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Comparing Random Effects Models, Ordinary Least Squares, or Fixed Effects with Cluster
Robust Standard Errors for Cross-Classified Data

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COMPARING APPROACHES FOR CROSS-CLASSIFIED DATA

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We would like to thank Tasha Beretvas and Tiffany Whittaker for their helpful comments on prior versions of this manuscript. A previous version of this paper was presented at the annual convention of the American Educational Research Association, April 12, 2021 (online).

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Supporting materials for this article are available on the Open Science Framework at <https://osf.io/s7hr3/>.

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Abstract

Cross-classified random effects modeling (CCREM) is a common approach for analyzing cross-classified data in psychology, education research, and other fields. However, when the focus of a study is on the regression coefficients at level one rather than on the random effects, ordinary least squares regression with cluster robust variance estimators (OLS-CRVE) or fixed effects regression with CRVE (FE-CRVE) could be appropriate approaches. These alternative methods are potentially advantageous because they rely on weaker assumptions than those required by CCREM. We conducted a Monte Carlo Simulation study to compare the performance of CCREM, OLS-CRVE, and FE-CRVE in models, including conditions where homoscedasticity assumptions and exogeneity assumptions held and conditions where they were violated, as well as conditions with unmodeled random slopes. We found that CCREM outperformed the alternative approaches when its assumptions are all met. However, when homoscedasticity assumptions are violated, OLS-CRVE and FE-CRVE provided similar or better performance than CCREM. When the exogeneity assumption is violated, only FE-CRVE provided adequate performance. Further, OLS-CRVE and FE-CRVE provided more accurate inferences than CCREM in the presence of un-modeled random slopes. Thus, we recommend two-way FE-CRVE as a good alternative to CCREM, particularly if the homoscedasticity or exogeneity assumptions of the CCREM might be in doubt.

Translational Abstract

Cross-classified data involve observations that are nested within more than one type of higher-level unit, which are not themselves hierarchically nested. For example, if students are included within neighborhoods and schools that are not perfectly nested within each other, data on those students has a cross-classified structure. In psychology, education, and other research areas, a common approach for analyzing cross-classified data is cross-classified random effects modeling (CCREM). However, CCREM is based on strict assumptions. If its assumptions are violated, the estimates from CCREM may be systematically biased and uncertainty assessments from CCREM may be misleading. When the focus of a study is on an individual-level predictor, ordinary least squares regression with cluster robust variance estimators (OLS-CRVE) or fixed effects regression with CRVE (FE-CRVE) offer practical alternatives to CCREM. These alternatives are beneficial in not relying on all the assumptions required by CCREM. In this study, we compared the performance of CCREM, OLS-CRVE, and FE-CRVE in conditions where key assumptions held true and conditions where those assumptions were violated. When individual-level errors have non-constant variance, OLS-CRVE and FE-CRVE provided similar or better performance than CCREM. When there is correlation between model predictors and random effects, only FE-CRVE showed adequate performance. Further, when the random slopes were incorrectly omitted from the model, OLS-CRVE and FE-CRVE provided a more accurate statistical inference than CCREM. Thus, if there are potential concerns regarding the assumptions of the CCREM, we recommend two-way FE-CRVE as an alternative to CCREM.

Keywords: cross-classified random effects model, two-way clustering, ordinary least squares, fixed effects estimation, cluster robust standard errors

Comparing Random Effects Models, Ordinary Least Squares, or Fixed Effects with Cluster Robust Standard Errors for Cross-Classified Data

In many areas of social science research, scholars encounter clustered data structures where the lower-level observations are nested within upper-level clusters. For example, individuals are clustered within their teachers or schools, and repeated measurements may be nested within individuals. Researchers often analyze such data using hierarchical linear models (HLM; Raudenbush & Bryk, 2002). Some clustered data have a more complex, cross-classified structure, where observations are nested within more than one dimension of clusters, which are not themselves hierarchically nested. For example, in models for educational data, students (level-1) may be nested within neighborhoods (level-2) and schools (level-2) at the same time. Students from the same neighborhood can be enrolled in different schools, and two students in the same school do not always live in the same neighborhood. In models of psychological response process data, response times (level-1) can be nested simultaneously in respondents (level-2) and items (level-2; e.g., Baayen et al., 2008). Further, in models of social psychology data such as the social relations model, interpersonal perception (level-1) is nested within two individuals (level-2) called actor and partner (Back & Kenny, 2010; Lüdtke et al., 2013).

If data follow a cross-classified structure, ignoring one of the dimensions of clustering and applying HLM can result in underestimation of standard errors (SEs) of level-2 predictors and overestimation of variance components in both levels (Meyers & Beretvas, 2006). As a generalization of HLM, the cross-classified random effects model (CCREM) has been developed for analyzing data structures that involve nesting in two or more dimensions (Goldstein & Sammons, 1997; Raudenbush, 1993; Van den Noortgate, De Boeck, & Meulders, 2003). Because multiple dimensions of clusters are considered in the model simultaneously, CCREM can

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examine the unique variance associated with each dimension (e.g., neighborhoods and schools; respondents and items). Furthermore, the associations of covariates related to each dimension can be analyzed jointly in the same model.

Considering its versatility, the CCREM may seem to be an ideal choice for modeling cross-classified data. However, the CCREM is premised on some strict assumptions, and the importance of these assumptions is not always recognized. In certain circumstances, alternative approaches to analyzing cross-classified data are available that avoid the strict assumptions of the CCREM. In particular, if the goal of an analysis is primarily to estimate the coefficients of level-1 predictors, potential alternatives to the CCREM include ordinary least squares with cluster robust variance estimators (OLS-CRVE) and two-way fixed effects models with CRVE (FE-CRVE). OLS-CRVE is also a viable alternative when focus is on level-2 predictors. Each of these approaches provide estimates of coefficients on predictors but do not provide estimates of variance components associated with the clustering dimensions. Instead, they account for dependence in the errors of observations that share a cluster in one or both dimensions using sandwich-type CRVE.

OLS is one of the most frequently used methods for estimating a multiple regression model (Wooldridge, 2010). The presence of clusters, however, violates the independence assumption underlying the classical OLS model. When the observations are grouped into independent clusters along a single dimension, OLS-CRVE can be applied in order to obtain asymptotically consistent SEs, without explicitly modeling the random effects of the clusters (McNeish et al., 2017; White, 1984). In its original form, OLS-CRVE requires a large number of clusters to perform adequately. However, small-sample adjustments to CRVE have been

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developed that provide good performance even when the number of independent clusters is limited (Bell & McCaffrey, 2002; Cameron et al., 2008; Pustejovsky & Tipton, 2018).

Cameron et al. (2011) extended one-way OLS-CRVE to handle clustering in multiple dimensions, as in cross-classified data. Cameron et al. (2011) showed in simulations that two-way OLS-CRVE performed well on a balanced cross-classified data structure, particularly when the homogeneity of errors assumption of the two-way random effects model is violated. However, CCREM was not considered in this study, and circumstances where CCREM and two-way OLS-CRVE perform differently are not widely understood. Petersen (2009) and Thompson (2011) also contributed to extending the idea of one-way clustering, comparing the performance of one-way and two-way OLS-CRVE in a simulation study.

Fixed effects (FE) modeling, an often-used approach in economics and political science, models dependency due to clustering by using a regression model with a fixed effect for each cluster—that is, a set of indicator variables for cluster affiliation. The fixed effects estimator (or so-called “within” estimator) is based on deviations from the means of each cluster (Angrist & Pischke, 2008; Wooldridge, 2010). With a cross-classified data structure, FE can account for more than one dimension of clusters by including cluster affiliation indicators for *each* dimension of clustering. CRVE can also be applied to correct SEs for remaining within-cluster dependence in the errors, beyond the dependence taken into account by the inclusion of fixed effects (Cameron et al., 2011; Cameron & Miller, 2015). Compared to CCREM and two-way OLS-CRVE, two-way FE-CRVE requires the weakest assumptions. However, two-way FE-CRVE is only useful for investigating level-1 predictors—those that vary *within* clusters—because all level-2 variation is removed through the inclusion of fixed effects.

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In psychological research, researchers have used CCREM much more frequently than the alternative approaches. From a search of empirical studies using cross-classified data published between January 2012 and December 2021, we identified 193 papers that applied CCREM, two-way OLS-CRVE, or two-way FE-CRVE (see Supplementary Section S1 for details). Of those, 177 (92%) articles used CCREM¹, 6 (3%) articles conducted two-way OLS-CRVE and 10 (5%) articles used two-way FE-CRVE. However, if the researcher focuses more on the coefficients of the level-1 (or level-2) covariates than on random effects, it would be worth considering models that require weaker assumptions than CCREM.

