Implicit and Explicit Spatial-Numerical Representations Diverge in Number-Form Synesthetes

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Abstract

In number-form (NF) synesthesia—a condition in which people report vivid, automatic and consistent mental layouts for numerical sequences—numbers and space are closely linked. These explicit associations are similar to the implicit associations demonstrated by the Spatial-Numerical Association of Response Codes (SNARC) effect. Thus, NF synesthesia offers a unique opportunity to investigate spatial-numerical associations. We tested implicit and explicit representations in NF synesthetes using a multiple case-study design. Over two sessions, synesthetes participated in a semi-structured interview focusing on the nature of their associations, as well as SNARC and number line estimation tasks. Contrary to our hypotheses, only one synesthete demonstrated SNARC effects congruent with her reported form, whereas two others exhibited SNARC effects that were the opposite of their explicit NFs. While this inconsistency between implicit and explicit representations may indicate separate underlying cognitive mechanisms, factors such as task-specific constraints and strategic variability must also be considered.

Keywords: spatial-numerical associations; number-form synesthesia; SNARC; sequence-space; implicit processing

1. Introduction

1.1 Numbers and Space in the General Population

From numbered city blocks to measuring cups for cooking, numbers and space are intertwined in many aspects of our daily lives. In the numerical cognition literature, these associations are generally referred to as spatial-numerical associations (SNAs). While the majority of empirical research on the integration of numerical and spatial thinking has been conducted in the last several decades, observational studies of SNAs in fact date back to the late 19th century. Galton (1880, 1881) was the first to describe cases of people with “the power of visualising numerals” (p. 252, 1880), or the ability to use internal spatial layouts to represent and manipulate numbers. He describes the ability as consisting of “the sudden and automatic appearance of a vivid and invariable ‘Form’ in the mental field of view, whenever a numeral is thought of, and in which each numeral has its own definite place” (p.88, 1881). While these descriptions refer to somewhat atypical spatial-numerical representations, this is the first known documentation of spatial associations for numbers and shaped much of the later research on SNAs.

The concept of an internal, spatial conception of number was further articulated nearly a century later by Restle (1970), who formally postulated the mental number line (MNL). Building on findings from Moyer and Landauer (1967), Restle found that when comparing two quantities—a single number and the sum of two numbers—participants’ error rates and response times varied as a function of the distance between the two quantities and the magnitude of the numbers being compared. Specifically, greater distances and numerical magnitudes yielded faster and more accurate responses, while the opposite was true for smaller differences and numerical magnitudes. Since this pattern of responses is similar to what one might expect when using a physical number line to make numerical judgements, Restle hypothesized that people were using a mental analog of the physical number line when making numerical judgements and operations (see also Dehaene, Dupoux, & Mehler, 1990).

The first demonstration of a clearly spatial link between numbers and sides of space came another two decades later, when Dehaene, Bossini, & Giraux (1993) simply asked people to determine whether a given number was even or odd. If participants were in fact using a mental analog of a number line to make these judgements, this should theoretically be reflected in their response times. Indeed, across a range of experimental manipulations, participants were consistently faster to respond to small numbers on the left side of space and large numbers on the right, even though the parity judgement was irrelevant to numerical magnitude. Referred to as the Spatial Numerical Association of Response Codes (SNARC) effect, this finding has been taken as evidence of an internal, linear representation of number that has smaller magnitudes extending to the left and larger magnitudes extending to the right (for reviews: Hubbard, Piazza, Pinel, & Dehaene, 2005; Wood, Willmes, Nuerk, & Fischer, 2008). While the directionality of this effect is widely believed to be culturally mediated, the link between numerical magnitudes and sides of space is evident across cultures and contexts (for reviews: Göbel, Shaki, & Fischer, 2011; Toomarian & Hubbard, 2018a).

As SNAs have become a topic of increasing intrigue and debate, several paradigms have been employed to elucidate the nature of the putative MNL. The classic SNARC effect is one commonly used measure, elicited by either a parity (even/odd) judgement or magnitude judgement. Evidence of a SNARC is derived from the slope of the regression line when regressing the difference in right-left reactions times (dRT) on numerical magnitude (Fias, Brysbaert, Geypens, & D’Ydewalle, 1996; Lorch & Myers, 1990). Negative regression slopes indicate an internal representation of numbers that is spatially oriented from left-to-right.

Another common paradigm is the number line estimation (NLE) task, in which participants are typically asked to place a given number on a number line with labeled endpoints (i.e. a bounded number line). Performance on this task is typically measured by the acuity of responses for each number. If the MNL is indeed a common cognitive representation that influences and aids in numerical judgements, then there should be correspondence between internal representations of quantity (the MNL) and tasks such as NLE, which utilizes a structurally-similar external representation. Thus, this task can theoretically be used in conjunction with the SNARC to test for consistency between internal and external spatial-numerical representations.

1.2 Numbers and Space in Synesthetes

An additional approach to better understand spatial-numerical associations is to study those who consciously experience these spatial-numerical associations, such as those initially described by Galton. For the majority of people, spatial-numerical associations are a largely *implicit* phenomenon, meaning they are not consciously experienced. The SNARC effect highlights a relationship that emerges independent of task instructions, and without conscious effort. Indeed, this implicit association can be discretely modulated and primed (e.g. Bächtold, Baumüller, & Brugger, 1998; Hung, Hung, Tzeng, & Wu, 2008). While tasks such as number-line estimation essentially require integration of spatial and numerical abilities, participants may not be consciously aware of this integration when completing the task. However, for roughly 10% of the population (Sagiv, Simner, Collins, Butterworth, & Ward, 2006; Seron, Pesenti, Noël, Deloche, & Cornet, 1992), associations between numbers and space are never far from their minds. When Galton (1880, 1881) reported descriptions of people's vivid and distinct mental forms for numbers and other ordered sequences, he was not simply describing random representational idiosyncrasies; rather, he was likely describing a variant of what is now recognized as a form of synesthesia (Cytowic, 1989; Simner & Hubbard, 2013).

Sequence-space synesthesia is a condition in which people have *explicit* associations between ordinal sequences (e.g. days, months, hours, letters, numbers) and a specific location in three-dimensional space (Eagleman, 2009; Jonas & Jarick, 2013). In our particular variant of interest—number form (NF) synesthesia—people report vivid, automatic and consistent mental layouts for numerical sequences (Galton, 1880, 1881; Sagiv et al., 2006; Seron et al., 1992). These synesthetic associations are not restricted to horizontal, linear forms but rather, can take any number of shapes and characteristics (Galton, 1880; Jonas & Jarick, 2013; Seron et al., 1992). Additionally, these mental representations are unique and differ from synesthete to synesthete, variability that may be attributable to differences in developmental visuospatial experiences (Price & Pearson, 2013). While the exact nature of the form differs between synesthetes, number forms have been found to be consistent within individuals over time (Piazza, Pinel, & Dehaene, 2006; Seron et al., 1992).

