A Contradiction between Necessity and Possibility Inferences from General Conditionals

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Abstract

The author investigated the relationship between possibility and necessity inferences from general conditionals (e.g., *If a card is red, then it is round*). An experiment examined how people reason about what cases are possible or necessary under a general conditional. Participants were asked to infer whether each truth-table cases (pq , $p\neg q$, $\neg pq$ and $\neg p\neg q$) ($\neg$ = not) must be in the set of cases under a true general conditional, and whether the combinatorial sets of the four truth-table cases are possible under the conditional. The test order of necessity and possibility questions was varied with the NP (necessity/possibility questions) and PN (possibility/necessity questions) group. The findings are as follows. The test order did not affect the relationship between possibility and necessity inferences for each of $p\neg q$, $\neg pq$ and $\neg p\neg q$ cases. It affected the relationship between necessity inferences for pq cases and possibility inferences for the sets including only $\neg pq$ or $\neg p\neg q$ cases. The NP group more often showed the contradiction between these two inferences than the PN group. The existing main accounts of conditionals are unable to explain the contradiction.

Alternatively, the author proposes an inference dissociation account for it.

Keywords: conditionals; possibility inferences; necessity inferences; contradiction; the inference dissociation account

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People often engage in modal reasoning about what is possible (or necessary) in science and everyday life. They often use the expressions ‘necessarily’ and ‘possibly’. Modal reasoning from conditionals (*if p, then q*) is reasoning about what may be the case or not, and what must or must not be the case given a conditional. A basic conditional is one “for which general knowledge, the meaning or reference of its clauses, or knowledge of its context, has no effect on the interpretation of the relation between its if-clause and its then-clause” (Johnson-Laird, 2011, p. 121). Goodwin (2014) classified basic conditionals into general and specific conditionals. A general conditional refers to a set of truth-table cases (e.g., *If a card is red, then it is round*. Here, a card is drawn from a set of cards). A specific conditional refers to a single conditional event (e.g., *If the card is red, then it is round*).

The present study concerns the modal implication of general conditionals: What cases are necessary, possible or impossible given a true general conditional. Two example problems are presented in Appendix A. The example problems include possibility and necessity questions. The possibility question is to infer whether each of the 15 sets of four truth-table cases (pq , $p\neg q$, $\neg pq$, $\neg p\neg q$) is possibly the set of cases under a general conditional. The necessity question is to infer whether the set of cases under a general conditional necessarily includes the four respective cases. The present study aims to examine how people make such possibility and necessity inferences from singular general conditionals.

The example problems concern the implication of general conditionals: What cases does a general conditional imply? Logically, the material implication of a conditional (*if p, then q*) is $\neg p$ or q (that is, pq or $\neg pq$ or $\neg p\neg q$) (here, $\neg$ = not). Thus, a true conditional implies that $\neg p$ or q is true and $p\neg q$ is false. Thus, pq , $\neg pq$ and $\neg p\neg q$ all are possible, but not necessary. That $p\neg q$ is false does not imply that $p\neg q$ is impossible. This is the possibility implication of the material conditional. For a general conditional, any combinatorial set of the three cases is possible, and any combinatorial set including $p\neg q$ is false. An important caveat is that a general conditional implies that q follows from given p , but not the necessity that there must be pq cases in the set under the material conditional. The example problems are unable to test the material implication account, because they don’t contain the “false” option in the response set. Thus, the present study does not test the material implication account.

Psychologically, there are two main accounts for the implication of conditionals: the deterministic and probabilistic accounts (Goodwin, 2014). The deterministic accounts such as the

mental model theory, claim that a true conditional tolerate no exceptions ($p \rightarrow q$), thereby being deterministic (Goodwin, 2014; Johnson-Laird, 2006; Johnson-Laird & Byrne, 2002). The probabilistic accounts claim that a true conditional is believable (highly probable and tolerating exceptions), thereby being probabilistic (de Finetti, 1995; Evans, Handley, & Over, 2003; Fugard, et al, 2011; Goodwin, 2014; Liu, et al, 1996; Oaksford & Chater, 2009; Oaksford, Chater, & Larkin, 2000; Oberauer & Wilhelm, 2003; Pfeifer, 2013a, b).

