Structural equation modelling with complex sampling designs and non-random attrition: A tutorial using Mplus

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Abstract

Complex sampling designs involving features such as stratification, cluster sampling, and unequal selection probabilities are often used in large-scale longitudinal surveys to improve cost-effectiveness and ensure adequate sampling of small or potentially under-represented groups. However, complex sampling designs create challenges when there is a need to account for non-random attrition; a near inevitability in social science longitudinal studies. In this article we discuss these challenges and demonstrate the application of weighting approaches to simultaneously account for non-random attrition and complex design in a large UK-population representative survey. Using an auto-regressive latent trajectory model with structured residuals (ALT-SR) to model the relations between relationship satisfaction and mental health in the *Understanding Society* study as an example, we provide guidance on implementation of this approach in both R and Mplus. Two standard error estimation approaches are illustrated: pseudo-maximum likelihood robust estimation and Bootstrap resampling. A comparison of unadjusted and design-adjusted results highlights that ignoring the complex survey designs when fitting structural equation models can result in misleading conclusions.

Keywords: structural equation modelling; complex survey designs; missingness; attrition; weighting

Complex sample designs are characterised by the presence of features such as stratification, cluster sampling, and unequal selection probabilities that lead to deviations from simple random sampling from an underlying target population (Lumley, 2004). Complex sampling designs are often adopted in large-scale longitudinal studies because they offer a number of significant scientific and practical advantages (Stapleton, 2006). Stratification, i.e., the division of the population into relatively homogenous subgroups and sampling a predetermined number of units from each, is often employed because it can increase precision. Cluster sampling may be employed to reduce face-to-face interviewing costs because, for example, interviewer travel can be reduced when participants can be recruited from the same neighbourhoods (Smith et al., 2009). This results in a larger sample size being possible for any given budget, and in practice this tends to have the overall effect of increasing precision. Finally, in many longitudinal studies it can be advantageous to oversample sub-populations that are rarer or of special interest to ensure adequate statistical power for analyses involving these groups (Connelly & Platt, 2014; Lynn, 2009a; Lynn et al, 2018; Plewis et al., 2007). Similarly, it is known that some sub-populations are more vulnerable to attrition (Eisner et al., 2018; Watson & Wooden, 2009) and baseline oversampling or refreshment sampling may be necessary to counteract their loss to the study over time (Corry et al., 2017; Deng et al., 2013). This will result in some members of the population having greater selection probabilities than others.

Many large-scale longitudinal studies that are widely used for secondary data analysis follow a complex sampling scheme employing some or all of these sampling design components. In the UK, for example, complex sampling designs characterise openly accessible longitudinal datasets such as the Millennium Cohort Study (MCS; Connelly & Platt, 2014), the English Longitudinal Study of Ageing (ELSA; Steptoe et al., 2013); and the UK Household Longitudinal Study (Lynn, 2009b). These datasets are invaluable resources for understanding population and human developmental dynamics and have advanced understanding in a diversity of areas, such as child development, mental health, education, labour market participation, and healthy ageing (Benzeval, 2020; Platt et al., 2020). They are accordingly very widely used; some having generated thousands of scientific publications.

Analysing data from complex sampling designs using standard statistical analysis methods that assume a simple random sample can, however, result in substantial bias in inferences (Hahs-Vaughn & Lomax, 2006; Kaplan & Ferguson, 1999; Muthen & Satorra, 1995; Vieira et al., 2016). The application of conventional structural equation modelling (SEM) techniques to clustered data can result in standard error estimates being too small and the over-rejection of models based on global χ^2 tests of model fit (Muthen & Satorra, 1995). This happens because such data violate the assumption of independence of observations, i.e., individuals within the same clusters are liable to be more similar to one another. Ignoring stratification typically has the opposite effect on standard errors (i.e., shrinking variance); however, this is most often outweighed by the effect of clustering (Stapleton, 2006) and bias cancellation between stratification and clustering effects certainly cannot be relied upon. Ignoring unequal probabilities of selection can impact parameter estimates in SEMs. In particular, it leads to biased estimates when selection probabilities are related to the substantive variables in the model (Asparouhov, 2005; Kaplan & Ferguson, 1999).

To overcome these issues, several approaches have been proposed to account for complex sampling designs when fitting SEMs, many of which are now implemented in widely used SEM software and modules (Asparouhov & Muthén, 2010; Muthen & Satorra, 1995; Oberski, 2014; West et al., 2018; Wu & Kwok, 2012). Among the approaches that have been suggested, 'design-based' approaches are more popular because of their relative ease of interpretation. Design-based approaches involve using information about the sampling method (clustering, stratification, and unequal selection probabilities) to adjust estimates of both parameters and their variances, as well as fit statistics.

Design-based approaches are relatively widely used in substantive applications of SEM (Davidov et al., 2006; Murray et al., 2021; Patalay et al., 2017; Speyer et al., 2020). The most popular design-based treatment is to employ pseudo-maximum likelihood (PML) SEM estimation to obtain design-adjusted parameter estimates and standard errors. PML uses weighted estimation (replacing sample covariances with their weighted equivalents) and Taylor Series Linearisation to obtain adjusted standard errors (Asparouhov, 2005; Oberski, 2014). From a user's perspective, this method only requires the specification of a weighting variable (recording the selection probabilities of observations), and cluster and stratification variables (recording the clusters and strata to which observations belong).

An alternative set of methods for estimating the variance of parameters in SEMs fitted to complex sample data are the replication weight methods (Asparouhov & Muthén, 2010; Stapleton, 2008). These methods can utilise the same information (weights, clusters, and strata) utilised in PML for variance estimation but use resampling methods for the calculation of SEs. For example, Mplus (Asparouhov & Muthén, 2010) uses the following formula to calculate bootstrapped standard errors:

$$\sqrt{\sum_{r=1}^{R} C_r ¿¿¿}$$

(1)

where θ is a model parameter, $\hat{\theta}$ is the estimate for that parameter using the original weight variable, and $\hat{\theta}_r$ is the estimate for that parameter using the rth replicate weight and where R is the number of replicates drawn. The value of C_r varies depending on the specific resampling method used, with variants including the bootstrap, Jackknife, balanced repeated replicates (BRR), and Fay methods (Asparouhov & Muthén, 2010). In the bootstrap method, resampling occurs at the primary sampling unit (PSU) level and weights are calculated as:

$$w_i f \frac{K_h}{K_h - 1}$$

(2)

where for *H* strata, w_i is the original individual-level weight, f is the number of times the PSU that the individual belongs to was drawn in the bootstrap sample, and K_h is the number of PSUs in stratum *h*. The standard errors can then be calculated as:

$$\sqrt{\sum_{r=1}^{R} \frac{1}{R-1} ¿¿¿}$$

(3)

The Jackknife, BRR and Fay methods use similar principles and are comprehensively described in (Asparouhov & Muthén, 2010).

