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1 A Tutorial on Bollen and Brand's Approach to Modeling Dynamics While Attending to
2 Dynamic Panel Bias

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Abstract

Recent studies demonstrate that when researchers are interested in dynamics they are better off using a statistical model described in Bollen and Brand (2010) rather than the often employed random-coefficient or multi-level model (Moral-Benito, Allison, & Williams, 2019; Xu, DeShon, & Dishop, 2020). Their Monte Carlo studies, however, were methodologically advanced papers. Here, we present a beginner, hands-on tutorial describing the technique. We provide code in snippet form that any researcher can apply to his or her longitudinal data, introduce fundamentals of dynamic modeling, and generalize the basic model in Bollen and Brand (2010) to situations that cover broader inferences than those discussed in the simulation papers.

Keywords: Dynamics, dynamical, modeling, longitudinal, HLM, RCM, panel bias, unobserved heterogeneity

Word count: 141

A Tutorial on Bollen and Brand’s Approach to Modeling Dynamics While Attending to Dynamic Panel Bias

Psychologists are increasingly interested in dynamics. Terms such as “dynamic,” “dynamics,” and “dynamical” are growing exponentially in published literature (DeShon, 2012). Dynamic models receive great attention in premier methodological outlets (e.g., Wang, Zhou, & Zhang, 2016; Voelkle & Oud, 2015). Substantive researchers in domains as far-reaching as leadership, training, well-being, and diversity (Blume, Ford, Surface, & Olenick, 2019; Dalal & Hulin, 2008; Sonnentag & Frese, 2012) call for a better understanding of how these processes operate as dynamic systems. One need only skim titles among various outlets in social science to see that these appeals are beginning to penetrate the field. Dynamics refers to a perspective in which a researcher captures how a system evolves from a given state at time t to another state at $t + 1$ as governed by its transition rules and external inputs.

Although researchers can use any number of models to make inferences about dynamics, such as the Arrelano-Bond approach or vector autoregressive modeling (Canova & Ciccarelli, 2013; Piening, Baluch, & Salge, 2013), random coefficient modeling (also referred to as multi-level, hierarchical linear, or mixed-effects modeling; RCM, MLM, HLM) is often used in the applied psychology literature (e.g., Drescher, Korsgaard, Welp, Picot, & Wigand, 2014; Park, Sturman, Vanderpool, & Chan, 2015; Sun, Song, & Lim, 2013). Yet, a recent Monte Carlo investigation (Xu, DeShon, & Dishop, 2020) demonstrated that HLM performs poorly when researchers are interested in dynamic inferences on data structures characterized by few time points and many units (short T and large N). When dynamic inferences are of interest – and researchers collect data with few time points and a large sample – the authors suggested Bollen and Brand’s (2010) technique due to its superior performance. Moral-Benito, Allison, and Williams (2019) provide similar results and recommendations. Both papers simulated model performance and were therefore fairly complex.

This paper is a user-friendly, hands-on tutorial of Bollen and Brand’s structural equations modeling (SEM) approach in lavaan, an R package for SEM. Its target is both graduate students and career academics who are interested in dynamics but have not yet explored the topic or run their own analyses. Our goal is to gently guide the reader through lavaan code, concepts of dynamic modeling, and a simple model (alongside extensions) that can be employed on longitudinal data (with short T and large N). We focus on a single model rather than reviewing the array of possible models for several reasons. First, other papers and books discuss the merits and pitfalls of different statistical models for longitudinal data (Grimm, Ram, & Estabrook, 2016; Hamaker, Kuiper, & Grasman, 2015; Orth, Clark, Donnellan, & Robins, 2020; Singer, Willett, Willett, & others, 2003; Voelkle, Brose, Schmiedek, & Lindenberger, 2014; Zyphur et al., 2019), and we want to avoid simply reiterating what they have already done. Second, focusing deeply on one model rather than narrowly on many models gives us more time to introduce code and slowly advance as different points of interest arise. Getting researchers to understand the backbone of how ideas are represented in code is a major piece to this paper, and we draw our inspiration from McElreath’s Bayesian statistics book that took a similar approach (2018). Throughout, we will repeatedly present images, equations, and code to facilitate understanding. Finally, there many great discussions of modeling techniques in top statistical journals (e.g., Hunter, 2018; Voelkle & Oud, 2015), but they can be difficult to parse without an intense background in methodology. Our goal is to speak more directly to beginners – to provide them with a simple model that can be used as a foundation for dynamics.

We begin with a brief overview of dynamics, and we discuss why the Bollen and Brand’s (2010) approach is an appropriate method for capturing dynamic patterns. There are a variety of questions available to researchers armed with longitudinal data, and each can be explored with an appropriate model (e.g., growth, time-invariant covariates analysis, etc.). When the interest is dynamics, or how a system evolves over time with inertia such that future states depend on prior states (Wang et al., 2016), specific challenges arise that, if

ignored, can result in spurious inferences. We cover those aspects next. In the core of the paper, we provide an exhaustive example in R to help readers implement the analytic technique in their own research. Throughout, we use examples from applied psychology to keep the discussion coherent, but the ideas apply to other areas of psychology as well. Data, R code, and Mplus code are available at the following repository (<https://github.com/anonymous4articles/bollen-brand-tutorial>).

Modeling Dynamics

Dynamic investigations are at the forefront of recent, innovative, applied psychological research. Jones et al. (2016), for example, studied pregnant women across a variety of industries and explored reciprocal relationships between revealing behaviors and physical health. Mathhews, Wayne, and Ford (2014) investigated working adults across several months and found that the effect of work family conflict on subjective well-being reversed direction when examined at different lags. Zablah et al. (2016) noted discrepancies in the strength of cross lag effects between customer satisfaction and front-line-employee satisfaction. Meier and Spector (2013) examined various directional and lag effects between work stressors and counter productive work behaviors, and they found that “the effect sizes did not depend on the length of the time lag” (p. 535). These studies highlight some of the fundamental notions of dynamics, including self-similarity or inertia – the tendency for a construct to retain its state over time – and reciprocal relationships – the tendency for constructs to form causal loops in which one construct initiates subsequent events that further change the original construct at a later period (Dishop et al., 2020a).

The state space representation of dynamics is typically applied in psychological and organizational research (DeShon, 2012) in which a system is governed by its initial states, transition rules, and external inputs across time. In this framework, a dynamic system evolves by repeatedly driving a state at time t to another state at time $t + 1$ through the application of the transition rules and external inputs (Boulding, 1955; Luenberger, 1979).

Equation 1 offers a heuristic.

$$\mathbf{States}_t = \mathbf{States}_{t-1} + \mathbf{Inputs}_t \quad (1)$$

States are viewed as functions of both their prior values alongside inputs to the system. Said differently, the states are driven by transition rules and updating mechanisms. As an example, the state could represent job satisfaction and, as expressed in the right-hand side of the equation, be driven by its prior values alongside inputs such as job autonomy and coworker support. Alternatively, the state could represent performance, with prior values constraining its movement and additional variables such as resources, motivation, and attention acting as inputs. The state need not be limited to a single outcome. Multiple states could act simultaneously with reciprocal influence in a dynamic system. More complicated updating mechanisms could exist (e.g., there are no constants in Equation 1). What matters for our purposes is that this heuristic represents the underlying system, or the actual updating mechanism from t to $t + 1$. It is the process governing state movement, which we only have indirect access to in empirical research.

Dynamic statistical models, then, are applied after data are collected with the hope of making inferences about the mechanisms just described. The goal of using one is to approximate the transition rules governing state movement. They commonly include two features. One is that dynamic statistical models incorporate lagged dependent variables as predictors (Harvey, 1990; Keele & Kelly, 2006), and these “lagged DVs” are often called “predetermined” by economists. The other is that dynamic statistical models treat time implicitly through the order of measurement occasions (Voelkle & Oud, 2015). Static models, conversely, use time as a predictor. An example dynamic statistical model to capture a simple reciprocal relationship between, say, performance and affect is depicted in the heuristic below.

$$\begin{array}{rcccl}
\mathbf{States}_t & & \mathbf{States}_{t-1} & & \mathbf{Inputs}_t \\
\begin{bmatrix} y_{i_t} \\ x_{i_t} \end{bmatrix} & = & \begin{bmatrix} b_1 y_{i_{(t-1)}} \\ b_1 x_{i_{(t-1)}} \end{bmatrix} & + & \begin{bmatrix} b_2 x_{i_t} \\ b_2 y_{i_t} \end{bmatrix} + \text{Error} \\
& & & & \\
& & & &
\end{array} \tag{2}$$

y_{i_t} is the value of the y for the i th unit at time t , $y_{i_{(t-1)}}$ is the value of y for the i th unit at time $t - 1$, x_{i_t} is the value of x for the i th unit at time t , and $x_{i_{(t-1)}}$ is the value of x for the i th unit at time $t - 1$. The model also includes coefficients relating prior to current states (e.g., the b_1 coefficients), as well as those relating contemporaneous inputs to current states (e.g., the b_2 coefficients). Each matrix could be expanded to include constants, additional variables, or more complex relationships (see Driver & Voelkle, 2018), but such extensions are not necessary for this introduction. Finally, statistical models include errors, and they make assumptions about the behavior of those errors. For now, we leave the equations as heuristics and do not specify the errors. See the Discussion for a full specification and list of assumptions.

In words using performance (y) and affect (x) as an example, this statistical model says that a given person's performance is a function of her prior performance, her current affect, and some sort of error. Moreover, her affect is a function of her prior affect, her current performance, and error. The model therefore embodies several principles of dynamics. Both states have inertia: performance is self-similar such that it retains something about itself from period to period, and the same is true of affect. With a principle like inertia embodied in the equations, we convey that performance cannot leap unbounded from a low to high value. There is continuity in the variable over time. To move it, an input must outwork the constraint of its prior level. Both states are influenced by inputs. In the current example,

inputs are free to vary from one moment to the next, and their influence is contemporaneous. One could easily extend the model to include longer lags or additional inputs. Both states are influenced by error, which represents the cumulative effect of unmeasured causes. Such unmeasured determinants will become important to our discussion later on. Notice also that time is not included as a predictor in the model. As stated by many others, time per se does not cause anything. Cause-and-effect explanations, instead, specify that changes happen over time. In sum, the system (which includes affect and performance) evolves according to the update mechanisms captured in the matrices. For broader discussions of dynamics concepts, see Dishop et al. (2020b), Cronin and Vancouver (2019), and Monge (1990).

