Bayesian hierarchical diffusion modeling of the inter-stimulus spacing effect
in the Eriksen flanker task

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Abstract

We introduce a formal model of the inter-stimulus spacing effect on flanker interference, based on the previous model by White, Ratcliff, and Starns (2011). In the flanker task, participants identify whether the central target, flanked by congruent or incongruent distractors, points to the right or left. We manipulated the inter-stimulus spacing to test the constructed model against the observed data. When the spacing was manipulated, the previous model was not consistent with the data in the congruent condition. Thus, we introduced two extensions to the model: First, the input from the stimuli was calculated as a ratio of target information to total information from all stimuli; second, the shape of the attentional spotlight was represented as a distribution narrower than the normal distribution used in the previous model. The model with these two extensions captured the data pattern, and comparisons with Bayes factors favored the extended model with both extensions, compared to the model with either one of the extensions. When the information accumulation rate within a trial was calculated as the ratio of target-related information to information from all inputs, rather than as the sum of inputs, an attentional shrinking mechanism was found to be involved in the flanker task when the inter-stimulus spacing was manipulated. Furthermore, when the attentional distribution was represented by a narrower distribution, the extended model better accounted for the inter-stimulus spacing effect of flanker interference than did the commonly used normal distribution model.

(243 words)

Keyword

Visual attention, Flanker task, Bayesian statistical modeling

1 Introduction

2 The metaphor of attention as a spotlight has a long history (for example,
3 Hernandez-Peon, 1964). Since the metaphor of the attentional spotlight was advanced
4 by Posner, Snyder, and Davidson (1980), a variety of characteristics of the spotlight
5 have been proposed (for a review, see Cave & Bichot, 1999). A classic and long-
6 standing approach to the study of visuospatial attention is the Eriksen flanker task
7 (Eriksen & Eriksen, 1974). In the typical experiment, participants identify a central
8 stimulus (i.e., target) while ignoring flanking stimuli (i.e., flankers) that contain
9 information congruent or incongruent with the target. For example, if the target to be
10 identified is a left-pointing arrow, congruent flankers are left-pointing arrows, whereas
11 incongruent flankers are right-pointing arrows, conflicting with the target. Responses
12 are faster when the target is surrounded by congruent flankers than when the target is
13 surrounded by incongruent flankers. Eriksen and Hoffman (1973) estimated the breadth
14 of the attentional spotlight to be one degree of visual angle because the interference
15 effect from the flankers diminished around this distance.

16 Within the spotlight, not all stimuli are processed equally. A stimulus at the
17 center of the spotlight is processed most effectively, and processing becomes less
18 effective for stimuli farther from the center (Eriksen & Eriksen, 1974; Eriksen &
19 Hoffman, 1972; Eriksen & Hoffman, 1973). Eriksen and Eriksen (1974) found slower
20 responses to the target with flankers closer to the target than with those farther from the
21 target. This suggests that flankers are processed more effectively when they are closer to
22 the focus of the attentional spotlight, resulting in more interference with target
23 processing. LaBerge (1983) also demonstrated that responses to a probe in a peripheral
24 position slowed when the task was to categorize a central letter. These results imply that

1 stimuli near the focus of the attentional spotlight receive more through processing,
2 compared to stimuli far from the focus.

3 White, Ratcliff, and Starns (2011) proposed a mathematical model that describes
4 the behavior of the attentional spotlight during a trial. Mathematical details of the model
5 will be described in the Modeling section below. The key assumption of this model is
6 that attentional resources are distributed over the stimuli at the beginning of a trial in the
7 form of a normal distribution, with flanker stimuli closer to the center of the spotlight
8 processed more effectively and automatically. As time goes on during the trial, this
9 attentional spotlight narrows down to the target, diminishing interference from the
10 flankers. After the spotlight sufficiently shrinks to exclude the flankers, a correct
11 response is likely to occur. However, on some trials, responses precede this shrinkage,
12 with the result that quick and incorrect responses may be driven by incongruent flankers
13 on which sufficient attentional resources are still allocated. Thus, in the flanker task,
14 incorrect responses on incongruent trials tend to be faster than correct responses
15 (Gratton, Coles, Sirevaag, Eriksen, & Donchin, 1988). Although other models can
16 account for the flanker interference effect (e.g., the Dual-Stage Two-Phase model of
17 Hübner, Steinhauser, & Lehle, 2010), the shrinking spotlight model has been shown to
18 provide a good fit to data from a variety of manipulations of the flanker task (White et
19 al., 2011; White, Servant, & Logan, 2018).