As compared to adjustments to parameter and standard error estimation, adjustment of global and local fit statistics for complex sampling designs has been less comprehensively studied (Bollen et al., 2013). A common approach is to apply a scaling correction to the overall χ^2 test. Such a correction is necessary because the distribution of the test statistic for the likelihood ratio test (comparing the analysis model to a baseline unrestricted model) normally used to evaluate overall model fit is no longer χ^2 distributed under PML with complex sampling designs. Its specific distribution depends on the sampling weights, stratification, and clustering. The Satorra-Bentler correction (Satorra & Bentler, 1994) involves dividing the original (unadjusted) χ^2value by an estimate of the average generalised design effect (Rao & Scott, 1984). The correction used in Mplus is based on similar principles but uses a correction factor based on the first and second derivatives of the PMLs for the analysis and unrestricted models (Asparouhov & Muthen, 2006; Asparouhov & Muthén, 2005). Mplus offers both PML estimation (or equivalent estimation) and resampling based methods for accounting for complex survey designs, making these solutions widely available for SEM users (Asparouhov & Muthén, 2010; Oberski, 2014).

Accounting for complex sampling alone; however, does not guarantee unbiased inferences from longitudinal data. In particular, a major additional source of bias is that longitudinal studies also almost invariably suffer attrition, i.e. the dropout of participants from longitudinal studies over time (Eisner et al., 2018; Lynn, 2018). As well as reducing statistical precision through decreasing the available longitudinal sample size, attrition can also introduce substantial parameter bias when related to the outcomes under study (e.g., Haviland et al., 2011). Two methods are commonly recommended and widely used for dealing with attrition in longitudinal studies: full information maximum likelihood estimation (FIML) and multiple imputation (MI) (e.g., Enders, 2013). These methods can provide unbiased inferences provided that data are missing at random (MAR: i.e., missing conditional on observed but not unobserved values; Rubin, 1976). However, these methods are not straightforward to apply in the context of longitudinal models with complex sampling designs (e.g., Silverwood et al., 2020).

The FIML approach to addressing missingness is an 'implicit imputation' method which is based on estimating unbiased parameter estimates by assuming an underlying distribution for the data (see e.g., Enders, 2010). The major difficulty with FIML is that in the context of longitudinal studies with non-random attrition it can be necessary to include very many auxiliary variables i.e., variables that are not of interest in the main analysis but can help achieve MAR or improve power because they are correlated with missingness and/or missing variables (Collins et al., 2001). In the saturated correlates method, for example, a saturated model for the auxiliary variables is added to allow the information for these variables to be used without changing how the main analysis parameters need to be interpreted. This means that correlations between every auxiliary variable and 1) every other auxiliary variable 2) every variable predictor, and 3) every residual term for the outcomes must be added to the model. Especially for already complex models this can significantly limit the number of auxiliary variables that can be included in the model because in practice large number of auxiliary variables can lead to estimation and convergence problems, especially if the auxiliary variables are themselves incomplete (Enders, 2010; Howard et al., 2015). It also necessitates adjustments to incremental fit statistics because the baseline model (to which the hypothesised model is compared) is affected by the inclusion of the auxiliary variables (see e.g., Enders, 2010)

Though it can be practically more difficult to implement than FIML because it involves multiple stages and a number of methodological choice points, MI has often been suggested as a good alternative solution to missing data problems because it easily accommodates very large numbers of auxiliary variables (Asendorpf et al., 2014; Azur et al., 2011; Van Buuren, 2011). MI approaches to dealing with attrition involve fitting a model for the distribution of missing data given the observed data. MI uses this model to substitute missing values with estimates of what those missing values would have been had they been observed, as predicted from data that is available (e.g., based on previous waves) and including a random component. In order to achieve correct standard errors, the estimation of the missing data is implemented several times to create several imputed datasets. The main analysis of interest is then conducted in each of those datasets. The resulting parameter estimates are averaged across the datasets and the standard errors associated with those parameters are calculated by combining the within- and between-imputed dataset variance in those parameters (where the within-dataset variance is based on the estimated within-dataset sampling variances). This latter stage of combining results across imputed datasets is referred to as 'pooling' and has been shown to result in statistically valid inferences given uncertainty in the data (Rubin, 2004; Yuan, 2011).

MI is; however, challenging to implement for complex sampling designs because of the need for the imputation model to be at least as complex as the analysis model. While MI techniques have been proposed to account for multi-level data and unequal selection probabilities (De Silva et al., 2020; Grund et al., 2016, 2018; Quartagno et al., 2019; Zhou et al., 2016c, 2016a, 2016b), techniques that can accommodate weighting, stratification, and cluster sampling in the same data are not widely accessible nor have they been comprehensively tested and validated. As such, a recent review concluded that there are currently no solutions that can be considered 'optimal' for analysing complex survey data using MI (Kleinke et al., 2020). Further, as a general technique it has been noted that MI can be technically demanding to implement for applied users and thus more vulnerable to mis-specification than some other missing data techniques (Seaman et al., 2012).

Attrition-adjusted design weights offer a simpler solution to dealing with non-random attrition in longitudinal studies from a dataset user's perspective. A weighting approach to correcting for attrition bias (e.g., see Seaman & White, 2013) involves deriving estimates of the probability of responding at a given wave or set of waves and using these estimates to create weights (usually the inverse of the estimated probability of responding). These are then used to up-weight those with a low probability of responding and down-weight those with a high probability of responding. The probability estimates used to calculate the weights are typically based on logistic or probit regression models in which being a respondent is predicted by information from previous waves and/or linked data (e.g., administrative data) that is available for all, irrespective of whether they were a respondent or not at the relevant wave/set of waves. The resulting attrition weights are then combined with the design weights to form attrition-adjusted design weights and these are incorporated into the design-based techniques outlined above in place of the design weights (Lynn & Watson, 2021).