The purpose of the present study is to explore the performance of two-way OLS-CRVE and two-way FE-CRVE as alternatives to CCREM, for researchers who want to focus on estimating regression coefficients (i.e., fixed effects) in models of cross-classified data. We focus on unbalanced, two-dimensional cross-classified data—a commonly encountered structure in the analysis of psychological and educational data. We first review three approaches to handling two-way clustering: CCREM, two-way OLS-CRVE, and two-way FE-CRVE. Next, we demonstrate these methods in an empirical data analysis, illustrating how they can yield different coefficient estimates and SEs. We then report a Monte Carlo simulation designed to assess how

¹ Among 50 randomly selected studies from 177 CCREM, individuals (primarily students) nested within schools and neighborhoods were the most frequently used cross-classified data. Other examples include individuals nested within primary and secondary schools, immigrant children nested within receiving and origin countries, measurements nested within items and individuals, or measurements cross-classified within other individuals (see Supplementary Table S1.2).

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the three methods perform across a range of conditions, including conditions where key assumptions of the CCREM are violated. We conclude with a discussion of limitations and implications for applied data analysis involving cross-classified structures.

Methods for Analyzing Clustered Data

In this section, we describe three distinct approaches for analyzing cross-classified data. We first review corresponding methods for one-way (hierarchical) clustering, including hierarchical linear models, OLS with CRVE, and fixed effects with CRVE. We then describe the corresponding extensions to two-way (cross-classified) clustering. We highlight assumptions, benefits, and drawbacks of each approach, sometimes arguing by analogy to the one-way setting due to lack of existing literature addressing two-way clustering. For ease of exposition, our development focuses on a cross-classified data structure in which student-level observations are nested in neighborhoods and schools. However, the methods and principles we discuss apply equally to other cross-classified data structures.

One-Way Clustering

We begin by considering a data structure with a single dimension of clustering, where individual observations are nested within groups. For concreteness, suppose that we have data on N students nested within G neighborhoods. Students are indexed as $i = 1, \dots, N$ and neighborhoods as $g = 1, \dots, G$, where the neighborhood index is understood to be a function of the student index (i.e., $g = g(i)$), and there is a total of n_g students in neighborhood g . Let Y_{ig} denote the outcome and let $\mathbf{X}_{ig}^W$ denote a $1 \times F_W$ row-vector of the level-1 predictors, both corresponding to student i in neighborhood g . Let $\mathbf{X}_g^G$ denote a $1 \times F_G$ row vector of neighborhood-level predictors.

Hierarchical Linear Model

Adopting the notation of Raudenbush and Bryk (2002, pp. 42-43), a hierarchical linear model (HLM) relates the outcome to the level-1 predictors as

$$\text{Level-1:} \quad Y_{ig} = \beta_{0g} + \mathbf{X}_{ig}^W \boldsymbol{\beta}_{1g} + u_{ig}, \quad u_{ig} \sim N(0, \sigma^2) \quad (1)$$

where β_{0g} is the intercept for neighborhood g , $\boldsymbol{\beta}_{1g}$ is a $F_W \times 1$ vector of parameters for neighborhood g , and u_{ig} is the level-1 error of student i in neighborhood g , which is assumed to be independent and normally distributed with a mean of 0 and homoscedastic variance of σ^2 .

Further, the predictors $\mathbf{X}_{ig}^W$ and error u_{ig} are assumed to be independent.

In the HLM framework, the level-2 model specifies how the group-specific parameters vary across level-2 groups. The simplest such model allows the intercept terms to vary across neighborhoods, while assuming that the remaining parameters are fixed. This model is given by

$$\begin{aligned} \text{Level-2:} \quad \beta_{0g} &= \gamma_{00} + \mathbf{X}_g^G \boldsymbol{\gamma}_{01} + b_{0g}, \quad b_{0g} \sim N(0, \tau_{G00}), \\ \boldsymbol{\beta}_{1g} &= \boldsymbol{\gamma}_{10} \end{aligned} \quad (2)$$

where γ_{00} is the average intercept, $\boldsymbol{\gamma}_{01}$ is an $F_G \times 1$ vector of coefficients for $\mathbf{X}_g^G$, and $\boldsymbol{\gamma}_{10}$ is a $F_W \times 1$ vector of coefficients for $\mathbf{X}_{ig}^W$. Possible elaborations of this model include allowing some of the slopes to vary across groups or allowing interactions between level-1 and level-2 predictors.

The level-2 random effects, b_{0g} , are assumed to be mutually independent and to follow a normal distribution with mean 0 and variance τ_{G00} . Like the level-1 errors, the level-2 random effects are assumed to have homogeneous variance across the clusters. The level-2 random effects are independent of the level-1 errors and independent of the covariates (i.e., the exogeneity assumption). Consider an example where the level-1 predictors include an indicator for whether a student receives individualized tutoring. The exogeneity assumption would be

violated here if students in higher-performing schools (with larger b_{0g} 's) are more likely to receive tutoring. HLMs are often estimated using restricted maximum likelihood (REML). If the number of clusters is small, REML yields more accurate SE estimates and variance components than maximum likelihood (ML; Maas & Hox, 2004).

One-Way OLS-CRVE

An alternative to HLM is to combine ordinary least squares (OLS) regression with CRVE. Combining Equations (1) and (2), the random intercept HLM can be represented as:

$$Y_{ig} = \gamma_{00} + \mathbf{X}_{ig}^W \boldsymbol{\gamma}_{10} + \mathbf{X}_g^G \boldsymbol{\gamma}_{01} + e_{ig}, \quad (3)$$

where $e_{ig} = b_{0g} + u_{ig}$ is the total error term for student i in neighborhood g . Compared to HLM, which distinguishes between level-1 random error (u_{ig}) and level-2 random error (b_{0g}), this representation involves a combined random error at a single level, implying that the errors are not independent.

Let $\mathbf{Y}_g$ be the $n_g \times 1$ vector of outcomes for students in neighborhood g , $\mathbf{X}_g$ be the $n_g \times (1 + F_G + F_W)$ matrix that collects all of the covariates across all levels of the model (including the intercept term), for the n_g students in neighborhood g , and $\mathbf{e}_g$ be the $n_g \times 1$ vector of combined errors for students in neighborhood g . Denote the corresponding vectors or matrices stacked across clusters $g = 1, \dots, G$ as $\mathbf{Y}$, $\mathbf{X}$, and $\mathbf{e}$, respectively. Letting $\boldsymbol{\gamma}$ denote the $(1 + F_G + F_W) \times 1$ vector collecting all the regression coefficients, Equation (3) can then be written compactly as $\mathbf{Y}_g = \mathbf{X}_g \boldsymbol{\gamma} + \mathbf{e}_g$ for $g = 1, \dots, G$, or as $\mathbf{Y} = \mathbf{X} \boldsymbol{\gamma} + \mathbf{e}$.

The OLS estimator for $\boldsymbol{\gamma}$ is

$$\hat{\boldsymbol{\gamma}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} = \left(\frac{1}{N} \sum_{g=1}^G \mathbf{X}_g' \mathbf{X}_g \right)^{-1} \left(\frac{1}{N} \sum_{g=1}^G \mathbf{X}_g' \mathbf{Y}_g \right). \quad (4)$$

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Letting $\mathbf{B} = \left(\frac{1}{N}\mathbf{X}'\mathbf{X}\right)^{-1}$ and $\bar{n} = \frac{1}{G}\sum_{g=1}^G n_g$, the variance of the OLS estimator is

$$Var(\hat{\boldsymbol{\gamma}}) = \frac{1}{N}\mathbf{B}\mathbf{M}\mathbf{B}, \quad \text{where} \quad \mathbf{M} = \frac{1}{\bar{n}G}\sum_{g=1}^G \mathbf{X}'_g \boldsymbol{\Omega}_g \mathbf{X}_g \quad (5)$$

and $\boldsymbol{\Omega}_g = E[\mathbf{e}_g \mathbf{e}'_g | \mathbf{X}_g]$ is the $N_g \times N_g$ covariance matrix of errors in cluster g .

In one-way OLS-CRVE, the correlation of errors within a cluster are accounted for when estimating the variance of $\hat{\boldsymbol{\gamma}}$. Specifically, $\boldsymbol{\Omega}_g$ in the central matrix $\mathbf{M}$ is estimated using $\hat{\mathbf{e}}_g \hat{\mathbf{e}}'_g$, where $\hat{\mathbf{e}}_g = \mathbf{Y}_g - \mathbf{X}_g \hat{\boldsymbol{\gamma}}$ is the vector of residuals for cluster g . Thus, the estimated central matrix for CRVE is $\hat{\mathbf{M}}^{CR} = \frac{1}{\bar{n}G}\sum_{g=1}^G \mathbf{X}'_g \hat{\mathbf{e}}_g \hat{\mathbf{e}}'_g \mathbf{X}_g$. (See Supplementary S2.1 for other ways of representing this calculation.) The cluster-robust variance matrix is then

$$\mathbf{V}^{CR}(\hat{\boldsymbol{\gamma}}) = \frac{1}{N}\mathbf{B}\hat{\mathbf{M}}^{CR}\mathbf{B} \quad (6)$$

As G increases to infinity, $\hat{\mathbf{M}}^{CR}$ converges to the true $\mathbf{M}$ (Cameron et al., 2011) because the estimated central matrix involves taking an average across clusters. Therefore, $\mathbf{V}^{CR}$ will be a consistent estimator for the sampling variance of the OLS estimator $\hat{\boldsymbol{\gamma}}$, provided the number of clusters is sufficiently large.

