Is the tilt-illusion in the stimuli?

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ABSTRACT

Interference between visual stimuli has been assessed mainly in regard to how such interactions may reduce stimulus power. The question of if, or to what extent, interference may change stimuli in other ways has been largely ignored. The present report asks if interference may alter the amplitude spectra of elongated contours so as cause their perceived orientations to be changed. Computations indicate that for two stimuli differing in orientation by between 5 and 40 degrees interference is largest for orientations between the peak orientations of the two stimuli which may cause their orientation tuning functions to be "pushed" apart. Since interference takes place in the stimuli and is independent of vision and the visual system it is possible, therefore, for the physics of stimuli to cause illusory tilt.

Key words: Interference, orientation, Gabor functions, visual illusion.

INTRODUCTION

Interference may occur when two signals are combined. Denoting the amplitude operator by $|\cdot|$ we get for two sine functions, $\sin(x + \rho_1)$ and $\sin(x + \rho_2)$ with ρ_1 and ρ_2 representing their phases, that $|\sin(x + \rho_1) + \sin(x + \rho_2)| < |\sin(x + \rho_1)| + |\sin(x + \rho_2)|$ when $\rho_1 \neq \rho_2 + 2n\pi$, with n being an integer (including 0). That is, the amplitude of the sum is smaller than the sum of the amplitudes when the functions are out-of-phase. In such cases the full amplitudes of the two functions cannot both be contained in the combined function. Thus, in such circumstances the amplitude of the first function, the second function, or both will be reduced when they are combined. Since by inverse Fourier Transform visual stimuli can be generated as combinations of sine functions interference may potentially reduce the stimulus power of any visual stimuli when they are combined (Skottun, 2017, 2018a, 2018b). This leads to the question of whether or not interference may also alter visual stimuli in ways other than by reducing their power. [FOOTNOTE 1]

The "Tilt-Illusion" refers to the phenomenon by which the perceived orientation of a contour is altered by the adding of a contour oriented to a somewhat different orientation. For two contours differing moderately in orientation their orientation difference tends to be perceived as larger than it actually is (Clifford, 2014). This has been widely attributed to mechanisms in the visual system (Blakemore et al., 1970; Schwartz et al., 2009; Coen-Cagli et al., 2012; Clifford, 2014; Huang et al., 2017; Kaneko et al., 2017; Nematzadeh et al., 2017; Westheimer, 2017). The question of if, or to what extent, interactions in the stimuli may cause illusory tilt has remained unexplored.

RESULTS

Interference between oriented stimuli was calculated based on 2-D Gabor functions (Fig. 1). Amplitudes at any given orientation was assessed by multiplying the amplitude spectrum with an oriented filter followed by summation. Figure 2a shows the amplitude as a function of orientation for two Gabor functions differing in orientation by 20 degrees (i.e., $\Delta\theta = 20 \text{ deg}$). (See Methods for details.)

Interference between two stimuli entails that the sum of amplitudes in the summed stimulus is smaller than the sum of the amplitudes in the two stimuli determined separately (Skottun, 2017, 2018a, 2018b). Were we to simply determine the sum of the amplitudes in the combination of the two Gabor functions we would obtain a measure of the combined amplitudes as a function of orientation. This, however, is not what we are seeking (e.g., this would give a monomodal function for small orientation differences). Rather what

Fig. 1

Figure 1. Illusory tilt induced with Gabor functions. **a**, A Gabor function with a vertical carrier. **b**, A function with a carrier tilted 20 degrees counterclockwise. **c**, The functions in panels a and b combined. The function on the right may appear slightly tilted clockwise. (The ability to see illusory tilt may depend on viewing conditions.) In the analyses the functions were overlapping but have here been displaced horizontally in order for them to be visually separable.

we are after is the amplitude as a function of orientation linked to each of the two stimuli *in the combined stimulus*.

