What Proportion of People Are Gifted? Assessing the Implications of Conceptions of Giftedness

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Abstract

There are several influential conceptions of giftedness that drive scholarship and educational practice. They generally define giftedness as some combination of high ability, high performance, or superior levels of other characteristics, operationalized by stated or implied cutoff levels (e.g., “top 10%”) of these features that must be met in order to be considered gifted. In many cases, the authors of the conceptions of giftedness also describe the proportion of people who are expected to qualify as gifted. However, such proportions are not logically derived from the conceptions. In this paper, we quantitatively operationalize some conceptions of giftedness and compute the implied giftedness rate under each using numerical calculations or simulation. Results reveal that the proportion of individuals who would meet the standard of giftedness under some conceptual models is unrealistically high, and that there is tremendous variability in the giftedness rate both between and within conceptions. The implication of this work is that the rigor and internal self-consistency of models of giftedness requires improvement.

Keywords: giftedness, identification; internal validity, numerical analysis, simulation; talent

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A growing focal point in social science research has been the broad flexibility individual researchers have in asking research questions as well as in collecting, analyzing, and reporting data. This flexibility has been termed the “garden of forking paths” (Gelman & Loken, 2014) or “researcher degrees of freedom” (Simmons, Nelson & Simonsohn, 2011). Such large flexibility is problematic because it can harm veracity, generalizability, and trust (Makel & Plucker, 2017). These problems have parallels in the study of giftedness and gifted education. A researcher setting out to study giftedness, or an educator seeking to better serve advanced students, faces a similar dilemma: there is no consensus on exactly what the term “gifted” means. The gifted education literature is replete with varying conceptual definitions of the term (c.f., Sternberg & Davidson, 2005), from those based solely on high IQ to modern multifactorial definitions.

There are also vast degrees of freedom to contend with when it comes to identifying giftedness. Limiting one’s attention to the pragmatic issue of educational policy (as opposed to approaching the question from a scientific or philosophical orientation) provides little clarity. With no federal mandate, large variation between (and sometimes within) states exists; the process of identifying gifted students is filled with inconsistencies (McClain & Pfeiffer, 2012).

Regardless, committing to a single conception of giftedness may not assure that a uniform number of students is identified as gifted because there are many ways that a given conceptual definition can be operationalized. This is because most conceptual definitions are not only underspecified with respect to important issues, such as the nature of the comparison group (i.e., “top 10% compared to whom?”), but are also silent on issues of measurement. Even a straightforward “high-IQ” definition of giftedness can result in identifying different individuals

depending on the instruments used. Assessing IQ using the Weschler Intelligence Scales for Children (WISC) may yield a different set of individuals being designated as “gifted” than if the Stanford Binet is used. Under repeated testing, students may even be identified as gifted one year but not the next (Lohman & Korb, 2006). Other seemingly irrelevant details, such as how much time has passed since the instrument was last normed, can play a striking role in the results of IQ testing due to the Flynn effect (Kanaya, Scullin, & Ceci, 2003; Truscott & Frank, 2001), which can influence the size and identity of the population of identified students. Of course, the influence of measurement error on classification decisions ensures that there will always be false positive and false negative diagnostic errors (McBee, Peters, & Waterman, 2014). Identification decisions will vary for this reason even when the same instruments are repeatedly administered. But these pragmatic, operational sources of variation in the proportion of students that are identified as gifted by schools are not the topic of this paper. Leaving aside concerns about measurement error and test norming, this paper is about the consequences of the conceptions of giftedness themselves, and how they affect the giftedness rate.

One way that conceptions of giftedness differ is in whether the meaning of the term stays constant or varies over the lifespan. For example, the talent development perspective (c.f., Subotnik, Olszewski-Kubilius, & Worrell, 2011) views ability (or potential) as the primary hallmark of giftedness in early childhood, transitioning to demonstrated achievement and then to eminence by midlife. A high-IQ adult with no outstanding achievement of note might be considered gifted under some conceptions but not others. According to some perspectives, giftedness is what you are; in others, it is what you do. Such differences in criteria across the lifespan can strongly influence the expected proportion of individuals who are considered gifted.

Knowing and agreeing on how many students are gifted matters for at least three reasons. First, the number of gifted students is relevant to education policy and advocacy efforts. Policy situations that involve 0.1% of students will not receive the same attention as policy situations that involve 15% of students. Although the comparison is unfortunate, we think the effectiveness of communication campaigns that discuss the incidence rates of things like cancer (e.g., “About 1 in 8 (12%) women in the US will develop invasive breast cancer during their lifetime”¹) or “About 1 man in 7 will be diagnosed with prostate cancer during his lifetime”²) or autism (e.g., “one in 68 children is diagnosed with an autism spectrum disorder”³) illustrate the power and persuasiveness wielded by communicating how prevalent diagnoses are. We do not think giftedness should be considered akin to cancer or autism spectrum disorder diagnosis. But the importance of identification of giftedness and providing appropriate services is not completely foreign to the analogy.

Second, knowing “how many” gifted students there are helps serve as a benchmarking assessment of how well identification practices are (or are not) working. Several recent studies (e.g., Card & Guiliano, 2015; Grissom & Redding, 2016; Lu & Weinberg, 2016) have described differences in rates of gifted identification for different demographic groups. The identification of “excellence gaps” of high performance on standardized tests (Plucker, Hardesty, & Burroughs, 2013; Plucker & Peters, 2016) has also garnered substantial attention. Both sets of research rely on comparison of demographic groups to assess equity of practice and performance. Although such results are extremely important to consider, they serve as an indirect assessment of whether conceptions of giftedness are being implemented appropriately. After all,

¹ <http://www.cancer.org/cancer/breastcancer/detailedguide/breast-cancer-key-statistics>

² <http://www.cancer.org/cancer/prostatecancer/detailedguide/prostate-cancer-key-statistics>

³ <http://www.adcouncil.org/Our-Campaigns/Health/Autism-Awareness>

how can the equity of a gifted identification system be assessed if the conception of what “giftedness” constitutes isn’t clear?

Third, and perhaps most important from a scientific perspective, knowing how many students are gifted helps assess the internal validity of different conceptions; it helps scholars evaluate and compare different conceptions of giftedness. Conceptions can be evaluated in many ways. Empirical research can be used to search for falsifying evidence that weakens conceptual propositions. But it is also important to interrogate conceptions for their internal, logical consistency. If a particular conception indicates that 10% of the population of students are gifted, but the logical consequences of the conception suggest that 10% is a completely implausible rate, then a substantial shortcoming of the conception has been identified. This, too, should be viewed as falsifying evidence that weakens the standing of the conception compared with competing alternatives.

