

Developmental Changes in Nonsymbolic and Symbolic Fractions Processing:

A Cross-Sectional fMRI study

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Data Availability Statement

Data that support the findings of this study are available on request from the corresponding author.

Research Highlights

- 2nd-graders, prior to formal fractions instructions, already recruit a right parietal-frontal network when comparing nonsymbolic ratios.
- 5th-graders, who have received formal fractions instruction recruit this same network not only for nonsymbolic ratios, but also for symbolic fractions.
- These findings support the RPS account by showing how symbolic fraction processing builds on neural substrates for nonsymbolic ratios.

Abstract

A substantial body of research has found that human and nonhuman animals are capable of processing the magnitudes of nonsymbolic ratios. Lewis, Mathews and Hubbard (2015) hypothesized that this ability may depend on a neurocognitive architecture called the *ratio processing system* (RPS). They further hypothesized 1) that the RPS might serve as a neurocognitive startup tool—an evolutionarily conserved cognitive architecture—and 2) that it can be recycled to support the acquisition of symbolic fractions knowledge. We tested these two predictions of the RPS account by comparing neural signatures of the RPS in 2nd-graders, who have not yet received formal symbolic fraction instruction, and 5th-graders, who have. During fMRI scanning, children performed ratio comparison tasks in which they determined which of two ratios or symbolic fractions was larger. Both cohorts showed behavioral and neural evidence of processing symbolic and nonsymbolic fractions magnitudes, with performance modulated by the numerical distance between stimuli. Consistent with our predictions, 2nd-grade children reliably recruited a right parietal-frontal network for nonsymbolic ratio comparisons but not symbolic fractions, and 5th-grade children recruited a bilateral parietal-frontal network for both nonsymbolic and symbolic fractions that overlapped with, but extended beyond, that found for 2nd-graders. These results present the first neuroimaging evidence that neural substrates for nonsymbolic ratios exist prior to formal learning and that this nonsymbolic foundation may be *recycled* to process symbolic fractions. These findings open the door for pedagogical strategies that focus on supporting this recycling process to improve students' understanding of symbolic fractions.

Keywords:

Ratio processing system, neural distance effects, cross-sectional fMRI investigation, neuronal recycling hypothesis, symbolic fractions acquisition.

1. Introduction

Basic competence with symbolic numbers is a foundational competence required for everything from purchasing groceries, to calculating monthly expenses to investing. Much like reading, this competence is so critical that societies world-wide strive to educate people so that it operates fluidly. At the same time, it is such an evolutionarily recent invention that it is difficult to imagine how human brains could have evolved specifically to process symbolic numbers. Dehaene and Cohen's (2007) *neuronal recycling hypothesis* offers a plausible account for how such recent cultural inventions might be supported by recycling ancient primitive cognitive architectures whose functionality has been exapted to support formal learning.

For example, several theorists have argued that knowledge of whole number symbols can be grounded on neurocognitive systems dedicated to individuating and processing sets of nonsymbolic elements or *numerosities*. These accounts suggest that this phylogenetically ancient *approximate number system* (or ANS) can be reoriented in development to support processing of modern number symbols (e.g., Gallistel & Gelman, 2000; Piazza, 2011). However, these ANS accounts have been largely limited to whole number learning and *a priori*, the ANS appears ill-suited for supporting other types of numbers such as fractions or decimals (e.g., Dehaene, 2011; Feigenson et al., 2004; Gallistel & Gelman, 2000; Matthews et al., 2016; but see Chesney & Matthews, 2018; Clarke & Beck, 2021; for notable exceptions that cast the ANS as compatible with processing rational numbers).

In contrast to the putative limitations of the ANS, a recent body of research has described a nonsymbolic *ratio processing system* (or RPS), a neurocognitive system that processes the magnitude of nonsymbolic analogs to fractions (Bhatia et al., 2020, 2022; Jacob et al., 2012; Jacob & Nieder, 2009b; Lewis et al., 2016; Matthews et al., 2016; Meng et al., 2019; Starling-

Alves et al., 2022). Investigations centering the RPS stand to expand the neuronal recycling hypothesis to the realm of fractions (Matthews et al., 2016; Sidney et al., 2017).

We tested two key propositions of Lewis et al.'s (2016) RPS account. Specifically, we hypothesized (1) that children should have substantial neural sensitivity to nonsymbolic ratios, even prior to formal schooling, and (2) that processing fractions symbols should leverage this nonsymbolic foundation, recruiting RPS systems over development (Lewis et al., 2016). To test these hypotheses, we compared developmental differences in neural signatures of the RPS among 2nd-graders, who have not yet received formal fractions instruction, and 5th-graders, who have had a few years of instruction.

1.1 A cognitive primitive for symbolic fractions

Converging lines of research have recently proposed that there is a primitive ability to process nonsymbolic ratio magnitudes, such as ratios instantiated by juxtaposing two line-segments (e.g., Bonn & Cantlon, 2017; Jacob et al., 2012; Matthews & Chesney, 2015; Matthews et al., 2016). Lewis, Matthews & Hubbard (2016) dubbed this system the RPS and further proposed that the RPS, as an ancient system sensitive to rational number magnitudes, might be “recycled” to support understanding of symbolic fractions and related concepts.