20 However, it remains unclear whether the shrinking spotlight model of White et
21 al. (2011) can account for the well-known effect of inter-stimulus distance on flanker
22 interference. As Eriksen and Eriksen (1974) demonstrated in their seminal study, the
23 interference from flankers decreases as the distance between stimuli increases. This
24 indicates that not all the stimuli inside the attentional spotlight are processed equally.

1 This and other research (Pinker & Downing, 1985; LaBerge, 1983) has supported the
2 notion that a stimulus at the center of the attentional spotlight receives the most
3 thorough and effective processing. Stimuli located farther from the center are still
4 processed, though less effectively as the distance from the most focused region
5 increases. This gradual decrease in processing efficiency, sometimes termed the
6 attentional gradient (LaBerge & Brown, 1989), is one of the critical features of
7 visuospatial attention and should be incorporated in any model of flanker interference.
8 Therefore, one purpose of the present study is to examine whether the shrinking
9 spotlight model can accommodate the inter-stimulus distance effect in flanker
10 interference.

11 A secondary goal of the present study is to extend the shrinking spotlight model
12 with a Bayesian hierarchical modeling approach. A Bayesian approach has two main
13 advantages. First, a hierarchical modeling approach that can incorporate inter-individual
14 and inter-trial variability is more feasible with the Bayesian approach than with
15 traditional maximum likelihood estimation methods. Second, the application of Bayes
16 factors enables us to evaluate models based on marginal likelihoods, that is, the
17 probability that the data are observed within a model, averaged over all parameters.
18 (Other benefits are discussed elsewhere; see Wagenmakers et al., 2018.) By taking
19 advantage of these benefits, the present research explores how visuospatial attention
20 selects information in the Eriksen flanker task.

21 In order to apply Bayesian hierarchical modeling, we modified the shrinking
22 spotlight model formulated by White et al. (2011) as described below. We conducted an
23 experiment in which participants completed a typical flanker task with varying inter-
24 stimulus distances, and we applied the modified model to the data. This version of the

model did not adequately describe the obtained data. We then investigated two possible accounts of this discrepancy, one based on input speed and the other on the shape of the attentional spotlight, and we proposed and tested corresponding new models. In order to determine the best explanation of the inter-stimulus effect on flanker interference, we used Bayes factors to conduct comparisons of the original and proposed models.

Modeling

A Bayesian hierarchical extension of the shrinking spotlight model

In order to examine whether the extension of the White et al. model can account for the inter-stimulus effect on flanker interference, we constructed a Bayesian hierarchical version of the shrinking spotlight model (White et al., 2011). We refer to this version of the model as the *addition model with the normal distribution* to distinguish it from modified models to be mentioned later, which we call *ratio models*. The addition model with the normal distribution assumes that the attentional resource at a certain moment during a trial is distributed as a normal distribution:

$$f(x|\sigma_t) = \text{Normal}(x|0, \sigma_t) = \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{x^2}{2\sigma_t^2}\right).$$

The distribution centers on 0, x in formulae denotes an x-coordinate, and its standard deviation is expressed by σ_t , which corresponds to the width of the attentional distribution at a certain moment in the trial. Another assumption in the shrinking spotlight model is that the standard deviation σ_t decreases linearly with time as a certain trial goes on. For each trial, we calculated an attentional distribution averaged across time t by integrating the above shrinking spotlight function from $\sigma_t = \sigma_0$ to $\sigma_t = 0$. To solve this approximately, we used the Riemann sum method by discretely

dividing the time into m parts. The attentional distribution resulting from this calculation is represented as:

$$g(x|\sigma_0) = \frac{1}{\sigma_0} \int_0^{\sigma_0} f(x|\sigma_t) d\sigma_t \cong \sum_{k=1}^m f(x|\sigma_0 \times \frac{k}{m}).$$

