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From single decisions to long-term choice patterns: Extending the dynamics of decision-making

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1. Abstract

Decision-making is usually studied on a trial by trial basis and each decision is assumed to represent an isolated choice process. These assumptions are also reflected in sequential sampling models which conceive of the decision-process as an accumulation of information about the attractiveness of the options at hand. Real-life decisions however are usually embedded in a rich context of previous choices at different time scales. A fundamental yet neglected question is therefore how the dynamics of choice processes unfold on a long-term time scale across several decisions.

Here, we present a neural-inspired attractor model that integrates the short-term mechanism of accumulation models with the long-term dynamics of coupled neural systems. The model represents a class of models that incorporate inherent long-term dynamics. We use the model to predict long-term patterns, namely oscillatory switching, perseveration and dependence of perseveration on the delay between decisions. Furthermore, we predict RT effects for specific trials. We validate the predictions in two new studies and a reanalysis of existing data from a novel decision game in which participants have to perform delay discounting decisions.

Applying the validated reasoning to a well-established choice questionnaire, we illustrate and discuss that taking long-term choice patterns into account may be necessary to accurately analyse and model decision processes.

2. Author Summary

On a normal day, we make a myriad of decisions, some of which turn out to be in our favour, some turn out to be wrong. When psychologists study decision making, they usually focus on each decision isolated from all the other decision. We show that decision should not only be studied in isolation, because the context of a decision, i.e. a previous decision, plays an

important role. We present a computational model of decision making that takes this context into account and predicts patterns of decision making that are typical when the context of a decision plays a role. We show in data from three studies, that these predicted patterns can be found in human decisions when the study also considers the context. Finally, we illustrate and discuss, how the study of decision could draw false conclusions when it does not consider the context of a decision.

Introduction

Decision-making is a central part of our human existence, often asking for a trade-off between different interests: Choosing between hearty or healthy meal options, between current joys and pension plans, or – as folk-wisdom names it – between one bird in the hand or two in the bush. Studies of decision-making have provided profound insight on which information decision-makers base such choices [1–5]. In such studies, participants usually have to choose between two options which offer a trade-off. For example, in delay discounting studies, participants have to choose between one option which is small, but delivered soon, and another option that is large, but delivered later.

The behavioural and neural data from decision-making studies support models that conceive of the decision process as a noisy process of information accumulation leading to a final choice [6–10] (so-called sequential sampling models or random walk model). These models can explain many effects found in empirical data in concordance with their neural underpinnings [7] and their process dynamics [11]. However, these models neglect the inherent dynamics over longer time-scales than single decisions and provide no means to reason about patterns on long-term time-scales that extend across multiple decisions [12–17]. Such patterns are typical for non-

linear coupled models, such as leaky integrator models [18] or parallel constraint satisfaction models [19,20].

Here, we present a simple neural-inspired attractor model of decision-making [12,18,21] that combines the long-term perspective of non-linear coupled models with the within-decision dynamics of sequential sampling models. The model has already been used to successfully predict hysteresis, i.e. path-dependence in sequences of decisions [12]. Here we will first show three further patterns across multiple decisions that the model predicts, namely oscillatory switching (i.e., the probability distribution for switching back and forth between choices), perseveration (sticking to a choice more often than switching), and dependency on the temporal distance between decisions (higher perseveration when decisions closely follow each other). Furthermore, we will present two predictions for response times (RT) in specific trials in sequences of decisions. We then present empirical data from three experiments that validate the predictions of the model.

3. Model

In the main part of this article, we will present the model on a conceptual level and present the results of computational simulation of the model, the details of which can be found in the appendix.

We predicted long-term choice patterns across sequences of choices and RT effects for specific trials in these sequences using a neural-inspired attractor model (see Figure 1a). This model takes the attractiveness of each option as input to two simple neural network units representing these options and competing with each other. A unit representing the option with a higher attractiveness hence receives a stronger input and has a higher chance of reaching an activation threshold first, which elicits the choice (similar to sequential sampling models).

Notably, noise in the system might still push the less attractive unit across the threshold, leading to a choice of this option. After eliciting a choice, the input is switched off and the units' activation falls back to resting level following the intrinsic dynamics of the system.

Figure 1. The neural attractor model. a) Sketch of the neural-inspired attractor model. Each units' activation represents the attractiveness of one option and inhibits the activation of the alternative option. When an option's activation reaches the choice-threshold, the choice of this option is elicited. b) Activation flow across six consecutive choices. After choosing the (here more attractive) small-near option in the first trial, the system perseverates with this choice and only occasional switches to the alternative can be observed, which are caused by noise in the system.