Our paper demonstrates Bollen and Brand's SEM technique with respect to relationships like those captured in the heuristics above. The models exemplify some of the core notions of dynamics, including inertia, or the notion that a construct retains its condition over time until something changes it, and reciprocal influence, or a pattern of feedback across multiple variables. Moreover, the models generalize to a variety of more complex models that could incorporate longer lags or additional variables. Finally, they capture relationships often explored in the applied psychology literature, albeit with the less appropriate HLM model. For example, Gabriel et al. (2014) explored the reciprocal relationship between fit perception, affect, and job satisfaction with concurrent and single lag predictors. Similarly, Piening, Baluch, Alina, and Salge (2013) examined the relationship between job satisfaction, customer satisfaction, and financial performance with employee HR perceptions in the prior year. Exploring lagged DVs, concurrent predictors, and reciprocal relationships is also increasingly common in the experience sampling literature.

The strength of dynamic models is that they are amenable to various lag and directional effects inherent in a dynamic system. This freedom, however, also comes with estimation problems in the form of dynamic panel bias. The Bollen and Brand technique deals with these problems more effectively than HLM. Before describing how, we unpack the

nature of the problem.

Dynamic Panel Bias

Dynamic panel bias is a set of statistical challenges that culminate in the application of models such as those discussed above. First, including a lagged dependent variable as a predictor forces a correlation between it and (some but not necessarily all of) the model errors, violating the well-known independence assumption. As a function of the statistical model, the lagged dependent variable cannot be entirely independent of prior error. See Baltagi (2008), Baum and Christopher (2006), Hurlin (2018), and Kiviet (1995) for mathematical treatments of this issue. Conceptually, the insight is that if error affects y_{it} , then it is unlikely to be independent of $y_{i(t-1)}$. Error at time t influences, in part, the outcome at t . At the subsequent period, the outcome then becomes a predictor, leading to a correlation between it and error at the previous step. This is known as dynamic bias, and it causes increasingly biased parameter estimates as the number of time points decreases (Marriott & Pope, 1954; Nickell, 1981). Dynamic bias exists even as N , the size of one's sample, increases to infinity.

Second, if the units (the things that make up the i 's in a typical model – individuals, teams, or cells) are not perfectly exchangeable, then the outcome is affected by what is known as unobserved heterogeneity. Unobserved heterogeneity represents all unmeasured variables that are stable over time within units but that vary from one unit to another. If, for example, a study models job satisfaction as an outcome with job autonomy and coworker support as time-varying predictors, then variables such as culture and cognitive ability would be unmeasured, time-invariant influences. Unobserved heterogeneity captures these time-invariant variables that are relevant for an outcome but not explicitly measured in the model. If unobserved heterogeneity is ignored, then serial correlation is introduced into the errors. If unobserved heterogeneity is modeled as independent but in fact correlates with the predictors in the model, then omitted variables bias is introduced into the parameter

estimates (Wooldridge, 2010).

Dynamic panel bias, therefore, refers to the set of aforementioned challenges in dynamic modeling. Introducing a lagged dependent variable as a predictor – a cornerstone of dynamic modeling (Keele & Kelly, 2006) – forces a correlation between it and the model errors. Moreover, the modeler must be sensitive to unobserved heterogeneity and its relation to the predictors. Choosing a model that can correctly specify these aspects, therefore, is necessary for understanding dynamic patterns.

Handling Dynamic Panel Bias via Bollen and Brand’s SEM Approach

Mitigating dynamic panel bias is the crux of the entire fixed versus random effects modeling literature (e.g., Mundlak, 1978). Although the terms “fixed” and “random” effects are used in different ways across statistics (Gelman & Hill, 2006), we use them here to describe the relationship between unobserved heterogeneity and the model predictors (Mundlak, 1978), where random effects refers to independence and fixed effects refers to a model in which unobserved heterogeneity is allowed to correlate with the model predictors. Bollen and Brand’s default SEM approach is shown in Figure 1.

Insert Figure 1 Here

The model uses an autoregressive outcome term (ρ) that represents self-similarity or inertia such that y at t is related to y at $t + 1$. Unobserved heterogeneity is represented as a latent variable (η) and, consistent with the fixed effects approach, is allowed to correlate with the model predictors. Finally, the model conditions on the first observation of the outcome (y_1) to avoid the initial condition problem (Anderson & Hsiao, 1982). Although the default approach includes concurrent effects from the independent variable and constrains the

parameter estimates to be equal across time, those restrictions can easily be freed in SEM. Bollen and Brand's base model is attractive because it provides researchers the opportunity to specify aspects that are inherent in a dynamic system. These include a lagged dependent variable, a correlation between the model predictors and unobserved heterogeneity, different time lag associations, and various directions of influence. As we show in this paper, researchers are free to cover all of these aspects with a simple understanding of how to implement the model(s) in R.

Why use Bollen and Brand's technique over other statistical models currently in the literature? Xu et al. (2020) report a Monte Carlo study comparing HLM with Bollen and Brand's SEM approach for recovering the parameters in a dynamic process. Across different sample sizes, time lengths, and within-unit variance, HLM produces upwardly biased autoregressive estimates when the lagged dependent variable is not within-unit centered and downwardly biased estimates when the lagged variable is within-unit centered. Bollen and Brand's approach, conversely, provides lower bias in both the autoregressive and predictor coefficients across various sample sizes and time structures. Although HLM is appropriate in some longitudinal contexts (e.g., growth trajectories or static relations over time), these authors report that HLM should not be used when the study questions concern dynamics.

Implement in R

Having introduced the Bollen and Brand model, we now run an exhaustive demonstration to help future work incorporate it. The "modeling" section below walks through several syntax examples that build upon one another. We begin with a simple, single variable trajectory and work up to a reciprocal, cross-lagged system. Moreover, the code in the "modeling" section is presented without applying it to data to familiarize readers with lavaan's syntax. Understanding what code represents before applying it to data is a useful way to learn new modeling strategies (McElreath, 2016), and walking through a few blank examples will drill home some of the concepts discussed above (e.g., unobserved

heterogeneity, fixed effects, conditioning on the first observation, inertia, reciprocal relationships). The examples mirror the equations above to help readers understand how one transitions from mathematical expressions to box-and-arrow diagrams and syntax.

Modeling Syntax

We use the lavaan package throughout the modeling presentation below. In the R environment, this package is installed with `install.packages("lavaan")` and loaded with `library(lavaan)`. A few key operators that are inherent to lavaan are used throughout, and these include `=~`, which is used for latent variables and should be read as, “is measured by,” `~`, which is used for regressions and should be read as, “is regressed on.” Finally, `~~` represents variance/covariance and should be read as, “is correlated with.” We begin with a model that captures the dynamics of a single variable (y) and then progressively add an additional predictor and cross-lagged effects to cover dynamic reciprocal effects. Again, this “modeling syntax” section is presented without evaluating data to familiarize readers with the code; data application comes later. Note that each figure shows the models across four time points but the code snippets use six. This difference is simply a result of space: the maximum number of time points we could fit within a code snippet while keeping it readable was six and box-and-arrow diagrams become unnecessarily complex beyond four time points. Mplus code for each of the code snippets below is available at the following repository (<https://github.com/anonymous4articles/bollen-brand-tutorial>).

Single Variable Dynamic Panel

A fixed effects representation of a single variable dynamic model is shown in Figure 2. An autoregressive term (ρ) captures the self-similarity or inertia of the variable over time. The latent variable (η), which represents unobserved heterogeneity, is related to y at each time point beyond the initial observation. The unobserved heterogeneity term is also allowed to correlate with the model predictor, which in this case is simply the first observation of y .

Insert Figure 2 Here

The lavaan code for the model represented in Figure 2 – with six time points rather than four – is as follows:

```
DP_mod1 <- "  
  
eta_y =~ 1*y2 + 1*y3 + 1*y4 + 1*y5 + 1*y6  
  
y2 ~ rho_y*y1  
y3 ~ rho_y*y2  
y4 ~ rho_y*y3  
y5 ~ rho_y*y4  
y6 ~ rho_y*y5  
  
y1 ~~ y1  
eta_y ~~ eta_y  
y1 ~~ eta_y  
  
"
```

where `DP_mod1` is a string that captures the layout of the model. Remember, we are not using data yet so this string has not been applied to anything, all we did was assign it to `DP_mod1`. The first part of the string

```
"  
  
eta_y =~ 1*y2 + 1*y3 + 1*y4 + 1*y5 + 1*y6  
  
"
```

279 is how we model unobserved heterogeneity. `eta_y` is the latent variable representing
280 unobserved heterogeneity, and it is measured by each observation of y except for the initial
281 observation (y_1). In this case, each factor loading is constrained to 1 (Bollen & Brand, 2010).
282 The next section of the `DP_mod1` string is a set of regressions that capture autocorrelation.

```
"  
  
y2 ~ rho_y*y1  
y3 ~ rho_y*y2  
y4 ~ rho_y*y3  
y5 ~ rho_y*y4  
y6 ~ rho_y*y5  
  
"
```

283 For example, y_2 is regressed onto y_1 , and `rho_y` represents the estimated coefficient relating
284 those values. This pattern of code is similar for the remaining time points in the syntax.
285 Notice that `rho_y` is the same for every regression and therefore the autocorrelation estimate
286 is constrained to be equal across time. To remove this constraint, we could change that part
287 of the string to:

```
"  
  
y2 ~ rho_y1*y1  
y3 ~ rho_y2*y2  
y4 ~ rho_y3*y3  
y5 ~ rho_y4*y4  
  
"
```

```
y6 ~ rho_y5*y5
```

```
"
```

where `rho_y` is replaced with `rho_y1` and so on so that each estimate is freely estimated. Also note that the coefficients can contain any label, we could have used `jackie`, `tito`, `jermaine`, `marlon`, and `michael` rather than `rho_y1`, `rho_y2`, `rho_y3`, `rho_y4`, and `rho_y5`. What matters is that the label is the same for any coefficients that are constrained to be equal and different for any free parameter estimates.