In this paper, we review some of the most commonly used conceptions of giftedness. They are: (1) high cognitive ability only, (2) multiple criteria definitions, (3) Renzulli's Three Ring Model (1978, 2005) and (4) the position statement of the National Association for Gifted Children (NAGC; 2010). Although there are many others (c.f. Sternberg & Davidson, 2005), we use these as examples of how conceptions can be analyzed from a quantitative perspective and quantify the conceptions to provide concrete estimates of the proportion of individuals each conception identifies as gifted. We hope that the results of these analyses illustrate why quantitative or psychometric *analysis* must accompany quantitative or psychometric *arguments* when conceptual or theoretical ideas about giftedness are being considered.

Quantifying Conceptions of Giftedness

For any conception, the giftedness rate depends on four features⁴:

- 1) Domains: the number of traits, domains, or criteria to be considered (e.g., intelligence, academic achievement, leadership, psychomotor, creativity, etc.)
- 2) Cutoffs: The required level(s) of these traits or criteria needed in order to be described as gifted (which necessitates knowledge of the comparison group)
- 3) Distribution: The statistical distribution of the traits or criteria, encompassing both the shape of the distributions (e.g., Normal? skewed?) as well as their correlations.
- 4) Decision rule: the method for combining information on the relevant criteria to reach a determination regarding whether the individual is or is not gifted.

Regarding the decision rule, point (4) above, we note that identification decisions made on the basis of multiple pieces of information often use “and”, “or”, or “mean” combination rules or a composite of these (McBee et al., 2014). The “and” rule requires a minimal standard to be fulfilled on every criterion or in every domain (e.g., scores above cutoff values on tests of both math and verbal achievement) whereas the “or” rule requires individuals to meet the cutoff on only one assessment (e.g., a single score exceeding the cutoff value on either a math or verbal test). The “mean” combination rules take the average performance of the multiple criteria. “and” combination rules reduce the number of students identified as gifted whereas “or” rules can inflate the number of students identified as gifted (the magnitude of reduction/inflation is determined by the correlation of the performance across criteria).

Calculating the expected giftedness rate under a particular conception of giftedness involves representing the conception’s positions with respect to these four factors. This becomes

⁴ Factors such as equally applying identification rules across all students matter as well, but such considerations are outside the state conceptions of giftedness.

complicated when several layers of apparently clear verbal statements are operationalized, making estimating the expected number of students who should be identified as gifted difficult. To avoid natural variations in implementation of theories that arise when applied in the real world (e.g., variation in interpretation, selection of instruments, local population differences, sampling error, measurement error), we use numerical computation or simulation in this paper to assess the logical or mathematical implications of each conception. We are not critiquing practicality or ease of implementation; we are assessing internal validity. Readers will find additional detail regarding our methodology in the Technical Appendix to this piece. All computations were implemented in the R statistical programming language (R Core Team, 2016); the code for producing the results in this paper is available via our project's Open Science Framework page at <https://osf.io/6e7g9/>. All figures in this paper were produced using the ggplot2 library (Wickam, 2009). The next section of the manuscript provides introduction and quantitative calculations of the giftedness rate under several conceptions of giftedness.

Model 1: Giftedness as General Cognitive Ability

The connection between giftedness and IQ goes back at least to Lewis Terman, who in addition to developing the Stanford-Binet intelligence test (1916), also began the longitudinal genetic study of genius in 1921, which followed individuals who had scored extremely highly on an IQ test throughout their lives. The use of IQ tests to identify students as gifted continues today with 45 state definitions including intelligence as at least a part of their definitions of giftedness and 16 states requiring IQ scores for gifted identification (McClain & Pfeiffer, 2012). Although not without its detractors (e.g., Kaufman, 2013), some researchers in the field have advocated for a return to the centrality of IQ in defining giftedness (e.g., Warne, 2015).

Calculating the expected giftedness rate under a ‘high-IQ’ conception of giftedness is straightforward because the four questions have simple answers: the number of domains to be considered is one (IQ), the cutoff is directly specified, the distribution is a univariate Normal distribution (Warne, Godwin, & Smith, 2013), and no combination rule is required since there is only a single criterion. Variability in the giftedness rate under IQ-based conceptions is therefore a function of the cutoff IQ presumed to be indicative of giftedness. For example, Terman (1926) identified students with an IQ of 140 or higher as gifted. Others conceptions require different IQ cutoffs. Integrating under the normal distribution of IQ scores from the cutoff to positive infinity yields the expected giftedness rate. This operation can be performed with a single line of R code (see Technical Appendix). As Figure 1 displays, the giftedness rates under an IQ-only model would be approximately 9.1% at a cutoff of 120, 2.2% at a cutoff of 130, and 0.4% at a cutoff of 140. Of course, incidence rates may vary substantially across individual schools or school districts, but such variation would not influence the population total of gifted students in the population. The results for this model, assuming a cutoff at the 95th percentile (approximately equal to an IQ of 125), are included in the first column of Table 1.

Insert Figure 1 about here

Insert Table 1 about here

Model 2: Multiple Criteria/Multiple Domains

Background. Many contemporary conceptions of giftedness broaden the term beyond intelligence alone to include the consideration of other factors, such as school achievement in specific subjects, motivation, or creativity. This type of conception is sometimes referred to as a multiple criteria model (Krisel & Cowan, 1997). According to the NAGC *State of the States* report, most state policies for identifying gifted students consider both IQ and academic achievement. Some conceptions of giftedness place more emphasis on demonstrated academic achievements than on indication of high general ability (c.f., Peters et al., 2014; Subotnik et al., 2011). Many states or school districts strongly weight high achievement in specific academic subjects in their gifted education identification policies, either alone or in combination with other factors such as IQ (McClain & Pfeiffer, 2012; NAGC, 2015).

Method. Two specific examples will be considered. In the first, giftedness is defined by two criteria or domains (e.g., academic achievement in reading and mathematics); in the second by three criteria or domains (e.g., IQ, math achievement, and reading achievement). In both examples, we will set the cutoff at the 95th percentile, implying that a student must demonstrate achievement in the top 5% relative national norms to “qualify” with respect to each domain. We have chosen to hold the cutoffs constant for this analysis both because its effect on the giftedness rate is intuitively obvious and because we wish to focus on the impact of the combination rule and the correlation(s) between the criteria. Thus, regarding the four criteria, we manipulate the number of domains, the correlations between them, and the combination rule, while holding the cutoffs constant.

