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Estimating age-based developmental trajectories using latent change score models based on
measurement occasion

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Computer code for the statistical analyses can be found at: <https://github.com/EduardoEstradaRs/>

Abstract

Accelerated longitudinal designs (ALDs) are designs in which participants from different cohorts provide repeated measures covering a fraction of the time range of the study. ALDs allow researchers to study developmental processes spanning long periods within a relatively shorter time framework. The common trajectory is studied by aggregating the information provided by the different cohorts. Latent change score models (LCS) provide a powerful analytical framework to analyze data from ALDs. With developmental data, LCS models can be specified using measurement occasion as the time metric. This provides a number of benefits, but has an important limitation: It makes it not possible to characterize the longitudinal changes as a function of a developmental process such as age or biological maturation. To overcome this limitation, we propose an extension of an occasion-based LCS model that includes age differences at the first measurement occasion. We conducted a Monte Carlo study and compared the results of including different transformations of the age variable. Our results indicate that some of the proposed transformations resulted in accurate expectations for the studied process across all the ages in the study, and excellent model fit. We discuss these results and provide the *R* code for our analysis.

Keywords: Accelerated longitudinal designs; Latent change score models; Developmental processes; Longitudinal data analysis, Dynamic modeling

Estimating age-based developmental trajectories using latent change score models based on measurement occasion

Studying developmental processes requires following up individuals across a relatively large age span (e.g., from childhood to adolescence). Carrying out a study that encompasses such a long period is often not feasible because of financial and logistic constraints. In these situations, *accelerated longitudinal designs* (ALD) have been proposed as a method to capture development using a limited number of measures per person (Bell, 1953, 1954; Duncan, Duncan, & Hops, 1996; McArdle, 1994; also called *cohort-sequential designs*, Nesselroade & Baltes, 1979; or *cross-sequential designs*, Schaie, 1965).

Bell (1953) introduced the methodological concept of *convergence*. This idea consists of integrating longitudinal information provided by several independent age groups. Such procedure relies on short-term repeated measures from independent age cohorts, rather than long-term repeated measures obtained from a single cohort age group, or one-time measurement from different age groups in a cross-sectional age design. The integration from these cohorts leads to a common or converging growth pattern.

Using an approach based on age convergence, independent age cohorts overlap with each other temporally. The resulting overlap provides critical information about the age span covered by the independent age cohorts. Nesselroade & Baltes (1979) popularized this concept under the name *cohort sequential design*. They proposed conjoining independent limited short-term longitudinal data from several age cohorts as a means to unearth an underlying overall growth trajectory. Thus, a focal idea of the accelerated longitudinal design is to approximate a temporally lengthy longitudinal study by integrating multiple short-term studies of different age cohorts simultaneously. In this study, we focus on ALDs and examine the performance of a set of

statistical models for recovering growth trajectories from data gathered using this research design.

Accelerated longitudinal designs have been widely implemented in psychological research. For example, Duncan et al. (1996) collected alcohol usage data from several adolescent age cohorts. Survey data were collected from four age groups from 12, 13, 14, and 15 years of age at the initial measurement occasion. The four age cohorts were annually measured on alcohol usage three times, thus approximating a single group longitudinal design spanning from age 12 through 17. In the accelerated longitudinal design by Miyazaki & Raudenbush (2000), seven age cohorts were measured on their attitude toward behavioral delinquency on five occasions annually. The initial age of the youngest age cohort was age 11, and this cohort was assessed at 12 through 15, while that of the oldest age cohort was at age 17 and this cohort was repeatedly assessed at 18 through 21. This ALD approximates a single cohort longitudinal design spanning age 11 through 21.

One research area with particular potential for ALD is cognitive development. From childhood to early adulthood, this development is characterized by rapid growth during early childhood and a progressive deceleration during adolescence. At some point between 20 and 30 years of age (the exact moment depends on the individual and the specific ability; McArdle, Ferrer, Hamagami, & Woodcock, 2002) most abilities reach a peak and start a slow decrease (e.g., working memory capacity or reaction time; Kail, 1991; Kail & Ferrer, 2007; Kail & Park, 1992; Kail & Salthouse, 1994; Salthouse, 2009) whereas others such as crystallized intelligence stop changing or continue to increase at a very slow rate (Cattell, 1987; Horn & Cattell, 1966, 1967; McArdle et al., 2002).

As with in any longitudinal phenomenon, cognitive development can be modeled by

using different metrics of time (Hoffman, 2012, 2015). In the context of the development of cognitive abilities, researchers are typically interested in characterizing the level as a function of biological age—measurement occasion has little relevance in these studies. To describe the phenomenon under investigation and detect between-individual differences, it is useful to align the individuals' data points along a common time metric of age, rather than occasion. In fact, during the period from childhood to early adulthood, the development of most cognitive abilities can be described through an exponential equation of the general form

$$y = u - (u - i) * e^{-(r * age)}, \quad [1]$$

where y is the ability or trait level, u is the upper bound or asymptote of the trajectory, i is the initial level when $age=0$ (therefore initial level $y_0 = i$), and r is the rate of change, which determines the speed to which the difference $(u-i)$ is reduced over time. With a positive r parameter, the expression in Equation 1 leads to an exponential trajectory with decelerated change¹. Figure 1 depicts 12 such trajectories with varying values for $u = \{25, 30 \text{ and } 35\}$ and $r = \{.1, .2, .4, .6\}$, and an initial level $y_0 = 10$. This type of exponential trajectories are common in empirical studies investigating the development of cognitive abilities (Ferrer & McArdle, 2004; Kievit et al., 2018; Li et al., 2004; McArdle et al., 2002; McArdle, Hamagami, Meredith, & Bradway, 2000).

INSERT FIGURE 1 HERE

One model suited to capture and describe this type of exponential function is the Latent Change Score model (LCS; Ferrer & McArdle, 2010; McArdle, 2001, 2009; McArdle & Hamagami, 2001). An LCS model represents the process of interest as a dynamical system in which the changes, instead of the levels, are the focus and are modeled as latent variables.

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¹ Note that, although r is positive, e is raised to a negative value due to the negative sign before the product $(r * age)$.

Although other modeling frameworks (e.g., hierarchical linear models; Bryk & Raudenbush, 1987; Raudenbush & Bryk, 2002) can also be used to represent developmental trajectories, including exponential growth, we focus here on LCS models for several reasons. First, they are implemented within a SEM framework. This provides great modeling flexibility, and a wide variety of estimation procedures and software are available. Second, LCS can easily incorporate a measurement structure that allows defining latent variables as a common cause for the manifest variables. This makes it possible to model longitudinal change in the latent variables, instead of the observed ones. Third, they allow an easy specification of dynamic parameters capturing the influences of the previous states of the system on later states (such as self-feedback and couplings).

The classic LCS model assumes that all measures are taken at the exact same discrete time points (i.e., the time intervals between measures are constant). This model can be specified in at least two different ways, depending on the time metric selected. With developmental data, one specification involves using *age* as the time metric and creating discrete age bins. For example, if one participant was measured at ages 7.4, 9.1 and 11.3, these are rounded to, say, 7, 9 and 11. This model would include as many time points as age bins are in the sample (e.g., 15 time points, matching years 5 to 20) and the observed scores are included in the corresponding age bin. Depending on the sampling, this procedure could lead to highly a sparse data set. For instance, if the study comprises 15 years, and only three measures are taken per person, 80% of the possible data points would be missing. Previous literature has shown that this specification allows recovering the parameters generating exponential trajectories typical of developmental data under varying conditions (Estrada & Ferrer, 2019; Hamagami & McArdle, 2001).

If the goal of the study is to examine development as a function of age (as described in

Equation 1), relevant questions include “What is the expected mean level at age 10?”, or “Does this individual show normative development?” (Estrada, Ferrer, Shaywitz, Holahan, & Shaywitz, 2018). In these situations, this LCS specification may be preferable. This approach, however, implies several problems. First, the very low, even zero, coverage for some of the moments in the covariance matrix can lead to high rates of convergence failure in some scenarios. Second, the creation of age bins (and in some studies, phantom variables representing the variable at time points when it was not measured) implies an additional complication in data handling and model specification. Third, it is very common that the time lags between observations are not constant between individuals, as well as between occasions for each individual. Recent research has shown that age binning with non-constant time lags leads to marked bias in the parameters capturing the measurement error variance (Estrada & Ferrer, 2019).

A second specification consists of using *measurement occasion* as the time metric. That is, instead of using age bins, the model is specified using the available time points (e.g., t_1 , t_2 and t_3) as the time metric. This approach benefits from the advantages of the SEM framework (modeling flexibility, variety of estimation procedures and software implementations, addition of dynamic parameters, possibility of specifying a measurement model) while offering several additional advantages. First, it implies much less sparsity in the data. In fact, it leads to a complete data set, as long as all participants are measured at all time points, because potential data incompleteness is due to non-expected participant attrition only. This data completeness implies a total coverage of the covariance matrix, which leads to less convergence and estimation issues. Second, an occasion-based LCS model is easier to specify, as it does not require the creation of age bins, phantom variables, or any other data manipulation.² Researchers have often

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² An occasion-based LCS model may still require bins or phantom variables when there is unbalance in the timing of each occasion or unequal intervals between occasions.

used occasion-based models to investigate developmental data (for example, Estrada, Ferrer, Román, Karama, & Colom, 2019; Ferrer, 2019; Green, Bunge, Briones Chiongbian, Barrow, & Ferrer, 2017; Hoffman, 2012; McArdle & Bell, 2000; Wendelken et al., 2017; Wendelken, Ferrer, Whitaker, & Bunge, 2016).

In spite of these benefits, using an occasion-based LCS in its traditional specification has an important limitation: It makes it not possible to characterize how a given developmental trajectory unfolds over age—or biological maturation, or any other developmental process. The standard LCS model based on measurement occasion falls short to capture such a developmental process. In this paper, we propose a simple method to overcome this limitation. In particular, we show how, by including a variable representing individual differences in age in the model, an occasion-based LCS model can indeed capture the intended developmental process. For this, we propose a general extension of the occasion-based LCS model so it can provide accurate expectations of the developmental trajectory at any given age.

Occasion-Based Latent Change Score Model

In this paper, we focus on the LCS model, with an emphasis on the proposed extension. For a general description of LCS models, their technical underpinnings, and potential uses, we refer to available literature (Ferrer et al., 2007; Ferrer & McArdle, 2004, 2010; McArdle, 2001, 2009; McArdle & Hamagami, 2001). A recent paper by Kievit et al., (2018) provides an intuitive step-by-step mathematical, visual, and narrative presentation of the general LCS model.

Figure 2 depicts a path diagram of a LCS model with three measurement occasions. We focus on a three-occasion model because it is common that ALDs use three time points in their sampling schedules (Estrada et al., 2019; Evans, 2006; Román et al., 2018). In fact, recent literature has shown that this specific sampling schedule leads to optimal bias and efficiency in

the parameters estimates in ALDs, particularly when the measures are separated two years apart (Estrada & Ferrer, 2019).