If new options are presented while the recently winning unit still shows residual activation, then this option gains a relative advantage in the next choice process (see Figure 1b). We used numerical simulation to derive predictions of long-term choice patterns from the model (see the appendix for more details). The first pattern predicted by the model is noise-induced oscillatory switching as measured by the distribution of switches between options in a sequence of repeating choices (see Figure 2a). First, if the more attractive option is chosen initially, it is either chosen consistently or – if a switch occurs due to noise – chosen again quickly. This leads to 0 switches or an even number of switches - back and forth - between the attractive and the unattractive option. Second, if the unattractive option is chosen initially – due to noise – subjects quickly return to the attractive option. This leads to one switch or an odd number of switches. For options that are equally attractive, there is random distribution of switches across sequences of choices that is not biased to an even or odd number of switches. Though this prediction might

seem trivial and could also be made by linear sequential sampling models, it could be seen as a basic sanity check of the presented attractor model. The central patterns of interest are the next two ones. The second pattern predicted by the model is perseveration as measured by Markov Analysis (see Figure 2b): A choice of an option increases the probability of repeating this choice independent from the relative attractiveness of the options. The third pattern is a temporal dependency of perseveration: Separating two decision by a short delay compared to a longer delay should lead to weaker perseveration compared to stronger perseveration (see Figure 2b).

Figure 2. Model predictions for sequences of repeating decision situations. a) The model predicts patterns of oscillatory switching that depend on the options' attractiveness and the initial choice in sequences of choices. If the attractive option is chosen initially, the model predicts that zero (staying with the attractive option) or an even number of switches across the sequence are more likely than an odd number of switches. If the unattractive option is chosen initially, the model predicts one (switching back to the attractive option) or an odd number of switches. b) The model predicts choice perseveration as identified by markov analysis: Choosing an option in one trial leads to a higher probability of staying with this choice in the next trial. This perseveration effect is stronger for short inter trial intervals (ITI) than for long ITI. Error bars mark 95% confidence intervals.

We further predicted RT effects for specific trials in sequences of choices. First, when choosing between two options in trials with clearly given preferences (attractive SN or attractive LN), choosing the preferred option should be easier than choosing the unpreferred option (Figure 3 left). Second, in trials with clearly given preferences and choices of this preferred option, it should be easier to choose the preferred option repeatedly (staying with the preferred option) than to come from the unpreferred option (switching to the preferred option, Figure 3 right).

Figure 3. Model predictions for RT effects (in cycles) for specific trials in sequences of choices Left: When choosing between two options in trials with clearly given preferences (an attractive SN or attractive LN option), choosing the preferred option should be easier than choosing the unpreferred option. Right: In trials with clearly given preferences and choices of this preferred option, it should be easier to choose the preferred option repeatedly (staying with the preferred option) than to come from the unpreferred option (switching to the preferred option).

4. Experiment 1

To study the effects predicted by the model empirically, we used a gamified decision task in which participants had to navigate an avatar in a virtual world to collect coins, choosing between small coins near to their avatar and large coins farther away within limited time [22]. This task was used previously to study sequences of choices [12] and offers the benefit that the spatial nature of the task allows to present the options at changing position, masking the sequential nature of the choices. We expected to find the predicted patterns of oscillatory switching depending on the initial choice (Hypothesis I) and perseveration overall (Hypothesis II)ⁱ and we expected the predicted RT effects, namely shorter RT for choosing the preferred option (Hypothesis III) and shorter RT for staying with the preferred option (Hypothesis IV).

1.1. Methods

1.1.1. Participants

15 participants of the Technische Universität Dresden took part in the experiment. All participants had normal or corrected-to-normal vision. They gave informed consent to the study and received either payment or class credit.

1.1.2. Setup and Procedure

Participants' moved an avatar on a playing field divided into 20 x 20 fields (see Figure 4 and Figure 5.a). To move the avatar, participants clicked with the mouse in one of four horizontally or vertically adjacent movement fields, as signalled by a white border surrounding the fields. Their task was to collect as much credits as possible within the allotted time limit. On each trial, two options were presented as coins, with each option's credit *value* printed as a red number on the coin. The horizontal and vertical distance of each option (field by field) to the avatar represented the *distance* of the option. One option was near but small (SN), the other option was far but large (LF). Options' *values* ranged from one to ten credits and *distance* ranged from two to fifteen fields. To collect an option, participants navigated the avatar to the chosen coin. When the avatar reached the coin, both options disappeared and the value of the selected option was shown again to the participant as a feedback that the trial was complete and the option's value was collected. After each trial, participants had to wait for the next trial in an inter trial interval (ITI) of 1.3 seconds. Within this interval, the mouse cursor was locked in the centre of the field containing the avatar. After the ITI, the next trial started. At all times, participants could see the remaining time within one block and the collected credits in Euro (1 credit = 1/10 € cent) on a panel at the right side the playing field.