To date, the RPS account has been supported by several behavioral studies with different age ranges. One line of research has shown indirect evidence for co-processing of symbolic and nonsymbolic ratios by using tasks indicating rapid, cross-format mapping of ratio magnitudes (e.g., Kalra et al., 2020; Matthews & Chesney, 2015). Kalra et al. (2020) conducted cross-format fraction comparison tasks, whereby 2nd- and 5th-grade children compared which of two ratios (i.e., line ratios vs. symbolic fractions) was larger. They observed a classic *distance effect* in which performance improved as the distance between ratios increased in both grades (e.g.,

Buckley & Gillman, 1974). These cross-format distance effects showed that children can semantically process both nonsymbolic and symbolic ratios as analog magnitudes. These cross-format distance effects parallel those found in adult populations (Binzak et al., 2019; Matthews & Chesney, 2015). Additionally, reaction times in these experiments were rapid enough that the authors suggested that participants possibly compare nonsymbolic line ratios to symbolic fractions without first converting nonsymbolic stimuli to symbolic form. These consistent findings across children and adults suggest the possibility that nonsymbolic and symbolic ratios may be compared through a shared system, consistent with the RPS account.

Additional findings suggest that more precise RPS acuity is correlated with increased competence with fractions and other symbolic mathematics domains among both children and adults (e.g., Hansen et al., 2015; Matthews, Lewis, & Hubbard, 2016; Möhring, Newcombe, Levine, & Frick, 2015; Park & Matthews, 2021; Wong, 2019). For instance, previous studies revealed that children's performance on nonsymbolic proportional match-to-sample tasks predicted their fractions knowledge (e.g., Hansen et al., 2015; Möhring et al., 2015; Wong, 2019; but see Bhatia et al., 2020). Studies with adults found RPS ability predicts both fractions knowledge and algebra performance (Matthews et al., 2016; Park & Matthews, 2021) even when controlling for domain general skills such as working memory. These findings imply a potential link between the RPS and symbolic fraction processing, consistent with the claim that the RPS serves as a cognitive primitive for supporting symbolic fraction knowledge.

1.2 Neural substrates for processing ratio magnitudes

Neuroimaging studies have also found that the same neural substrates may be used to support both nonsymbolic and symbolic representations of ratio magnitudes (Ischebeck, Schocke, & Delazer, 2009; Jacob & Nieder, 2009b, 2009a; Mock et al., 2018; but see Bhatia et

al., 2022). For example, Jacob & Nieder (2009b) used a functional MRI adaptation paradigm—in which activation increases as the distance between the habituated ratio and deviant ratio increases—to investigate the neural distance effect for nonsymbolic ratios. They observed that both the intraparietal sulcus (IPS) and prefrontal cortex (PFC) regions exhibited distance effects for nonsymbolic ratio magnitudes (represented by line and dot ratios). Notably, these regions have often been implicated in analog processing of numerical magnitude (Dehaene et al., 1998; Matejko & Ansari, 2019; Nieder & Dehaene, 2009; Piazza et al., 2004; Sokolowski et al., 2021; Sokolowski, Fias, Mousa, et al., 2017). Moreover, Ischebeck et al. (2009) observed similar activation of the prefrontal-parietal network during symbolic fraction comparisons in adults. These parallel findings implicate the use of common neural substrates for processing both nonsymbolic and symbolic ratio magnitudes, consistent with the RPS account.

Although this conclusion about common neural substrates for symbolic and nonsymbolic ratios was initially driven by findings from between-subject paradigms, more recent studies have employed within-subjects designs among adult participants (Mock et al., 2018; Binzak et al., submitted; but see Bhatia, Longo, Chesnokova, & Prado, 2022). Mock (2020) and Binzak et al. (submitted) used magnitude comparison paradigms in which participants compared ratio magnitudes in symbolic and nonsymbolic formats. Results revealed overlapping regions of neural activation, regardless of whether comparisons were made within or across symbolic and nonsymbolic formats. Task-specific activation was particularly apparent in the right inferior parietal lobules (including the IPS), and right prefrontal regions. Moreover, activation for cross-notation comparisons was characterized by neural distance effects, suggesting shared magnitude-dependent processing of nonsymbolic and symbolic representations at the neural level. These

results further support the RPS hypothesis by demonstrating use of common neural substrates for nonsymbolic ratios and symbolic ratios (fractions) within individuals.

1.3 Present study

Although recent studies have demonstrated that symbolic and nonsymbolic ratios are processed by similar substrates in adults, it is not clear *how* fractions in such dissimilar formats come to be processed by the same substrate, because adults already have had years of experience with fractions symbols. Indeed, studying adult participants exclusively cannot settle the question of whether a pre-existing cortical system for nonsymbolic ratio magnitude serves as the basis for symbolic fractions; answering this question requires a developmental approach. To our knowledge, there have been no neuroimaging studies investigating how children process nonsymbolic vs. symbolic ratios.

The aim of the current study was two-fold: 1) to explore similarities between nonsymbolic and symbolic ratio processing in young children at both the behavioral and neural levels, and 2) to explore developmental differences in behavioral and neural signatures for ratio processing during early years of fraction instruction. Therefore, we specifically aimed to compare the neural activations between children who have not yet received formal fractions instruction (2nd-graders) and children who have received a few years of fractions instruction (5th-graders). We used fMRI to investigate neural activation during ratio comparison in multiple formats among these two groups of primary school children. We used a paradigm nearly identical to Kalra et al. (2020) to better understand a possible shared system across nonsymbolic and symbolic ratios at a neural-level. Ratio comparisons were presented in three different formats: 1) nonsymbolic notations comparing two line-ratios, 2) mixed notations comparing

symbolic and nonsymbolic ratio formats and 3) symbolic notations comparing symbolic fractions.