The original mental model theory (I referred to it as MMT hereafter) (Johnson-Laird, 2006; Johnson-Laird & Byrne, 2002) has the following prediction. A true general conditional, *if p then q*, implies that any combinatorial set of three cases ($pq, \neg pq, \neg p \rightarrow q$) is possible, and any combinatorial set including $p \rightarrow q$ is impossible. Thus, there are seven possible sets under the conditional: ($pq, \neg pq, \neg p \rightarrow q$), ($pq, \neg pq$), ($pq, \neg p \rightarrow q$), ($\neg pq, \neg p \rightarrow q$), (pq), ($\neg pq$), ($\neg p \rightarrow q$). Among these sets, whenever p occurs, q necessarily follows. Bell & Johnson-Laird (1998) used MMT to explain modal reasoning. It postulates that individuals construct possibility models under the premises in which they make explicit only what is true. A conclusion is possible if it holds in at least one model, whereas it is necessary if it holds in all the models. A conclusion is not possible if it is false in all the models, whereas it is not necessary if it is false in at least one model. The mental model theory predicts the following relationship between possibility and necessity inferences from a general conditional. Given the seven possible sets (that is, possibility models) under the conditional, each of $pq, \neg pq$, and $\neg p \rightarrow q$ is present in only some of the possible sets, and thereby is possible, but not necessary; whereas $p \rightarrow q$ is absent in all the possible sets, and thereby is impossible. No previous studies have tested these predictions.

Moreover, Johnson-Laird et al. (2015) brought forward a revised version of MMT of conditionals. The revised version doesn't deal with general conditionals. Thus, the present study doesn't test the revised version of MMT.

The suppositional account is a main probabilistic account (Evans, 2007; Evans & Over, 2004; Evans, Over, & Handley, 2005; Over & Cruz, 2017; Over & Evans, 2003; Fugard, et al, 2011; Politzer, Over, & Baratgin, 2010). It explains the subjective meaning of conditionals as hypothetical thinking. A conditional (*if p, then q*) expresses people's subjective belief or confidence in a conditional relying on the *Ramsey test* that people hypothetically add its antecedent to their stock of knowledge and judge the likelihood of the consequent. People judge the probability of a conditional assertion in accordance with the conditional probability of the consequent given the antecedent,

$P(q|p)$. Thus, people generally interpret a true conditional as believable (highly probable and tolerating exceptions) rather than logically valid or necessary (Adams, 1998; Evans, Ellis, & Newstead, 1996; Evans & Over, 2004; Evans, 2007, p56). Such interpretation based on $P(q|p)$ is modulated by some pragmatic factors such as background knowledge so that the probabilistic threshold of a conditional being believable may not be high (Over & Cruz, 2017; Singmann, Klauer, & Over, 2014; Skovgaard, Singmann, & Klauer, 2016). Thus, for a true or believable general conditional, the suppositional account does not rule out the possibility of $p \neg q$, and predicts that people should infer four possibilities ($pq, p \neg q, \neg pq, \neg p \neg q$) under the conditional. Thus, the four cases all are possible, but are not necessary.

Psychological research has examined possibility inferences from a conditional by using the possibility judgment or list task. In such tasks, people tend to produce the three possibilities that a material implication of the conditional gives rise to, and to judge $p \neg q$ cases impossible (Barrouillet, Gauffroy, & Lecas, 2008; Barrouillet, Grosset, & Lecas, 2000; Goodwin, 2014; Handley, Evans & Thompson, 2006; Schroyens, 2010). These findings conform to MMT.