Our calculations assume that the scores follow a multivariate normal distribution, implying that each individual component follows a univariate normal distribution. The multivariate normal distribution describes the correlations between the scores as well as their

means and standard deviations; we can manipulate these correlations to examine how the giftedness rate changes depending on the strength of the relationship of scores across criteria. For example, we would expect a relatively strong correlation between reading and language arts scores, but a lower correlation between IQ and creativity.

The combination rule plays a key role in determining the giftedness rate. The “and” combination rule requires qualifying scores on all the criteria simultaneously, whereas the “or” rule requires a qualifying score in only one criterion. Because these two rules define the upper and lower bounds of the identification rate, we will not provide calculations under the “mean” rule (McBee et al., 2014). The giftedness rate under various conditions of this conception of giftedness can be calculated by numerically integrating the multivariate normal distribution in various ways according to the combination rule; see the Technical Appendix for further details and R code.

Results. Results from Models 2A and 2B are summarized in the second and third columns of Table 1. The second column provides giftedness rates when two domains are considered. When a top 5% cutoff is used, under the “and” rule, the proportion of students who meet the criteria for giftedness ranges from around 0.25% if the two variables are scores are uncorrelated to 5% if they are perfectly correlated. Under the “and” rule, a low correlation between criteria means a lower proportion will qualify, because it becomes relatively less common to exceed the cutoff on both criteria simultaneously as the correlation between criteria weakens.

By contrast, the proportion identified as gifted under the “or” rule ranges from nearly 10% (9.75%) when the criteria are perfectly uncorrelated to 5% when they are perfectly correlated. This is because the “or” rule provides people with multiple opportunities to qualify –

students need not be strong in both areas in order to be gifted. The highest giftedness rate occurs under the “or” rule with a zero correlation between criteria, because in this case each additional criterion considered creates a completely independent opportunity to exceed the cutoff. Figure 2 displays the impact of combination rule and the correlation between criteria in two-assessment multiple criteria definitions.

Insert Figure 2 about here

A similar pattern is observed for the three-criterion model presented in the third column of Table 2. Again, the giftedness rate depends strongly on the correlation between the criteria and the combination rule. The addition of the third criterion makes the “and” rule more restrictive by providing an additional hurdle that must be cleared to qualify as gifted. Its impact is stronger when the correlation between the domains is low. Conversely, adding a third criterion makes the “or” rule more permissive, because each additional criterion provides another opportunity to qualify. Under the “or” rule, the less correlated the criteria are, the higher the identification rate.

In summary, the giftedness rate under a multiple criteria system can vary tremendously, even when fixing the qualifying score across criteria to the 95th percentile. With two criteria, the giftedness rate could vary from 0.25% to 9.75% depending on the combination rule and correlation between criteria. With three criteria, the giftedness rate could vary from 0.01% to 14.3%.

Under multiple criteria models, the devil truly is in the details. Let us imagine a hypothetical conception of giftedness stating “*gifted students comprise the top 5% of*

*demonstrated achievement in the core academic subjects of reading and mathematics.*⁵” Clearly, this conception is based on an “and” rule and specifies cutoff scores at the 95th percentile, seemingly with respect to national norms, although the specific reference group is not mentioned. The giftedness rate under such a definition will therefore depend entirely on the correlation between reading and mathematics scores. A correlation of $r=0.50$ would imply a giftedness rate of 1.2%; a correlation of $r=.80$ would imply a rate of 2.5%; these correlations likely provide reasonable upper and lower bounds for the strength of the relationship between reading and mathematics achievement. Therefore, for the conception to remain internally consistent, it would need to explicitly state or implicitly assume that the giftedness rate falls within this range of 1.2%-2.5%. Any other expectation regarding the giftedness rate would be illogical given the mathematical implications of this definition of giftedness.

Model 3: Renzulli’s Three-Ring Model

Background. In 1978, Joseph Renzulli published a paper questioning what makes a student gifted and called for a re-examination of the definition of giftedness. In the nearly four decades since, this paper has been cited over 1,800 times and has grown extremely popular in schools (Mathews, 2014; McClain & Pfeiffer, 2012). Under Renzulli’s model, there are three relevant clusters (or domains) of traits: above average ability, creativity, and task commitment. According to Renzulli (2005), these clusters combine and interact with each other to enable students to exhibit gifted behaviors and to facilitate creative accomplishment. Students who exhibit, or have the potential to exhibit, sufficient levels of ability, creativity, and task commitment in any of 11 general domains (which have at least 45 specific performance areas)

⁵ This is hardly hypothetical; the Illinois state definition of giftedness reads, “*A child shall be considered gifted and talented in any area of aptitude, and, specifically, in language arts and mathematics, by performing in the top 5% locally in that area of aptitude.*” (Illinois Public Act 094-0151)

will demonstrate gifted behaviors. They are mathematics, philosophy, religion, life science, visual arts, social sciences, language arts, physical sciences, law, music, and movement arts. Gifted behaviors always occur within a domain and are performed by certain students at certain times in certain contexts. Individual students are identified as “gifted” if they are expected to perform gifted behaviors a sufficient proportion of the time to merit educational opportunities not typically provided in regular classrooms.

Renzulli also developed an identification system to be used along with the Three Ring model of giftedness. This identification system still has its foundation with performance on standardized academic measures with “students who score at or above the 92nd percentile on either verbal or numerical sections of the achievement test should automatically be placed in the Talent Pool” (Renzulli, 2005, p. 43). Doing this is explained by “all traditionally bright youngsters will automatically be selected, and they will account for approximately 50% of our Talent Pool” (Renzulli, 2005, p. 45).

Students not identified via test scores are identified by training teachers to nominate students based on independent ratings on various scales that either correspond to the Three Ring model or other general academic performance domains (e.g., Leadership, Reading, Music, etc.). Although there are also additional rounds of “safety valve” nominations (e.g., parent nominations), because the overall talent pool is recommended to be about 15% (that can vary across local schools) that is then narrowed down via subsequent decisions of a review and selection team, this assures that the proportion of students who are gifted is less than 15%.

However, we are interested in understanding how many students qualify as gifted with respect to *theoretical* conceptions of giftedness, not according to affiliated but atheoretical operational definitions. Although practical, the aforementioned identification model is not a

quantitative application of the Three Ring Model itself. One virtue of the Three Ring conception is that it describes the conditions under which outstanding accomplishments could be expected to arise – that is, when ability, task commitment, and creativity are all present with sufficient intensity. Thus, our analysis attempts to determine the proportion of students who achieve outstanding accomplishment in all three areas based on the logical quantitative consequence of Three Ring model.

