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Title:

Evidence for preference consistency across risky, ambiguous, and vague gambles

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Abstract

Humans have variable preferences over the uncertainty of potential outcomes during decision-making. To date, most uncertainty research has focused on how individuals form subjective values of risky and ambiguous choices where uncertainty results from known or unknown outcome variability, rather than exploring situations where outcome values are uncertain. Here we examine ‘vagueness,’ defined as a specific form of uncertainty in which a player is informed only that the outcome of a decision lies somewhere within a known range of values. We use a model-based approach and an experimental gambling task to examine whether individuals evaluate uncertainty in outcome values differently from uncertainty in the probability of the outcomes. Our results show that subjective valuation of vagueness, risk, and ambiguity depend upon individuals’ distinct preferences for the first and second moments of gamble outcomes, indicating that individuals have preferences for uncertainty in mean outcome values and outcome variability, respectively, that guide decision-making across types of uncertain situations.

Significance Statement

Uncertainty describes situations where there is unknown information about future events. To date, most research has focused on risk, where the likelihoods of potential outcomes are known as in a coin flip or ambiguity, where likelihoods of potential outcomes are unknown. However, in addition to uncertainty about likelihoods, decision-makers also face unknown outcome values, like when an investor considers next year’s stock market returns. We examine this latter form of uncertainty as ‘vagueness’, where a decision maker knows only that an outcome lies somewhere within a range of values. We show that humans have distinct preferences for uncertainties in the value and likelihood of outcomes, respectively, that together explain choices across all three types of uncertainty.

Main text:

Most decisions, even those as simple as deciding what to have for lunch, involve uncertainty and are guided by individual preferences for this uncertainty. Modern portfolio theory (i.e., mean-variance analysis (1)) models how individuals assign value to uncertain options as a function of the mean and variability of possible outcomes. This is a common approach to studying preferences and decision-making when possible outcome values and their probabilities are both known. However, a number of other types of uncertainty exist where outcome values and their probabilities are partially or fully *unknown*. Here, we consider one such possibility, ‘*vagueness*,’ which we define as an uncertain situation where the decision maker knows only that an outcome lies somewhere within a known range of values. To understand how individuals make choices when uncertainty exists in outcome value and outcome probability, we tested a set of gambles where information about outcomes and probabilities was manipulated, and modeled preferences for risky, ambiguous, and vague gambles. Our results show that subjective valuation of uncertain choices may be reduced to individuals’ preferences for uncertainty in the first two moments of gamble outcomes indicating that individuals have dissociable preferences for uncertainty in outcome mean values and outcome variability, respectively, that guide decision-making across types of uncertain situations.

Ninety-eight participants (male/female = 60/37, age = 35.1 ± 10.1 ; one individual did not complete the demographics survey; **Table S1**) took part in the current study. Previous uncertainty research has focused on risk (known outcome values, known probabilities, (2)) and ambiguity (known outcome values, unknown probabilities, (3)), two extreme cases of uncertain situations where decision makers have good information about possible outcome values. Vagueness, a third extreme case, captures uncertain situations where an individual lacks information about outcome values. To investigate how individuals subjectively value choices with these types of uncertainty (i.e., uncertainties in value and probability of outcomes, respectively), we asked participants to make a series of choices between two gambles. There were four different types of gambles (**Fig. 1a**): (i) certain gambles, where the indicated payoffs are guaranteed if chosen; (ii) risky gambles, where the value and probability of indicated payoffs are known; (iii) ambiguous gambles, where the value of payoffs is known, but probability is unknown, and (iv) vague gambles, where a range of values is provided (i.e., the range between the upper and lower bounds indicated as ‘ $V_{\min}$ to $V_{\max}$ ’, respectively; bottom-left of **Fig. 1a**), so that the amount is predetermined, but unknown. Each trial involved a combination of two of the four gamble types.

The task included both trials where a gamble with an uncertain outcome (either type ii, iii, or iv) was paired with a certain gamble (three different gamble pairings), and trials where a risky gamble was paired with a gamble with ambiguity or vagueness (two different gamble pairings). These five gamble pairings were intermixed (see **Methods** for the number of trials for each gamble pairing), and each participant faced trials in one of four pseudo-randomized orders. Participants completed 170 unique trials and received the outcome of one of their choices, in addition to a fixed compensation for participating (see **Methods** for gamble development and experimental procedure details).

As a first step toward exploring subjective valuation of different types of uncertainty, we measured individuals' preferences for uncertain vs. certain gambles (**Fig. 1b**), where the uncertain gambles were risky, ambiguous, or vague. Per previous studies (3-8), we hypothesized that participants' decisions would be influenced by the level of uncertainty within each type of gamble. Indeed, we found this to be the case such that participants' choices varied by the level of uncertainty within each gamble type. These initial descriptive data are consistent with previous work showing increased probability of choosing uncertain options with higher high-payoff probabilities (seen in certain vs. risky trials (4, 8)) and decreased probability of choosing uncertain options with greater ambiguity (seen in certain vs. ambiguous trials (7, 8)), and in certain vs. vague trials, provide new data showing a quadratic relationship between the probability of choosing the uncertain option and the level of vagueness (**Fig. 1b**). In particular, participants' choice patterns under vagueness indicate that increasing payoff vagueness does not necessarily decrease subjective value.