The RPS hypothesis suggests that the human brain comes equipped with the ability to process nonsymbolic ratios from infancy (e.g., McCrink & Wynn, 2007). As such, both 2nd- and 5th-grade children should be able to compare nonsymbolic and symbolic ratios, replicating Kalra et al. (2020). Furthermore, we expected that nonsymbolic ratio comparisons would be the easiest and that symbolic fraction comparisons would be the hardest. Secondly, we hypothesized that children have a neural architecture for ratio processing that functions similarly to that of adults—even among children who have received little instruction on symbolic fractions. Thus, we expected that children would engage the same frontal and parietal regions of the brain for nonsymbolic and symbolic formats as seen in adults (e.g., Ischebeck et al., 2009; Mock et al., 2018). The IPS is the region most consistently implicated in processing nonsymbolic whole number magnitudes (e.g., Bhatia et al., 2022; Dehaene, Piazza, Pinel, & Cohen, 2003; Matejko & Ansari, 2019; Piazza et al., 2004; Sokolowski, Fias, Mousa, et al., 2017; Sokolowski et al., 2021) and we therefore hypothesized that it would also process nonsymbolic ratios prior to instruction.

Regarding developmental differences, we hypothesized that if the RPS plays a role as a foundation for building symbolic fractions concepts, then symbolic fraction processing should shift to recruit the same regions engaged in nonsymbolic ratio processing as children receive instruction and practice with symbolic fractions. Thus, we predicted that older children (i.e., 5th-graders), exposed to years of fractions instruction would show overlapping activation for both symbolic and nonsymbolic ratios (as seen in adults), whereas younger children (i.e., 2nd-graders) would show activation in those regions only for nonsymbolic ratios.

2. Methods

2.1 Participants

We recruited 47 2nd- ($M_{Age} = 7.68$, $SD_{Age} = 0.43$) and 45 5th-grade ($M_{Age} = 10.68$, $SD_{Age} = 0.47$) children from several public schools in a mid-sized Midwestern city as a part of a larger study of the development of children's fractions abilities (see Kalra et al., 2020). All participants were right-handed native English-speakers with normal or corrected to normal vision (See Supplementary Methods for demographic information). Parents or guardians gave written consent, and children gave verbal assent. All protocols were approved by the biomedical research ethics committee of the Institutional Review Board. Participants received monetary compensation and small gifts for their participation.

One 2nd-grade child was excluded due to an ADHD diagnosis, and two children failed to complete the scan due to excessive movement (one 2nd-grade and 5th-grade). Another two 5th-grade children were excluded due to technical issues during scan acquisition. We additionally excluded runs with head movement greater than 2.5 mm and runs in which children showed chance level behavioral performances in the scanner. A participant's full set of data was excluded if three or more out of the six functional runs were removed. Due to this filtering process, an additional seventeen 2nd-grade and nine 5th-grade children were excluded. After these exclusions, the final analytic sample consisted of twenty-eight 2nd-graders and thirty-three 5th-graders.

2.2 The Ratio Comparison Task

Participants completed ratio comparison tasks in the MRI scanner (Binzak et al., submitted) using stimuli presented by E-prime software (Psychology Software Tools, Shapsburg, PA). On each trial, participants compared two ratios made either from juxtaposed line segments

forming nonsymbolic ratios or from symbolic fractions (see Figure 1). Comparison pairs were presented in each of three types: 1) Fraction vs. Fraction (Symbolic, hereafter Sym), 2) Line ratio vs. Fraction (Mixed), and 3) Line ratio vs. Line ratio (Nonsymbolic, hereafter Nonsym). Stimuli were presented side-by-side in a light gray color on a black background. Participants selected the larger ratio by pressing the corresponding button with either their index (indicating the left ratio was judged larger) or middle finger (indicating the right). Children completed at least one run of practice trials outside the scanner to become familiar with the task before the real scan.

To manipulate task difficulty, we varied the numerical distance between comparison stimuli among trials. Numerical distance is defined as the difference between the magnitudes of the compared ratios $[|\text{fraction A} - \text{fraction B}|]$. Fraction pairs were organized into three distance bins for the purposes of analysis (Jacob & Nieder, 2009a; Kalra et al., 2020): near (.048-.233), medium (.262-.446), and far (.514-.750) (See Table 1). As seen in previous studies with whole numbers and fractions (e.g., DeWolf et al., 2016; Ischebeck et al., 2009; Mock et al., 2018), we predicted that participants would exhibit distance effects in both their neural and behavioral responses. Specifically, we expected near comparisons to yield the lowest accuracy, the longest response times, and the greatest neural activation. In contrast, the far condition should yield the highest accuracy, lowest response times, and least neural activation.

Each participant completed six runs of 36 trials per run for a total of 216 trials. Each individual run included an equal number of trials from each notation condition (Nonsym, Mixed, and Sym), and the trials in each notation were evenly distributed among the distance bins (near, medium, and far). All notations and distances were counterbalanced across the six runs. The number and the order of control conditions were counter-balanced across participants (see

Stimuli section). Within each run, stimulus presentation order was random for each participant. All runs were counterbalanced across participants.

Each trial began with a fixation cross, presented for 1250 – 1750 ms (with the range corresponding to randomly selected jitter, $1500\text{ms} \pm 250\text{ms}$) followed by the presentation of ratio stimuli. Participants could respond upon stimulus onset, and stimuli remained on-screen until the trial timed out after 4000ms even if participants failed to respond.

2.2.1 Stimuli

Symbolic Fractions. We used the set of 27 irreducible proper fractions composed of single digit components. We selected 36 pairs from the 352 possible unique pairings with stimuli balanced across pairs that 1) both shared a common denominator, 2) the numerically larger fraction had a larger numerator and a smaller denominator than the smaller fraction, 3) the numerically larger fraction had a larger numerator and a larger denominator, and 4) the larger fraction had a smaller numerator and denominator than the smaller fraction (*incongruent* pairs). However, in the far distance bin, it was impossible to include incongruent numerator pairs because no qualifying pair existed with distance greater than .306 given the set of fractions with single digit components.