In order to determine this the *Relative Amplitude Sum* (RAS) for a given filter orientation of ϕ was calculated as $\sum F_{1+2}(\phi)/(\sum F_1(\phi) + \sum F_2(\phi))$, where $\sum F_1(\phi)$, $\sum F_2(\phi)$, and $\sum F_{1+2}(\phi)$, are the sums of the filtered amplitude spectra for Stimulus 1, Stimulus 2 and the two combined, respectively. A Relative Amplitude Sum value below 1.0 indicates interference (RAS = 1 - interference)(Skottun, 2017). Relative Amplitude Sum as a function of orientation for the two functions in Fig. 2a is shown in Fig. 2b for five different phase differences between the stimuli; 0, $\pi/4$, $\pi/2$, $3\pi/4$, and π . In this plot two things are apparent:

- (1) The interference (reduction from RAS = 1) increases with difference in phase up to π . And, (2) The largest interference (largest reduction from RAS = 1) is found at orientations between the peak orientations of the two functions.

In order to estimate the tuning functions of each of the two Gabor functions when they are presented together it is necessary to take interference into account. This was done by multiplying the original tuning functions with the Relative Amplitude Sum functions. That is, the functions in Fig. 2a were multiplied with the functions in Fig. 2b. An example of this is shown in Fig. 2c for an Relative Amplitude Sum function obtained with a phase difference of π . As can be seen, the resulting functions (solid lines) are shifted outwards (i.e., away from each other) relative to the original curves (dashed lines).

Since the amount of interference depends on the phase difference between the two Gabor functions (Fig. 2b) in order to obtain an estimate of the effect of interference for all phase angles the procedure leading to the results in Fig. 2c was repeated for all five phase angles. After this the five resulting curves were added. The resulting functions are shown in Fig. 2d with solid lines. As can be seen, these functions are farther apart than the original functions shown with dashed lines. Thus, interference between stimuli with different but overlapping orientation tuning functions may cause their amplitude spectra to be shifted in regard to orientation.

In Fig. 3a is shown the magnitude of this shift for the tuning function of Stimulus 1 as a function of orientation difference between the carriers of the two Gabor functions ($\Delta\theta$). These results are for the average orientation (central tendency) calculated as $\sum \phi H(\phi)/\sum H(\phi)$ where $H(\phi)$ denotes the adjusted tuning function (the kind of function depicted with solid lines in Fig. 2d). This plot shows that for $\Delta\theta$ between 5 and 40-50 degrees interference causes the tuning functions to be shifted away from each other.

Fig. 2

Figure 2. The effect of interference. **a**, Example of amplitude as a function of orientation of two Gabor functions ($\sigma = 10$ elements; carrier frequency = 20.4 cycles/image; and a total image size of 256 elements) differing in orientation by 20 degrees (i.e., $\Delta\theta = 20 \text{ deg}$ in this example). **b**, Relative Amplitude Sum (i.e., $1 - \text{interference}$) as a function of orientation for the same two Gabor functions. Results are shown for five different relative phase values: 0 , $\pi/4$, $\pi/2$, $3\pi/4$, and π . Since $\text{RAS} = 1 - \text{interference}$ these functions thus show interference as a function of orientation. **c**, The orientation tuning functions in Panel a after they have been multiplied with the RAS function in Panel b for a phase difference of $\rho = \pi$ (solid lines). For comparison are shown the un-adjusted functions (dashed lines). As can be seen, multiplying with the RAS function causes the tuning functions to be shifted away from each other. **d**, In order to incorporate interference at all phase angles the tuning functions (functions in Panel a) were multiplied with each of the five RAS functions (each of the functions in Panel b) and then summed. The functions have been normalized to facilitate comparisons. As can be seen the adjusted tuning functions (solid lines) are farther apart than the original tuning functions (dashed lines).

For larger and smaller orientation differences there is however little or no effect. In Fig. 3b are re-plotted psychophysical data from Fig. 1b of Clifford (2014). These data and the results in Fig. 3a show clear similarities.

The observed shift in orientation results from interference reducing the amplitudes most severely at orientations between the peaks in the two tuning functions. This causes these functions to be "pushed" away from each other, as it were. The reason little or no orientation shift exists for large orientation differences is that in order for there to be interference the two stimuli need to have non-zero amplitudes at the same components (i.e., overlapping amplitude spectra, Skottun, 2018b). When the stimuli differ substantially in orientation there is little overlap between the amplitude spectra.