Although the evidence for a “positive manifold” of substantial positive correlations across many diverse tasks involving storage, manipulation, and the application of information has been taken as evidence of a dominant general factor of intelligence (Spearman, 1904; Warne, 2015), the evidence of a domain-general creativity is mixed (Plucker & Makel, 2010). Regardless, Renzulli clearly states that he believes creativity to vary across context (Renzulli, 2005). Similarly, as acknowledged by Renzulli, the notion of a domain-general or trait-level “task commitment” is unlikely given that current theories of motivation, such as the expectancy-value theory (Wigfield & Eccles, 2000) are explicitly situational. Our interpretation of the Three-Ring conception is that gifted behaviors operate within each separate domain. The specific areas listed by Renzulli are a non-exhaustive list that is subsumed within the 11 general areas (Renzulli, personal communication, February 28, 2016). Thus, our analysis focused on these 11 areas.

Quantitative analysis of this model is complicated because the model explicitly situates the three Rings of ability, creativity, and motivation within domains – and lists 11 domains as specific examples. Because the giftedness rate depends on the correlation between the three Rings within the 11 domains, we were forced to consider how giftedness is manifested both across and within domains. Our general approach was to simulate data on the Three Ring

“scores” within a set of eleven domains following a multilevel factor analysis model. Any individual having all three Ring scores above the cutoffs in at least one of the eleven domains was considered to be gifted. Renzulli’s Three Ring conception is therefore quantified with respect to the four determinants of the giftedness rate as follows:

- (1) *number of domains or criteria*: eleven
- (2) *cutoff(s)*: all three Rings within a domain must be at the 85th percentile or higher
- (3) *distribution*: multivariate normal with correlations derived from a multilevel factor analysis model
- (4) *combination rule*: composite; “and” rule within domains, “or” rule between domains.

Method. We used Monte Carlo simulation for this analysis because exact analytical solutions would have been intractable considering the complexity of the combination rule, which is essentially eleven sets of “ands” each joined by an “or.” A similar method was presented in McBee et al. (2014) for analyzing the performance of Georgia’s multiple criteria identification system (see the Technical Appendix and the R code on the project’s OSF page for more detail on how Monte Carlo simulation was performed).

A graphical depiction of the simulation logic for the Three Ring model is presented in Figure 3. The model is a multilevel confirmatory factor analysis (CFA). The Three Rings are represented as both domain-general traits as well as specific, within-domain characteristics. Giftedness as “overlapping rings” can only occur within domains, meaning that a person with a high level of domain-general ability, creativity, and task commitment will not be identified unless these manifest themselves inside one of the eleven domains. Although Figure 3 depicts only four domains for the sake of visual clarity, our actual simulation considered all eleven discussed by Renzulli (2005).

Insert Figure 3 about here

Table 2 displays the three sets of parameters that we manipulated to investigate different conditions. The parameters were: Ring stability (stable vs. unstable); domain-general correlations between ability, task commitment (TC), and creativity (low vs. high); and within domain-specific correlations between ability, TC, and creativity (low vs. high).

In this model, individuals have domain-general endowments of ability, creativity, and task commitment. These domain-general Rings create correlations between the Ring traits across domains -- so a person's ability in mathematics will be related to his ability in law, science, and the other domains under the guise of "Ring stability." Ring stability refers to the strength of the factor loadings of the domain-specific Rings on the domain-general Ring. These loadings create a positive manifold of positive correlation amongst the Rings across domains. High Ring stability indicates a strong positive manifold and highly correlated levels of ability across domains, creativity across domains, and task commitment across domains. Conversely, low Ring stability indicates a weak manifold, and therefore large variation in Rings across domains. In our model, Ring stability was manipulated by altering the factor loadings relating the within-domain rings to their between-domain counterparts.

The correlation between the Rings was manipulated at two levels: at the overarching between-domains level and also separately within domains. The Ring correlations at the domain general level are represented in Figure 3 by the curved arrows between ability, creativity, and TC in the upper half of the model. These correlations represent the strength of relationship between these three traits independent of domain. The Ring correlations at the within-domains (domain

specific) level are represented by the curved arrows between ability, motivation, and TC within each domain separately, illustrated in the lower half of Figure 3. These values may differ across domains, because the correlation between these three traits may be different in, for example, mathematics than they are in visual arts, physical science, or philosophy. Ability and task commitment may be uncorrelated across domains, but within a specific domain, they may be highly correlated if an individual's ability provides rewarding positive feedback that encourages further participation in the domain.

The simulation was repeated under two conditions of Ring stability, two levels of between-domain inter-Ring correlation, and two levels of within-domain inter-Ring correlation. These conditions were fully crossed, resulting in a 2x2x2 factorial design with a total of eight conditions. The specific parameters chosen for these conditions are provided in Table 2 and were chosen to help provide reasonable upper and lower bound estimates.

Insert Table 2 about here

Results. The results from Model 3 are displayed in the fifth column of Table 1 as well as in Figure 4. The proportion of students who are gifted according to the Three Rings varies tremendously across conditions. The lowest giftedness rate (7.6%) was found in the stable Ring / weak correlation condition. The highest rate (47.4%) was found in the unstable Ring / high correlations condition.

A few general patterns were observed. First, the giftedness rate tended to be higher in the unstable Ring condition. As discussed, the Three Rings model is a set of “ands” each joined by an “or”, and low correlations increase the identification rate under an “or” rule. The unstable

Ring condition creates a lower correlation between the domains, thus increasing the overall giftedness rate. Second, the strength of the domain-general correlations between Rings had little impact on the giftedness rate. However, the within-domain correlation (which relates to the “and” part of the Three-Ring model, where increasing correlations between rings increases the probability that all three of them are above the cutoff) had a strong influence on the giftedness rate (either doubling or quadrupling the rate of giftedness depending on Ring stability).

Renzulli’s proposition that the talent pool would be expected to include 15% of the students is a plausible value for this model. This rate would occur in the “unstable Ring” condition when the within-domain correlations are all approximately equal to $r = 0.3$, or in the “stable Ring” condition when the within-domain correlations are all $r = 0.5$. The latter set of conditions strikes us as realistic. The higher these domain-specific correlations become, the more sensitive the giftedness rate becomes to Ring stability. Low levels of Ring stability coupled with high domain-specific correlations lead to inordinately high giftedness rates approaching 50% – higher, we believe, than what Renzulli intended when he proposed the Three-Ring model.