At each repeated occasion t , a latent variable representing changes (Δy_t) in a latent process (y_t) is specified. Thus, at each occasion, the latent process is a function of the initial unobserved level, y_0 , plus the accumulation of changes up to that occasion. One general specification to capture such changes is the so-called *dual LCS* model, in which changes are a function of: a) an additive linear effect captured by the latent variable y_a , and b) a proportional effect from the latent process itself at the previous occasion, captured by the parameter β . The means of the latent initial level and slope (μ_{y_0} and μ_a) capture, respectively, the mean level in the process at the first occasion and the average additive component at every repeated occasion. The variances of these latent variables ($\sigma^2_{y_0}$ and σ^2_a) denote individual differences in such initial level and additive change. These two components are typically allowed to be correlated ($\sigma_{y_0,a}$ expressed as a covariance, or $\rho_{y_0,a}$ as a correlation).

INSERT FIGURE 2 HERE

In addition to the standard LCS specification, Figure 2 includes both a linear and a nonlinear effect of age³ at the first measurement occasion as a predictor of the latent level y_0 . Therefore, the latent initial mean $\mu_{y_0} = \tau_{y_0} + \gamma_L Age_1 + \gamma_{NL} Age_1^{\kappa}$. Similarly, the variance $\sigma^2_{y_0}$ is partitioned in three components: variance explained by $\gamma_L Age_1$, $\gamma_{NL} Age_1^{\kappa}$, and residual variance; and only the latter is correlated with y_a . The variance of the measurement error, or observed variability not due to the process, is captured by the parameter σ^2_e .

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³ In this description, we consider age as the relevant metric for characterizing the developmental process under investigation.

Sources of variance in the Latent Change Score model

In the LCS model depicted in Figure 2 (as well as in the standard LCS model; Kievit et al., 2018; McArdle, 2001, 2009), potential variability in the trajectories is due to two between-person sources of variance. First, the initial level y_0 captures the mean μ_{y0} (fixed effect) and variance σ^2_{y0} (random effect) in the latent level of y at occasion 1. This latent initial level is regressed on age_1 , and therefore the variance at occasion 1 is allowed to be explained by between-individual differences in age. The second between-person component is the latent additive component y_a , which also has a mean μ_a and variance σ^2_a (fixed and random effects, respectively). In Figure 2, this latent component is not regressed on any other variable, and therefore is an exogenous source of between-individual variance. Because, in a univariate LCS, the asymptote y_u equals $(y_a / -\beta)$, the variance σ^2_a captures between-individual differences in the asymptote (i.e., maximum level) of the trajectories.

All the within-individual changes are characterized as a function of the two elements of the dual system, β (proportional to the previous level) and y_a (constantly added at every occasion). In other words, within-person variation at the latent level is due to passage of time only, and there are no additional within-person sources of variance. This implies that time-specific, within-person variations around the individuals' model-implied trajectories are considered measurement residuals.⁴

Relation between LCS models and exponential trajectories for change

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⁴ It is possible to obtain a stochastic LCS by including an additional source of variance in the latent variable Δy . This variance, usually constrained to be stationary, and called “dynamic error”, and captures variations around the model-implied trajectory that are not measurement error, but prediction error. In contrast to measurement errors, these variations are carried on to later states of the system. Including this additional source of variance is infrequent in the discrete-time LCS literature, although it is more common in the dynamic systems literature (e.g., Ji & Chow, 2019).

The exponential trajectory in Equation 1 is directly related to, and can be re-specified as, a dynamic system described by a dual LCS model. In this section, we explain the relation between both specifications. Our goal is to highlight how the parameters in the LCS model are directly related to the coefficients in Equation 1. These equivalences are the basis for evaluating the accuracy of our parameter estimates in the Results section.

In Equation 1 [$y = u - (u - i) * e^{-(r * age)}$], the first derivative with respect to age represents the instantaneous rate of change of y with respect to *age*. This derivative results in the first-order ordinary differential equation (*continuous time*),

$$d(y) / d(age) = r (u - y) = ru - ry \quad [2]$$

In Equation 2, the terms ru can be replaced with a single coefficient y_a . This parameter expresses a linear additive component, equivalent to the additive component described in the LCS from the previous section and Figure 2. In turn, the parameter r can be replaced by $-\beta$, which has the same meaning: auto-effect or self-feedback of the latent variable on itself. Therefore, Equation 2 can be re-expressed as

$$d(y) / d(age) = y_a + \beta y \text{ (with negative value for } \beta). \quad [3]$$

Equation 3 describes a system in which the instantaneous rate of the latent change is a function of two elements (a dual LCS system). This system is defined in continuous time (CT). However, the traditional LCS model described in the previous section is defined in discrete time (DT). Therefore, the parameters in Equation 3 must be rescaled to a DT metric with a particular time lag Δt . Specifically, the DT self-feedback is $\beta_{DT} = e^{\beta_{CT} \cdot \Delta t} - 1$, the mean and variance of the additive component y_a in DT are $\mu_{a,DT} = \mu_{a,CT} \cdot \left(\frac{e^{\beta_{CT} \cdot \Delta t} - 1}{\beta_{CT}} \right)$, and $\sigma^2_{a,DT} = \sigma^2_{a,CT} \cdot \left(\frac{e^{\beta_{CT} \cdot \Delta t} - 1}{\beta_{CT}} \right)^2$; and the DT covariance between the latent additive component and latent level at the first

occasion is $\sigma_{y0-a,DT} = \sigma_{y0-a,CT} \cdot \frac{e^{\beta_{CT} \cdot \Delta t} - 1}{\beta_{CT}}$. The rest of the parameters in the LCS system (mean and variance of the latent level at first occasion, μ_{y0} and σ_{y0}^2 , and measurement error variance σ_e^2) do not change as a function of the time lag. For more details about the correspondence between LCS in discrete and continuous time, as well as an extension to multivariate systems, see Voelkle & Oud (2015), and Voelkle, Oud, Davidov & Schmidt (2012).

Note that the two key elements describing the change in the LCS model in discrete time depicted in Figure 2 are the latent additive component y_a and the proportional effect (or self-feedback β). Focusing on the LCS model as a dynamic system, change from $t-1$ to t is described by the discrete-time difference equation $\Delta y / \Delta t = y_a + \beta y$ (with negative value for β , and for a specific Δt). The first element adds a constant quantity at each repeated occasion. In contrast, the second element adds a quantity that is proportional to the score at the previous occasion. The combined effect of the linear and proportional components of change leads to a non-linear trajectory which can be accelerated or decelerated. Such trajectory is exponential in a univariate system.⁵

The self-feedback β captures the influence of the process on its own state at the following occasion. Positive values imply that the change is accelerated. In other words, scores further from the initial point i lead to even larger subsequent changes. This leads to an “explosive” trajectory because, after a given level is reached, even small increases in time lead to dramatic changes in y . In contrast, negative values of β lead to deceleration or damping of the change: scores further from i lead to smaller subsequent changes, with the trajectory tending to the asymptote u . When modeling data that follow an exponential function with overall increase, such

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⁵ Other more complex nonlinear trajectories are also possible with extensions of the LCS model (Hamagami & McArdle, 2019).

as those representing cognitive development from childhood to early adulthood, the self-feedback can be also interpreted as the rate at which the difference between the initial level and the asymptote is reduced.

In contrast, the additive component y_a is a multiplication of the rate of change r ($= -\beta$) by the asymptote u (see Equations 2 and 3). The sign of y_a indicates whether the asymptote is above or below zero. For example, with $\beta < 0$, and scores > 0 (e.g., when measuring reading ability with a cognitive test) the linear additive component y_a is positive. Whether the overall change is positive or negative and, thus, the trajectory implies general increase or decrease, depends on the difference $d = u - i$. Positive values of d imply growth, either accelerated or decelerated, whereas negative values imply decline.

Because the latent level at the first occasion y_0 is exactly i in Equation 1, the parameters in the occasion-based LCS model include all the information needed to adequately capture the longitudinal change defined by such equation (assuming cohort convergence). In other words, given any initial level y_0 , the parameters from the three-occasion LCS model will produce accurate expectations for the level at the second and third occasions (y_2 and y_3). However, in order to produce accurate expectations *for the first occasion*, individual differences in age must be included in the model. Table 1 summarizes the correspondence between the parameters in Equation 1 and the standard LCS model.

INSERT TABLE 1 HERE

Specification of the age effects into the occasion-based LCS model

The present work is concerned with an LCS specification in which measurement occasion is used as the time metric. In such model, the time metric of interest (age) must be introduced as

a predictor, capturing individual differences at the first time point of the study.

If the relation between *age* and level of *y* is given by Equation 1, the optimal way to describe the nonlinear development of *y* would be to include a nonlinear regression in the LCS model and estimate the coefficients *u*, *i*, and *r* that best capture the relation between *age*₁ and *y*₀. However, such exponential parameters involve complex nonlinear constraints and are not always easy (or possible) to specify within a SEM model. One option is using SEM in continuous time, as well as other approaches implementing longitudinal modeling of latent variables (Driver, Oud, & Voelkle, 2017; Hunter, 2018; Ji & Chow, 2019; Ou, Hunter, & Chow, 2017; Voelkle & Oud, 2015; Voelkle et al., 2012). Alternatively, we propose a solution that leads to a trajectory matching the true developmental process. This solution is relatively simple and within the scope of any SEM user.

Our proposed approach consists of including a nonlinear transformation of age into the occasion-based LCS model *with a linear link*. There is no clear rationale to determine which transformation can best approximate exponential trajectories such as those represented in Figure 1. Indeed, it is reasonable to think that different transformations could correspond to different instances of Equation 1. Because of this, we applied a *ladder of re-expressions* proposed by Mosteller and Tukey (1977; see also Miller, 1984) to investigate a wide range of nonlinear functions of age. Based on this idea, we included the following general equation in our occasion-based LCS models:

$$y_0 = \tau_{y0} + \gamma_L Age_1 + \gamma_{NL} Age_1^{\kappa},$$

where τ_{y0} is the regression intercept for the initial latent level, γ_L and γ_{NL} capture, respectively, the linear and nonlinear effects of age at first occasion, and κ is one of the elements in the *ladder* $\kappa = \{-1, -1/2, -1/5, 1/4, 1/3, 1/2, 2, 3, Ln, e^{\cdot}\}$ (Mosteller & Tukey, 1977). Note that all the

elements in κ are powers of Age_1 , except for the last two: Ln is its natural logarithm, and e^{-} represents the Euler's irrational number e raised to $(-Age_1)$. Each of the elements in this ladder implies a different nonlinear transformation of Age_1 . Importantly, these transformations are easy to perform and include in the model. One option to do so is defining the transformed variable within the model syntax (this feature is implemented in some SEM programs such as *Mplus*, Muthén & Muthén, 1998). Another option is to compute the transformation of age, and then include the new variable in the model as a linear predictor of y_0 .