--- Figure 1 about here ---

In order to calculate the amount of attentional resource allocated to each stimulus, we set L as the width of each stimulus (1.1 degrees in the present study), and D_j as the distance between stimuli on the j -th trial (5 levels: 0.28, 0.55, 0.83, 1.22, 1.82 and 2.42 degrees), as shown in Figure 1. As defined in White et al. (2011), the amount of attentional resource allocated to the target stimulus on a certain trial is:

$$a_{target,j} = \int_{-\frac{L}{2}}^{\frac{L}{2}} g(x|\sigma_0) dx$$

Similarly, the amount of attentional resource allocated to one of the inner flanker stimuli is described as:

$$a_{inner,j} = \int_{\frac{L}{2}+D_j}^{\frac{3L}{2}+D_j} g(x|\sigma_0) dx$$

The amount of attentional resource allocated to one of the outer flanker stimuli is described as:

$$a_{outer,j} = \int_{\frac{3L}{2}+D_j}^{\frac{5L}{2}+D_j} g(x|\sigma_0) dx$$

The core of the model is the Ratcliff diffusion model (Ratcliff, 1978), in which reaction time and accuracy are represented as a joint probability distribution with four parameters: A (boundary separation), β (starting point), δ (drift rate), and τ (non-

decision time). The simultaneous (joint) probability distribution is often termed the Wiener distribution. Boundary separation represents response thresholds, thus describing how cautious the response is. Starting point quantifies the a-priori bias of a participant towards one response boundary (in this experiment, either left-pointing or right-response). Non-decision time reflects processes that are not directly relevant to the process of information accumulation (e.g., response execution, stimulus encoding). We fixed these three parameters across congruency and inter-stimulus distance conditions.

Most importantly, the drift rate δ is determined by the sum of the perceptual input from each stimulus weighted by the amount of attentional resources allocated to that stimulus. We quantify the amount of information that each stimulus is afforded as $p_i (> 0)$, and the direction of target and flanker stimuli as $R_{target,ij}$, $R_{flanker,ij}$ respectively (+1 when the stimulus is a right-pointing arrow; -1 when the stimulus is a left-pointing arrow). The perceptual inputs from target and flanker stimuli are described as $p_i R_{target,ij}$, $p_i R_{flanker,ij}$, and $p_i R_{outer,ij}$. The amount of attentional resources allocated to all the flanker stimuli is the sum of $a_{inner,j}$ and $a_{outer,j}$, such that $a_{flanker,ij} = 2a_{inner,ij} + 2a_{outer,ij}$. The drift rate δ is formulated as:

$$\delta_{ij} = p_i(R_{target,ij}a_{target,ij} + R_{flanker,ij}a_{flanker,ij})$$

When $\delta_j > 0$, this drives a “right-pointing” response, which corresponds to the upper bound of the Wiener distribution. On the other hand, when $\delta_j < 0$, this drives a “left-pointing” response. The joint distribution of reaction time and response is expressed as:

$$RT_{ij}, Response_{ij} \sim \text{Wiener}(A_i, \tau_i, \beta_i, \delta_{ij}).$$

For the prior distributions of each parameter, we used the weakly informative prior distribution. The model specified above was constructed according to the prior and likelihood structures that follow:

$$p_i \sim \text{LogNormal}(\mu_p, \sigma_p)$$

$$A_i \sim \text{LogNormal}(\mu_\alpha, \sigma_\alpha)$$

$$\Phi^{-1}\left(\frac{\tau}{\ln RT}\right) \sim \text{Normal}(\mu_\tau, \sigma_\tau)$$