To formally evaluate the effects of the different types of uncertainty on individuals' choices, we used mean-variance analysis to model the subjective value of uncertain options (7, 9, 10). For certain, risky, and ambiguous gambles, we estimated risk preferences (ρ), ambiguity aversion (β), and sensitivity to utility differences ('inverse temperature'; λ) using standard existing models (see **Methods** for model details). To investigate subjective value and preferences for vague gambles, we hypothesized that individuals' subjective valuation of a vague gamble is a function of a subjective expected value (subjective mean), and a measure of the variability of the potential outcomes (dispersion) (1). Subjective mean was defined as a weighted average, using vagueness value weight ε ('value weight' hereafter), of the highest and lowest possible outcomes; thus, one can think of ε as a normalizing parameter such that $\varepsilon = 0.5$ indicates that an individual equally weights $V_{\min}$ and $V_{\max}$, $\varepsilon > 0.5$ indicates an *optimistic* assessment of value and $0 \leq \varepsilon < 0.5$ indicates a *pessimistic* assessment of value. Dispersion was defined as the squared distance between $V_{\min}$ and $V_{\max}$ ($[V_{\max} - V_{\min}]^2$; see **Methods** for alternative models). Analogous to risk preference in the mean-variance model, the vagueness dispersion preference (α ; 'dispersion preference' hereafter) is a parameter that captures the extent to which individuals are sensitive to the distance between outcomes, such that $\alpha = 0$ indicates dispersion neutrality, $\alpha > 0$ indicates dispersion tolerance, and $\alpha < 0$ indicates dispersion aversion. **Figure 2** illustrates the role of each parameter in the calculation of subjective mean and outcome variability for each gamble type.

Regardless of gamble pairing (i.e., combinations of certain, risky, ambiguous, or vague gambles), all choices were used in the individual parameter estimation procedure (see **Methods** for model structure details and estimation procedures). To test whether our suggested model explains participants' choices better than alternative models (see **Methods** for more information about rationale for dispersion), we also conducted formal model-comparison using integrated Bayesian Information Criteria (iBIC, lower values indicate better model fit). As hypothesized, the dispersion model best explained participants' behavioral choices compared against seven alternative (nested and extended) models (**Fig. S1**; see **Methods** for alternative model details). Note that

simulations confirmed that the dispersion model parameters are recoverable and also uniquely identifiable (see **Methods** for model recovery details).

Consistent with previous studies on decision-making about risky and ambiguous gambles (4, 5, 10, 11), participants on average exhibited both risk aversion (vs. risk-neutrality: $t(97) = -6.22$, $P = 1.27\text{e-}08$; **Fig. 3a**) and ambiguity aversion (vs. ambiguity-neutrality: $t(97) = 8.32$, $P = 5.63\text{e-}13$; **Fig. 3a**; plotted as ambiguity tolerance ($-\beta$)). In the subjective valuation of vague gambles, participants showed optimism about outcome mean values (vs. neutrality: $t(97) = 5.27$, $P = 8.37\text{e-}07$, indicating $\varepsilon > 0.5$; **Fig. 3a**). This significant ‘optimism’ result indicates that, on average, participants valued the vague gamble more than its expected value, when, as in the model, expected value was calculated as the average of $V_{\max}$ and $V_{\min}$. Moreover, participants were averse to dispersion in the vague gambles (vs. neutrality: $t(97) = -52.43$, $P = 5.54\text{e-}73$; **Fig. 3a**), indicating that, holding expected value constant, individuals become less likely to choose vague gambles as outcome variability increases. On average, therefore, during vagueness, participants are optimistic and dispersion averse. Note that optimism and dispersion aversion have opposite effects on the total subjective value of the option; that is, as a function of increasing levels of vagueness, optimism linearly increases the subjective value, while dispersion aversion monotonically decreases (proportionate to squared distance) the subjective value of the vague gamble. The willingness to tolerate more payoff dispersion in return for higher subjective expectation of mean outcome is consistent with observed real world investment behavior (12), and the combination of positive ε (linear) and negative α (non-linear) effects of vagueness on subjective utility can account for the quadratic relationship observed in the model free data shown in **Fig. 1b**.

We next explored regularities among these preferences. First, a correlation between value weight (ε) and dispersion preference (α) was computed (**Fig. 2**, green). Based on the formal model-comparison result (**Fig. S1**), outcome variability of a vague gamble is computed in the form of dispersion rather than variance, suggesting that the first and second moments of gamble outcomes are independently processed. However, empirical individual estimates of value weight and dispersion preference were positively correlated ($r = 0.21$, $P = 0.036$; **Fig. 3bi**). Further investigation on individuals’ model-fits and their estimated parameters showed that there exist individual differences in the strategy of processing vagueness. Particularly, the significant correlation between value weight and dispersion preference was only observed in the participants whose behavioral choices were better explained by models which captured outcome variability with variance instead of dispersion (**Fig. S5**). See **Methods** section ‘**Relationship between value weight and dispersion preference**’ for a discussion of the model structure and the characteristics of vague gambles.

We next looked for consistencies among types of uncertainty. Notice that ambiguity aversion (β) and value weight (ε) are similar in that they describe the extent to which one weights the high value option when computing subjective mean (**Fig. 2b**, red). Estimated ambiguity tolerance ($-\beta$) was positively correlated with value weight (ε) ($r = 0.28$, $P = 0.0045$; **Fig. 3bii**), indicating that participants who discount the probability of receiving the high-payoff in an ambiguous gamble also have a pessimistic assessment of receiving

the larger payoff in a vague gamble. Next, since both risk preference (ρ) and dispersion preference (α) indicate utility or disutility of dispersion in gambles (**Fig. 2b**, blue), we looked for a relationship between these preferences. Individuals' risk preference (ρ) was positively correlated with dispersion preference (α) ($r = 0.34$, $P = 0.00071$; **Fig. 3biii**), indicating that individual preferences for avoiding variability in outcomes is independent from whether gambles are risky or vague. Moreover, the preferences for uncertainty in mean outcome value (i.e., ambiguity aversion (β) and value weight (ϵ)) were not correlated with the preferences for outcome variability (i.e., risk and dispersion preferences, α and ρ), suggesting that individual preferences for the first and second moments of gamble outcomes are not associated with one another. Again, the formal model-comparison showed that vague gambles are subjectively valued through two dissociable processes of outcome mean and variability (c.f., see **Fig. S5** for individual differences in the computing strategy specifically for vagueness).

Finally, the inverse temperature parameter (λ) was positively correlated with risk preference (ρ) ($r = 0.24$, $P = 0.017$; **Fig. S2a, 2bi**) and dispersion preference (α) ($r = 0.28$, $P = 0.0054$; **Fig. S2bii**), indicating that participants who are less sensitive to subjective value differences between options are more averse to variability in outcomes. No relationship was observed between inverse temperature and ambiguity aversion ($r = 0.022$, $P = 0.83$) or value weight ($r = -0.0057$, $P = 0.96$).