However, no previous studies have examined what cases are necessary given general conditionals. For the example problems, how individuals make possibility and necessity inferences? Can they make necessity inferences in accordance with possibility inferences? MMT predicts that in possibility inference, $pq, \neg pq$, and $\neg p \neg q$ all are possible, and $p \neg q$ is impossible; in necessity inference, the set of cases under a conditional neither necessarily includes nor excludes $pq, \neg pq$, and $\neg p \neg q$ respectively, and necessarily excludes $p \neg q$. The suppositional account predicts that in possibility inference, $pq, \neg pq, \neg p \neg q$, and $p \neg q$ all are possible; in necessity inference, the set of cases under the conditional neither necessarily includes nor excludes the four respective cases. Both the suppositional account and MMT predict that there should be consistency between necessity and possibility inferences for each case. In particular, participants should judge that the sets including only $\neg pq$ or $\neg p \neg q$ cases are possible and pq cases are not necessary in the set of cases under the conditional. Thus, both predict that pq cases should be possible, but not necessary under the conditional. To examine the relationship between necessity inferences for pq cases and possibility inferences for the sets including only $\neg pq$ or $\neg p \neg q$ cases will test the suppositional account and MMT. The present study aims to examine the relationship rather than to accurately distinguish the existing accounts of conditionals.

Modal logic provides the rules that the relationship should adhere to. For the example problems,

it predicts that the relationship between possibility and necessity inferences from conditionals should adhere to the following rules. Asserting that $\neg p$ ($\neg pq$ or $\neg p\neg q$) is possible implies that p (pq or $p\neg q$) is not necessary, and vice versa: Asserting that p (pq or $p\neg q$) is not necessary implies that $\neg p$ ($\neg pq$ or $\neg p\neg q$) is possible. Asserting that pq is necessary implies that $\neg pq$ or $\neg p\neg q$ is not possible, and vice versa. However, no previous studies have tested these rules. The present experiment aimed to investigate whether people adhere to these rules in the implication inference of general conditionals.

Experiment

Method

Participants

96 college students (42 males, 54 females) from Shaanxi Normal University in China participated in the experiment. They had not studied any logic course.

Design and materials

I used a 2×2 mixed design. The kind of modal questions (necessity versus possibility questions) was the within-subjects variable. The test order of the two kinds of modal questions (necessity/possibility questions versus possibility/necessity questions) was the between-subjects variable. The test order of necessity/possibility questions formed the NP group where necessity questions preceded possibility questions. The test order of possibility/necessity questions formed the PN group where possibility questions preceded necessity questions.

Necessity questions asked participants to judge whether the set of cases under a true general conditional necessarily includes each truth-table cases (pq , $p\neg q$, $\neg pq$, and $\neg p\neg q$). Possibility questions asked participants to judge whether each of the combinatorial sets of the truth-table cases is possible under a true general conditional. For a general conditional, there were 15 sets (see the details in Appendix A). Among these sets, the sets including only $\neg pq$ or $\neg p\neg q$ cases were the target sets. There were three target sets: $(\neg pq, \neg p\neg q)$, $(\neg pq)$, and $(\neg p\neg q)$. The other sets were the non-target sets. Possibility inferences for the target sets would be compared with necessity inferences for pq cases.

The NP group aimed to examine whether preceding necessity inferences would constrain subsequent possibility inferences. If participants judged that the set of cases under a conditional necessarily includes pq cases, then they should judge the target sets impossible. The PN group aimed to examine whether preceding possibility inferences would constrain subsequent necessity

inferences. If participants judged the target sets possible, then they should judge that the set of cases under the conditional does not necessarily include pq cases. According to MMT, necessity inferences are based on the construction of mental models (that is, possible sets under the conditional), and the construction of fully explicit mental models demands working memory resource. In the PN group, the fully explicit models preceding the necessity questions formed the explicit model basis for the subsequent necessity inferences. This would improve the subsequent necessity inferences. In contrast, in the NP group, the fully explicit models followed the necessity questions. Thus, no preceding explicit models were available in the necessity inference, resulting in that the necessity inferences would be more likely to err. Thus, the PN group would show more consistency between necessity inferences and possibility inferences than the NP group.