Model 4A: NAGC Position Statement – Childhood / Ability-Centric Variant

Background. Founded in 1954, NAGC is an organization dedicated to supporting families, educators, and researchers who work with gifted and talented children. On its website, NAGC states that its primary goal and priority is to make “gifted learners a national priority”⁶ and that “Academically gifted and talented students in this country make up approximately six to ten percent of the total student population (three to five million students)”.⁷ In 2010, NAGC published a position statement titled *Redefining Giftedness for a New Century: Shifting the Paradigm*. This position statement was not specifically intended as a statement on identification

⁶ <http://www.nagc.org/about-nagc/goals-priorities>

⁷ <https://www.nagc.org/resources-publications/resources/gifted-education-us>

(Olszewski-Kubilius & Speirs-Neumiester, March 15, 2016, personal communication) but does articulate a clear conception of giftedness and is considered to be as one of the “definitions of giftedness” by NAGC.⁸ As a large and visible advocacy organization dedicated to helping meet the needs of gifted and talented students, we felt it would be appropriate to analyze a conception developed by an NAGC task force and approved by its Board of Directors. The document defined giftedness as follows:

Gifted individuals are those who demonstrate outstanding levels of aptitude (defined as an exceptional ability to reason and learn) or competence (documented performance or achievement in top 10% or rarer) in one or more domains. Domains include any structured area of activity with its own symbol system (e.g., mathematics, music, language) and/or set of sensorimotor skills (e.g., painting, dance, sports).

The NAGC position statement of giftedness specifies both symbol systems and sensorimotor skills as domains of giftedness. Although the position statement specifically lists six domains, these were only intended to be examples, not a comprehensive list of all possible domains (Olszewski-Kubilius & Speirs-Neumiester, March 15, 2016, personal communication).

We performed two versions of our analysis. The first version considered the case of childhood giftedness (Model 4A); the second, adult giftedness (Model 4B). This is necessary because the NAGC position statement describes a developmental trajectory for the nature of giftedness from childhood to adulthood where childhood giftedness is understood in terms of *ability* or potential, largely relative to academic subjects that all students are engaged in learning (e.g., math, language arts, and reading). On the other hand, giftedness in adulthood is embodied

⁸ <https://www.nagc.org/resources-publications/resources/definitions-giftedness>

through advanced *achievement* and the quality of contributions to a specialized domain. Mere ability no longer defines giftedness, instead, the quality of one's performance relative to other adults in the discipline or domain becomes the relevant criterion. According to the position statement, "As individuals mature through childhood to adolescence, however, achievement and high levels of motivation in the domain become the primary characteristics of their giftedness" (NAGC, 2010, p. 1). The primary distinction between our childhood and adulthood analyses is how the "top 10% or rarer" reference group was determined, although some additional differences will also be described.

Method. The NAGC conception of giftedness defines the gifted as those in the top 10% of performance in any domain with its own symbol system or psychomotor domain. Therefore, the NAGC conception is essentially an "or" combination rule with many criteria. We can analyze this model using the same mathematics that were used for the "or" multiple-criteria system. By positing a multivariate normal distribution for the involved criteria, the giftedness rate can be calculated by numerically integrating the distribution to calculate probability. The cutoff ("top 10% or rarer") is specified in the definition. All that is needed to do the calculation is to decide on the number of and plausible correlations between the criteria.

A complicating factor in determining a plausible set of criteria is that two overarching categories of skills and abilities are posited: cognitive-academic and sensorimotor. Clearly, criteria are nested within these domains, and the correlation between criteria within these umbrella categories should be larger than the correlations for criteria between categories. So, for example, the correlation between mathematics achievement and reading achievement should be higher (due to both being within the "symbol systems" domain) than the correlations between math achievement and gymnastics or drawing ability.

Ad hoc “guesstimation” of the correlations between the variables is indefensibly arbitrary and is also unlikely to result in a mathematically acceptable correlation matrix, as correlation and covariance matrices must be positive definite. Our method for arriving at an acceptable and defensible correlation matrix was based on the confirmatory factor analysis (CFA) model, which assumes that responses on a set of observed variables are caused by a smaller number of unobservable latent variables. Observed variables that are caused by the same latent variable will be correlated with one another as a consequence of their mutual cause. We assumed, in accordance with *g* or CHC theory (Carroll, 1993), that performance across multiple symbolic domains will be partially driven by a single latent *cognitive ability* variable. Similarly, we assumed that performance in sensorimotor domains is driven by a single latent variable that could be called *athleticism*. The operation of these latent variables creates correlations between the specific domains that they influence. Additional correlation between families of domains could exist if the cognitive ability and athleticism latent variables are themselves correlated. See online supplement for additional detail regarding our use of the CFA model to generate reasonable values for correlations between the criteria. We manipulated three quantities to create different correlation matrices: the total number of criteria to be considered; the factor loadings that alter the within-domain correlations between criteria; and the inter-factor correlation that controls the strength of the relationship between symbol systems and sensorimotor skills. The NAGC conception is understood in terms of the four factors as follows:

- (1) *number of domains or criteria*: unbounded by the definition
- (2) *cutoff(s)*: 90th percentile
- (3) *distribution*: multivariate normal with correlations derived from a two-factor model
- (4) *combination rule*: “or”

Further detail regarding the method can be found in the Technical Appendix.

Results. The analysis was computed under many different conditions, including the number of domains (6, 10, and 20), the factor loadings (Λ , ranging from 0.10 to 0.90), and the inter-factor correlation between cognitive ability and athleticism; ranging from 0 to 0.6). We chose these values as reasonable upper and lower bounds of the quantities in question. The results are presented in Figure 5.

Insert Figure 5 about here

When only six domains (e.g., three symbol systems and three sensorimotor) were considered, the percentage of the population who is gifted ranged from about 27% if the correlations between factors is high and the factor loadings are high ($r = .6$ and $\Lambda_i = .9$) to 47% if the correlations between factors and the factor loadings are low ($r = .0$ and $\Lambda_i = .1$). For ten domains, the giftedness rate ranged from 32% to 65%. And with twenty domains, the rate varied from 38% to 87%.

This pattern is consistent with the properties of an “or” combination rule. The more domains that one considers “worthy” for inclusion as part of giftedness, and the lower the correlations between those domains, the more students who qualify as gifted. The giftedness rate approaches 100% as the number of qualifying domains is increased. Given the NAGC’s statement that roughly 10% of students are gifted, the very high rate of giftedness implied by the NAGC definition of giftedness indicates that this conception suffers from poor self-consistency.

Model 4B: NAGC Position Statement – Adulthood / Achievement-Centric Variant

Background. Adulthood involves specialization into a narrow area of expertise.

