alpha is often not applicable. Specifically, it assumes that all items are equally good indicators of a unidimensional construct (McNeish, 2018; Sijtsma, 2009), but these assumptions of an essentially tau-equivalent model are frequently not met.

In order to distinguish between test-takers of broadly varying ability, LSTs should cover a broad range of difficulty (i.e., broad test information curve). In principle, LSTs offer numerous possibilities to construct simple as well as very difficult items. The majority of previous studies relied on items, which were rather simple, compared to what is needed in applied settings. For example, it was found that simple LSTs were too easy for highly able adult samples (Birney et al., 2012). Thus, it needs to be evaluated whether LSTs can cover a difficulty range that allows researchers to differentiate between individuals at high ability levels.

Unfortunately, many studies including LSTs have not reported measurement models (e.g., Andrews et al., 2006; Bateman & Birney, 2019; Birney & Bowman, 2009; Daniel, 2016). The few studies examining the psychometric properties fitted Rasch models and found varying proportions of items with misfit to the model (e.g., Bertling, 2012; Birney et al., 2006; Zeuch et al., 2011). Confirmatory factor analysis can be helpful in testing competing measurement models of LSTs and thus informing researchers about the underlying factor structure of LSTs.

Predicting Difficulty of LST Items

Two item attributes have been discussed in the literature to be essential for determining item difficulty: relational complexity and number of steps. In addition, we propose a third item attribute—vector sums—which integrates complexity across steps.

Highest Relational Complexity. Based on the concept of relational complexity, the difficulty of an item is determined by the most complex step (Birney et al., 2006, 2012). In

other words, the step that requires the highest number of vectors being considered simultaneously is the limiting factor in the item solution process. As the most complex step is considered the most important feature of the item in RC theory, the common LST notation focuses on this characteristic: Items that only require consideration of one vector are called binary. If the highest complexity requires two vectors, they are called ternary, and so on.

Number of Steps. Another account of item difficulty is on serial processing which could be approximated with the number of steps (Birney & Bowman, 2009). Notwithstanding, the theory predicts that the highest relational complexity is the main attribute for predicting item difficulty. Previous evidence seems to backup this model (Birney et al., 2006; Zeuch et al., 2011), but there are some methodological shortcomings of these studies that might have precipitated this result. For example, Zeuch et al. (2011) only differentiated between items requiring four vs. less than four steps, which apparently restricts the amount of variance considerably. Another study using a mix of the highest complexity and steps as characteristics in a linear-logistic test model (rather than using only highest complexity) led to an increase in R^2 of about 5% (Bertling, 2012). In yet another study with children, the number of necessary steps accounted for more variance in the item difficulty variation than the complexity (Zhang et al., 2009). Thus, the empirical evidence paints a more nuanced picture, as relational complexity is not the single contributor to task difficulty.

Vector Sums. We argue that solely focusing on the most complex step falls short, because this assumption presupposes that the cognitive load of different steps are basically independent. One explanation why it is not sufficient to consider only the highest complexity step is, that it is easier to “take back and redo” a step with lower complexity. Particularly in items requiring multiple steps, it is an unrealistic assumption that participants see the solution path right away and solve the target cell without ado. Consequently, trial and error processes are inevitable, and having to retrace some steps for solving an item might be rather the rule

than the exception. In other words, items with more steps require more strategic and systematic storing and processing of information (Birney & Bowman, 2009).

Stimulus Type. It is an open question if item difficulty of LSTs is affected by stimulus type. For other fluid intelligence tasks the most prominent distinction below a g-factor concerns the stimulus material, that is, verbal, numerical, and figural (Wilhelm, 2005).

Whether or not stimulus content leads to differences in item difficulty or item discrimination is an empirical question. The majority of studies administering LSTs used figural stimuli (e.g., Bateman et al., 2019; Birney et al., 2006; Zeuch et al., 2011), with one exception in a study with children that used numbers as stimuli (Zhang et al., 2009). To our knowledge, there has been no study in which LSTs have been administered using more than one content domain.

Rotation. In solving LSTs, participants might have preferences for row- or column-based processing. There is no study examining the influence of processing directions in Sudokus or LSTs, but there is similar research regarding spatial biases in visuospatial attention tasks (Eviatar, 1995; Rinaldi et al., 2014). Thus, the rotation of the stimulus might affect the difficulty of an item, for instance, an item, which is rotated by 90 degrees, could exhibit different psychometric properties. Hearne et al. (2017, 2019) suggested to generate “new” items by simply rotating LSTs, but it remains unclear if there is a bias in the order the information is extracted from LSTs. If rotation of an item does not yield different psychometric properties, this would be an advantage for the rational construction of parallel test versions especially in conjunction with various stimulus domains.

