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Comparing set-to-number and number-to-set measures of cardinal number knowledge in
preschool children using latent variable modeling

Yi Mou¹, Bo Zhang², Manuela Piazza³, Daniel C. Hyde²

¹Department of Psychology, Sun Yat-sen University

²Department of Psychology, University of Illinois at Urbana-Champaign

³Center for Mind/Brain Sciences, University of Trento

Contact: Dr. Yi Mou (mouyi5@mail.sysu.edu.cn) with questions

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Abstract

Children's understanding of the cardinal numbers before entering school provides the foundation for formal mathematics learning. Two types of tasks have been primarily used to measure children's knowledge of cardinal numbers: set-to-number and number-to-set tasks. However, there has been a continued debate as to whether the two types of tasks measure the same conceptual construct, allowing comparison and interchangeable use, or whether they measure different but related constructs. To answer this question, we analyzed the relation between task and item level performance on representative set-to-number (e.g., How-Many?) and number-to-set (Give-N) tasks in a large group of 3- to 4-year-old preschoolers ($N = 204$, median age = 3y 10m). By constructing and comparing models with different latent variable structures, we found that the best-fitting model was a bi-factor model, where performance on set-to-number and number-to-set tasks is best explained by both overlapping and some distinct aspects of cardinal number knowledge. Further analyses ruled out the idea that differences between tasks were due solely to non-numerical, general cognitive or language factors. Together these results suggest that set-to-number and number-to-set tasks have some commonalities but also retain at least some significant conceptual distinctness. Based on these results, we suggest these two types of tasks should no longer be used indiscriminately to inform theory or educational assessment of numerical abilities in preschool children.

Keywords: cardinal number, bi-factor model, Give-N task, How-Many? task

Comparing set-to-number and number-to-set measures of cardinal number knowledge in preschool children using latent variable modeling

Many children start to learn about cardinal numbers before they receive formal instruction in school. This cardinal number knowledge forms the foundation for acquiring more complex numerical and mathematics education. It is also used as a metric of immediate school readiness and predictive of mathematics achievement throughout formal schooling (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; vanMarle, Chu, Li, & Geary, 2014). In addition to its translational importance, the emergence of cardinal number knowledge has been used as a test case for prominent and opposing theories of continuity and change in cognitive development (Carey, 2009; Gelman & Gallistel, 1978; Le Corre, Van de Walle, Brannon, & Carey, 2006; Piaget, 1952). Due to its theoretical and translational importance, researchers have been seriously investigating the nature and development of early cardinal number knowledge for several decades (Barner, 2017; Gelman & Gallistel, 1978; Le Corre et al., 2006; Le Corre & Carey, 2007; Mix, Sandhofer, Moore, & Russell, 2012; Mou, Berteletti, & Hyde, 2018; Piantadosi, Tenenbaum, & Goodman, 2012; Sarnecka & Carey, 2008; Sarnecka, Goldman, & Slusser, 2015; Spelke, 2017; vanMarle et al., 2014; Wynn, 1990, 1992).

In most of this work, one of two types of tasks is employed: a number-to-set task (e.g., Give-N, Give-a-number; Le Corre et al., 2006; Sarnecka & Carey, 2008; Wynn, 1992) or a set-to-number (e.g., How-Many?, What's-On-This-Card?; Gelman, 1993; Le Corre et al., 2006; Wynn, 1992). Despite extensive use and broad application, many fundamental psychometric properties of these tasks remain unclear. Here we ask one basic psychometric

question about these tasks: do set-to-number and number-to-test tasks assess the same mental construct (or not)?

It is clear from both the developmental trajectory and theories of conceptual development that the cardinal number knowledge is complex and multifaceted. For children in industrialized cultures, learning of cardinal numbers starts as children begin to learn to say the number words in order, around 2 years of age. Over the next several years, children typically show stage-like progression in their understanding of cardinal numbers. They progressively come to understand the meanings of the first few smaller numbers words, “one”, then “two”, then, “three” sequentially and in a seemingly piecemeal fashion without necessarily understanding how to use counting to determine cardinal number or the numerical relationships between number words. Months after learning the smaller cardinal number words up to about four, children appear to make a rapid conceptual gain, generalizing their understanding to larger number words (six or more; Carey, 2009; Le Corre et al., 2006; Wynn, 1990, 1992). Thus, children’s learning of smaller number words (less than four or five) seems qualitatively different from learning of larger number words, and correspondingly, researchers have proposed distinct cognitive mechanisms underlying these learning processes (see Barner, 2017; Baroody et al., 2017; Carey, 2009; Spelke, 2017 for discussion on this issue). More recent work suggests that the developmental trajectory of cardinal number knowledge may extend even further, as children need additional time to understand increasingly larger numbers (Cheung, Rubenson, & Barner, 2017).

Most theoretical work implicates a variety of components required to have a comprehensive understanding of the cardinal numbers (e.g., Carey, 2009; Fuson, 1988;

Spelke, 2011). More specifically, most agree that children have to understand that a set of items has an exact numerical value (or a cardinality); sets that can be put into perfect one-to-one correspondence have the same number of items (equinumerosity); each number word has an exact numerical value (a cardinal meaning); that the list of number words can be employed generatively to determine the cardinality of any set (the cardinality principle), and each number (N) has a unique successor ($N + 1$, the successor function; Barner, 2017; Cheung et al., 2017; Carey, 2009; Fuson, 1988; Gelman, 1993; Le Corre & Carey, 2007; Sarnecka, Goldman, & Slusser, 2015; Sarnecka & Wright, 2013; Wynn, 1990, 1992).

Number-to-set and set-to-number tasks are both designed to measure cardinal number knowledge. However, it cannot be assumed that both types of tasks measure the same aspects of cardinal number knowledge given its multifaceted nature. Number-to-set tasks require children to create a set of objects for a given number, whereas the set-to-number tasks require children to determine the number of a given set (see Baroody, Lai, & Mix, 2017; Fuson, 1988; Odic, Le Corre, & Halberda, 2015). Although there are many subtle variants, one typical example of a number-to-set task is the Give-N task, where children are presented a collection of items (e.g., toy fish) and asked to give the experimenter a particular number (N) of them (e.g., “Can you give me five fish?”; Le Corre et al, 2006; Sarnecka & Carey, 2008; Wynn, 1990, 1992). Other variants include asking a child to pick the set from two or more options that matches the number word requested (e.g., Benoit, Lehalle, Molina, Tijus, & Jouen, 2013; vanMarle et al., 2014; Wynn, 1992). In contrast, one typical example of the set-to-number task is the How-Many? task, where children are presented a set of items (real objects or items on a card) and are asked to count the items to determine the cardinality with

questions like “how many objects are there?” (the How-Many? Task; Gelman, 1993; Le Corre et al., 2006; Wynn, 1992). Set-to-number tasks also have several variants. Another common version of this task presents the same situation but avoids mentioning number explicitly by asking, “What is on this card?” (the What-is-on-this-card? Task; Gelman, 1993; Le Corre et al., 2006). In yet another variant, the experimenter counts the objects and asks children if they counted correctly (Gelman & Meck, 1983; Sarnecka & Carey, 2008).