Previous studies have suggested that attrition weighting can be an effective method of addressing attrition bias provided that data are MAR (Rubin, 1976) i.e., that attrition can be explained based on observed variables (Lewin et al., 2018; McGuigan et al., 1997). Attrition weights are, therefore, commonly provided with longitudinal datasets employing complex sampling designs as part of their data releases (Connelly & Platt, 2014; Lynn & Kaminska, 2010; Schmidt & Woll, 2017; Trappmann et al., 2019; Vandecasteele & Debels, 2006; Watson & Wooden, 2012). There are; however, some disadvantages to these methods. In some cases, for example, a weight is not available for a particular combination of waves and the analyst must either rely on a suboptimal weight or derive their own from the information available (Lynn & Watson, 2021).The former can result in a loss of bias reduction or efficiency and the latter somewhat negates the 'ease of implementation' advantage of the approach. Further, weighting does not allow the full potential analysis sample to be included in the analysis and is, therefore, often less efficient than other techniques. For example, if there was 70% attrition between baseline and a given wave of interest then only 70% of the sample would have weights defined for that analysis. Similarly, in the cases where attrition is non-monotonic (i.e., participants may miss some waves but return later), the weights provided by most longitudinal studies tend to discard participants who were missing at some earlier wave but provided data at subsequent waves. While this is not an inherent limitation of weighting, it reflects how attrition weighting is typically implemented in practice. Further, unlike FIML and MI, weighting deals only with respondent-level missingness and not item-level missingness. Finally, it has been noted that when the predictors in the model for missingness (i.e., the model predicting the probability of being a respondent) are related to the probability of responding but are not related to the outcome of interest in the main analysis, weighting can increase parameter variance without reducing bias (Alanya et al., 2015). The reduction in estimation efficiency due to these drawbacks when using weights may not be a major problem for the longitudinal studies with large sample sizes when weighed against their advantages compared with alternative methods, though it may be more of a problem in smaller studies.

Taken together, given the challenges of FIML and MI for analysing data from longitudinal surveys with complex sampling designs, weighting in a design-based method such as PML is likely to be the method of choice for many users. However, while missing data techniques and adjustment for complex survey designs in SEM are individually discussed in a number of places, there is little explicit discussion regarding their combined treatment. In the current tutorial we, therefore, illustrate the application of weighting techniques, using attrition-adjusted design weights, for the analysis of longitudinal SEMs Mplus software.

Method

Data

Data for the present illustration come from the *Understanding Society* Study. *Understanding Society* data have been downloaded from the UK Data Archive more than 40,000 times, making it one of the most-used data sets in the archive. More than 4,000 resultant publications are known of, with the citation rate for those journal articles in Scopus published between 2013 and 2018 being 2.6 times the average for similar discipline papers Analysis of the study data has informed many policy areas including, for example, the Casey Review on opportunity and integration (Casey, 2016)the Taylor Review on modern working practices (Taylor et al., 2017)and the Homelessness Reduction Act 2017. As a much used and highly influential large, UK-wide, high-quality longitudinal study with a complex sampling design involving stratification, clustering and unequal selection probabilities, it provides a good case for the present illustration. A more comprehensive description of the study and how to access the data can be found on the study website at: https://www.understandingsociety.ac.uk/. The first wave of data collection for *Understanding Society* took place in 2009. Participants were sampled using a multi-stage representative probability sample of households (Lynn, 2009b).The main sample comprises four subsamples: the general population sample (GPS, which includes the General Population Comparison sample; GPC), the former British Household Panel Survey (BHPS; from wave 2), the Ethnic Minority Boost Sample (EMBS; an oversample of households from areas with high proportions of ethnic minorities) and the Immigrant and Ethnic Minority Boost Sample (IEMBS; a sample of immigrants and ethnic minority individuals from wave 6 on).

The current illustration uses data on adult participants (aged over 16) who were recruited at wave 1 and who remained in the study until wave 9, i.e., only the GPS and EMBS samples. Of those in the sampling frame at baseline, only persons in households that provided data at wave 1 were re-contacted at wave 2. In subsequent waves, however, it was possible for members who had not been included at a given wave to return to the study at a later stage, provided that they remained eligible (e.g., they did not migrate out of the UK). Thus, there were some participants who missed some waves but returned to the study later. Where participants clearly indicated that they did not want to participate again ('adamant refusals'), no further contact was attempted. Information about participants was gathered through three instruments at each wave: a household enumeration survey (one per household to identify the members of the household), a household questionnaire (one per household to collect household-level information), and an individual questionnaire (one per individual). Data come from the latter instrument and, therefore, exclude participants who have only household-level information. The sample size used in the current illustration is n=19,551.

Measures

We focus on the mental health and partner relationships data. The links between romantic relationships and mental health outcomes is an active area of research and large, longitudinal population-representative studies can be valuable for illuminating their reciprocal links (Braithwaite & Holt-Lunstad, 2017).

Mental health was measured using the *General Health Questionnaire* 12-item version (GHQ-12). This was available at all 9 waves of the study but we here focus on waves 1,3,5,7, and 9 where partner relationship data were also available. It comprises 12 items referring to concentration, sleep, playing a useful role, feeling capable of making decisions, feeling constantly under strain, having problems overcoming difficulties, enjoying day-to-day activities, feeling able to face problems, feeling unhappy or depressed, losing confidence, feeling worthless, and general happiness. Responses to each item were on a four-point Likert-type scale from 'better than usual' to 'much less than usual'. Responses to the 12 items were summed to provide a single score with a range of 0 to 36.

Partner relationship satisfaction was measured using the Revised Dyadic Adjustment Scale (RDAS; Busby et al., 1995). A composite score was derived as the sum of four items that ask a respondent to rate different aspects of their relationship on a six-point scale. Aspects of the relationship rated are: how often they consider terminating their relationship with their partner, how often they regret getting married to/living with their partner, how often they quarrel with their partner, and how often their partner gets on their nerves. These items were administered to adults who were married or co-habiting with their partner.

Analyses

To illustrate the use of design-based techniques to address missingness, we use an auto-regressive latent trajectory model with structured residuals (ALT-SR; Curran et al., 2014) to examine the longitudinal within-person links between mental health and relationship satisfaction. These models are helpful for studying reciprocal within-person relations between constructs over time (Berry & Willoughby, 2017; Mund & Nestler, 2019; Murray et al., 2019), for example, how changes in individuals' relationship satisfaction leads to or is impacted by changes in their mental health status. ALT-SRs involve fitting a cross-lagged panel model (CLPM) structure to the residuals from a parallel process growth curve model. This allows the between- and within-person relations between the two constructs to be dis-aggregated and the pooled within-person effects of one construct on another to be estimated. For most applications this is an advance on the traditional cross-lagged panel model in which the parameter estimates are a difficult-to-interpret blend of within- and between-person effects (Berry & Willoughby, 2017). In the current ALT-SR application, intercept, and slope factors were specified for both mental health and relationship happiness with the factor loadings for the intercept factors fixed to 1 and the factor loadings for the slope factors fixed proportional to the distance between waves (data collections were equidistant). All slope and intercept factors were allowed to covary. Further, structured residuals were created by creating residual variables that loaded perfectly on the corresponding observed variable (loadings were fixed to 1 and observed variable residual variances were fixed to 0). This aspect of the model allows the within-person effects to be modelled. A cross-lagged panel structure was then fitted to these residuals. The cross-lagged component of the model is shown in Figure 1 and includes lag-1 autoregressive and cross-lagged effects as well as within-wave (residual) covariances between mental health and relationship happiness. Each autoregressive, cross-lagged and within-wave residual covariance parameter was fixed equal to the corresponding autoregressive, cross-lagged and within-wave covariance effect at other waves. Model fit is evaluated based on conventional SEM criteria with <.05 for RMSEA and SRMR and >.95 for TLI and CFI considered to indicate a well-fitting model (Hu & Bentler, 1999; Schermelleh-Engel et al., 2003).