In each group, there were two kinds of problem contents (cards and balls: if a card is round then it is red, or if a ball is white then it is metal), and two denial representations (implicit vs. explicit denial of p or q) of truth-table cases. For example, for $p \neg q$ cases (round nonred cards), a round blue card is an implicit denial for *red*, whereas a round nonred card is an explicit denial for *red*. The two kinds of problem contents and the two denial representations of cases combined into the following two versions of booklets. Version 1 consisted of two problem contents: implicit-cards and explicit-balls. Version 2 consisted of two problem contents: implicit-balls and explicit-cards. Within each version, the two kinds of problem contents were counterbalanced. Within each necessity question, the four items were counterbalanced. Within each possibility question, the 15 items were counterbalanced. For example, in the NP group, one presentation order of Version 1 is presented as Appendix A. The two problems were respectively presented on two sheets. Here is the English version translated from the original Chinese version.

Procedure

I conducted the experiment in a quiet classroom. Participants were randomly arranged to the two experimental conditions. Every participant received a booklet and a pen. They took about 15 minutes to complete the tasks.

Results

Participants' responses showed no obvious differences between the two kinds of problem contents, and also showed no obvious differences between the two denial representations of cases. Thus, their responses were collapsed. The overall results of necessity inferences are shown in Table 1. The overall results of possibility inferences are shown in Table 2. In Tables 1 and 2, there is a

mode response in the response set of each item. For an item, when the rate of the mode response was significantly larger than the overall rate of the other responses, the mode response was predominant for the item. The chi-squared tests revealed the predominant responses that are the italic data points in the two tables.

Table 1

The results (frequencies and percentages) of necessity inferences

Cases	The NP group			The PN group		
	necessarily	necessarily	neither	necessarily	necessarily	neither
	includes	excludes		includes	excludes	
<i>pq</i>	<i>68 (70.8%)</i>	3 (3.1%)	25 (26.0%)	50 (52.1%)	1 (1.0%)	45 (46.9%)
<i>p¬q</i>	5 (5.2%)	<i>73 (76.0%)</i>	18 (18.8%)	1 (1.0%)	<i>91 (94.8%)</i>	4 (4.2%)
<i>¬pq</i>	8 (8.3%)	14 (14.6%)	<i>74 (77.1%)</i>	3 (3.1%)	2 (2.1%)	<i>91 (94.8%)</i>
<i>¬p¬q</i>	26 (27.1%)	4 (4.2%)	<i>66 (68.7%)</i>	1 (1.0%)	2 (2.1%)	<i>93 (96.9%)</i>

Note: The italic data points are the predominant responses.

Table 2

The results (frequencies and percentages) of possibility inferences

		The sets of truth-table cases															
Groups	Responses																
		pq			pq			pq			$p\neg q$			$p\neg q$			
		pq	pq	pq	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	
		$\neg pq$	$\neg pq$	$\neg pq$	pq	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	$p\neg q$	
		$\neg p\neg q$				$\neg p\neg q$	$\neg p\neg q$	$\neg p\neg q$	$\neg p\neg q$	$\neg p\neg q$	$\neg p\neg q$	$\neg p\neg q$	$\neg p\neg q$	$\neg p\neg q$	$\neg p\neg q$	$\neg p\neg q$	
The NP group	Possible	fre	83	82	89	88	24	22	24	24	12	9	10	10	62	63	74
		%	86.5	85.4	92.7	91.7	25.0	22.9	25.0	25.0	12.5	9.4	10.4	10.4	64.6	65.6	77.1
	Impossible	fre	13	14	7	8	72	74	72	72	84	87	86	86	34	33	22
		%	13.5	14.6	7.3	8.3	75.0	77.1	75.0	75.0	87.5	90.6	89.6	89.6	35.4	34.4	22.9
The PN group	Possible	fre	91	88	91	95	11	13	12	11	8	4	5	5	71	70	71
		%	94.8	91.7	94.8	99.0	11.5	13.5	12.5	11.5	8.3	4.2	5.2	5.2	74.0	72.9	74.0
	Impossible	fre	5	8	5	1	85	83	84	85	88	92	91	91	25	26	25
		%	5.2	8.3	5.2	1.0	88.5	86.5	87.5	88.5	91.7	95.8	94.8	94.8	26.0	27.1	26.0

Note: The italic data points are the predominant responses.