The question of whether number-to-set and set-to-number tasks measure the same construct has been raised repeatedly over the years but still remains unanswered. On the one hand, performance on the two tasks is often highly correlated and often not statistically different (Benoit et al., 2013; Le Corre et al., 2006; Le Corre & Carey, 2007). Based on these sorts of results, it has been proposed that the tasks measure the same conceptual construct and can be used interchangeably (Le Corre & Carey, 2007). However, it should be noted that null results, or a lack of differences in a summary score between tasks, are not strong evidence for a common construct. On the other hand, systematic differences in performance between the two types of tasks are often found despite high correlations (Baroody et al., 2017; Fuson, 1988; Schaeffer, Egges, & Scott, 1974; Wynn, 1992). For example, summary scores are typically higher on set-to-number than number-to-set tasks (Baroody et al., 2017; Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989; Opfer, Thompson, & Furlong, 2010; Sarnecka & Carey, 2008; Le Corre et al., 2006; Wynn, 1992). Furthermore, recent work suggests there may be item level differences (i.e., item refers to a given test number in a task, e.g., “three”; Baroody et al., 2017). Further research is needed to understand how potential item level differences between tasks correspond to known differences in the developmental trajectory of

learning the first few smaller numbers compared to learning larger numbers mentioned above (Wynn, 1992).

Some propose that the differences in performance on the two types of tasks reflect deeper conceptual differences between them (see Baroody et al., 2017; Fuson, 1988; Frye et al., 1989; Mix et al., 2012; Sarnecka & Carey, 2008). For example, it may be that the process of ascertaining the cardinality of an existing set is conceptually distinct from the process of producing a set that matches the exact numerical meaning of the requested number word (see Fuson, 1988). One specific theory along these lines proposes that cardinal number knowledge includes the conceptual abilities to subitize, or quickly recognize, the cardinal number of objects for labeling or producing small sets of 3 or less (i.e., verbal subitizing), to use a counting strategy to determine how many items are in a set by using the last word in the count (i.e., count-cardinal concept), and to precisely produce a collection of a given number (i.e., cardinal-count concept; Baroody et al., 2017; also see Fuson, 1988). These conceptual components are thought to build on one another as cardinal number knowledge develops. Accordingly, set-to-number tasks measure the earlier developing count-cardinal concept and number-to-set tasks measure the more advanced cardinal-count concept (Baroody et al., 2017; also see Fuson, 1988).

Another possible reason for systematic differences in performance between tasks is that number-to-set tasks may tap more explicitly into children's conceptual understanding of cardinal meaning of number words, while set-to-number tasks may be solved by some children through procedural knowledge of the counting routine (Frye et al., 1989; Mix et al., 2012; Sarnecka & Carey, 2008). Under this sort of view, set-to-number tasks may simply test

if a participant can count items and use the last number word as the answer, which may not require a deeper understanding of cardinal number (see Wynn, 1992 for a discussion). Thus, differences between tasks may be due to differences in procedural versus conceptual knowledge of cardinal numbers, where set-to-number tasks may rely more on procedural knowledge and overestimate cardinal number knowledge.

Finally, performance differences may not reflect differences in numerical constructs of tasks at all, and instead may arise from a variety of non-numerical factors such as language, general cognitive, or other task demands (see Baroody et al., 2017; Le Corre et al., 2006). Working memory, inhibition, conflict processing, and linguistic abilities are known to be associated with the acquisition of cardinal number knowledge (Cirino, 2011; LeFevre et al., 2010; Mou, Berteletti, & Hyde, 2018; Piazza, Pinel, & Cohen, 2003; vanMarle, Chu, Li, & Geary, 2014). Creating a set through counting in number-to-set tasks requires children to keep a target number in working memory, compare the number words being produced through counting to the target number, and stop the count when the target number matches the count. Set-to-number tasks also involve these non-numerical cognitive abilities, as they require participants to verbally enumerate each item presented, remember which items have already been counted, only count each item once, and stop/inhibit the counting procedure after all items have been enumerated. However, participants in number-to-set tasks need to count items and generate a set simultaneously, whereas those in set-to-number tasks just need to count the items already generated by the experiment (Le Corre et al., 2006; Sarnecka & Carey, 2008). Thus, number-to-set tasks may be more demanding on working memory, executive functions, and language processing compared to set-to-number tasks (Baroody et

al., 2017; Frye et al., 1989; Le Corre et al., 2006; Wynn, 1992). No studies to date have rigorously measured and tested the contribution of general cognitive and linguistic abilities to explaining the relationship between set-to-number and number-to-set tasks, despite the fact that these abilities can be reliably measured in preschool children (e.g., Dunn & Dunn, 2007; Willoughby et al., 2012).

In sum, the existing evidence is ambiguous as to whether set-to-number and number-to-set tasks measure the same construct or not. Some studies highlight similarities, while others highlight differences (Baroody et al., 2017; Benoit et al., 2013; Le Corre et al., 2006; Wynn, 1992). To the extent that there are differences, it still remains unclear if these differences reflect true differences in the assessment of conceptual-numerical aspects of the task or whether differences can be attributed to non-numerical cognitive and linguistic demands of the tasks. Understanding the relation between these tasks is important, as a majority of inferences in the literature regarding preschool cardinal number knowledge and its relation to later mathematical and cognitive development hinge on data from them. If the two types of tasks capture the common aspects of cardinal number knowledge, then existing results and educational assessments using either task can be readily compared and future work should use these tasks interchangeably. If the two tasks capture unique aspects of cardinal number knowledge, then more care needs to be taken in comparing studies in the existing literature and with task choice for research and educational assessment in the future.

Here, we attempt to address these issues by examining the psychometric properties of set-to-number and number-to-set tasks through comparison of models representing different theoretical alternatives. Specifically, we measured early number understanding using

comparable electronic versions of a Give-N and a How-Many? task to represent number-to-set and set-to-number tasks, respectively. We also measured general cognitive abilities including working memory, conflict processing, response inhibition, and linguistic abilities to consider general sources outside of the numerical knowledge domain (Baroody et al., 2017; Negen & Sarnecka, 2012).