The final piece of the string in the `DP_mod1` command represents the (co)-variances.

```
"
```

```
y1 ~~ y1
```

```
eta_y ~~ eta_y
```

```
y1 ~~ eta_y
```

```
"
```

Again, the Bollen and Brand technique conditions on the first observation of the outcome variable (in this case we only have one variable), so y_1 can be thought of as the lone exogenous variable. The `y1 ~~ eta_y` piece captures the fixed effects approach, where unobserved heterogeneity is allowed to correlate with the model predictor(s).

Two Variable Concurrent Dynamic Panel

We now present a model with a second, exogenous variable x that has a concurrent effect on y . This representation was shown in Figure 1, but we repeat it here for clarity. Similar to the previous model, an autoregressive term captures self-similarity of the outcome variable y , a latent variable representing unobserved heterogeneity is related to every observation of y except for the initial observation, and unobserved heterogeneity is allowed to

304 correlate with the model predictors, which, in this case, includes y_1 and all observations of x .

305

306 Insert Figure 1 Here

307

308 The code for the model represented in Figure 1 is:

```
DP_mod2 <- "  
  
eta_y =~ 1*y2 + 1*y3 + 1*y4 + 1*y5 + 1*y6  
  
y2 ~ rho_y*y1 + b1*x2  
y3 ~ rho_y*y2 + b1*x3  
y4 ~ rho_y*y3 + b1*x4  
y5 ~ rho_y*y4 + b1*x5  
y6 ~ rho_y*y5 + b1*x6  
  
x2 ~~ x3 + x4 + x5 + x6  
x3 ~~ x4 + x5 + x6  
x4 ~~ x5 + x6  
x5 ~~ x6  
  
eta_y ~~ x2 + x3 + x4 + x5 + x6  
y1 ~~ eta_y + x2 + x3 + x4 + x5 + x6  
  
"
```

where our unobserved heterogeneity term, **eta_y** has the same properties as the previous model: it is measured by each observation of the outcome y , except for the initial observation, and each factor loading is constrained to be one. The regressions of the outcome variable, y ,

```
"
y2 ~ rho_y*y1 + b1*x2
y3 ~ rho_y*y2 + b1*x3
y4 ~ rho_y*y3 + b1*x4
y5 ~ rho_y*y4 + b1*x5
y6 ~ rho_y*y5 + b1*x6
"
```

still contain an autocorrelation term, **rho_y**, but they also include an additional contemporaneous effect from x . The concurrent effect of x on y is constrained to be equal across time by using the same **b1** label for each coefficient. If we wanted to allow the coefficient to change across time we would change the label to **b1**, **b2**, **b3**, etc. at each respective time point. Keep in mind **b1** does not refer to “time point one.” Rather, it is the *first* coefficient relating scores of x to y (across time).

The largest change to the model syntax is in the (co)-variance piece, which is now more complicated because y_1 is no longer the lone exogenous variable in the model.

```
"
x2 ~~ x3 + x4 + x5 + x6
x3 ~~ x4 + x5 + x6
x4 ~~ x5 + x6
x5 ~~ x6
"
```

```

eta_y ~~ x2 + x3 + x4 + x5 + x6
y1 ~~ eta_y + x2 + x3 + x4 + x5 + x6

"

```

Each exogenous variable covaries with all other exogenous variables and unobserved heterogeneity (fixed effects). The (co)-variances are also written succinctly, but they do not have to be. The following code

```

"
x2 ~~ x3 + x4 + x5 + x6

"

```

could also be written as

```

"

x2 ~~ x3
x2 ~~ x4
x2 ~~ x5
x2 ~~ x6

"

```

where both capture the notion that x_2 needs to covary with the other exogenous variables. The same is then true for `eta_y` and y_1 . Because the first observation of the outcome variable, y_1 , is viewed as an exogenous variable in Bollen and Brand's technique, we do not model the first observation of x here.

To summarize, this model includes a contemporaneous effect from x and therefore requires additional (co)-variance syntax for the exogenous variables. Other than the additional (co)-variance syntax and the instantaneous effects of x on y at each time point, the model is similar to the single variable dynamic panel model. We model autocorrelation on the outcome variable, allow unobserved heterogeneity to correlate with the predictors (fixed effects), and condition on the first observation of the outcome variable.

We now move to a model where the effect of x on y is lagged.

Two Variable Lagged Dynamic Panel

Applications of longitudinal models in the literature sometimes capture a contemporaneous effect of x on y , which is what was presented above. By definition, this representation assumes that x instantly causes y . In many cases, however, it may also be appropriate to capture effects that occur at different lags, which is to say that $x_{(t-n)}$ causes y_t , where n is any value allowed by the data structure. Figure 3 presents a model consistent with Bollen and Brand's approach that captures a lag-one effect.

Insert Figure 3 Here

All aspects are the same as the previous model, but now the effect of x is a lag-one predictor of y rather than a concurrent effect. The code to implement such a model is here:

```
DP_mod3 <- "
eta_y =~ 1*y2 + 1*y3 + 1*y4 + 1*y5 + 1*y6
```

```

y2 ~ rho_y*y1 + b1*x1
y3 ~ rho_y*y2 + b1*x2
y4 ~ rho_y*y3 + b1*x3
y5 ~ rho_y*y4 + b1*x4
y6 ~ rho_y*y5 + b1*x5

x1 ~~ x2 + x3 + x4 + x5
x2 ~~ x3 + x4 + x5
x3 ~~ x4 + x5
x4 ~~ x5

eta_y ~~ x1 + x2 + x3 + x4 + x5
y1 ~~ x1 + x2 + x3 + x4 +x5
"
```

347 where almost all aspects are identical to the previous model. The **b1** coefficients, however,
 348 now apply to a lagged value of x to represent the influence of $x_{(t-1)}$ on y_t . That is, in this
 349 model there is a lag-one influence of x on y . The only other difference is that x_1 is included
 350 in this model and x_6 is not. Again, this is because we are modeling a lagged relationship and
 351 are uninterested in a contemporaneous effect of x_6 on y_6 . In this syntax, the lag effect of x is
 352 constrained to be equal across time – to remove this constraint our regression syntax would
 353 be as follows:

```

"
```

```

y2 ~ rho_y*y1 + b1*x1
y3 ~ rho_y*y2 + b2*x2
y4 ~ rho_y*y3 + b3*x3
y5 ~ rho_y*y4 + b3*x4
```

```
y6 ~ rho_y*y5 + b4*x5
```

```
"
```

where distinct labels (i.e., b_2 , b_3 , etc.) are used to represent different estimates across time.

In summary, this model still captures unobserved heterogeneity and its relationship with the exogenous variables, autocorrelation, and conditions on the first observation of y , but it also models a lag effect from x to y . Generalizing this model to even longer lags is straightforward, all we need are different values of x in the regression portion of the string syntax. The same can be said for modeling longer autocorrelation effects on y . Moreover, in many cases it may also be appropriate to model the self-similarity of both y and x . We turn to that model next.

Extensions of the Two Variable Lagged Dynamic Panel

Figure 4 represents a similar case to the model above, but now we include autoregression estimates of x . Doing so changes a number of other aspects about the model. First, our only exogenous variables are now x_1 and y_1 because all other time points of x and y are either predicted by themselves or the other variable (or both). Moreover, we need an additional unobserved heterogeneity term for x as it can be thought of as a “dependent” variable due to the autocorrelation paths. This new unobserved heterogeneity term for x will also correlate with the model predictor(s) in the fixed effects representation.

Insert Figure 4 Here

The syntax for this model is shown below.

```

DP_mod4 <- "

eta_y =~ 1*y2 + 1*y3 + 1*y4 + 1*y5 + 1*y6
eta_x =~ 1*x2 + 1*x3 + 1*x4 + 1*x5


y2 ~ rho_y*y1 + b1*x1
y3 ~ rho_y*y2 + b1*x2
y4 ~ rho_y*y3 + b1*x3
y5 ~ rho_y*y4 + b1*x4
y6 ~ rho_y*y5 + b1*x5


x2 ~ rho_x*x1
x3 ~ rho_x*x2
x4 ~ rho_x*x3
x5 ~ rho_x*x4


y1 ~~ y1
x1 ~~ x1
eta_y ~~ eta_y
eta_x ~~ eta_x
y1 ~~ x1 + eta_y + eta_x
x1 ~~ eta_y + eta_x
eta_y ~~ eta_x


"

```

375 autocorrelation on y and a lagged effect of x on y :

```
"  
  
y2 ~ rho_y*y1 + b1*x1  
y3 ~ rho_y*y2 + b1*x2  
y4 ~ rho_y*y3 + b1*x3  
y5 ~ rho_y*y4 + b1*x4  
y6 ~ rho_y*y5 + b1*x5  
  
"
```

376 but we also include an autoregressive estimate for x that is constrained to be equal across
377 time:

```
"  
  
x2 ~ rho_x*x1  
x3 ~ rho_x*x2  
x4 ~ rho_x*x3  
x5 ~ rho_x*x4  
  
"
```

378 where `rho_x` represents the coefficient. In this case, we only go up to x_5 and not x_6 because
379 x_6 is not used to predict y .