Compared to HLM, one-way OLS-CRVE involves somewhat weaker assumptions. Because the level-2 random effects are not modeled explicitly, the distributional assumptions for random effects are not required, and level-1 errors do not need to be homoscedastic. However, OLS-CRVE does not provide estimates of variance components for cluster-level errors. In terms of estimation, the OLS estimator does not use the covariance structure of the errors to estimate the regression coefficients, leading to less precise estimates than the REML estimator of HLM in unbalanced data (McNeish et al., 2017). Finally, both HLM and OLS require the assumption of exogeneity between covariates and errors.

One-Way FE-CRVE

Fixed effects (FE) models represent a further alternative to HLM. Like OLS, they involve a single-level estimating equation and so avoid some of the assumptions and complexity of HLM. Moreover, unlike OLS and HLM, the FE model does not require the exogeneity assumption to provide unbiased parameter estimates.

The one-way FE model is closely related to the HLM, but omits the intercept and the level-2 predictors. Just as in Equation (1),

$$Y_{ig} = \mathbf{X}_{ig}^W \boldsymbol{\gamma}_{10} + b_{0g} + u_{ig}. \quad (7)$$

Note that the FE model only includes the student-level covariates ($\mathbf{X}_{ig}^W$). Compared to the error e_{ig} in the OLS estimating equation, the error here is divided into the cluster-level intercept b_{0g} and the level-1 error u_{ig} . There are two key differences between FE and HLM. First, FE treats the cluster-level intercepts not as random effects, but as unique parameters—called “fixed effects”—without imposing any further distributional assumptions. This distinction allows the FE estimator to remain unbiased even when the exogeneity assumption is violated. Viewed as unique parameters, the b_{0g} become equivalent to the coefficients for cluster affiliation indicator (or dummy) variables. Second, the FE estimating equation does not involve level-2 predictors (i.e., $\mathbf{X}_g^G$ is not included) because such predictors are fully collinear with the cluster-specific intercepts (Greene, 2018, p. 396).

One way to estimate the FE model is via OLS estimation of Equation (7), including indicator variables for each cluster (Greene, 2012). However, this approach becomes infeasible when the number of clusters is large. An alternative is to use the deviations-from-mean estimator or “within” estimator (Angrist & Pischke, 2008; Wooldridge, 2010). Let $\bar{Y}_g$ denote the mean

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outcome, $\bar{\mathbf{X}}_g^W$ denote the average level-1 covariate vector, and $\bar{u}_g$ denote the average level-1 error term, all for neighborhood g . The within estimator is based on the equation:

$$\dot{Y}_{ig} = \dot{\mathbf{X}}_{ig}^W \boldsymbol{\gamma}_{10} + \dot{u}_{ig}, \quad (8)$$

where $\dot{Y}_{ig} = Y_{ig} - \bar{Y}_g$, $\dot{\mathbf{X}}_{ig}^W = \mathbf{X}_{ig}^W - \bar{\mathbf{X}}_g^W$ and $\dot{u}_{ig} = u_{ig} - \bar{u}_g$. The within estimator removes all between-cluster variability, including the influences of cluster-invariant variables (i.e., level-2 variables). Further, the within estimator allows cluster-level effects that are correlated with the covariates. This is a weaker assumption than what is needed for OLS-CRVE or HLM because it allows for the possibility of correlation between the b_{0g} 's and the covariates.

The within estimator of $\boldsymbol{\gamma}_{10}$ and the cluster-robust variance estimator are the same as the estimators presented in Equations (4) and (6) but using $\dot{\mathbf{X}}_g^W$ in place of $\mathbf{X}_g$ and $\dot{Y}_g$ in place of Y_g . The FE estimator is consistent for $\boldsymbol{\gamma}_{10}$ if either the number of clusters G or the number of observations per cluster n_g goes to infinity (Greene, 2012, p. 362). Findings from previous simulation studies indicate that the FE estimator shows higher power and better confidence interval coverage rates than OLS-CRVE when the data included a small number of clusters (Huang, 2016; McNeish & Stapleton, 2016). However, just as with OLS, using CRVE to estimate the sampling variance of the FE estimator requires a large number of clusters.

There is a close relationship between the FE model and the HLM with group mean-centered predictors (Allison, 2009; Hamaker & Muthén, 2020; Raudenbush & Bryk, 2002). In fact, the coefficients for group-mean centered predictors in a random-intercepts HLM are *numerically identical* to the FE estimates. Thus, group-mean centering and related techniques provide a way to address the exogeneity assumption while still retaining the other advantages of HLM, such as the ability to incorporate level-2 predictors and estimate substantively meaningful variance components (Hamaker and Muthén, 2020; McNeish & Kelley, 2019). However, we do

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not elaborate further on group mean centering because the present study focuses on estimation of coefficients of level-1 predictors, where the bias and accuracy of group-mean centered HLM and FE will be equivalent.

Two-Way Clustering

Each of the methods that we reviewed in the context of one-way clustering has a direct extension to two-way, cross-classified data structures. Let us now consider a data structure where N students are cross-classified within neighborhoods $g \in \{1, 2, \dots, G\}$ and schools $h \in \{1, 2, \dots, H\}$, where g and h are implicitly functions of i . Let $Y_{i(gh)}$ denote the outcome for student i within neighborhood g and school h and let $\mathbf{X}_{i(gh)}^W$ denote a $F_W \times 1$ vector of level-1 predictors for the student i in neighborhood g and the school h . The notations g and h in parentheses indicate that both indices are at level-2. Let $\mathbf{X}_g^G$ be a $1 \times F_G$ row vector of level-2 covariates for neighborhood g and $\mathbf{X}_h^H$ be a $1 \times F_H$ row vector of level-2 covariates for school h .

Cross-Classified Random Effects Model

The CCREM extends HLM in order to allow for dependence in both dimensions of clustering. The level-1 equation for CCREM is

$$\text{Level-1:} \quad Y_{i(gh)} = \beta_{0(gh)} + \mathbf{X}_{i(gh)}^W \boldsymbol{\beta}_{1(gh)} + u_{i(gh)}, \quad (9)$$

where $\beta_{0(gh)}$ is the intercept for neighborhood g and school h , $\boldsymbol{\beta}_{1(gh)}$ an $1 \times F_W$ vector of regression coefficients corresponding to the predictors $\mathbf{X}_{i(gh)}^W$, and $u_{i(gh)}$ is the level-1 error, which is assumed to be normally distributed and homoscedastic, with $u_{i(gh)} \sim N(0, \sigma^2)$. The level-2 equations describe how the parameters of Equation (9) vary across neighborhoods and schools. In the simplest such model, the intercepts are allowed to vary but the remaining slope coefficients are assumed to be constant across level-2 units, so that

$$\begin{aligned} \beta_{0(gh)} &= \gamma_{00} + \mathbf{X}_g^G \boldsymbol{\gamma}_{01} + \mathbf{X}_h^H \boldsymbol{\gamma}_{02} + b_{0g0} + c_{00h}, \\ \text{Level-2:} \quad \boldsymbol{\beta}_{1(gh)} &= \boldsymbol{\gamma}_{10} \end{aligned} \tag{10}$$

The terms $\boldsymbol{\gamma}_{01}$ and $\boldsymbol{\gamma}_{02}$ are $F_G \times 1$ and $F_H \times 1$ vectors of slope coefficients for $\mathbf{X}_g^G$ and $\mathbf{X}_h^H$, respectively. Finally, b_{0g0} and c_{00h} are random effects for neighborhoods and schools, respectively, which are assumed to follow normal distributions with means of 0 and variances of τ_{G00} and τ_{H00} , respectively. Just as in HLM, random effects are assumed to be independent of covariates at all levels. The variance components in CCREM can be described in terms of intra-unit correlation coefficients (IUCCs). For example, the conditional IUCC, or proportion of error variance, due to the neighborhood dimension is defined as

$$IUCC_{neighborhood} = \frac{\tau_{G00}}{\tau_{G00} + \tau_{H00} + \sigma^2}. \tag{11}$$

An advantage of CCREM is that it explicitly models and provides inference for regression coefficients (the $\boldsymbol{\gamma}$'s) for multiple dimensions of clusters. For example, neighborhood-level characteristics (e.g., average household size) and school-level characteristics (e.g., school Title I status) can be examined simultaneously. CCREM also provides estimates of variance components for multiple dimensions of random effects, which describe the variation in the outcome attributable to each dimension of clustering. If all its assumptions are met, the CCREM estimators of regression coefficients are expected to be more efficient compared to any other estimator, just as HLM estimators are more efficient than ordinary least squares regression (McNeish et al., 2017).

Less widely recognized is that CCREM makes stronger assumptions than some alternative approaches. First, CCREM is assumed to be correctly specified in terms of which student-level parameters are allowed to vary or to be constant across clusters. For the simple specification in Equation (10), we assume that none of the slope coefficients vary across clusters;

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we shall call this the “random intercepts” assumption. Another assumption is that the variance of level-1 errors is constant (homoscedasticity assumption). Further assumptions are that the random effects for each dimension are mutually independent and that the random effects at each level follow normal distributions. Finally, just as in HLM, the CCREM relies on the assumption that the covariates are independent of the random effects (exogeneity assumption).