Figure 1*Example Items of a Latin Square Task*

- A**
	?			
	1			
	5			
	4			
3	2			
1
2
3
4
5
- Item with **numerical stimuli**. This item can be solved in **one step** when considering **one vector** (second column).
- B**
A			?	B
				D
C		B	E	A
			C	
A
B
C
D
E
- Item with **verbal stimuli**. This item can be solved in **one step** when simultaneously considering information from **two vectors** (first row and fourth column).
- C**
■		⚡		
	■		☀	
				⚡
◇	▶		?	
⚡
☀
■
▶
◇
- Item with **figural stimuli**. This item can be solved in **one step** when simultaneously considering information from **three vectors** (last row as well as third and fifth column).
- D**
5	3		?	2
		3		
		2		
1	5		2	3
		5		
1
2
3
4
5
- Item with **numerical stimuli**. This item can be solved in **two steps**. The first step requires consideration of information of **one vector** (row 4). Using the information of the first step, the target cell can be solved by considering **two vectors** (row 1 and column 3). [Alternative solution: first step with two vectors (row 4 and column 3) and second step with 1 vector (row 1)]

The above-described item attributes can be reconciled with the binding hypothesis of working memory (WM) (Oberauer, 2019; Oberauer et al., 2008). This conceptualizes WM as a system for building, maintaining, and updating arbitrary bindings, or put differently, a system of relational integration (Wilhelm et al., 2013). In terms of the binding hypothesis, maintaining bindings demands the limited resources of the WM (Oberauer, 2019). Thus, all proposed item attributes depend on the capacity to store bindings in WM. This is, because the number of consecutive steps requires the maintenance and recall of these bindings as well as the capacity to maintain and process information in parallel, which is akin to sequential coordination (Hagendorf & Sá, 1996). As the combination and integration of information in LSTs play a crucial role, it makes sense to understand the complexity of these tasks as WM capacity requirements in terms of WM load (Larson et al., 1988). We argue that the focus should not be on the most complex step and the number of steps alone, but also on the number of vectors within each step. Thus, we propose to examine the sum of vector complexities across steps as a necessary item characteristic determining performance in LSTs.

Item Classification

A general problem of these items is the possibility of multiple solution paths (see Figure 1, Panel D). Especially, for items with an increasing number of steps, there might be more than one order of steps to arrive at the solution or even steps of different complexity, for example, overall more steps but of a lower complexity each. In other words, the higher the number of steps, the higher the probability that there is more than one way to find the correct solution.

In previous studies, item attributes were coded based on the solution path with the lowest value of the most complex step. This approach assumes that participants would rather take more simple steps (remember more stimulus-cell bindings) instead of solving a step with a higher relational complexity. Based on this classification system, the example items in

Figure 1 would be classified as binary 1 step (A), ternary 1 step (B), and quaternary 1 step (C) and binary 3 steps (D).

As described in the previous section we theorize that item difficulty is determined by the amount of information, which needs to be held in working memory while solving the task. Therefore, we assume that it is beneficial to find the shortest solution path possible. In line with this, we use the shortest solution path possible for the item classification. Based on the classification system used in the current investigation, the example items in Figure 1 are classified as 1-step vector 1 (A), 1-step vector 2 (B), 1-step vector 3 (C), and 2-steps vector sum 3 (D).

The Current Investigations

The aim of the current investigations was to empirically test psychometric properties of LSTs using data from convenience samples obtained in four independent pilot studies. We used a large pool of items varying in stimulus types and, theoretically, covering a wide difficulty range, even in samples of high ability individuals. In most previous studies, the highest complexity for LSTs was 3 vector steps (quaternary). We used 5 x 5 Latin squares (see Perret et al., 2011, for an example with a sample of children). That way, the WM load can be increased further than in the previously used 4 x 4 LSTs without implementing additional rules. Parallel versions of tests were created, which should be of equivalent difficulty based on the classification system using the shortest solution paths. The samples of the four pilot studies varied in their general ability (general public, university students, members of a society for the highly gifted) and worked on test batteries on different test media and contexts (computerized proctored, online unproctored, paper-pencil proctored).

We sought to study the psychometric properties of LSTs in terms of item difficulties, reliabilities, and unidimensional measurement models. Based on theoretical considerations regarding characteristics influencing item difficulty, we expected to cover a broad difficulty

range with the administered item sets. In addition, the following three ancillary research questions were investigated:

- a) RC theory and the extended classification system predict equivalent psychometric characteristics of items with different stimulus types (numbers, letters, or figures) and rotations. We examined if this assumption holds up empirically.
- b) Which item characteristics contribute to explaining variance in item difficulties? We expected the number of bindings that need to be established and memorized to have the highest impact on item difficulty, followed by the complexity of these bindings.
- c) How do LSTs relate to other measures of reasoning ability? Based on previous empirical findings, we expected to find medium to high correlations as an indication of construct validity.

Method

To ensure sufficient data quality, the following cleaning procedures were applied to all studies. Participants were excluded if 1) they did not solve a single LST correctly because it was assumed that they did not understand the task, 2) if they worked on less than 25% of LST items presented in the study, 3) are probably careless or unengaged responders, which was established by the total response time and the longest string of same answer option chosen (see description of the specific studies for more details). A summary of the number of participants excluded in each study is provided in the appendix (Table A.1). Items were scored dichotomously with 1 for correctly solved and 0 for incorrect responses and no response (based on the assumption that participants could not have solved these items). Items were excluded if a) they were extremely easy ($\geq .97$), b) they had at least one substantial negative ($< -.10$) inter-item correlation, and c) they deteriorated model fit and had high residual correlations with other items ($> .23$).