NF synesthesia offers a unique lens through which we can investigate SNAs, and to better understand the relation between implicit and explicit spatial-numerical associations. NF synesthetes are typically consciously aware of their number-space associations, as demonstrated by their ability to describe and draw their forms in detail. This provides an ideal set-up for comparison between implicit and explicit number representations. Some researchers have hypothesized that explicit NFs and the implicit SNARC may rely on shared neural mechanisms of co-activation, specifically in regions of parietal cortex underlying spatial and ordinal/numerical processing (Hubbard, Piazza, Pinel, & Dehaene, 2005; Tang, Ward, &

Butterworth, 2008, but see Eagleman, 2009)¹. A unitary mechanism for implicit and explicit spatial-numerical processing would predict correspondence between behavioral measurements of internal and external representations.

1.3 Correspondence Between Representations

Previous research has primarily focused on assessing the correspondence between explicit synesthetic reports and implicit SNAs. Observing SNARC effects that go in non-canonical directions (e.g., right-to-left or top-to-bottom) for synesthetes whose NFs go in non-canonical directions has been taken as evidence corroborating synesthetes' subjective reports. In one such study, Jarick et al. (2009) tested two synesthetes with vertical (bottom-to-top) forms for the numbers 1-10, and horizontal forms for larger numbers (10-20 left to right; 21-40, 41-60, etc. right to left). They employed two different tasks both aimed at measuring implicit spatial-numerical associations- the classic parity-judgement SNARC task (Dehaene et al., 1993) and an implicit cuing task (Fischer, Castel, Dodd, & Pratt, 2003). Across both tasks, Jarick et al. found that synesthetes with non-canonical number forms exhibited SNARC effects only when responses were oriented congruently with their forms, but not when responses were oriented incongruently. The authors argue that this congruency is effectively an empirical verification of the synesthetes' number forms and that explicit synesthetic forms likely underlie implicit SNARC effects.

These findings of compatibility have also been found using the Size Congruity Effect (SiCE) in synesthetes (Gertner, Henik, Reznik, & Cohen Kadosh, 2013), an effect which tests automatic processing of number (Henik & Tzelgov, 1982). When presented with pairs of numbers that differ in physical and numerical size, people are faster when these two properties are congruent (e.g., the larger value is also physically larger). Gertner et al. (2013) found that synesthetes exhibited a SiCE only when their internal number forms were congruent with the task orientation. The same has been shown for distance effects in NF synesthetes (Gertner, Henik, & Cohen Kadosh, 2009).

Prior to these studies demonstrating general consistency between implicit and explicit spatial-numerical representations in NF synesthetes, Piazza, Pinel and Dehaene (2006) reported an investigation of the SNARC in a synesthete ("SW") with a curvilinear, right-to-left number form. While SW performed better on number comparison tasks that were congruent with his highly irregular, non-canonical number form than on those that were incongruent, he lacked a SNARC effect in parity judgement tasks, both when stimuli were presented centrally on the screen and when presented peripherally. If synesthetic number forms can be considered explicit mental number lines, one might have predicted that SW's right-to-left association would have yielded a "reverse" SNARC effect, which would indicate faster responses to smaller numbers on the right and larger numbers on the left. However, SW effectively had no evidence of an association one way or the other, as his SNARC slope did not differ significantly from zero.

In another case study of an NF synesthete, Hubbard, Piazza, Pinel & Dehaene (2009) tested "DG," who had spatial associations for 58 different ordinal sequences, including integers, on a variety of numerical tasks. His non-canonical number form—curved in a C-shape from bottom-to-top for 1-10, then horizontal—led to several task-specific predictions about his

¹ Eagleman proposed that number forms are reified, such that they are neurally encoded as graspable objects in ventral visual stream. Although intriguing, this view is inconsistent with common synesthetic phenomenology in which synesthetes report that their number forms wrap around their bodies or form landscapes that they can traverse (e.g. Hubbard, Ranzini, et al., 2009; Smilek, Callejas, Dixon, & Merikle, 2007).

performance. The overall prediction was consistency between his form (bottom-to-top/left-to-right) and task performance. While his explicit reports of non-canonical SNAs were largely congruent with the data from implicit tests, such as a numerical cued-detection task, there were some inconsistencies. For instance, DG did not have a significant horizontal or vertical SNARC effect, despite the prediction that he would have a vertical SNARC. This was the case for both parity and magnitude judgements.

It is difficult to generalize the findings of this and other single-case studies of NF synesthetes to all NF synesthetes, particularly given the inconsistent results. Indeed, in the intervening decade or so since these studies were conducted, our understanding of the SNARC effect and SNAs in general has become much more nuanced. We now know that the SNARC effect is known to vary widely between individuals in the general population. Recent estimates place the percentage of people who have significantly negative slopes at around 70% (Cipora & Wood, 2017, supplementary material), with a notable portion of participants having null or even positive SNARC slopes. For this reason, many recent studies of the SNARC in non-synesthetes have transitioned away from examining group effects and toward investigating individual differences.

This revelation of high interindividual variability in the SNARC draws into question the validity of comparing the SNARC effects of single synesthetes to a group of non-synesthetic controls. If the non-synesthetic control group is inhomogeneous, then using it to compare against one synesthete is inherently problematic. Specifically, we cannot confidently attribute any potential differences to having synesthesia and not just characteristic variability. One approach to addressing this issue is to test a group of similar NF synesthetes against a group of similar controls. Jonas et al. (2014) used this approach by testing the parity SNARC in a group of NF synesthetes whose forms all extended in the canonical left-to-right direction, as well as a group of non-synesthetic controls. While all participants were reported to be native English speakers, it is unclear whether they were matched for age, gender, or experience. In a series of two experiments, they found that the SNARC for synesthetes did not differ from non-synesthetic controls, contrary to expectation. Similar behavior for the (parity) SNARC paradigm but different explicit/subjective experiences with spatial sequences suggests a dissociation between the cognitive representations supporting the SNARC and the cognitive phenomena that give rise to NF synesthesia. While this approach is an improvement over previous single-case/control designs, we suggest that unless control groups are reported to be precisely matched across multiple dimensions known to influence SNAs (e.g. age, education level, languages spoken)—which would be practically very difficult—the comparison to a standard control group in the case of SNAs is not theoretically sound.