$$\beta_i \sim \text{Uniform}(0,1)$$

$$\mu_p \sim \text{Normal}(0,100)$$

$$\mu_\alpha \sim \text{Normal}(0,100)$$

$$\mu_\tau \sim \text{Normal}(0,100)$$

$$\sigma_p \sim \text{StudentT}(4,0,10)$$

Experiment

Methods

Participants

Twenty-eight undergraduate students participated in the experiment for partial fulfillment of course credit. The sample size was determined based on a rule of thumb: In the authors' previous experience with the Eriksen flanker task (e.g., Kobayashi & Ogawa, 2021), approximately 25 participants were enough to observe flanker interference. We recruited the participants via an online system used in a university department; this produced 28 participants. All participants gave written informed consent and reported having normal or corrected-to-normal vision. None was aware of

the purpose of the study. Ethical approval for this study was obtained from Kwansei Gakuin University Committee for Regulations for Behavioral Research with Human Participants.

Materials and Apparatus

All experimental sessions were conducted in a dimly lit room. Experimental stimuli were presented on a 24-inch display (BenQ XL2420T), and the experiment was programmed with PsychoPy (Peirce et al., 2019). Participants were seated at a viewing distance of 57 cm from the display. A chin rest was used to ensure that the distance was fixed throughout the experiment.

In the flanker task, arrow symbols served as experimental stimuli; one of these was a target, and the others were flankers. The display contained one target and four flankers. The target arrow pointed either right or left. All of the flanker arrows pointed in the same direction, which was congruent with the target arrow orientation on the half of the trials, and incongruent on the other half. All stimuli were of the same size, subtending 1.1 x 1.1 degrees.

Stimuli were aligned equidistantly in a row at the center of the screen. Six conditions of inter-stimulus spacing were employed: 0.28, 0.55, 0.83, 1.22, 1.82, and 2.42 degrees.

Procedure

In the experiment, the participants engaged in a flanker task. A schematic illustration of the trial sequence is shown in Figure 2. A trial started with a 500-ms fixation cross, followed by one central target flanked by four flanker arrows. The participants identified the orientation of the target arrow while ignoring the flanker

arrows. The target arrow pointed either to the left or right. The participants were asked to respond by pressing the “F” key on a keyboard if the target pointed to the left and the “J” key if it pointed to the right, as quickly and accurately as possible. The trial ended when either the participant responded or 2,000 ms elapsed without a response. Each participant completed 960 trials. On half of these trials, the flanker arrows contained information neither congruent nor incongruent with the target. These neutral condition trials are not discussed further in this report.

--- Figure 2 about here ---

The target arrow pointed right on half of the trials and left on the other half, and flanker arrows were either congruent or incongruent with the orientation of the target, resulting in four types of stimuli display: right-pointing target, right-pointing flankers; right-pointing target, left-pointing flankers; left-pointing target, left-pointing flankers; and left-pointing target, right-pointing flankers. In each display type, 120 trials were presented.

Data analysis

Responses faster than 250 ms were excluded as outliers. This criterion led to the elimination of 40 trials, meaning that 0.30% of the data were eliminated. Trials with response times longer than 3 standard deviations from the mean of the participant in each condition were also to be excluded, but no trial met this criterion. Participants failed to respond within 2,000 ms on 65 trials, and the data from these trials were not recorded. This data preprocessing resulted in 13,375 valid trials.

We estimated parameter values with the Markov Chain Monte Carlo (MCMC) method. We used R 3.6.3 and the rstan R package (Stan Development Team, 2019) to obtain MCMC samples from four independent chains of 5,500 iterations. The initial 500 samples of each chain were discarded as warmups. We confirmed that the parameter estimation was converged since all R-hat values were below 1.01. For the calculation of the natural logarithms (ln) of marginal likelihoods (i.e., the negative free energy) and the Bayes-factor value of each model, we used the bridgesampling R package (Gronau and Singmann, 2020).

Results

In this section, we first summarize the reaction time and accuracy data and then analyze it with the addition model with the normal distribution, which is based on the original model proposed by White et al. (2011).