Across three forms of uncertainty, we show that individuals are averse to outcome variability. Equally important, preferences in individuals' subjective valuation of mean outcomes are common across ambiguous and vague gambles, such that the most optimistic individuals are those who have the highest ambiguity tolerance. Taken together, these provide an initial model-based examination of how individuals subjectively respond to uncertainty in outcome values (i.e., vagueness) and indicate that commonalities exist in preferences for outcome mean values and outcome variability, respectively.

Most individuals (optimistically) overestimated the mean of vague gambles, a finding which is consistent with research on unrealistic optimism in daily decision-making when outcomes are uncertain (13, 14). Sharot et al. (15) suggested that optimism bias persists because individuals have an 'optimistic update' during learning (i.e., are more sensitive to information that is better than expected). Moreover, in that work, levels of optimistic information updating are positively associated with self-reported optimism scores (16), indicating that individuals who are strongly optimistic are more likely to have persistent optimistic biases due to continued optimistic updating (see **Table S2** for further discussion).

Participants discounted the subjective mean of vague gambles as a function of the dispersion of possible outcome values. Individual differences in this dispersion preference were positively correlated with risk preference, which suggests that the two characteristics may share similar underlying mechanisms. However, we note that formal model-comparison suggested that the model separately identifying risk and dispersion preferences explains participants' behavioral choices better than the model coding preferences for risk and dispersion under vagueness with a single parameter. Moreover,

the estimated risk and dispersion preferences only shared 12% of their individual differences, indicating that each type of preference has a significant separable contribution to the subjective valuation of uncertain options. Together, these results suggest that preferences for avoiding variability in outcomes under risk and vagueness have both common and dissociable contributions to subjective valuation.

The present work has implications for considering cognitive processes that are used to make choices across different types of uncertainty. Efforts to understand how individuals assign value to uncertain decisions have primarily focused on two categories of models: expected utility theory (2) and mean-variance analysis (modern portfolio theory, (1)). Expected utility theory defines subjective value as the expectation of an individual's valuation of gamble outcomes while risk preferences are captured by a parameter measuring the curvature of the valuation function. Within this framework, ambiguity aversion has typically been represented as a negative bias on the perceived probability of outcomes (9). Between these two types of uncertainty (risk and ambiguity), previous studies have generally focused on results showing that preferences for risk and ambiguity are not correlated and that the processes underlying each form of uncertainty are dissociable (5, 6, 17). The current results, adopting a mean-variance framework, suggest that the previously identified dissociation of risk and ambiguity preferences, may reflect distinct transformations over the variance and mean value of uncertain outcomes, respectively. That is, the separate correlations of ambiguity preference with value weight and risk preference with dispersion preference, suggests that across types of uncertainty, individuals apply two general classes of value transformation, one biasing mean value of uncertain outcomes and a second weighting the variance of uncertain outcomes.

To summarize, the current study suggests that human decision-making across multiple types of uncertainty are characterized by preferences over outcome variability and biases over outcome mean. Real life decisions are likely to involve a mixture of types of uncertainty in outcome value and probability, and individuals may perceive the same uncertain situations quite differently. While a financial economist's expertise may allow her to perceive a mutual fund as a risky investment, for someone choosing a retirement plan at a first job, potential returns from the same fund may seem vague. In such contexts, preferences over the variability of future outcomes are often given primary consideration; however the current work suggests a second factor, optimism or pessimism over outcome values, that is critical to decision-making under uncertainty.

Methods:

Participants. Individuals ($N = 187$) were initially recruited through Amazon Mechanical Turk (MTurk), an online crowdsourcing service. A number of studies have shown that MTurk allows a more demographically diverse sample than standard American college samples (18) and that MTurk behavioral economics results follow those reported in standard laboratory settings (19-21); however, we also acknowledge potential issues with MTurk samples (20) that have been previously noted (e.g., potential less control over participants, choice stability) and that stake sizes have been related to participants' choice behaviors ((4), though also see (20-23) where magnitude of payments does not affect participants' choices). We have taken several steps to minimize the possible impact of these issues (e.g., within-subject design estimating preferences across gamble types, requiring strict evidence of task comprehension, catch trials during practice and throughout the task). These efforts are discussed in the relevant sections below.

73 individuals failed to correctly answer six task comprehension questions (see **Fig. S3** for the quiz details) and did not continue with the task. An additional 11 participants began but did not complete the task. After task completion, five participants were excluded from analysis for failing more than four out of 10 catch trials (catch trials were included where the utility of one gamble is always larger than the other gamble, regardless of individuals' preferences; see **Experimental procedures** for details). These criteria yielded 98 individuals (male/female = 60/37, age = 35.07 ± 10.05 ; one participant did not complete demographics survey; **Table S1**) for the present study. All participants provided written informed consent following an explanation of study procedures. The study was approved by the Institutional Review Board (IRB) of Virginia Tech, and all experimental procedures followed relevant institutional guidelines and regulations. In addition to participants' demographics, we collected two self-report questionnaires measuring real-life responses to uncertain situations: Revised Life Orientation Test (LOT-R, (16)) and Barratt Impulsiveness Scale (BIS-11, (24)). Note that results from both questionnaires were similar to those of typical individuals from previously reported large sample (LOT-R: 14.3 ± 4.3 (16); BIS-11: 62.3 ± 10.3 (25); see **Table S1**).

Experimental procedures. Participants made a series of 170 unique choices between two gambles. The task included four different types of gambles (**Fig. 1a**): certain, risky, ambiguous, vague gambles. Each gamble pairing comprised two different types of gambles: 'certain vs. risky gambles' (30 trials), 'certain vs. ambiguous gambles' (29 trials), 'certain vs. vague gambles' (29 trials), 'risky vs. ambiguous gambles' (41 trials), or 'risky vs. vague gambles' (41 trials). The five different gamble pairings were intermixed, the presented position of gamble types was counter balanced within each gamble pairing, and trial order followed one of the four pseudo-randomized trial orders per participant. To evaluate participants' understanding of the task, a brief quiz was administered following all instructions, and only participants who correctly answered all six questions continued in the choice task. Participants were paid at the end of the study, based on the actual outcome of one randomly selected gamble choice (\$0.00-\$0.90), in addition to a base \$0.50 compensation (in range of other comparable MTurk studies (18, 20, 22, 23)).