Nonsymbolic Ratios. Nonsymbolic ratios were composed of pairs of juxtaposed gray lines. To build the nonsymbolic ratios, we used our symbolic fraction pairs as the reference, keeping the same holistic fractional magnitudes and distances between pairs across notation conditions. To minimize the probability that participants used each line-length (i.e., the numerator or denominator component) as a cue to make the comparison decision, we created two sets of line ratios. One set (numerator controlled, see Table 2) was controlled to minimize the correlation between the numerator length and overall ratio magnitude. The numerator length was randomly generated between 33-336 pixels, and the corresponding denominator length was then

determined. The other set (denominator controlled, see Table 2) was controlled to minimize the correlation between denominator length and overall ratio magnitude. The denominator length was randomly generated to be between 130-300 pixels, and corresponding numerator length was then determined.

2.3 Fraction Instruction

Because US 2nd-graders typically have not yet received formal instructions on fractions, and because the nonsymbolic line ratios were an unfamiliar format, all children received a brief PowerPoint lesson¹ (also used by Kalra et al., 2020) introducing the concept of ratios/fractions prior to experimental runs. This included instruction on the idea that fractions get larger as numerator sizes increase, as denominator sizes decrease, and as the two components become closer to the same value. To introduce nonsymbolic and symbolic ratios in a child-friendly manner, we used cartoon characters to depict how height comparisons can make a ratio. Children were instructed that, “Joey is half as tall as Sara. When we think of Sara’s and Joey’s heights together, we can call it a RATIO. And we use numbers to talk about ratios”. Cartoon characters were eventually replaced by lines, so that children could gain some familiarity with the types of line ratios that would be presented as stimuli. When we introduced line ratios, the corresponding symbolic fractions were also presented simultaneously.

2.4 Data Acquisition

Participants were scanned in a General Electric 3-Tesla scanner (GE Medical Systems, Waukesha, WI) equipped with a 32-channel array head coil (Nova Medical) at the University of Wisconsin–Madison. Foam padding was used to limit head motion. Structural images were

¹ Two 5th graders (one excluded due to large movement) did not receive a power point instruction because we decided to provide it for 5th graders after we learned 5th graders are not familiar with nonsymbolic format of ratios.

collected by using motion-corrected 3D T1-weighted (T1w) MPnRAGE with 1mm isotropic resolution (TR = 4.876ms, TE = 1.82ms, Flip angle = 4°, FOV = 224mm X 224mm, in plane resolution: 256 X 256 pixels, the number of axial slices = 176) (Kecskemeti et al., 2016).

Functional images were acquired with a 3D T2-weighted (T2w) echo-planar imaging sequence (TR = 2000ms, TE = 22ms, Slice thickness = 3mm, Flip angle = 75°, FOV = 224mm X 224mm, 128 X 128 matrix). Each volume consisted of 38 slices (1.75mm X 1.75mm voxel size) with a 52ms inter slice interval. The first 5 volumes of each functional run, during which participants waited for the task to begin, were also collected to allow for T2 equilibrium effects. In total, 110 volumes were acquired for each functional run.

2.5 Imaging analysis

2.5.1 Preprocessing

All images were analyzed using Brain Voyager QX 2.8.2 (Brain Innovation, Maastricht, Netherlands). Each individual's data were preprocessed using the following procedure. The first five volumes of each functional run were discarded to account for the stabilization of magnetic saturation. Functional images were corrected for differences in slice time acquisition by using sinc interpolation with ascending and interleaved order and 3D motion by using trilinear sinc interpolation, followed by high-pass temporal filtering (GLM-Fourier with a cut-off of 2 sines/cosines per cycle). The preprocessed functional images were co-registered to the T1 anatomical images through parameter-based initial alignment followed by manual fine tuning. These co-registered data were transformed into Talairach Space (Talairach & Tournoux, 1988). These co-registrations and Talairach transformation processes were visually inspected by the analysis team and problematic cases were rerun and resolved by the team. Functional images were smoothed by applying an 8mm full width at half maximum (FWHM) Gaussian kernel. A

hemodynamic response function in BV (Brain Innovation, Maastricht, Netherlands) was used to model the expected BOLD signal for each distance condition and notation (distances near, medium, and far; notations Nonsym, Mixed, and Sym).

2.5.2 Motion Comparison

Prior to group analysis, we checked for possible differences in head motion between grades. We regressed movement parameters against grade using fixed effects models. We found no differences between 2nd- and 5th-graders in terms of translational ($\beta = -.040$, $t = -.404$, $p = .688$) or rotational ($\beta = -.014$, $t = -.747$, $p = .457$) movements.

2.5.3 Whole-Brain Analyses

We calculated random effects GLMs separately for 2nd and for 5th graders. We first contrasted near and far distance conditions to identify the regions of the brain showing greater activity in near distances relative to far distances—that is, those exhibiting *neural distance effects* (NDEs). Next, to isolate the regions involved in processing each notation, we conducted the same NDE analysis for each notation separately (i.e., nonsymbolic, mixed, and symbolic). To identify overlapping regions across the NDEs in different notations, we then conducted a whole brain conjunction analysis (e.g., Matejko & Ansari, 2019) across different notations in each grade—for example, $[(\text{Sym}_{\text{near}} - \text{Sym}_{\text{far}}) \cap (\text{Nonsym}_{\text{near}} - \text{Nonsym}_{\text{far}}) \cap (\text{Mixed}_{\text{near}} - \text{Mixed}_{\text{far}})]$.

Whole brain contrasts were thresholded at an uncorrected p-value of .005 (cf., Matejko & Ansari, 2019 with children), then corrected for multiple-comparisons using Brain Voyager's cluster-level statistical threshold estimator which resulted in a false positive rate (α) below .05 (Goebel et al., 2006). Brain Voyager's procedure uses Monte Carlo Simulations (typically 1000 iterations) to estimate the spatial smoothness of the uncorrected whole brain map, which allows calculating cluster-level false- positive rates and identifying the minimum cluster threshold size

that yields $\alpha < .05$ (Forman et al., 1995; Goebel et al., 2006). Critically, this approach avoids false positive results due to invalid cluster inferences (Eklund, Nichols, & Knutsson, 2016).