Figure 4. Sketch of one trial in the delay discounting game. During the inter-trial-interval of 1.3 seconds, the mouse cursor is locked in the avatar's current position. After the ITI, the two options are presented as coins with values written on top in red font. Participants have to steer the avatar field-by-field to the option they want to collect. When they reach the option's field, the option's value is collected and the collected value is presented to indicate that the choice is completed. At all time, participants could see the remaining time within one block and the collected credits in Euro on a panel at the right side of the playing field.

The experiment consisted of three blocks, with one block lasting eight minutes. This amount of time allowed participants to work through a complete design matrix of trials (see below) at least one time. Between blocks, participants were informed about the credits collected and were instructed to rest briefly before the self-paced start of the next block.

Before the start of the experimental blocks, participants worked through a test block of one minute to get used to the virtual environment and handling of the mouse.

1.1.3. Design

In the game, participants had to choose between consecutively presented similar choices: distances of the SN option ($D_N = [2, 5]$) and the LF option ($D_F = D_N + [1, 4, 7]$) were orthogonally varied in random order across trials; credit values of the SN option ($V_S = [10, 20, 30, 40, 50]$) and the LF option ($V_L = [100, 90, 80, 70, 60]$) were combined pairwise (10 and 100, 20 and 90, and so on) and kept constant within each sequence. Sequence length varied randomly between 4, 5, and 6 choices. This yielded 30 sequences of an average length of 5 trials; 150 trials in total for a complete block. We created more blocks than participants could perform within the limited time frame (3 x 8 minutes) to be sure that participants had enough trials to work through.

1.2. Results

Hypothesis I stated that in sequences of trials that start with a choice of the attractive option, we expect no switches or an even number of switches. In contrast, in sequences of trials that start with a choice of the unattractive option, we expect an odd number of switches. For trials with no clearly attractive option, we expected a random distribution of switches unbiased to an odd or even number of switches.

To identify sequences with attractive SN or LF options or with no clearly attractive option, we calculated for each participant the individual indifference point [e.g. 22] and then sorted sequences according to distance of the ratio of the SN and LF option's value to the indifference point. For example, if a participant had an indifference point of 0.5, this means that an SN option with a value $\frac{1}{2}$ of the LF option (e.g. SN: 5 credits, LF: 10 credits) is equally attractive to this participant because of discounting. For this participant, an SN option's value with only 0.3 of the LF option's value (e.g. SN: 3 credits, LF: 10 credits), would lead to a distance to the indifference point of -0.2. For each participant, we calculated the median of distances to the indifference point and categorized all sequences according to their distance to the indifference point as attractive SN, neutral, and attractive LF. For these categories, we counted the number of sequences that started either with the choice of the SN option or the LF option and that yielded either an even or an odd number of switches between options. For statistical analysis, we subtracted the number of odd switches from the number of even switches. The resulting difference score was analysed in a repeated measures analysis of variance (RMANOVA). This revealed a significant main effect of the initially chosen option (SN vs. LF), $F(1,14) = 7.827, p = .014, \eta^2_p = .359$, a significant main effect of attractiveness (SN vs. neutral vs. LF), $F(2,28) = 11.420, p < .001, \eta^2_p = .449$, and – as predicted – a significant interaction,

$F(2,28) = 270.036, p < .001, \eta^2_p = .951$. Hence, we found the predicted pattern of a largely higher number of even switches for sequences in which the attractive option was chosen initially.

Hypothesis II stated that we should find a perseveration bias, which is that the probability to choose the option again that has been chosen previously, is higher than the probability to switch between options. We performed a markov analysis across all trials, in which we counted the number of SN and LF choices with respect to the previously chosen option. This markov analysis takes into account the baseline number of choices of the SN/LF option so that perseveration cannot simply occur because of a general biased to the SN/LF option. For statistical analysis, we analysed whether the difference of switch stay probabilities was positively different from zero, using a dependent samples t -test, which revealed a significant difference from zero, $t(14) = 16.938, p < .001, d = 4.373$. Hence, a clear perseverative bias was present as predicted by the model. A graphical summary of the results is given in Figure 5a.

Figure 5. Choice results of studies 1 and 2. a) Results of study 1 show oscillatory switching in relative switch frequencies (left sub-panel) and choice perseveration (right sub-panel) as predicted by the model. B) Results of study 2 confirm the results of study 1. Error bars mark 95% confidence intervals.