Consider an example where student academic performance is influenced by student involvement in a reading intervention and neighborhood-level resources. However, the data do not include a measure of neighborhood-level resources, which are likely to be positively correlated with neighborhood-level average academic performance, even after controlling for student demographic characteristics and school characteristics. In addition, neighborhood-level resources might influence whether students participate in the intervention. Thus, the neighborhood-level resources might affect both whether students participate in the intervention and students' academic performance, creating correlation between the random effects and the predictor of participation in the intervention. In such a case, the exogeneity assumption will be violated because neighborhood-level resources are not included in the model.

Based on findings in the HLM literature, the validity of coefficient estimates and significance tests in the CCREM may depend on these assumptions being met (T. A. Snijders & Bosker, 2012, p. 230). If distributional assumptions of level-1 errors and level-2 random effects in HLM are violated, the SEs of coefficient estimates can be biased and the Type I error can be inflated (Luo & Kwok, 2009; Rasbash & Goldstein, 1994; Raudenbush & Bryk, 2002). If the exogeneity assumption is violated in HLM, then not only will SEs be affected, but the coefficient estimates themselves may be biased (Schielzeth et al., 2020). To our knowledge, the consequences of violating these assumptions in the CCREM have not been studied.

Two-Way OLS-CRVE

As an extension of one-way CRVE, Cameron et al. (2011) proposed two-way CRVE that provides cluster-robust inference for cross-classified data structures. Considering the same data structure as in CCREM, Equations (9) and (10) can be written as

$$Y_{i(gh)} = \gamma_{00} + \mathbf{X}_g^G \boldsymbol{\gamma}_{01} + \mathbf{X}_h^H \boldsymbol{\gamma}_{02} + \mathbf{X}_{i(gh)}^W \boldsymbol{\gamma}_{10} + e_{i(gh)}, \quad (12)$$

where $e_{i(gh)} = b_{0g0} + c_{00h} + u_{i(gh)}$ is the combined error for student i within neighborhood g and school h . The combined errors for students $i(gh)$ and $j(kl)$ will be dependent if they share the same neighborhood (i.e., $g = k$) or school ($h = l$), but will be independent if the students are in different neighborhoods and different schools ($g \neq k$ and $h \neq l$).

Following the same notation as previously, let $\mathbf{Y}$ be the $N \times 1$ vector of outcomes for observations, let $\mathbf{X}$ be the $N \times (1 + F_W + F_G + F_H)$ matrix that stacks $\mathbf{X}_{i(gh)} = (1, \mathbf{X}_g^G, \mathbf{X}_h^H, \mathbf{X}_{i(gh)}^W)$ across observations, and let $\boldsymbol{\gamma} = (\gamma'_{01}, \gamma'_{02}, \gamma'_{10})'$ be the full set of regression coefficients. Just as with one-way clustering, the OLS estimator of Equation (12) is

$$\hat{\boldsymbol{\gamma}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}. \quad (13)$$

However, the variance of $\hat{\boldsymbol{\gamma}}$ is more complex because the errors are dependent in multiple dimensions. Specifically, $\text{Var}(\hat{\boldsymbol{\gamma}}) = \frac{1}{N} \mathbf{B} \mathbf{M} \mathbf{B}$, where $\mathbf{B} = \left(\frac{1}{N} \mathbf{X}'\mathbf{X}\right)^{-1}$ and

$$\mathbf{M} = \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^N \mathbf{X}_{i(gh)} \mathbf{X}_{j(kl)}' \times \text{Cov}(\mathbf{e}_{i(gh)}, \mathbf{e}_{j(kl)}). \quad (14)$$

The two-way CRVE involves using products of residuals to estimate the covariances between errors for pairs of observations that are dependent because they share one or both clustering dimension. It is given by

$$\hat{\mathbf{V}}^{CR} = \frac{1}{N} \mathbf{B} \hat{\mathbf{M}}^{CR} \mathbf{B} \quad (15)$$

where the middle matrix is

$$\hat{\mathbf{M}}^{CR} = \frac{1}{N} \mathbf{X}'(\hat{\mathbf{e}}\hat{\mathbf{e}}' \circ \mathbf{S}^{GH})\mathbf{X}, \quad (16)$$

the notation $\circ$ means element-wise multiplication, and $\mathbf{S}^{GH}$ is an $N \times N$ matrix with entry i, j equal to 1 if they share a cluster in either of two dimensions and equal to 0 otherwise. The matrix $\mathbf{S}^{GH}$ can also be represented as $\mathbf{S}^{GH} = \mathbf{S}^G + \mathbf{S}^H - \mathbf{S}^{G \cap H}$, where $\mathbf{S}^G$ and $\mathbf{S}^H$ have entries of 1 only if observation i and j share the same cluster in the G and H dimension, respectively, and where $\mathbf{S}^{G \cap H}$ is a matrix that has the entry of 1 only if observation i and j share the same cluster in *both* dimensions. By substituting into Equation (16), we can see that the central matrix in two-way CRVE is comprised of a sum of three one-way CRVEs, the first clustering on neighborhood, the second clustering on school, and the third clustering on the intersection of neighborhood and school:

$$\hat{\mathbf{M}}^{CR} = \frac{1}{N} [\mathbf{X}'(\hat{\mathbf{e}}\hat{\mathbf{e}}' \circ \mathbf{S}^G)\mathbf{X} + \mathbf{X}'(\hat{\mathbf{e}}\hat{\mathbf{e}}' \circ \mathbf{S}^H)\mathbf{X} - \mathbf{X}'(\hat{\mathbf{e}}\hat{\mathbf{e}}' \circ \mathbf{S}^{G \cap H})\mathbf{X}]. \quad (17)$$

Thus, the two-way CRVE estimator is equivalent to the sum of the one-way CRVE estimators for each clustering dimension, minus the one-way CRVE for the intersection of the clustering dimensions. The matrix $\hat{\mathbf{M}}^{CR}$ is an asymptotically consistent estimator of the central matrix as the number of clusters in *both* G and H dimensions go to infinity (Cameron et al., 2011).

The primary advantage of two-way OLS-CRVE over CCREM is that OLS-CRVE involves fewer assumptions. Specifically, because the level-2 random effects are not modeled explicitly, the distributional assumptions for random effects are not required. Level-1 errors and level-2 random effects need not be independent. Two-way OLS-CRVE also avoids the complexity of modeling random effects and covariance structures as in CCREM. Researchers can implement this method using several readily available R packages, including the lfe (Gaure, 2013, 2014) and the fixest packages (Berge, 2018).

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Compared to the CCREM, potential drawbacks of two-way OLS-CRVE may be similar to those of one-way OLS-CRVE versus HLM. Two-way OLS-CRVE does not provide variance component estimates for the random effects in each dimension of clustering. The estimation of the coefficients in two-way OLS-CRVE may be less efficient than CCREM, particularly if the clusters vary widely in size or are unbalanced. Finally, both CCREM and OLS-CRVE require the exogeneity assumption, i.e., that the errors are not correlated with the covariates. Violation of this assumption could lead to bias in the OLS estimator $\hat{\mathbf{Y}}$ and, consequently, to mis-calibrated hypothesis tests and CIs.

Two-Way FE-CRVE

The logic of the one-way FE can be extended to handle multiple dimensions of clustering, which provides a way to avoid the exogeneity assumption and further distributional assumptions of the CCREM. The two-way FE model can be written as

$$Y_{i(gh)} = \mathbf{X}_{i(gh)}^W \mathbf{Y}_{10} + b_{0g0} + c_{00h} + u_{i(gh)}, \quad (18)$$

where $u_{i(gh)}$ is the level-1 random error of student i from neighborhood g and school h , $\mathbf{X}_{i(gh)}^W$ is the $1 \times F_W$ vector of level-1 predictors, and b_{0g0} and c_{00h} are cluster-invariant errors in the neighborhoods and schools, respectively. Compared to the CCREM, the key difference in FE is that b_{0g0} and c_{00h} are estimated as fixed effects of specific dimensions of clusters, rather than as random effects. Treating the b_{0g0} and c_{00h} as fixed effects accounts for cluster-invariant factors that lead to dependence between observations that share a clustering dimension. Because the intercepts are treated as fixed effects, level-2 predictors ($\mathbf{X}_g^G$ and $\mathbf{X}_h^H$) are not included in Equation (18).

Similar to one-way FE, two-way FE uses a generalization of the within-estimator to calculate the coefficient estimates of fixed effects $\hat{\mathbf{Y}}_{10}$, called the Method of Alternating

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Projections (Gaure, 2013b). This method avoids the computational problems that arise from treating the fixed effects as indicator variables and using OLS to estimate Equation (18). It can therefore be applied even when the model includes a large number of clusters in one or both dimensions. Section S2.2 of the supplementary materials provides further computational details.