380 Turning to the latent variables representing unobserved heterogeneity, the syntax now
381 includes an `eta_x` and an `eta_y` because we are predicting both x (with autoregression) and
382 y (with autoregression and lag effects from x).


```

"
eta_y =~ 1*y2 + 1*y3 + 1*y4 + 1*y5 + 1*y6
eta_x =~ 1*x2 + 1*x3 + 1*x4 + 1*x5
"

```

383 As stated, x_6 is not included because it does not predict anything. Finally, the unobserved
 384 heterogeneity terms (for both x and y) are allowed to correlate with the model predictors

```

"
y1 ~~ y1
x1 ~~ x1
eta_y ~~ eta_y
eta_x ~~ eta_x
y1 ~~ x1 + eta_y + eta_x
x1 ~~ eta_y + eta_x
eta_y ~~ eta_x
"

```

385 and with each other. Here, y_1 and x_1 are the only exogenous variables, again, because later
 386 points of x are predicted by autoregression.

387 Two Variable Reciprocal Dynamics

388 Having described models with autoregression, unobserved heterogeneity, and lagged
 389 effects, we now present a model that captures reciprocal influence. This mutual influence or
 390 cyclical structure aligns with intuitive notions of dynamics and establishes a system where
 391 co-movement and progression are of more interest than “independent” versus “dependent”
 392 effects. Figure 5 shows a reciprocal dynamic model consistent with Bollen and Brand’s

technique. Here, current values of x influence future values of y , which, in turn, influence future values of x and thus establish a dynamic system (DeShon, 2012).

Insert Figure 5 Here

The major change to this model from the one presented previously is that cross-lagged effects now occur on both x and y . The code to implement the model is below.

```
DP_mod5 <- "

eta_y =~ 1*y2 + 1*y3 + 1*y4 + 1*y5 + 1*y6
eta_x =~ 1_x2 + 1*x3 + 1*x4 + 1*x5 + 1*x6

y2 ~ rho_y*y1 + b1*x1
y3 ~ rho_y*y2 + b1*x2
y4 ~ rho_y*y3 + b1*x3
y5 ~ rho_y*y4 + b1*x4
y6 ~ rho_y*y5 + b1*x5

x2 ~ rho_x*x1 + b2*y1
x3 ~ rho_x*x2 + b2*y2
x4 ~ rho_x*x3 + b2*y3
x5 ~ rho_x*x4 + b2*y4
x6 ~ rho_x*x5 + b2*y5

x1 ~~ x1
```

```

y1 ~~ y1

eta_x ~~ eta_x
eta_y ~~ eta_y

x1 ~~ y1
eta_x ~~ eta_y

x1 ~~ eta_x + eta_y
y1 ~~ eta_x + eta_y

"

```

400 Both variables are now predicted not only by previous values of themselves but also prior
 401 values of the other variable,

```

"

y2 ~ rho_y*y1 + b1*x1
y3 ~ rho_y*y2 + b1*x2
y4 ~ rho_y*y3 + b1*x3
y5 ~ rho_y*y4 + b1*x4
y6 ~ rho_y*y5 + b1*x5

x2 ~ rho_x*x1 + b2*y1
x3 ~ rho_x*x2 + b2*y2
x4 ~ rho_x*x3 + b2*y3
x5 ~ rho_x*x4 + b2*y4
x6 ~ rho_x*x5 + b2*y5

```

```
"
```

402 where the coefficients `rho_y`, `rho_x` (autoregression), and `b1`, `b2` (cross effects) are constant
 403 across time. Again, those constraints are easily allowed to vary over time by creating
 404 different labels for each time point.

405 In the unobserved heterogeneity latent variable syntax,

```
"
eta_y =~ 1*y2 + 1*y3 + 1*y4 + 1*y5 + 1*y6
eta_x =~ 1_x2 + 1*x3 + 1*x4 + 1*x5 + 1*x6
"
```

406 we now include the sixth observation of x because it is being predicted in our model. All
 407 factor loadings are constrained to be one. Finally, the (co)-variance syntax

```
"

x1 ~~ x1
y1 ~~ y1

eta_x ~~ eta_x
eta_y ~~ eta_y

x1 ~~ y1
eta_x ~~ eta_y

x1 ~~ eta_x + eta_y
```

```
y1 ~~ eta_x + eta_y
```

```
"
```

is similar to prior examples and allows correlations between unobserved heterogeneity and the exogenous variables (x_1 and y_1 here). Notice that we also allow the unobserved heterogeneity on x to correlate with the unobserved heterogeneity on y .

Summarizing The Exemplar Models

The statistical models above can be used to investigate a dynamic process. We started with a single variable and worked up to a two-variable system with cross-lags and autoregression. In all cases, we modeled unobserved heterogeneity and allowed it to correlate with the model predictors, conditioned on the first observation, and imposed equality constraints on the estimated coefficients across time. All of these aspects could be modified with the freedom of structural equations modeling, but we recommend careful thought for any modeling choice. The introductory models are amenable to many circumstances, and they offer beginners a place to start when pursuing research on dynamics.

Example With Simulated Data

Having described the model syntax and its components, we now implement two examples on data. The first uses simulated and the second empirical data. The data used in this first example contain two variables, affect and performance, on 400 simulated subjects across eight time points. We generated the data such that affect and performance contain many of the aspects described so far. Both have autocorrelation and influence one another in a one-lag structure, where affect at time t influences performance at time $t + 1$, which then influences affect at time $t + 2$, and so on. The data generating equations for x (affect) and y (performance) are, respectively

$$\text{Performance}_{it} = 0.10\text{Performance}_{i(t-1)} + 0.25\text{Affect}_{i(t-1)} + \text{Unob-Het-Y}_i + e_{it} \quad (3)$$

$$\text{Affect}_{it} = 0.15\text{Affect}_{i(t-1)} - 0.35\text{Performance}_{i(t-1)} + \text{Unob-Het-X}_i + e_{it} \quad (4)$$

where “Unob-Het-Y” is unobserved heterogeneity for performance and “Unob-Het-X” is unobserved heterogeneity for affect. The true effects relating performance at t to performance at $t - 1$ and affect at $t - 1$ are, respectively, 0.10 and 0.25. The true effects relating affect at t to affect at $t - 1$ and performance at $t - 1$ are, respectively, 0.15 and -0.35.

We apply three models to the data. The first examines a concurrent relationship between affect and performance, the second a lag-one effect of affect on performance, and the third models the entire lag-one reciprocal structure where affect and performance are no longer viewed as “independent” or “dependent” but, instead, are considered dynamic, cyclical sources of influence. We do not present these various models to make a conclusion about model accuracy. The third is the only one that aligns with the data-generating process. Rather, we move through these models to show how the code changes as we become interested in modeling different aspects. In this demonstration, we maneuver through many pieces of dynamic modeling, including stationarity checks, data restructuring, estimating, and evaluating output. The data are contained in a data frame called, “data_f” with four columns: “id,” “Time,” “Aff,” and “Per.” A snippet of the data set is shown below.

```
head(data_f)
```

```
##    id Time      Aff      Per
## 1  1     1 -1.595250  7.891421
## 2  1     2 -3.862064 10.163081
## 3  1     3 -6.494902 10.607205
```

448 ## 4 1 4 -6.493717 12.535556

449 ## 5 1 5 -7.492986 9.578426

450 ## 6 1 6 -2.772642 7.116707

451 Stationarity

452 Modeling techniques make assumptions about the data generating processes they
 453 attempt to capture, and often an important assumption in dynamic analysis is stationarity,
 454 or the stability of the statistical characteristics of the data through time. Almost all models
 455 used in the applied psychological literature – apart from growth models – assume that the
 456 data they model are realizations of a stationary process. In simple terms, this means that we
 457 expect the properties (mean and variance) of a series at time t to be the same at time $t + 1$
 458 (advanced readers note that we are referring to “weak” stationarity).

459 Acknowledging stationarity is critical for analyzing dynamic relationships because two
 460 series that are non-stationary will be related regardless of the data generating process
 461 (Braun, Kuljanin, & DeShon, 2013; Kuljanin, Braun, & DeShon, 2011). That is, two (or
 462 more) independent trajectories that share no causal relationship will appear related in the
 463 observed data when they have trend. The first step in a dynamic analysis, then, is to check
 464 whether there is evidence of trend (i.e., non-stationarity).

465 The most common method for checking stationarity is the Augmented Dickey-Fuller
 466 (ADF) test (Dickey & Fuller, 1979) in which the null hypothesis is that the time series
 467 contains a time-*dependent* error term. If the series is stationary, it will contain a
 468 time-*invariant* error term and thus the ADF significance test will be rejected. This test was
 469 designed in the econometrics literature where single time-series are more prevalent than
 470 panel data (repeated measures designs with many N), but equivalent tests are now available
 471 for data structures more consistent with those found in the psychological and organizational
 472 literatures. Complete explanations of stationarity tests for both time-series and panel data

are available in Jebb and Tay (2017) and Braun et al. (2013). Because our interest here is on panel data, we use a panel version of the ADF test.

We model two variables in our example, affect and performance, so we need to assess stationarity on both. To do so, we load two libraries

```
library(tseries)
library(plm)
```

and then restructure our data set into a format that the ADF test can read.

```
data_dickey_structure <- plm.data(data_f, index = c("id", "Time"))
```

The previous code took our data set, `data_f` and assigned it to a new data set, `data_dickey_structure` that is amenable to the ADF test. The two variables following the `index` command above, “id” and “Time,” need to be replaced with whatever column names refer to the individual participants and the time (in our case they are called “id” and “Time”).