Typically, adults participate in only a very small number of the vast number of available domains of excellence. Although the available number of domains increases in adulthood as compared to childhood, limitations in time and attention necessarily limit an individual's participation to a relatively small number of domains. For example, a person might be an eminent lawyer, a reasonably accomplished chess player, an intermediate runner, and a novice pianist. But that person does not participate in vast numbers of other domains. The person is not a pilot, race car driver, ballet dancer, mathematician, triathlete, or poet. Developing skills in those areas would limit the opportunity to become accomplished in others.

Although the text of the NAGC position statement is vague on this point, we confirmed that the intended point of comparison for the "top 10%" consists of all of those individuals who participate in the domain, not all individuals regardless of participation (Olszewski-Kubilius & Speirs-Neumiester, March 15, 2016, personal communication). The importance of this distinction can be illustrated by considering chess. We focus on chess because chess skill can be measured using a uniquely well-elaborated rating system. Most people do not play chess seriously; probably only a minority of U.S. adults even knows the rules. The amount of skill required to be in the top 10% of chess ability could mean a relatively low level of skill if the comparison group includes all people, even those who do not know the rules. Or it could mean a very high level of skill if the comparison group includes only active, ranked chess players. Because all children attend school and take all basic subjects, using the entire student population as a reference group is appropriate. However, as illustrated above, because adults only participate in a small number of the possible domains, using only those who participate in that domain as a reference group is appropriate for the adulthood model.

Method. We applied the NAGC definition of giftedness to adults by asking, “what proportion of the population is in the top 10% of performance in symbol-system or sensorimotor domains *compared to the active practitioners of those domains*? To answer this question, we first estimated the population at risk for participation in a domain, as well as the distribution of developed skill and expertise in the domain, using information from the 2015 United States Chess Federation ratings database. Full details of the method are available in the Technical Appendix.

The key insight from the analysis of the chess ratings is an estimate of the discrepancy between the top 10% standard for ranked chess players versus the top 10% standard for the general population. Figure 6 displays this difference. Armed with this insight, we were able to estimate upper and lower bounds for the giftedness rate among adults under the NAGC conception. The general approach we used is similar to the NAGC childhood analysis with a few key distinctions. The first difference is the aforementioned higher standard. Based on the U.S. Chess Federation Elo data, we set the cutoff for the 90th percentile within a domain ($z = 1.645$) to be equal to the 99.99272nd percentile for the general public, which was equivalent to a cutoff of $z = 3.798$. In other words, what an expert practitioner of a domain would consider high performance is more than two standard deviations beyond what would be considered high performance for non-specialists.

Insert Figure 6 about here

As discussed in the Technical Appendix, we approximated the empirical distribution of Elo ratings for ranked players using a mixture of three truncated normal distributions. These

distributions corresponded to beginners ($M = 100$, $SD = 226$), intermediate players ($M = 676$, $SD = 210$), and experts ($M = 1529$, $SD = 399$). We estimated the distribution of Elo scores for the whole U.S. population of available chess players by assuming that 75% would be beginners, nearly 25% intermediate players, and a tiny fraction experts – where the expert proportion was assumed to equal twice the number of ranked expert players divided by the total size of the risk set. These assumptions are admittedly arbitrary, though we hope defensible, and the final result is robust to other assumptions about the beginner-to-intermediate ratio. (However, the result is quite sensitive to the assumed proportion of expert players in the general population).

Regardless, variation in the composition of the distribution of non-ranked chess players does not change the general point of this analysis – that what counts as excellent performance for those that intensely practice within a domain is far, far superior to what would be considered an excellent performance for those that do not specialize in the domain.

We hope that readers do not misunderstand; this analysis is leveraged on strong and potentially erroneous assumptions (see Appendix), as it is premised on the notion that we can estimate what the entire U.S. population's distribution of chess ratings would be if they had them and also that chess provides a representative example of the relationship between expert and novice performance that can be generalized to other domains. The point is not to take the precise numbers literally, but rather to elucidate the importance of keeping in mind the point of comparison for the “top 10% or higher.” This distinction has a tremendous impact on the giftedness rate.

The first difference between our NAGC adult (4B) and child (4A) models is that the benchmark for what is considered “gifted” changes from high ability in childhood to high achievement in adulthood. Because high achievement is now benchmarked against specialized

practitioners of the domain, the z-score cutoff for determining giftedness rises from $z = 1.645$ to $z = 3.798$.

The second difference is that many more domains of specialization are open to adults than to children. The analysis presented here considered 100 and 200 domain conditions. This increase in the number of disciplines demanded a different analytic approach, because the numerical integration function in R that we used to calculate the giftedness rate can accept a covariance matrix no larger than 20×20 in size. This restriction led us to abandon the analytical approach in favor of Monte Carlo simulation, which was based on a sample size of $n = 500,000$ per condition. The large number was used to control Monte Carlo error in the estimated giftedness rate; under low base rates even numerically small Monte Carlo error can comprise a large percentage of the estimated values.

The third change compared with the childhood NAGC analysis is that we believe that there are a proportionally smaller number of sensorimotor domains open to adults as compared with cognitive domains with their own symbol systems. Whereas the childhood analysis considered half of the domains to be sensorimotor, this analysis considered only 20% to be psychomotor with the remained 80% ‘symbol-system’ domains that load on the cognitive factor. Despite being a lower proportion of possible domains, this still represents a higher raw number of sensorimotor domains than used in the childhood giftedness example. To summarize, the simulation procedure was as follows:

- (1) Given the specified number of domains, the factor loadings, and the inter-factor correlation, calculate the covariance matrix Σ according to the confirmatory factor analysis model equations.

- (2) Randomly sample either 100 or 200 domain scores for $n = 500,000$ people from a multivariate normal distribution based on the covariance matrix computed in step (1).
- (3) Implement the “or” combination rule by comparing the maximum score across domains to the z-score cutoff of 3.798. If the maximum domain score exceeds the cutoff, designate the person as gifted.
- (4) Calculate the gifted proportion by dividing the sum of the number of gifted people by the number of people simulated ($n = 500,000$).

Results. Results from this analysis are presented in Figure 7. Please note that the maximum value on the y-axis of the Figure is 2%, not 100%. The two panels represent 100 and 200 domains. The most striking result of this analysis is the much lower rate of adult giftedness. When considering 100 domains, the giftedness rate ranged from 0.4% to 0.75%, with the slightly higher rate found under low factor loadings. For 200 domains, the giftedness rate ranged from .63% to 1.45%. The correlation between symbol system and sensorimotor domains had almost no effect, likely as a consequence of the relatively small number of sensorimotor domains included in this analysis. The estimated Monte Carlo error for this analysis was $\pm .007$, or 5.4% of the smallest value computed in the simulation.