1.4 Present Study

In the present study, we aimed to resolve the discrepancies evident in earlier studies testing the correspondence between implicit and explicit spatial-numerical representations in NF synesthetes. While previous research has paved the way for investigating SNAs in NF synesthetes, there are important limitations and opportunities for deeper understanding which we address in the current study. First, previous studies have primarily highlighted instances of congruence between implicit and explicit measures of SNAs in synesthetes, effectively ignoring occasions when that correspondence is imperfect. Additionally, the majority of previous investigations of the SNARC in NF synesthetes have been single case studies, in which one synesthete has been studied thoroughly but has not been directly compared to other synesthetes undergoing the same protocol. Multiple-case study designs have the advantage of allowing in-depth investigations, but also permit developing a richer understanding of the similarities and

differences across NF synesthetes, and informing the degree to which we may generalize our findings to SNAs more broadly (see Hubbard & Ramachandran, 2005).

In this study, we employed a multiple-case study design in a group of synesthetes with both canonical and noncanonical NFs. This approach simultaneously offers the opportunity for the in-depth investigations afforded by single-case design while also allowing generalizations to be made by comparing across synesthetes. Additionally, we used two magnitude-relevant tasks rather than just one to allow us to more directly and thoroughly explore the extent of correspondence between implicit and explicit SNAs.

Lastly, little is known about how NF synesthetes represent numbers beyond integers, specifically rational numbers. Understanding the representation of fractions on mental number lines is emerging as a critical topic in numerical cognition given the importance of learning fractions for later higher-order math skills (Booth & Newton, 2012; Siegler et al., 2012) (Siegler et al.). Recent evidence suggests that fractions are represented on the non-synesthetic number line (e.g. Bonato, Fabbri, Umiltà, & Zorzi, 2007; Toomarian & Hubbard, 2018b), but the quality of and access to these representations is influenced by stimulus characteristics and task demands. Whether this extends to NF synesthetes is an open empirical question. We predicted that synesthetic number forms would be weaker for fractions than for integers, as they are not as commonly encountered in daily life. However, the fact that many number types are represented in synesthetic NFs, as well as the prevalence of “zooming” for many synesthetes, suggested that fractions might indeed be represented on NFs. Testing this question in a sample of NF synesthetes will help to develop tighter links between numerical cognition and synesthesia research. To investigate this, we used both fraction and whole number stimuli in our comparison tasks. Therefore, the current study builds on previous research investigating SNAs in NF synesthetes by addressing the aforementioned limitations and extending the reach of the investigation.

2. General Methods and Procedures

All sessions took place in a quiet testing room. For computer tasks, participants were seated in a comfortable chair at a distance of approximately 68 cm from the computer monitor. Experiments were programmed with E-prime 2.0.10 (Psychology Software Tools, Sharpsburg, PA) on a Dell Optiplex 390 Desktop PC (3.1 GHz, 4 GB RAM) running Windows 7.0 64-bit operating system. Stimuli were presented on a Dell UltraSharp U2212H 21.5" flat-screen monitor at a resolution of 1024×768 and a refresh rate of 60 Hz. Left button presses corresponded to the ‘D’ key, and right button presses to the ‘K’ key on a standard keyboard (distance = 8.5 cm).

Recruitment of all NF synesthetes was conducted as part of a separate, ongoing recruitment process for synesthetes of all subtypes. As part of that process, participants were recruited via announcements made in large undergraduate classes, mass emails, and paper flyers posted around the local community. All communications briefly described various synesthetic experiences and individuals who believe they may qualify are invited to the lab for an initial semi-structured interview session. All participants were screened to confirm their synesthetic experiences using a broad synesthesia questionnaire designed to validate their synesthetic experience(s) and detail the nature of their associations (Ramachandran & Hubbard, 2001). This questionnaire asks about the phenomenology of any and all synesthetic experiences, the perceptual reality of these experiences, and any connections between these experiences and the physical environment. During this initial screening, all potential subtypes were noted, including sequence-space/number-form synesthesia. All verified synesthetes in this database who indicated

that they experienced synesthetic number-forms were re-contacted for the current study, yielding a sample of eight sequence-space synesthetes. All four participants were female; the age of each participant is listed in Table 1. Participants were compensated \$12/hour in cash for participation. The study protocol was approved by the university institutional review board (#2014-0691).

2.1 Session 1 Methods

In the first session, participants took part in an additional semi-structured interview that specifically focused on the nature of their sequence-space associations. As a part of the “Sequence-Space/Number-Form Questionnaire” participants were asked to provide details about basic phenomenology, the nature of their visualizations, the development of these associations (e.g. age of onset), distinct representations for whole numbers and fractions, and basic demographic and background information (Appendix A). All participants provided hand-drawn depictions of their representations for the number ranges 1-10, 1-100, and fractions between 0-1, in addition to their verbal descriptions. These self-reported descriptions aided in our characterization of each participant’s number forms, as drawing a number-form often required transforming multidimensional representations into two-dimensional space. For all tasks, participants were instructed to respond to the best of their ability.

Participants also completed an unconstrained number placement task, in which participants were shown a number at the top of an otherwise blank, black screen, and instructed to click on the position in space that best corresponded to the location of that number on their number-form. Participants first responded to a subset of 20 positive integers (range: 1-40, modeled after Hubbard et al., 2009), followed by the set of 27 irreducible, single-digit fractions between 0 and 1 (modeled after Toomarian & Hubbard, 2018b; see Appendix B). For both stimulus sets, the largest and smallest numbers were presented first and second, respectively, to provide cognitive landmarks/reference points. All other numbers presented in a random order. Since the stimuli were not presented in order, this task served as a basic validation and quantification of each synesthete’s implicit number form.

2.2 Session 2 Methods

All eight participants who took part in the first session were invited to return for an additional session. Of those eight, four participants returned—SY1, SY2, SY4, and SY8. The participants from the first session who did not return either did not respond to our inquiries or had moved away from the study location. The second session lasted approximately one hour and was comprised of two types of tasks: magnitude comparison and number-line estimation.

2.2.1 Magnitude comparison

Participants completed two magnitude comparison tasks, from which we extracted individual SNARC effects. In both tasks, participants were instructed to compare the magnitude of a centrally presented number (the target) to a reference number provided at the start of the task. In each trial, participants saw a central fixation cross (600 ms), followed by a blank screen (1000 ms) and the target number (3000 ms or until a response was detected). In the first task, participants compared whole numbers (1-9) to the reference 5. There was a total of 416 trials, with the eight unique target numbers randomly presented 26 times in each of two blocks. In one block, smaller numbers corresponded to a left button press and larger number to a right button press, while the other block had this correspondence reversed (i.e. smaller/right, larger/left). Ten practice trials preceded each block, which included visual feedback.