We then applied confirmatory factor analysis to analyze data from these two types of tasks (Brown, 2006). More specifically, we built, tested, and compared models with different latent variable structures corresponding to different alternative hypotheses regarding the relationship between tasks (see Data Analysis section for further details on rationale and approach). If the two tasks measure the same construct, then a model that includes a single, common latent factor should best explain performance within items and across tasks (i.e., single-factor model; see Data Analysis for details). This would provide direct support for the idea that the tasks are interchangeable. On the other hand, if the tasks measure independent constructs or distinct aspects of cardinal number knowledge, then a model with a latent variable for each task will better explain performance within items and across tasks (i.e., a distinct-factors model). Finally, if the tasks measure some overlapping and some distinct aspects of cardinal number knowledge, then a model with both a common latent variable (indicating the overlapping aspects of the tasks) and distinct, task-specific latent variables (indicating the distinct aspects) will best explain performance (i.e., a bi-factor model). Both the distinct factors and the bi-factor model would provide some indirect evidence that cardinal number knowledge may involve multiple, conceptual constructs.

In addition, we tested the extent to which general, non-numerical cognitive, linguistic,

and maturational variables explained variance in the latent variables in the best-fitting model. This would allow us to further infer whether and to what extent similarities and/or differences between tasks were due to task demands rather than number-specific knowledge.

Method

Participants

Two hundred and four children (110 girls) participated in this study ($\text{Mean}_{\text{age}} = 3\text{y } 10\text{m } 17\text{d}$; $\text{SD}_{\text{age}} = 61\text{d}$; $\text{Median}_{\text{age}} = 3\text{y } 10\text{m } 6\text{d}$; $\text{Range}_{\text{age}} = 2\text{y } 10\text{m } 24\text{d} - 4\text{y } 3\text{m } 18\text{d}$). There was only one child younger than 3 years old (2y 10m 24d), and the next youngest child was 3y 7m 19d old. The youngest child was included as we had no a priori reason for removal. Nevertheless, removing the youngest child does not change the findings of this study. Ten additional children participated in this study but were excluded from data analyses due to evidence of a neurological or developmental disorder ($n = 3$; parent reports), for not understanding instructions in English ($n = 1$), inaccurate birthdate information ($n = 2$), or not completing both cardinal number tasks ($n = 4$).

Children were recruited from a dynamic database of families with children of all ages who were living in and around a midwestern college town in the United States and who had previously expressed interest in participating in cognitive development research through one of our community outreaches or recruiting events. Families from this pool with children around 3-4 years of age were called and asked if they were willing to make a lab visit to participate in our study. Participants were comprised of those whose parent or guardian was called and agreed to bring them into the lab for the study. We obtained additional demographic information about race/ethnicity, parent education, and family income from 146 of the families who participated

in this study via parental self-report (See Table 1 for details). Written informed consent for the child's participation was obtained from the parent or legal guardian.

Procedure

Data were collected on the first visit of a larger, multisession experimental study of numerical and cognitive abilities that was conducted over several years. Several recent publications addressing other issues have already been published from the larger dataset (citations redacted for purposes of double-blinded review process). Given the resource intense nature of assessment and the sample size, it was necessary use multiple experimenters for data collection, including 2 postdoctoral researchers, 3 post-baccalaureate research assistants, and advanced undergraduate experimenters in the lab. All numeracy assessments were given to each child by the same experimenter, but the experimenter could vary between children and for the non-numerical assessments. All experimenters received extensive, coordinated training and continual supervision according to a common testing protocol. Moreover, numerical and cognitive tasks were computer-based to increase consistency of the procedure and minimize experimenter-child interactions. Experimenters gave only minimal verbal instructions. Feedback was given by the computer program automatically (see individual task descriptions for specifics).

Given the number and complexity of assessments administered to each child, we decided to present them in a fixed order, as order effects and systematic fatigue were possible and a full counterbalancing was not possible with this many tasks (see Geary, vanMarle, Chu, Rouder, Hoard, & Nugent, 2018 for similar approach). We did so to maximize consistency of assessment across children in order to directly compare all children, even if at the cost of not

being able to understand potential order effects.

Cardinal number knowledge assessments

Give-N (GN). A version of the Give-N task was used as our number-to-set task. This task was a modified computer version of the classic Give-N task (Wynn, 1990, 1992). In this task, children were asked to generate a number of objects as requested. The task started with a trial with the target number “one”, in which ten identical items (e.g., butterflies) were presented in a row on the top of the computer screen and a voice generated from the computer program said “one”. Children were asked to give one item by pressing the space bar on the keyboard to move an item from the top to the middle of the screen. One press moved one item. Children received feedback from the experimenter for this trial. Then, the test numbers (i.e., test items) from “two” to “eight” were presented in a random order and children did not receive feedback for these numbers. Next, two more test blocks were administered, each containing test numbers from “one” to “eight” and these numbers were presented in a random order within a block (24 trials in total were involved in the 3 blocks). Each block had a different type of item (butterflies, fish, and birds). Children were encouraged to count aloud when they pressed the button, and they were allowed to restart a trial if they thought they gave a wrong number. Children’s responses to each item were recorded, and both the overall accuracy of the task (percent correct out of 24 trials) and the accuracy for each item (out of 3 trials per item) were used for data analyses.

We used accuracy as a summary score for this task (as well as for the following How-Many? task), rather than the highest number the child could understand (or “knower level”, as some other studies have), for two reasons. First, the operational definitions of the “highest

number known” vary widely across studies and research groups (e.g., Le Corre et al., 2006; Lee & Sarnecka, 2011; Piantadosi, Jara-Ettinger, & Gibson, 2014; Wynn, 1992), making it challenging to definitively compute and compare across studies. Accuracy, in contrast, can be objectively computed and directly compared across studies. Second, a summary score of “highest number known” does not allow the examination of variance at the item level within a task or between tasks critical to our modeling approach, but accuracy does.

How-Many? (HM). A version of the How-Many? task was used as our set-to-number task. This task was a modified computer-based version of the classic How-Many? task (Le Corre et al., 2006; Wynn, 1992). In this task, children were presented with a number of objects on a computer screen and these objects were randomly placed within a rectangle envelope grid. Children were asked “How many X (where X could be apples, butterflies or fish)? Can you count them?” Children were encouraged to use their fingers to point and count the items one by one. The last number word the child produced was taken as their answer, and the experimenter recorded this number by pressing the corresponding number key on the keyboard. Many ask the “How many?” question after the counting process occurs in an attempt to elicit a distinct cardinal value answer. We decided not to do this, as asking “How-many?” after counting seems confusing to some children in our pilot testing of the task. Others have also raised this concern and provided empirical support for the idea that pragmatics of asking the questions again may lead some children to think their first answer was not correct and/or they must give a different answer (Gelman, 1993; Greeno, Riley, & Gelman, 1984; Le Corre et al., 2006; Sarnecka & Carey, 2008).