For the necessity questions, in the NP group, the predominant inference for pq cases was “necessarily includes”. In the PN group, the rate of “necessarily includes” inferences did not differ from the rate of the other inferences. In both groups, the predominant inference for $p\neg q$ cases was “necessarily excludes”, as is predicted by MMT rather than the suppositional account. The predominant response for $\neg pq$ or $\neg p\neg q$ cases was “neither” (equivalent to “possible”), as is predicted by MMT and the suppositional account.

For the necessity questions, the test order of modal questions yielded a significant effect on necessity inferences. The rate of positive inferences for pq cases was lower in the PN versus NP group ($\chi^2(1) = 7.1, p < .01$). The rate of negative inferences for $p\neg q$ cases was higher in the PN versus NP group ($\chi^2(1) = 13.6, p < .01$). The rate of “neither” inferences for $\neg pq$ or $\neg p\neg q$ cases was higher in the PN versus NP group (for $\neg pq$, $\chi^2(1) = 12.5, p < .01$; for $\neg p\neg q$, $\chi^2(1) = 26.7, p < .01$). Thus, in the PN group, preceding possibility inferences reduced subsequent positive inferences for pq cases, but increased both subsequent negative inferences for $p\neg q$ cases and “neither” inferences for $\neg pq$ and $\neg p\neg q$ cases. This result is consistent with the prediction of MMT that the preceding possible sets under a conditional premise can improve the subsequent necessity inferences in the PN group, as compared with the NP group.

For the possibility questions in both groups, most participants judged the sets including $p\neg q$ cases impossible, as is consistent with the prediction of MMT rather than the suppositional account. Most participants judged the sets without $p\neg q$ cases possible, and judged the target sets including only $\neg pq$ or $\neg p\neg q$ cases possible. The overall response pattern is consistent with the prediction of MMT rather than the suppositional account. Moreover, possibility inferences showed no obvious differences between the NP and PN group on the whole. The test order of modal questions did not affect possibility inferences for the target sets.

Next, I examined the relationship between possibility and necessity inferences in each group.

In both the two groups, for $p\neg q$, $\neg pq$ and $\neg p\neg q$ cases, the predominant possibility inferences were consistent with the respective predominant necessity inferences. The predominant impossible inferences for the sets including $p\neg q$ cases were consistent with the predominant “necessarily excludes” inference for $p\neg q$ cases. The predominant possible inferences for the target sets were consistent with the predominant “neither” inferences for $\neg pq$ and $\neg p\neg q$ cases. Thus, there was the consistency between possibility and necessity inferences for each of $p\neg q$, $\neg pq$ and $\neg p\neg q$ cases.

In both the two groups, most participants judged the target sets possible. For the target sets,

possibility inferences showed no obvious differences between the two groups. Thus, in the NP group, preceding necessary inferences for *pq* cases did not reduce subsequent possible inferences for the target sets, as compared with the PN group. The two inferences were contradictory. Subsequent possibility inferences about whether at least one of the target sets are possible were independent of preceding necessity inferences about whether *pq* cases are necessary or not ($\chi^2(1) = 1.0, p > .05$). The marginal proportion (83.3%) of possible inferences for at least one of the three target sets was larger than the marginal proportion (29.2%) of not “necessarily includes” inferences for *pq* cases (McNemar test, $\chi^2(1) = 46.6, p < .001$). The rate of the contradiction with both necessary inferences for *pq* cases and possible inferences for at least one of the three target sets was 57.3%. The rate of the consistency with both “neither” inferences for *pq* cases and possible inferences for at least one of the three target sets was 26.0%. Overall, the majority of the NP group showed the contradiction between necessary inferences for *pq* cases and possible inferences for the target sets.

For the necessity questions, participants generated fewer necessary inferences for *pq* cases in the PN group (52.1%) than in the NP group (70.8%) ($\chi^2(1) = 7.1, p < .01$). In the PN group, preceding possible inferences for the target sets suppressed subsequent necessary inferences for *pq* cases as compared with preceding necessary inferences for *pq* cases in the NP group. Necessity inferences about whether *pq* cases are necessary or not depended on possibility inferences about whether at least one of the target sets are possible ($\chi^2(1) = 10.1, p < .01$). 43.7% of participants showed the consistency between “neither” inferences for *pq* cases and “possible” inferences for at least one of the target sets.