As we discussed previously, the traditional occasion-based LCS model is unable to characterize the longitudinal changes as a function of age; in order to do so, *age* must be included in the model. However, the effect of age needs only to be considered at the initial point of each individual's trajectory. Such an effect is nonlinearly carried over to following occasions and multiplicatively interacts with a self-feedback effect of the discrete dynamic model. Therefore, this age adjusted latent change score model enables the linear and nonlinear effects of age to dynamically exert influence over following measurement occasions. In other words, the dynamic parameters β and y_a include all the necessary information to capture any exponential trajectory, *given any initial level* (and assuming cohort convergence). Therefore, the occasion-based LCS model only requires the inclusion of age_1 as a predictor of the initial level y_0 .

In the next sections, we describe this procedure and examine whether such inclusion of age in an occasion-based LCS model allows one to generate trajectories that match those brought about by an age-based process. Moreover, we investigate which of the examined transformations can be used for this purpose with greater efficacy and generality. In particular, we evaluate whether the examined models are able to recover the generating population's mean trajectory and covariance structure.

Methods

Monte Carlo study

We generated repeated measures for a latent process y , unfolding according to an age metric, from 5 to 20 years. The longitudinal change in the process was consistent with Equation 1. Based on previous empirical studies we imposed a set of simulation parameters defining twelve realistic trajectories for y . Specifically, we used estimates obtained from the Woodcock-Johnson reading cluster measured in a large and representative sample from the US: $u \approx 31$, $y_0 \approx 12$, $r \approx .26$ (Ferrer et al., 2007; Ferrer, Shaywitz, Holahan, Marchione, & Shaywitz, 2010; Shaywitz, Shaywitz, Fletcher, & Escobar, 1990). In our simulation, the upper bound u was set to one of the values in $u = \{25, 30, 35\}$, the rate of change r was set to one of the values in $r = \{.1, .2, .4, .6\}$, and y_0 was always set to 10.

These sets of values were used to specify dynamic changes through a continuous time system $y_{i, Age} = y_{0,i} + dy_i$, where $dy_i / d_{Age} = y_{a,i} + \beta \cdot y_i$. Specifically, change was defined as the first-order ordinary differential equation:

$$d \begin{bmatrix} y_i \\ y_{a,i} \end{bmatrix} / dt = \begin{bmatrix} \beta & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_i \\ y_{a,i} \end{bmatrix}.$$

Therefore, the instantaneous rate of change was a function of a constant additive component $y_a = u \cdot r$, and a self-feedback $\beta = -r$ capturing a damping influence from the level already achieved. The latent level y_0 at $t = 0$ (5 years of age) and the latent additive component y_a followed a bivariate normal distribution as:

$$\begin{bmatrix} y_0 \\ y_a \end{bmatrix} \sim N \left(\boldsymbol{\mu} = \begin{bmatrix} \mu_{y0} \\ \mu_a \end{bmatrix}, \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{y0}^2 & \sigma_{y0,a} \\ \sigma_{y0,a} & \sigma_a^2 \end{bmatrix} \right)$$

Table 2 reports the parameters for the twelve generated growth trajectories. Figure 1 depicts the resulting trajectories. Because the LCS models were fitted to a time lag of 2 years, Table 2 includes the values for $\Delta t=2$ for the parameters affected by the time lag. This facilitates the comparison of the generating parameters and the model estimates.

INSERT TABLE 2 HERE

All the trajectories in Figure 1 and Table 2 start at a common initial value ($y_0 = 10$), and an exponential growth towards an upper bound or asymptote u . However, they vary in the value for such asymptote, and in the rate of change from the common initial point, according to the corresponding parameters.

Sampling schedule: Accelerated longitudinal design. For each trajectory in Table 2, we generated 100 samples. Each of them comprised $n=500$ complete individual trajectories (i.e., yearly data points from 5 to 20 years). Each individual trajectory was then sampled at three time points only. We generated 11 cohorts, with different age at the initial measurement occasion (t_1) for each cohort. The second and third measurement occasions (t_2 and t_3) were separated two years apart each. Thus, the total time span from the first to the third measure would correspond to five years in an empirical study. In order to generate data sets similar to those obtained in empirical studies, each individual was sampled at any random week within the corresponding year. Therefore, the time lags varied between individuals, as well as between occasions within each individual (although the average time lag was always two years).⁶ Previous research has

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⁶ For each of the twelve trajectories, we also generated conditions with $n=200$ and with fixed time lags –i.e., all individuals measured at the first week of the corresponding years, for all years. However, the resulting 36 conditions did not provide any additional information, and therefore were not included in this manuscript. The results from these conditions are available from the authors upon request.

shown that, in accelerated longitudinal data with multiple cohorts and when age is used as the underlying time metric, this sampling schedule leads to an optimal recovery of the true generating parameters (Estrada & Ferrer, 2019).

At any time t , the manifest scores were given by $Y_{it} = y_{it} + e_{it}$, where the measurement error $e_{it} \sim N(0, \text{diag}[\sigma_e])$. Note that: a) the latent variable y_a is not linked to any manifest observation, b) the measurement error e is stationary, with zero mean, constant variance and no auto-correlation, and c) no dynamic error is assumed (there is no random *diffusion* in the process and the innovations do not carry over the future states of the system). Table 3 reports the sampling schedule and Figure 3 depicts the observed and latent trajectories for one sample, for trajectory condition 5.

INSERT TABLE 3 HERE

INSERT FIGURE 3 HERE

Figure 3 shows that the mean trajectory for the group corresponds to the simulation values in Table 2 and figure 1, and is the same for the observed and latent trajectories. The variances in the initial level and in the additive component lead to variability in the individual trajectories. The measurement error variance leads to random differences between the latent and observed scores at any particular time point.

Data analysis procedure

For each of the generated samples, we estimated a set of LCS models using measurement occasion (t_1 , t_2 and t_3) as the time metric. In each model, we included the linear effect of age on the latent level at the first occasion ($Age_1 \rightarrow y_0$), as well as the effect of one of the nonlinear transformations in κ ($Age_1^\kappa \rightarrow y_0$), described above (see Figure 2). The estimates from the $k=100$ samples in each condition were averaged, so we obtained one set of estimates for each model and

type of trajectory.

To estimate the expected mean trajectory, we used the estimated parameters (averaged across the $k=100$ replications in each condition) to compute the model expectations for the level of y , for the different ages in the study. We computed these expectations for $Age_1 = \{5.1, 6, 7, 8, 9, 10, 11, 12, 13, 14, \text{ and } 15\}$ ⁷. The expectations were computed as:

$$\begin{aligned} E(y_1) &= E(y_0) = \tau_{y0} + \gamma_L Age_1 + \gamma_{NL} Age_1^k \\ E(y_2) &= E(y_1) + \mu_a + \beta E(y_1) \\ E(y_3) &= E(y_2) + \mu_a + \beta E(y_2) \quad (\text{note that } \beta < 0). \end{aligned} \tag{4}$$

The extended LCS model described above constrains the covariance structure to be stationary with respect to age (i.e., the expected variances for y_1, y_2 and y_3 , as well as their covariances have the same value regardless of Age_1). In order to allow a different covariance structure for different ages, we fitted again all the models (one separate model for each transformation of age), with two important modifications: a) We applied a multi-group specification and used Age_1 (rounded to integer) as the grouping variable (i.e., cohort, treated as categorical), and b) The residual variance $\sigma^2_{y0,c}$ was freely estimated for each cohort c . This way, each cohort had a different expected covariance structure.

In other words, within each sample, individuals with Age_1 ranging from, say, 5 to 5.99 were grouped into cohort 1 and had expectations for the covariances of the latent variable y at ages 5, 7 and 9. The following group (2) had expectations for ages 6, 8, and 10, and so forth.

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⁷ All the LCS models were fitted in a time scale in which five years of age corresponded to $time=0$. Some of the transformations of age yield infinite values—either positive or negative—for $time=0$. Because of this, the first age in the series is 5.1 years of age (or $time=0.1$).

These expectations were averaged across replications and models, so each model produced 11 matrices of size 3x3, one for each cohort. The covariances included in the expectations for different cohorts were averaged.⁸ Appendix 1 includes examples of how to compute the models' expectations for the mean trajectory for different values of Age_1 . Appendix 2 includes the expectations for the variance-covariance structure of the latent variables y_1 , y_2 and y_3 , based on our proposed LCS model. The last section of the *R* code provided allows automatic computation of such expectations.

The main goal of our analysis was to evaluate the accuracy of the expectations from the different LCS models, for each of the trajectories described in Table 2, and to assess the extent to which they were biased with respect to the true age-based mean trajectory and covariance structure. In order to compare the performance of each model with a reference, for each sample we estimated: a) A model in which the mean level of y_0 was predicted for all ages. In other words, $\mu_0 = \tau_0$, with no effect of Age_1 (Model 1), and b) A model capturing only the linear effect of Age_1 on y_0 (Model 2). These two models generally provided the worst and second worst possible sets of expectations, respectively.

For each sample, we added one last model intended to produce the best possible expectations (Model 13). In order to do so, we performed a two-step procedure. In step one, we fitted Equation 1 as a nonlinear regression to each sample, with Age_1 and Y_1 as the predictor and outcome variables, respectively. We registered the estimates for u , d and r , and then used such estimates to compute the predicted individual values of $\hat{Y}_{1i}$, given the observed Age_{1i} . In step two, we included $\hat{Y}_1$ in the LCS model, instead of Age_1 , as a predictor of y_0 : $y_0 = \tau_{y0} + \gamma_{NL} \hat{Y}_1$. The regression coefficient γ_{NL} was freely estimated, and always took values close to one. This method

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⁸ This approach is tenable only if the individuals within cohorts differ in Age_1 . Otherwise, Age_1 would be constant within cohorts, and the covariance matrix would not be positive definite.

allowed considering the true exponential relation between Age_1 and y_0 , while specifying only linear links in the SEM-LCS model.

Table 4 reports a summary of the transformations of Age_1 in all the LCS models. The LCS models were estimated with the *R* package *lavaan* v0.6-1 (Rosseel, 2012). The nonlinear regressions for the first step of model 13 were estimated with the *R* package *minpack.lm* v1.2-1 (Elzhov, Mullen, Spiess, & Bolker, 2016). The *R* code for all the analyses in this study can be found in the following repository: <https://github.com/EduardoEstradaRs>

INSERT TABLE 4 HERE

Evaluation of bias in the mean trajectory. For each of the 12 possible trajectories, we computed the expected y level at $Age = \{5.1, 6, 7, 8, 9, 10, 11, 12, 13, 14, \text{ and } 15\}$. Then, we compared these expected values with those expected from the 13 LCS models. For each data point, we computed $bias = E(y)_g - E(y)_{LCS}$, where $E(y)_g$ is the expected y level of the true *generating* process, and $E(y)_{LCS}$ is the expected y level from the occasion-based *LCS* model. Note that each starting age provided three data points for t_1 , t_2 and t_3 . We computed the total (*Root*

Mean Square) bias for each model in each condition as
$$RMSB = \sqrt{\sum_{p=1}^P \left(E(y)_g - E(y)_{LCS} \right)^2 / P},$$

where P is the total number of data points compared. We also obtained the fit indices *AIC* (Akaike, 1974) and *BIC* (Schwarz, 1978; Sclove, 1987) from all the samples for each model and trajectory.