We started the task with a practice trial, in which one apple was presented on the

computer screen and children were asked with the “how many” question. Children received feedback for this trial (e.g., “there is one apple”). After the practice trial, three test blocks were given, each containing 8 test numbers (i.e., test items) from “one” to “eight” and these numbers were presented in a random order in a block. Each block contained a different type of item (apples, butterflies, and fish). Participants did not receive feedback from the experimenter in these test blocks. The child’s responses to each item were recorded by entering the response into the computer, and both the overall accuracy of this task (percent correct out of the 24 test trials) and the accuracy for each item (out of 3 trials for each item) were included for data analyses.

Vocabulary assessment

Peabody Picture Vocabulary Test IV. Children’s receptive vocabulary was assessed with the standardized vocabulary task, *Peabody Picture Vocabulary Test IV* (Dunn & Dunn, 2007), and the standard test procedure was conducted. The raw scores of this test were recorded and included for data analyses.

General cognitive assessments

Children’s general cognitive abilities, including working memory, conflicting processing, and response inhibition, were assessed with the following tasks in the following order.

Working Memory. Children’s visual working memory was measured using a modified version of the classic N-back memory task (e.g., Kirchner, 1958). In this task, images of objects (e.g., sandwich, doll, etc.) were sequentially presented on the screen and children were asked to press a button when the same image was repeated. Among the total 60 images presented in the task, six images were repeated immediately (1-back) and six additional

images were repeated after one novel image (2-back). Each image was presented for 2 seconds and images were separated by 500 msec. The experimenter and the child together completed the first sequence of images until the first repeated image presented. During the first sequence, the experimenter said “*different, different...*,” and “*Same! So we press the button!*” when the first repeated image was shown. Then, children completed the remaining test trials without receiving feedback from the experimenter. A sound effect obtained off the internet was also produced by the computer as feedback for correct responses (ascending chimes) and no additional feedback was given for errors or misses. Given that this and other general cognitive tasks were administered at the end of the test session, feedback was given to maintain children’s interest and attention. Children’s percent correct for the 12 repeated images were used to score their performance.

Conflict Processing. Children’s executive processing of conflicting information was measured with a modified version of the Simon task (Willoughby et al., 2012). In this task, an arrow was presented on the computer screen and children judged the direction the arrow pointed in (left or right) by pressing the corresponding keys on the keyboard. There were 3 blocks, with the first two serving to solidify the instructions and a third test block to measure spatial conflict processing. In block 1, arrows were presented in the center of the screen (8 trials). In block 2, arrows were on the left or the right side of the screen and always pointed congruently with the location in which they were presented (e.g., an arrow that was presented on the left of the screen always pointed to left) (14 trials). In the test block, arrows were presented on the left or the right side, but pointed either congruently or incongruently (e.g., an arrow that was presented on the left side of the screen pointed to the right) with the

location they were presented (20 trials). In the incongruent trials, children received conflicting information from the location and the pointing direction of the arrow, and were required to resolve the conflicting information and focus on where the arrow pointed.

Different sound effects found on the internet were associated with correct (repeating sonar-like sound) and incorrect responses (descending slide whistle) to provide feedback. To focus on children's ability to process conflicting information, only the accuracy of the (third) test block was included in the analyses. However, a review of accuracy for the first and the second practice blocks were .93 (SD = .13) and .91 (SD = .11), respectively, suggesting that children understood the instructions of the task well.

Response Inhibition. Children's response inhibition was measured with a go/no-go task (Durstun et al., 2002; Willoughby et al., 2012). In this task, pictures of cartoon animals were sequentially presented on the computer screen, and children were asked to press a button on the keyboard for every animal they saw (i.e., the *go* trials) and not press when a snake was presented (i.e., the *no-go* trials). There were 15 snake pictures among 60 pictures in total, and the snake pictures were separated by 1, 3, or 5 pictures of other animals. Each picture was presented for 2 seconds and the pictures were separated by 250 ms blank screen. Different sound effects found on the internet were associated with correct (ascending chimes) incorrect responses (cartoon cork pop sound for pressing button when the snake was shown). The percent correct on the *no-go* trials (out of 15 trials) was used to score children's performance.

Data analysis

We first examined the descriptive statistics and correlations on accuracy between the two

cardinal number knowledge tasks. We then used a latent variable model comparison approach to examine whether the two tasks are better explained by equivalent or different latent constructs. To do this, we built and compared three models with different latent structures (see Fig 1). The first was a single-factor model, corresponding to the hypothesis that performance in the number-to-set and set-to-number tasks is best explained by a common construct. In the single-factor model, each item of the two number tasks (i.e., each test number in the number-to-set and the set-to-number tasks) loads onto a single general latent factor. The value of an item loading a latent factor quantifies the strength of the relationship between the performance on that particular item and the latent factor (see Brown, 2006). The second was a distinct-factors model, corresponding to the hypothesis that performance in the number-to-set and set-to-number tasks is best explained by two distinct but developmentally correlated constructs. In the distinct-factors model, each item of the two number tasks loads onto a task-specific latent factor. Furthermore, a correlation between the two latent factors was assumed and further estimated. The third was a bi-factor model, which included a common (or shared) latent variable as well as distinct latent variables for each task. In the bi-factor model, each item of the two number tasks loads onto a single general factor, and items within each task additionally load onto task-specific factors. The general factor represents the common variance shared across all items of the two tasks (i.e., the commonality between the two tasks); the specific factors represent the additional variance shared among items within each task (i.e., the unique part of each task) after accounting for variance associated with the general factor. This model corresponded to the hypothesis that performance in the two tasks is best explained by both shared and distinct variance of the number-to-set and set-to-number

tasks (Reise, 2012; Zhang, Sun, Cao, & Drasgow, in press). The bi-factor model provided an even better fit to the data than the distinct-factors model (or the single-factor model). Armed with the bi-factor model, we further tested the nature of the latent variables by regressing general cognitive and linguistic abilities on them.

In these analyses, we reported the following goodness-of-fit indexes to evaluate how different models fit the observed data: chi-square statistic, Root Mean Square Error of Approximation (RMSEA), Standardized Root Mean Square Residual (SRMR), Tucker-Lewis index (TLI) and Comparative Fit Index (CFI). These are the standard indices to be reported when evaluating model fit (Lacobucci, 2010; West, Taylor, & Wu, 2012). According to the rule of thumb suggested by Hu and Benler (1999), CFI and TLI greater than .95, SRMR less than .08 and RMSEA less than .06 would indicate good fit. Unacceptable goodness-of-fit metrics are suggestive of model misspecification and should weigh into both model evaluation and model comparison (Bentler, 2007). When comparing models, we considered both the chi-square difference test and the model fit changes. Specifically, TLI difference greater than .01, CFI difference greater than .01, RMSEA difference greater than .015 and BIC difference greater than 10 were indicative of meaningful differences in fit (Chen, 2007; Gignac, 2007; Raftery, 1995). Satorra-Bentler's corrected chi-square was computed by specifying the estimator as MLM to handle data non-normality (Satorra & Bentler, 2001).