After completing the task with whole numbers, participants compared single-digit, irreducible fractions to the reference $\frac{1}{2}$, in a task exactly replicating the fractions comparison

task used in Toomarian and Hubbard (Toomarian & Hubbard, 2018b). Each of the 26 fraction targets (those listed in Appendix B, except $\frac{1}{2}$) appeared eight times, yielding 416 total trials, with response side counterbalanced across two blocks. A total of 10 practice trials with visual feedback preceded each block. Participants did not receive feedback on test trials. All participants completed the whole number comparison task first, followed by the fraction comparison task. For both tasks, two synesthetes (SY1 and SY4) first responded to small numbers on the left and large on the right, whereas the other two synesthetes (SY2 and SY8) first responded to small numbers on the right and large on the left. All participants switched directions for the second block of the task. Magnitude relevant tasks were used rather than parity judgement to be able to draw conclusions across both the whole number and fractions SNARC effects, as there is no equivalent of parity for fractions.

We predicted that synesthetes with left-to-right (canonical) number forms would exhibit a classic SNARC effect (faster for small numbers on the left and large numbers on the right) for whole numbers. The classic SNARC effect reflects the directional organization (left-to-right) that would be congruent with their reported forms. We predicted that synesthetes who reported a non-canonical directionality (right-to-left) of their number forms would exhibit a reverse SNARC effect. Additionally, because participants self-reported very robust explicit number forms for whole numbers, particularly for positive integers 1-10, but weaker associations for fractions, we predicted a correspondingly weaker trend for implicit fractions representations. However, we hypothesized that the directionality of the SNAs for fractions would still correspond to the reported, explicit form.

2.2.2 Number line estimation

In the number line estimation task, participants were presented with a number at the top of the screen and asked to use a computer mouse to click on the appropriate position for that number on the bounded number line below. This number-to-position task included two separate blocks, one consisting of proper fraction stimuli to be placed on a 0-1 number line and one consisting of improper fractions to be placed on a 0-5 number line. Critically, in order to investigate whether performance differed when the bounded number lines were congruent with participants' reported forms, each number line task was administered in both the canonical direction (0 on left end, 5 on right end) and the non-canonical (5 on left end, 0 on right end) direction. Participants completed the NLE task in the order corresponding to their canonicity, such that they all first completed the task in the orientation that was congruent with their reported form, followed by the orientation that was incongruent with their form. This resulted in SY1, SY2, and SY4 completing the left-to-right orientation first, while SY8 did the right-to-left orientation first. Responses yield a measure of percent absolute error (PAE) for each participant (Booth & Siegler, 2006, 2008). Smaller PAE values indicate higher acuity. We predicted that PAE would be smallest, indicating greater acuity, on the version of the task congruent with the direction of each synesthete's number form. PAE for the 0-1 task may be slightly greater than the 0-5 version, since most of the participants reported weaker associations for numbers in that range.

3. Results

3.1 Session 1 Results

Key insights from the Sequence-Space/Number-Form Questionnaire are detailed in Table 1. Demographic information, questionnaire responses, and descriptions of synesthetic number-forms for the four participants who did not return for session 2 are provided in the supplementary

materials. All synesthetes reported having a spatial form for calendars in some way, though the units of the calendar were not consistent (e.g. not all had forms for weeks). Three synesthetes reported the ability to zoom in/out of their spatial representations, either in their mind's eye or projected in space. Two synesthetes reported being aware of at least one immediate family member who also had synesthesia of some kind. Conjuring mental images was generally very easy for our sample, though reports of navigation and memory ability varied widely (e.g. for places or dates). All synesthetes recall having these associations since early childhood, with nearly all citing that these associations have existed for as long as they can remember. All synesthetes reported that their forms were elicited whenever presented with or thinking about numbers (as well as other stimuli for which they have forms), with the forms often fading shortly after removal of the stimulus or shifting attention.

Descriptions of each synesthete's number form for fractions and whole numbers are listed in Table 2, based on the synesthetes' drawings and self-reported descriptions. Of the four synesthetes, two reported ascending left-to-right NFs, one reported a slightly descending left-to-right NF, and one reported a right-to-left NF. See supplemental materials for reports of other descriptions of NFs, such as a vertical bottom-to-top NF. These forms have been classified in relation to the canonical, left-to-right oriented number line. For the purposes of this study, in which we are focusing on horizontal orientation, synesthetes who reported vertical or right-to-left number orientations are considered to have a non-canonical organization of their number form.

Responses from the unconstrained number placement task were plotted to obtain a two-dimensional visualization of each participant's number form. The output from each participants' number placement task was compared with the drawings provided in the interview portion of the session. Figure 1 shows this comparison for two participants (see Supplementary Material for all plots). As predicted, these computer mouse-placement tasks largely validated the written/drawn reports of synesthetes' forms, though some multidimensional aspects of the reported forms were diminished (i.e. 3D forms were reduced to 2D). The resulting form effectively served as a qualitative, rapid test-retest reliability of each synesthetes' number form.

Table 1. Descriptions of select questionnaire responses for each participant. “Age”: age at test; “Zoom”: reported ability to zoom in or out on reported number form; “Family”: immediate family members who are known to experience synesthesia of any type; “Mental Images”: self-reported ease (scale: 1-9) of constructing mental images.

Synesthete	Age	Onset age	Other Forms	Zoom	Family	Mental Images	Other Notes
SY1	20	Elementary school/unknown	Calendar (months, days, years); Temperature	Yes	No	Easy (2)	Finds math “relaxing” and enjoyable; very good memory for places/dates/time; spatial form for years begins at 2000; no fraction or decimal form, but has form for negative numbers
SY2	19	Unknown/all ways	Grapheme-color; Calendar; Temperature Alphabet;	Yes	Yes- father	Easy (1)	Some numbers “weighted”/bolded/“special”; fractions “nicer” than decimals; prime numbers “spiky”
SY4	18	Unknown/all ways	Calendar (months, days, years)	No	Yes- mother, brother	Easy (1)	Finds math difficult, including arithmetic; thinks NF hinders math ability; Year 2000 has same spatial position as 0 on NF
SY8	57	4-5 yrs	Calendar (years, weeks, days); Time; Alphabet	Yes	No	Difficult (7.5)	Extensive mathematical training; reliant on finger counting in early years; calendar is a ribbon; bad at facial recognition

Table 2. Descriptions of synesthetic number forms.

Synesthete	Fraction Form	Whole Number Form	Horizontal Canonicity
SY1	Horizontal left to right	Horizontal, ascending left to right, stacked by decade	Canonical
SY2	Horizontal left to right	Horizontal, ascending left to right, stacked by decade	Canonical
SY4	Horizontal left to right	Horizontal, slightly descending left to right	Canonical
SY8	Horizontal right to left	Horizontal, ascending right to left, stacked by decade	Non-canonical

Note. Descriptions of synesthetic number forms for each participant. Canonicity refers to left-to-right horizontal orientation.