Furthermore, we conducted a two factor random effects ANOVA with Grade (2nd vs. 5th) as a between-subjects factor, numerical distance (near vs. far) as a within-subject factor and their interactions to identify the regions showing differences in the distance effect in between 2nd- and 5th- graders. Significant regions were identified by whole brain analysis, and then the mean beta values for each subject were extracted from each cluster. Next, we identified the clusters where 5th-graders showed larger distance effects than 2nd-graders did and vice versa. We repeated these analyses for each of the three notations (Nonsym, Mixed, and Sym) (*Supplementary Results*).

Next, we explored *notation effects*, indicated by brain regions that were sensitive to a given notation relative to other notations. We first conducted random effects analysis contrasting nonsymbolic (Nonsym) and symbolic (Sym) notations as a within-subject factor. Additionally, to identify the regions that were specifically sensitive to mixed (Mixed) comparisons as opposed to same notation comparisons, we performed the same random effects analysis searching the regions showing greater activations in Mixed relative to Nonsym and Sym (*Supplementary Results*).

2.5.4 Region of Interest (ROI) Analyses

Finally, we performed *a priori*-specified ROI analysis on the bilateral IPS. We used a coordinate set based on Houdé et al.'s (2010) meta-analysis of number processing in children. The IPS coordinates in Talairach space were converted from MNI space using the mini2tal script (http://eeg.sourceforge.net/mrdoc/mri_toolbox/mni2tal.html). With these converted coordinates, we centered a 10 mm X 10 mm X 10 mm cube around each IPS coordinate and extracted the

average beta values for each subject at each cross section of 3 notations (Nonsym, Mixed, and Sym) X 3 distance bins (near, medium, far). With these extracted beta values, we conducted a mixed-effects regression in order to evaluate the distance effects in the IPS.

3. Results

3.1 Behavioral analysis

Error rates. Children in both grades were capable of discriminating ratio magnitude accurately (2nd: $M_{err} = .046 - .194$; 5th: $M_{err} = .023 - .231$) and rapidly (2nd: $M_{rt} = 1057-1987$ ms; 5th: $M_{rt} = 988- 1057$ ms) across all notations (see Table 3 for results disaggregated by notation). To investigate distance effects on error rates, we conducted mixed effects logistic regressions to account for within-subject correlation among trials using the ‘gImer’ function of lme4 package in R software (Bates et al., 2015).

We first regressed error (incorrect: 0 or correct: 1) against notation (3 levels, Nonsym = 0, Mixed = 1, Sym = 2), distance bin (3 levels, Far = 0, Med= 1, Near = 2), and grade (2 levels: 5th-graders =0, 2nd-graders = 1), such that higher level exhibits higher error rates (Table 4; Figure 2). We used a backward difference coding scheme to allow binary comparison of variables at different levels as specified by our hypotheses (i.e., Nonsym < Mixed < Sym, and Far < Med < Near) (Kalra et al., 2020). As predicted, we found a significant effect of grade, whereby 2nd-graders were more likely to make errors than 5th-graders (*Odds Ratio (OR)* = 1.542, $p = .001$). We found significant distance effects: the likelihood of making an error on medium distance trials was higher than for far distance trials ($OR = 2.87$, $p < .001$), and the likelihood of making an error on near distance trials was higher than for medium distance trials ($OR = 3.37$, $p < .001$). As predicted, the likelihood of making an error on Mixed was higher than for Nonsym ($OR = 2.47$, $p < .001$). However, contrary to our predictions, the likelihood of making an error in Sym was not

statistically higher than that of Mixed ($OR = 1.11, p = .147$). The same pattern of results was found within each grade (Supplementary results).

Reaction times. For reaction times, we conducted mixed effects linear regressions using the ‘lmer’ function of lme4 package in R (Bates et al., 2015). As with the analysis of error rate, we regressed reaction times against notation (3 levels, Nonsym = 0, Mixed = 1, Sym = 2), distance bin (3 levels, Far = 0, Med = 1, Near = 2), and grade (2 levels: 5th-graders = 0, 2nd-graders = 1) (Table 4; Figure 3), using a backward difference coding scheme. As with error rates, we expected 5th graders would perform better than 2nd graders, but the analysis showed that 5th-graders were not significantly faster than 2nd-graders ($\beta = -98.56, t = -1.548, p = .127$). However, for other effects, the results were consistent with the error rate analyses. We found a significant distance effect for reaction times, whereby participants responded slower on near distance trials than for medium ones ($\beta = 161.7, p < .001$) and on medium distance trials than on far ones ($\beta = 216.21, p < .001$). As for notation effects, both grade cohorts responded faster for Nonsym than for Mixed notations ($\beta = 264.9, p < .001$), and responded faster for Mixed than for Sym notation ($\beta = 218.19, p < .001$). Together, we observed notation and distance effects across 2nd- and 5th-graders in both accuracy and reaction time. These results indicate that children’s performance was modulated by holistic distance between ratios suggesting their sensitivity toward ratio magnitudes even prior to instructions.

Prior work shows that participants sometimes use heuristic strategies (e.g., Fazio et al., 2016; Morales et al., 2020; Schneider & Siegler, 2010) for fraction comparisons. For example, the *gap strategy* heuristic is based on the fact that larger fractions often have a smaller difference between its numerator and denominator (Denominator – Numerator = Gap) compared to smaller fractions. To rule out possible effects of the gap strategies, we performed additional mixed

effects analysis that factors in the gap of ratios. The results confirm that distance effects are still significant ($p < .001$ in both grades) even after controlling for possible use of gap (Supplementary Results).