To compare models, corresponding chi-square difference tests were carried out following procedures proposed by Asparouhov and Muthén (2006). No residual correlations were allowed in these models. All the analyses were performed in RStudio (Version 0.99.903; RStudio team, 2015), and the factor analyses were performed using the R package "lavaan"

(Rosseel, 2012).

The requirement for inclusion was that children complete both the Give-N and How-Many? tasks. However, a small proportion of data were missing for working memory (4.4%), conflict processing (6.4%), response inhibition (3.4%) and vocabulary (2.5%) tasks, respectively. The missing data were handled with a Full Information Maximum Likelihood (FIML) procedure, which has been shown to produce more accurate estimates of model parameters than other common strategies such as listwise deletion, pairwise deletion, or similar pattern imputation (Enders & Bandalos, 2001; Enders, 2001).

Results

Descriptive statistics

At the task level, accuracy on Give-N and How-Many? were highly correlated, $r = .77$, $p < .001$ (Table 2). Children were overall more accurate on the How-Many? task than the Give-N ($t(203) = 4.572$, $p < .0001$, Cohen's $d = .403$), replicating previous work (Le Corre et al., 2006; Wynn, 1992). Accuracy on the two tasks was also highly correlated at the item level for all items except test number “one” (see Table 3). The lack of a correlation for “one” across tasks was likely due to a ceiling effect and associated reduced variance compared to the other numbers¹. Item level accuracy on How-Many? was better than Give-N on items 2, 3, 4 and 5 (the α level of significance was adjusted with Bonferroni correction to control the familywise type I error rate; $\alpha = .05/8$). Performance on both the Give-N and How-Many? correlated with children's scores of the general cognitive and the linguistic tasks (see Table

¹ Removing number “one” from the two tasks does not change the conclusions of the model comparison analyses.

2).

Model evaluation and comparisons

Three models with different latent variable structures were fitted and compared to identify the one that best characterized the relationship between the two cardinal number knowledge measures (Fig 1). Standardized factor loadings for each model are displayed in Table 4.

The single-factor model tested the idea that both tasks measure the same construct, where all the 16 items of the two counting tasks (8 items for each task) loaded on a common latent factor (the single-factor model in Fig 1). The single-factor model did not fit the data well ($\chi^2 = 367.206$, $df = 104$, RMSEA = .111, SRMR = .081, TLI = .792, CFI = .820), suggesting that the two tasks do not likely measure the same construct.

The distinct-factors model tested the idea that the two tasks measure different constructs that may or may not be correlated with one another. For this model, the Give-N items and How-Many? items loaded on two separate latent factors, and the correlation between the two factors was freely estimated (the distinct-factors model in Fig 1). The fitness for the distinct-factors model was also inadequate ($\chi^2 = 274.184$, $df = 103$, RMSEA = .090, SRMR = .078, TLI = .864, CFI = .883), but was nevertheless significantly better than that of the single-factor model ($\Delta\chi^2_1 = 67.242$, $p < .0001$, Δ RMSEA = .021, Δ TLI = .072, Δ CFI = .063, Δ AIC = 129.225, Δ BIC = 125.906).

The bi-factor model tested the idea that the two tasks measure some common and some unique aspects of cardinal number knowledge. For this model, all of the 16 items loaded on a common latent factor and items from Give-N and How-Many? were also allowed to load on

two specific latent factors (i.e., the Give-N and the How-Many? specific latent factors) (the bi-factor model in Fig 1). Following common practice of bi-factor modeling (Reise, 2012), all the three latent factors are set orthogonal to each other. The bi-factor model fit the data relatively well ($\chi^2 = 175.071$, $df = 88$, $RMSEA = .070$, $SRMR = .056$, $TLI = .926$, $CFI = .946$), and significantly better than the distinct-factors model ($\Delta\chi^2_1 = 75.005$, $p < .0001$, $\Delta RMSEA = .020$, $\Delta TLI = .062$, $\Delta CFI = .063$, $\Delta AIC = 129.832$, $\Delta BIC = 80.06$) and the single-factor model ($\Delta\chi^2_{15} = 170.726$, $p < .0001$, $\Delta RMSEA = .045$, $\Delta TLI = .134$, $\Delta CFI = .126$, $\Delta AIC = 259.057$, $\Delta BIC = 205.966$).

Item-level analysis of bi-factor model

Given that the bi-factor model was the best fitting model, we further analyzed it to better understand the relationship between the two tasks. A closer look at the loading matrix of (Table 4) revealed that 14 out of 16 items (i.e. test numbers) have significant loadings greater than 0.5 on the common latent factor. In addition, all Give-N items showed stronger loadings on the common latent factor than the corresponding How-Many? items. The difference in loading strength of corresponding items on each task achieved significance on items 1, 2, 3, 4, and 5 ($\chi^2_1 > 10.82$, $ps < .001$). As for the two task-specific latent factors, all Give-N items had significant loadings on the specific Give-N latent factor except for item 3; all How-Many? items had significant loadings on the specific How-Many? latent factor except for item 1 and 2.

We further examined the proportion of variance of all items that was explained by the common and task specific latent factors (after controlling for the common latent factor) by computing Omega Hierarchical (ω_H) and Omega Hierarchical Subscale (ω_{HS}) scores

(Rodriguez, Reise, & Haviland, 2016). Results show that 85.8% of variance of the 16 items was explained by the common latent factor ($\omega_H = .858$), 9.8 % variance of the Give-N items was explained by the Give-N specific latent factor ($\omega_{HS,GN} = .098$), and 28.7% variance of the How-Many? items was explained by the How-Many? specific latent factor ($\omega_{HS,HM} = .287$).

Contribution of general cognitive abilities, language, and age

To test whether variance explained by the latent factors in the bi-factor model could be attributed to non-numerical, general cognitive and linguistic abilities rather than number-specific knowledge, we specified a latent multiple regression where scores of general cognitive and linguistic tasks and age were regressed on the common and two task-specific latent factors. Standardized regression coefficients of the general cognitive and linguistic variables and age and their associated p values are presented in Table 5. The overall model showed a good fit, $\chi^2 = 284.451$, $df = 153$, $RMSEA = .065$, $SRMR = .053$, $TLI = .920$, $CFI = .939$. All of the cognitive and linguistic variables measured, including working memory, conflict processing, response inhibition, and vocabulary, were significant predictors of the common latent factor; age was not. The five non-numerical variables accounted for 38.6% of the variance of the common latent factor (see Table 5). In contrast, none of the five non-numerical predictors were related to the specific latent factors (see Table 5).