Summary statistics

--- Figure 3 about here ---

Figure 3 shows mean reaction times across participants in the flanker task by inter-stimuli spacing condition. The left and right panels show the data from trials with correct and incorrect responses, respectively. On correct trials, the overall reaction time was shorter in the incongruent condition than in the congruent condition. The responses on incongruent trials became shorter as the inter-stimulus spacing increased, while the responses on congruent trials remained stable in all the spacing conditions. Further, the reaction time was shorter on incorrect trials than on correct trials, consistent with the

assumption of the shrinking spotlight model of White et al. (2011) that attentional resources are distributed over a wide region at the beginning of a trial. Thus, input from incongruent flankers drives a response in the direction opposite to the target.

--- Figure 4 about here ---

As shown in Figure 4, accuracy rates in the flanker task were high, over 85% in all conditions. Performance was more accurate on congruent trials than on incongruent trials, and this difference was consistent across distance conditions (99.24 % vs. 93.98%, averaged across distance conditions). On incongruent trials, accuracy rates increased as the inter-stimulus distances became larger. Overall, the presence of incongruent flankers interfered with target identification performance, and the extent of this interference decreased as the flankers were located farther away from the target.

Fit of the addition model with the normal distribution

--- Figure 5 about here ---

Figure 5 shows predicted reaction times based on expected a posteriori (EAP) estimates of marginal posterior distributions of parameters from the addition model with the normal distribution. The model captured the pattern in the incongruent condition; for example, reaction times were longer in the smaller spacing conditions than in the larger spacing conditions, and the change was gradual. However, the model predicted shorter reaction times for smaller inter-stimulus distances, but the reaction times in the congruent condition did not vary with inter-stimulus distance. Thus, the extension of the

shrinking spotlight model with the Bayesian hierarchical approach did not fit well with the data obtained in the present flanker task in which the inter-stimulus distance was varied.

Two reasons for the inaccuracy of the model prediction appear to be possible. First, the simple expression of the drift rate parameter as the sum of the perceptual inputs may have been inappropriate. Second, the assumption of the normal distribution as the shape of the attentional spotlight may have been inadequate. In order to investigate whether these reasons could account for the inaccuracy, we constructed three additional models: (1) a ratio model with a normal distribution; (2) an addition model with a generalized normal distribution as the shape of the attentional spotlight; and (3) a ratio model with a generalized normal distribution.

Ratio model with the normal distribution

--- Figure 6 about here ---

In the original version of the model, the drift rate was expressed as the sum of the perceptual inputs. As schematized in Figure 6, the sum of the attentional resources allocated to all the stimuli was greater in the smaller distance conditions than in the larger distance conditions. On congruent trials, this sum of perceptual inputs should have driven the target direction response to be greater in the smaller distance condition than in large distance conditions; however, the data were inconsistent with this prediction. Based on the finding of greater interference with flankers that are larger or with higher contrast than the target stimulus (Eriksen and Schutz, 1979), we constructed

a model in which the drift rate is calculated as the ratio of target-congruent information to the total perceptual input:

$$\delta_{ij} = \frac{p_i R_{target,ij} a_{target,ij}}{p_i (R_{target,ij} a_{target,ij} + R_{flanker,ij} a_{flanker,ij})}$$

Here, the definitions of p , R , and a are equivalent to those in the original addition model with the normal distribution. But when the ratio is used as the drift rate, the model predicts that reaction times on congruent trials would not vary with the distance between stimuli.

Addition model with the generalized normal distribution

In order to examine the possibility that using the normal distribution as the shape of the attentional spotlight does not fully capture the characteristics of the attentional spotlight, we constructed an addition model with the generalized normal distribution. The generalized normal distribution is an extension of the normal distribution with the shape parameter κ ; when κ is 2, the generalized normal distribution equals the normal distribution. Larger κ produces a broader distribution, and smaller κ results in a more compressed shape. The model assumes that the attentional resources at a certain moment during a trial are distributed as:

$$h(x|\sigma_t, \kappa) = \frac{\kappa \exp\left(-\left(\frac{|x|}{\sigma_t}\right)^\kappa\right)}{2\sigma_t \Gamma\left(\frac{1}{\kappa}\right)},$$

where Γ represents the gamma distribution. We replaced the normal distribution in the original model with the generalized normal distribution as follows:

$$m(x|\sigma_0, \kappa) = \frac{\sigma_0}{2} \int_0^{\sigma_0} h(x|\sigma_t, \kappa) d\sigma_t \cong \sum_{i=1}^m h(x|\sigma_0 \times \frac{i}{m}, \kappa)$$

$$M(x|\sigma_0, \kappa) = \int_{-\infty}^x m(s|\sigma_0, \kappa) ds$$

The drift rate was formulated as the sum of the perceptual inputs weighted by the amount of attentional resources. The prior and likelihood structures were the same as in the addition model with the normal distribution, except we used the prior with the median value 2:

$$\kappa \sim \text{LogNormal}(2, 2)$$

Ratio model with the generalized normal distribution

We also constructed a ratio model with the generalized normal distribution. Here, the drift rate was calculated as the ratio of the target information to the total input, unlike drift rate in the addition model above. The prior and likelihood structures were the same as in the ratio model with the normal distribution.

Model comparison

In order to evaluate whether the revised versions of the model are more consistent with the data, we conducted model comparisons using the negative natural logarithms (ln) of marginal likelihoods (i.e., the free energy) and Bayes factors. Marginal likelihoods represent the probability that the data are observed within a model, averaged over all parameters. When the values of free energy are smaller, the model is a better predictor of the observed data, relative to the other models with larger free energy values. Bayes factors are calculated as the ratio of marginal likelihoods of the two models, making it possible to quantify the extent to which the data favor one model over another. First, we calculated the free energy values of each model. Then, we compared each version of the model to the addition model with the normal distribution. After

determining whether any version fits better than the addition model with the normal distribution, we conducted additional comparisons between those versions of the model.

--- Figure 7 about here ---

Predicted reaction times based on the estimated parameter values are shown in Figure 7. As can be seen, the two ratio models captured the patterns in the data better than the two addition models. Furthermore, the two models with the normal distribution predicted a steeper decline in reaction times as a function of the inter-stimulus distance than the actual data. Based on the free energy values, the ratio model with the generalized normal distribution provided a better fit than the other three models. More specifically, the fit of the addition model with the normal distribution (the original model) was worse (76512.83) than the addition model with the generalized normal distribution (76400.36), the fit of the addition model with the generalized normal distribution was worse than the ratio model with the normal distribution (76353.58), and the fit of the ratio model with normal distribution was worse than the ratio model with the generalized normal distribution (76320.23).

A model comparison with Bayes factors is shown in Table 1. The fit improved when ratio models were used, as can be observed in the comparisons between the ratio model with the normal distribution and the addition model with the normal distribution (76282.61 vs. 76443.42; $BF = 69.13 \times 10^{69}$), and between the ratio model with the generalized normal distribution and the addition model with the normal distribution (76245.34 vs. 76449.42; $BF = 13.88 \times 10^{65}$). Evidence in favor of the ratio models over the addition model with the normal distribution is thus extremely strong. Of the

two addition models, the addition model with the generalized normal distribution fit the data better than the addition model with the normal distribution (76395.34 vs. 76443.42; $BF = 75.90 \times 10^{24}$). Importantly, of the two ratio models, the ratio model with the generalized normal distribution fit the data better than the ratio model with the normal distribution (76245.34 vs. 76282.61; $BF = 15.24 \times 10^{16}$), providing very strong evidence in favor of the ratio model with generalized normal distribution over the other ratio model.

--- Figure 8 about here ---

Reaction times simulated per individual based on estimates of the ratio model with the generalized normal distribution are shown in Figure 8. The shape parameter of the generalized normal distribution, κ , was estimated at 0.52, which means that the model estimated the shape of the attentional spotlight to be more compressed than the normal distribution. The estimated shapes of the generalized normal distribution and the normal distribution with the same variance are shown in Figure 9.

--- Figure 9 about here ---

Overall, the present analysis indicates that the data in this experiment can be described by a model in which the rate of information accumulation is modeled as the ratio of the amount of target information to all of the perceptual input, and the shape of the attentional spotlight is assumed to be a generalized normal distribution with a shape more compressed than the normal distribution.