This lack of overlap explains the fall-off in orientation shift (induced tilt) for large $\Delta\theta$ s. Since interference between two stimuli requires that their amplitude spectra overlap it is of some relevance that the Tilt-Illusion is spatial frequency selective (Georgeson, 1973) as is the present interference effect (Fig. 3c).

In the case of the little or no shift in orientation found at the very smallest angles the reason for this is that the interference in these cases is broadly tuned in regard to orientation. Combined with the fact that the orientations of the carriers of the two stimuli are very close together this entails that interference mainly reduces the amplitudes uniformly with little shift in orientation.

Lest there be any confusion, it should be pointed out that the physical images of the stimuli are not being altered by combining them. What is being altered is their stimulus power. (For a discussion of "stimulus power" see Skottun, 2018a.)

DISCUSSION

Visual illusions are generally assumed to reflect mechanisms in the visual system. The present results suggest that it may be possible to generate visual illusions from interactions between elements *in the stimuli*. The shift in orientation tuning demonstrated here exists entirely in the stimuli and does not depend on the stimuli being seen or that they activate any visual system. The present analyses describe what one might call the "physics of the stimuli". [FOOTNOTE 2] What is shown here is that this may create illusory tilt. Illusory tilt is clearly a visual illusion and thus involves vision. This, however, does not mean that vision or the visual system *creates* the illusion. According to the present analyses vision is required to see the illusion but does not create it. This should not be taken to mean that factors in the visual systems may not also influence the illusory tilt as interference in the stimuli and visual factors are not mutually exclusive.

Fig. 3

Figure 3. Induced tilt as a function of relative orientation and spatial frequency difference. **a**, Interference induced shift as a function of $\Delta\theta$. The shifts are for the central tendency of the tuning function for Stimulus 1 (i.e., not the combined shift in the two functions together). **b**, Psychophysical data re-plotted from Fig. 1b of Clifford (2014). In both panels positive shift values indicate that the tuning functions of the two stimuli have been shifted away from each other. **c**, Induced tilt as a function of the carrier spatial frequency of Stimulus 2 relative to that of Stimulus 1 for $\Delta\theta = 20 \text{ deg}$.

It may very well be that psychophysical performance is not fully attributable to the physics of the stimuli. In fact, given the many nonlinearities in the visual system this is to be expected. However, in order to explore discrepancies between the physics of the stimuli and psychophysical performance one will need to determine the physical properties of the stimuli. The present report provides an example of how this may be done.

As for the question posed in the title – "Is the tilt-illusion in the stimuli?" – what can be said based on the present analyses is that interference in visual stimuli has the potential to create illusory tilt.

METHODS

Stimuli. The stimuli were pairs of Gabor functions. A Gabor function consists of a sinusoidal carrier multiplied with a Gaussian. Two-Dimensional Gabor functions were generated as:

$$f(x, y) = \exp(-((x - l/2)^2 + (y - l/2)^2)/(2\sigma^2)) \cos(\omega((x - l/2)\cos(\theta) - (y - l/2)\sin(\theta)) - \rho),$$

where l is the size of the image (which was 256 along both the x and y dimensions), σ is the standard deviation of the Gaussian, θ is the orientation of the carrier, ω is the spatial frequency of the carrier and ρ denotes its phase. In order to alter the orientation a value $\Delta\theta$ was added to θ for one of the Gabor functions giving $\theta_2 = \theta_1 + \Delta\theta$ where θ_1 denotes the orientation of one function and θ_2 denotes the orientation of the other. The phase, ρ , was held constant at 0 for one of the functions. For the other it was set to one of the following values: 0, $\pi/4$, $\pi/2$, $3\pi/4$, and π .