Finally, we constructed and compared several additional models to further test whether the two tasks drew differentially on general cognitive or linguistic demands. To do this, we set up a baseline model where working memory, conflict processing, response inhibition, vocabulary, and age were regressed on each of the two factors in the distinct-factors model. All parameters in the baseline model were freely estimated. For each of these 5 non-

numerical variables we then constructed a constrained distinct-factors model, where the regression coefficients for each of the non-numerical variables were constrained to be the same for each task's latent factor while all other parameters were freely estimated. We then compared the constrained models to the baseline model using chi-square difference tests. The results showed that none of the five comparisons yield significant results: working memory ($\Delta\chi^2 = .015, p = .901$); conflict processing ($\Delta\chi^2 = .360, p = .549$); response inhibition ($\Delta\chi^2 = .738, p = .390$); vocabulary ($\Delta\chi^2 = .600, p = .438$); and age ($\Delta\chi^2 = 1.845, p = .174$). These results further demonstrate that there were no significant relationships between the non-numerical variables and the two task-specific latent factors.

Discussion

The main purpose of the current study was to investigate whether set-to-number and number-to-set tasks measure the same construct or not. Various arguments have been made regarding the relationship between these tasks. Some have argued that the two types of tasks measure the same construct and can be used interchangeably (e.g., Le Corre & Carey, 2007). Under this view, any observed differences in performance between tasks should be attributed to difficulty differences arising from more general cognitive or linguistic demands of the task, not conceptual differences between them (see Le Corre et al., 2006 for discussion). Le Corre and colleagues (2006) state that data collected from these two types of tasks in their study provide "overwhelmingly consistent pictures of what children understand about how counting represents number" (Le Corre, et al., 2006, p. 151). Others claim that the tasks measure conceptually distinct aspects of cardinal number knowledge and data from them should not be directly compared (Baroody et al., 2017; Fuson, 1988; Wynn, 1992). Until now, no studies

have employed rigorous psychometric analyses to answer the question of whether data from one of these tasks can be flexibly compared with data from the other type of task in the existing literature or whether researchers and practitioners should use these tasks interchangeably in the future. To address these outstanding questions, we developed several statistical models with latent variable structures that mapped on to existing theoretical models of the relationship between tasks and statistically compared these models to decide between them.

Our study showed that two tasks are highly overlapping in construct, with over 85% of the variance on all items across both tasks being associated with a common latent factor. A single-factor model that considered the two tasks as equivalent performed more poorly than other models that considered them to measure at least some conceptually distinct constructs (i.e., the distinct-factors and the bi-factor models). A bi-factor model with both a shared latent factor and task-specific latent factors performed better than a model where each task had only a task-specific latent factor (and those task-specific latent factors were correlated). Based on these model comparison results, we conclude that set-to-number and number-to-set tasks have substantial overlap but do not appear to be conceptually interchangeable.

Further analysis of the bi-factor model showed that around 1/3 of variance of the common latent factor was explained by the combination of general non-numerical, cognitive, linguistic, and maturational variables (i.e., age), suggesting a substantial portion of shared variance between tasks is due to non-numerical cognitive factors. However, the majority of the variance in the general latent factor could not be explained by these non-numerical factors. Furthermore, no substantial variance in task-specific latent factors was explained by

non-numerical factors. These findings are consistent with previous studies showing that general cognitive and linguistic abilities are associated with numeracy in preschool children, but that number-specific abilities make a larger contribution (Cirino, 2011; LeFevre et al., 2010; Mou et al., 2018; vanMarle et al., 2014). These findings extend previous work by partialling out variance between tasks due to general and number-specific variance, providing evidence against accounts that performance differences are due to general task demands alone. Instead, based on these results, it appears that number-to-set and set-to-number tasks measure both common and unique aspects of numerical knowledge.

We also found that item loadings on the common latent factor were significantly stronger for smaller number items (1-5) on the Give-N task compared to the corresponding items on the How-Many? task. Children typically learn the cardinal meaning of the first few numbers (e.g., from 1-4 or 5) slowly and sequentially (often referred to as subset-knowers stages; see Le Corre et al., 2006; Wynn, 1990, 1992). Sometime after learning the numbers up to 4 or 5 in a piecemeal fashion, children appear gain a deeper insight into how counting can be employed to derive cardinality for smaller and larger numbers (see Le Corre et al., 2006; Wynn, 1990, 1992). Although the proposed mechanisms differ between accounts, it is generally accepted that the process by which children learn the meanings of smaller numbers is qualitatively different than the process whereby they learn larger numbers (> 4 or 5; e.g., Baroody et al., 2017; Le Corre & Carey, 2007; Wagner, Kimura, Cheung, & Barner, 2015; Wynn, 1992). Once children have generalized knowledge of how counting works to larger numbers, they should rarely make mistakes on the smaller or larger numbers, at least within the range of numbers we tested (1-8), regardless of how they are tested (see Le Corre et al.,

2006; Carey, 2009 for review). On the other hand, children who have not yet understood how counting works (i.e., subset-knowers) should fail on all larger number items regardless of how they tested because they lack the conceptual resources to understand the concept of a large, exact cardinal number even if they have some additional cognitive resources such as subitizing to represent small exact quantities or a magnitude system for representing approximate numerical values (Carey, 2009; Hyde, 2011; Spelke, 2017). Thus, in principle, individual small number items are more likely to be uniquely diagnostic of cognitive development compared to larger number items because individual large number items are more likely to be jointly passed or failed depending on level of cognitive development.

One explanation for the differences in the strength of the loadings on the common latent factor between tasks might be that cardinal number knowledge is a unitary, shared conceptual construct and the Give-N task is more reflective of this conceptual construct than the How-Many? task. An alternative account is that cardinal number knowledge is comprised of multiple concepts that build on one another and the common factor between tasks in the bi-factor model represents the later emerging ability to produce a set given a cardinal number, or what has been called the cardinal-count concept. According to some views of cardinal number knowledge development (Baroody et al., 2017; Fuson, 1988), the cardinal-count concept requires understanding of the earlier emerging count-cardinal concept, which is tested with set-to-number tasks such as our How-Many? task. Under this account, then, it would make sense that items from both tasks load on the common factor and it would also make sense that items from the Give-N task loads more strongly on the common factor than items from the How-Many? task.