Now we run the ADF test on both affect

```
adf.test(data_dickey_structure$Aff)
```

```
##
```

```
## Augmented Dickey-Fuller Test
```

```
##
```

```
## data: data_dickey_structure$Aff
```

```
## Dickey-Fuller = -11.855, Lag order = 14, p-value = 0.01
```

```
## alternative hypothesis: stationary
```

and performance.


```
adf.test(data_dickey_structure$Per)
```

```
491 ##
492 ## Augmented Dickey-Fuller Test
493 ##
494 ## data: data_dickey_structure$Per
495 ## Dickey-Fuller = -13.291, Lag order = 14, p-value = 0.01
496 ## alternative hypothesis: stationary
```

497 In both cases, we used our new data structure, `data_dickey_structure` and tested one of
 498 our variables by using `$` with the `adf.test` command. The null hypothesis of the ADF test
 499 is non-stationarity, and in both outputs above our p -value was lower than 0.05, providing
 500 evidence for stationarity variables in the data. We can now proceed with the next modeling
 501 aspects. We discuss what to do if the ADF test is not satisfied in the discussion section.

502 SEM Wide Formatting

503 Structural equations modeling in lavaan requires wide data. Our `data_f` set is
 504 currently in long format, but we can use the following command to make it wide:

```
df_wide <- reshape(data_f, timevar = "Time", idvar = "id", direction = "wide")
```

505 where we assign the output to a new data set, `df_wide`. The `reshape` command takes our
 506 original data set, `data_f`, acknowledges which columns represent the subject and time values
 507 (“id” and “Time” in our case), and then repositions the data to a wide structure. This syntax
 508 automatically assigns time values to the variables by adding a period and then the time
 509 label. For instance, there are now columns listed as “Per.1,” “Per.2,” “Per.3” and so on for
 510 eight time points rather than one “Per” column. We use those labels in our model syntax.

511 Modeling

512 To start modeling we need to load lavaan.

```
library(lavaan)
```

513 Our first example models a concurrent effect from affect to performance across time while
514 also staying true to the Bollen and Brand technique (Figure 2). Remember, this is simply a
515 statistical model. The true data-generating process was a cross-lag system shown in
516 equations 5 and 6. The string syntax is here:

```
DP_mod1 <- "  
  
eta_y =~ 1*Per.2 + 1*Per.3 + 1*Per.4 + 1*Per.5 + 1*Per.6 + 1*Per.7 + 1*Per.8  
  
Per.2 ~ rho_y*Per.1 + b1*Aff.2  
Per.3 ~ rho_y*Per.2 + b1*Aff.3  
Per.4 ~ rho_y*Per.3 + b1*Aff.4  
Per.5 ~ rho_y*Per.4 + b1*Aff.5  
Per.6 ~ rho_y*Per.5 + b1*Aff.6  
Per.7 ~ rho_y*Per.6 + b1*Aff.7  
Per.8 ~ rho_y*Per.7 + b1*Aff.8  
  
Aff.2 ~~ Aff.3 + Aff.4 + Aff.5 + Aff.6 + Aff.7 + Aff.8  
Aff.3 ~~ Aff.4 + Aff.5 + Aff.6 + Aff.7 + Aff.8  
Aff.4 ~~ Aff.5 + Aff.6 + Aff.7 + Aff.8  
Aff.5 ~~ Aff.6 + Aff.7 + Aff.8  
Aff.6 ~~ Aff.7 + Aff.8  
Aff.7 ~~ Aff.8
```

```

Per.1 ~~ Aff.2 + Aff.3 + Aff.4 + Aff.5 + Aff.6 + Aff.7 + Aff.8

eta_y ~~ Per.1 + Aff.2 + Aff.3 + Aff.4 + Aff.5 + Aff.6 + Aff.7 + Aff.8

"

```

517 where the structure is identical to the demonstrations shown earlier but now we are using
 518 actual variable names and the time points go up to eight. To evaluate the string on the data
 519 we use the following command:

```
DP_mod1_fit <- sem(DP_mod1, data = df_wide)
```

520 where the results get stored in `DP_mod1_fit`. To view the output we use,

```
summary(DP_mod1_fit, fit.measures = T, standardized = T)
```

521 where we request fit indices and standardized estimates, respectively, by using the
 522 `fit.measures = T` and `standardized = T` commands within our `summary` syntax. There
 523 are many other `summary` modifications available for viewing different fit indices or
 524 standardized estimates, those can be found in the lavaan documentation. For space concerns
 525 we do not print the entire output here.

526 The model fit statistics are $\chi^2(66) = 461.20$, $p < 0.05$; RMSEA = 0.12; CFI = 0.93;
 527 SRMR = 0.041, and the standardized coefficient estimates are as follows. Performance has a
 528 positive autoregressive effect ($B = 0.12$, $SE = 0.023$, $p < 0.05$). The concurrent relationship
 529 between affect and performance is also positive ($B = 0.045$, $SE = 0.020$, $p < 0.05$), such
 530 that low affect at t is associated with low performance at t . The autoregressive coefficient on
 531 performance does not mean that performance increases across time. It indicates, instead,
 532 that performance has inertia.

Recall that the true data-generating process is a cross-lagged system. The model just presented does not capture all of those aspects but it does provide a helpful contrast. Before coming to the full, cross-lagged model we present one more intermediate step where we model a lag-one relationship between affect and performance (see Figure 3). The string syntax is below

```
DP_mod2 <- "

eta_y =~ 1*Per.2 + 1*Per.3 + 1*Per.4 + 1*Per.5 + 1*Per.6 + 1*Per.7 + 1*Per.8

Per.2 ~ rho_y*Per.1 + b1*Aff.1
Per.3 ~ rho_y*Per.2 + b1*Aff.2
Per.4 ~ rho_y*Per.3 + b1*Aff.3
Per.5 ~ rho_y*Per.4 + b1*Aff.4
Per.6 ~ rho_y*Per.5 + b1*Aff.5
Per.7 ~ rho_y*Per.6 + b1*Aff.6
Per.8 ~ rho_y*Per.7 + b1*Aff.7

Aff.1 ~~ Aff.2 + Aff.3 + Aff.4 + Aff.5 + Aff.6 + Aff.7
Aff.2 ~~ Aff.3 + Aff.4 + Aff.5 + Aff.6 + Aff.7
Aff.3 ~~ Aff.4 + Aff.5 + Aff.6 + Aff.7
Aff.4 ~~ Aff.5 + Aff.6 + Aff.7
Aff.5 ~~ Aff.6 + Aff.7
Aff.6 ~~ Aff.7

Per.1 ~~ Aff.1 + Aff.2 + Aff.3 + Aff.4 + Aff.5 + Aff.6 + Aff.7

eta_y ~~ Per.1 + Aff.1 + Aff.2 + Aff.3 + Aff.4 + Aff.5 + Aff.6 + Aff.7
```

```

"

DP_mod2_fit <- sem(DP_mod2, data = df_wide)
summary(DP_mod2_fit, fit.measures = T, standardized = T)

```

538 and it is similar to that described in the “Two Variable Lagged Dynamic Panel” section.
 539 Model fit statistics are $\chi^2(66) = 213.31$, $p < 0.05$; RMSEA = 0.075; CFI = 0.97; SRMR =
 540 0.025, and the standardized parameter estimates are 0.087 ($SE = 0.021$, $p < 0.05$) for the
 541 autoregressive effect of performance and 0.29 ($SE = 0.019$, $p < 0.05$) for the lag-one effect of
 542 affect.

543 At this point, we hope the reader is in a position to comfortably view syntax that
 544 models the two-variable system. In the model below, we incorporate cross-lag and
 545 autoregressive effects on both affect and performance, and because we predict both (i.e.,
 546 both are considered “outcomes”), we include two unobserved heterogeneity terms: **eta_y**
 547 represents all of the individual differences that influence performance that are not captured
 548 by affect, and **eta_x** represents all of the individual differences that influence affect that are
 549 not captured by performance. We do so (see Figure 5) with the following syntax:

```

DP_mod3 <- "

eta_y =~ 1*Per.2 + 1*Per.3 + 1*Per.4 + 1*Per.5 + 1*Per.6 + 1*Per.7 + 1*Per.8
eta_x =~ 1*Aff.2 + 1*Aff.3 + 1*Aff.4 + 1*Aff.5 + 1*Aff.6 + 1*Aff.7 + 1*Aff.8

Per.2 ~ rho_y*Per.1 + b1*Aff.1
Per.3 ~ rho_y*Per.2 + b1*Aff.2
Per.4 ~ rho_y*Per.3 + b1*Aff.3

```

```
Per.5 ~ rho_y*Per.4 + b1*Aff.4
```

```
Per.6 ~ rho_y*Per.5 + b1*Aff.5
```

```
Per.7 ~ rho_y*Per.6 + b1*Aff.6
```

```
Per.8 ~ rho_y*Per.7 + b1*Aff.7
```

```
Aff.2 ~ rho_x*Aff.1 + b2*Per.1
```

```
Aff.3 ~ rho_x*Aff.2 + b2*Per.2
```

```
Aff.4 ~ rho_x*Aff.3 + b2*Per.3
```

```
Aff.5 ~ rho_x*Aff.4 + b2*Per.4
```

```
Aff.6 ~ rho_x*Aff.5 + b2*Per.5
```

```
Aff.7 ~ rho_x*Aff.6 + b2*Per.6
```

```
Aff.8 ~ rho_x*Aff.7 + b2*Per.7
```

```
Aff.1 ~~ Aff.1
```

```
Per.1 ~~ Per.1
```

```
Aff.1 ~~ Per.1
```

```
eta_x ~~ eta_x
```

```
eta_y ~~ eta_y
```

```
eta_x ~~ eta_y
```

```
Aff.1 ~~ eta_y
```

```
Per.1 ~~ eta_x
```

```
Aff.1 ~~ eta_x
```

```
Per.1 ~~ eta_y
```

```

"

DP_mod3_fit <- sem(DP_mod3, data = df_wide)

summary(DP_mod3_fit, fit.measures = T, standardized = T)

```

The fit indices for this model are $\chi^2(108) = 124.78$, $p = 0.13$; RMSEA = 0.020; CFI = 0.99; SRMR = 0.035 and the estimated coefficients are, respectively, 0.079 ($SE = 0.021$, $p < 0.05$) for the performance autoregressive effect, 0.11 ($SE = 0.020$, $p < 0.05$) for the affect autoregressive effect, 0.25 ($SE = 0.019$, $p < 0.05$) for the one-lag effect of affect on performance, and -0.35 ($SE = 0.019$, $p < 0.05$) for the one-lag effect of performance on affect. These estimates are consistent with the true coefficients used to generate the data (0.15, 0.10, 0.25, and -0.35, respectively).