Discussion

In the present study, we extended the shrinking spotlight model (White et al., 2011) to account for the inter-stimulus distance effect in flanker interference. The modeling revealed that the rate of information accumulation during the flanker task can be better captured by calculating the drift rate as a ratio of the target information to the total input information rather than as the sum of the perceptual inputs. Our model comparisons also revealed that the inter-stimulus distance effect was described better when the attentional spotlight was interpreted in the shape of the generalized normal distribution with a compressed form, rather than as a normal distribution. In addition, we implemented the models as Bayesian hierarchical models, which incorporate both individual differences and similarities, thus allowing more flexible modeling.

One of the features of our ratio model with the generalized normal distribution is that the target selection is driven by the information accumulation defined as the ratio of the task-relevant information to the total perceptual input. This is consistent with a previous study that found greater interference when the target stimulus was smaller or lower in contrast than the flanker stimuli (Eriksen & Schutz, 1979). The perceptual input, p , may reflect size and contrast; the perceptual input for the target stimuli $pR_{target,j}$ would be smaller for a smaller target stimulus, or for a target with lesser contrast, leading to a smaller drift rate δ . Smaller drift rates reflect slow information accumulation on a certain trial, meaning that longer reaction times would emerge when drift rates are larger. Our modeling approach implies that the information used in the flanker task is better represented as the ratio rather than as the sum of the input information.

Another interesting feature of our extended model is that the shape of the attentional spotlight in our model is described as the generalized normal distribution with a shape more compressed than the normal distribution. Although most previous studies have assumed that the normal distribution is the shape of the attentional spotlight (Pinker & Downing, 1985; LaBerge, 1983), it is also possible that the more compressed distribution is appropriate as a metaphor of the attentional spotlight. The attentional breadth is believed to select a small region of the visual field, and the attentional resources outside this region decrease gradually as the distance from the center increases, without a sharp boundary (Downing, 1988; Eriksen & St. James, 1986). Given this attentional gradient, the normal distribution seems plausible. However, the normal distribution does not fit well with other characteristics of the focused visuospatial attentional spotlight.

The normal distribution is the probability distribution with the largest entropy among distributions with the same variance. Entropy can be interpreted as the average level of “surprise,” such that a probability distribution with a large entropy is a distribution in which any random variable from the distribution is likely and thus less surprising. Therefore, assuming a normal distribution as the attentional spotlight implies that all information inside it is selected evenly. But attention is a mechanism by which we select and focus on information among irrelevant distractors; thus the entropy characteristic of the normal distribution is not consistent with the function of attention. Further research is needed to examine whether the assumption of a compressed distribution also applies to other attentional tasks that might involve a shrinking mechanism, such as the Navon task, the breadth-of-attention task, and the useful field of view task.

1 The use of Bayesian statistical modeling is an important feature of this study.
2 Although more and more studies have adopted this approach in various fields of
3 cognitive psychology (van de Schoot et al., 2017; see also Lee, 2011; Muto, 2021b), the
4 use of statistical modeling in visual attention research has been limited (Kobayashi and
5 Ogawa, 2020). Instead of using conventional null-hypothesis testing methods, we have
6 expressed a hypothesis as a mathematical model based on a previous model by White et
7 al. (2011). Specifically, we hypothesized that the mechanisms described in the shrinking
8 spotlight model could be applied to a situation in which inter-stimulus spacing is varied
9 between trials. We then constructed other models based on previous research in visual
10 attention to compare with the original model. The model comparison allowed us to
11 examine hypotheses that are difficult to investigate with experimental manipulation
12 only. We do not claim that conventional null hypothesis significance testing is
13 impracticable to explore hypotheses. Rather, we believe that the statistical modeling
14 approach will prove useful in expanding our understanding of cognitive mechanisms
15 and should be utilized hand-in-hand with the traditional approach. By avoiding data loss
16 by means of accounting for within- and between-participant variability (Muto, 2021a),
17 Bayesian hierarchical modeling enables more flexible, thorough, and candid exploration
18 of experimental results. The work presented here is a contribution to the growing
19 number of statistical modeling approaches that are revealing the cognitive processes that
20 underly visuospatial attention.