Evaluation of bias in the covariance structure. For each of the 12 generating populations (one for each trajectory) we took one measurement of the continuous latent process y at the central point of each year from 5 to 20 years (i.e., 15 measures). We computed the covariance matrix for these time points. Then, we compared this true covariance matrix with the expectations from our models, averaged across the $k=100$ replications. For any given statistic θ ,

the residual was computed as $Res = \theta_g - \theta_{LCS}$. The Root Mean Square Residual for the whole

matrix was computed as $RMSR = \sqrt{\sum_{m=1}^M (\theta_g - \theta_{LCS})^2 / M}$, where M is the total number of

moments (i.e., covariances) available for comparison. Appendix 3 includes one example of these expected, generating and residual matrices for one model and trajectory.

Results

Bias in the model expectations for the mean trajectory

All the occasion-based LCS models converged for all samples and trajectories. In the first set of analyses, we evaluated the extent to which the models produced biased expectations for the mean of y at each age. Table 5 reports the (*Root Mean Square*) Total Bias for each model across all conditions. This statistic was originally computed in the metric of the variable y (upper section of Table 5). For an easier interpretation, the lower section of Table 5 includes *relative RMSB*, computed as the ratio $RMSB_m / RMSB_g$, where $RMSB_g$ is the bias of the generating model (model 13) for each trajectory.

INSERT TABLE 5 HERE

As described in Table 5, the baseline specification (model 1) led to the least accurate expectations. This was expected because this model predicts the same value of y_0 regardless of the age at the first measurement occasion. Model 2, including a linear effect of age only, was the second most biased model. In contrast, model 13, using $\hat{Y}_1$ as predictor of y_0 (i.e., the generating model) led to the least biased expectations. Models 1, 2, and 13 were used here as reference models. Given this and the fact that they have little interest in applied scenarios, we focus on the results from the remaining models.

The upper section of Table 5 describes the total raw bias. The results in this section indicate that, holding r constant, the models are more biased for higher values of u (or y_a). That

is, the bias is higher for trajectory 3 than for 1, for trajectory 6 than for 4, and so on. In other words, the higher the difference between the initial point and the upper bound, the more difficult it is to approximate the nonlinear trajectory. This result was also found when fitting model 13, although this model always yielded the most accurate expectations. This finding may be due to the fact that a higher $u-i$ difference implies more degree of nonlinearity, and therefore, a lower ability of the nonlinear transformations to adapt to the true exponential trajectory. However, the most important finding in Table 5 is the fact that models 8 and 9 appear to be the least biased across all conditions. For conditions with lower rates of change (i.e., conditions 1 to 3, with $r=-\beta=.1$, and conditions 4 to 6, with $r=-\beta=.2$) model 9 achieved the most accurate expectations. In contrast, for conditions with higher rate of change (conditions 7 to 9, with $r=-\beta=.4$, and conditions 10 to 12, with $r=-\beta=.6$) model 8 achieved the best results. These results were consistent regardless of u values. In the conditions with better performance of model 9 (trajectories 1 to 6), model 8 was generally the second or third best. In contrast, the opposite did not hold: model 8 led to high biases in some of the conditions from 7 to 12.

In order to visualize these results concerning the bias, we included figures depicting the trajectories for conditions 2, 5, 8 and 11, (Figures 4 to 7, respectively), and the expectations computed from models 1, 8, 9, and 12 in each trajectory. In these figures, low bias (or high accuracy in the expectations) is represented by a close correspondence between the true age-based generating process (grey line) and the expectations from the occasion-based LCS model for the three occasions (black lines connecting the age-specific expectations, with red, green and blue dots marking first, second and third occasion, respectively). Inspection of Figures 4 to 7 illustrates that, if the expectation for t_1 is accurate, the expectations for t_2 and t_3 are also accurate. This is the case for model 8 (including the effect of $Age_1^{(1/2)}$) in all trajectories, and for model 9

(Age_1^2) in trajectories 1 to 6. The linear model yields large biases because it is not possible to approximate the curved trajectory of y_0 with just a linear effect of Age_1 . Even if the expectations for y_2 and y_3 are corrected due to the rest of the LCS model parameters, the bias in y_0 leads to high overall misfit. In fact, the higher the value of r , the more biased the linear expectations. The exponential function (model 12) offers relatively good expectations, although they are never as good as the expectations from models 8 and 9.

INSERT FIGURES 4, 5, 6 AND 7 HERE

Table 6 reports the parameter estimates and relative biases, RB , (averaged across the $k=100$ samples) for all 13 models, estimated for the trajectories 2, 5, and 11. These three conditions were included to illustrate the parameter estimates for the different models. The estimates from the nine remaining conditions are available upon request to the authors. Table 6 also includes the expected values for the model parameters, computed from the generating trajectory and rescaled for a time lag of two years, which were used for the relative bias computations.

INSERT TABLE 6 HERE

Several aspects in Table 6 are worth noting. First, the estimates are very consistent across models, with the exception of parameters related to the latent initial level (μ_{y0} or τ_{y0} , σ_{y0}^2 , $\sigma_{y0,a}$, γ_L and γ_{NL}). These parameters, however, are entirely dependent on the specific transformation of age in each model, and thus cannot be compared across models or against a reference. In contrast, the parameters β , μ_a , σ_a^2 , and σ_e^2 , show little variability across models, and their estimates are very similar to the true generating values. Their RB values ranged between -.08 and .10 (with one exception of .29 for σ_a^2 in the baseline model for condition 2). According to previous literature, values of relative bias below |10%| are usually considered acceptable (Flora

& Curran, 2004; Rhemtulla, Brosseau-Liard, & Savalei, 2012). These excellent results are consistent with the small bias for the mean trajectory expectations, and for the covariance structure (see below).

Interestingly, the small bias for the error variance σ_e^2 , was always positive: this parameter was slightly overestimated in all the models and conditions. The reason for this appears to be the fact that the different measures did not have a time lag of exactly two years. In other words, the randomness in the time lags introduced additional variance in the trajectories, and this additional variance was correctly identified as not related to the latent trajectory. Importantly, despite this slight overestimation, the parameters that define the trajectory were consistent across conditions and matched the expected values derived from the nonlinear generating functions expressed in Equations 1 to 3.

Bias for the mean trajectory quantified as model fit

Table 7 reports the *AIC* and *BIC* fit indices, averaged across the 100 samples for each model and condition. The results in Table 7 are consistent with those in Table 5. Again, as expected, the best fitting model is model 13 (the generating process). Among the rest of the models, the best fit is always achieved by models 8 or 9. Specifically, model 8 is the best in conditions 7 to 12 (with higher r). In conditions 1 to 6 (with lower r), the best model is either model 8 or 9, with very little difference between them.

INSERT TABLE 7 HERE

The results in Table 7 are consistent with the results in Table 5: model 8 ($Age^{1/2}$) is an excellent option in all the simulated conditions, as it is better able to recover the trajectory (Table 5), and fit the data (Table 7) than other models. Model 9 (Age^2) is also a good option in the

conditions with low rate of change r (trajectories 1 to 6). Again, for any given value of r , the trajectories with higher values of the asymptote u lead to worse model fit: condition 3 led to worse fit than conditions 1, 6 worse than 4, and so on. As expected, models 1 and 2 achieved the worst and second worst fit, respectively. Model 13 achieved the best fit in all conditions.

Bias for the covariance structure

Table 8 reports the Root Mean Square Residual (*RMSR*) for all trajectories and models. The top section in Table 8 reports the absolute values, whereas the bottom section reports the same values divided by the average covariance in each condition. The most important finding in Table 8 is the fact that some models were able to recover the covariance structure with great accuracy. For example, the maximum *RMSRs* (i.e., worse results) for models 8 and 9 were 3.21 and 4.30, respectively, both for trajectory 12. These residuals represent only 22% and 30% of the average covariance in such condition. In the rest of the conditions, the *RMSRs* were lower for these two models.

INSERT TABLE 8 HERE

In general, model 13 yielded the best results regarding the recovery of the population covariance. However, other models were as good as model 13, or even better in some conditions. For example, model 9 achieved slightly lower values of *RMSR* for the trajectories 1, 2, and 6, and very similar values for the trajectories 4 and 5. The results in Table 8 are consistent with those in Tables 4 and 6 regarding the adequacy of models 8 and 9 for recovering the population statistics across all conditions. Specifically, model 8 obtained the best results for the trajectories 3, 10, 11, and 12, and model 9 was more accurate in the remaining trajectories.

Comparing across trajectories, the covariance structure appeared to be more difficult to

recover in those trajectories with higher rates of change r : *RMSR* was higher for trajectory 12 than for 10, for trajectory 9 than for 7, and so forth. In fact, all models had very low *RMSR* values in the trajectories with low rates of change, particularly in conditions 1 to 3. Higher values of the asymptote u also lead to higher residuals: trajectories 10 to 12 had higher *RMSR* than 7 to 9, which in turn had higher *RMSR* than 4 to 6, and so forth.

Discussion

Summary of findings

The main goal of this study was to examine the extent to which an occasion-based LCS model is able to recover the expectations for the mean level and the covariance structure of an age-based developmental trajectory. For this, we extended the classic occasion-based LCS specification by adding a link from age to the initial level of the process. Our results indicate that the proposed procedure achieves such goal. A large set of exponential trajectories, describing the development of a cognitive ability based on a maturational process, can be recovered by means of an occasion-based LCS model. In order to do so, a simple transformation of age needs to be introduced at the first measurement occasion in the model. If the correct transformation is used, the resulting occasion-based LCS model achieves excellent results, including predicted values for the age-span of the study, as well as expectation bias and model fit.

Among the nonlinear transformations of *age* that we examined, three deserve special attention. The two-step model (model 13) was the most accurate across all conditions: It achieved the lowest bias in the mean trajectory, and was the best model regarding the covariance structure in 9 conditions, and very close to the best model in the other 3. These results were expected, as this model included a predictor based on a previous non-linear regression that captures the true function that generated the data ($\hat{Y}_1 \rightarrow y_0$). Based on our results, we hold that

this procedure is an excellent option to recover developmental trajectories such as the ones described in this study. However, this approach entails several complications. First, it requires two separate analyses: a) step 1, estimate a nonlinear regression and compute the individual predictions; and b) step 2, estimate an LCS-SEM including those predictions. Each of these steps requires its specific programming, but researchers using *R* can easily obtain the libraries for both steps, and include all the analysis in the same script. See the *R* code available in <https://github.com/EduardoEstradaRs>. Second, fitting two separate models implies more potential difficulties in terms of non-convergence and biased estimates for particular samples (especially if the model assumptions are not met) as well as compounded uncertainty. Third, if the generating process is not described by the function used for the nonlinear regression of the first step, this approach may lead to more biased predictions than the rest of the models.