3.2 Neuroimaging analysis

3.2.1 Neural distance effects across all notations in 2nd- and 5th-graders: whole-brain analysis

To explore the brain regions that are sensitive to the holistic distance in each grade, we first performed a random effects GLM contrasting near and far distances for 2nd-graders vs. 5th-graders collapsed across all notations. The contrast near > far in 2nd-graders revealed greater activation in the near condition in several regions including bilateral superior parietal lobules (SPL) as well as the IPS, the middle frontal gyrus (MFG) and the insula, ($p < .05$; see Figure 4 and Table S2 in Supplementary). Overall, neural distance effects were found in similar regions for 2nd- and 5th-graders, but activation covered a larger area for 5th-graders. Specifically, the regions showing neural distance effects in 5th graders included bilateral SPL and inferior parietal lobules (IPL) with the IPS, insula, and several frontal regions ($p < .05$; see Figure 4 and Table S2 in Supplementary).

3.2.2 Neural distance effects in each notation among 2nd- and 5th-graders.

To unpack the results above, we performed identical whole-brain analyses contrasting near > far within each notation. For 2nd-graders, the neural distance effects were mainly in response to Nonsym and Mixed ($p < .05$, corrected; see Figure 5 and Table S3 in Supplementary). We found neural distance effects for Nonsym in multiple regions of the brain, including the bilateral parietal lobules and right inferior frontal gyrus (IFG). The parietal lobules have previously been implicated in processing fractions magnitudes (e.g., Ischebeck et al., 2009; Jacob & Nieder, 2009b, 2009a; Mock et al., 2018; Wortha et al., 2020) and also whole numbers (Ansari, 2008;

Sokolowski et al., 2021; Sokolowski, Fias, Bosah Ononye, et al., 2017). The IFG has been frequently implicated in domain-general abilities such as cognitive control (for review see Aron, Robbins, & Poldrack, 2004; for meta-analysis see Levy & Wagner, 2011). The distance effects for Mixed were broader than those demonstrated for Nonsym, covering broader frontal and parietal regions of the brain including the IFG. We suspect that this was likely due to increased processing demands imposed by translation costs for converting between nonsymbolic and symbolic fractions. For example, such patterns also have observed in the tasks which require to interpret semantic meanings of symbolic numbers (Ansari, 2008; Houdé, Rossi, Lubin, & Joliot, 2010; Lussier & Cantlon, 2016). As for Sym, we found a small cluster of the right IFG showing neural distance effect ($p < .05$, uncorrected), but this cluster was not significant after cluster correction likely due to low power. These results showed that 2nd-graders' frontal-parietal network was particularly sensitive to nonsymbolic formats of ratio magnitudes.

In contrast to our findings with 2nd-graders, we found neural distance effects in all notations for the 5th grade cohort ($p < .05$; see Table S4 in Supplementary). Among 5th-graders, even Sym comparisons recruited both frontal-parietal regions, with Nonsym and Mixed comparisons recruiting broader parietal and frontal regions. These contrasting results across cohorts show a developmental difference between 2nd- and 5th-graders, particularly regarding neural responses to symbolic fractions. These results are somewhat consistent with prior findings with whole numbers showing that neural distance effects were found in the parietal lobules in adults, but not in children (Ansari et al., 2005; Cantlon et al., 2009). This finding may indicate that children's brains become more sensitive to symbolic fractions sometime after children start to receive formal fraction instruction. As noted in the behavioral analysis, neural distance effects could possibly be due to children's use of gap strategies. We therefore confirmed that the neural

distance effects for symbolic fractions were mainly due to the holistic distance by performing a whole-brain analysis using gap rather than distance as the contrast. This analysis revealed no significant activation due to gap distance (see Supplementary Results; see also Karla, Binzak et al., 2020).

3.2.3 Conjunctions of neural distance effects across all notations

Next, to validate our findings regarding overlap in the brain regions principally involved in processing fraction magnitudes across nonsymbolic and symbolic notations, we performed conjunction analyses of neural distances effect across all notations. Since 2nd-graders did not show statistically significant distance effects in Sym, the conjunction analyses were performed only with 5th-graders. We identified a few regions that exhibited neural distance effects for all notations, including the right IFG, insula and bilateral parietal lobules. The parietal effects were larger effects on the right, consistent with the findings of our whole-brain analysis and previous studies with fractions and whole numbers ($p < .05$; see Table S5 in Supplementary and Figure 6) (Kersey & Cantlon, 2017; Mock et al., 2018; Sokolowski, Fias, Mousa, et al., 2017; Yeo et al., 2017). These results with children show that common fronto-parietal regions are sensitive to fraction magnitudes regardless of notation in 5th graders—similar to prior findings with adult samples (e.g., Ischebeck et al., 2009; Mock et al., 2018).

3.2.4 Neural sensitivity to ratio magnitudes in 2nd- and 5th-graders: ROI analysis

To examine whether our a priori region of interest – the IPS – was sensitive to symbolic and nonsymbolic ratio magnitudes at a regional-level, we tested for neural distance effects in the IPS coordinates from Houde et al. (2010) using three distance bins in each notation. We first conducted a linear mixed-effects regression examining fixed effects by regressing extracted mean beta values against grade (2nd = 0, 5th = 1), hemisphere (left = 0, right = 1), distance bin

(far = 0, medium = 1, near = 2) and notation (Nonsym = 0, Mixed = 1, Sym = 2). We had hypotheses regarding the change of brain engagement between levels (i.e., 2nd-graders > 5th-graders, Sym > Mixed > Nonsym, and near > med > far).