The versions of the parameter and standard error estimation for the above-described ALT-SR are illustrated. First an (inappropriate) unadjusted model is fitted for comparison against the adjusted versions. These models were fitted using robust estimation that included adjustment for non-normality but not the complex survey design. Second, the model is fitted using PML, with standard errors first estimated using robust (sandwich) variance estimation to adjust for the complex sampling design and second using a Bootstrap resampling-based technique. It is recommended that both standard error estimation methods are used in applications as a sensitivity check Code is provided as an Appendix.

Attrition weight selection

Care must be taken in the selection of the correct weight for a given analysis to ensure that bias is effectively addressed (Lynn & Kaminska, 2010; Lynn & Watson, 2021). In addition to design weights, the *Understanding Society* data release includes both cross-sectional analysis weights and longitudinal analysis weights, each of which includes adjustments for attrition. Further, as data were collected at multiple levels (e.g., household level, adult proxy and main interview, adult main interview only, and self-completion module), different weights are appropriate depending on which level the data come from. The most appropriate weight for the current analysis is the longitudinal weight for participation in the individual interviews up to wave 9. This is the 'i_indscus_lw' weight variable in the *Understanding Society* data release. It is derived by making a series of ten adjustments to the initial design weight, each based on a logistic regression model of the conditional probability of responding to the next survey instrument in the sequence, where the sequence consists of wave 1 household enumeration, wave 1 individual interview, and then the individual interview at each wave up to wave 9. Covariates for the first nonresponse model (wave 1 household enumeration) are primarily characteristics of small geographical areas derived from government neighbourhood statistics and 2001 Census data. Covariates for the second model (wave 1 individual interview) are primarily household characteristics taken from the wave 1 household enumeration and household questionnaire. Covariates for all subsequent models include a wide range of indicators from previous wave individual interviews, in addition to household and area characteristics.

Mplus implementation of PML

In Mplus, PML is implemented using by specifying TYPE= COMPLEX under the ANALYSIS command and supplying WEIGHTING, STRATIFICATION, and CLUSTER variables under the DATA command. The stratification variable in UKHLS is 'w_strata' and the cluster variable (recording the PSUs to which cases belong) is 'w_psu' where w indicates the wave of interest. These do not change over waves since they refer to the point of sampling; however, they are provided with the data for each wave for ease. The weighting variable depends on the data used (which modules and combinations of waves) and the goal of the analysis (the target population and type of inference, i.e., longitudinal versus cross-sectional). As described above, for the current analysis an appropriate weight would be 'i_indscus_lw'. With the above arguments supplied, the model is then specified as it would be for any other analysis in the MODEL command. Mplus model syntax is described in detail at: https://www.statmodel.com/language.html.

Mplus implementation of resampling for standard error estimation

In Mplus, implementation of resampling methods is achieved by supplying the stratification, cluster, and weight variables as described in the previous section, alongside specifying values for the BOOTSTRAP and RESPSE commands in the ANALYSIS section. REPSE specifies the type of resampling method to be used. Mplus offers several options: bootstrapping, Jackknife, Fay and BRR methods (Asparouhov & Muthén, 2010). In the current illustration we use the bootstrap method with 500 draws.

Results

The unadjusted model fit well by conventional criteria [χ^2 (35) = 267.43 p<.001; RMSEA=.018; SRMR=.038; CFI=.987; TLI=.987]. The design-adjusted models fit slightly better [χ^2 (35) = 85.67, p<.001; RMSEA=.009; SRMR=.035; CFI=.993; TLI=.991]. The cross-lagged parameter model results from the models fit using Mplus are shown in Table 1, with unadjusted model results are also shown for comparison. Only the autoregressive and cross-lagged parameters are shown, with full model results available in Supplementary Materials. In the unadjusted model there were significant within-person autoregressive effects for both relationship happiness and mental health as well as a significant within-person cross-lagged effect of mental health on relationship happiness. In the design-adjusted analyses; however, only the autoregressive effects were significant, i.e., design adjustment resulted in changes in significance for the main effects of substantive interest in this analysis. Standard errors were larger, reflecting that the unadjusted analyses under-estimate the uncertainty. Standard errors were similar across the two variance estimation methods in the design-adjusted setting.

Discussion

In this tutorial, we have noted that complex sampling designs are characteristic of many open and widely used longitudinal datasets. When statistical analyses, including SEM, are conducted using these kinds of datasets techniques must be employed that take into account the presence (as relevant) of unequal selection probabilities, stratification, and clustering to avoid biased parameter estimates, standard errors, and model fit statistics. These same studies are also vulnerable to the effects of drop-out and associated attrition bias, making it important to apply analysis techniques that can simultaneously address complex sampling designs and attrition. Commonly used and recommended missing data techniques to address attrition, such as FIML with auxiliary variables and multiple imputation can be difficult to apply in practice with complex longitudinal survey data. Attrition-adjusted design weights, however, may offer a more practical solution for most applications.

Using an autoregressive latent trajectory model with structured residuals (ALT-SR), we provided an illustration of fitting longitudinal models to complex survey data in *Mplus* (Muthén & Muthén, 2015). We illustrated parameter estimation using pseudo-maximum likelihood robust estimation and standard error estimation based both on PML and resampling techniques (Asparouhov & Muthén, 2010; Oberski, 2014). Our example came from the *Understanding Society* study and examined the within-person cross-lagged relations between relationship satisfaction and mental health (see Braithwaite & Holt-Lunstad, 2017 for a recent review of research in this area). We selected an ALT-SR model for our example because of the growing popularity of this model in longitudinal SEM applications (e.g., Berry & Willoughby, 2017; Curran et al., 2014; Mund & Nestler, 2019; Murray et al., 2019; Oh et al., 2020); however, the principles discussed and the implementation illustrated is applicable to SEMs more broadly.