Two-way FE controls dependence in the clustering dimensions that are specified in the model. However, it will often be sensible to combine the FE model with two-way CRVE in order to account for any remaining dependence in the errors within a clustering dimension, which might not be fully captured by the cluster-specific intercepts (Cameron & Miller, 2015, pp. 329-330). For example, such dependence might occur if data include only neighborhood and school identifiers, but where students are actually nested within classrooms nested within schools. In that case, dependency due to the classroom is not captured by the fixed effects. As another example, the random slope for the neighborhood covariates might be mistakenly omitted even when the impact of neighborhood covariates varies in the outcome variable. Furthermore, in repeated measures data, errors within a cluster may be auto-correlated, a form of dependence not captured by the fixed effects. Across these examples, the assumption that the errors $u_{i(gh)}$ are independent and homoscedastic might still be violated.

Two-way FE-CRVE requires fewer assumptions than either CCREM or OLS-CRVE. Two-way FE-CRVE does not assume any distribution for level-2 random effects, instead treating the error from each cluster as a parameter to be estimated. Moreover, two-way FE-CRVE does not involve assuming that the random effects are independent of covariates, which relaxes the exogeneity assumption required in both CCREM and two-way OLS-CRVE. The fixed effects estimator is still consistent and unbiased even when the cluster-specific effects and covariates are correlated, whereas CCREM and two-way OLS-CRVE no longer provides consistent and

unbiased coefficients. However, estimates of two-way FE-CRVE may be less efficient than CCREM when the assumptions of the latter model hold—just as HLM may be more efficient than fixed effects models with one-way clustering (McNeish & Kelley, 2019, p. 26; Wooldridge, 2010). The cluster-specific dummy variables consider heterogeneity in both dimensions, so researchers only need to focus on the level-1 covariates of interest. However, FE-CRVE does not provide a way to estimate the associations of cluster-level covariates because the cluster coefficient absorbs any effect specific to that cluster. For the same reason, level-2 random effects are not estimated in two-way FE-CRVE. Differences in the assumptions of each method are summarized in the supplementary materials (see Supplementary Table S2.1).

The benefits of FE and CCREM can be melded through the use of adaptive centering methods (Raudenbush, 2009), which are a generalization of group-mean centering in HLM. Just as group-mean centering leads to coefficient estimates identical to those from one-way FE, using adaptively centered predictors in CCREM leads to coefficient estimates identical to those from two-way FE, while still retaining the ability to incorporate level-2 covariates and estimate variance components (Raudenbush, 2009). However, because of the equivalence of the point estimators for level-1 coefficients, we do not consider adaptively centered CCREM in the empirical application or simulation study presented in subsequent sections. We offer further observations about the technique in the discussion section.

Mixing fixed effects and random effects

Another alternative to the CCREM is to treat one clustering dimension as fixed effects and another clustering dimension as random effects. This approach can be expressed in the same way as Equation (18), but in this case, the cluster-invariant error b_{0g0} is interpreted as a random effect. This approach is effective when researchers want to focus on one clustering dimension

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while controlling the other clustering dimensions. For example, if the focus is on the neighborhood effects, one can model the neighborhood clustering dimension with random effects (b_{0g0})—and potentially also include neighborhood-level predictors in the model—while treating the school clustering dimension as fixed effects (c_{00h}).

This approach interpolates between CCREM and two-way FE. One benefit is that it does not require any special software and can be implemented using routines for estimating HLMs. The clustering dimension treated as fixed effects is simply modeled with indicator variables. For the clustering dimensions modeled by random effects, level-2 covariates can be incorporated, although covariates from the clustering dimensions treated as fixed effects still cannot be estimated. Further, this fixed-and-random approach requires weaker assumptions than CCREM (but stronger assumptions than FE). Specifically, no distributional assumptions are required for the dimension modeled by fixed effects, and covariates may be correlated with the cluster-specific effects in this dimension. Still, distributional assumptions are needed for the clustering dimension modeled by random effects. When all CCREM assumptions are met, this approach provides less efficient estimates than CCREM, but more efficient estimates than two-way FE.

Previous Research

Few studies have examined alternatives to the CCREM that can accommodate a cross-classified data structure. As part of the paper that introduced multi-way CRVE, Cameron et al. (2011) showed that when the homoscedasticity assumption was violated, two-way FE-CRVE provides the rejection rates closer to the nominal level than two-way OLS-CRVE. However, their simulation study limited consideration to a panel data structure where individuals are cross-classified by year and state (balanced data) and did not include CCREM. With other cross-classified data structures (such as neighborhood-by-school), many of the cross-classification

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cells may be rarely observed or even empty. Thus, the performance of CCREM needs to be compared to that of other methods with unbalanced data structures, particularly under conditions with heteroskedasticity and other forms of model misspecification. Furthermore, other data-generating conditions, such as IUCC, have been considered specifically in simulation studies of CCREM (Beretvas & Murphy, 2013; Meyers & Beretvas, 2006) but not for the alternatives of OLS or FE.

Empirical Example

We compared CCREM, two-way OLS-CRVE, and two-way FE-CRVE in an empirical application based on Garner and Raudenbush (1991). The data include 2,310 students from 17 schools living in 524 neighborhoods, who left school between 1984 and 1986 in Scotland (see Supplementary Table S3.1 for sample demographics). The outcome variable is educational attainment, a composite score of two national examinations. The level-1 covariates included seventh-grade verbal reasoning quotient (VRQ), seventh-grade reading test score (Reading), and demographic variables (Dad's occupation, Dad's education, Mom's education, Dad's unemployment, and individual's gender). The sole level-2 covariate is neighborhood social deprivation. The original analysis applied HLM to examine the impact of neighborhood social deprivation on students' educational attainment while ignoring the school dimension. Raudenbush (1993) expanded this analysis to CCREM, considering school membership as a second dimension of clustering.

Table 1 presents the coefficient estimates and corresponding SEs using CCREM, two-way OLS-CRVE, and two-way FE-CRVE. Most coefficient estimates of level-1 covariates were similar across all the methods. This similarity might be due to the large sample size or low IUCC ($\rho_{IUCC} = .009$ for both neighborhood and school dimensions). However, for the Dad Education

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predictor, two-way FE-CRVE produced a smaller coefficient estimate (.106) than CCREM (.144) or two-way OLS-CRVE (.142). This difference in the coefficient estimates might be due to the difference between within-cluster and between-cluster effects on the covariate. Because between-cluster effects are absorbed by the fixed effects, only within-cluster effects are estimated in two-way FE-CRVE (see details in Supplementary Table S3.2).

The methods differed in terms of the SEs of the coefficient estimates for Dad Unemployed and Male. For these covariates, the SEs from two-way FE-CRVE were greater than those from the other methods. These differences in SEs led to different statistical significance levels for the Dad Unemployed predictor (significant in CCREM and two-way OLS-CRVE, but not significant with two-way FE-CRVE). Only CCREM showed statistically significant coefficient estimates for the Male predictor, while the other two approaches had insignificant results. These results highlight that the statistical inference of the covariates may vary depending on the method chosen by the researcher.

Further, the coefficient estimates of covariates in two-way FE-CRVE are not the same as those in CCREM and two-way OLS-CRVE. Because the FE approach removes all between-cluster variability, the coefficient estimates in FE indicate the partial association between the predictor and the outcome, after accounting for all between-cluster variation in the level of the outcome. In contrast, the coefficient estimates from the other two approaches are pooled effects of the covariate. Thus, the coefficient estimates across FE and the other two approaches are similar only when the between-cluster effects of the covariates are the same as the within-cluster effects (i.e., only if the within-cluster effects are similar to the pooled effects of the covariate). We also note that Neighborhood Deprivation, the cluster-invariant variable, was not explicitly estimated with FE-CRVE. FE controls all neighborhood-level differences in the level of the

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outcome, including variation explained by Neighborhood Deprivation. On the other hand, the coefficient estimates from the other approaches included Neighborhood Deprivation.

We also examined an alternative way of estimating Neighborhood Deprivation by modeling the neighborhood dimension with random effects while controlling the school dimension with fixed effects (see the results in the supplementary materials, Table S3.2). The level-1 covariates in FE+RE showed similar coefficient estimates to those of CCREM, but differed in a few instances (e.g., Dad Education) compared to the FE-CRVE estimates. This intermediate approach provided an estimate of the level-2 Neighborhood Deprivation covariate while controlling both dimensions with fewer assumptions than CCREM.

The results from this one example are not necessarily indicative of the general performance of these methods. Therefore, we conducted a Monte Carlo simulation study to investigate how the approaches perform across a broad range of conditions.

Simulation Study

We conducted a Monte Carlo simulation study to assess the performance of CCREM, two-way OLS-CRVE, and two-way FE-CRVE for analyzing cross-classified data structures. We considered unbalanced designs relevant to psychological and educational research, such as data on students who are cross-classified by neighborhoods and schools. We assessed performance for estimating level-1 and level-2 coefficients in terms of the parameter bias, root mean squared error (RMSE), and 95% CI coverage.