Hypothesis III stated that choosing the preferred option should be faster than choosing the unpreferred option. We performed a t-test on the dependent variable RT with the independent variable *choice of preferred-unpreferred option* in trials with one attractive option, which yielded a significant difference, $t(14) = 3.62, p = .003, 95\% \text{ CI } [.033 \text{ sec}, .131 \text{ sec}]$ (Figure 6 top-left).

Hypothesis IV stated that choosing in trials, where participants stay with the preferred option, they are faster than in trials where participants switch to the preferred option (when they chose the unpreferred option in the previous trial). We performed an t-test on dependent variable RT with the independent variable *stay with/switch to preferred option* in trials with one clearly attractive option, which yielded a significant difference, $t(14) = 4.76, p < .001, 95\% \text{ CI } [.065 \text{ sec}, .171 \text{ sec}]$ (Figure 6 top-right).

Figure 6. Results of studies 1 and 2 for RT (in seconds) for trials in which participants choose the preferred vs. the unpreferred option (left) and for trials in which participants stayed with the preferred option vs. switch from the unpreferred to the preferred option (right).

1.3. Discussion

Experiment 1 found the dynamic long-term patterns in sequences of decisions as predicted by the attractor model, namely oscillatory switching and perseveration. It also found the predicted RT effects for choosing the preferred option and for staying with the preferred option. However, one could criticize two features of Experiment 1. First, the sequences were relatively short, so that speaking of long-term dynamics could be seen as a far-fetched argument. Second, the sequences consisted of identical choice option in distance and values. While distance might have been occluded by the repositioning of options from trial to trial, the constancy of values could obviously be detected by participants. We hence performed experiment 2 to mitigate these two arguments.

5. Experiment 2

Experiment 2 aimed to replicate the findings of experiment 1, but used longer sequences of choices and aimed to mask the similarity of these choices by superficially varying option's values. Despite these changes, we expected to find the predicted patterns of oscillatory switching (Hypothesis I) and perseveration (Hypothesis II) and the RT effects for choosing the preferred option (Hypothesis III) and for staying with the preferred option (Hypothesis IV).

1.4. Methods

1.1.4. Participants

20 participants of the Technische Universität Dresden took part in the experiment. All participants had normal or corrected-to-normal vision. They gave informed consent to the study and received either payment or class credit.

1.1.5. Procedure and Setup

The procedure and setup was similar to Experiment 1, with the following exceptions.

Experiment 2 was split in two runs of the game. In the first run, we measured the indifference points of each participant across choice option varied along the different distances and values (see Design). In the second run, we used these estimated indifference points to adjust our manipulation of the options' relative attractiveness in the sequences of choices. We presented sequences of 30 trials for each relative attractiveness configuration. Since the sequences were much longer than in Experiment 2, we made each single trial within a sequence superficially different by randomly varying the value of the LF option. Beneath this superficial difference in absolute value, however, we kept the subjective values of the SN and the LF option constant: We chose the SN value relative to the random LF value according to the estimated indifference point plus or minus an additional amount to influence the SN options' attractiveness. Relative to the indifference point, a sequence could then consist of SN options that were more attractive, equally attractive, or less attractive.

1.1.6. Design

In run 1, we measured the indifference points of each participant. We presented choices with 3 intervals ($D_N = 2, 3$, or 4 fields, chosen randomly; $D_F = D_N + 2, 4$, or 7 fields) and 8 differences in value ($V_L = [50 - 99]$ credits, chosen randomly; $V_S = 20, 40, 60, 70, 80, 85, 90$, or 95% of V_L).

In run 2, we used this estimated indifference point to adjust our manipulation of the options' relative attractiveness for sequences of 30 trials. Within a sequence, we kept the distances of the two options constant, chosen from 3 soon times ($D_N = 2, 3$, or 4 fields, randomly chosen for each sequence) and 3 intervals ($D_F = D_N + 2, 4$, or 7 fields, systematically chosen

across sequences). Within each sequence, we randomly chose the LF option's value ($V_L = [50 - 99]$ credits). We kept the subjective values of the SN and the LF option constant relative to the indifference point by choosing the SN value relative to the random LF value. Each sequence could hence consist of SN options that were more attractive (indifference point -20%), equally attractive (indifference point $\pm 0.001\%$) or less attractive (indifference point +20%). This resulted in 12 sequences of 30 trials each; 360 trials for a complete block.

1.5. Results

Hypothesis I stated that in sequences of trials that start with a choice of the attractive option, we expect no switches or an even number of switches. In contrast, in sequences of trials that start with a choice of the unattractive option, we expect an odd number of switches. For trials with no clearly attractive option, we expected a random distribution of switches unbiased to an odd or even number of switches.