Task development. The gambles used in our task were designed to capture individuals' preferences for different types of uncertainty. Each gamble pairing was separately designed to have a matched range of expected value difference between the two gambles ($EV_{\text{difference min}} = -55$, $EV_{\text{difference max}} = 38.5$). Two catch trials were included for each gamble pairing where the subjective utility of one gamble was always larger than the other gamble, regardless of individuals' preferences. Each gamble pairing is detailed below.

Certain vs. risky gambles. Each risky gamble was presented as a pie with pie slices proportional to the outcome probabilities and numerical indications of the high- and low-payoff. The low-payoff was always zero. Three levels of high-payoff values (25¢, 48¢, or 90¢) and three probabilities of the high-payoff (25%, 50%, and 75%) were paired to form nine unique gambles. Three certain gambles were represented as circles with a payoff indicated. Values for certain gambles were set below, close to, and above the expected value of each risky gamble. Pairing each risky gamble with three appropriate certain gambles yielded a total of 27 unique gamble pairs. To match the range of expected value differences across the 3 levels of high payoff values, one additional set of gambles was added with the largest expected value difference ($EV_{\text{difference}} = 38.5$). Two additional sets of gambles where the certain gamble has a higher payoff than the high-payoff of the risky gamble were used as catch trials; in the other pairs, the value in the certain gamble was below the high-payoff of the risky gamble.

Certain vs. ambiguous gambles. Each ambiguous gamble had high- and low- payoffs, and a level of ambiguity. As in risky gambles, the low-payoff was always zero. Three levels of high-payoff values (25¢, 48¢, or 90¢) and three levels of ambiguity ($A = [25\%, 50\%, \text{ and } 75\%]$) were paired to form nine unique gambles. Similar to risky gambles, each ambiguous gamble was presented as a pie with pie slices proportional to known outcome probabilities for the high- and low- payoffs, with ambiguity represented as a partially covered area of the pie. In the 25% ambiguity case, the actual probability of the high-payoff lies between 37.5% (lower-bound = $50\% - A/2$) and 62.5% (upper-bound = $50\% + A/2$). In the same manner, 50% ambiguity indicates that actual probability of the high-payoff lies between 25%-75%, and 75% ambiguity indicates that actual probability of the high-payoff lies between 12.5%-87.5%. As was the case with certain vs. risky gambles, three certain gambles had payoffs below, close to, and above the expected value calculated under the assumption that the perceived probability of both the high- and low-outcomes is 50%. Pairing each ambiguous gamble with three appropriate certain gambles yielded a total of 27 unique gamble pairs. Two additional sets of gambles where the certain gamble has a higher payoff than the high-payoff of the ambiguous gamble were used as catch trials; in the other pairs the value in the certain gamble was below the high-payoff of the ambiguous gamble.

Certain vs. vague gambles. Each vague gamble has lower- (V_{min}) and upper- (V_{max}) bounds that determine a level of vagueness. Three median values ($M = [15¢, 25¢, \text{ or } 48¢]$) and three levels of dispersion (Vagueness = $[25\%, 50\%, \text{ and } 75\%]$) were paired to form nine unique gambles. The range ' V_{min} to V_{max} ' indicated to participants that the

payoff could be anywhere within the range $V_{\min} = (100\% - \text{Vagueness}) \times M$ and $V_{\max} = (100\% - \text{Vagueness}) \times M$; participants were instructed that for these gambles, an outcome lies somewhere within the indicated range of values. Three certain gambles were designed as guaranteed payoffs that lie within the value range indicated for each of the vague gambles; pairing these with the vague gambles yielded a total of 27 unique gamble pairs. Two additional sets of gambles where the certain gamble has a payoff larger than $V_{\max}$ or a payoff smaller than $V_{\min}$ were used as catch trials; in the other pairs, the value in the certain gamble was between the upper- and lower- bound values of the vague gamble.

Risky vs. ambiguous gambles. The nine unique risky gambles from the certain vs. risky trials crossed with and the nine unique ambiguous gambles from the certain vs. ambiguous trials produce 81 possible gamble sets for this gamble pairing. Thirty-nine gamble sets were selected to maximize spread of the expected value differences between the two gambles in a pair without producing an over-long experiment. Two additional catch trials were created that satisfy one of the following two cases; i) [high-payoff of the Risky gamble $\geq$ high-payoff of the ambiguous gamble], and [probability of the high-payoff of the risky gamble $\geq$ the highest possible probability of the high-payoff of the ambiguous gamble], or ii) [high-payoff of the risky gamble $\leq$ high-payoff of the ambiguous gamble], and [probability of the high-payoff of the risky gamble $\leq$ the lowest possible probability of the high-payoff of the ambiguous gamble].

Risky vs. vague gambles. The nine unique risky gambles from the certain vs. risky trials were used for this gamble pairing. The vague gambles had the same three levels of vagueness, but new sets of expected values (three levels) were designed, so that expected value differences between gamble pairs matched the range of other gamble pairings ($EV_{\text{difference}} = -55-38.5$). Again, 39 gamble sets were chosen to maximize spread of the examined expected value differences without producing an over-long experiment. Two additional sets of gambles where the lower-bound of the value range was larger than high-payoff of the risky gamble were used as catch trials.