Figure 1. Samples from two synesthetes (top: SY 3, bottom: SY 5) of the correspondence between drawn number forms (left) and corresponding output from the unconstrained number placement task (right). Data plots for all other synesthetes available as supplementary material.

3.2 Session 2 Results

3.2.1 Magnitude comparisons results

Practice trials were excluded prior to analysis, as well as trials for responses that occurred less than 300 ms or greater than 2000 ms after stimulus onset, consistent with previous analyses of this task. Across all participants, this resulted in 0.5% of trials being excluded for the fractions task and 0.4% of trials being excluded for the whole number task. Overall accuracy was high for comparisons of both fractions (mean=0.947, SD=0.223) and whole numbers (mean=0.976, SD=0.152). Overall reaction times were calculated only on correct trials (fractions: mean=664.19, SD=224.84; whole numbers: mean=503.14, SD=137.57). Accuracies and reaction times for each synesthete are listed in Table 3.

Table 3. Results of individual SNARC effect analyses.

<i>Synesthete</i>	<u>Accuracy</u>		<u>RT</u>		<i>B</i>	<i>SE B</i>	<i>Int.</i>	<i>t</i>	<i>p</i>	<i>R</i> ²
	<i>Mean</i>	<i>SD</i>	<i>Mean</i>	<i>SD</i>						
<i>Whole SNARC</i>										
<i>SY1</i>	1.00	--	459.45	77.52	15.66	3.00	-80.23	5.23	0.002*	0.81
<i>SY2</i>	0.96	0.20	508.26	124.51	7.77	4.37	-59.65	1.78	0.13	0.34
<i>SY4</i>	0.97	0.16	505.00	150.29	-14.23	5.74	91.04	-2.48	0.047*	0.51
<i>SY8</i>	0.98	0.15	538.91	166.29	-25.18	6.15	90.73	-4.10	0.006*	0.74
<i>Fraction SNARC</i>										
<i>SY1</i>	0.99	0.10	560.01	111.01	19.73	34.60	-20.00	0.57	0.57	0.01
<i>SY2</i>	0.93	0.26	659.51	207.16	-182.77	105.32	1.83	-1.74	0.095	0.11
<i>SY4</i>	0.92	0.26	696.92	275.07	121.63	86.01	2.19	1.41	0.17	0.08
<i>SY8</i>	0.94	0.23	741.05	236.51	-277.28	113.60	165.64	-2.44	0.022*	0.20

Note. * $p < 0.05$; whole number t -statistics based on 6 degrees of freedom; fraction t -statistics based on 24 degrees of freedom. SD=standard deviation; B=slope; SE B=standard error of the slope; Int= y -intercept

For each of the magnitude comparison tasks, the difference between the left and right hand median reaction times (dRTs) were regressed onto numerical magnitude for individual participants, yielding a regression slope. These slopes—one for fractions and one for whole numbers—were used as quantitative measurements of the strength and direction of each synesthetes' implicit SNARC and are listed in Table 3. We did not test for a group level SNARC effect due to the small size of the group ($n=4$), but rather tested each participant individually.

Three of the four synesthetes has a significant SNARC effect for whole number comparisons, meaning the slope of the regression line significantly differed from zero. See Figure 2 for plots of the whole number SNARC effect. SY4 demonstrated a significantly negative SNARC slope for whole numbers, indicating a canonical implicit representation of number, which is congruent with her explicit reports. Notably, however, the other three synesthetes demonstrated a response-coding for whole numbers that was incongruent with the direction of their reported spatial number forms. SY1 explicitly reported an internal number form extending in the canonical direction, but had a significantly positive SNARC slope, indicating faster responses to large numbers on the left and small numbers on the right. Similarly, SY2 had a positive SNARC slope despite reporting a canonically-orientated number form, though because the effect was not significant, she effectively had no evidence of a SNARC. Conversely, SY8

described her form as being in the non-canonical right-to-left orientation but exhibited a classic SNARC effect (negative slope) for both whole numbers and fractions. None of the other three synesthetes had a significant SNARC effect for fractions.

Figure 2. Individual SNARC slopes for each of the synesthetes from session 2. Shaded regions indicate 95% confidence intervals. SY2 was the only synesthete whose SNARC slope did not significantly differ from zero.

3.2.2 Number line estimation results

The PAE was calculated for each of the stimulus ranges in each of the number line orientations ($PAE = [answer - correct\ answer / numerical\ range] \times 100$). Smaller PAE values indicate higher acuity; mean PAE scores are presented in Table 3. Data from the number-to-position tasks largely support the contradictory findings from the SNARC analyses detailed above. We expected that synesthetes would show larger PAE scores (indicating lower acuity) for the number line orientation incongruent with their explicit forms. Descriptively, this was the case for SY2 and SY4, but not for SY1 or SY8, who had greater acuity (smaller PAE) for the conditions incongruent with their reported forms. These were the same synesthetes who showed incongruencies in the SNARC task.

Table 4. Results from the number line estimation tasks for synesthetes in session 2.

<i>Canonical number line</i>	<i>Non-canonical number line</i>
------------------------------	----------------------------------

<i>Syn</i>	NLE 0-1	NLE 0-5	Avg. PAE	<i>Syn</i>	NLE 1-0	NLE 5-0	Avg. PAE
1	5.04	5.99	5.365 (5.92)	1	2.90	4.54	3.46 (2.94)
2	2.31	7.86	4.199 (5.18)	2	4.10	11.81	6.73 (8.48)
4	5.17	9.92	6.79 (7.81)	4	7.36	10.21	8.34 (12.45)
8	3.47	11.07	6.06 (10.33)	8	13.60	7.87	11.65 (17.94)

Note. NLE= number line estimation, *PAE* = *percent absolute error*; *Avg. PAE* is the average *PAE* across both NLE; standard deviations are listed in parentheses

4. Discussion

In this study, we investigated implicit and explicit spatial-numerical associations in a group of number-form synesthetes. Synesthetes exhibited a wide range of distinct number forms, which differed in shape, extent, and direction. In the subgroup that underwent additional testing, we found marked incongruencies between their reported number forms and the results of two cognitive behavioral paradigms, namely a magnitude-based SNARC task and a number line estimation task. Only one of the synesthetes demonstrated a SNARC effect for fractions. We discuss these findings below, in light of previous research on NF synesthesia and studies of SNAs in the general population.