Example With Empirical Data

The second example uses empirical data from Rowland, Wenzel, and Kubiak (2018), who collected observations on an undergraduate sample. Students completed mindfulness training, underwent an experience sampling protocol in which they responded to surveys roughly six times per day for forty days, participated in weekly laboratory visits, and completed follow-up assessments. The researchers evaluated patterns across all of these data sources. To make things simple, we focus only on two variables: anger and satisfaction. Both were measured six times a day (using a beeper protocol) during the ESM procedure. Participants were asked to indicate how they felt at the current moment by adjusting a visual slider from 0 (not at all) to 100 (very much) on two items: satisfied and angry. The data used below were aggregated to the day-level, meaning that we took the average of participant i 's six anger scores on a given day and saved that value. We then selected data over the first eight days to analyze. See the supplemental repository for the code that we used to clean the data. Note that we are limiting ourselves to keep this tutorial coherent.

Rowland et al. (2018) conducted a superb study, and they were kind enough to post their data on OSF. This document is not a re-analysis of their data, but a tutorial on programming with respect to Bollen and Brand's model. There are additional aspects to their data that more advanced readers may wish to explore.

We follow the same routine as the prior example. We assess stationarity, restructure the data, and then estimate three different models: a concurrent model, a lag-one model, and a reciprocal model. The data are contained in a data frame called, "rowland_data" with four columns: "id," "dayno," "sat," and "ang," which refer, respectively, to the participant ID, observation period, satisfaction rating, and anger rating.

```
head(rowland_data)
```

```
##   id dayno      sat      ang
## 1  1     1 39.00000 39.50000
## 2  1     2 59.00000 40.50000
## 3  1     3 57.83333 46.66667
## 4  1     4 84.33333 29.00000
## 5  1     5 65.50000 25.66667
## 6  1     6 72.40000 43.00000
```

Stationarity

First, we need to ensure that both variables, anger and satisfaction, are stationary.

```
data_dickey_structure <- plm.data(rowland_data, index = c("id", "dayno"))
adf.test(data_dickey_structure$sat)
adf.test(data_dickey_structure$ang)
```

Indeed, both satisfaction and anger are stationary.

590 SEM Wide Formatting

591 Second, we need to change the data to wide prior to modeling.

```
rowland_wide <- reshape(rowland_data, timevar = "dayno",
                        idvar = "id",
                        direction = "wide")
```

592 Modeling

593 The first example models a concurrent effect from anger to satisfaction (similar to
594 Figure 2).

```
dp_mod1 <- "

eta_y =~ 1*sat.2 + 1*sat.3 + 1*sat.4 + 1*sat.5 + 1*sat.6 + 1*sat.7 + 1*sat.8

sat.2 ~ rho_y*sat.1 + b1*ang.2
sat.3 ~ rho_y*sat.2 + b1*ang.3
sat.4 ~ rho_y*sat.3 + b1*ang.4
sat.5 ~ rho_y*sat.4 + b1*ang.5
sat.6 ~ rho_y*sat.5 + b1*ang.6
sat.7 ~ rho_y*sat.6 + b1*ang.7
sat.8 ~ rho_y*sat.7 + b1*ang.8

ang.2 ~~ ang.3 + ang.4 + ang.5 + ang.6 + ang.7 + ang.8
ang.3 ~~ ang.4 + ang.5 + ang.6 + ang.7 + ang.8
ang.4 ~~ ang.5 + ang.6 + ang.7 + ang.8
ang.5 ~~ ang.6 + ang.7 + ang.8
ang.6 ~~ ang.7 + ang.8
```

```

ang.7 ~~ ang.8

sat.1 ~~ ang.2 + ang.3 + ang.4 + ang.5 + ang.6 + ang.7 + ang.8
eta_y ~~ sat.1 + ang.2 + ang.3 + ang.4 + ang.5 + ang.6 + ang.7 + ang.8

"

dp_mod1_fit <- sem(dp_mod1, data = rowland_wide)
summary(dp_mod1_fit, fit.measures = T, standardized = T)

```

595 The model fit statistics are $\chi^2(66) = 122.804$; $p < 0.05$; RMSEA = 0.083; CFI = 0.94;
 596 SRMR = 0.080, and the standardized coefficient estimates are as follows. Satisfaction had a
 597 positive autoregressive effect ($B = 0.12$, $SE = 0.03$, $p < 0.05$). The concurrent relationship
 598 between anger and satisfaction was negative ($B = -0.47$, $SE = 0.04$, $p < 0.05$), such that low
 599 anger at t was associated with high satisfaction at t .

600 The second example demonstrates a lagged, two-variable dynamic panel. Anger will
 601 now be evaluated as a lag-one predictor of satisfaction.

```

dp_mod2 <- "

eta_y =~ 1*sat.2 + 1*sat.3 + 1*sat.4 + 1*sat.5 + 1*sat.6 + 1*sat.7 + 1*sat.8

sat.2 ~ rho_y*sat.1 + b1*ang.1
sat.3 ~ rho_y*sat.2 + b1*ang.2
sat.4 ~ rho_y*sat.3 + b1*ang.3

```

```

sat.5 ~ rho_y*sat.4 + b1*ang.4
sat.6 ~ rho_y*sat.5 + b1*ang.5
sat.7 ~ rho_y*sat.6 + b1*ang.6
sat.8 ~ rho_y*sat.7 + b1*ang.7

ang.1 ~~ ang.2 + ang.3 + ang.4 + ang.5 + ang.6 + ang.7
ang.2 ~~ ang.3 + ang.4 + ang.5 + ang.6 + ang.7
ang.3 ~~ ang.4 + ang.5 + ang.6 + ang.7
ang.4 ~~ ang.5 + ang.6 + ang.7
ang.5 ~~ ang.6 + ang.7
ang.6 ~~ ang.7

sat.1 ~~ ang.1 + ang.2 + ang.3 + ang.4 + ang.5 + ang.6 + ang.7
eta_y ~~ sat.1 + ang.1 + ang.2 + ang.3 + ang.4 + ang.5 + ang.6 + ang.7

"

dp_mod2_fit <- sem(dp_mod2, data = rowland_wide)
summary(dp_mod2_fit, fit.measures = T, standardized = T)

```

Model fit statistics are $\chi^2(66) = 224.84$, $p < 0.05$; RMSEA = 0.14; CFI = 0.83; SRMR = 0.095, and the standardized parameter estimates are 0.24 ($SE = 0.05$, $p < 0.05$) for the autoregressive effect of satisfaction and 0.21 ($SE = 0.05$, $p < 0.05$) for the lag-one effect of anger.

The last example demonstrates a reciprocal dynamic panel. Both anger and satisfaction will have autoregression, and they will both act as lag-one inputs to the other state.

```
dp_mod3 <- "  
  
eta_y =~ 1*sat.2 + 1*sat.3 + 1*sat.4 + 1*sat.5 + 1*sat.6 + 1*sat.7 + 1*sat.8  
eta_x =~ 1*ang.2 + 1*ang.3 + 1*ang.4 + 1*ang.5 + 1*ang.6 + 1*ang.7 + 1*ang.8  
  
sat.2 ~ rho_y*sat.1 + b1*ang.1  
sat.3 ~ rho_y*sat.2 + b1*ang.2  
sat.4 ~ rho_y*sat.3 + b1*ang.3  
sat.5 ~ rho_y*sat.4 + b1*ang.4  
sat.6 ~ rho_y*sat.5 + b1*ang.5  
sat.7 ~ rho_y*sat.6 + b1*ang.6  
sat.8 ~ rho_y*sat.7 + b1*ang.7  
  
ang.2 ~ rho_x*ang.1 + b2*sat.1  
ang.3 ~ rho_x*ang.2 + b2*sat.2  
ang.4 ~ rho_x*ang.3 + b2*sat.3  
ang.5 ~ rho_x*ang.4 + b2*sat.4  
ang.6 ~ rho_x*ang.5 + b2*sat.5  
ang.7 ~ rho_x*ang.6 + b2*sat.6  
ang.8 ~ rho_x*ang.7 + b2*sat.7  
  
sat.1 ~~ sat.1  
ang.1 ~~ ang.1  
sat.1 ~~ ang.1  
  
eta_x ~~ eta_x  
eta_y ~~ eta_y
```

```

eta_x ~~ eta_y

sat.1 ~~ eta_x
sat.1 ~~ eta_y
ang.1 ~~ eta_x
ang.1 ~~ eta_y

"

dp_mod3_fit <- sem(dp_mod3, data = rowland_wide)
summary(dp_mod3_fit, fit.measures = T, standardized = T)

```

The fit indices for this model are $\chi^2(108) = 295.85$, $p < 0.05$; RMSEA = 0.12; CFI = 0.82; SRMR = 0.10, and the estimated coefficients are, respectively, 0.25 ($SE = 0.05$, $p < 0.05$) for the satisfaction autoregressive effect, 0.16 ($SE = 0.05$, $p < 0.05$) for the anger autoregressive effect, 0.23 ($SE = 0.05$, $p < 0.05$) for the lag-one effect of anger on satisfaction, and 0.09 ($SE = 0.04$, $p < 0.05$) for the lag-one effect of satisfaction on anger.

Some Extensions

There are other aspects worth mentioning that researchers might encounter when running dynamic models. We do not have the space to review them fully, and they may not be required for beginner applications, but it is necessary to know they exist.