Analyses

All analyses were conducted using R 4.1.2 (R Core Team, 2021). The main R package used for confirmatory factor analysis was *lavaan 0.6-10* (Rosseel, 2012). A weighted-least-squares approach (WLSMV) was used for model estimation since responses were dichotomously scored. Since these investigations were pilot studies within a new test development, we deemed the following fit indices indicative of an acceptable fit: comparative fit index (CFI) $\geq .90$ and root mean square error of approximation (RMSEA) $\leq .07$ (Bentler, 1990; Browne & Cudeck, 1992). Pairwise deletion was used to handle missing values.

To investigate which item characteristics influence whether a person can solve an item, we computed logistic mixed models in which the response for each item was predicted by fixed effects (vector sums vs. steps and highest vector complexity) and random intercepts for person and test version. As vector sums are highly related to the number of steps ($r = .76$) and the highest vector is part of the sum, a model including steps and highest vector complexity—like in previous studies—will be compared to a model including vector sums as predictor of the single item responses. As this analyses uses data from Studies 2 to 4, it will be reported after the results for the individual studies. All R scripts, data files, and supplementary materials are available at https://osf.io/st6nv/?view_only=bcaa0c7d4f9b467980b94b73977fdb57.

Study 1

Research Aims

Apart from the main research question on psychometric properties, Study 1 was set out to answer the question whether LSTs measure the same underlying ability irrespective of stimulus type.

Methods

Procedure and Material

Participants were recruited via social media, flyers, and newspaper ads. The LSTs were administered as part of a larger study on the relations of cognitive abilities (XXX). Testing took part in form of group-testing with up to 8 participants each. The test battery was administered on two days, and participants completed the LST tasks at the end of the second day. Among others, three computerized LST tasks were presented with different stimuli—numbers, letters, or figures. Each task comprised 12 items presented in order of assumed increasing difficulty. Items were created by randomly using letters from A-Z, numbers from 1-9, or a five figures out of a set of figures. This is in contrast to the following studies, in which the same five stimuli (letters from A-E, numbers from 1-5, and same five figures) were used in all items. We applied trial-wise response deadlines and the elapsed time was visualized using a progress bar that was shown at the top of the monitor. However, there was virtually no time pressure as the response windows were 2.5 times larger than the mean response times in an unspeeded pilot study.

Sample

A total of $N = 309$ participants completed the LSTs. Two of them were excluded as their median response time was less than 2 seconds, and three more participants were excluded for giving identical responses in six or more consecutive trials. Sample information for the remaining participants can be found in Table 1.

Table 1*Sample Information for All Studies*

Study	<i>N</i>	Sample characteristic	Age	female [%]
1	304	public	18-54 ($M = 23.76$, $SD = 5.47$)	64
2	314	60% Mensa members ^a	18-81 ($M = 35.75$, $SD = 12.46$)	55
3	1,329	University students	15-29 ($M = 20.50$, $SD = 1.64$)	30
4	1,042	University students	17-30 ($M = 19.40$, $SD = 1.52$)	35

Note. The *N* in this table refers to the analytical samples after excluding participants.

^aMensa is a society for highly intelligent people.

Results

Data from Study 1 was used to investigate whether LSTs measure the same underlying ability irrespective of the administered stimulus type. The three tasks using numerical, verbal, or figural stimuli fitted the unidimensional models well (see Table 2). The correlated factors model ($\chi^2(374) = 399$, CFI = .96, RMSEA = .02 [.00; .03]) revealed that factor correlations were high (.84, .85, and .84). As an alternative model, we additionally fitted a nested factor model including a g-factor loading on all items and two method factors only loading on items using numerical or verbal stimuli respectively using figural stimuli as a reference factor (Eid et al., 2017). The model fitted the data well ($\chi^2(406) = 1,078$, CFI = .97, RMSEA = .01 [.00; .03]). While the g-factor had significant variance and loadings, the factor variance and loadings of the two stimulus-specific factors were not significant.

Discussion

The majority of studies administering LSTs used figural stimuli (e.g., Bateman et al., 2019; Birney et al., 2006; Zeuch et al., 2011), with one exception that used numbers as stimuli (Zhang et al., 2009). To our knowledge, there has been no study in which LSTs have been administered using more than one content domain. The results of the Study 1 indicate that performance in LSTs does not depend on stimulus type as the correlations between tests using

figural, numerical, and verbal stimuli were very strong. The design of the test versions was not strictly parallel, that is, item attributes differed across stimulus type. If the three versions would have used identical items with different stimuli, correlations would be supposedly closer to unity. A nested factor model reveals a strong g-factor, which indicates that LST items with differing material measure the same construct. Because of this we administered LSTs with varying stimuli in the following studies.