Insert Figure 7 about here

These results are once again consistent with the general behavior of the “or” combination rule. Adding domains and reducing the correlation in performance across domains leads to an increase in the giftedness rate. The absolute rates are relatively low simply because of the very high standard of performance required to meet a within-discipline level of excellence (top 10%

of participating individuals with the assumption that the vast majority of individuals do not participate in any given domain). Although the NAGC conception (and the organization itself, as far as we know) takes no position on the rate of giftedness among adults, we think these estimates of adulthood giftedness seem somewhat too high. The results indicate that the implied rate adult giftedness is somewhere between approximately 1 in 66 to 1 in 200. We do not believe that 1 in 200 (much less 1 in 66) U.S. adults occupy the top 10% of performance in the domains that they practice.

Because the identity of the reference group is not explicitly identified in the NAGC statement, one could argue that we read too far afield when performing this analysis. If we had not adjusted the cutoff for giftedness to its much higher standard based on within-domain performance, it is clear that we would have estimated the adult giftedness rate to be virtually 100%.

Discussion

In this paper, we provide calculations of the giftedness rate under four of the most common conceptions of giftedness. As illustrated in Table 1, when excluding adult giftedness, the proportion of students qualifying as gifted can range from less than 0.5% (under an “and”-based multiple criteria conception) to as high as nearly 90% (under the NAGC “childhood” variant when low correlations between criteria are assumed). Moreover, a large range of plausible giftedness rates exists even within conceptions, largely depending on the correlation between the criteria that are deemed capable of providing evidence of giftedness.

It is important to note that the correlations between criteria are a more-or-less fixed property of the social and educational universe, not something under the control of the field’s theoreticians, unless more precise assessments with higher specificity and sensitivity can be

developed. The wide variation within conceptions described in this paper exists largely because these correlations (or factor loadings) are largely unknown. In the absence of reliable estimates for these values, we estimated the giftedness rates under what we hope are reasonable upper and lower bounds for them. Reliable empirical data on these correlations would allow us to produce much more precise estimates of the giftedness rate implied by each conception. Calculating these correlations from school records is further complicated by the two-stage process that many schools use in their gifted identification process (McBee, Peters, & Miller, 2016). When a nomination is required before identification assessments are administered to students, correlations calculated between these assessments will be suppressed due to range restriction (Crocker & Algina, 1986). Empirically calculated correlation coefficients would therefore be downwardly biased unless these data are collected from *all* students in the school, not just a subset of potentially gifted students.

Our perspective is that the high giftedness rate calculated under the childhood version of the NAGC conception is especially problematic. It is particularly troubling that the NAGC explicitly describes the giftedness rate as 6-10% of the population⁹ while also identifying the “all children are gifted” trope as one of the top myths about giftedness that must be challenged¹⁰. It is surprising and ironic to note that this is the logical consequence of its own conception of giftedness. The Renzulli Three Ring model is also quite permissive under some conditions, but it is at least plausible for this model to create its intended giftedness rate of approximately 15%.

Regarding the NAGC conception, it is understandable why such an organization would be hesitant to exclude any domain from the purview of giftedness; doing so will invoke conflicts

⁹ <https://www.nagc.org/resources-publications/resources/frequently-asked-questions-about-gifted-education>

¹⁰ <https://www.nagc.org/resources-publications/resources/myths-about-gifted-students>

of values that are subjective and difficult to reconcile. It is easy to avoid such disagreements when one perceives there to be no cost to adding additional domains. But our analysis reveals that there is a cost – the internal validity of the conception is substantially weakened. Our analysis suggests either that, unless one is willing to bestow the gifted label on a large fraction of children, then (a) the set of domains that one considers as evidence of giftedness must be rather circumscribed, (b) the standard for each criterion must be higher, (c) or an “or” combination rule should not be used in a theoretical definition. A definition asserting that roughly 10% are gifted, but that being gifted means being in the top 10% in any of a large set of subjects, skills, and endeavors, is stating a mathematical impossibility. Both of these statements cannot be true.

An alternative perspective would be to consider these results of the NAGC conception analysis (both versions) to be broadly compatible with a talent development worldview. From that perspective, although large numbers of young children may possess the raw ingredients of giftedness, few will achieve adult eminence (or even expertise) due to many of them frequently lacking experiences required to develop elite performance or expertise. Subotnik, Olszewski-Kubilius, and Worrell (2011) described factors such as the availability of advanced learning experiences, psychosocial support, and continual motivation to engage in deliberate practice as requirements to developing talent. Such requirements produce a “leaky pipeline” of talent development in which many have potential for greatness but few actually achieve it. The large number of gifted children suggested by the NAGC conception applied to childhood might be considered a positive feature of the model rather than a shortcoming. However, it would suggest that current identification practices in the U.S., which frequently select less than 10% of students for gifted education programs, are missing huge numbers of potentially qualified students. And indeed, such a stance would be compatible with recent work on the impact of the nomination

stage of the identification process which suggests that common identification practices may realistically be missing up to 70% of the qualified students (McBee et al., 2016) as well as recent work indicating the large number of students performing above grade level (Peters, Rambo-Hernandez, Makel, Matthews, & Plucker, in press). There may be a great many un-identified but highly talented individuals among us.

Implications

The movement to include more domains under the giftedness umbrella makes conceptual sense; individuals can pursue myriad domains (as demonstrated in model 4b). However, if the purpose of gifted identification is to select students to receive a particular educational service, this expansion of domains considered may need to be encumbered by the number of domains served. If the intent of gifted identification is to specify which students need special services in particular domains or subject areas, then conceptions must clearly articulate all these components. This does not mean that domains that are not included are not important. Rather, that in a world of limited resources, only so many domains can receive differentiated learning options in school. However, if the intent of gifted identification is to provide students (and their families) with information (and not necessarily services) about areas of talent, then broadening the domains used in identification may have merit.

As states, districts, and schools consider how giftedness should be conceptualized, the debate is often limited to which conceptualization should be implemented. As we show in this paper, vast variability can exist within conceptualizations as well. Thus, even if all schools relied on the same conceptualization of giftedness, implementation differences could lead to vastly different numbers (and types) of students identified as gifted. Each of the four features to quantify conceptions (domains, cutoffs, distributions, and decision rules) has meaningful

implication for determining who and how many students are identified as gifted. And the “who” and “how many” are two hallmarks of assessing whether schools are using appropriate identification methods. With such variability within conceptions, schools need to focus beyond simply *which* conception is selected, but also *how* it is implemented. Without doing so, their values and the validity of their practices may not match their intentions.