The fixed effects results showed that the IPS activation increased as the distance decreased, with an especially marked increase between far and medium bins ($p < .001$; Table 5). We also found that IPS engagement increased with notation in the order of Sym > Mixed > Nonsym (Mixed > Nonsym; $p = .055$, Sym > Mixed; $p = .014$). However, there were no significant differences between hemispheres or grade levels.

To better understand the results, we tested the effect of distance in the IPS ROIs at each notation, hemisphere, and grade separately. We used the ‘anova’ function from the ‘car’ package in R that performs the Wald chi-square test which shows the main effect across different levels of distance (near, med, and far) (see Figure 7). In 2nd-graders, only the right IPS showed significant distance effects in Nonsym ($\chi^2 = 7.27, p = .026$) and Mixed ($\chi^2 = 7.27, p = .026$) notations. In 5th-graders, the bilateral IPS showed significant distance effects for all notations (left: $\chi^2 = 8.21, p = .017$ for Mixed, $\chi^2 = 8.20, p = .017$ for Mixed; right: $\chi^2 = 6.91, p = .031$ for Nonsym, $\chi^2 = 17.80, p < .001$ for Nonsym, $\chi^2 = 14.31, p < .001$ for Sym) with the exception of the left IPS, which did not show significant distance effects in Nonsym ($\chi^2 = 2.88, p = .236$). These results are consistent with the whole-brain analyses: both 2nd- and 5th-graders showed right lateralized distance effects for nonsymbolic ratios.

4. Discussion

In this study, we compared behavioral performance and neural signatures for children who have not yet received formal fractions instruction (2nd-graders) and children who have received a few years of such instruction (5th-graders) to test the hypothesis that fractions

knowledge builds on preexisting human capacities to process nonsymbolic ratios. Our results were consistent with two key predictions of Lewis et al.'s (2015) RPS account:

1. Holistic ratio processing prior to formal instruction: Children who had not yet received prior fractions instruction (i.e., 2nd-graders) could quickly and accurately compare nonsymbolic ratios, and reliably recruited parietal-frontal networks to do so.
2. Neuronal recycling of the RPS for symbolic fraction processing: Children with prior fractions instruction (i.e., 5th-graders) recruited similar frontal-parietal regions for both nonsymbolic ratios and symbolic fractions

Moreover, our data were consistent with the prospect that educational experiences may drive a developmental shift from a primarily right lateralized system that processes nonsymbolic ratios to a bilateral system that integrates across formats. This too was consistent with the RPS account advanced by Lewis et al. Below, we recap our findings, highlighting key predictions of the RPS account tested by examining similarities and differences between our cohorts. We further discuss how the patterns discovered provide new insights into the cognitive architecture for processing nonsymbolic rational number values and how that architecture develops with the acquisition of symbolic fractions knowledge.

Holistic ratio processing prior to formal instruction

One key tenet of the RPS account is that children have perceptual sensitivity to nonsymbolic ratio analogs even prior to receiving formal instruction on symbolic numerical ratios and fractions. Our in-scanner behavioral data replicated previous findings demonstrating such sensitivity to nonsymbolic ratios among young children, and our fMRI data allowed us, for the first time, to identify the neural systems associated with nonsymbolic ratio processing.

Behavioral evidence

We found that, in-scanner, 2nd-grade children could effectively compare nonsymbolic ratio magnitudes before formal instruction on rational numbers. These results replicated Kalra et al.(2020)'s findings with the larger out-of-scanner cohort: 2nd-graders were capable of comparing rational number magnitudes accurately and rapidly in all three notations, although 5th-graders were faster and more accurate overall. We also similarly found behavioral distance effects in all notations for 2nd- and 5th-graders, suggesting that even 2nd graders can compare holistic ratio magnitude information in multiple notations. These results were consistent with previous studies that have shown that young children even as young as 4-years-old can process nonsymbolic ratios holistically (Kalra et al., 2020; Park et al., 2021; Szkudlarek & Brannon, 2021; See also McCrink & Wynn, 2007). These findings provide strong evidence in support of the RPS hypothesis that children are equipped with preexisting neurocognitive architectures for interpreting nonsymbolic ratio magnitudes.

Also as predicted, we found significant differences in performance by notation for both grade cohorts. Children were fastest and most accurate with nonsymbolic (line vs. line) ratio comparisons, and slowest and least accurate with symbolic (fraction vs. fraction) comparisons. The fact that nonsymbolic ratio comparisons were fast and accurate supports the idea of a pre-existing (i.e., ontogenetically early) mechanism sensitive to nonsymbolic ratio magnitudes that may later be recycled to process symbolic fractions, as the RPS account predicts. Notably, children were just as accurate with mixed notation comparisons (line-fraction) as they were with within notation symbolic (fraction-fraction) comparisons. This is noteworthy because it was reasonable to expect that mixed notation comparisons should impose additional translation costs (Lyons et al., 2012). For example, if children first converted a nonsymbolic ratio to a symbolic

fraction and then determined which of two was larger, performance should presumably be worse (lower accuracy, slower reaction times) than for within format symbolic comparisons. However, this was not the case. This result replicated past findings and was consistent with our neural findings that nonsymbolic and symbolic ratios may utilize a common internal magnitude code (see Kalra et al., 2020; Matthews & Chesney, 2015).

Neuroimaging evidence

Critically, our fMRI data also provided neural evidence of holistic processing of ratio magnitudes. Consistent with the predictions of the RPS account, both 2nd and 5th graders showed significant neural distance effects for nonsymbolic ratio (line-line) processing in bilateral² frontal-parietal regions including the IPS, the middle frontal gyrus, and the bilateral insula. These brain regions have previously been demonstrated to respond during nonsymbolic numerical processing, with analogs to both rational numbers (e.g., Mock et al., 2018; Vallentin & Nieder, 2010) and whole numbers (e.g., Houdé et al., 2010; Sokolowski, Fias, Mousa, et al., 2017). Our results further buttress the argument that nonsymbolic ratio magnitudes are processed holistically in ways similar to other types of analog magnitudes and by engaging similar sets of brain regions (Bueti & Walsh, 2009; Cohen Kadosh et al., 2008; Walsh, 2003).