4.1 General Observations Across Synesthetes

On the whole, the findings detailed above illustrate the complex interplay between implicit and explicit numerical representations. While we expected that synesthetes' reported forms and their subsequent implicit spatial-numerical associations would be congruent, evidence from multiple tasks suggest that the picture is more nuanced than a direct correspondence between internal conceptualizations and external representations. Whereas previous research on NF synesthetes has generally demonstrated such congruency, whether in canonical or non-canonical orientations, this study is the first to demonstrate clear dissociations between implicit and explicit spatial-numerical associations in a group of NF synesthetes.

Such a dissociation may be in line with the dissociation between implicit and explicit processing described in studies of people with spatial neglect. Priftis et al. (2006) showed that patients with left neglect had impaired performance on a number line bisection task but were seemingly unaffected on tests of the SNARC. The authors suggested that this difference was due to the difference in task demands, with number line bisection requiring direct access to the MNL while the SNARC is implicit. Similarly, Ninaus et al. (2017) found that performance on a number line bisection task was not correlated with individual SNARC scores, further supporting the hypothesis that these two tasks may be supported by disparate underlying cognitive mechanisms. While in this study we saw some degree of continuity across the NLE task and the SNARC task (in that the pattern of congruency was generally preserved across tasks), the possibility that these tasks do not directly measure internal numerical conceptualizations might account for the differences we observe between synesthetes' internal number forms and their explicit performance on these tasks.

Another possible explanation for the heterogeneity in our findings is the notable individual variability of implicit SNAs in the adult population more broadly. Recent studies of SNAs in nonsynesthetes have shown inconsistencies in SNAs (for both whole numbers and fractions), with much of this variability being previously masked by group-level reporting of data. For instance, Cipora et al. (2017) highlighted that for many studies reporting a group-level

negative SNARC slope, between 60-80% of individual SNARC slopes were negative. Other studies with nonsynesthetes have begun exploring the factors that may impact both the strength and direction of individual SNAs, such as level of mathematics ability, visuospatial reasoning, fluency in other languages, etc. (Cipora & Nuerk, 2013; Cipora, Patro, & Nuerk, 2015; Georges, Hoffmann, & Schiltz, 2017; Hoffmann, Mussolin, Martin, & Schiltz, 2014; Viarouge, Hubbard, & McCandliss, 2014). These factors may be influencing outcomes of these tasks measuring implicit SNAs, and thus it is possible that these same factors would be influencing the synesthetes in this study as well. This possibility should be explored in future studies of NF synesthesia.

This study is also the first investigation of the fractions SNARC effect in number form synesthetes. We have previously used this same task to elicit a fractions SNARC in non-synesthetic adults (Toomarian & Hubbard, 2018b), while also showing that the fractions SNARC correlated with the SNARC for whole numbers. In this study, two synesthetes did not report their NF extending to fractions, and only one synesthete (SY8) had a SNARC effect for fractions; notably, it was in the direction contrary to the orientation of her reported form. Our results demonstrate that while synesthetes may report weak explicit representations of fractions on their NFs, they generally do not demonstrate an implicit spatial representation for fraction magnitudes. This may be due to the fact that fractions are less commonly encountered in daily life (relative to whole numbers), and thus are less reliably/robustly represented on synesthetic NFs.

4.2 Discussion of SY8

Our results highlight the heterogeneity of synesthetic number forms and implicit spatial-numerical associations, suggesting that additional factors are likely at play in determining individual spatial-numerical associations. Here, we would like to more thoroughly discuss the unique case of SY8, who described a right-to-left oriented number line but whose performance on the magnitude judgement and number line tasks indicated the opposite. There are several considerations to take into account in interpreting these results. First, it should be noted that the current study only tested for implicit SNAs in the horizontal dimension. Thus, for someone like SY8, whose form went right-to-left but also included vertical stacking, we cannot get the complete picture of their internal conceptualizations from these horizontally-focused tasks. Ideally, follow-up studies would incorporate paradigms similar to those used by Hubbard et al. (2009), which included vertically-oriented tasks.

Secondly, the dissociation between implicit and explicit representations in the case of SY8 may be partially explained by mathematical expertise, as she is a professor of mathematics. While it is unclear what may have influenced or encouraged the non-canonical directionality of her number form (she is monolingual and has no extensive experience with cultures that orient numbers differently), she reported that she consistently represents numbers in the opposite orientation of her internal conceptualization as a requirement of her profession. As a mathematician, she regularly switches to represent numbers going from left-to-right, such as when using Cartesian graphing coordinates and working with functions. Interestingly, she reported being able to make this translation with ease, potentially indicating a high level of cognitive flexibility. Galton's early case studies also included a report from a presumed synesthete with extraordinary calculation abilities (e.g. rapid multiplication of two fifteen-digit numbers) (1880). He reported that "when I am multiplying together two large numbers, my mind is engrossed in the operation and the idea of locality in the series for the moment sinks out of prominence" (p. 253, 1880). This would suggest that more advanced mathematical thinking may be separable from more basic numerical and ordinal thought processes.

This possibility has been empirically examined by Cipora et al. (2016), who investigated the SNARC in a group of professional mathematicians, as well as a group of engineers and non-engineer controls. While the engineers and controls both demonstrated a group-level SNARC effect, the mathematician group did not. This difference was not related to greater overall response variance or intelligence. Based on this finding, we might have expected SY8 to exhibit a flat SNARC slope in the current study, if not a positive one. However, Cipora et al. noted that while a significant effect failed to emerge at the group level, 9 of the 14 mathematicians had a negative SNARC slope. Thus, SY8's implicit SNA may indeed be consistent with those of other professional mathematicians. The authors propose that greater abstract reasoning and/or cognitive flexibility may account for the difference in SNAs between mathematicians and the other two groups. Future studies should directly investigate the link between cognitive flexibility and congruency of implicit and explicit spatial-numerical representations in NF synesthetes.

One final consideration relevant to SY8 is her age; at 57 years old, she was 37 years older than the average age of the other synesthetes from session 2. Given that most studies of the SNARC have been conducted on young adult populations, she is likely much older than the majority of participants in other studies of the SNARC. In a recent investigation of the SNARC effect across the lifespan, Ninaus et al. (2017) found that SNARC effect was stronger with increasing age, which supports the theory that directionality of the SNARC is primarily influenced by culturally congruent sensorimotor experience. SY8 had nearly 40 additional years of exposure to left-to-right numerical organizations, particularly through her work, which might be influencing her canonical SNARC effect. Of course, such an explanation would imply a marked differentiation between her explicit number form and the SNARC effect, since it implies that her lived experience with external influences overrides her own spatial number form. An additional consideration is that evidence from grapheme-color synesthetes has suggested that synesthetic associations decline in consistency with age (Meier, Rothen, & Walter, 2014). While this has not been systematically tested in NF synesthetes, it is possible that overlearned (canonical) cultural representations supersede internal, idiosyncratic representations in aging NF synesthetes.