Data Generation

We generated cross-classified data using a CCREM for data on students cross-classified into one of G neighborhoods and H schools. We generated data involving a single student-level predictor, a single neighborhood covariate, and a single school covariate. In the most general

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specification, we allowed the relationship between the student-level predictor and the outcome to vary across neighborhoods, with a random slope b_{1g0} . Thus, the combined level-1 and level-2 equation was

$$Y_{i(gh)} = \gamma_{00} + \gamma_{01}X_g^G + b_{0g0} + \gamma_{02}X_h^H + c_{00h} + \gamma_{10}X_{i(gh)}^W + b_{1g0}X_{i(gh)}^{Within} + u_{i(gh)}. \quad (19)$$

Data generation entailed four steps: generating ID variables, generating covariates, generating level-2 random effects and level-1 errors, and finally, combining the components of the dependent variable.

We first generated ID variables for H schools with a constant number of students per school. We then generated neighborhood IDs for each student, following a process whereby students are assigned to 10% of all possible neighborhood-school combinations—a degree of sparsity similar to that observed in Garner and Raudenbush's (1991) data. (See Supplementary Materials Section S4.1 for further details.)

Just as the errors in a null model of CCREM are divided into random effects of neighborhoods, schools, and individuals, level-1 covariates in CCREM can be decomposed into neighborhood-, school-, and individual-level variation. After assigning neighborhood and school IDs to all students, we generated a level-1 covariate that included between-neighborhood and between-school variation, $X_g^W \sim N(0, 20)$ and $X_h^W \sim N(0, 20)$, and within-cell variation, $X_{i(gh)}^{Within} \sim N(0, 60)$. Then, we added three components, taking $X_{i(gh)}^W = X_{i(gh)}^{Within} + X_g^W + X_h^W$ for the level-1 covariate. Thus, the proportion of variance in this covariate is .2 for both the neighborhood and school dimensions. We also generated level-2 covariates X_g^G and X_h^H for each neighborhood and school, respectively, following standard normal distributions. We generated these level-2 covariates independent of each other and of the level-1 covariate.

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Third, to examine the performance of the estimation methods when the exogeneity assumption is violated, we generated neighborhood random effects b_{0g0} using a process that allowed for correlation between the level-1 covariate. We simulated the neighborhood random effects to have mean 0, variance τ_{G00} , and a correlation of r_g with the neighborhood-level component of the level-1 covariate, according to

$$b_{0g0} = r_g \times \sqrt{\frac{\tau_{G00}}{20}} \times X_g^W + v_{0g0} \quad (20)$$

where v_{0g0} follows a normal distribution with a mean of 0 and a standard deviation (SD) of

$\sqrt{(1 - r_g^2)\tau_{G00}}$. We simulated the school random effects c_{00h} to have mean zero and variance τ_{H00} . We varied τ_{H00} while fixing τ_{G00} at .05. Further, we generated the random slope b_{1g0} to have a mean of 0 and a variance of τ_{G10} , independent of the random intercept and of the covariates.

Fourth, we generated level-1 errors as $u_{i(gh)} \sim N(0, v_{i(gh)})$ to be either homoscedastic or heteroskedastic, the latter of which violated an assumption of the CCREM. For homoscedastic conditions, we set the variance $v_{i(gh)} = \sigma^2 = 1 - \tau_{G00} - \tau_{H00}$, so that the total variance of the outcome was constant at 1. For heteroskedastic conditions, we generated level-1 errors $u_{i(gh)}$ based on a normal distribution with mean zero and variance that was a function of the level-1 covariate, as $v_{i(gh)} = \sigma^2 \times \exp\left(\frac{15X_{i(gh)} - 50}{15^2}\right)$. This ensured that the *average* variance across the distribution of $X_{i(gh)}$ remained equal to σ^2 . More details are available in the supplementary materials (S4.2). Finally, we combined the covariates, random effects, and error terms according to Equation (19) to generate the dependent variable $Y_{i(gh)}$.

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We generated the data using custom-written R code, available in the accompanying supplementary materials. For each simulated dataset, we estimated the CCREM, two-way OLS-CRVE, and two-way FE-CRVE using R 4.0.4 (R Core Team, 2021). As in the empirical example, we estimated the CCREM using the `lmer()` function in `lme4` package (Bates et al., 2015) and `lmerTest` package for the significance test via Satterthwaite's degrees of freedom method (Kuznetsova et al., 2017). We estimated two-way OLS-CRVE and two-way FE-CRVE using the `felm()` function from the `lfe` package (Gaure, 2013a). Significance tests were conducted using cluster-robust Wald tests.

Simulation Design

As listed in Table 2, we manipulated five design factors that might influence the performance of the estimation methods: (1) CCREM assumptions, (2) effect size of coefficients for level-1 covariates, (3) the number of level-2 clusters in each of the two dimensions, (4) the average number of level-1 students within level-2 clusters (schools), and (5) IUCC for between-school variance. For several of these factors, we selected conditions based on evidence from empirical studies.

CCREM assumptions

We examined four distinct conditions related to CCREM assumptions. First, we generated a random intercept model to meet both the homoscedasticity and exogeneity assumptions, using a constant level-1 variance and setting the variance of the random slope (τ_{G10}) and correlation between covariates and neighborhood effects (r_g) to zero. Second, we generated data with heteroskedastic errors, while maintaining the random intercepts ($\tau_{G10} = 0$) and exogeneity ($r_g = 0$) assumptions. Third, we generated data where the exogeneity assumption is violated by setting a positive correlation between covariates and neighborhood effects (r_g),

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while maintaining the random intercepts ($\tau_{G10} = 0$) and homoscedasticity assumptions. In our empirical example, the correlation between continuous predictors and neighborhood random effects varies from .056 to .078. Castellano et al. (2014) examined values of .2, .4, and .6 for this correlation, based on 1982 High School and Beyond (HSB) survey data measuring students' mathematics achievement scores clustered within schools (Raudenbush & Bryk, 2002). The correlation between SES and the school random intercept was .36. Although the HSB data example is one-way cluster data, we used the median value of 0.4 to generate a substantial violation of the exogeneity assumption. Finally, we generated a random slope model that meets the homoscedasticity and exogeneity assumptions, by setting a positive random slope variance ($\tau_{G10} = .05$). Under this condition, the generated data is fitted to the random intercept model, so the CCREM is mis-specified.

Average effect size on coefficients for level-1 and level-2 covariates

We used values for the regression coefficients (γ_{10} , γ_{01} and γ_{02}) chosen to correspond to correlations of $r = .1$, $.3$, or $.5$. For example, for the smallest effect size, the coefficient for the level-1 covariate $X_{i(gh)}^W$ is $\gamma_{10} = .1 \times \left(\frac{1}{10}\right) = .01$ where the SDs of covariate $X_{i(gh)}^W$ and outcome variable $Y_{i(gh)}$ are 10 and 1, respectively.

Number of cross-classified dimensions

In empirical datasets collecting neighborhood and school clusters, the number of neighborhoods (G) and schools (H) are generally different from each other, as students attending a given school may come from several different neighborhoods. Among 177 CCREM studies published in the last decade, we randomly selected 50 studies and examined the number of clusters (see Supplementary Table S1.3). We set the sample size conditions for the simulations based on the data structures of these studies. The first and third quantiles of the smaller

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clustering dimension were 25 and 120, respectively. Also, the first and third quantiles of the larger clustering dimension ranged from 86 to 465. The ratio of the two clustering dimensions had a median of 4 and a mode of 3. Therefore, we selected a small, medium, and large number of groups on each dimension to follow a ratio of 3.5 neighborhoods per school. Accordingly, we set the numbers of neighborhoods and schools as 70 and 20 (small), 245 and 70 (medium), or 525 and 150 (large).

Number of students per school

In the empirical studies, the first and third quantiles of level-1 units per cluster are 27 and 126, respectively (see Supplementary Table S1.3). Previous simulation studies used relatively smaller sample sizes of 20 and 40 students per school (Meyers & Beretvas, 2006) or 10 students per neighborhood (Luo & Kwok, 2009). Based on this, we set an average school sample size of 30 or 100 students per school.

IUCC

Based on applied studies and multilevel modeling textbooks, conditional IUCC values for CCREM ranged from .01 to .24 (Meyers & Beretvas, 2006). Past simulation studies have examined conditional IUCC values between .05 and .15, or .15 and .3 (Beretvas & Murphy, 2013; Meyers & Beretvas, 2006). To encompass IUCCs in different situations, we considered three conditional IUCC conditions: .05 (small), .15 (medium), or .25 (large). Considering that IUCCs differed across clustering dimensions in most empirical studies, we manipulated only the neighborhood IUCC while fixing the school IUCC at 0.05.

Performance Criteria

We assessed the performance of the CCREM, two-way OLS-CRVE, and two-way FE-CRVE estimators for level-1 and level-2 coefficients based on three criteria: (1) parameter bias,

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(2) RMSE, and (3) 95% CI coverage (Morris et al., 2019). Because REML is an iterative algorithm that does not always converge, we also tracked convergence rates of the CCREM (see Supplementary Figure S4.1). We conducted 1,000 replications per condition, which was sufficient to achieve Monte Carlo SE of RMSE estimates of no more than .0036, or less than 5%, on average. We conducted ANOVA to determine to what extent the performance of two alternative methods, OLS-CRVE and FE-CRVE, differ from that of CCREM, using partial eta-squared (η_p^2) to measure effect size. We also examined which simulation conditions substantively affected each performance criterion.