Table 2

Model Fit of Confirmatory Factor Analyses

Study	Version	Stimuli	<i>N</i>	Items	χ^2	<i>df</i>	CFI	RMSEA	[90%CI]	ω (α)
1		Figural	304	10	44	35	.92	.03	[.00; .05]	.50 (.47)
1		Verbal	304	9	35	27	.94	.03	[.00; .06]	.53 (.52)
1		Numerical	304	10	44	35	.92	.03	[.00; .05]	.46 (.44)
2		Figural	314	14	69	77	1.00	.00	[.00; .02]	.76 (.75)
3	A	Numerical	340	12	91	54	.90	.04	[.03; .05]	.59 (.57)
3	B	Numerical	449	14	154	77	.89	.05	[.04; .06]	.69 (.67)
3	C	Numerical	450	14	117	77	.89	.03	[.02; .05]	.66 (.60)
4	A	Verbal	343	12	96	54	.90	.05	[.03; .06]	.67 (.64)
4	B	Verbal	361	12	102	54	.89	.05	[.04; .06]	.69 (.64)
4	C	Verbal	338	12	60	54	.98	.02	[.00; .04]	.64 (.60)

Note. CFI = Comparative Fit Index; RMSEA = Root Mean Square Error of Approximation; ω = omega

categorical following Kelley & Pornprasertmanit (2016); α = traditional alpha (i.e., not based on tetrachoric correlations). In Study 1 all participants worked on all test versions.

Study 2

Research Aims

The aim of Study 2 was to examine whether LSTs cover a broad difficulty range, also for participants with a high ability level. Furthermore, we examined the measurement model of the test and its convergent validity. Lastly, we investigated the relation between item characteristics and difficulties.

Methods

Procedure and Material

Participants were recruited via student mailing lists, SurveyCircle (a platform where requests for study participations can be posted), and a mailing list for Mensa members in Germany (Mensa is a society for highly intelligent people). The study was conducted online with SosciSurvey (Leiner, 2019) without time restrictions. An online assessment was essential for this study, as the main target group of the study—Mensa members—are distributed across the country. After three practice trials participants had to solve 16 figural LSTs ordered in increasing difficulty (see Q-Matrix in the online Supplement Table S.1). In addition to the LST, we also administered a newly developed reasoning test that was based on the code-breaking game *Mastermind* (see also Best, 2001), in which participants had to deduce a color code based on given premises. The test consists of 18 items with 3-5 colors at 3-4 positions with up to 4 premises.

Sample

The online sample comprised 356 participants. In addition to the participants excluded for the reasons listed in Table A.1, five participants were excluded because they reported to be too young (i.e., < 18 years) for legally consenting to participate in the study. After the data pre-processing, observations from 314 participants were available for the data analyses (see Table 1).

Results

We were able to develop items covering a wide range of empirical difficulty without ceiling effects (see Figure 2). This was achieved by obtaining varying item complexities using only the single rule that stimuli can appear only once in each vector.

The unidimensional model exhibited close fit to the data from Study 2 (see Table 2). Standardized loadings ranged from .28 to .62 (see Table S2 in the online Supplement).

In addition to the LST, a newly developed deductive reasoning test based on the game *Mastermind* was presented in Study 2. A unidimensional measurement model for the Mastermind test was fitted to the data. The latent LST factor shared a substantial amount of variance with the latent factor of the Mastermind test (see Table 3).

Figure 2

Item Difficulties

Note. $N = 314$. s = step(s), v = vector(s), e.g., s2v1,3 refers to an item, which can be solved in two steps considering 1 and 3 vectors respectively. Error bars show interval of ± 2 standard errors. Item order as presented in the test. Grey dashed line represents guessing probability.

Table 3*Correlations Between LST and Other Ability Measures*

Study	Model	<i>N</i>	<i>r</i> (<i>SE</i>)
2	Mastermind	314	.60 (.06)
3A	Figural sequences	430	.48 (.04)
3B	Figural sequences	449	.52 (.03)
3C	Figural sequences	450	.55 (.03)
4AB	Equation solving	704	.49 (.05)
4C	Equation solving	338	.22 (.08)

Discussion

Again, we found clear evidence that LSTs are unidimensional. Additionally, there were no ceiling effects even in a sample of highly able individuals. That is, the results demonstrated that using a 5x5 matrix made it possible to construct sufficiently difficult items that differentiate well at the upper tail of the ability distribution. The convergent validity between the LST and the Mastermind test had the expected strength. Study 2 therefore contributed information to a pragmatic aspect of LSTs.

Study 3**Research Aims**

The aim of Study 3 was to replicate the unidimensional measurement models and describe the determinants of item difficulty while overcoming some limitations of the previous study (i.e., proctored instead of unproctored online study). To extend the nomological net of LSTs, the correlation with an established figural sequences task was examined (Schipolowski et al., 2020).