Computational models of subjective valuation. To formally incorporate preferences over different types of uncertainty into our analysis, we used a mean-variance analysis (1) approach to estimate each individual's preferences for uncertainty. Models for certain (equation 1), risky (equation 2), and ambiguous (equation 7) gambles followed standard models. Expected value (EV) and variance (VAR) for certain, risky and ambiguous gambles are specified below (equations 2, 3, 5 and 6):

$$U_{\text{Certain}} = EV_{\text{Certain}} = V_{\text{payoff: Certain}} \quad (\text{eq. 1})$$

$$EV_{\text{Risky}} = P_{\text{high-payoff}} \times V_{\text{high-payoff: Risky}} + (1 - P_{\text{high-payoff}}) \times V_{\text{low-payoff: Risky}} \quad (\text{eq. 2})$$

$$VAR_{\text{Risky}} = P_{\text{high-payoff}} \times (1 - P_{\text{high-payoff}}) \times (V_{\text{high-payoff: Risky}} - V_{\text{low-payoff: Risky}})^2 \quad (\text{eq. 3})$$

$$U_{\text{Risky}} = EV_{\text{Risky}} + \rho \times VAR_{\text{Risky}} \quad (\text{eq. 4})$$

$$EV_{\text{Ambiguous}} = (P_{\text{high-payoff}} - \beta (A/2)) \times V_{\text{high-payoff: Ambiguous}} \quad (\text{eq. 5})$$

$$VAR_{\text{Ambiguous}} = (P_{\text{high-payoff}} - \beta (A/2)) \times (1 - (P_{\text{high-payoff}} - \beta (A/2)))$$

$$U_{\text{Ambiguous}} = EV_{\text{Ambiguous}} + \rho \times \text{VAR}_{\text{Ambiguous}} \times (V_{\text{high-payoff:Ambiguous}})^2 \quad (\text{eq. 6})$$

$$U_{\text{Ambiguous}} = EV_{\text{Ambiguous}} + \rho \times \text{VAR}_{\text{Ambiguous}} \quad (\text{eq. 7})$$

In the equations above, U is the utility of the gamble, P is the probability of earning a payoff, $V_{\text{high-payoff}}$ and $V_{\text{low-payoff}}$ represent the high- and low- payoffs, respectively, A indicates the level of ambiguity, ρ is risk preference and β is ambiguity aversion. Estimates of ρ indicate risk aversion ($\rho < 0$), risk neutrality ($\rho = 0$), or risk seeking ($\rho > 0$). Similarly, estimates of ambiguity aversion indicate ambiguity aversion ($\beta > 0$), ambiguity neutrality ($\beta = 0$), or ambiguity tolerance ($\beta < 0$) (see **Fig. S4** for β estimation using a standard power utility function instead of a mean-variance approach).

In the current study, we investigated several models for the subjective valuation of vague gambles. Note that, in classic mean-variance analysis, variance captures the amount of outcome variability. However, with vague gambles, we lack probability information needed to calculate the mean and variance. Model 1 (eq. 8-10) is a variation of the mean-variance analysis where we replaced the mean and variance with the subjective mean (eq. 8) and dispersion (eq. 9), respectively. Analogous to the literature on ambiguity (9), the subjective mean of a vague gamble is defined as a weighted sum of the upper- and lower-bounds of possible outcome values (eq. 8). This vagueness value weight, ε ('value weight' hereafter), captures individual differences in sensitivity to the high vs. low outcome. Equation 9 defines the dispersion using a standard distance formula that squares the difference between $V_{\text{max:Vague}}$ and $V_{\text{min:Vague}}$ so that the measure is nonzero.

$$EV_{\text{Vague}} = \varepsilon \times V_{\text{max:Vague}} + (1 - \varepsilon) \times V_{\text{min:Vague}} \quad (\text{eq. 8})$$

$$\text{VAR}_{\text{Vague}} = (V_{\text{max:Vague}} - V_{\text{min:Vague}})^2 \quad (\text{eq. 9})$$

$$\text{Model 1: } U_{\text{Vague}} = EV_{\text{Vague}} + \alpha \times \text{VAR}_{\text{Vague}} \quad (\text{eq. 10})$$

The estimated coefficient ε indicates whether an individual is optimistic ($0.5 < \varepsilon \leq 1$), neutral ($\varepsilon = 0.5$), or pessimistic ($0 \leq \varepsilon < 0.5$) about the outcome of a vague gamble. Similarly, α is the vagueness dispersion preference ('dispersion preference' hereafter) that indicates whether individuals are averse ($\alpha < 0$), neutral ($\alpha = 0$), or tolerant ($\alpha > 0$) to dispersion.

We hypothesized that dispersion is a plausible description of how individuals evaluate outcome variability, because, unlike determining variance, it is not necessary to first calculate the subjective mean to find dispersion (26). An alternative modeling strategy would define outcome variability using a standard variance formula under the assumption that 'mean' is calculated as in equation 8 and that the probability of $V_{\text{max:Vague}}$ is ε (eq. 11). That means that, unlike Model 1, the calculation of the subjective mean and the calculation of outcome variability are linked.

$$\text{VAR}_{\text{Vague_alternative}} = \varepsilon \times (1 - \varepsilon) \times (V_{\text{max:Vague}} - V_{\text{min:Vague}})^2 \quad (\text{eq. 11})$$

Both Model 2 and Model 3 below use $\text{VAR}_{\text{Vague_alternative}}$ (eq. 11) as the measure of outcome variability.

Model 2 (eq. 12) allows us to estimate an analog to the risk preference ρ in a standard risky gamble model (eq. 4)—we use ρ_{Vague} to indicate this risk preference for vague gambles:

$$\text{Model 2: } U_{\text{Vague}} = EV_{\text{Vague}} + \rho_{\text{Vague}} \times \text{VAR}_{\text{Vague_alternative}} \quad (\text{eq. 12})$$

Model 3 (eq. 13) assumes that valuations in vague gambles are calculated following the mean-variance analysis, but that risk preferences for vague gambles are the same as for risky gambles, $\rho = \rho_{\text{Vague}}$.

$$\text{Model 3: } U_{\text{Vague}} = EV_{\text{Vague}} + \rho \times \text{VAR}_{\text{Vague_alternative}} \quad (\text{eq. 13})$$

Model 4 (eq. 14) formalizes a model where individuals are not sensitive to outcome variability in vague gambles.

$$\text{Model 4: } U_{\text{Vague}} = EV_{\text{Vague}} \quad (\text{eq. 14})$$

Four variations of Model 1-4 that assume individuals have unbiased expectations ($\varepsilon = 0.5$) were also evaluated (labeled as Model $1_{\varepsilon=0.5}$, Model $2_{\varepsilon=0.5}$, Model $3_{\varepsilon=0.5}$, and Model $4_{\varepsilon=0.5}$ respectively).