Limitations

We attempted to minimize any possible discrepancy between our understanding and the intent of those who developed the conception by contacting them for clarification on some quantification issues. All who we contacted were extremely helpful and forthcoming. It is our sincere hope that their message has not been meaningfully altered by our delivery. However, if this is the case despite our best intentions and efforts, such discrepancy further illustrates the vagueness and ambiguity that are inescapable features of verbal theories. Conceptions need to be unambiguously understandable if they are going to be implemented with fidelity.

Another limitation of this paper is that we only assessed a few of the many conceptions of giftedness. In particular, we note that the implications of Subotnik, Olszewski-Kubilius, and Worrell’s (2011) comprehensive umbrella conception of talent development were not directly assessed here, but doing so would be fascinating. Given the complexity, nuance, and psychological grounding of this longitudinal model of talent development, we suspect that developing a quantitative analog would be a difficult task characterized by an exceptional number of degrees of freedom. However, this effort could yield important insights into the nature of talent development.

Finally, our calculations assume either univariate or multivariate normality for the distribution of “scores” on the various criteria. If this assumption is violated then our results are

incorrect. The assumptions we made regarding upper and lower bounds for factor loadings and correlations have a strong effect on the results. In the absence of reliable empirical estimates of these parameters, we tried to bracket the true values of these parameters. Readers may disagree with our choices, but we have tried to be transparent about them. Interested readers may download our R code from the project page on the Open Science Framework (<https://osf.io/6e7g9/>) and alter the assumed correlations at will to replicate or extend our findings to assess alternative assumptions or models.

Conclusion

Many of the common conceptions in gifted education that make intuitive sense can have unanticipated effects on the number of individuals identified as gifted. We hope that the results of these analyses illustrate the value that quantitative or psychometric analysis can contribute to theoretical arguments. It is one thing to assert that a certain percentage of students is gifted according to some theoretical perspective; it is quite another to show that this percentage naturally arises from the conception's features. In addition, this type of analysis powerfully reveals ambiguities in the conceptions themselves. Such ambiguities must be clarified if conceptions of giftedness are going to be uniformly implemented in schools and evaluated for their ecological validity.

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Table 1

Summary Table estimating the proportion of individuals estimated to be identified as gifted under various conceptions of giftedness

	Model 1	Model 2A	Model 2B	Model 3		Model 4A	Model 4B
	IQ	M/C Two Criteria	M/C Three Criteria	Renzulli's Three Ring		NAGC Childhood	NAGC Adulthood
Brief Definition	Cutoff = gifted (top 5%)	2 Domains (top 5%)	3 Domains (top 5%)	Top 15% in all 3 rings in any general performance area		Top 10% in Symbol or Sensori-motor domain	Top 10% of those practicing a domain
Low Estimate	5%	OR Rule: 5% AND Rule: .25%	OR Rule: 5% AND Rule: .01%	7.60%	High loadings	6 domains: 27% 10 domains: 32% 20 domains: 38%	100 domains: 0.4% 200 domains: 0.6%
High Estimate	5%	OR Rule: $\approx$ 9.75% AND Rule: 5%	OR Rule: $\approx$ 14.3% AND Rule: 5%	47.70%	Low loadings	6 domains: 47% 10 domains: 65% 20 domains: 87%	100 domains: 0.75% 200 domains: 1.45%
Estimated Range	5%	.25- $\approx$ 9.75%	$\approx$ 0- $\approx$ 14.3%	7.6-47.7%		27%-87%	.4%-1.45%

Table 2

Parameters for the Renzulli Three-Ring Model Simulation

Factor Loadings					
		Unstable Condition		Stable Condition	
Ability		0.4		0.8	
Task Commitment (TC)		0.2		0.5	
Creativity		0.2		0.5	
Domain-General Correlations					
		Low Condition		High Condition	
		Ability	TC	Ability	TC
Task Commitment (TC)		0.1		0.5	
Creativity		0.1	0.1	0.5	0.5
Domain-Specific Correlations					
		Low Condition		High Condition	
		Ability	TC	Ability	TC
Task Commitment (TC)		0.2		0.8	
Creativity		0.2	0.2	0.8	0.8

Note: Factor loadings describe the strength of the correlation between the many domain-specific Ring traits and their domain-general counterparts. These are represented by the straight arrows in Figure 4. The domain-general correlations describe the strength of the correlations between ability, creativity, and task commitment independent of domain; these are represented in Figure 4 by the curve arrows on the upper half of the figure. The domain-specific correlations describe the strength of the correlations between ability, motivation, and task commitment within domains. They are represented by the multiple sets of curved arrows on the lower half of Figure 4.

Figure 1

Giftedness rates by cutoff under an IQ-only conception of giftedness

Note: The three shaded areas, beginning at IQ scores of 120, 130, and 140, encompass 9.1%, 2.3%, and 0.4% of scores, respectively.

Figure 2

Giftedness rate under two-assessment multiple criteria definition by combination rule and correlation between criteria.

Note: Cutoff on both assessments is set to the 95th percentile.

Figure 3

Graphical depiction of simulation approach for the Renzulli's 'Three Ring' model

Note: Only four of the eleven domains are shown here for clarity. Simultaneously being in the top 15% of all three 'rings' within a domain produces giftedness in that domain. Analysis computes probability of being gifted in at least one domain. The curved double-headed arrows in the between domains part of the model represent the correlations between domain-general traits. The straight arrows linking the between-domain traits to specific within-domain characteristics are factor loadings. The curved, double-headed arrows in the within-domains part of the model describe the correlations between the Ring characteristics within domains.

Figure 4

Proportion of gifted students according to Renzulli's 'Three-Ring' model by ring stability across domains (factor loadings), domain-general ring intercorrelations, and domain-specific ring intercorrelations

Note: See Table 2 for specific parameters used in each condition. Calculations based on a Monte Carlo simulation with $n=20,000$ repetitions and the eleven domains from Renzulli (2005). Estimated Monte Carlo error ± 0.16 .

Figure 5

Gifted proportion according to NAGC position statement (childhood / ability-centered version) by number of domains, the correlation between domains, and the correlation between factors.

Figure 6

Histogram of simulated Elo scores for the general population

Note: The dotted red reference line denotes the estimated 90th percentile Elo score for the general population (738); the dashed blue reference line indicates the observed 90th percentile Elo score for ranked chess players (1831).

Figure 7

Gifted proportion according to NAGC position statement (adulthood / achievement-centered version) by number of domains, the correlation between domains, and the correlation between factors.

Note: Scale of y-axis is extremely truncated; the maximum value is 2%, not 100%. Estimated Monte Carlo error $\pm .009$.