Although number-to-set and set-to-number tasks share much in common, our results clearly show that they do retain some conceptual distinctness. The bi-factor model showed that each of the two tasks had unique psychometric properties (i.e., the task-specific latent factors) that cannot be explained by a common latent factor nor general cognitive, linguistic, or maturational variables. That is, the How-Many? task and the Give-N task continued to explain unique variance (28.7% and 9.8%, respectively) after controlling for common variance explained between both tasks. Again, this finding is consistent with view that cardinal number knowledge is made up of at least two critical concepts: the count-cardinal concept tested by set-to-number tasks (i.e., How-Many? task), and the cardinal-count concept that builds off the count-cardinal concept and is tested by number-to-set tasks (i.e., Give-N).

We can only still speculate as to what aspects of cardinal number knowledge are distinct between tasks. Modern theories of early number development illustrate a complex and challenging developmental process involving the coordination of many cognitive and linguistic subcomponents (Barner, 2017; Le Corre et al., 2006; Le Corre & Carey, 2007; Mix et al., 2012; Mou et al., 2018; Sarnecka & Carey, 2008; vanMarle et al., 2016; Wynn, 1992). Thus, the landscape of potential conceptual differences between tasks is vast. Descriptively, number-to-set tasks require participants to produce a set of items based on a given cardinal number word, while set-to-number require children to enumerate a set to produce a cardinal number word, likely through a counting procedure (at least for larger sets beyond a child's subitizing range; Baroody et al., 2017; Fuson, 1988; Le Corre et al., 2006; Mix et al., 2012). One straightforward difference may be that set-to-number tasks may draw first and more heavily on procedural knowledge of how to count, whereas number-to-set tasks may draw

first and more heavily on understanding how the meanings of particular number words, or a procedural/conceptual distinction (Fuson, 1988; also see Baroody et al., 2017; Frye et al., 1989; Mix et al., 2012; Rittle-Johnson & Alibali, 1999; Sarnecka & Carey, 2008; Wynn, 1990).

Some propose that the procedural knowledge of counting is necessary to bring about deeper conceptual development or change (see Carey, 2009, Fuson, 1988; Siegler & Crowley, 1991; Wynn, 1990). For example, it may be that cardinal number knowledge is made of at least two critical concepts: the cardinal-count concept tested by number-to-set tasks (i.e., Give-N) that is based, in part in, the count-cardinal concept tested by set-to-number tasks (i.e., How-Many? task) (e.g., Baroody et al., 2017; Fuson, 1988). However, others argue that the relationship between procedural and conceptual knowledge of cardinal number is more complex (e.g., Cowan, Dowker, Christakis, & Bailey, 1996; Gelman & Meck, 1986). What is certainly clear is that these components of cardinal number knowledge are highly correlated. Nevertheless, the developmental trajectories of each could be measurably distinct. Future work should continue test this idea using intense microgenetic or longitudinal measurements from both tasks.

Limitations and future directions

It must be noted that there are many variants of set-to-number and number-to-set tasks employed in the literature (Baroody et al., 2017; Benoit et al., 2013; Gelman, 1993; Ginsburg & Baroody, 2003; Odic et al., 2015; Sarnecka & Carey, 2008; vanMarle et al., 2014). For example, children in our How-Many? task were only asked “How many?” at the beginning of the trial and the last number in the count list was taken as the answer if no other response was

given, but others ask the question after children have counted the objects (e.g., Le Corre et al., 2006; Wynn, 1992). Opinions in the field differ as to whether asking How-Many? after the initial answer overestimates, underestimates, or accurately estimates cardinal number knowledge (e.g., Frye et al., 1989; Fuson, 1988; Gelman, 1993; Greeno et al., 1984, Sarnecka & Carey, 2008). The one main empirical study to directly compare performance across How Many? task versions showed that children performed worse when asked “How Many?” before compared to after or before and after counting (Frye et al., 1989). However, asking the question after counting and eliciting a correct response from the child does not guarantee that they understand cardinal numbers, as they may still use the “last word rule” to provide a correct answer (see Frye et al., 1989; Sarnecka & Carey, 2008 for discussion).

Others have claimed that asking the question How-Many? after children provide an initial answer may confuse them and cause some to change their response because they think the experimenter is questioning their original answer (e.g., Gelman, 1993; Greeno et al., 1984). It was also our experience during piloting before beginning our study that some children were confused by asking the question after, even though some have reported that asking How Many? after the initial answer does not raise issues and/or may lead to better performance (Baroody et al., 2017; Frye et al., 1989; Fuson, 1988). We acknowledge that our choice to not ask the How-Many? question after counting could have had consequences for our estimates of cardinal number knowledge. For example, consistent with one empirical study on the matter (Frye et al., 1989), asking the How Many? question only before counting could have led children to perform worse and led us to underestimate their knowledge.

To summarize, there are many other subtleties in the way different groups conduct and

score these tasks (e.g., Le Corre et al., 2006; Lee & Sarnecka, 2011; Piantadosi et al., 2014; Wynn, 1992). To compound the issue, there are zealous allegiances to particular versions of these tasks despite little empirical support for the position that one particular version better characterizes cardinal number knowledge (see Le Corre et al., 2006 for a discussion or Frye et al., 1989 for one empirical comparison). We developed and employed what we think are representative and comparable versions of the two tasks, as a first step in rigorously comparing the psychometric properties of each type. Similar versions to what we employed have been used in previously published work (e.g., Gelman, 1993; Piantadosi et al., 2014; Wynn, 1990, 1992, see Frye et al., 1989 for empirical comparison). Furthermore, we presented our tasks in a fixed order to all children. We completely acknowledge that the conclusions we draw may be limited to idiosyncrasies of our particular versions and procedures of set-to-number and number-to-set tasks and other variants could produce different results (e.g., estimation rather than counting versions of these tasks; see Odic et al., 2015). In order to know whether this is the case or not, further rigorous psychometric studies and comparisons in the spirit of our efforts would need to be conducted.

Finally, we chose to sample broadly across a wide range of cardinal number knowledge understanding, from children who understood very little about number words and counting to children who had mastered the cardinal principle, as there were not clear theoretical reasons to narrow our sample to a particular point on the developmental continuum. We acknowledge that our results may have been different if we chose to sample in a different manner. For example, our results may have been different if we were more restrictive in terms of the stage of conceptual development of the participants or if we broadened our age range. Future work

may wish to broaden the age range or restrict sampling to particular stages of development for particular theoretical reasons. Relatedly, with such a large range of abilities, children undoubtedly used different sorts of strategies to solve the tasks. Recording and analyzing strategy was not our focus, but would be an interesting direction for future work.