21 There is opportunity for further development in modeling behavioral data in the
22 flanker task. For example, we modeled the shrinking spotlight as the averaged
23 attentional distribution in which the width of the distribution σ_t changes from its initial
24 value σ_0 to 0. Although this averaged distribution was successful in predicting how the

1 data behaved in the current experiment, because of technical difficulties, we did not
2 model how fast this shrinking proceeds; representing a non-linear drift rate analytically
3 is difficult. Further research should be undertaken to model the information
4 accumulation process that takes the shrinking speed into account. This model
5 improvement would allow us to investigate how the shrinking speed changes with the
6 manipulation of inter-stimuli distance.

8 **Conclusion**

9 The Eriksen flanker task has been a famous and popular approach in
10 investigating the mechanisms by which attention selects visuospatial information.
11 However, attempts to mathematically describe the processes involved in the task require
12 further development. In this investigation, we modeled the processing involved in
13 flanker interference to account for the inter-stimulus distance effect by extending the
14 shrinking spotlight model (White et al., 2011) using a Bayesian hierarchical approach.
15 Our modeling approach suggests that the target selection process is driven by the
16 amount of information in the target stimulus relative to all the stimuli. We also propose
17 that representing the shape of the attentional spotlight as the generalized normal
18 distribution better captures the data from the flanker experiment. Despite the
19 exploratory nature of its model selection process, this study offers insight not only on
20 the nature of the selection process in the flanker task, but also on the way Bayesian
21 statistical modeling can contribute to theoretical development in the field of visual
22 attention.

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1 **Figures & Tables**

2 ● Figure 1

3 The values used in the models. α is the amount of attentional resource allocated to a
4 stimulus, L is the width of each stimulus (1.1 degrees in the present study), and D_j is
5 the distance between stimuli on the j -th trial (5 levels: 0.28, 0.55, 0.83, 1.22, 1.82 and
6 2.42 degrees)
7

1 ● Figure 2

- 2 The structure of a trial. A fixation cross was presented for 750 ms at the beginning of a
3 trial. Then, a target and 4 flanker stimuli were presented until a participant made a
4 response. If an incorrect response was made, a participant was presented a feedback,
5 which told that they just made an incorrect response.
6

1 ● Figure 3

2 Mean reaction time observed in the experiment. The left panel shows the data in
3 incorrect trials, and right panel shows the data in correct trials. The dotted lines show
4 the congruent condition, and the solid lines show the incongruent condition.
5

1 ● Figure 4

2 Mean accuracy observed in the experiment. The dotted line shows the congruent
3 condition, and the solid line shows the incongruent condition.
4

1
2 ● Figure 5

3 Mean reaction time simulated from the addition model with the normal distribution. The
4 dotted lines show the congruent condition, and the solid lines show the incongruent
5 condition. The points were the data observed in the experiment.
6

1 ● Figure 6

2 Compared to the larger distance conditions, the amount of attentional resources
3 allocated to each stimulus is larger in smaller distance conditions.
4

1 ● Figure 7

2 Mean reaction time simulated from (1) the ratio model with the normal distribution; (2)
 3 the addition model with the generalized normal distribution; and (3) the ratio model
 4 with the generalized normal distribution. The dotted lines show the congruent condition,
 5 and the solid lines show the incongruent condition. The points were the data observed in
 6 the experiment.

7

8

1 ● Figure 8

- 2 Mean reaction time within each participant simulated from the ratio model with the
 3 generalized normal distribution. The dotted lines show the congruent condition, and the
 4 solid lines show the incongruent condition. The points were the data observed in the
 5 experiment.
 6

1 ● Figure 9

- 2 The estimated shape of the generalized normal distribution, simulated from the
3 parameter value estimated from the ratio model with the generalized normal
4 distribution.
5

1 ● Table 1
2

Table 1
A model comparison with Bayes factors.

	Normal	Generalized Normal	BF
Addition	76443.42	76395.34	75.90×10^{24}
Ratio	76282.61	76245.34	15.24×10^{16}
BF	69.13×10^{69}	13.88×10^{65}	

3