Relationship between value weight and dispersion preference. Note that Model 1 compared with Models 2 or 3 leads to a hypothesis about the relationship between parameters ε (vagueness value weight) and α (dispersion preference). As described above, if outcome variability preferences were perfectly dependent on mean outcome value preferences, then dispersion preference $\alpha = \rho_{\text{Vague}} \times [\varepsilon \times (1 - \varepsilon)]$ (eq. 10-12; or $\alpha = \rho \times [\varepsilon \times (1 - \varepsilon)]$ for Model 3). Given that individuals are on average risk averse ($\rho < 0$), a positive correlation is expected between α and ε , particularly for optimistic individuals ($\varepsilon > 0.5$). On the other hand, if outcome variability preferences were perfectly independent from mean outcome value preferences, dispersion preference α should not be correlated with ε . We directly examined the impact of individuals' mean-variability computation-dependence on the correlation between the value weight and the dispersion preference (see **Fig. S5** for results). In particular, preference independence between mean outcome value and outcome variability was measured from individuals' log-likelihood difference between Model 1 and 2. As expected, participants who showed higher dependence (median split) showed significant and positive correlation between value weight (ε) and dispersion preference, whereas participants who showed higher independence did not show such a pattern.

Parameter estimation and model comparison. All 170 trials for each participant were used for model parameter estimation (also see **Fig. S5** showing that estimated parameters were consistent when only the gamble pairings with a certain gamble were used for estimation). The Softmax choice rule was used to analyze individual subjective values of gambles with individuals' choices:

$$P(\text{option 1}) = [1 + \exp(-\lambda \times (U_{\text{option 1}} - U_{\text{option 2}}))]^{-1} \quad (\text{eq. 15})$$

where λ , the inverse temperature, indicates sensitivity to the difference between gamble utilities. For the trials where both gambles had some type of uncertainty (risky vs ambiguity; risky vs vague), $P(\text{option 1})$ was the probability of choosing the ambiguous (or vague) option over the risky option. Otherwise, $P(\text{option 1})$ was the probability of choosing the uncertain over the certain option.

All of the participants' model parameters (λ , ρ , β , ε , and α) were estimated using random effects under a hierarchical Bayesian model structure (27, 28). For all parameters, the group-level distributions were assumed to be Gaussian with free hyperparameters of group-level mean (μ), SD (σ), and a standard normal distribution ($Normal(0, 1)$; eq. 16) following the non-centered parameterization method (eq. 17) (29). Particularly for inverse temperature λ , we applied an exponential transformation to constrain the parameter to positive numbers, and for the value weight ε , we used a logistic function as follows (eq. 18) to constrain the value between 0 and 1:

$$\varepsilon_Normal \sim Normal(0, 1) \quad (\text{eq. 16})$$

$$\varepsilon_BeforeTransform = \mu + \varepsilon_Normal \times \sigma \quad (\text{eq. 17})$$

$$\varepsilon = [1 + \exp(-\varepsilon_BeforeTransform)]^{-1} \quad (\text{eq. 18})$$

The group-level μ and σ (hyperparameters) for all the parameters were estimated using uninformative priors where $\mu \sim Normal(0, 10)$ and $\sigma \sim Cauchy(0, 2.5)$ with lower bound of zero. We used Markov chain Monte Carlo (MCMC) sampling with the No-U-Turn variant of Hamiltonian Monte Carlo technique implemented in the Stan (30) and its interface to R (29). Four chains were run where each chain was initiated at random values and had 5000 samples drawn, discarding the first 2000 samples for warm-up. We visually inspected the chain convergences and good mixing, and also confirmed all values of the potential scale reduction factor (Rhat) were less than 1.05 for all variables (31).

To assess which of the proposed subjective valuation models of vagueness best explains participants' behavioral choices, we computed the integrated Bayesian Information Criteria (iBIC); small iBIC values indicate a better fit. The fit for Model 1 (variability of outcomes defined as a dispersion) was superior compared with alternative models (**Fig. S1**).

Model recovery. To examine whether the parameters included in our suggested model (Model 1) can be identified from other parameters and capture individual differences if they exist, we conducted a model recovery analysis. First, we sampled 98 sets of parameters (λ , ρ , β , ε , and α for 98 simulated participants; 'true parameters' hereafter) using known mean and SD for the group-level distributions; mean and SD (hyperparameters) estimated for each parameter from the actual behavioral data were used, so that the simulation data follow a sensible parameter range: $\lambda \sim lognormal(-1.75, 0.44)$, $\rho \sim Normal(-0.0064, 0.011)$, $\beta \sim Normal(0.30, 0.40)$, $\varepsilon_BeforeTransform \sim Normal(0.23, 0.49)$, and $\alpha \sim Normal(-0.0017, 0.0010)$. Second, simulated sets of parameters and the actual 170 gambles (payoff and probability information) were used to generate 170 behavioral choices between pairs of gambles. Third, we used the same

parameter estimation procedures described above to re-estimate (recover) the parameters ('estimated parameter' hereafter) that best explain the simulated behavioral choices. Following the three steps, we confirmed that all true parameters were significantly correlated with their estimated parameters (λ : $r = 0.94$, $P = 2.06\text{e-}47$; ρ : $r = 0.98$, $P = 2.85\text{e-}74$; β : $r = 0.90$, $P = 5.43\text{e-}36$; ε : $r = 0.78$, $P = 1.66\text{e-}21$; and α : $r = 0.51$, $P = 8.67\text{e-}08$). These significant positive correlations indicate that the model parameters can be identified and capture individual differences if they exist.

Code availability. Code associated with this work is available from the corresponding author upon request.

Data availability. Data associated with this study are available from the corresponding author upon request.