Oriented Filter. In order to assess the amplitude as a function of orientation the 2-D amplitude spectra were multiplied with an orientation selective filter after which it was summed (i.e., inner products between stimulus and filter were determined). The filter is given by the following *Mathematica* code:

```
filter = Table[0, {l}, {l}];
For[y = 1, y ≤ l, y ++,
  For[x = 1, x ≤ l, x ++,
    If[x ≠ l/2, ang = Abs[N[ArcTan[(y - l/2)/(x - l/2)]]], Continue];
    filter[[x, y]] = Exp[-(Abs[ang])^2/(2σf2)]];
]
```

where σ_f is the orientational width of the filter which was set to 0.05 radians corresponding to 2.9 degrees. (The variable *ang* is given a value in first line of code inside the inner loop.) The above code gives a vertical

filter. In the tests this orientation was held constant and the stimuli were rotated. That is, the relative orientation, ϕ , was varied by altering θ_1 and θ_2 while holding constant the difference, $\Delta\theta$, between θ_1 and θ_2 . The orientation, ϕ , was altered in 1 degree steps.

Analyses. Analyses were carried out using *Mathematica* (Wolfram Research). For each relative orientation, ϕ , the amplitude spectra of the two stimuli were separately multiplied with the oriented filter and the amplitudes for each stimulus were added. For the same ϕ this was also done for the two stimuli combined. Thus, for each relative filter orientation, ϕ , three amplitude sums were obtained: $\sum F_1(\phi)$, $\sum F_2(\phi)$, and $\sum F_{1+2}(\phi)$, where $F_1(\phi)$, $F_2(\phi)$, and $F_{1+2}(\phi)$ are the filtered amplitude spectra of $f_1(x, y)$, $f_2(x, y)$, and $f_1(x, y) + f_2(x, y)$, respectively. By varying ϕ the orientation tuning of the Gabor functions were determined from $\sum F_1(\phi)$ and $\sum F_2(\phi)$ (giving results as those shown in Fig. 2a) and the orientation tuning of the Relative Amplitude Sum (RAS) as a function of orientation was calculated as $\sum F_{1+2}(\phi) / (\sum F_1(\phi) + \sum F_2(\phi))$ (giving functions of the kind shown in Fig. 2b). For each $\Delta\theta$ value RAS functions were obtained for five different phase angles. That is, the RAS was determined with each of the phase values, $\rho = 0, \pi/4, \pi/2, 3\pi/4$, and π , for one of the carriers while the phase of the other was held constant at 0. (Note that θ_1 and θ_2 denote the orientations of the Gabor carriers and ϕ denotes angle of the filter relative to θ_1 .)

In order to estimate the effect of interference upon the orientation tuning functions each of the five RAS functions were multiplied with the tuning function for each stimulus giving five adjusted tuning functions of the kind shown with solid lines in Fig. 2c. These were then added to give a single adjusted tuning function, $H(\phi)$, incorporating interference at all five phase angles (as shown in Fig. 2d).

In order to quantify the shift in orientation the central tendency (mean angle) of the combined adjusted function for Stimulus 1 was calculated as $\sum(\phi H(\phi)) / \sum H(\phi)$, where $H(\phi)$ represents the adjusted tuning function for this stimulus. This value was then subtracted from the central tendency (mean angle) of the un-adjusted function for Stimulus 1 (calculated in the same manner, i.e. from the function labelled *Stim.1* in Fig. 2a) giving the induced tilt resulting from interference (i.e., the results shown in Fig. 3a).

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FOOTNOTES

1. Interference can also be demonstrated based on norms and wavelet coefficients (Skottun, 2018a). This entails that interference is not specific to the Fourier transform.
2. It has been suggested that interference does not take place in the stimuli but rather in filters which correspond to visual neurons. In order to counter this misconception it may be useful to consider the adding of two sine gratings of equal frequency, amplitude, and orientation but which differ in spatial phase by π (i.e., 180 degrees). In this case the two gratings cancel each other resulting in a uniform field without variations in luminance across space. This canceling does not depend on any filters or neurons but takes place in the stimulus independently of whether or not it is seen or activate any visual system. (For the adding of out-of-phase sine functions see Fig. 1 of Skottun, 2017). Also, the fact that interference can be demonstrated with norms of space domain stimuli demonstrates that interference is not dependent on filtering (Skottun, 2018a. See also Footnote 1).

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