Methods

Procedure and Material

Participants took part in voluntary, preparatory math courses at three southern German universities in fall of 2019. At the end of the course, participants took one of three booklets of a paper-pencil test battery including 20 LST items with numeric stimuli. Items of all three versions were constructed to be parallel to each other on theoretical grounds, specifically, that is, they had the same numbers of steps and vector complexities. Items 5, 10, and 15 were identical across the booklets and served as linking items (see Q-matrix in the online Supplement Table S.2). As in past studies (e.g., Birney et al., 2006), in the current investigation items had to be reclassified as an additional solution path was found after further inspection.

In addition to the LST, reasoning ability was also measured with a test based on the figural subtest of the *Berlin Test of Fluid and Crystallized Intelligence* (BEFKI 11-12+; Schipolowski et al., 2020), for which convergent validity with other measures of cognitive ability has been repeatedly shown (e.g., for other gf tasks, Schroeders & Wilhelm, 2010; for declarative knowledge and vocabulary, Goecke et al., 2020, Schipolowski et al., 2020; for WMC, Weiss et al., 2020). Participants had to infer rules according to which a sequence of geometric drawings change their shading and form and to select missing drawings out of several response alternatives. Participants were given a time limit of 14 minutes for 16 items. The test was administered via paper-pencil. For the analyses, item parcels with similar mean scores were created after excluding four items that were below guessing probability.

Sample

A total of 1,732 newly enrolled university students worked on the test battery. In addition to the exclusions listed in Table A.1, 376 participants were not included in the analysis because they took notes, contrary to the instructions. We did not allow note taking as such notes relieve the burden on working memory and thereby substantially modify the task participants solve. Preventing note taking was, therefore, deemed relevant to ensure the tests

captured the intended cognitive requirements and processes. 1,329 participants were included in the analysis (see Table 1) with 430 students working on version A, 449 on version B, and 450 on version C.

Results

Linking items (5, 10, and 15) had comparable solving probabilities across all three groups, suggesting that group differences are not attributable to ability differences (see Figure 3). Most items with the same item attributes (number of steps, highest vector complexity, and vector sums) displayed comparable solving probabilities—with few exceptions (i.e., items 6, 9, and 13). After exclusion of some items based on the criteria described in the methods section, the unidimensional model fit of Study 3 was acceptable (see Table 2).

A unidimensional measurement model for the figural sequences test with parcels as indicators was fitted to the data ($CFI = .99/.99/1.00$, $RMSEA = .04/.04/.00$). The LST factor shared a substantial amount of variance with the factor capturing individual differences in the ability to solve figural sequences (see Table 3).

Figure 3*Item Difficulties*

Note. $N_A = 430$, $N_B = 449$, and $N_C = 450$. Error bars show interval of ± 2 standard errors. The grey dashed line represents guessing probability.

* Reclassified items. These items do not closely match with the items at the same position from the other two versions.

Discussion

Study 3 added to the evidence that LST tests measure a unidimensional cognitive ability. Descriptively, parallel items having the same number of highest complexities and number of steps differ in their solving probabilities, indicating that these characteristics are not sufficient to predict item difficulty. Using sums of the vector complexity of each step does not solve this issue. If all items are included, the unidimensional model does not fit the data of study 3 well. This misfit most likely stems from a strong speed effect, that is, participants did not get a chance to work on all items as test time was too restrictive.

Study 4

Research Aims

The last study in this series replicated Study 3 with the crucial addition of testing the equivalence of rotated items, which has not been done in previous research. Furthermore, the nomological net was extended by examining the relationship of LSTs with solving equations, an indicator for numerical reasoning ability (Schroeders & Wilhelm, 2010; Wilhelm, 2000).

Methods

Procedure and Material

Study 4 was part of voluntary, preparatory online math courses at three different southern German universities in fall 2020. The LST test in this study had 16 items with letters as stimuli (see Q-matrix in the online Supplement Table S.3). All items were presented on a single page with a time limit of 20 minutes. Again, participants were assigned randomly to three parallel versions. Items of version B were created by rotating items of version A by 90 degrees. Items from version C were not rotated versions of other tests but had the same attributes as items from the versions A and B regarding steps, vectors, number of to-be-inferred letters, and number of empty cells in the grid.

A *solving equations* test was used for measuring reasoning ability in the numerical content domain, which has been previously used in the context of college admission (Formazin et al., 2011). In this test, participants had to solve a system of equations containing several statements about variables (e.g., “ $A = 2 \times B$. $A + B = 15$ ”). Participants had to infer the correct numerical value for each variable. In total, 14 items were administered in Study 4. The first half of the items was presented before the LSTs, the second half after the LSTs. Both test halves were constructed to be comparable in difficulty with a response time of 10 minutes, excluding instruction time. The test was administered online.

Sample

Analyses were based on data from 1,042 participants, who worked on the online test (see Table A.1 for exclusions). 343 participants worked on version A, 361 on version B, and 338 on version C. A subsample of $n = 838$ participants worked on the solving equations test. Sample information can be found in Table 1.