Taken together, it is clear that several factors converge and likely interact to build an individual SNA profile. The case of SY8 highlights the confluence of factors such as occupation and cultural experience in shaping this profile. Longitudinal studies of NF synesthetes that extend beyond childhood would help to further elucidate the role of these factors.

4.3 Theoretical Implications

The considerations noted above highlight the power and necessity of in-depth, mixed-methods, single-case approaches to investigating cognitive phenomena. There is much to be gained from examining complex questions from an individual perspective, while also looking across individuals for commonalities and differences, a strength of the multi-case study approach. As described by Hubbard and Ramachandran (2005), the multiple-case study approach in studies of synesthesia affords fine-grained analyses of individuals with heterogeneous synesthetic profiles while also allowing for conclusions to be drawn about the group. In the case of this paper, this methodological approach led to the group-level insights that 1) NF synesthetes have generally inconsistent implicit and explicit representations of numbers, 2) the heterogeneity in number forms and implicit/explicit congruency may be more strongly related to other factors besides having NF synesthesia, and 3) NF synesthetes do not have robust implicit associations for fractions. In addition to these group-level findings, we were able to dive deeper into the phenomenology by investigating possible influences at the single-case level, such as our analysis of SY8.

The current results challenge the simple prediction made by Hubbard et al. (2005) that explicit number-forms and implicit SNAs depend on the same mechanisms. In the decade since this explanation was first proposed, it has become clear that individual differences, strategy choice and other factors play critical roles in both explicit number forms and in non-synesthetic SNAs. Indeed, more recent studies (e.g. Arend, Yuen, Ashkenazi, & Henik, *in press*) have also noted that observed behavioral results with synesthetes may reflect the influences of both implicit MNL representations and explicit synesthetic NFs. We concur, but go even further: Recent evidence from numerical cognition argue that the MNL is not a unitary phenomenon, but instead reflects both spatial-numerical mental representations and response-selection (Basso Moro, Dell'Acqua, & Cutini, 2018). Given this complexity, we must consider many more possible mappings between numbers and space to fully account for the wide variability in these implicit and explicit measures across both synesthetes and non-synesthetic controls. While researchers have attempted to account for this variability by measuring myriad possible influences, we still do not have a strong understanding of why this intra- and inter-individual variability exists. Thus, it is important to consider alternative explanations—beyond individual cognitive differences—that may account for this pattern of results.

Emerging thinking in numerical cognition suggests that the SNARC, and other similar tasks that are used to investigate SNAs are accessing multiple spatial representations of number. For instance, some authors have argued that the SNARC effect is not an index of a spatial MNL, but rather that it indexes processes like order in working memory (e.g. Abrahamse, Van Dijck, & Fias, 2016), or processes associated with linguistic “markedness” of number words (e.g. Nuerk, Iversen, & Willmes, 2004). Most recently, Shaki and Fischer (2018) have contended that—contrary to the majority of their own prior work (see Fischer & Shaki, 2014, for a review)—SNAs emerge as artifacts of either explicit magnitude processing or explicit spatial- directional processing; they do not reveal spatial-conceptual links. This was supported by two novel experiments combining implicit or explicit magnitude judgements (i.e. either a parity or magnitude-based go/no-go task combined with arrows) with implicit spatial-directional cues (i.e. using a central response key for various response rules) (see also Fischer & Shaki, 2016). They revealed a SNARC only in the conditions with explicit magnitude processing, suggesting that previously reported evidence of a SNARC may have been the result of task-related bias (see also Karolis, Iuculano, & Butterworth, 2011). While this proposal is certainly compelling, more research is needed to replicate and extend these findings to other paradigms.

Similarly, in the NLE task, the assumption that performance is indicative of implicit number magnitude representations has been challenged on several fronts (e.g. Huber, Moeller, & Nuerk, 2014; Slusser & Barth, 2017). One consideration is that the imposed linear representation makes it difficult to determine whether a small PAE is indicative of an internal linear representation (as generally assumed) or rather, indicative of the participants simply learning how to do the task correctly. Huber, Moeller & Nuerk (2014) present two pieces of evidence to support this claim: 1) adults can be easily trained to do both linear and logarithmic versions of NLE well, and 2) young children who represented numbers logarithmically and those who represented them linearly on a paper-and-pencil number line task did not differ on a separate computerized, logarithmic training version of NLE. As this violates the expectation that those with presumed logarithmic internal representations would perform better on the logarithmic training task, the leading alternative explanation is that the task is not appropriately capturing implicit SNAs. It is also possible that NLE performance is influenced by task-specific strategies, such as proportional reasoning (e.g. Slusser, Santiago, & Barth, 2013; Slusser & Barth, 2017) or using reference points or “benchmarks” (e.g. Barth & Paladino, 2011; Siegler & Opfer, 2003).

These strategies may have been differentially relied upon by our synesthetes, leading to incongruencies between our measures and their reported forms.

Ultimately, the most likely possibility is that there is no single factor that determines implicit and explicit representations of number, but rather that multiple factors are responsible for response profiles in a person at any given point in time. Alibali and Sidney (2015) suggested that people choose from a range of possible strategies as a function of task constraints and context. We suggest that observed behavioral patterns may reflect a complex weighting of different representations and strategies, and that these weightings may differ across individuals and task contexts. For instance, the general consistency we observed between the unconstrained number placement task and synesthetes' reported/drawn number forms may be because were minimal task constraints, and the context in which numbers were presented was simple. However, the SNARC task requires a speeded choice, and the NLE tasks impose a visual structure that likely interfered with our synesthetes' number forms. In line with the dynamic strategy choice account, these additional task features constrained the extent to which NF synesthetes accessed their own number forms, choosing instead to rely on different processing strategies.

4.4 Conclusion

Taken together, the results presented in this study build upon what was previously known about NF synesthetes while also highlighting more general issues that must be reckoned with in the field. Similar to past reports of NF synesthesia, the participants in this study demonstrated a wide array of NFs. While there was notable inter-individual variability in NFs, the synesthetes' drawn forms bore great similarity to the forms derived via an unconstrained number placement task. We found that it was very uncommon for NF synesthetes to have robust implicit representations for fractions, which unsurprisingly resulted in weak implicit SNAs for fraction stimuli. Contrary to our hypotheses, performance on classic indices of implicit SNAs—the SNARC and NLE task—generally did not match up with explicit reports of synesthetic number forms. While this may indicate separate underlying cognitive mechanisms for implicit and explicit spatial-numerical representations, task-specific constraints and strategic variability could also account for these differences, and thus must be seriously considered in future investigations. To more thoroughly explore these issues, future studies should consider testing synesthetic associations using paradigms that are less susceptible to strategic effects, and which are cognitively more direct, such as spatial cuing paradigms or implicit measures of spatial processing like spontaneous eye movements. We also recommend that studies of SNAs in which participants are presumed to be non-synesthetic systematically inquire about potential synesthetic experiences to ensure homogeneity of the group. These steps will help to further refine our understanding of explicit SNAs in synesthetes and non-synesthetes alike.