We showed that results were similar across the two standard error estimation methods used (robust estimator and bootstrap resampling); however, these results differed in important ways from the results from unadjusted analyses that ignored the sampling design. For example, the naïve models ignoring the sampling design suggested that there was a significant negative effect of mental health on later relationship happiness; however, that effect was not significant in the design adjusted analyses.

The parameter estimate for the autoregressive effect of relationship happiness was also smaller and less significant in the design-adjusted analyses compared with the unadjusted analyses. These results thus further reinforce the message that ignoring survey design features when fitting SEMs can make a considerable difference to the conclusions drawn from complex survey data (Kaplan & Ferguson, 1999; Stapleton, 2006; Wu & Kwok, 2012).

It is, however, important to highlight the limitations of weighting approaches to addressing attrition. First, they provide unbiased estimates only under data that are missing at random (MAR), therefore, if data are missing over and above that which can be predicted from available data then they will not provide completely unbiased results (Rubin, 1976). This situation is known as 'missing not at random' (MNAR). Methods for dealing with missing not at random (MNAR) data in SEM fit to complex survey data has received little attention and it is, therefore, not clear whether and how they should be implemented. Indeed, an important future direction will be evaluation of techniques that aim to address MNAR data such as pattern mixture, random coefficient, and selection models in the context of complex survey designs.

Second, weighting techniques are less efficient than other methods of dealing with missing data. In large datasets such as the one used in the present example where statistical power is seldom an issue this may matter little; however, for smaller datasets it can become a problem. Reasons for their poorer efficiency include the fact that they do not include all available cases, they address respondent-level but not item-level missingness, and they (typically) do not account for non-monotonic attrition where participants miss a wave/waves but return to a study at a later stage. Further, if the predictors used in the model to derive the weights are related to the probability of missingness but not to the variables used in the main analysis then weighting can increase parameter variance without any bias reduction (Alanya et al., 2015). Finally, weights may not always be available for a particular combination of waves or instruments, necessitating either the use of a suboptimal weight (which sacrifices either bias reduction or efficiency) or the construction of a weight by a user. This latter technique somewhat negates a commonly cited advantage of weighting techniques, namely their ease of implementation.

For these reasons, further development and evaluation of other missing data techniques that can be implemented in complex survey data remains an important (active) area of research. Several suggestions have, for example, been advanced based on the use of multiple imputation based techniques (De Silva et al., 2020; Grund et al., 2016; Oberski, 2014; Zhou et al., 2016a, 2016b); however, at present these are either not widely available in SEM software; have not been thoroughly evaluated, or only address some aspects of complex sampling (e.g., weighting but not clustering and stratification or clustering but not weighting). It has been suggested, for example, that multiple imputation could be combined with design-adjusted analysis ; however, this approach requires further evaluation (Oberski, 2014). Other areas where further evaluation is needed is in methods for assessing design-adjustments to global fit indices in the context of nested model comparisons such as measurement invariance testing (e.g., Svetina et al., 2020) and adjustments to local fit statistics (Bollen et al., 2013). .

Conclusion

Addressing non-random attrition in longitudinal surveys with complex sampling designs is important to ensure unbiased parameter and variance estimates. Design-weighted analyses using pseudo-maximum likelihood estimation and a robust variance estimator or resampling-based variance estimator can provide a practical solution for most common longitudinal SEM applications. These techniques can be implemented in Mplus using the code provided in the current tutorial.

References

Alanya, A., Wolf, C., & Sotto, C. (2015). Comparing multiple imputation and propensity-score weighting in unit-nonresponse adjustments: A simulation study. *Public Opinion Quarterly*, *79*(3), 635–661.

Asendorpf, J. B., Van De Schoot, R., Denissen, J. J., & Hutteman, R. (2014). Reducing bias due to systematic attrition in longitudinal studies: The benefits of multiple imputation. *International Journal of Behavioral Development*, *38*(5), 453–460.

Asparouhov, T. (2005). Sampling weights in latent variable modeling. *Structural Equation Modeling*, *12*(3), 411–434.

Asparouhov, T., & Muthen, B. (2006). Comparison of estimation methods for complex survey data analysis. *Mplus Web Notes*, 1–13.

Asparouhov, T., & Muthén, B. (2010). Resampling methods in Mplus for complex survey data. *Structural Equation Modeling*, *14*(4), 535–569.

Asparouhov, T., & Muthén, B. (2005). Multivariate statistical modeling with survey data. *Proceedings of the Federal Committee on Statistical Methodology (FCSM) Research Conference*, 14–16.

Azur, M. J., Stuart, E. A., Frangakis, C., & Leaf, P. J. (2011). Multiple imputation by chained equations: What is it and how does it work? *International Journal of Methods in Psychiatric Research*, *20*(1), 40–49.

Benzeval, M. (2020). Understanding families lives across the lifecourse: The value of panel studies. Understanding Society: The UK Household Longitudinal Study. In *Handbook on Demographic Change and the Lifecourse*. Edward Elgar Publishing.

Berry, D., & Willoughby, M. T. (2017). On the practical interpretability of cross-lagged panel models: Rethinking a developmental workhorse. *Child Development*, *88*(4), 1186–1206.

Bollen, K. A., Tueller, S. J., & Oberski, D. (2013). Issues in the structural equation modeling of complex survey data. *Proceedings of the 59th World Statistics Congress*, 2013.

Braithwaite, S., & Holt-Lunstad, J. (2017). Romantic relationships and mental health. *Current Opinion in Psychology*, *13*, 120–125.

Busby, D. M., Christensen, C., Crane, D. R., & Larson, J. H. (1995). A revision of the Dyadic Adjustment Scale for use with distressed and nondistressed couples: Construct hierarchy and multidimensional scales. *Journal of Marital and Family Therapy*, *21*(3), 289–308.

Casey, L. (2016). *The Casey Review: A review into opportunity and integration*.

Collins, L. M., Schafer, J. L., & Kam, C.-M. (2001). A comparison of inclusive and restrictive strategies in modern missing data procedures. *Psychological Methods*, *6*(4), 330.

Connelly, R., & Platt, L. (2014). Cohort profile: UK millennium Cohort study (MCS). *International Journal of Epidemiology*, *43*(6), 1719–1725.

Corry, N. H., Williams, C. S., Battaglia, M., McMaster, H. S., & Stander, V. A. (2017). Assessing and adjusting for non-response in the Millennium Cohort Family Study. *BMC Medical Research Methodology*, *17*(1), 16.