In this experiment, we used sequences of options in the experimental block that were already adapted to participants indifference points as measured in the measurement block. To establish comparability to experiment 1 in the number of switches, we split up the sequences of 30 equal configurations of options into 5 subsequences of 6 consecutive options. For these subsequences, we counted the number of sequences that started either with the choice of the SN option or the LF option and that yielded either an even or an odd number of switches between options. Again, for statistical analysis, we subtracted the number of odd switches from the number of even switches. The resulting difference score was analysed in a RMANOVA. This revealed neither a significant main effects of the initially chosen option (SN vs. LF), $F(1,19) = 3.845, p = .065, \eta^2_p = .168$, nor a significant main effect of attractiveness (SN vs. neutral vs. LF), $F(2,38) = 3.358 p = .065, \eta^2_p = .150$, but – as predicted – a significant

interaction, $F(2,38) = 48.794, p < .001, \eta^2_p = .720$. Hence, we found the predicted pattern of a largely higher number of even switches for sequences in which the attractive option was chosen initially.

Hypothesis II stated that we should find a perseveration bias, which is that the probability to choose the option again that has been chosen previously, is higher than the probability to switch between options. We again performed a Markov analysis across all trials, in which we counted the number of SN and LF choices with respect to the previously chosen option. For statistical analysis, we analysed whether the difference of switch and stay probabilities was positively different from zero, using a dependent samples t -test, which revealed a significant difference from zero, $t(19) = 10.953, p < .001, d = 2.449$. Hence, a clear perseverative bias was present as predicted by the model. A graphical summary of the results is given in Figure 5b.

Hypothesis III expected faster RT for trials in which participants chose the preferred option. We performed a t -test on the dependent variable RT with the independent variable *choice of preferred-unpreferred option* in trials with one attractive option. It yielded no significant difference, $t(19) = 1.01, p = .328, 95\% \text{ CI } [-.033 \text{ sec}, .095 \text{ sec}]$, though the effect pointed in the expected direction (Figure 6 bottom-left).

Hypothesis IV stated that trials in which participants stayed with the preferred option should be faster than when participants switched to the preferred option after choosing the unpreferred option in the previous trial. We performed a t -test on the DV RT with the independent variable *stay with/switch to preferred option* in trials with one clearly attractive option. It yielded a significant difference, $t(19) = 3.48, p = .0025, 95\% \text{ CI } [.028 \text{ sec}, .113 \text{ sec}]$ (Figure 6 bottom-right).

1.6. Discussion

Experiment 2 replicated Experiment 1 and the predictions by the attractor model for choice patterns and partly for RT, despite longer sequences and superficially different options. Though RT data did not yield the significant difference for choices of the preferred option, the descriptive difference pointed in the right direction.

With these two successful validations, only the third of the model's predictions still remains to be validated: a modulation of perseveration by the length of the delay between two consecutive decisions. To this end, we reanalysed data from a study that is already published ([23], <https://osf.io/23ax5/>).

6. Reanalysis of existing data

The attractor model predicts that perseveration will be affected by the delay between two consecutive decisions. We reanalysed data from a previously published study that had used the same paradigm, though with a different mode of varying the presented option's values. The original study had used sequences of ascending or descending differences between the two offered options. Each sequence had been presented one time with a short delay (the inter trial interval between two trials) or a long delay.

Though the setup using ascending or descending sequences is different to the repeated presentation of options in experiments 1 and 2, it still allows for an analysis of the consequences of different delays, when the analysis focusses on the mid-sequence trials for the level of perseveration that should occur. For these trials, the model's prediction leads to the hypothesis that perseveration (as indicated by markov analysis) is stronger in trials with a short delay compared to trials with a long delay between trials.

1.7. Methods

1.1.7. Participants

59 participants of the Technische Universität Dresden took part in the experiment. All participants had normal or corrected-to-normal vision. They gave informed consent to the study and received class credit for their participation.

1.1.8. Procedure and setup

The procedure and setup was similar to Experiment 1. However, participants worked on sequences of eight trials in which the distance of the LF option was increased or decreased by one field from trial to trial (for exact values, see design). Furthermore, the inter trial interval (ITI) between two trials was varied across sequences (1 second or 2.5 seconds, compared to 1.3 second in experiments 1 & 2). Participants were not informed about both, the sequences and the varying ITI. Finally, participants were randomly split into two groups, a positive mood and a negative mood group, which affected the background in the game and pictures that participants were shown between blocks.