Analyses

The majority of analyses in Study 4 were analogous to those in the first three studies. In addition, we investigated whether the performance on LSTs was independent of item rotation. First, we descriptively compared item difficulties. If rotated items with all other item attributes being equal would differ in their difficulty, it would be indicative for different solution processes (e.g., an effect of direction of information extraction). Second, we tested the equivalence of different test versions with multiple-group confirmatory factor analysis (Schroeders & Gnambs, 2018; Wu & Estabrook, 2016). If items are not equivalent (i.e., differences in their association with the latent variable or their difficulty), persons with the same ability will score differently. The LSTs are deemed to be measurement invariant if absolute fit is acceptable and relative fit in comparison to the preceding level of invariance testing is small ($\Delta\text{CFI} < .01$; Chen, 2007; Cheung & Rensvold, 2002).

Results

Difficulties of versions A and B, which contained rotated versions of the same items, are highly similar (Figure 4). For three items of version C (items 2, 4, and 8), item difficulties deviated substantially from the other two versions, although item attributes were the same across all three versions. An ancillary research question was whether rotating the items in a manner that columns and rows are swapped (i.e., rotation by 90 degrees) would influence item difficulty. A multi-group factor analysis supports the idea that test versions A and B are invariant and strictly parallel (for more detail see online Supplement, Table S.8). Although Version C contained items that have the same attributes regarding the number of steps and

relational complexity, model fit deteriorates significantly when introducing the according parameter constraints.

For version A an acceptable model fit for the unidimensional model was achieved by excluding items 11, 12 and 13, which had negative inter-item correlations (see Table 2). Excluding the same items also led to an acceptable model fit for versions B and C.

Figure 4

Item Difficulties

Note. s = step(s), v = vector(s), e.g., s2v1,3 refers to an item, which can be solved in two steps considering 1 and 3 vectors respectively. Error bars show interval of ± 2 standard errors. Item order as presented in the test. Grey dashed line represents guessing probability. $N_A = 343$, $N_B = 361$, and $N_C = 338$.

In addition to the LST test, an equation solving task was presented in Study 4. A unidimensional measurement model was fitted to the data. Item parcels were used as indicators. The measurement model for equation solving as well as the correlated factor models had an acceptable model fit (CFI = .92 - 1.00, RMSEA = .02 - .06). LST versions A

and B were pooled, because they were invariant. The LST factor shared a substantial amount of variance with the factor of the equation solving task (see Table 3), supporting the convergent validity of the LST.

Discussion

Hearne et al. (2017, 2019) suggested to generate “new” items by simply rotating LSTs, but there has not been an empirical investigation whether rotation affects item difficulty. The main result of Study 4 concerns the equivalence of assessment properties of the test versions A and B. Descriptive statistics as well as measurement invariance results indicate that performance on LSTs was independent of their rotation. In other words, it does not make a difference whether operations were performed across columns or rows (i.e., vectors). This is advantageous for the economic construction of parallel test versions. Interestingly, constructing parallel tests based on item characteristics from the design matrix (and even additional ones such as the number of empty cells in the grid) does not result in parallel items. In consequence, additional item characteristics that have not been considered yet influence performance on LSTs.

Prediction of Item Difficulty

Data from Studies 2 to 4 including 2,685 participants and 7 test versions were used in the logistic mixed effects models to predict item difficulties. The predictors were recoded so that their least complex instance was coded with 0, so that the intercept represents the odds for items with the lowest numbers of steps (1) and the minimum of highest complexity (2) or the lowest vector sums (2). Effects were all in the expected direction: With increasing complexity and steps the probability of solving an item decreases (see Table 4). The amount of explained variance by vector sums as single predictor (Model 1) compared to the combination of highest vector complexity and steps (Model 2) was of similar magnitude. In Model 2 the influence of the steps was more pronounced than the highest vector complexity. Conditional R^2 , which is the proportion of total variance explained through fixed and random effects, was about .27

and the marginal R^2 , which is the proportion of variance explained by fixed effects, was about .13 in both models. The variance for the test version was very small (0.01) indicating that the prediction does not vary across item samples. A model including an interaction term between highest vector complexity and steps does not significantly differ from the model without interaction term, that is, the interaction term was non-significant and did not explain variance beyond the main effects.

Table 4

Parameter Estimates and R^2 From Logistic Mixed Effects Models

Predictors	Model 1			Model 2		
	Odds Ratio	95%-CI	<i>p</i>	Odds Ratio	95%-CI	<i>p</i>
(Intercept)	7.07	6.44 – 7.76	<.001	9.18	8.27 – 10.19	< .001
vector sums	0.68	0.67 – 0.69	<.001			
highest vector complexity				0.63	0.61 – 0.65	< .001
steps				0.41	0.39 – 0.42	< .001
Marginal R^2 /Conditional R^2		.127 / .268			.134 / .275	

General Discussion

Across four studies, we examined the psychometric properties of the LST across different stimulus types, studied their relation to other measures of reasoning ability, and investigated the influence of item attributes on task difficulty. The key findings were as follows: First, we showed that LSTs measured the same ability, independent of their stimulus domain, justifying the use of LSTs with different stimulus types in the subsequent studies. Second, we demonstrated that LSTs can be adjusted to cover a broad range of difficulty across the reported studies, indicating that LSTs are suitable to measure reasoning ability within a broad ability spectrum. Third, we found that LSTs are substantially correlated with a number of other reasoning measures. Finally, our results indicated that the psychometric

properties did not differ if matrices were rotated, this is, when columns and rows were swapped.