Curran, P. J., Howard, A. L., Bainter, S. A., Lane, S. T., & McGinley, J. S. (2014). The separation of between-person and within-person components of individual change over time: A latent curve model with structured residuals. *Journal of Consulting and Clinical Psychology*, *82*(5), 879.

Davidov, E., Yang-Hansen, K., Gustafsson, J.-E., Schmidt, P., & Bamberg, S. (2006). Does money matter? A theory-driven growth mixture model to explain travel-mode choice with experimental data. *Methodology*, *2*(3), 124–134.

De Silva, A. P., De Livera, A. M., Lee, K. J., Moreno-Betancur, M., & Simpson, J. A. (2020). Multiple imputation methods for handling missing values in longitudinal studies with sampling weights: Comparison of methods implemented in Stata. *Biometrical Journal*.

Deng, Y., Hillygus, D. S., Reiter, J. P., Si, Y., & Zheng, S. (2013). Handling attrition in longitudinal studies: The case for refreshment samples. *Statistical Science*, *28*(2), 238–256.

Eisner, N. L., Murray, A. L., Eisner, M., & Ribeaud, D. (2018). A practical guide to the analysis of non-response and attrition in longitudinal research using a real data example. *International Journal of Behavioral Development*, 0165025418797004.

Enders, C. K. (2010). *Applied missing data analysis*. Guilford press.

Enders, C. K. (2013). Dealing with missing data in developmental research. *Child Development Perspectives*, *7*(1), 27–31.

Grund, S., Lüdtke, O., & Robitzsch, A. (2016). Multiple imputation of multilevel missing data: An introduction to the R package pan. *Sage Open*, *6*(4), 2158244016668220.

Grund, S., Lüdtke, O., & Robitzsch, A. (2018). Multiple Imputation of Missing Data for Multilevel Models: Simulations and Recommendations. *Organizational Research Methods*, *21*(1), 111–149. https://doi.org/10.1177/1094428117703686

Hahs-Vaughn, D. L., & Lomax, R. G. (2006). Utilization of sample weights in single-level structural equation modeling. *The Journal of Experimental Education*, *74*(2), 161–190.

Haviland, A. M., Jones, B. L., & Nagin, D. S. (2011). Group-based trajectory modeling extended to account for nonrandom participant attrition. *Sociological Methods & Research*, *40*(2), 367–390.

Howard, W. J., Rhemtulla, M., & Little, T. D. (2015). Using principal components as auxiliary variables in missing data estimation. *Multivariate Behavioral Research*, *50*(3), 285–299.

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling: A Multidisciplinary Journal*, *6*(1), 1–55.

Kaplan, D., & Ferguson, A. J. (1999). On the utilization of sample weights in latent variable models. *Structural Equation Modeling: A Multidisciplinary Journal*, *6*(4), 305–321.

Kleinke, K., Reinecke, J., Salfrán, D., & Spiess, M. (2020). *Applied Multiple Imputation*. Springer.

Lewin, A., Brondeel, R., Benmarhnia, T., Thomas, F., & Chaix, B. (2018). Attrition bias related to missing outcome data. *Epidemiology*, *29*(1), 87–95.

Lumley, T. (2004). Analysis of complex survey samples. *J Stat Softw*, *9*(1), 1–19.

Lynn, P. (2009a). *Methods for longitudinal surveys*. Wiley Online Library.

Lynn, P. (2009b). Sample design for understanding society. *Underst. Soc. Work. Pap. Ser*, *2009*.

Lynn, P. (2018). Tackling panel attrition. In *The Palgrave Handbook of survey research* (pp. 143–153). Springer.

Lynn, P., & Kaminska, O. (2010). *Weighting strategy for Understanding Society*. Understanding Society at the Institute for Social and Economic Research.

Lynn, P., & Watson, N. (2021). Issues in weighting for longitudinal surveys. *Advances in Longitudinal Survey Methodology*, 447–468.

McGuigan, K. A., Ellickson, P. L., Hays, R. D., & Bell, R. M. (1997). Adjusting for attrition in school-based samples: Bias, precision, and cost trade-offs of three methods. *Evaluation Review*, *21*(5), 554–567.

Mund, M., & Nestler, S. (2019). Beyond the cross-lagged panel model: Next-generation statistical tools for analyzing interdependencies across the life course. *Advances in Life Course Research*, *41*, 100249.

Murray, A. L., Eisner, M., & Ribeaud, D. (2019). Within-person analysis of developmental cascades between externalising and internalising problems. *Journal of Child Psychology and Psychiatry*.

Murray, A. L., Hall, H. A., Speyer, L. G., Carter, L., Mirman, D., Caye, A., & Rohde, L. (2021). Developmental trajectories of ADHD symptoms in a large population-representative longitudinal study. *Psychological Medicine*, 1–7.

Muthen, B. O., & Satorra, A. (1995). Complex sample data in structural equation modeling. *Sociological Methodology*, 267–316.

Muthén, L. K., & Muthén, B. (2015). Mplus. *The Comprehensive Modelling Program for Applied Researchers: User's Guide*, *5*.

Oberski, D. (2014). lavaan. survey: An R package for complex survey analysis of structural equation models. *Journal of Statistical Software*, *57*(1), 1–27.

Oh, Y., Greenberg, M. T., & Willoughby, M. T. (2020). Examining longitudinal associations between externalizing and internalizing behavior problems at within-and between-child levels. *Journal of Abnormal Child Psychology*, 1–14.

Patalay, P., Moulton, V., Goodman, A., & Ploubidis, G. B. (2017). Cross-domain symptom development typologies and their antecedents: Results from the UK millennium cohort study. *Journal of the American Academy of Child & Adolescent Psychiatry*, *56*(9), 765–776.

Platt, L., Knies, G., Luthra, R., Nandi, A., & Benzeval, M. (2020). Understanding society at 10 years. *European Sociological Review*.

Plewis, I., Calderwood, L., Hawkes, D., Hughes, G., & Joshi, H. (2007). Millennium Cohort Study: Technical report on sampling. *London: Centre for Longitudinal Study, Institute of Education*.

Quartagno, M., Carpenter, J. R., & Goldstein, H. (2019). Multiple Imputation with Survey Weights: A Multilevel Approach. *Journal of Survey Statistics and Methodology*.

Rao, J. N., & Scott, A. J. (1984). On chi-squared tests for multiway contingency tables with cell proportions estimated from survey data. *The Annals of Statistics*, 46–60.

Rubin, D. B. (1976). Inference and missing data. *Biometrika*, *63*(3), 581–592.

Rubin, D. B. (2004). *Multiple imputation for nonresponse in surveys* (Vol. 81). John Wiley & Sons.