1.1.9. Design and data selection

The options' values ranged from one to ten credits, with the values of the two options in each trial adding up to eleven credits to keep the overall value of each trial constant (smaller/larger reward pairs were: 1/10, 2/9, 3/8, 4/7, 5/6). The SN option was two or three fields away from the avatar and the LF was an additional 1, 2, 3, 4, 5, 6, 7, or 8 fields further away from the avatar than the SN option.

To stay with the type of analyses we used hitherto, we selected the inner four trials of each sequence, i.e. the 3rd, 4th, 5th, and 6th trial. Within these trials, one could expect participants

to stay with or to switch away from the option that they had initially chosen at the start of the sequence of trials, while one could assume participants to stay with the chosen trial in the first 2 and the last 2 trials of a sequence. This logic holds for both, ascending and descending trials so that we can collapse across this variable. We further collapsed across the mood groups to increase the power of the applied markov analysis.

Within this setup, participants completed 601.81 trials on average (SD=116.45 trials). Considering the selection of the inner 4 trials of each sequence (1/2 of the trials of each sequence) and the 2 ITIs, this results in an average of 150 trials per participant in the markov analysis.

1.8. Results

Our hypothesis stated that we should find a perseveration bias, which is that the probability to choose the option again that has been chosen previously, is higher than the probability to switch between option. This perseveration bias should be larger for the short ITI compared to the long ITI. We again performed a Markov analysis for all included trials, in which we counted the number of SN and LF choices with respect to the previously chosen option. For statistical analysis, we analysed whether the difference of switch and stay probabilities was different for the short and the long ITI, using a dependent samples *t*-test, which revealed a significant difference, $t(58) = 3.041$, $p = .004$, $d = .396$. Hence, a difference in perseverative bias was present as predicted by the model.

1.9. Discussion

The reanalysis shows that the delay between two consecutive choices influences the strength of perseveration as predicted by the model. Even though the reanalysis used data from the same paradigm, the setup of trials was different to the one in experiments 1 and 2 and hence

not optimal to find the predicted difference. This might explain why the effects is relatively small, though still highly reliable in the studied sample.

7. General Discussion

We proposed to extend the current trial-oriented focus of sequential sampling models to longer time-scales to capture patterns of decision making across multiple decisions [12–17]. For this purpose, we considered the inherent process dynamics that are typical for non-linear coupled models, such as leaky integrator models [18], parallel constraint satisfaction models [24], and – as a very simple instance – neural-inspired attractor models [12]. We hence used the latter, simple model which incorporates the basic process of sequential sampling models but extends their short-term focus to a perspective that examines choice dynamics across multiple consecutive decisions and hence to long-term time-scales. This extension takes into account how choices influence each other and how such interactions give rise to patterns in sequences of decisions that reflect effects of memory traces of previous choices, which are modulated by noise in the system. Using computational simulation, we derived three typical patterns of decision making and two RT effects. We validated these predictions in two experimental studies that were specifically designed to study long-term patterns, and via reanalysing data from a third study. All studies used a specific decision game that allows investigating the dynamics of delay discounting in an immersive setting and occluding sequences of similar choices.

Though we showed that the predicted patterns can be found in this specific setup, the model's assumptions about inherent dynamics are generic. The model and its more complex counterparts could hence be applied to other areas of decision-making: Applications of similar models and phenomena of long-term patterns at longer time-scales can be found, for example, in areas of perceptual decision-making [16,18,25–27]. Especially perseveration and even higher

order sequence effects have been extensively studied in the context of perceptual decision making [e.g. 28–30] and have been modelled based on attractor dynamics [31]. However, such effects are often neglected in empirical and modeling work of value-based decision making, where trials of interest are analyzed in isolation. Beside a first approach for risky decision-making [13], we show here for the first time, that the same phenomena as for perceptual decision making can also be found for value-based decision making, i.e., delay discounting. The common assumption across both fields applications is that the visual dominance of the different percepts (in perceptual decision making) or the different attractiveness of the options (in value-based decision making) governs the decision process, but the information about this dominance or attractiveness is noisy and sticky which can lead to switches or perseveration in sequences of decisions and thereby may lead to choices of a weaker or suboptimal option [12]. With our work, we contribute to the astonishing insight that these phenomena occur across the wide range from perceptual to value-based decision making and that models that are usually applied to the former area can be used equally to predict patterns in the latter area. Hence, our work could be seen as a generalization of the work from perceptual decision making to value-based decision making and perseveration is one example of first-order sequence effects (for a more extensive overview of sequence effects and the assumed underlying mechanisms in perceptual decision making, see [28]).