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Author Contributions:

D.C., K.K., S.B.B., and P.H.C. designed the experiments. D.C. and K.K. analyzed data. All authors discussed the analyses and results. D.C., K.K., and S.B.B. drafted the initial manuscript. All authors revised and approved the final manuscript.

Competing financial interests:

The authors declare no competing financial interests.

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Figure Legends:

Figure 1:

Figure 1. Four types of uncertainty. (a) Participants made a series of choices between two gambles that have different types of uncertainty. The value and probability information in the certain and risky gambles is fully known to participants. Certain gambles guarantee the indicated prize. Risky gambles have high- and low- payoffs, each of which has a corresponding probability. As depicted here, the size of each ‘piece of pie’ indicates the chance of winning the indicated amount if the gamble is selected. Note that each pie piece is marked in different colors; the color scheme is retained throughout the task within subject, but counter balanced between subjects. Ambiguous and vague gambles have partial information. Vague gambles have a predetermined but unknown prize value; a decision maker is informed only that an outcome lies somewhere within the indicated range of values. In the example, the prize is at least 24¢ but no more than 71¢. Ambiguous gambles have fully known prizes, but probabilities are partially occluded. In the example, the chance of winning 25¢ is at least 37.5% ($50 - 12.5\%$) but no more than 62.5% ($50 + 12.5\%$). (b) To examine whether each type of uncertainty affected individuals’ decisions, we calculated the probability of choosing uncertain over certain options under different levels of risk (probability of the high-payoff = [25, 50, and 75%]), ambiguity ($A = [25, 50, \text{and } 75\%]$), and vagueness ($V_{\max} - V_{\min} = [8-22, 23-34, \text{and } 47-71]$; divided into terciles with comparable bin sizes). Participants’ choices varied significantly as a function of the levels of uncertainty (Risk: $F(2, 194) = 28.30$, $P = 1.64e-11$; Ambiguity: $F(2, 194) = 11.61$, $P = 1.72e-05$; Vagueness: $F(2, 194) = 17.12$, $P = 1.43e-07$), indicating individual differences in sensitivity to all three types of uncertainty. Error bars show s.e.m.

Figure 2:

Figure 2. Optimistic value weighting and dispersion aversion explain subjective valuation of vague value. The model used to value certain, risky and ambiguous gambles followed the literature on assigning value to uncertain gambles with mean-variance analysis: ρ indicates risk preference and β indicates ambiguity aversion. The last line represents the best model for vague gambles (see **Methods** and **Fig. S1** for model comparison) where ϵ indicates the weight between the lower- and upper- bounds of the range of vague value and α indicates individual preference for dispersion. As depicted, the subjective expectation of the ambiguous and vague gambles weight possible outcomes (through β) and vagueness (through ϵ), respectively. The effect of outcome variability is captured by risk preference ρ for all gambles other than vague gambles where it is replaced by dispersion preference α .

Figure 3:

Figure 3. Attributes in computing subjective mean and response to outcome variability are shared across subjective valuation of different types of uncertainty. **(a)** Estimated parameters showed that individuals are risk averse ($\rho < 0$), ambiguity averse ($\beta > 0$), optimistic ($\epsilon > 0.5$), and dispersion averse ($\alpha < 0$). **(b)** Individuals' preferences for different types of uncertainty were estimated using the best explaining model (see **Methods** for model details), and correlations between estimated parameters were calculated to examine the amount of variability the attributes share across different types of uncertainty. **(bi)** Vagueness value weight and dispersion preference were positively correlated, suggesting that individuals who are more optimistic about vague outcomes are more dispersion tolerant. **(bii)** Ambiguity tolerance ($-\beta$) was positively correlated with value weight, which means that participants who were ambiguity tolerant were more optimistic about vague outcomes. **(biii)** Risk preferences were positively correlated with dispersion preferences, indicating that participants were consistent in disliking outcome variability across these gamble types. Each point represents an individual participant, red- and grey lines indicate regression results between indicated parameters.

Supporting Information

for

Evidence for preference consistency across risky, ambiguous, and vague gambles

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Tables:

Table S1. Demographics and self-reported questionnaire scores

	All participants ^a
Age	35.1 ± 10.1
Sex	60 male; 37 female
Education	Some High School (N = 1) Finished High School (N = 7) Some College (N = 30) College Degree (N = 43) Master's Degree (N = 12) Graduate Degree (N = 3) Other (N = 1)
Optimism (LOT-R)	16.6 ± 3.7
Impulsivity (BIS-11)	56.6 ± 10.2
General Risk	4.9 ± 2.2

Revised Life Orientation Test, LOT-R (1); Barratt Impulsiveness Scale, BIS-11 (2); 11-likert scale response to “How willing are you to take risks, in general?”, General Risk (3); ^aOne participant did not complete the questionnaires, so a total of 97 participants’ demographics and self-reported questionnaire scores are reported. Data are represented as mean ± SD.

Table S2. Correlations between model parameters and self-report measures of individuals' attitude toward uncertainty.

	Pearson's correlation ^a
Optimism (LOT-R)	
Inverse temperature	$r = 0.0045, P = 0.97$
Risk preference	$r = -0.019, P = 0.86$
Ambiguity aversion	$r = 0.081, P = 0.43$
Value weight	$r = -0.045, P = 0.66$
Dispersion preference	$r = -0.078, P = 0.45$
Impulsivity (BIS-11)	
Inverse temperature	$r = 0.12, P = 0.25$
Risk preference	$r = 0.14, P = 0.16$
Ambiguity aversion	$r = -0.054, P = 0.60$
Value weight	$r = 0.090, P = 0.38$
Dispersion preference	$r = 0.12, P = 0.23$
General Risk	
Inverse temperature	$r = 0.14, P = 0.18$
Risk preference	$r = 0.21, P = 0.041$
Ambiguity aversion	$r = 0.15, P = 0.15$
Value weight	$r = -0.10, P = 0.31$
Dispersion preference	$r = 0.18, P = 0.080$