Satorra, A., & Bentler, P. M. (1994). *Corrections to test statistics and standard errors in covariance structure analysis.*

Schermelleh-Engel, K., Moosbrugger, H., & Müller, H. (2003). Evaluating the fit of structural equation models: Tests of significance and descriptive goodness-of-fit measures. *Methods of Psychological Research Online*, *8*(2), 23–74.

Schmidt, S. C., & Woll, A. (2017). Longitudinal drop-out and weighting against its bias. *BMC Medical Research Methodology*, *17*(1), 164.

Seaman, S. R., & White, I. R. (2013). Review of inverse probability weighting for dealing with missing data. *Statistical Methods in Medical Research*, *22*(3), 278–295.

Seaman, S. R., White, I. R., Copas, A. J., & Li, L. (2012). Combining multiple imputation and inverse-probability weighting. *Biometrics*, *68*(1), 129–137.

Silverwood, R. J., Calderwood, L., Sakshaug, J. W., & Ploubidis, G. B. (2020). A data driven approach to understanding and handling nonresponse in the Next Steps cohort. *London: UCL Centre for Longitudinal Studies: CLS Working Paper*, *5*, 2020.

Smith, P., Lynn, P., & Elliot, D. (2009). Sample design for longitudinal surveys. *Methodology of Longitudinal Surveys*, 21–33.

Speyer, L. G., Hall, H. A., Ushakova, A., Murray, A. L., Luciano, M., & Auyeung, B. (2020). Longitudinal effects of breast feeding on parent-reported child behaviour. *Archives of Disease in Childhood*.

Stapleton, L. M. (2006). An assessment of practical solutions for structural equation modeling with complex sample data. *Structural Equation Modeling*, *13*(1), 28–58.

Stapleton, L. M. (2008). Variance estimation using replication methods in structural equation modeling with complex sample data. *Structural Equation Modeling: A Multidisciplinary Journal*, *15*(2), 183–210.

Steptoe, A., Breeze, E., Banks, J., & Nazroo, J. (2013). Cohort profile: The English longitudinal study of ageing. *International Journal of Epidemiology*, *42*(6), 1640–1648.

Svetina, D., Rutkowski, L., & Rutkowski, D. (2020). Multiple-group invariance with categorical outcomes using updated guidelines: An illustration using M plus and the lavaan/semTools packages. *Structural Equation Modeling: A Multidisciplinary Journal*, *27*(1), 111–130.

Taylor, M., Marsh, G., Nicol, D., & Broadbent, P. (2017). *Good work: The Taylor review of modern working practices*. Department for Business, Energy & Industrial Strategy London.

Trappmann, M., Bähr, S., Beste, J., Eberl, A., Frodermann, C., Gundert, S., Schwarz, S., Teichler, N., Unger, S., & Wenzig, C. (2019). Data resource profile: Panel Study Labour Market and Social Security (PASS). *International Journal of Epidemiology*.

Van Buuren, S. (2011). Multiple imputation of multilevel data. *Handbook of Advanced Multilevel Analysis*, *10*, 173–196.

Vandecasteele, L., & Debels, A. (2006). Attrition in panel data: The effectiveness of weighting. *European Sociological Review*, *23*(1), 81–97.

Vieira, M. de T., Salgueiro, M. de F., & Smith, P. W. (2016). Investigating impacts of complex sampling on latent growth curve modelling. *Journal of Applied Statistics*, *43*(7), 1310–1321.

Watson, N., & Wooden, M. (2009). Identifying factors affecting longitudinal survey response. *Methodology of Longitudinal Surveys*, *1*, 157–182.

Watson, N., & Wooden, M. P. (2012). The HILDA survey: A case study in the design and development of a successful household panel survey. *Longitudinal and Life Course Studies*, *3*(3), 369–381.

West, B. T., Sakshaug, J. W., & Aurelien, G. A. S. (2018). Accounting for complex sampling in survey estimation: A review of current software tools. *Journal of Official Statistics*, *34*(3), 721–752.

Wu, J.-Y., & Kwok, O. (2012). Using SEM to analyze complex survey data: A comparison between design-based single-level and model-based multilevel approaches. *Structural Equation Modeling: A Multidisciplinary Journal*, *19*(1), 16–35.

Yuan, Y. (2011). Multiple imputation using SAS software. *J Stat Softw*, *45*(6), 1–25.

Zhou, H., Elliott, M. R., & Raghunathan, T. E. (2016a). A two-step semiparametric method to accommodate sampling weights in multiple imputation. *Biometrics*, *72*(1), 242–252.

Zhou, H., Elliott, M. R., & Raghunathan, T. E. (2016b). Multiple imputation in two-stage cluster samples using the weighted finite population Bayesian bootstrap. *Journal of Survey Statistics and Methodology*, *4*(2), 139–170.

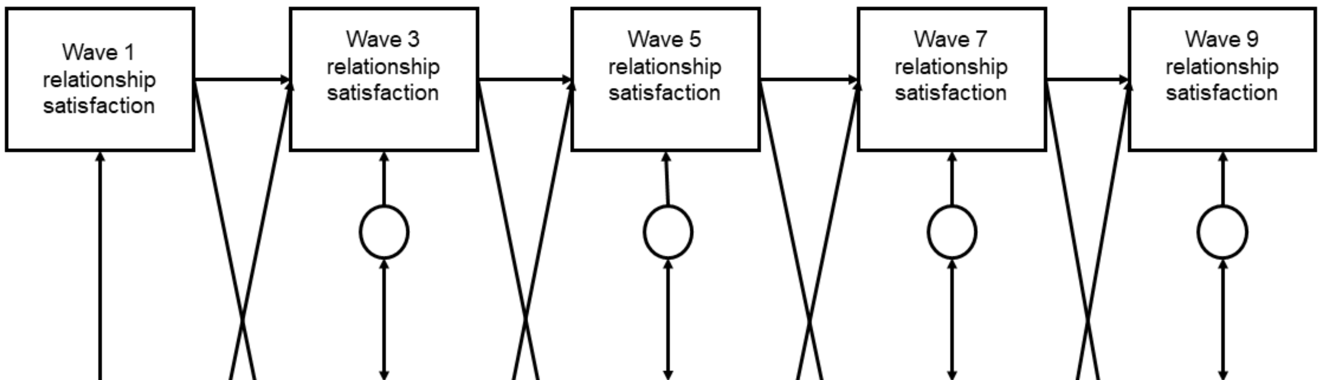
Zhou, H., Elliott, M. R., & Raghunathan, T. E. (2016c). *Synthetic multiple-imputation procedure for multistage complex samples*. De Gruyter Open.