The decisive difference between dynamic (attractor) models and common sequential sampling models is their inherent dynamics [27], which allows for the long-term patterns to emerge. The creators of common sequential sampling models removed these – usually non-linear – dynamics on purpose. For example, the ballistic linear accumulator model [32] was conceptualized as a simplification of the leaky accumulator model [18]. The purpose of the

simplification was to allow for quantitative fitting of response time data. Our findings indicate that, while such simplifications might be necessary for technical reasons, one should not lose sight of phenomena for which simplified models lose predictive power [27]. While sequential sampling models are able to extract general biases in their respective parameters via data fitting, they have two shortcomings that our model overcame: First, they are not able to fit perseveration as a bias that depends on the choice in the previous trial (the bias parameter favours one specific option across all trials) and, second, they are not able to *predict* biases that depend on the previous decision, since it is not specified what happens in the break between decisions.

Our results also show how widening the focus from single-trial decision making to patterns across trials, since it allows for asking new questions. Instead of post-hoc data fitting that characterizes the research process in this field, we used the model to predict the patterns that we then validated in the data. Our model's structure combining accumulation dynamics within a trial with dynamics across trials allowed new predictions to emerge that led us to design an experiment that would normally not be executed in the area of value-based decision making.

1.10. Open questions

It is an open question, whether our model draws an adequate picture of the decision making process for the delay discounting decisions studied here, since these decisions are multi-attributive while our model only receives one input per option. Though it is an open question whether the decision system indeed uses a single variable, i.e., the subjective value used in our model or bases its decision indeed on the single attributes, two points indicate that this obvious limitation does not question the general point we make here. First, our model represents a class of models with inherent dynamics which includes more complex, multi-attributive models, e.g. the PCS model [24]. For this model, one can expect the same effects that we showed here.

Second, and more practically, an increasing number of works using sequential sampling models to model delay discounting (e.g. [8,9]) are based in the same assumption of one input per option. Our work proceeds exactly on this path and extends the work based in sequential sampling models, so that the number of inputs to the model is not central to the main point of our study.

One might ask whether the incorporation of the long-term perspective has substantive implications for other research. If we consider that long-term phenomena like perseveration influence each trial in the sequence, then ignoring these phenomena might lead to inaccurate conclusions. For example, the monetary choice questionnaire [33] has been used to study differences between control groups and groups with substance use disorder, i.e. heroin addicts [34,35]. Higher k -values (the curvature parameter of the hyperbolic discounting model, see [36]) in such a study are usually interpreted as the heroin addict group showing more impulsiveness. However, since the questionnaire doesn't factor in perseveration effects, the conclusion of a difference between the two populations that lies in the discounting itself might be premature. We simulated choices in the questionnaire with our model for a k -value of 0.01 (resulting in 44% of choices for the SS option according to the questionnaire). We performed two runs of the simulation with exactly the same model configuration. The only difference was that in the first run we used the short delay between decisions, as used in the other simulations here, whereas in the second run, we used a long delay to eradicate any perseveration effects. The latter run hence implements the assumption of independent single decisions. When delays between decisions were long enough to eradicate any perseveration, the model choose exactly as the original k -value of 0.01 predicts. However, for the short delay, the k -value rises to a value of 0.015 due to the specific order of choices influencing each other. Hence, the fact that a heroin population shows higher k -values might not (or not only) be attributable to more impulsive decision making,

but instead could be caused by hastier transitions from item to item leading to perseveration. These two interpretations, a change in the decision process and weighing of the options (as usually assumed) vs. a change in perseveration (as shown here), are largely different and lead to different conclusions about disorder-related changes in decision making that go beyond a purely academic question.

1.11. Conclusion

We presented a model that predicts decision patterns across multiple decisions based upon accumulation dynamics within and between decisions. Experimental results validated these predictions. Integrating long-term patterns of decision-making may therefore substantially further our understanding of the behavioural and neural processes that underlie optimal and suboptimal decision-making. Considering the stability of general patterns in a long-term decision-making perspective may also improve upon the explanatory power of single decisions that suffer from a high level of unexplained noise instead. We speculate that the incorporation of a long-term perspective into decision models and empirical studies will support an understanding of deficient decision-making and the underlying circuitry in the healthy and the dysfunctional brain with respect to the stability of decision patterns.

8. Authorship Statement

All authors contributed to the conception of the study and model. SS, SL, and MD designed the study. SS and MD executed the study and analysed the data. SS, SL, and MD drafted the manuscript and TGr, SSi and TGo revised the manuscript critically.

9. Data availability statement

Primary data (csv format), analysis files (Matlab) and model code (Matlab) is available via the Open Science Framework (osf.io/d2yhb).