Simulation Results

For each performance criterion, we depicted results for the level-1 covariate using boxplots to facilitate comparison across the estimation methods. We illustrated variation across conditions using small multiples (faceted) plots, focusing on factors that had stronger influences on performance. Within each plot, boxplots depict the median and range of a performance criterion by method, across all simulation conditions not represented in the rows or columns of the figure. The supplementary materials (Section S6) include results for the level-2 covariate.

Parameter Bias

Parameter bias describes the difference between an estimator and a true value on average. Rules of thumb for acceptable levels of parameter bias are varied across prior simulation studies (Burton et al., 2006, p. 4, 287; Harwell, 2020, p. 3), but lower parameter bias indicates better performance. For interpretation, we focused on the magnitude of the parameter bias compared to the size of the coefficient (see Supplementary Table S5.1). Figure 1 depicts the range of biases by method. When all assumptions were met, the parameter bias of all three methods was very close to zero. When the homoscedasticity assumption was violated, the parameter bias of the

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three methods was still very low, and differences between methods were also minimal. Under both of these conditions, the parameter bias was less than 1% of the effect size.

In contrast, when the exogeneity assumption was violated, the parameter bias of the three methods differed. Both CCREM and OLS-CRVE showed larger parameter bias than FE-CRVE, with the bias of two-way OLS-CRVE consistently larger than that of CCREM. ANOVA results showed that the methods differed substantially in parameter bias ($\eta_p^2 = .995$; Table S5.2). The number of students per school also showed a large effect on parameter bias ($\eta_p^2 = .747$). Specifically, a larger number of students per school resulted in less parameter bias for CCREM. The average parameter bias relative to the three effect size conditions (0.01, 0.03, or 0.05) was 0.30 for CCREM and 0.56 for OLS-CRVE. In contrast, two-way FE-CRVE had parameter bias very near zero across conditions, consistent with findings from previous studies on one-way clustering (McNeish & Kelley, 2019).

In the conditions with un-modeled random slopes, parameter bias was greater than under the first two conditions but less than under conditions where the exogeneity assumption is violated. The parameter bias in this condition is influenced by effect sizes ($\eta_p^2 = .142$). We note that Monte Carlo SEs were larger under this condition compared to other assumption conditions, which led to greater variation in bias estimates (for the detail, see Supplementary Table S5.3).

Root Mean Squared Error

RMSE measures the overall accuracy of an estimator on the same scale as the parameter, with smaller RMSE indicating greater accuracy (Burton et al., 2006). In Figure 2, we displayed the RMSE of each estimator, standardized by the square root of total sample size (i.e.,

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$RMSE \times \sqrt{N}$) because raw RMSE is strongly affected by sample size. Figures depicting unstandardized RMSE are available in the supplementary materials (Figure S5.1).

When all the assumptions were met, two-way OLS-CRVE showed a larger RMSE than the other two methods as the number of students per school increased ($\eta_p^2 = .936$, Table S5.4). Two-way FE-CRVE performed better than two-way OLS-CRVE and showed similar RMSE to CCREM when the number of students per school was large. RMSE also depended on the number of schools ($\eta_p^2 = .982$) and IUCC ($\eta_p^2 = .353$; Figure S5.2).

With a violation of the homoscedasticity assumption, CCREM still showed lower RMSE than two-way OLS-CRVE or FE-CRVE across all conditions. The RMSE of two-way OLS-CRVE was even higher than that of two-way FE-CRVE. The number of schools ($\eta_p^2 = .982$) and the in-school sample size ($\eta_p^2 = .936$) strongly affected RMSE, as expected. Although not as much as other conditions, the IUCC also had a strong influence ($\eta_p^2 = .353$; Table S5.4).

When the exogeneity assumption was violated, two-way FE-CRVE performed better than two-way OLS-CRVE and CCREM because all FE assumptions still held. In contrast, two-way OLS-CRVE showed greater RMSE than even CCREM. In this condition, the performance of three methods was sensitive to the number of schools ($\eta_p^2 = .959$), number of students per school ($\eta_p^2 = .927$), and IUCC ($\eta_p^2 = .067$). RMSE tended to decrease as the number of schools increased or the number of students per school decreased, and the relative advantage of FE-CRVE over CCREM also increased. When IUCC was high as .25, two-way OLS-CRVE showed a higher RMSE, whereas CCREM and two-way FE-CRVE showed little change (Figure S5.2).

The random slope condition showed the opposite result, with OLS-CRVE performing better than other conditions. As under other conditions, the number of schools ($\eta_p^2 = .959$) and the students per school ($\eta_p^2 = .927$) influenced the magnitude of RMSE.

Confidence Interval Coverage

CI coverage measures the proportion of 95% interval estimates that contain the true parameter. It is influenced both by the bias of the point estimator and by the accuracy of the corresponding SEs. Figure 3 depicts the CI coverage across conditions. Note that the scale of the Y-axis in Figure 3 differs across the assumption conditions.

When all assumptions were met, the CI coverage of CCREM and two-way FE-CRVE were within the acceptable range. Two-way OLS-CRVE showed CI coverage within the acceptable range except when the number of schools was 20 ($\eta_p^2 = .513$, Table S5.5). This lower CI coverage is consistent with prior research indicating that CRVE can be biased downward when the number of clusters is limited (Cameron et al., 2011; Cameron & Miller, 2015).

When the homoscedasticity assumption of CCREM was violated, the CI coverage of CCREM tended to be out of the acceptable range across the conditions. However, the CI coverage of two-way OLS-CRVE was in the acceptable range except when the number of schools was 20. Two-way FE-CRVE maintained CI coverage within the acceptable range, similar to the conditions where all assumptions were met. In both CRVE methods, CI coverage rates drew closer to the nominal level as the number of schools increased ($\eta_p^2 = .665$).

When the exogeneity assumption was violated, the CI coverages of CCREM and two-way OLS-CRVE were below the acceptable level. The CI coverage of these two methods grew substantially worse as the number of schools ($\eta_p^2 = .945$) or students per school increased ($\eta_p^2 = .497$). As the biased parameter estimates became more precise, CI coverage levels decreased for both methods. However, coverage of two-way FE-CRVE, which does not require the exogeneity assumption, did not change substantially across the different conditions.

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When the CCREM was misspecified due to the omission of a random slope, its CI coverage was drastically lower than nominal, indicating a strong potential for incorrect statistical inferences. Furthermore, the coverage of CCREM was lower when the number of students per school was 100 ($\eta_p^2 = .937$), as biased parameter estimates became precise. In contrast, two-way OLS-CRVE and FE-CRVE provided coverage levels within the acceptable range across all conditions.

Discussion

This paper investigated two-way OLS-CRVE and two-way FE-CRVE as potential alternatives to CCREM when analyzing cross-classified data. Many studies in psychology and other social sciences have analyzed cross-classified data by only using CCREM, yet CCREM invokes strong assumptions. If the goal of an analysis focuses on coefficients of level-1 predictors rather than on variance components, two-way OLS-CRVE and two-way FE-CRVE could be used. Extending a study by McNeish et al. (2017) on methods for analyzing hierarchical data structures, we described how OLS or FE models could be applied to analyze cross-classified data by pairing them with two-way CRVE to assess sampling uncertainty. Cameron et al. (2011) proposed and examined the performance of two-way CRVE on balanced panel data, but our simulations focused on unbalanced data, which is a more common scenario in psychological and educational applications. We also considered scenarios where specific assumptions of the CCREM were either met or violated, highlighting differences in performance of alternative methods that require fewer assumptions than CCREM.

Our simulation findings illustrate that inferences based on CCREM are generally quite sensitive to violations of modeling assumptions. When the homoscedasticity or exogeneity assumptions were violated, two-way FE-CRVE showed better performance than CCREM. Two-

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way FE-CRVE consistently showed similar or even better parameter bias for the coefficient estimates of level-1 covariates. This pattern is similar to findings on one-way methods, where one-way FE showed unbiased coefficient and SE estimates, similar to HLM (Huang, 2016; McNeish & Stapleton, 2016). When the data-generating process includes unmodeled random slopes, CCREM had lower RMSE than FE-CRVE but drastically mis-calibrated CI coverage. Under these conditions, two-way FE-CRVE had higher RMSE than CCREM but CI coverage was within the acceptable range than those of CCREM—especially for conditions with a larger number of clusters, where CRVE methods should perform better. This finding illustrates that the choice between CCREM and FE involves a trade-off between potential for bias and potential precision gains.

Although its performance was not as good as two-way FE-CRVE, two-way OLS-CRVE showed acceptable performance on parameter bias except for the case where the exogeneity assumption was violated. Notably, under conditions with random slopes, OLS-CRVE outperformed the other approaches, providing lower RMSE and adequate CI coverage. This result is surprising because OLS-CRVE had systematically higher RMSE than the other methods under all other conditions. The performance of OLS-CRVE under random slopes may be due to the different weights of the within-cluster and between-cluster effects of covariates used in each method. FE-CRVE uses only the within-cluster information while OLS-CRVE uses both the within and between-cluster information to estimate the coefficient. The presence of random slopes means that the within-cluster estimates contain more noise than under the random intercept model, whereas the between-cluster estimate is still precise. Compared to the other methods, OLS-CRVE puts relatively more weight on between-cluster information, and therefore performs better than FE-CRVE, which only uses the within-cluster effects, or CCREM, which

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uses a different weighting of between- and within-cluster effects. Because the CCREM was misspecified, its weighting was less efficient than OLS-CRVE.