In addition to model 13, two other models are worth noting. In contrast to model 13, these models do not require predictions from a preliminary nonlinear regression. Model 8 includes a linear predictor based on the square root of age at the first measurement occasion ($Age_1^{1/2} \rightarrow y_0$), whereas model 9 includes a predictor based on its square ($Age_1^2 \rightarrow y_0$). Regarding the mean trajectory, in the conditions with a low rate of change r (i.e., growth is slow, i.e., the difference between the initial state and the upper bound is reduced slowly) model 9 appears to offer the least biased expectations. However, this model incurs in high biases for conditions with $r \geq .4$. In contrast, model 8 achieved little bias in all the simulated conditions. It was the second best model with $r < .4$, and the best model with $r \geq .4$. Interestingly, it achieved less biased expectations than model 12 ($e^{(-Age_1)} \rightarrow y_0$) in all the simulated conditions. Regarding the covariance structure, model 9 is the best option in trajectories 1 to 9, whereas model 8 achieves lower residuals in trajectories 10, 11 and 12. However, the differences between them were small

in all conditions.

Based on these results, we recommend using both models 8 and 9 for recovering age-based trajectories by means of an occasion-based LCS model. However, we note that model 13 is also an excellent option, with the caveats discussed previously. Of course, these recommendations apply only to the specific conditions simulated in this report.

Theoretical and methodological considerations

Previous work has shown that choosing between age and occasion as the time metric may not be very relevant for characterizing the trajectory *within individuals* (Hoffman, 2012, 2015). This is because, for any particular case, every year is just one year further in the study, one year further from birth, or one year ahead in school. In fact, it has been shown that occasion-as-time and age-as-time models are algebraically equivalent with respect to their fixed effects (i.e., the mean trajectory; Hoffman, 2012, 2015). However, this issue had not been addressed in the context of LCS models, or any other model including dynamic parameters or a measurement structure that can account for measurement error and other sources of irrelevant variance. Furthermore, occasion-based LCS models are commonly used in developmental studies examining the dynamics of various cognitive processes in various phases of the life-span (Anblagan et al., 2018; Estrada et al., 2019; Kievit et al., 2018; Lövdén et al., 2014; Raz et al., 2008; Ritchie et al., 2015).

In this paper, we included one observed indicator for each repeated measure of the latent variable. It is important to note, however, that the LCS specification includes a measurement model and can separate variance in the latent process from error variance introduced by the measurement process. Indeed, our approach allowed us to estimate the age-based latent

covariance structure with great accuracy. This was possible because, with more than two repeated measures, there is enough information (i.e., degrees of freedom) to separate the variance in the latent variable y from the error variance. Together with the addition of dynamic parameters, the measurement model is one of the main advantages of the LCS specification, compared to standard hierarchical linear models. Because all our findings regard the latent variable y , our results should generalize to scenarios with multiple indicators (i.e., second-order growth model; McArdle, 1988), provided that longitudinal measurement invariance holds (Ferrer, Balluerka, & Widaman, 2008).

To the best of our knowledge, this paper is the first demonstration of how to recover the covariance structure from an age-based trajectory using an occasion-based model. Our proposed solution implies: a) creating groups using cohort as a categorical variable, b) using an LCS model with multiple groups, c) regressing the latent intercept, or initial level, on linear and nonlinear age at time 1, d) allowing only the residual variance of the latent intercept to differ across cohorts, and e) finding the right functional relation between age and initial level. Our results support the idea that such procedure, applied to data from an ALD, allows an accurate recovery of the age-based covariance structure.

In our simulation, we introduced variability in the specific moment during the year in which the individuals were measured. In other words, the time interval between observations was not constant within or between individuals. The LCS model in discrete time does not account for this variation, but rather assumes that all participants are measured at the exact same occasions. Interestingly, in our analyses, the model parameters, the age-related mean trajectory, and the covariance structure, were all accurately recovered in spite of this wrong assumption. These results provide evidence for: a) the bias estimation introduced by rounding time was very small,

and b) such variance was adequately identified by the models as “error variance” and, therefore, did not affect the recovery of the age-based trajectory. Naturally, these findings only apply to the conditions simulated here, and the random time lags can lead to substantial biases when other sampling schedules are applied. In those scenarios, an LCS model in continuous time is more adequate (Estrada & Ferrer, 2019). Similarly, if the research question does not involve dynamic features or a measurement structure, alternative multilevel specifications are available (Hoffman, 2015).

In econometrics, two-step regression analyses were devised to counteract biases in estimation of the regression effect of an explanatory variable on a dependent variable by partialing out an effect of an instrumental variable on a predictor. In such two-step regression models, a predictor is regressed on the instrumental variable and the resulting expected scores are brought into the main regression equation as a predictor to account for variance of the dependent variable. In the present work, model 13 represents an adaptation of this procedure for a different goal. However, an age effect on the latent level variable needs to be critically examined. Because age at the initial measurement and the latent level are positively correlated, the age effect on the latent level needs to be partialled out in order to make a quantitative adjustment of the self-feedback parameter and an additive constant parameter. When the effect of age on the latent scores is nonlinear, and software has computational limitations to handle such nonlinearity, a two-step approach might be inevitable, especially if biases of parameter estimates are reduced substantially.

It is important to note that the specification proposed in this paper relies on identifying an adequate functional relation between age and the latent level at the first occasion. In other words, the dynamic parameters in the model are able to reproduce the remaining points in the trajectory

only after individual differences in age are correctly transformed to differences in the latent level at the first occasion.

Why is it useful to estimate an occasion-based LCS model to describe age-based trajectories?

An occasion-based LCS is an appropriate modeling approach in a broad variety of scenarios involving developmental trajectories or longitudinal data in general. One positive feature is the possibility of specifying a measurement model for studying the development of latent constructs. In fact, because we modeled the development of y *at the latent level*, our findings can be generalized to scenarios in which y is a common latent factor underlying the scores in a set of manifest indicators. This only requires specifying a measurement model linking the general factor to the manifest variables. Another positive feature is the flexibility for freeing or constraining specific parameters in the model. This flexibility is not readily apparent in other approaches, such as multilevel models. Finally, LCS models can include multiple longitudinal variables simultaneously, as well as multiple groups.

Compared to age-based LCS models, an occasion-based model has several advantages. First, the decrease in the number of manifest variables leads to a substantial reduction of the covariance matrix. In turn, this facilitates model convergence and allows working with smaller samples. Second, because data sparsity is minimal, there is no reduction in the coverage of the covariance matrix. Third, there are no age bins without information and, therefore, no need to interpolate data. Finally, an occasion-based LCS model is easier to specify and estimate: its programming is considerably simpler, it involves a much-reduced set of moments in the model matrices, and requires less data pre-processing.

Previous work has demonstrated how to specify nonlinear change as a function of age in

longitudinal SEM, but most (if not all) of this work addressed scenarios in which age was the time metric of the model (c.f. Grimm & Ram, 2009; Grimm, Ram, & Hamagami, 2011). This is the first work showing how to describe a nonlinear trajectory in a model in which age is not the underlying time metric.

Limitations and future directions

In this paper we focused on scenarios in which the goal is describing development as a function of age. However, other time metrics beyond occasion and age are also possible. Depending on the question addressed, researchers may want to arrange the observed individual trajectories with respect to alternative metrics of time. These could include, for example, grade in school, time *since* a given event (e.g., starting a romantic relationship) or time *to* an event (e.g., dementia onset or death; Hoffman, 2012, 2015).

Several research directions for future studies are important based on our findings. For example, we did not investigate cohort differences in our analyses (i.e., age $\times$ change interactions). Therefore, we do not know whether our results can be generalized to scenarios in which there are between-cohort differences in the parameters describing the developmental trajectories (Miyazaki & Raudenbush, 2000). Especially when a study includes cohorts from a wide age range, it is reasonable to think that influences of various kinds can cause differences between cohorts in the developmental trajectories (e.g., reading or mathematic ability). This is very important because the key assumption of an accelerated longitudinal design is the fact that there are no cohort differences. If such assumption does not hold, separating longitudinal change from cohort differences is not possible and the interpretation of change becomes very difficult. Different strategies have been proposed for these scenarios in the context of mixed models (Fitzmaurice, Laird, & Ware, 2012; Miyazaki & Raudenbush, 2000).

In the context of LCS-SEM, different types of non-convergence can be controlled in different ways. For example, if an interaction cohort $\times$ initial state exists, one option would be to include a dummy variable representing each age cohort in the transformation function: $y_0 = \tau_{y0} + \gamma_L Age_1 + \gamma_{NL} Age_1^{\mathbf{k}} + \sum_{c=1}^{C_{cohort}} \omega_c D_c$, where ω is a cohort specific effect and D is a dummy variable representing a specific cohort in the model. If there is an interaction involving cohort $\times$ shape of change, the model will likely not yield accurate expectations just by regressing the initial level on age. In other words, having the correct expectations for time 1 may not lead to correct expectations for time 2 and 3 in the presence of cohort differences. In fact, to detect this type of cohort effect, one may need to regress the latent additive component y_a on age. However, it is unclear whether this effect captures between-individual differences in the rate of change r or the asymptote u . In other words, addressing the issue of cohort non-convergence requires more complex extensions of our proposed model. This line of work goes beyond the goal of our current analyses but should be considered in future research.

In a different vein, careful thought is necessary when deciding how to transform the latent scores using the initial age. Since there is limited information about the relation between the first observed score and age, $\sigma(Y_1, Age_1)$, we only expect to estimate one unknown parameter to adjust the level score to achieve model identification. Therefore, in order to perform more elaborate nonlinear transformations, certain unknown parameters implied in nonlinear equations need to be ignored or fixed at certain values.

The present project focused on trajectories following an exponential form. This function is quite common in developmental research, but it is important to note that our findings may not generalize to trajectories with different functional forms. We do not recommend a blind application of our models, even if they showed excellent performance in our simulated

conditions. If the developmental process follows a trajectory other than exponential, these transformations may lead to biased estimates and, possibly, wrong conclusions. In those scenarios, we recommend investigating what is the best model for the given data and research question.

Similarly, we studied univariate situations only. In bivariate or multivariate cases, an expected latent curve is no longer clear-cut exponential. Thus, it is uncertain that the conclusions in the present study apply to the bivariate system of change. For example, it would be interesting to examine whether the relation between age at first occasion and initial level should be the same for both variables. Future research should investigate bivariate and multivariate LCS systems where dynamical coupling effects influence the latent changes.

The model presented here can be extended by including additional predictors that account for individual differences in change. For example, a time-invariant demographic variable (e.g., sex, socio-economic status, or ethnicity) could be included as a predictor of the initial state and the linear additive component. Similarly, it would be possible to include time-varying predictors at each of the repeated occasions.

It must be noted that the LCS applied in this manuscript combines both the upper bound and the rate of change in a single latent variable (y_a). In other words, these two elements of the exponential trajectory cannot be modeled separately in the standard LCS. Although computing the upper bound is relatively simple, after the self-feedback and the additive component have been estimated (see Table 1), researchers interested in estimating each of the three components in Equation 1 directly can apply different modeling frameworks.

Conclusion

Accelerated longitudinal designs allow researchers to study developmental processes spanning long periods within a relatively shorter time framework. Latent change score models provide a convenient and powerful analytical framework to analyze data from such designs. In this paper, we extend a standard LCS model using measurement occasion as the underlying time metric as a way to capture age-related trajectories from childhood to early adulthood. The proposed extension allows describing a given developmental process and provides accurate expectations for the process across the full age range in the study.