10. Ethics statement

The study was performed in accordance with the guidelines of the Declaration of Helsinki and of the German Psychological Society. An ethical approval was not required since the study did not involve any risk or discomfort for the participants. All participants were informed about the purpose and the procedure of the study and gave written informed consent prior to the experiment. All data were analysed anonymously.

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12. Appendix: Computational Modelling and Simulation

The formalized model used for the simulations below built on models which have been used to study the neural activation dynamics of perceptual decisions in the past [18,25,compare 26]. Recently, we showed that it can also be applied to delay discounting, where it successfully predicted within and across trial effects [12,37].

These two units inhibit each other which opens a natural mapping of these two units to the architecture of more complex models, i.e. parallel constraint satisfaction networks [e.g. 19]. The two units in the model here could be seen as the two options in a parallel constraint satisfaction network that are fed by informative cues from lower network levels. By interactive activation and competition both systems lead to a settled decision state in the end of the decision process.

1.12. Mathematical model description

The dynamics of the model are defined by two coupled differential equations (one for each neural unit) representing non-linear neural activation dynamics [26,25,see also 21,38].

$$\tau \dot{u}_{SN} = -u_{SN} + h + w_r \cdot \sigma(u_{SN}) + w_i \cdot \sigma(u_{LF}) + I_{SN} + q\xi_{SN}(t)$$

$$\tau \dot{u}_{LF} = -u_{LF} + h + w_r \cdot \sigma(u_{LF}) + w_i \cdot \sigma(u_{SN}) + I_{LF} + q\xi_{LF}(t)$$

Here, τ denotes the timescale (defining the step size of the Euler solution), h denotes the resting level, w_i the (inhibitory) coupling strength of the two equations, and w_r the recurrent feedback, I_{SN} and I_{LF} the input representing the attractiveness of the two options, q the strength of the normally distributed noise ξ , and σ a sigmoid non-linearity, mirroring non-linear neural population dynamics:

$$\sigma(u) = \frac{1}{(1 + e^{-\beta u})}$$

Hence, interactions between the two units only happen to the extent that the activation u exceeds a soft threshold [39,40]. Note that the β -parameter is also called the gain-parameter, modulating the discreteness of neural activation states, which we varied across simulated subjects [for hypotheses concerning β and aging, see e.g. ,41]. For a list of the chosen parameter values, see Table 1. Importantly, these parameters are identical to the ones from our recent applications of this model to delay discounting [12,37] with exception of the strength q of the added normally distributed noise (the previous versions ran without any noise) and the random variation of β across simulated subjects (the previous versions ran only one prototypical subject).

Table 1. Basic parameters of the model

Parameter	Value
τ	10
h	-5
w_i	-7
w_r	6
q	0.1
β	1 ± 0.2 std across 20 simulated subjects

The dynamics of the system are governed by the inputs to the two units, that is I_{SN} and I_{LF} . To drive the units near to the activation threshold, these inputs consisted of a base Input I_{base} modified by the relative attractiveness of the two options

$$I = I_{base} \pm I_{attractiveness}$$

As in the recent application of the model I_{base} was set to 6. $I_{attractiveness}$ was set to 0.05, yielding e.g. for the condition of an attractive LF option $I_{SN} = 5.95$ and $I_{LF} = 6.05$.

1.13. Simulation design

We simulated three conditions, an attractive SN option, a neutral condition, and an attractive LF option by varying $I_{attractiveness}$ in the described way. We simulated sequences of trials within each condition, with each sequence consisting of 30 consecutive trials and replicated 4 times, leading to an overall number of 360 simulated trials (3 conditions x 4 replications x 30 trials). With this setup, we simulated 20 subjects, varying the gain parameter β between subjects by a standard deviation of 0.2.

1.14. Simulation procedure

The two coupled differential equations constitute a neural system with two units inhibiting each other so that only one unit can win the competition and elicit the final choice. This competition unfolds over time. We simulated the behaviour of the system by numerical integration (using the Euler method with the step size τ as described above) with each trial having a maximum length of 200 time steps (considering average simulated response times below 30 circles, this offers enough time for all trials to find a decision). Results were obtained using Matlab 2010a running on a Windows 8 platform.

In each trial, we presented to the network at time $t = 60$ the two options. A choice was elicited when one unit's output reached a threshold of 0.85. The simulation was run continuously from trial to trial, so that the intrinsic dynamics of the system could lead to the predicted perseveration phenomena.

To simulate the influence of ITI length on perseveration, we reran the original simulations with two different ITI lengths, one time presenting the options at $t = 55$ and one time at $t = 65$.

ⁱ For the modulation of perseveration by the delay between consecutive decisions, see experiment 3.