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In Defense of Intuitive Mathematical Theories as the Basis for Natural Number

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Abstract

Though there are holes in the theory of how children move through stages of numerical competence, the current approach offers the most promising avenue for characterizing changes in competence as children confront new mathematical concepts. Like the science of mathematics, children's discovery of number is rooted in intuitions about sets, and not purely in analytic truths.

Rips et al. present a thought-provoking assessment of the current debate on the origin of numerical concepts in language development. The article's main challenge is to the hypothesis that number word meanings are bootstrapped from systems of non-linguistic number representation. On many fronts the authors' arguments are compelling, and the points they raise offer important challenges to current models. I disagree, however, that problems with current developmental theories are the result of an inherently flawed approach, or that they are misled in their general trajectory. Instead, I argue that there is no better way to understand human knowledge of natural number than to witness its development in young children.

The problem is to determine the explananda to the theory of number knowledge. Rips et al.'s general thesis is that knowledge of natural number is defined in terms of an inferential system, and that therefore developmentalists should focus their efforts on evaluating how children come to manipulate numbers as syntactic objects, independent of their particular denotations. Stipulating that this particular knowledge should act as a metric of competence, however, is entirely arbitrary, and unprecedented in developmental psychology. In the study of human intuitive theories of biology, physics, and psychology, an implicit distinction is made between common sense understanding and scientific understanding. To untangle the two, developmentalists interested in human knowledge of biology, for example, have investigated children's initial intuitive theories, and how these theories react and change as children are exposed to new vocabulary and concepts (e.g., Carey 1985; Piaget 1929/2007).

Developmentalists have also looked back in scientific history to the earliest biological theories, and have examined parallels between conceptual change in ontogeny and

scientific history. This approach allows us to differentiate theories that come spontaneously to each child from those that are discovered scientifically and transmitted from generation to generation, whether explicitly in school or implicitly in how we talk and reason about biology in the presence of children. It is doubtful that we would benefit from stipulating what should count as “knowledge of biology,” since there is no such static object. However, we can make progress by asking how children initially reason about biological phenomena, how biological reasoning has changed over human history, and how children’s theories evolve as they are exposed to culturally transmitted scientific knowledge.

The same arguments can be made for mathematics. To stipulate what should count as “knowledge of number” would risk blurring the line between common sense understanding and scientific understanding (see Husserl’s discussion of psychologism in the study of logic; Husserl 1970). We can, however, ask about children’s intuitive knowledge, about the scientific history of mathematics, and about how children’s mathematical theories evolve as they are exposed to new vocabulary (e.g., the count list) or concepts (e.g., addition, subtraction, etc.). This approach does not differ from that of linguistics, which seeks to characterize core properties of language by separating properties that are exhibited by all human languages from those that vary from language to language. Using this approach, the best way to understand human knowledge (of number, biology, or language) is to distinguish the components that come naturally to each child from those that have evolved idiosyncratically over human history, and to study their interaction in child development. Developmental theories are perhaps short on details regarding how this happens, but ultimately are, I believe, on the right track.

What is this track, in the case of number? As Rips et al. note, there has been an

explosion of progress in understanding pre-linguistic systems of number. Also, we have made steady progress in our characterization of how children initially interpret number words. These studies have perhaps not convinced us of how core systems are implicated in number word learning, but they should convince us that children's first hypotheses about number make contact with representations of cardinalities, and that this connection only grows with age. It therefore remains possible that mental representations of number are always rooted in cardinalities, and that higher-order principles (such as commutativity) amount to beliefs about number, rather than being constitutive of number knowledge. Although cardinalities may indeed be irrelevant to defining natural number scientifically (i.e., in mathematics), we cannot assume a priori that concepts in the psychology of number take the same form as concepts in mathematics. To determine whether they do, we must examine human knowledge empirically, as it unfolds in development.

Progress in mathematics is owed mainly to the unbounded inferential power of its formal syntactic representations (many of which are beyond our intuitive grasp). Still, the science would arguably not exist if its basic truths did not satisfy human intuition, and were not relatable to our experience. Kant, hardly an empiricist, argued that our mathematical concepts are responsible to intuitions about objects and sets in the world: "The concept of twelve is by no means obtained by merely thinking the union of seven and five... We must go beyond these concepts, and have recourse to an intuition which corresponds to one of the two – our five fingers, for example..." (Kant 1781/1934, p. B15). This is not to say that our mathematical intuitions (e.g., pertaining to cardinalities) are derived from experience (*à la* Mill). Instead, the logic that governs experience may constrain the scientific theories that we formulate to explain it. To the extent that this is true, it should hardly be surprising if human

knowledge of number remains bound to intuitions about sets and cardinalities throughout development. Connecting “object talk” to formal symbols may be the intuitive basis not only for mathematics, but also for mental representations of number, as humans move from reasoning about physical objects as infants to reasoning about mathematical objects as adults.