Table 1:Coss-lagged parameters from unadjusted and design-adjusted ALT-SR

	Unadjusted			***PML***			***Resampling (Bootstrap)***		
Regressions	**Estimate**	**SE**	***p***	**Estimate**	**SE**	***p***	**Estimate**	**SE**	***p***
Relationship happiness on mental health	-.007	0.005	<.001	0.007	0.009	.400	0.007	0.009	.408
Mental health on relationship happiness	0.024	0.032	.467	0.079	0.060	.184	0.079	0.058	.176
Relationship happiness on relationship happiness	0.173	0.027	<.001	0.143	0.047	.003	0.143	0.047	.002
Mental health on mental health	0.080	0.011	<.001	0.064	0.019	.001	0.064	0.019	.001

Figures

Figure 1: Main analysis model of the relations between relationship satisfaction and mental health



Note. Rectangles represent observed variables and ellipses represent latent variables; single-headed arrows represent regression paths and double-headed arrows represent covariances. Only the autoregressive and cross-lagged parameters are shown for visual clarity.

APPENDIX: Code example

Mplus implementation of unadjusted and design-adjusted analyses

```
TITLE: unadjusted analysis of complex survey designs

DATA:
file is  ! file path goes here
VARIABLE:

NAMES ARE ! specify variable names

pidp      a_screlhappy     a_scdassat_dv   a_screlparei       a_screlparcd
a_screlparwt     a_screlpards      a_screlparrg       a_screlparar       a_screlparir
a_screlparks     a_scparoutint     a_scghq1_dv        a_psu    a_strata c_screlhappy
c_scghq1_dv      c_scdassat_dv     e_screlhappy       e_scghq1_dv        e_scdassat_dv
e_screlparei     e_screlparcd      e_screlparwt       e_screlpards       e_screlparrg
e_screlparar     e_screlparir      e_screlparks       e_scparoutint      g_screlhappy
g_scghq1_dv      g_scdassat_dv     g_screlparei       g_screlparcd       g_screlparwt
g_screlpards     g_screlparrg      g_screlparar       g_screlparir       g_screlparks
g_scparoutint    i_screlhappy      i_scghq1_dv        i_indscus_lw
i_scdassat_dv    i_screlparei      i_screlparcd       i_screlparwt
i_screlpards     i_screlparrg      i_screlparar       i_screlparir
i_screlparks     i_scparoutint     missPSU            missweight;

USEVARIABLES ARE

a_scdassat_dv c_scdassat_dv e_scdassat_dv g_scdassat_dv i_scdassat_dv
a_scghq1_dv c_scghq1_dv e_scghq1_dv g_scghq1_dv i_scghq1_dv;
! specify variables to use in analysis

MISSING ARE ALL(-999);  !    specify missing data codes

!! for design adjusted analyses ‘STRATIFICATION, CLUSTER and WEIGHT line should be !!
uncommented

!STRATIFICATION IS a_strata; !specify the stratification variable

!CLUSTER is a_psu;       !specify the cluster variable

!WEIGHT IS i_indscus_lw; !specify the attrition weight  variable

ANALYSIS:
ESTIMATOR=MLR; ! select robust maximum likelihood estimation

!! for resample-based standard errors, BOOSTRAP and REPSE lines should be !!uncommented!!

  BOOTSTRAP=500;  ! specify the number of bootstrap draws
  REPSE= BOOTSTRAP; !specify the resampling method

!!for design-adjusted analyses TYPE line should be uncommented !!

!TYPE= COMPLEX; ! specify type=complex to estimate models fit to complex survey data
```

```
MODEL:  ! specify ALT-SR model
!!!Bivariate growth curve!!!

iMH sMH| a_scghq1_dv@0 c_scghq1_dv@1 e_scghq1_dv@2 g_scghq1_dv@3 i_scghq1_dv@4; !
intercept and slope factor loadings for mental health

iRH sRH| a_scdassat_dv@0 c_scdassat_dv@1 e_scdassat_dv@2 g_scdassat_dv@3
i_scdassat_dv@4; ! intercept and slope factor loadings for relationship happiness

[iMH-sRH*];     !intercept and slope factor means (freely estimated)

iMH-sRH*;     !intercept and slope factor variances (freely estimated)

iMH with sMH-sRH;    !intercept and slope factor covariances
sMH with iRH-sRH;
iRH with sRH;

L_MH1 by a_scghq1_dv@1; ! create the residual factors for mental health
L_MH3 by c_scghq1_dv@1;
L_MH5 by e_scghq1_dv@1;
L_MH7 by g_scghq1_dv@1;
L_MH9 by i_scghq1_dv@1;

a_scghq1_dv-i_scghq1_dv@0; ! fix residuals of observed variables to 0

[L_MH1-L_MH9@0]; !estimate residual factor means
L_MH1-L_MH9*; !estimate residual factor variances


L_RH1 by a_scdassat_dv@1; !create residual factors for relationship satisfaction
L_RH3 by c_scdassat_dv@1;
L_RH5 by e_scdassat_dv@1;
L_RH7 by g_scdassat_dv@1;
L_RH9 by i_scdassat_dv@1;

a_scdassat_dv-i_scdassat_dv@0;

[L_RH1-L_RH9@0];
L_RH1-L_RH9*;

!!!relations between residual variables!!!

L_RH9 on L_RH7 (a1) !fix corresponding autoregressive parameters equal over time
L_MH7 (c1); !fix corresponding cross-lagged parameters equal over time

L_MH9 on L_MH7 (a2)
L_RH7 (c2);

L_RH7 on L_RH5 (a1)
L_MH5 (c1);

L_MH7 on L_MH5 (a2)
L_RH5 (c2);
```

```
L_RH5 on L_RH3 (a1)
L_MH3 (c1);

L_MH5 on L_MH3 (a2)
L_RH3 (c2);

L_RH3 on L_RH1 (a1)
L_MH1 (c1);

L_MH3 on L_MH1 (a2)
L_RH1 (c2);


L_RH9 with L_MH9 (r);
L_RH7 with L_MH7 (r);
L_RH5 with L_MH5 (r);
L_RH3 with L_MH3 (r);
L_RH1 with L_MH1 ;

iMH with L_MH1@0; !  set covariances between growth factors and residual factors to 0
iMH with L_RH1@0;

sMH with L_MH1@0;
sMH with L_RH1@0;

iRH with L_MH1@0;
iRH with L_RH1@0;

sRH with L_MH1@0;
sRH with L_RH1@0;

OUTPUT: STAND;
```