Conclusions

In sum, the present study clearly provides the strongest evidence to date that set-to-number and number-to-set tasks capture both unique and common aspects of children's cardinal number knowledge. Our results suggest that the tasks are not interchangeable and studies employing each should not be directly compared. We suggest that the best practice moving forward may be to measure with both types of tasks, analyze them separately, and compare them when assessing early conceptual understanding of cardinal number in research or educational settings (also see Baroody et al., 2017, Fuson, 1988 for alternative strategies). Given the evidence presented that the two tasks require some conceptually distinct skills, we also cannot assume that experience with number-to-set production and set-enumeration (i.e., set-to-number) is interchangeable either; instead, educators and researchers may begin to pursue the hypothesis that varied practice with both sorts of tasks best encourages learning about cardinal number in preschool-aged children. Together our results also provide indirect evidence for theories that propose that cardinal number knowledge is composed of multiple, conceptual components (Baroody et al., 2017; Carey, 2009; Fuson, 1983).

More broadly, the present study provides an example of how constructing and comparing models with different latent variable structures can be used to test existing hypotheses

regarding the relationship between tasks in academic and educational settings (also see Klem, Melby-Lervag, Hagtvet, Lyster, Gustafsson, & Hulme, 2015 for an example in language development).

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Table 1. The demographic information of the child participants' families (N = 146).

Race/Ethnicity (%)		The highest level of parental education (%)		Family yearly income (%)	
Caucasian	82.2	MD/PhD	13.7	\$180,000 or more	4.8
Latino/Hispanic	2.7	Master's level	27.4	\$170,000--179,999	2.1
African American	2.1	Bachelor's level	41.1	\$160,000-169,999	2.7
Asian/Pacific Islander	0.7	Associate's level	11.0	\$140,000-159,999	4.8
Other	1.4	High school	6.1	\$120,000-139,999	9.6
Multiple	10.3	Lower than high school	0.7	\$100,000-119,999	14.4
No report	0.7			\$80,000-99,999	14.4
				\$60,000-79,999	19.9
				\$40,000-59,999	17.8
				\$20,000-39,999	5.5
				\$10,000-&19,999	2.1
				No report	2.0

Table 2. Descriptive statistics and bivariate correlations among variables.

	Range	Mean	SD	1	2	3	4	5	6
1.Give-N	.04-1	.67	.28						
2.How-Many?	.21-1	.73	.22	.77*					
3.working memory	.08-1	.71	.23	.26*	.23*				
4.conflict processing	.35-1	.77	.17	.34*	.32*	.09			
5.response inhibition	0-1	.85	.17	.43*	.38*	.12	.32*		
6.vocabulary	30-130	82.86	19.09	.40*	.30*	.37*	.32*	0.16*	
7.age (days)	1054-1568	1412.06	60.94	.22*	.14 [†]	.10	.11	0.08	.29*

Note. * indicates significant correlations ($p < .05$); [†] indicates marginally significant

correlations ($p < .10$); For age, 1054 days equal 2y 10m 24d, 1568 days equal 4y 3m 18d; and 1412 days equal 3y 10m and 17d.

Table 3. Item correlations and difficulty differences between Give-N and How-Many?

	Give-N		How-Many?		<i>r</i>	<i>p</i>	<i>t</i>	<i>p</i>	Cohen's <i>d</i>
	<i>Mean Accuracy</i>	<i>SD</i>	<i>Mean Accuracy</i>	<i>SD</i>					
ITEM1	0.98	0.10	0.99	0.07	.039	.584	-0.599	0.550	-0.085
ITEM2	0.90	0.26	0.94	0.18	.629**	<.0001	-3.101**	0.002	-0.211
ITEM3	0.79	0.36	0.89	0.23	.590**	<.0001	-4.891**	<.0001	-0.374
ITEM4	0.65	0.43	0.75	0.34	.613**	<.0001	-4.207**	<.0001	-0.294
ITEM5	0.58	0.44	0.69	0.34	.532**	<.0001	-4.075**	<.0001	-0.291
ITEM6	0.53	0.41	0.59	0.37	.585**	<.0001	-2.411*	0.017	-0.168
ITEM7	0.52	0.40	0.53	0.40	.555**	<.0001	-0.374	0.709	-0.027
ITEM8	0.45	0.41	0.48	0.38	.615**	<.0001	-1.263	0.208	-0.086

Note. * indicates significant correlations ($p < .05$); ** indicates significant correlations ($p < .001$).

Table 4. Standardized factor loadings of the three latent variable models

	Single-factor Model	Distinct-factors Model		Bi-factor Model		
	Common	GN- specific	HM- specific	Common	GN- specific	HM- specific
GN.1	0.26	0.25		0.42	0.33	
GN.2	0.55	0.52		0.78	0.44	
GN.3	0.72	0.71		0.81	0.06	
GN.4	0.83	0.85		0.78	-0.36	
GN.5	0.84	0.87		0.75	-0.45	
GN.6	0.82	0.86		0.75	-0.44	
GN.7	0.83	0.86		0.75	-0.43	
GN.8	0.80	0.82		0.70	-0.43	
HM.1	0.11		0.11	0.08		0.08
HM.2	0.45		0.46	0.58		-0.04
HM.3	0.61		0.64	0.65		0.15
HM.4	0.75		0.78	0.70		0.35
HM.5	0.70		0.77	0.61		0.45
HM.6	0.76		0.81	0.63		0.51
HM.7	0.71		0.81	0.59		0.60
HM.8	0.67		0.76	0.56		0.55

Note. GN = Give-N task; HM = How-Many? task; Numbers following task abbreviations in the first column indicate the item level number tested (GN.1 = number “one” in the Give-N task). Numerical values in the remaining columns represent standardized factor loadings of each item (i.e., each test number) in the two tasks on the latent factors. The larger the loadings are, the more variance of the item is explained by the common and task-specific latent factors.

Table 5. Standardized regressions coefficients of 5 non-numerical predictors on the common and specific latent factors

Predictor	Common		GN-specific		HM-specific	
	Estimate	<i>p</i>	Estimate	<i>p</i>	Estimate	<i>p</i>
response inhibition	0.332**	<.0001	-0.028	0.786	0.049	0.615
vocabulary	0.232**	0.001	-0.011	0.908	-0.093	0.361
working memory	0.141*	0.028	0.025	0.772	0.048	0.594
conflict processing	0.196**	0.005	0.101	0.291	0.029	0.772
age	0.064	0.304	-0.144	0.081	-0.014	0.875

Note. * indicates significant correlations ($p < .05$); ** indicates significant correlations ($p < .001$).

Figure 1. Three latent variable models for data fitting (GN = Give-N; HM = How-Many?)