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Appendix 1. Example of the computation of the expectations from the extended occasion-based LCS model (Note: The *R* script provided includes code for computing these expectations).

Let us assume the researcher is using Model 8 to compute the expected mean level of y given initial ages of 6, 9 and 12 years. Let us further assume that we obtained the parameters estimates reported in the condition 2 of Table 6. First, we rescale the ages to the metric used in our analysis (5 years = 0). The expected mean levels at these three first occasions are:

$$E(\mathbf{y}_1) = E(\mathbf{y}_0) = \tau_{y0} + \gamma_L \mathbf{Age}_1 + \gamma_{NL} \mathbf{Age}_1^\kappa = 8.4 + .41 \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + 3.27 \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}^{(1/2)} = \begin{bmatrix} 12.08 \\ 16.58 \\ 19.92 \end{bmatrix}$$

Now we apply Equations 4 and 5 to compute the expected means at the second and third occasions:

$$E(\mathbf{y}_2) = E(\mathbf{y}_1) + \mu_a + \beta E(\mathbf{y}_1) = \begin{bmatrix} 12.08 \\ 16.58 \\ 19.92 \end{bmatrix} + 5.52 + (-.19) \begin{bmatrix} 12.08 \\ 16.58 \\ 19.92 \end{bmatrix} = \begin{bmatrix} 15.30 \\ 18.95 \\ 21.66 \end{bmatrix},$$

and

$$E(\mathbf{y}_3) = E(\mathbf{y}_2) + \mu_a + \beta E(\mathbf{y}_2) = \begin{bmatrix} 15.30 \\ 18.95 \\ 21.66 \end{bmatrix} + 5.52 + (-.19) \begin{bmatrix} 15.30 \\ 18.95 \\ 21.66 \end{bmatrix} = \begin{bmatrix} 17.92 \\ 20.87 \\ 23.06 \end{bmatrix}.$$

If the researcher is using Model 13, two steps are required for computing the expected levels of y . The first step is fitting the nonlinear regression to obtain $\hat{Y}_1$. Assuming that the obtained parameters are those from condition 2,

$$E(\mathbf{y}_1) = E(\mathbf{y}_0) = \hat{Y}_1 = u - (u - i) * e^{(-r * age)} = 30 - (30 - 10) * e^{(-.1 \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix})} =$$

$$30 - 20 \begin{bmatrix} .90 \\ .67 \\ .50 \end{bmatrix} = \begin{bmatrix} 11.90 \\ 16.59 \\ 20.07 \end{bmatrix}$$

Again, we apply Equations 4 and 5 to obtain:

$$E(\mathbf{y}_2) = E(\mathbf{y}_1) + \mu_a + \beta E(\mathbf{y}_1) = \begin{bmatrix} 15.16 \\ 18.96 \\ 21.78 \end{bmatrix}, \text{ and}$$

$$E(\mathbf{y}_3) = E(\mathbf{y}_2) + \mu_a + \beta E(\mathbf{y}_2) = \begin{bmatrix} 17.80 \\ 20.88 \\ 23.16 \end{bmatrix}.$$

Appendix 2. Expectations for the latent variables y_1 , y_2 , and y_3 , from the extended occasion-based LCS model (Note: The *R* script provided includes code for computing these expectations).

The initial level in the process y_0 is predicted by the linear and nonlinear effects of age at first occasion ($\gamma_L \cdot Age_1$, and $\gamma_{NL} \cdot Age_1$; see Figure 2). The residual variance of the initial level is freely estimated for each cohort c ($\sigma_{y0,c}^2$). The rest of the model parameters are constrained to have equal value across cohorts. The variances of age ($\sigma_{Age1L,c}^2$), nonlinear transformation of age ($\sigma_{Age1NL,c}^2$), and the covariance among them ($\sigma_{Age1L Age1NL,c}$), are computed from the data in each cohort. Note: The elements allowed to differ across conditions are depicted in red.

Variance of y_1

$$\sigma_{y1,c}^2 = \sigma_{y0,c}^2 + \left(\gamma_L \sigma_{Age1L,c}^2 + \gamma_{NL} \sigma_{Age1L Age1NL,c} \right) \gamma_L + \left(\gamma_{NL} \sigma_{Age1NL,c}^2 + \gamma_L \sigma_{Age1L Age1NL,c} \right) \gamma_{NL}$$

Variance of y_2

$$\begin{aligned} \sigma_{y2,c}^2 = & \left((\gamma_L + \gamma_L \beta) \sigma_{Age1L,c}^2 + (\gamma_{NL} + \gamma_{NL} \beta) \sigma_{Age1L Age1NL,c} \right) (\gamma_L + \gamma_L \beta) + \\ & \left((\gamma_L + \gamma_L \beta) \sigma_{Age1L Age1NL,c} + (\gamma_{NL} + \gamma_{NL} \beta) \sigma_{Age1NL,c}^2 \right) (\gamma_{NL} + \gamma_{NL} \beta) + \\ & \left((1 + \beta) \sigma_{y0,c}^2 + \sigma_{y0,a} \right) (1 + \beta) + (1 + \beta) \sigma_{y0,a} + \sigma_a^2 \end{aligned}$$

Variance of y_3

$$\begin{aligned} \sigma_{y3,c}^2 = & \left((\gamma_L + 2\gamma_L \beta + \gamma_L \beta^2) \sigma_{Age1L,c}^2 + (\gamma_{NL} + \gamma_{NL} \beta + \gamma_{NL} \beta^2) \sigma_{Age1L Age1NL,c} \right) (\gamma_L + 2\gamma_L \beta + \gamma_L \beta^2) + \\ & \left((\gamma_L + 2\gamma_L \beta + \gamma_L \beta^2) \sigma_{Age1L Age1NL,c} + (\gamma_{NL} + \gamma_{NL} \beta + \gamma_{NL} \beta^2) \sigma_{Age1NL,c}^2 \right) (\gamma_{NL} + 2\gamma_{NL} \beta + \gamma_{NL} \beta^2) + \\ & \left((1 + 2\beta + \beta^2) \sigma_{y0,c}^2 + (2 + \beta) \sigma_{y0,a} \right) (1 + 2\beta + \beta^2) + \\ & \left((1 + 2\beta + \beta^2) \sigma_{y0,a} + (2 + \beta) \sigma_a^2 \right) (2 + \beta) \end{aligned}$$

Covariance of y_1 and y_2

$$\begin{aligned} \sigma_{y_1 y_2, c} = & \left(\gamma_L \sigma_{Age_{1L}, c}^2 + \gamma_{NL} \sigma_{Age_{1L} Age_{1NL}, c} \right) (\gamma_L + \gamma_L \beta) + \\ & \left(\gamma_L \sigma_{Age_{1L} Age_{1NL}, c} + \gamma_{NL} \sigma_{Age_{1NL}, c}^2 \right) (\gamma_{NL} + \gamma_{NL} \beta) + (1 + \beta) \sigma_{y_0, c}^2 + \sigma_{y_0, a} \end{aligned}$$

Covariance of y_1 and y_3

$$\begin{aligned} \sigma_{y_1 y_3, c} = & \left(\gamma_L \sigma_{Age_{1L}, c}^2 + \gamma_{NL} \sigma_{Age_{1L} Age_{1NL}, c} \right) (\gamma_L + 2\gamma_L \beta + \gamma_L \beta^2) + \\ & \left(\gamma_L \sigma_{Age_{1L} Age_{1NL}, c} + \gamma_{NL} \sigma_{Age_{1NL}, c}^2 \right) (\gamma_{NL} + 2\gamma_{NL} \beta + \gamma_{NL} \beta^2) + \\ & (1 + 2\beta + \beta^2) \sigma_{y_0, c}^2 + (2 + \beta) \sigma_{y_0, a} \end{aligned}$$

Covariance of y_2 and y_3

$$\begin{aligned} \sigma_{y_2 y_3, c} = & \left((\gamma_L + \gamma_L \beta) \sigma_{Age_{1L}, c}^2 + (\gamma_{NL} + \gamma_{NL} \beta) \sigma_{Age_{1L} Age_{1NL}, c} \right) (\gamma_L + 2\gamma_L \beta + \gamma_L \beta^2) + \\ & \left((\gamma_L + \gamma_L \beta) \sigma_{Age_{1L} Age_{1NL}, c} + (\gamma_{NL} + \gamma_{NL} \beta) \sigma_{Age_{1NL}, c}^2 \right) (\gamma_{NL} + 2\gamma_{NL} \beta + \gamma_{NL} \beta^2) + \\ & \left((1 + \beta) \sigma_{y_0, c}^2 + \sigma_{y_0, a} \right) (1 + 2\beta + \beta^2) + \left((1 + \beta) \sigma_{y_0, a} + \sigma_a^2 \right) (2 + \beta) \end{aligned}$$

Appendix 3. Example of the true, expected and residual covariance for Trajectory 5 and Model 9

Population covariances (Trajectory 5). Note: The covariances that can be recovered are shaded															
	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10	y11	y12	y13	y14	y15
y1	22.66														
y2	20.61	18.90													
y3	18.93	17.51	16.34												
y4	17.55	16.36	15.39	14.59											
y5	16.43	15.43	14.61	13.94	13.39										
y6	15.50	14.66	13.97	13.41	12.95	12.57									
y7	14.75	14.03	13.45	12.97	12.58	12.26	11.99								
y8	14.13	13.52	13.02	12.61	12.28	12.00	11.78	11.60							
y9	13.63	13.10	12.67	12.32	12.03	11.80	11.60	11.45	11.32						
y10	13.21	12.76	12.38	12.08	11.83	11.63	11.46	11.32	11.21	11.12					
y11	12.87	12.47	12.15	11.88	11.67	11.49	11.34	11.22	11.12	11.04	10.98				
y12	12.59	12.24	11.96	11.72	11.53	11.37	11.24	11.14	11.05	10.98	10.92	10.88			
y13	12.37	12.05	11.80	11.59	11.42	11.28	11.17	11.07	10.99	10.93	10.88	10.84	10.80		
y14	12.18	11.90	11.67	11.48	11.33	11.20	11.10	11.02	10.95	10.89	10.84	10.81	10.78	10.75	
y15	12.03	11.77	11.56	11.39	11.25	11.14	11.05	10.97	10.91	10.86	10.81	10.78	10.75	10.73	10.71
Model-implied covariances (Model 9)															
	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10	y11	y12	y13	y14	y15
y1	23.34														
y2		19.36													
y3			16.75												
y4				14.75											
y5					13.50										
y6						12.53									
y7							12.08								
y8								11.61							
y9									11.28						
y10										11.06					
y11											11.04				
y12												10.94			
y13													10.94		
y14														10.83	
y15															10.88
Residual covariances															
	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10	y11	y12	y13	y14	y15
y1	.68														
y2		.45													
y3			.40												
y4				.16											
y5					.11										
y6						-.04									
y7							.09								
y8								.02							
y9									-.04						

y10	.04		.08		-.06					
y11		.18		.02		.06				
y12			.18		.03		.06			
y13				.06		.13		.13		
y14					.07		.08		.08	
y15						.26		.21		.17

Table 1. Correspondences between the parameters in the exponential function (Equation 1) and the standard LCS model.