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Appendix A. Number-Space Synesthesia Questionnaire

The following questions were presented as a 13-page questionnaire, with several inches of blank space following each question for adequate elaboration. If additional space was allotted for a question, it is noted below.

1. Number-form Synesthesia: Basic Questions

Number-form synesthesia is a phenomenon in which people automatically and consistently associate numbers with specific locations in space. Do you experience number-form synesthesia? Yes / No

Some people with number-form synesthesia also visualize other ordered sequences (e.g., letters of the alphabet, weekdays, months, temperature) spatially. Please list all sequences that you visualize spatially.

Are your synesthetic experiences very apparent or do you have to make a special effort to attend to them?

Can you ignore your number-form synesthesia? Yes / No

If **yes**, please describe how and when you can do this.

Can you completely stop your number-form synesthesia? Yes / No

If **yes**, please describe how and when you can do this.

How, if at all, do your number-form synesthetic associations change when you are using other languages you may know?

Do you have a form for historical year numbers (e.g., 1996)? Yes / No (Please circle one)

If **yes**, is it identical to the form you have for numbers (e.g., does the year 1996 occupy the same location as the number 1,996)? Please describe.

Similarly, if you have any other forms that involve numbers (such as temperature or height), do those values have the same location as the numbers in your number form (e.g., is 67 degrees Fahrenheit in the same location as the number 67)? Please describe.

Please illustrate, to the best of your ability, the locations of the numbers 0 to 1 on your number form. [Full page allocated]

Please illustrate, to the best of your ability, the locations of the numbers 1 through 10 on your number form. [Full page allocated]

Please illustrate, to the best of your ability, the locations of the numbers 1 through 100 on your number form. [Full page allocated]

2. Visualization and Experience of Number-form Synesthesia

Would thinking about one stimulus (e.g., the number 7) still bring to mind its association (i.e., its spatial location)? Would it be the same when you dream?

Is your synesthesia present when you close your eyes? Yes / No

Do you visualize your associations in your mind, or do you see them superimposed on your visual field?

Can you pinpoint the synesthetic perception in space?

Do you experience something (ex. colors, objects, or patterns) projected somewhere in front of you?

Can you point to an exact point where your visualizations occur or is your experience relatively scattered?

Can you manipulate your number form in your mind's eye, (such as being able to "zoom in" or "zoom out" on particular sections of numbers, or rotate it), or is your number form static? Please describe.

Does the location change in different environments (e.g., so that it will occupy "free" space left in a specific setting and not interfere with the perception of other objects)?

How long does your number-form synesthesia typically last when you experience it?

Does your spatial form go away immediately after the stimulus is removed or the thought disappears, or does it fade out more slowly? Please describe.

Are there any conditions under which your number-form synesthetic associations are not as strong or reliable (e.g., if you are tired, on medication, etc.)?

Do certain emotions or how hard you focus affect your number-form synesthesia? Do the associations get stronger or weaker?

Does your number-form synesthesia interfere with other things or your day-to-day activities?

Does your number-form synesthesia ever distract you? Yes / No

If **yes**, please describe how and when this tends to happen.

3. Development of the Number-Form Synesthesia

Are you aware of any family members that experience this form of synesthesia? Yes / No. If **yes**, which family member(s)?

At what age did you begin to experience number-form synesthesia?

Have your number-form synesthetic experiences been consistent since they developed? (For example, is the number 1 always visualized in the same location?) Yes / No

If **no**, please describe how your associations have changed.

Can you think of any experiences that could have shaped your number-form associations?

Under what conditions do you experience number-form synesthesia?

Are there any particular triggers?

4. Whole Numbers and Fractions

Do Roman numerals or other nonsymbolic number representations have specific locations in space for you? Yes / No

Can you list different classes of numbers (e.g. whole numbers, fractions, negative numbers) for which you have a spatial association? Do you visualize them in separate forms or are they integrated?

If yes, can you describe your spatial locations for:

fractions (e.g., $3/5$)? Yes / No

decimal numbers (e.g., 0.3)? Yes / No

negative numbers (e.g., -4)? Yes / No

irrational numbers (e.g. π)?

imaginary numbers (e.g., i)? Yes / No

Do you find math difficult or easy? Please describe your general experience with math.

Do you feel that your number-form synesthesia has helped or hindered your math abilities? Please describe how.

Can you describe the process you would use to solve a simple equation? For example, how would you approach $7 + 5 = ?$

Does this approach to solving math problems change for fractions? For example, $1/4 + 3/5$?

Do you believe you could perform mathematics or simple number operations if your synesthesia went away? Yes / No **If no**, please explain why you feel this way.

Does your number-form synesthesia aid you when doing daily tasks involving fractions (for example, measurements when baking, dividing a cake, etc.)?

How similar or different are your associations for fractions and decimals from other whole numbers?

5. Background Information

What languages other than English, if any, can you speak, read, and/or write? (Please indicate age of acquisition and proficiency for each)

Have you ever lived or spent a significant amount of time in a country other than the United States? Yes / No

If yes, which one(s)?

Are you artistic? Yes / No

If yes, what kind of art do you engage in (e.g., painting, sculpture, photography, etc.)?

Would you describe yourself as having a good memory? Do you have a better for certain types of information over others (e.g., names, locations, etc.)? Please explain.

Do you have a good memory for dates and/or times? Yes / No

Do you have a good sense of direction? Do you have difficulty navigating new areas or using maps? Please describe.

Do you consider yourself to have high mental visual imagery? On a scale from 1 to 9, where 1 is very easy and 9 is very difficult, how easy or difficult is it for you to create detailed visual images in your head?

1	2	3	4	5	6	7	8	9
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Very easy

Very difficult

Please explain your answer:

Appendix B. Fraction stimuli used in unconstrained number placement task and fraction comparison task

1/2 (unconstrained number placement task only)

1/3

1/4

1/5

1/6

1/7

1/8

1/9

2/3

2/5

2/7

2/9

3/4

3/5

3/7

3/8

4/5

4/7

4/9

5/6

5/7

5/8

5/9

6/7

7/8

7/9

8/9

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