However, the marginal proportion (77.1%) of possible inferences for at least one of the three target sets was significantly larger than the marginal proportion (47.9%) of not “necessarily includes” inferences for *pq* cases (McNemar test, $\chi^2(1) = 21.8, p < .01$). 33.3% of participants showed the contradiction between preceding possible inferences for the target sets and subsequent necessary inferences for *pq* cases.

Overall, in the PN group, there were the two main response patterns. Some participants showed the consistency between “neither” inferences for *pq* cases and “possible” inferences for the target sets. Some participants showed the contradiction between possible inferences for the target sets and necessary inferences for *pq* cases. Thus, there were obvious individual differences.

In sum, necessity inferences for *pq* cases were more consistent with possibility inferences for the

target sets in the PN versus NP group. More participants showed the consistency between “neither” inferences for pq cases and possible inferences for the target sets in the PN versus NP group. Conversely, more participants showed the contradiction between necessary inferences for pq cases and possible inferences for the target sets in the NP versus PN group. Thus, the test order of modal questions modulated the relationship between necessity inferences for pq cases and possibility inferences for the target sets. These results are consistent with the prediction of MMT. However, the substantial participants showed the contradiction in both groups, as is contrary to the prediction of MMT, the suppositional account and the rules of modal logic. They all predict that the contradiction should not occur.

Discussion

The present results have multiple implications for the existing accounts for conditionals.

For the possibility questions, in both groups, the overall inference pattern for the 15 sets supports the prediction of MMT rather than the suppositional account. For the necessity questions, in both groups, the predominant necessary inference for pq cases diverged from the prediction of MMT and the suppositional account. Most participants judged that the sets including $p \neg q$ cases are impossible and $p \neg q$ cases are necessarily excluded, as is consistent with the prediction of MMT rather than the suppositional account. This result means that they interpreted the conditionals as deterministic (tolerating no exceptions), as supports the deterministic accounts rather than the probabilistic accounts. This is consistent with the previous findings (Goodwin, 2014; Handley, Evans & Thompson, 2006).

The test order of modal questions did not affect the relationship between possibility and necessity inferences for each of $p \neg q$, $\neg pq$ and $\neg p \neg q$ cases. In both the NP and PN group, most participants showed the consistency between possibility and necessity inferences for each of $p \neg q$, $\neg pq$ and $\neg p \neg q$ cases. This is consistent with the prediction of MMT and the rules of modal logic.

The test order of modal questions affected the relationship between necessity inferences for pq cases and possibility inferences for the target sets. The contradiction between necessary inferences for pq cases and possible inferences for the target sets more often occurred in the NP group and in the PN group. In both the NP and PN group, the substantial participants showed the contradiction. This is contrary to the prediction of MMT, the suppositional account and the rules of modal logic. What caused the contradiction? I explain it as follows.

The contradiction means that necessary inferences for pq cases were not based on the possibility

models (the sets including pq or $\neg pq$ or $\neg p\neg q$), as is contrary to the prediction of MMT. It also means that necessary inferences for pq cases were not based on the rule of modal logic: the assertion that $\neg p$ (the target sets including only $\neg pq$ or $\neg p\neg q$) is possible implies that p (pq and $p\neg q$) is not necessary. Thus, the contradiction suggests that necessity and possibility inferences were two separate processes with different reasoning grounds.

The overall pattern of possibility inferences for the 15 sets is predicted by MMT. This suggests that given a true conditional, participants judged what is possible according to the principle of semantic compatibility in MMT. This principle claims that the semantic interpretation of reasoning premises is imagining the possibilities compatible with the premises (Johnson-Laird, 2001). According to the principle, it is easy to infer that the target sets compatible with a general conditional are possible. Thus, such inference may be a process of semantic intuition.

However, MMT cannot explain the predominant necessary inference for pq cases. Neither can the other three accounts. I propose a heuristic account that such necessary inference may be an evidence inference process using some heuristics. I specify the account as follows.