Exponential function for decelerated growth (Equation 1 and Figure 1)		Standard LCS model	
Parameter	Meaning	Parameter / Variable	Meaning
i	Initial level of the process when $age = 0$	y_0	Latent level at the first time point, with mean μ_{y_0} and variance σ_{y_0}
r	Rate of change, or speed to which the difference ($u-i$) is reduced over time (for decelerated growth). In Equation 1, and for decelerated growth, this parameter is positive. With a negative value for r : - The product ($r*age$) becomes positive. - u becomes the asymptote when $age \rightarrow -\infty$ - The trajectory experiences accelerated decrease.	β	Self-feedback: influence of the process on its own state at the following occasion. This parameter has the opposite sign to r : $\beta = -r$ in the continuous-time differential equation. Therefore, decelerated growth is represented by β with negative sign. (Note that the LCS model implies a discrete-time difference equation)
u	Upper bound or asymptote of the process when $age \rightarrow \infty$	y_u	Upper asymptote for the latent level. This is not directly modeled in the LCS. It can be computed as $y_u = y_a / -\beta$, where
		y_a	Latent additive component of change, with mean μ_{y_a} and variance σ_{y_a} (or μ_a and σ_a for simplicity).
Additional parameters in the LCS model			
		$\sigma_{y_0,a}$	Covariance between initial level and additive component.
		σ_e	Measurement error variance.

Table 2. Simulation parameters defining the different developmental trajectories

Trajectory	Simulation parameters (continuous time)							Implied statistics		Values for $\Delta t=2$			
	β (= $-r$)	μ_{y0}	μ_a (= $\mu_u * r$)	σ_{y0}^2	σ_a^2	$\sigma_{y0,a}$	σ_e^2	$\rho_{y0,a}$	μ_u	β	μ_a	σ_a^2	$\sigma_{y0,a}$
1	-.1	10	2.5	25	.07	.95	2	.7	25	-.18	4.53	.24	1.72
2	-.1	10	3.0	25	.11	1.14	2	.7	30	-.18	5.44	.35	2.06
3	-.1	10	3.5	25	.14	1.33	2	.7	35	-.18	6.34	.47	2.40
4	-.2	10	5.0	25	.29	1.89	2	.7	25	-.33	8.24	.80	3.12
5	-.2	10	6.0	25	.42	2.27	2	.7	30	-.33	9.89	1.15	3.75
6	-.2	10	7.0	25	.57	2.65	2	.7	35	-.33	11.54	1.56	4.37
7	-.4	10	10.0	25	1.17	3.79	2	.7	25	-.55	13.77	2.22	5.22
8	-.4	10	12.0	25	1.69	4.55	2	.7	30	-.55	16.52	3.20	6.26
9	-.4	10	14.0	25	2.30	5.30	2	.7	35	-.55	19.27	4.35	7.30
10	-.6	10	15.0	25	2.64	5.68	2	.7	25	-.70	17.47	3.58	6.62
11	-.6	10	18.0	25	3.80	6.82	2	.7	30	-.70	20.96	5.15	7.94
12	-.6	10	21.0	25	5.17	7.96	2	.7	35	-.70	24.46	7.01	9.27

Table 3. Sampling schedule applied to all the samples in the study. Shaded cells represent one measurement occasion for all cohort members during the corresponding year.

Age in years															% of cases entering in each cohort
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
■		■		■											18.8%
	■		■		■										18.8%
		■		■		■									9.4%
			■		■		■								9.4%
				■		■		■							6.3%
					■		■		■						6.3%
						■		■		■					6.3%
							■		■		■				6.3%
								■		■		■			6.3%
									■		■		■		6.3%
										■		■		■	6.3%
											■		■		6.3%

Table 4. Summary of the transformations of Age_1 in each LCS model

Model	Age_1 linear effect	Age_1 nonlinear effect
1 (Worst model)	no	no
2 (Linear)	$\gamma_L \cdot Age_1$	no
3		$\gamma_{NL} \cdot Age_1^{(-1)}$
4		$\gamma_{NL} \cdot Age_1^{(-1/2)}$
5		$\gamma_{NL} \cdot Age_1^{(-1/5)}$
6		$\gamma_{NL} \cdot Age_1^{(1/4)}$
7		$\gamma_{NL} \cdot Age_1^{(1/3)}$
8	$\gamma_L \cdot Age_1$	$\gamma_{NL} \cdot Age_1^{(1/2)}$
9		$\gamma_{NL} \cdot Age_1^2$
10		$\gamma_{NL} \cdot Age_1^3$
11		$\gamma_{NL} \cdot Ln(Age_1)$
12		$\gamma_{NL} \cdot e^{(-Age_1)}$
13 (Best model, 2 steps)	no	$\gamma_{NL} \cdot Y_1$

Table 5. Total Bias for the expectations of each model, for all simulation conditions. The models with lower bias in each condition are shaded.

[illegible]

Table 6. Occasion-based LCS model parameter estimates and relative bias (averaged across 100 samples) for the simulation conditions two, five and eleven.

Model	Parameter			Relative Bias = $(\hat{\theta} - \theta) / \theta$										
	β (= -r)	μ_{y0} (or τ_{y0})	μ_a (= $\mu_u * r$)	σ^2_{y0}	σ^2_a	$\sigma_{y0,a}$	σ^2_e	γ_L	γ_{NL}	β (= -r)	μ_a (= $\mu_u * r$)	σ^2_a	$\sigma_{y0,a}$	σ^2_e
Trajectory 2														
Expected value with $\Delta t=2$	-0.18	.	5.44	.	0.35	2.06	2.00	.	.					
1. Baseline	-0.18	16.17	5.38	35.56	.45	1.93	2.13	.	.	.00	-.01	.29	-.06	.06
2. Linear	-0.18	10.94	5.49	18.72	.37	2.15	2.14	1.25	.	.00	.01	.06	.04	.07
3. Age ⁻¹	-0.19	11.12	5.51	18.65	.37	2.18	2.14	1.22	-.07	.06	.01	.06	.06	.07
4. Age ^(-1/2)	-0.19	11.78	5.52	18.59	.37	2.20	2.14	1.16	-.64	.06	.01	.06	.07	.07
5. Age ^(-1/5)	-0.19	14.29	5.52	18.53	.37	2.21	2.14	1.07	-3.08	.06	.01	.06	.07	.07
6. Age ^(1/4)	-0.19	6.64	5.52	18.44	.37	2.21	2.14	.78	4.73	.06	.01	.06	.07	.07
7. Age ^(1/3)	-0.19	7.51	5.52	18.42	.37	2.20	2.14	.69	3.93	.06	.01	.06	.07	.07
8. Age ^(1/2)	-0.19	8.40	5.52	18.40	.37	2.20	2.14	.41	3.27	.06	.01	.06	.07	.07
9. Age ²	-0.18	9.98	5.50	18.34	.37	2.17	2.14	1.94	-.07	.00	.01	.06	.05	.07
10. Age ³	-0.18	10.23	5.50	18.36	.37	2.16	2.14	1.64	.00	.00	.01	.06	.05	.07
11. Ln(Age)	-0.19	11.26	5.52	18.48	.38	2.21	2.14	.98	.85	.06	.01	.09	.07	.07
12. e ^(-Age)	-0.19	12.19	5.52	18.44	.37	2.20	2.14	1.08	-3.06	.06	.01	.06	.07	.07
13. True model	-0.19	-.01	5.50	18.34	.37	2.17	2.14	.	1.00	.06	.01	.06	.05	.07
Trajectory 5														
Expected value with $\Delta t=2$	-0.33	.	9.89	.	1.15	3.75	2.00	.	.					
1. Baseline	-0.33	19.54	9.88	46.86	1.17	3.44	2.18	.	.	.00	.00	.02	-.08	.09
2. Linear	-0.33	12.66	9.99	17.66	1.17	3.67	2.19	1.64	.	.00	.01	.02	-.02	.10
3. Age ⁻¹	-0.34	13.18	10.03	17.30	1.18	3.74	2.19	1.57	-.20	.03	.01	.03	.00	.10
4. Age ^(-1/2)	-0.34	14.91	10.05	16.90	1.19	3.79	2.19	1.42	-1.70	.03	.02	.03	.01	.10
5. Age ^(-1/5)	-0.34	21.19	10.06	16.54	1.19	3.81	2.19	1.20	-7.83	.03	.02	.03	.02	.10
6. Age ^(1/4)	-0.34	2.21	10.06	16.07	1.19	3.80	2.19	.51	11.52	.03	.02	.03	.01	.10
7. Age ^(1/3)	-0.34	4.38	10.06	16.01	1.19	3.80	2.19	.29	9.51	.03	.02	.03	.01	.10
8. Age ^(1/2)	-0.34	6.59	10.05	15.91	1.19	3.79	2.19	-.37	7.82	.03	.02	.03	.01	.10
9. Age ²	-0.34	10.46	10.01	15.80	1.18	3.71	2.19	3.23	-.16	.03	.01	.03	-.01	.10
10. Age ³	-0.34	11.06	10.01	15.98	1.17	3.70	2.19	2.53	-.01	.03	.01	.02	-.01	.10
11. Ln(Age)	-0.34	13.47	10.06	16.31	1.19	3.81	2.19	.97	2.12	.03	.02	.03	.02	.10
12. e ^(-Age)	-0.34	15.71	10.05	16.13	1.19	3.79	2.19	1.23	-7.45	.03	.02	.03	.01	.10
13. True model	-0.34	-.02	10.02	15.72	1.18	3.73	2.19	.	1.00	.03	.01	.03	-.01	.10
Trajectory 11														
Expected value with $\Delta t=2$	-0.70	.	20.96	.	5.15	7.94	2.00	.	.					
1. Baseline	-0.71	24.66	21.22	46.30	5.32	7.65	2.16	.	.	.01	.01	.03	-.04	.08
2. Linear	-0.71	18.53	21.16	23.05	5.28	7.54	2.16	1.46	.	.01	.01	.03	-.05	.08
3. Age ⁻¹	-0.71	19.86	21.25	20.52	5.33	7.73	2.16	1.28	-.54	.01	.01	.03	-.03	.08
4. Age ^(-1/2)	-0.71	24.12	21.30	17.71	5.36	7.82	2.16	.91	-4.26	.01	.02	.04	-.02	.08

5. $\text{Age}^{(-1/5)}$	-.71	39.44	21.31	15.57	5.37	7.84	2.16	.39	-19.26	.01	.02	.04	-.01	.08
6. $\text{Age}^{(1/4)}$	-.71	-6.38	21.28	13.38	5.35	7.76	2.16	-1.22	27.40	.01	.02	.04	-.02	.08
7. $\text{Age}^{(1/3)}$	-.71	-1.10	21.26	13.18	5.34	7.74	2.16	-1.73	22.46	.01	.01	.04	-.03	.08
8. $\text{Age}^{(1/2)}$	-.71	4.34	21.24	12.94	5.32	7.70	2.16	-3.22	18.21	.01	.01	.03	-.03	.08
9. Age^2	-.71	13.93	21.15	14.79	5.27	7.53	2.16	4.77	-.33	.01	.01	.02	-.05	.08
10. Age^3	-.71	15.35	21.14	16.35	5.27	7.52	2.16	3.22	-.02	.01	.01	.02	-.05	.08
11. $\text{Ln}(\text{Age})$	-.71	20.43	21.30	14.37	5.36	7.81	2.16	-.16	5.15	.01	.02	.04	-.02	.08
12. $e^{(-\text{Age}^1)}$	-.71	25.95	21.25	13.18	5.33	7.72	2.16	.46	-18.28	.01	.01	.03	-.03	.08
13. True model	-.71	-.02	21.20	12.67	5.30	7.63	2.16	.	1.00	.01	.01	.03	-.04	.08

Table 7. *AIC* and *BIC* fit indices, averaged across the 100 samples for each model and condition.