Psychometric Properties of LSTs

One important finding of the current studies is evidence for the general assumption that task performance is relatively independent from stimulus type and rotation. This is important as influences based for example on the Gestalt of items (Primi, 2001) introduce variance presumably unrelated to reasoning ability. The finding that LSTs can be used interchangeably with different stimulus types suggests that test takers abstract from the content domain. Similarly, rotating the item seems to have no impact on item difficulty suggesting that only the bits of information themselves, but not their orientation are considered for solving the task.

The present sets of items covered a broad range of difficulties. Even though only one rule needs to be followed (i.e., each stimulus can only appear once in each row and column), the items have been shown to be quite complex. In contrast to other reasoning tasks, where many more rules might need to be explained to test takers, the LSTs function decently with exactly one easy to grasp rule. This avoids misconceptions about the task and limits possible test taking strategies that are often a nuisance in predicting item difficulties prior to testing (Marrero & Gámez, 2004; Vigneau et al., 2006).

Fit indices for unidimensional measurement models of the LSTs reached acceptable to good fit after item selection in 7 out of 10 cases across all studies. Although for three models CFI values were just below the limit of acceptance with values of .89, the RMSEA indicated appropriate fit of all models. A number of items had to be excluded in the process, which shows that there is still a long way to go before unsupervised automatic item generation can be used for this task. Until then, careful considerations about the selection of items and about the speed-accuracy trade-off (Garrett, 1922; Phillips & Rabbit, 1995; Thurstone, 1937; Ulitzsch et al., 2020) are necessary in test construction. Because reliabilities barely reached

acceptable levels and measurement models needed considerable item selection and did not always exceed typical fit cutoff values, further investigations are suggested to shed light at item functioning.

The current collection of studies is at the border of psychometric investigation and intelligence research. These two research strands have always been intertwined or even inseparable. Unfortunately, the construction of gf tasks is still more an art than a science (Kyllonen & Christal, 1990), making it hard to learn more about underlying processes. Development of new tasks that are closely tied to WMC and can be constructed in a systematic manner constitute a vital part of furthering intelligence research.

Predicting Item Difficulty of LSTs

From a theoretical perspective, we believe working memory is of clear relevance in solving LSTs. Competing theories distinguish between WM and reasoning components (Birney et al., 2012). However, we deem WM capacity as the basis of reasoning operations (e.g., Wilhelm, 2005). If the WM load of the items and other attributes influencing their difficulty could be determined more precisely, it would increase the usability of these items. Previous research evidenced that besides the complexity of the given problems, also the number of steps influences the performance on LST items (Bateman, 2020; Birney & Bowman, 2009; Zeuch, 2010). From the perspective of the binding hypothesis of WMC (Oberauer, 2019; Oberauer et al., 2008) both item attributes reflect different cognitive components. The two relevant processes are the establishment of arbitrary bindings, which is limited by binding capacity, and the maintenance of bindings (Oberauer et al., 2019). While the former is certainly more associated with the complexity of an item and the latter with the steps, it is impossible to disentangle them properly as new bindings need to be constituted, while at the same time maintaining others. Using the binding hypothesis as a theoretical basis exceeds RC theory as the latter focuses solely on building bindings (i.e., integrating relations).

In accordance with our hypotheses and the theoretical derivation based on the binding hypothesis, steps had a stronger effect on item responses than highest vector complexities (see Model 2 in Table 4). Vector sums seem to be a potential alternative in regards of explained variance but have the disadvantage of not clearly separating parallel (complexity) and serial (steps) processing. Across studies using different item sets with different stimulus types and different samples, the amount of variance accounted for by theoretically meaningful item attributes was rather small. Previous research on difficulty prediction reports explained variances larger than 60% (Birney et al., 2006; Gabales & Birney, 2011; Zeuch et al., 2011). However, all of these analyses are based on IRT model parameters and therefore shaped by the model assumptions like equal item discrimination, which would correspond to equal factor loadings in our measurement models, which is not given. Furthermore, item categorization in previous studies was based on the solution path, which keeps the highest vector complexity at a minimum compared to the shortest path (the path which keeps the steps at a minimum) in the current investigation. Comparing and modelling different item categorizations constitutes a research desideratum for future research.