The previous studies on truth judgment for specific conditionals demonstrate that pq as affirmative evidence render a conditional true, $p\neg q$ as negative evidence render the conditional false, both $\neg pq$ and $\neg p\neg q$ as irrelevant evidence neither render it true nor render it false (that is, uncertain judgments (the third value) for the truth or falsity of the conditional). These form the defective truth-table or the de Finetti truth-table (Baratgin, Over & Politzer, 2013, 2014; Evans & Over, 2004). However, the de Finetti truth-table predicts only truth judgments for specific conditionals from truth-table cases. It does not predict truth-table cases from the truth value (true, false or uncertain) of specific conditionals. Moreover, it doesn't deal with general conditionals. Thus, the present results of general conditionals could not test the de Finetti's truth-table.

Because pq cases are the affirmative evidence for a true general conditional, people probably infer that there must be the affirmative evidence (pq cases) in the set of cases under the conditional. This is the principle that a true conditional mentally entails affirmative evidence. People may intuitively associate a true conditional with the affirmative evidence (pq cases) in that such evidence matches and supports the conditional. Thus, the evidence matching may be heuristic. The evidence matching heuristic means that pq cases are the affirmative evidence that can match and support the conditional, resulting in the inference that the set of cases necessarily included pq cases.

The evidence matching heuristic is different from the matching heuristic (Evans, 2006, 2007).

The latter is that in the Wason selection task with a conditional statement (e.g. If there is an A on one side of the card, then there is a 3 on the other side of the card), people tend to select the cards with the items (A or 3) named explicitly in the statement. The item (p or q) matching is independent of whether the items are negative. The evidence matching heuristic refers to the evidence (pq cases) matching rather than the item (p or q) matching.

Overall, the contradiction resulted from that necessity and possibility inferences were the two separate independent processes with different reasoning grounds. Possibility inference should be a process of semantic intuition on the basis of the principle of semantic compatibility. Necessity inference should be an evidence inference process based on the evidence matching heuristic. The evidence inference process was dissociated with the process of possibility inference, as is contrary to the prediction of MMT that necessity inferences should be based on the construction of possibility models. I refer to this account as the inference dissociation account.

Although the contradiction occurred in both the NP and PN group, the contradiction was modulated by the test order of modal questions. The contradiction more often occurred in the NP versus PN group. In the NP group, the majority showed the contradiction. This means that the preceding necessity inferences were not based on the imagination of the possibility models (the possible sets). Thus, participants didn't spontaneously imagine the possibility models. This result diverges from the prediction of MMT that necessity inferences should be based on the imagination of the possibility models. In the PN group, the preceding possible sets under a conditional premise improved the subsequent necessity inferences so that more participants showed the consistency between "neither" inferences for pq cases and possible inferences for the target sets. This result is consistent with the prediction of MMT that the explicit full possibility models can improve the corresponding necessity inferences. However, only 43.7% of participants showed the consistency between "neither" inferences for pq cases and "possible" inferences for at least one of the target sets. The majority didn't show the consistency. 33.3% of participants showed the contradiction without considering the explicit "possible" inferences for the target sets. Overall, necessity inferences for pq cases depended on whether the explicit possibility models were available in the necessity inference. When the explicit possibility models were unavailable, most participants probably based their necessity inferences on the evidence matching heuristic rather than the imagination of the possibility models. Thus, MMT can explain the partial results rather than the overall pattern.

Conclusion

Modal reasoning from general conditionals shows the following pattern. The test order of modal questions does not affect the relationship between possibility and necessity inferences for each of $p \rightarrow q$, $\neg p q$ and $\neg p \rightarrow q$ cases. In the two groups, most individuals show the consistency between possibility and necessity inferences for each of $p \rightarrow q$, $\neg p q$ and $\neg p \rightarrow q$ cases. The test order affects the relationship between necessity inferences for $p q$ cases and possibility inferences for the target sets. The contradiction between both inferences more often occur in the necessity/possibility versus possibility/necessity condition. The existing accounts of conditionals all are unable to explain the contradiction. Alternatively, I propose the inference dissociation account for it.