Similar to Cameron et al. (2011), we found that CI coverage of OLS-CRVE was acceptable when the number of clusters was 70 or greater. Considering that the CI coverage of CCREM was not acceptable when its assumptions are violated, with more than 70 clusters, two-way OLS-CRVE can be recommended as another alternative for researchers who want to avoid the complex model and strict assumptions of CCREM.

Additionally, this study compared how the performance of methods differed depending on data-generating conditions. The number of clusters in two dimensions substantially affected the performance of the three methods. Similar to findings of prior studies, we found that when their assumptions were all satisfied, CI coverage of three methods generally trended towards the nominal .95 levels as the number of clusters increased (Cameron et al., 2011). However, when the exogeneity assumption was violated and random slopes were unmodeled, the CI coverage of CCREM and two-way OLS-CRVE actually decreased as the number of clusters increased. For biased parameter estimators, CI coverage can decrease as precision increases. The sample size within the clusters had a small impact on the performance, as in Cameron et al. (2011). When the exogeneity assumption was violated, larger IUCC resulted in higher parameter bias and RMSE, particularly for OLS-CRVE.

Limitations

Findings from this simulation study have several limitations, some of which point towards directions for further methodological research. We focused on comparing the performance of three methods that require different assumptions. However, we only considered violations of the homoscedasticity, exogeneity, and random intercepts assumptions. Further

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assumptions of the CCREM include that the random effects are normally distributed, that the level-2 random effects are independent of level-1 errors, and that the random effects of each dimension of clustering are independent (Meyers & Beretvas, 2006). Shi et al. (2010) noted that the omitted correlation between random effects would result in inflated random effects and proposed to include random interaction terms in CCREM to address this possibility. Future research could examine the extent to which violation of such assumptions leads to differences in the performance of CCREM, OLS-CRVE, and FE-CRVE, providing further guidance for applied researchers who suspect that the CCREM assumptions might not hold.

This study also examined data structures with a relatively large number of clusters in each dimension and where the covariates were normally distributed, without outliers or high leverage points. Such conditions are favorable for the performance of CRVE, where SEs, hypothesis tests, and CIs involve asymptotic approximations as the number of clusters in each dimensions grows large. However, researchers may also need to analyze cross-classified data with a smaller number of clusters—particularly in primary studies where sample sizes are limited by financial or time constraints. McNeish and Stapleton (2016) compared the effects of small-sample corrections on several one-way clustering methods. In future research, it would be useful to investigate small-sample corrections for tests and CIs based on CCREM or CRVE methods. Some methods, including the Kenward-Roger small-sample correction, are already applicable to CCREM (Kenward & Roger, 1997), but small-sample corrections for multi-way CRVE methods remain to be developed. Potential avenues for further development include extensions of bias-reduced linearization (Bell & McCaffrey, 2002; Pustejovsky & Tipton, 2018) or cluster-wild bootstrap techniques (MacKinnon et al., 2021).

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Also, our simulations focused on models with un-centered, standardized covariates generated based on normal distributions. Group-mean centering is often recommended in the literature on HLM (Enders & Tofighi, 2007; Raudenbush & Bryk, 2002), and adaptive centering methods (a generalization of group-mean centering) are available for cross-classified models. These centering methods have the benefit of relaxing the exogeneity assumption, leading to coefficient estimates and SEs that are identical to the results of FE approaches (Raudenbush, 2009, pp. 477-481). Compared to FE models, adaptive centering methods for CCREM have the advantage of allowing for inclusion of level-2 predictors and estimation of variance components, both of which are useful in practice (cf. Allison, 2009; Hamaker & Muthén, 2020; McNeish & Kelley, 2019). However, since variance components are not the focus of the present study, adaptive centering was not considered in the simulation study. Furthermore, two-way FE methods can be combined with CRVE techniques, which provide robustness to violation of the other modeling assumptions required by CCREM. It is not clear how such robustness could be obtained with adaptively centered CCREM. More generally, adaptive centering methods need further development for cross-classified data. Future research should investigate their implementation with CCREM and compare their performance relative to FE-CRVE methods.

Lastly, we focused on three methods for handling cross-classified data with two dimensions, but this was not an exhaustive set of possible analytic approaches. For example, only one dimension of cross-classified data can be modeled using HLM or one-way FE while using two-way CRVE to account for dependence in both dimensions. Alternately, two-dimensionally cross-classified data could be modeled by using random effects for one dimension and FE for the other dimension. This approach does not require the strong assumptions of the CCREM for the clustering dimension controlled by FE, while still providing level-2 random

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effects and estimated coefficients of the level-2 covariates for the other clustering dimension. In our empirical application, this approach yielded similar results to CCREM and OLS-CRVE. To determine conditions under which this similarity will hold more broadly, further simulation work should consider such methods in addition to CCREM, two-way OLS-CRVE and FE-CRVE.

Implications for Data-Analysis Practice

Findings from this study suggest some guidelines for applied researchers to consider when analyzing unbalanced, cross-classified data. To choose among the methods that we have examined, researchers must first consider which parameters are of interest. When research questions center on the magnitude of variance components, then only CCREM is available (possibly with adaptive centering). When interest is in coefficients of level-2 predictors, then researchers can choose between CCREM and OLS-CRVE. Finally, if interest centers on coefficients of level-1 predictors, then all three methods are available. In the latter two situations, the choice of method hinges on whether CCREM assumptions are met. As theory would suggest, CCREM performs the best when all of its assumptions are met. However, we recommend using two suggested methods, two-way OLS-CRVE and two-way FE-CRVE, when there is concern that the homogeneity, exogeneity, or random intercepts assumptions are violated. These two methods account for dependence in multiple clustering dimensions, similar to CCREM, but are more robust to violations of the homoscedasticity assumption or omission of random slopes. If there is concern about the plausibility of the exogeneity assumption, then we strongly recommend using two-way FE-CRVE as an alternative or supplement to CCREM. Indeed, reporting two-way FE-CRVE and CCREM results side by side can serve as a useful sensitivity analysis, allowing researchers to gauge the implications of exogeneity assumption violations. More generally, for analyses focused on estimating coefficients of level-1 covariates, two-way

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FE-CRVE is a useful alternative for researchers who want to avoid the complexity and strict assumptions of the CCREM. We hope that researchers will consider these suggestions and broaden the scope of methods they apply for analysis of cross-classified data.

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Table 1

Comparison of the Modeling Results

	CCREM		Two-way OLS-CRVE		Two-way FE-CRVE	
	Est.	SE	Est.	SE	Est.	SE
Fixed effects						
Intercept	0.086	0.028**	0.081	0.033*		
VRQ	0.028	0.002***	0.028	0.003***	0.026	0.003***
Reading	0.026	0.002***	0.026	0.002***	0.026	0.002***
Dad Occupation	0.008	0.001***	0.008	0.001***	0.006	0.002**
Dad Education	0.144	0.041***	0.142	0.047**	0.106	0.049*
Mom Education	0.059	0.037	0.062	0.044	0.057	0.037
Dad Unemployed	-0.121	0.047*	-0.115	0.047*	-0.112	0.067
Male	-0.056	0.028*	-0.055	0.036	-0.063	0.043
Neighborhood Deprivation	-0.157	0.026***	-0.148	0.029***		
Random effects (SD)						
Neighborhood	0.062					
School	0.066					
Residual	0.676					

Note. * $p < .05$, ** $p < .01$, *** $p < .001$

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Table 2

Simulation Conditions

Experimental Factors	Levels
CCREM assumptions	All assumptions met Homoscedasticity assumption violated Exogeneity assumption violated Random slopes
Effect size of coefficients for level-1 and level-2 covariates	.1 (small), .3 (medium) or .5 (large)
Number of level-2 clusters (neighborhoods $G \times$ schools H)	70×20 (small), 245×70 (medium) or 525×150 (large; 3.5:1 ratio)
Number of level-1 students per school (J)	30 (small) or 100 (large)
School IUCC	.05 (small), .15 (medium) or .25 (large)

Note. Neighborhood IUCC is fixed at .05.

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Figure 1

Parameter Bias of the Coefficient Estimates for the Level-1 Covariate using CCREM, Two-Way OLS-CRVE, and Two-Way FE-CRVE for Each Assumption Scenario.

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Figure 2

RMSE of the Coefficient Estimates for the Level-1 Covariate using CCREM, Two-Way OLS-CRVE and Two-Way FE-CRVE for Each Assumption Scenario.

Note. RMSE is scaled by the square root of total sample size.

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Figure 3

95% CI Coverage of the Coefficient Estimates for the Level-1 Covariate using CCREM, Two-Way OLS-CRVE and Two-Way FE-CRVE for Each Assumption Scenario.