Additional Item Attributes Explaining Item Difficulty

Beyond the number of vectors and the steps necessary to solve an item, it is possible to think of additional item attributes contributing to item difficulty, for example, the requirement to combine different strategies to solve different items. Perret et al. (2011) distinguished two types of ternary (vector-2) items: secant items, where the target cell is located at the intersection of the vectors that are needed for the solution (e.g., Panel B, Figure 1) vs. non-secant items, where the target cell is not in the intersection (e.g., vector-2 step of Panel D, Figure 1). The two strategies that are related to these item types are: (a) focus on the cell for secant items (see Figure 1, Panel A and B) vs. (b) focus on the stimulus for non-secant items (see Figure 1, Panel C and vector-2 step of Panel D). These strategies differ in their complexity: While for 1-vector items only the focus-on-cell strategy is possible, for all items

including steps with more than two vectors, only the focus-on-stimulus strategy is possible. Vector-2 items, however, can be solved with both strategies depending on the layout of the item.

The number of distractors (or stimuli) that are shown in the matrix, but are not needed for solving the target cell, might play an additional role. One hypothesis is that it might be easier to see the correct solution if there are no distracting stimuli. In contrast, Birney et al. (2006) argued that the opposite might be true: items might be easier if more cells are filled. However, they did not find a relation between the number of cells filled in a priori and item difficulty. Even more interesting than the number of distractors could be the number of impasses or opportunities to go astray (Engle, 2002). Often—especially in LSTs with a higher number of steps—there are cells that can be filled in but are not required for the solution. Therefore, these unnecessary steps can take up capacity and bury the path to the solution of the target cell.

The number of steps is meant to represent storage capacity (Birney et al., 2012). But the relationship is presumably more complex. In some cases, there might be two steps within the same item that can be solved independently from each other, but in a subsequent step the information derived from both steps are needed to solve the target cell. In other cases, the number of stimuli that are not included in the matrix initially but need to be derived by the test-taker, might be one for each step (the first step is always solely based on initially available information). In the same vein, an interesting item attribute to consider could be how many derived stimuli are required for each step.

Additionally, the chunking used by participants to solve a target cell in an LST might be more complex than what has been assumed so far. Previous studies acknowledged that information from the same vector can be chunked, but maybe participants also chunk information based on the same stimulus (e.g., “1s are present in every row except for the bottom row”). Such a stimulus chunking strategy could explain why items with a high

complexity based on the vector chunking assumption actually showed high solving probabilities in the current investigation (e.g., item 4 in study 2). Taken together, a variety of item attributes of LSTs and their interplay (i.e., interactions) have not yet been considered in the literature and should be systematically probed in future research.

Limitations and Desiderata for Future Research

One limitation of the current investigation is that the order of items is confounded with the number of steps. For reasons of test efficiency, items with more steps tend to be presented later. Future studies could randomize the position to quantify the influence of item position and eventually rule it out as an influencing factor (see Schweizer et al., 2009). Additionally, items were excluded following a data-driven approach. Regarding item selection, larger numbers of items with all combinations of item attributes would strengthen our understanding of unique and combined effects on item difficulty. Future studies with sufficient testing time should create such permutations. Our investigation yields first evidence about the equivalence of LSTs with different stimulus types. Future research could investigate the potential differences in more detail by using strictly parallel items. Furthermore, factor saturations as indicated by omega indicate that there is still specific variance in the items that is not explained by the factor. Lastly, the nomological net of LSTs needs further investigation.

Conclusion

Based on four large studies, we conclude that Latin Square Tasks qualify as psychometric sound indicators of reasoning ability with a theory-based construction rationale. Due to the simple construction principles of LSTs that potentially allow for automated item generation, they can be considered a valuable addendum to the huge collection of reasoning measures (for an overview see Chapter 6 in Carroll, 1993). We suggest a theoretical perspective on LSTs which is inspired by the binding hypothesis of WMC, interpreting both the number of steps and the highest relational complexity as decisive item attributes, which directly transfer to working memory load. Finally, we want to emphasize that current

theoretical approaches are too simplistic to exhaustively predict item difficulty, and a more precise prediction is necessary.

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Appendix

Table A.1

Exclusion Criteria and Number of Exclusions for Studies 1 Through 4

	No correct response	>75% missing	Total response time < 2 min	> 25% of consecutive trials with identical responses
Study 1	0 (0.0%)	0 (0.0%)	3 ^a (1.0%)	2 ^b (0.6%)
Study 2	0 (0.0%)	8 (2.2%)	28 (7.9%)	4 (1.1%)
Study 3	1 (0.1%)	24 (1.4%)	— ^c	2 (0.1%)
Study 4	6 (0.5%)	39 (3.0%)	7 ^d (0.5%)	2 (0.1%)

Note. ^a The criterion for fast responses was set to be two seconds or faster on average (median), as this study used trial-wise response windows.

^b The criterion for repetitive responses was set to be 6 consecutive trials, as there were fewer items in this study.

^c Study 3 was a paper-pencil test without response time recorded.

^d For 421 participants response time was not recorded because of technical errors. Based on the response time distribution and the other exclusion criteria, they were included in the analyses. Due to technical errors 200 participants could not be allocated to one of the three LST test versions and had to be excluded.