The best fitting models in each condition are shaded.

Model	Trajectory											
	1	2	3	4	5	6	7	8	9	10	11	12
<i>AIC</i>												
1. Baseline	7162.3	7361.6	7551.1	7314.8	7626.5	7930.2	7480.6	7877.3	8213.1	7497.0	7907.8	8254.6
2. Linear	6958.6	7050.0	7124.8	6936.4	7093.9	7264.4	7049.4	7337.3	7600.1	7131.5	7473.4	7780.3
3. Age ⁻¹	6958.3	7049.5	7123.1	6929.7	7079.6	7245.9	7010.7	7278.2	7531.0	7062.7	7379.4	7672.7
4. Age ^(-1/2)	6957.4	7047.4	7120.4	6919.9	7062.6	7221.2	6962.4	7201.1	7432.5	6975.9	7257.2	7521.5
5. Age ^(-1/5)	6956.7	7045.5	7118.2	6911.6	7048.8	7200.2	6921.3	7132.2	7340.9	6901.8	7145.5	7375.3
6. Age ^(1/4)	6955.9	7043.0	7115.6	6901.3	7032.3	7173.5	6872.1	7046.3	7219.7	6820.3	7009.6	7182.2
7. Age ^(1/3)	6955.8	7042.7	7115.3	6900.0	7030.3	7170.1	6866.5	7036.5	7205.2	6813.1	6996.6	7162.6
8. Age ^(1/2)	6955.7	7042.2	7114.8	6897.9	7027.2	7164.6	6858.8	7022.9	7184.6	6805.6	6982.8	7141.8
9. Age ²	6955.8	7041.5	7114.3	6897.5	7028.3	7163.2	6883.1	7070.2	7250.2	6893.4	7131.6	7358.7
10. Age ³	6956.3	7042.4	7115.3	6902.0	7036.1	7175.1	6912.4	7121.9	7322.0	6949.4	7218.8	7473.6
11. Ln(Age)	6956.3	7044.3	7116.9	6906.4	7040.4	7186.9	6895.8	7088.2	7279.9	6857.6	7073.8	7275.9
12. e ^(-Age1)	6956.1	7043.4	7116.6	6903.0	7035.4	7178.5	6874.4	7051.9	7227.5	6815.1	6998.7	7167.3
13. True model	6953.7	7039.3	7112.2	6893.6	7022.6	7156.3	6852.2	7013.8	7170.9	6798.4	6970.3	7125.8
<i>BIC</i>												
1. Baseline	7191.8	7391.1	7580.6	7344.3	7656.0	7959.7	7510.1	7906.8	8242.6	7526.5	7937.3	8284.1
2. Linear	6992.3	7083.7	7158.5	6970.1	7127.7	7298.1	7083.1	7371.0	7633.8	7165.3	7507.1	7814.1
3. Age ⁻¹	6996.3	7087.4	7161.1	6967.6	7117.5	7283.8	7048.7	7316.1	7568.9	7100.7	7417.3	7710.6
4. Age ^(-1/2)	6995.3	7085.3	7158.3	6957.8	7100.5	7259.1	7000.3	7239.0	7470.5	7013.8	7295.2	7559.4
5. Age ^(-1/5)	6994.6	7083.4	7156.2	6949.5	7086.7	7238.1	6959.2	7170.2	7378.8	6939.8	7183.4	7413.2
6. Age ^(1/4)	6993.9	7081.0	7153.6	6939.2	7070.2	7211.5	6910.0	7084.3	7257.7	6858.3	7047.6	7220.1
7. Age ^(1/3)	6993.8	7080.6	7153.2	6937.9	7068.2	7208.0	6904.5	7074.4	7243.1	6851.0	7034.5	7200.5
8. Age ^(1/2)	6993.6	7080.1	7152.7	6935.8	7065.1	7202.5	6896.7	7060.8	7222.5	6843.6	7020.8	7179.8
9. Age ²	6993.8	7079.5	7152.2	6935.4	7066.2	7201.1	6921.0	7108.1	7288.1	6931.3	7169.6	7396.6
10. Age ³	6994.2	7080.3	7153.2	6940.0	7074.0	7213.0	6950.3	7159.8	7359.9	6987.3	7256.7	7511.5
11. Ln(Age)	6994.2	7082.2	7154.9	6944.3	7078.3	7224.8	6933.8	7126.1	7317.9	6895.5	7111.7	7313.8
12. e ^(-Age1)	6994.0	7081.3	7154.5	6940.9	7073.4	7216.4	6912.3	7089.8	7265.4	6853.0	7036.7	7205.2
13. True model	6987.4	7073.1	7145.9	6927.3	7056.3	7190.0	6886.0	7047.5	7204.6	6832.1	7004.0	7159.5

Table 8. Root Mean Square Residual (*RMSR*) for the expected covariances from of each model, for all simulation conditions. The models with lower bias in each condition are shaded.

Model	Trajectory											
	1	2	3	4	5	6	7	8	9	10	11	12
Absolute <i>RMSR</i>												
1. Baseline	8.43	15.30	25.74	13.23	22.46	35.04	19.41	37.71	61.08	18.16	34.80	56.07
2. Linear	.48	.63	.51	.79	1.45	2.35	2.94	6.00	10.03	5.55	12.08	21.34
3. Age ⁻¹	.48	.67	.59	.91	1.73	2.79	3.27	6.47	10.72	5.74	12.25	21.63
4. Age ^(-1/2)	.47	.67	.61	.95	1.81	3.02	3.29	6.53	10.88	5.27	11.16	19.77
5. Age ^(-1/5)	.45	.63	.58	.83	1.58	2.75	2.61	5.23	8.75	3.73	7.86	13.70
6. Age ^(1/4)	.44	.53	.46	.51	.93	1.85	1.15	2.45	4.31	1.36	3.07	5.27
7. Age ^(1/3)	.43	.51	.44	.46	.82	1.69	.95	2.06	3.72	1.09	2.55	4.42
8. Age ^(1/2)	.43	.48	.41	.40	.63	1.41	.65	1.45	2.78	.70	1.77	3.21
9. Age ²	.41	.40	.35	.38	.20	.71	.41	.73	1.65	.85	2.27	4.30
10. Age ³	.41	.40	.36	.38	.27	.78	.62	1.43	3.03	1.75	4.73	8.12
11. <i>Ln</i> (Age)	.44	.59	.53	.68	1.30	2.37	1.93	3.91	6.63	2.52	5.36	9.12
12. <i>e</i> ^(-Age1)	.45	.53	.42	.48	.72	1.48	.79	1.68	3.07	.88	2.15	3.74
13. True model	.42	.41	.34	.38	.20	.74	.38	.59	1.43	.39	1.10	2.13
Average covariance in population	13.12	14.79	16.60	10.09	12.40	14.93	8.37	11.11	14.32	7.89	10.86	14.38
<i>RMSR</i> relative to average population covariance												
1. Baseline	.64	1.03	1.55	1.31	1.81	2.35	2.32	3.39	4.27	2.30	3.20	3.90
2. Linear	.04	.04	.03	.08	.12	.16	.35	.54	.70	.70	1.11	1.48
3. Age ⁻¹	.04	.04	.04	.09	.14	.19	.39	.58	.75	.73	1.13	1.50
4. Age ^(-1/2)	.04	.05	.04	.09	.15	.20	.39	.59	.76	.67	1.03	1.38
5. Age ^(-1/5)	.03	.04	.03	.08	.13	.18	.31	.47	.61	.47	.72	.95
6. Age ^(1/4)	.03	.04	.03	.05	.07	.12	.14	.22	.30	.17	.28	.37
7. Age ^(1/3)	.03	.03	.03	.05	.07	.11	.11	.19	.26	.14	.23	.31
8. Age ^(1/2)	.03	.03	.02	.04	.05	.09	.08	.13	.19	.09	.16	.22
9. Age ²	.03	.03	.02	.04	.02	.05	.05	.07	.12	.11	.21	.30
10. Age ³	.03	.03	.02	.04	.02	.05	.07	.13	.21	.22	.44	.56
11. <i>Ln</i> (Age)	.03	.04	.03	.07	.10	.16	.23	.35	.46	.32	.49	.63
12. <i>e</i> ^(-Age1)	.03	.04	.03	.05	.06	.10	.09	.15	.21	.11	.20	.26
13. True model	.03	.03	.02	.04	.02	.05	.05	.05	.10	.05	.10	.15

Figure 1. Developmental trajectories of cognitive abilities with varying values for the upper bound (u) and the rate of change (r)

Note: Values of the rate of change $r = \{.1, .2, .4, \text{ and } .6\}$ are represented by blue dotted, black, grey, and red dashed lines, respectively. For each value of r , three lines are depicted with values of $u = \{25, 30, \text{ and } 35\}$

Figure 2. Path diagram of a Latent Chance Score Model (LCS) using measurement occasion as the time metric, and age as predictor of the latent level at the first occasion

Note: The parameter $\sigma_{y0,a}$ captures the covariance between the linear additive component y_a and the residual variance of y_0 .

Figure 3. Latent and observed trajectories of y for one sample with the trajectory 5 ($u=25$, $r=.2$; $n=500$). The black lines represent the population mean over time.

Figure 4. Expectations from models 2, 8, 9 and 12 for the trajectory 2 ($u=30$ and $r=.1$)

Note: The grey line represent the true mean level in the generating process. The black lines represent the models' expectations for individuals entering the study at ages 5.1, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15.

Figure 5. Expectations from models 2, 8, 9 and 12 for the trajectory 5 ($u=30$ and $r=.2$)

Note: The grey line represent the true mean level in the generating process. The black lines represent the models' expectations for individuals entering the study at ages 5.1, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15.

Figure 6. Expectations from models 2, 8, 9 and 12 for the trajectory 8 ($u=30$ and $r=.4$)

Note: The grey line represent the true mean level in the generating process. The black lines represent the models' expectations for individuals entering the study at ages 5.1, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15.

Figure 7. Expectations from models 2, 8, 9 and 12 for the trajectory 11 ($u=30$ and $r=.6$)

Note: The grey line represent the true mean level in the generating process. The black lines represent the models' expectations for individuals entering the study at ages 5.1, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15.