Measurement Models

Our models did not incorporate measurement error, but it could easily be included with lavaan. Imagine that affect and performance were assessed with three indicators at each period. Their latent representations at time one would be:

```

'

Per.1 =~ flp_1*P_indicator1 + flp_2*P_indicator2 + flp_3*P_indicator3
Aff.1 =~ fla_1*A_indicator1 + fla_2*A_indicator2 + fla_3*A_indicator3
'

```

621 where P_indicator1, P_indicator2 and P_indicator3 are observed performance
622 measures at time one, and A_indicator1, A_indicator2, and A_indicator3 are observed
623 affect measures at time one, each with their own respective factor loadings. We would then
624 use latent performance and affect in the other pieces of the model:

```

'

# Prediction

Per.2 ~ rho_y*Per.1 + b1*Aff.1

Aff.2 ~ rho_x*Aff.1 + b2*Per.1


# Covariance


Aff.1 ~~ eta_y
Per.1 ~~ eta_x


Aff.1 ~~ eta_x
Per.1 ~~ eta_y

'

```

Model Comparisons

Researchers cannot run χ^2 difference tests directly on the models presented because they are not nested. For example, in the first example model we used the eighth observation of affect to predict performance, but in the final model we did not:

```
'
# First example uses Aff.8
Per.8 ~ rho_y*Per.7 + b1*Aff.8

# Last example does not use Aff.8
Per.8 ~ rho_y*Per.7 + b1*Aff.7

'
```

To make the models properly nested, we would need to add the following to the entire string syntax in our last example:

```
'
Aff.8 ~~ Aff.8

'
```

where the new term estimates a variance for affect at time 8. Doing so makes our covariance matrix equivalent across models and allows us to compare the two models using the familiar χ^2 difference test. In R, a χ^2 test can be conducted with:

```
anova(DP_mod1_fit, DP_mod3_fit)
```

For a straightforward, larger discussion of model comparisons see the discussion of

model search strategies in Bollen and Brand (2010).

Time-Varying Coefficients

As discussed throughout, researchers are free to allow covariate coefficients to vary across periods. Substantively, doing so implies that the influence of a predictor differs over various waves. In code, one could do so by using $b1*x1 + b2*x2 + b3*x3 + b4*x4$ instead of $b1*x1 + b1*x2 + b1*x3 + b1*x4$. An anonymous reviewer pointed out that, although researchers sometimes assess time-varying coefficients, the panel model literature often neglects the possibility of testing whether the latent time-invariant variables (unobserved heterogeneity) have different effects at different periods. That is, estimating the factor loadings of unobserved heterogeneity rather than setting all of them to 1. Removing this constraint permits a researcher to assess whether unobserved heterogeneity has the same effect across all waves of the data, which could be important when participants encounter life transitions (e.g., actively employed to retired). To allow unobserved heterogeneity to differ over periods, include the following.

```
'  
  
eta_y =~ 1*y1 + fl1*y2 + fl2*y3 + fl3*y4 + fl4*y5 + fl5*y6  
  
'
```

Again, the substantive meaning of such a model would be that the unmeasured, stable characteristics differ in their influence on the outcome across waves.

Cointegration

In the current paper, we assessed stationarity before moving to statistical modeling because Granger and Newbold (1974) demonstrated that regressions between independent non-stationary series will likely produce significant coefficient estimates and large R^2 values. Researchers are at risk, therefore, of making spurious inferences when they use regression-based models with data that contain trends, random walks, or non-stationary

variance across time – even if the series are independent/non causal. It is also possible, however, that two non-stationary processes are related. We then need a way to assess that relationship without biasing our results in the manner presented by Granger and Newbold (1974). In a later paper, Granger (1986) showed that, under very specific circumstances, we can garner evidence for a relationship between two non-stationary series. Formally, two series, x and y are said to be cointegrated if (1) both are integrated of the same order, $I(d)$, and (2) they share a linear combination that is $I(0)$. In other words, there is a linear combination of x and y that is stationary (requirement #2) and they both require the same number of steps to make them stationary (requirement #1).

The basis for cointegration is simple: do we have evidence for a relationship between series in a situation where we know spurious relationships are likely (i.e., non-stationarity)? Implementing cointegration techniques, however, is much more difficult. The analyst needs to be sensitive to the type of non-stationarity present in the series: trends or random walks. A random walk is defined as

$$y_t = 1 * y_{(t-1)} + e_t \quad (5)$$

where the variable is exactly what it was at the prior time point but also accumulates error across time. Such a series may increase, decrease, or change directions at any instant. A series with trend is simply one that increases or decreases over time. The steps required to demonstrate cointegration change if the analyst is dealing with trends or random walks, and the analyst also needs to adjust their approach depending on whether they have single-unit or panel data (Kao, 1999). We recommend, for now, ensuring that data are stationary – cointegration is a much more advanced topic.

Granger Causality

Statistical routines are often paired with follow up tests, and because prediction is a major emphasis in economics a common post-hoc evaluation of multivariate time series data is Granger-Causality (Granger, 1980). This test should not be thought of as a “causal” test but, instead, a statistical evaluation of the prediction ability of each piece of a model. We strongly recommend that – rather than relying on statistical routines alone – researchers state their assumptions, the justifications for those assumptions, and acquire excellent data to provide evidence for or against an idea (Freedman, 1991); we do not see Granger-Causality as a necessary component to dynamic modeling. It may be of interest to some, however, so we will discuss and demonstrate it here.

A variable, x , is said to Granger-Cause another variable, y , if the strongest prediction of y stems from the histories of both x and y rather than the history of y alone. Said another way, x Granger-Causes y if it adds predictive value above the history of y itself. We evaluate Granger-Causality by adding constraints to our original model to create a “granger-cause evaluation” model and then compare the models through fit tests. Formally, the “granger-evaluation model” constrains all of the paths from y on x to be zero (leaving only the autoregressive paths, or the history paths, of y) and is then compared to the original model through a chi-square difference test. The “granger-evaluation model” has fewer parameters to estimate because of the added constraints and is therefore a less complicated (more parsimonious) model. Less complicated models fit worse than their complicated (unconstrained) counterparts, so the “granger-evaluation model” will fit the observed data worse than our original model. If the chi-square difference test shows that the “granger-evaluation model” fits significantly worse, then we have evidence of Granger-Causality. In other words, if constraining the prediction of x on y to be zero leads to significantly worse fit then we have evidence that x “granger causes” y . Again, take this phrasing lightly.

Notice that we are only half-way there, the broader emphasis of this paper is modeling reciprocal, dynamic influence where the analyst might be interested in demonstrating Granger-Causality of both x on y and y on x . To create a “granger-evaluation model” for affect (x) we would update the syntax from our final example model as follows:

```
granger_cause_x_mod <- "

eta_y =~ 1*Per.2 + 1*Per.3 + 1*Per.4 + 1*Per.5 + 1*Per.6 + 1*Per.7 + 1*Per.8
eta_x =~ 1*Aff.2 + 1*Aff.3 + 1*Aff.4 + 1*Aff.5 + 1*Aff.6 + 1*Aff.7 + 1*Aff.8

Per.2 ~ rho_y*Per.1 + 0*Aff.1 # HERE IS THE CHANGE
Per.3 ~ rho_y*Per.2 + 0*Aff.2 # HERE IS THE CHANGE
Per.4 ~ rho_y*Per.3 + 0*Aff.3 # HERE IS THE CHANGE
Per.5 ~ rho_y*Per.4 + 0*Aff.4 # HERE IS THE CHANGE
Per.6 ~ rho_y*Per.5 + 0*Aff.5 # HERE IS THE CHANGE
Per.7 ~ rho_y*Per.6 + 0*Aff.6 # HERE IS THE CHANGE
Per.8 ~ rho_y*Per.7 + 0*Aff.7 # HERE IS THE CHANGE

Aff.2 ~ rho_x*Aff.1 + b2*Per.1
Aff.3 ~ rho_x*Aff.2 + b2*Per.2
Aff.4 ~ rho_x*Aff.3 + b2*Per.3
Aff.5 ~ rho_x*Aff.4 + b2*Per.4
Aff.6 ~ rho_x*Aff.5 + b2*Per.5
Aff.7 ~ rho_x*Aff.6 + b2*Per.6
Aff.8 ~ rho_x*Aff.7 + b2*Per.7

Aff.1 ~~ Aff.1
Per.1 ~~ Per.1
```

```

Aff.1 ~~ Per.1

eta_x ~~ eta_x
eta_y ~~ eta_y
eta_x ~~ eta_y

Aff.1 ~~ eta_y
Per.1 ~~ eta_x

Aff.1 ~~ eta_x
Per.1 ~~ eta_y

"

granger_cause_x_mod_fit <- sem(granger_cause_x_mod, data = df_wide)

```

708 where we constrained the b1 path of prior affect on performance to be zero so that we can
 709 ask the question, “does affect predict performance above and beyond performance?” To
 710 compare the two models we then use

```
anova(granger_cause_x_mod_fit, DP_mod3_fit)
```

711 to run the familiar chi-square difference test. To examine if performance Granger-Causes
 712 affect we would create another “granger-evaluation model” where, instead of constraining the
 713 b1 path to zero, we would constrain the b2 path to zero and subsequently compare the
 714 models with the chi-square difference test.

715 We feel it is appropriate to end this section by saying, again, that this test is simply
 716 another statistical tool to show that the data do not contradict our explanations. It should

be associated with prediction rather than causation, and it is not a required step in dynamic analysis.

Discussion

The goal of our paper was to introduce fundamentals of dynamic modeling. We described simple models that researchers can use to represent and explore dynamic systems in psychological science. The models were based on an approach introduced by Bollen and Brand (2010). Their technique explicitly incorporates unobserved heterogeneity, allows for both random or fixed effects representations, conditions on the first observation of the outcome variable(s), and is amenable to different lag and cross-lag effects. It also sufficiently handles dynamic panel bias. Our work is a launching point for dynamic modelers, but readers should prepare themselves for the next level of complexity that lies ahead. To navigate toward an advanced understanding of the topics introduced here, it is necessary to start mastering structural equations. Such equations are the foundation of sophisticated methods articles, and readers will encounter them as they continue to read about dynamic modeling. Below, we present the structural equations for the models presented in this paper. Doing so, hopefully, offers a bridge between our novice-oriented paper and more advanced articles on the topic.