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Appendix A: A version of questionnaires

Instruction

Please complete the following problems by the presentation order of items. Please tick your answers.

Problem one

There is a pack of cards. The shape of these cards is either round or square. The color of these cards is either red or blue. It is known that assertion “if a card is round, then it is red”, is true for the pack of cards.

1. Given the true assertion, please judge whether the pack of cards necessarily includes the respective cards below.

- A. Round red cards (*necessarily includes, necessarily excludes, neither*)
- B. Round blue cards (*necessarily includes, necessarily excludes, neither*)
- C. Square red cards (*necessarily includes, necessarily excludes, neither*)
- D. Square blue cards (*necessarily includes, necessarily excludes, neither*)

2. Given the true assertion, please judge whether it is possible that each of the following sets of cards is the pack of cards.

- (1) (Round red cards, round blue cards, square red cards, square blue cards) (*possible, impossible*)
- (2) (Round red cards, round blue cards) (*possible, impossible*)
- (3) (Round red cards, square red cards) (*possible, impossible*)
- (4) (Round red cards, square blue cards) (*possible, impossible*)
- (5) (Round blue cards, square red cards) (*possible, impossible*)
- (6) (Round blue cards, square blue cards) (*possible, impossible*)
- (7) (Square red cards, square blue cards) (*possible, impossible*)
- (8) (Round red cards) (*possible, impossible*)
- (9) (Round blue cards) (*possible, impossible*)
- (10) (Square red cards) (*possible, impossible*)
- (11) (Square blue cards) (*possible, impossible*)
- (12) (Round red cards, round blue cards, square red cards) (*possible, impossible*)
- (13) (Round red cards, round blue cards, square blue cards) (*possible, impossible*)
- (14) (Round red cards, square red cards, square blue cards) (*possible, impossible*)
- (15) (Round blue cards, square red cards, square blue cards) (*possible, impossible*)

Problem two

There is a box of balls. The color of these balls is either white or nonwhite. The material of these balls is either metal or nonmetal. It is known that assertion “if a ball is white, then it is metal”, is true for the box of balls.

1. Given the true assertion, please judge whether the box of balls necessarily includes the respective balls below.

- A. Nonwhite nonmetal balls (*necessarily includes, necessarily excludes, neither*)
- B. Nonwhite metal balls (*necessarily includes, necessarily excludes, neither*)
- C. White nonmetal balls (*necessarily includes, necessarily excludes, neither*)
- D. White metal balls (*necessarily includes, necessarily excludes, neither*)

2. Given the true assertion, please judge whether it is possible that each of the following sets of balls is the box of balls.

- (1) (White nonmetal balls, nonwhite metal balls, nonwhite nonmetal balls) (*possible, impossible*)
- (2) (White metal balls, nonwhite metal balls, nonwhite nonmetal balls) (*possible, impossible*)
- (3) (White metal balls, white nonmetal balls, nonwhite nonmetal balls) (*possible, impossible*)
- (4) (White metal balls, white nonmetal balls, nonwhite metal balls) (*possible, impossible*)
- (5) (Nonwhite nonmetal balls) (*possible, impossible*)
- (6) (Nonwhite metal balls) (*possible, impossible*)
- (7) (White nonmetal balls) (*possible, impossible*)
- (8) (White metal balls) (*possible, impossible*)
- (9) (Nonwhite metal balls, nonwhite nonmetal balls) (*possible, impossible*)
- (10) (White nonmetal balls, nonwhite nonmetal balls) (*possible, impossible*)
- (11) (White nonmetal balls, nonwhite metal balls) (*possible, impossible*)
- (12) (White metal balls, nonwhite nonmetal balls) (*possible, impossible*)
- (13) (White metal balls, nonwhite metal balls) (*possible, impossible*)
- (14) (White metal balls, white nonmetal balls) (*possible, impossible*)
- (15) (White metal balls, white nonmetal balls, nonwhite metal balls, nonwhite nonmetal balls) (*possible, impossible*)