^aOne participant did not complete the questionnaires, so a total of 97 participants' self-reported questionnaire scores were used in analyses. Participants were also asked to complete a survey for general demographic information, along with three self-report measures to assess their attitudes toward uncertainty. First, to examine whether individuals' optimistic valuation is associated with individuals' optimism and pessimism biases, we used the Revised Life Orientation Test (LOT-R) (1). Previously, it has been shown that the self-reported optimism measure is positively correlated with individuals' tendency to update the information that is better than expected (optimistic update) compare to the information that is worse than expected (4). Unlike this bias of optimistic update, individuals' optimistic valuation (vagueness value weight) was not associated with the self-reported optimism measure. This result suggests that optimistic valuation may have a distinct origin (e.g., enforcing exploration toward options that have larger uncertainty (17)) from optimistic information updating. Second, Barratt Impulsiveness Scale (BIS-11) (2) was administered to assess the association between individuals' impulsivity and ambiguity aversion; consistent with previous report (5), model parameters for risk preference and ambiguity aversion were not correlated with self-reported impulsivity measure. Third, 11-likert scale response to "How willing are you to take risks, in general?" (General Risk)—a simple self-report measure that highly predicts real-world risky behaviors (3)—was measured and correlated with estimated risk preference parameters. As expected, individuals' risk preference was positively correlated with the self-reported preference for risky choices.

Figure Legends:

Figure S1:

Figure S1. Dispersion model best explains participants' subjective valuation of vagueness. We hypothesized that (Model 1) subjective valuation of vague values involved a vagueness value weight (ϵ) between the lower- and upper- bounds of the range of vague value and dispersion preference (α). The value weight measures optimism or pessimism bias in computing the subjective mean, while the dispersion preference captures the disutility of outcome variability. Model comparison using integrated Bayesian Information Criteria (iBIC) showed that Model 1 (see eq. 10 in **Methods**) best explains participants' choices compared against 3 models (and 4 model variants where $\epsilon = 0.5$) that have alternative response to outcome variability. Both Models 2 and 3 assume that outcome variability is assessed using variance instead of dispersion. Model 2 (eq. 12) includes a risk preference ρ_{vague} estimated separately for vague gambles, while Model 3 (eq. 13) includes the risk preference ρ estimated with Risky gamble data. Model 4 (eq. 14) assumes that participants ignore outcome variability of vague gambles. The four model variants introduced an assumption that expectations were unbiased ($\epsilon = 0.5$) (see **Methods** for model specifications).

Figure S2:

Figure S2. Attributes in computing subjective mean and response to outcome variability are shared across subjective valuation of different types of uncertainty. (a) Estimated parameters showed that individuals have a moderate level of sensitivity to utility differences between the options. (b) Individuals' preferences for different types of uncertainty were estimated using the best explaining model (see **Methods** for model details), and correlations between estimated parameters were calculated to examine the amount of variability the attributes share across different types of uncertainty. Inverse temperature was positively correlated with (bi) risk preference and (bii) dispersion preference, indicating that participants who were less value sensitive dislike outcome variability for both risky and vague gambles. Each point represents an individual participant, red- and grey lines indicate regression results between indicated parameters.

Figure S3:

<p>1. If you choose this lottery, you could have a chance to win:</p> <ul style="list-style-type: none">☐ 0¢☐ 25¢☐ 40¢☐ 59¢☐ 72¢
<p>2. If you choose this lottery, your chance that you will win 48¢ could be approximately:</p> <ul style="list-style-type: none">☐ 0%☐ 25%☐ 50%☐ 75%☐ 100%	<p>3. If you choose this lottery, your chance that you will win 0¢ could be approximately:</p> <ul style="list-style-type: none">☐ 0%☐ 25%☐ 50%☐ 75%☐ 100%
<p>4. If you choose this lottery, the chance that you will win 90¢ could be approximately (check ALL that apply):</p> <ul style="list-style-type: none">☐ 0%☐ 25%☐ 50%☐ 75%☐ 100%	<p>5. If you choose this lottery, the chance that you will win 0¢ could be approximately (check ALL that apply):</p> <ul style="list-style-type: none">☐ 0%☐ 25%☐ 50%☐ 75%☐ 100%
<p>6. If you choose this lottery, you could have a 100% chance to win (check ALL that apply):</p> <ul style="list-style-type: none">☐ 0¢☐ 24¢☐ 45¢☐ 71¢☐ 90¢	

Figure S3. Quiz. Only participants who correctly answered all six questions continued in the choice task.

Figure. S4:

Figure S4. Using a standard power utility function gives a consistent result in ambiguity aversion parameter estimation. Adopting from a standard maxmin expected utility theory (6), ambiguity aversion is implemented as a biased perception of probability information. Here, we show that parameter estimation of ambiguity aversion is consistent independent from two different forms of utility function (defining risk and risk preference): mean-variance analysis (7) and a standard power utility function following expected utility theory (8).

Figure S5:

Figure S5. Relationship between value weight and dispersion preference. In vagueness gambles, both mean outcome value and dispersion calculation are affected by the levels of vagueness. As noted in the Methods section, Model 1 specifies the first and second moments of gamble outcomes are computed independently, whereas Model 2 and 3 specify that the dispersion (the second moment) is computed dependent on the mean outcome value. Although a formal model-comparison showed that Model 1 better explains participants' behavioral choices, we directly examined the impact of this computation-independence on the correlation between vagueness value weight and dispersion preference. **(a)** Particularly, model-fit (log-likelihood) difference between Models 1 and 2 were used as a measure of computation-independence (positive value indicates higher independence). **(b)** As expected, participants who showed higher dependence (median split) showed significant and positive correlation between value weight (ϵ) and dispersion preference, whereas participants who showed higher independence did not show such pattern.

Figure S6:

Figure S6. Estimated parameters were consistent when only the gamble pairings with a certain gamble were used for estimation (88 trials per participant). When a full five gamble pairings were used (x-axis), individuals' dispersion aversion (right most) were systematically over-estimated than the estimation results from a partial data (y-axis). Future study may investigate a potential interaction effect in subjective valuation of a vague gamble and its alternative option (risky (x-axis) or certain (y-axis) gamble). Each point represents an individual participant.

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