Full Specification & Assumptions

The dynamic models presented in this article were constrained versions of a more general lagged panel model. The full model, which we will work through slowly, is as follows.

$$y_{it} = \rho_t y_{i(t-1)} + \mathbf{B}_{yxt} \mathbf{x}_{it} + \mathbf{B}_{yzt} \mathbf{z}_i + \lambda_t \eta_i + \varepsilon_{it} \quad (6)$$

This equation asserts that the response person i gives to variable y at time t is a function of several things: his or her response to the variable at the last time point ($y_{i(t-1)}$), the concurrent values of any observed time-varying variables for the respondent, such as affect or

depletion (the $\mathbf{x}_{it}$ variables), the values of any observed time-constant variables for the respondent (the $\mathbf{z}_i$ variables), such as nationality or number of siblings, some unobserved time-constant person-specific effects (η_i), such as personality, and idiosyncratic noise that varies from period to period for each respondent (ε_{it}). The other terms in the model (ρ_t , $\mathbf{B}_{yxt}$, $\mathbf{B}_{yzt}$, and λ_t) house the coefficients that relate one or more predictors to the outcome.

A slightly more advanced description would be as follows. y_{it} is the value of the dependent variable for the i th case in the sample at the t th time period, ρ_t is the autoregressive coefficient of the effect of $y_{i(t-1)}$ on y_{it} , $\mathbf{x}_{it}$ is the vector of time-varying covariates for the i th case at the t th time period, $\mathbf{B}_{yxt}$ is the row vector of coefficients that give the impact of $\mathbf{x}_{it}$ on y_{it} at time t , $\mathbf{z}_i$ is the vector of observed time-invariant covariates for the i th case with $\mathbf{B}_{yzt}$ a row vector of coefficients at time t that give the impact of $\mathbf{z}_i$ on y_{it} . The η_i is a scalar of all other latent time-invariant variables that influence y_{it} , and λ_t is the coefficient of the time-invariant variable at time t (at least one of these λ_t is set to one to provide the units in which the latent variable is measured). The ε_{it} is the random disturbance for the i th case at the t th time period with $\mathbb{E}(\varepsilon_{it}) = 0$ and $\mathbb{E}(\varepsilon_{it}^2) = \sigma_{\varepsilon_t}^2$. It is also assumed that ε_{it} is uncorrelated with $\mathbf{x}_{it}$, $\mathbf{z}_i$, and η_i and that $COV(\varepsilon_{it}, \varepsilon_{is}) = 0$ for $t \neq s$. We also assume that $\mathbf{z}_i$ and η_i are uncorrelated (more on this below).

This full model can be constrained to reach the models explored in our code. Specifically, both fixed and random effects models can be expressed by constraining the general lagged panel model. Before getting there, though, note that a typical constraint (which can be applied to both fixed and random effects models) is to restrict the autoregressive coefficient ρ_t to be equal across time (i.e., $\rho_t = \rho$). Thus, the general lagged panel model would become

$$y_{it} = \rho y_{i(t-1)} + \mathbf{B}_{yxt} \mathbf{x}_{it} + \mathbf{B}_{yzt} \mathbf{z}_i + \lambda_t \eta_i + \varepsilon_{it} \quad (7)$$

where the only change is that ρ is now constant across time. Conceptually, doing so expresses the idea that the influence of the outcome on itself over successive periods is constant. From here, we move to the fixed effects model by imposing additional constraints.

Fixed Effects

(8)

$$y_{it} = \rho y_{i(t-1)} + \mathbf{B}_{yx} \mathbf{x}_{it} + \eta_i + \varepsilon_{it}$$

The most glaring change is the removal of $\mathbf{B}_{yz} \mathbf{z}_i$, or the observed, time-invariant variables and their coefficients. In a fixed effects model, unobserved heterogeneity η_i is allowed to correlate with the observed, time-varying covariates $\mathbf{x}_{it}$. Conceptually, this means that all unmeasured time-constant factors about the respondent are allowed to correlate with his or her observed, time-varying factors – a reasonable assumption. If we were to include observed time-constant variables $\mathbf{z}_i$ alongside unobserved heterogeneity η_i , then they would be perfectly collinear and we would not be able to separate their effects (Bollen & Brand, 2010). The fixed effects approach, therefore, chooses to fold $\mathbf{z}_i$ into unobserved heterogeneity. Doing so is a disadvantage in the sense that the impact of time-invariant variables cannot be estimated, but it is an advantage in the sense that it allows for the realistic assumption of dependence among unmeasured, stable factors and measured, time-varying effects. There are a few other, less obvious constraints embedded in the equation above. We also constrained the coefficients on the time-varying variables to be equal at all periods (i.e., $\mathbf{B}_{yxt} = \mathbf{B}_{yx}$). This restriction was implemented in our code with `b1*x1 + b1*x2 + b1*x3` and can be removed by giving the coefficients different names, e.g., `b1*x1 + b2*x2 + b3*x3`. Moreover, we constrained the coefficients (factor loadings) of unobserved heterogeneity to be equivalent across time (i.e., $\lambda_t = 1$). This restriction was implemented in our code with `eta_y =~ 1*y1 + 1*y2 + 1*y3` and can be removed by giving the coefficients different values. Finally, the error variances can also be set to be equal across time ($\sigma_{et}^2 = \sigma_e^2$).

The random effects model is also a restricted form of the general lagged panel model.

Random Effects

(9)

$$y_{it} = \rho y_{i(t-1)} + \mathbf{B}_{yx} \mathbf{x}_{it} + \mathbf{B}_{yz} \mathbf{z}_i + \eta_i + \varepsilon_{it}$$

Once again, the coefficients relating the time-varying variables to the outcome are assumed to be constant ($\mathbf{B}_{yxt} = \mathbf{B}_{yx}$; $\mathbf{B}_{yzt} = \mathbf{B}_{yz}$), and the coefficients on unmeasured time-stable factors are set to 1 for all t ($\lambda_t = 1$). Conceptually, this means that the random effects model assumes that all model predictors have effects on y_{it} that are the same over all time periods – an assumption that is shared with the fixed effects model. Moreover, both models assume that the error variances over periods are equal ($\sigma_{et}^2 = \sigma_e^2$). Where the random effects model differs from the fixed effects model is that it does not permit unobserved heterogeneity to correlate with the predictors. That is, η_i is assumed to be uncorrelated with $y_{i(t-1)}$, $\mathbf{x}_{it}$, and $\mathbf{z}_i$.

Consider how the structural equations align with the box-and-arrow diagrams presented earlier. The single variable dynamic panel (Figure 2) was a fixed effects model with no predictors other than unobserved heterogeneity η_i and the autoregressive effect of $y_{i(t-1)}$. The two variable concurrent dynamic panel (Figure 1) was a fixed effects model with unobserved heterogeneity η_i , autoregression $y_{i(t-1)}$, and a concurrent time-varying effect x_{it} . The two variable lagged dynamic panel (Figure 3) was a fixed effects model with unobserved heterogeneity η_i , autoregression $y_{i(t-1)}$, and a lagged time-varying effect $x_{i(t-1)}$. The cross lag dynamic panel included two fixed effects models, one for x and the other for y . x_{it} was predicted by unobserved heterogeneity with respect to x , η_{xi} and autoregression $x_{i(t-1)}$. y_{it} was predicted by unobserved heterogeneity with respect to y , η_{yi} , autoregression $y_{i(t-1)}$, and a lagged time-varying covariate $x_{i(t-1)}$. Finally, the cross lag dynamic system with reciprocal influence also included two fixed effects models. x_{it} was predicted by unobserved heterogeneity with respect to x , η_{xi} , autoregression $x_{i(t-1)}$, and a lagged time-varying covariate $y_{i(t-1)}$. y_{it} was predicted by unobserved heterogeneity with respect to y , η_{yi} ,

autoregression $y_{i(t-1)}$, and a lagged time-varying covariate $x_{i(t-1)}$. Across all, unobserved heterogeneity was allowed to correlate with the model predictors.

Conclusion

The goal of this article was to offer a steppingstone into dynamic modeling. Researchers unfamiliar with longitudinal modeling can use this paper as a resource to start learning about the major concepts in dynamics. Moreover, they will be able to see how each concept is (a) represented in box-and-arrow diagrams and (b) evaluated in code. Bollen and Brand's (2010) approach was presented because it is amenable to the aspects researchers may wish to capture when they investigate dynamics. It also uses a structural equations framework which many psychologists are familiar with. We began by describing dynamic statistical models as those which approximate the transition rules and updating mechanisms of a system. They capture how a system updates from t to $t + 1$ through the iterated application of inertia and inputs. We offered a beginner-friendly explanation of dynamic panel bias, which is the set of statistical challenges arising in the application of dynamic statistical models. We explained Bollen and Brand's technique and implemented it with many syntax examples, providing researchers a foundation from which to start modeling on their own. We closed with the structural form of a general, lagged dynamic panel and a discussion of model assumptions, an aspect which we saved for the end as it connects our novice article to some of the more advanced statistical literature. This work should appeal to those who are interested in dynamics and are familiar with introductory concepts of statistics but have not yet run their own analyses. We hope that it encourages newcomers into dynamics, refines the thinking of more advanced modelers, inspires future empirical research and theory and, along the way, advances knowledge regarding psychological phenomena.

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Figure 1. Two variable concurrent dynamic panel

Figure 2. Single variable dynamic panel

Figure 3. Two variable lagged dynamic panel

Figure 4. Cross lag dynamic panel

Figure 5. Cross lag dynamic system with reciprocal influence