Interestingly, we observed a developmental shift in response to nonsymbolic line-line comparisons. Although 2nd-graders' distance effects were characterized primarily by right lateralized parietal responses, 5th-graders neural distance effects were more balanced bilaterally at both whole-brain and regional levels. These developmental differences are similar to the findings from previous studies with numerosities. Prior work has found that young children engage primarily the right IPS to process numerosities (Ansari & Dhital, 2006; Cantlon et al.,

² Although the left region only included a small number of voxels

2006; see also meta-analysis of Kaufmann et al., 2011; Sokolowski, Fias, Mousa, et al., 2017), but adults engage both right and left IPS (e.g., Ansari & Dhital, 2006), suggesting that the right IPS plays an important role as a foundation for processing numerical information early in development and the left IPS engages later in development (Ansari, 2016). The developmental differences we found with nonsymbolic ratios mirror the right-to-bilateral shift observed in these prior studies, suggesting that the right IPS in particular may be specialized for nonsymbolic ratios prior to formal instruction.

Neuronal recycling of the RPS for symbolic fraction processing

The second key prediction of the RPS account is that symbolic fraction processing builds on neural foundations for processing nonsymbolic ratios. Consistent with the RPS model, symbolic fraction comparisons did not lead to significant neural distance effects in 2nd-graders, but did in 5th graders. Critically, 5th-graders engaged the same fronto-parietal network recruited for nonsymbolic ratios—including bilateral IPS and inferior parietal lobes—for symbolic fraction processing. This finding suggests the three years of instruction with symbolic fractions between 2nd- and 5th-grade may lead to a recycling of the inferior parietal lobules—machinery initially geared to process nonsymbolic ratios—for symbolic fractions.

It is also important that the brain areas identified in 5th-graders were consistent with those found in prior studies of adults fraction processing for both and nonsymbolic (e.g., Mock et al., 2018; Vallentin & Nieder, 2010) and symbolic stimuli (Cui et al., 2020; Ischebeck et al., 2009; Mock et al., 2018; Wortha et al., 2020). These similarities indicate that the neural processing of symbolic fractions was refined to such an extent by 5th grade in our sample that it produced adult-like activation patterns. Taken together, these findings demonstrate that years of fraction

instruction may help engage bilateral engagement of the inferior parietal lobule for nonsymbolic ratios processing and develop neural specialization for symbolic fraction processing.

Considerations regarding the development of symbolic fraction processing

Although the patterns emerging from our data were largely consistent with the RPS account, they also raise a number of questions. In particular, we were struck by the fact that we did not observe neural distance effects with symbols among 2nd-graders, which contrasts with: a) the fact that 2nd-graders did show behavioral distance effects with symbols and b) that they demonstrated neural distance effects with mixed notation comparisons. We consider each in turn below.

No neural responses toward symbolic ratios while 2nd graders can still compare.

One plausible explanation of our failure to find neural distance effects for symbolic comparisons in the younger age group may be due to the heterogeneity of early (pre-instruction) neural responses toward symbolic fractions. Several previous studies have noted distributed neural responses in processing various information including visual stimuli as faces and even mind reading (e.g., Cox et al., 2015; Norman et al., 2006; Scherf et al., 2007). Along with these studies, some prior work suggests that some cognitive processes may be characterized by more distributed processing in the developing brain and become localized to certain regions of the brain with increased experience (e.g., Jacobs & Jordan, 1992). Based on these previous results, it is plausible that the initial representations for symbolic fractions may be sparse and broadly distributed and only converge on a dedicated neural system after formal instruction on fractions.

Thus, it is possible that in our study, individual 2nd-graders may have indeed exhibited neural distance effects for symbolic fractions, but the effects may have been too widely distributed (within individuals) and anatomically heterogeneous (across individuals) to yield

localized neural distance effects in our group-level analyses. After more experience with symbolic fractions, these distributed responses may become more localized, both within and across participants, reflecting the patterns seen in the fifth graders in our study. To better understand the brain responses toward symbolic fraction in the early developing brain and developmental changes of its patterns, we are currently analyzing longitudinal data and using multivariate approaches to identify developmental changes in distributed and/or overlapping representations for symbolic fraction processing.

2nd graders' neural responses toward mixed comparisons are different from symbolic ones

Even if the hypothesis of distributed representations for early symbolic fraction processing is correct, a question remains as to why 2nd-graders exhibited neural distance effects for mixed notations but not for within notation symbolic ones. One possible scenario is that the brain responses to nonsymbolic ratios may be more coherent and stronger enough to lead a detectable neural distance effects in mixed notation. As we mentioned above, the neural activation for symbols can be sparse and distributed. However, sparse and distributed activation could still plausibly encode specific magnitudes, such that there was a distance dependent response for regions responsible for holistic processing of nonsymbolic ratios. This distance-dependent response may have strong enough to compensate for the weaker and distributed signals from symbolic fractions. This state of affairs would allow 2nd graders to exhibit localized neural distance effects between holistic magnitudes in mixed notations.

Another possible scenario is that symbolic notations may allow children to rely on a mix of strategies that are not necessarily rooted in holistic magnitude, whereas the mixed notation may force them to use semantic representations that are based on holistic distance. As many previous studies have noted (e.g., Fazio et al., 2016; Morales et al., 2020; Obersteiner et al.,

2013), symbolic fraction comparison may allow multiple strategies, such as choosing the stimulus with the smaller gap between numerator and denominator, which need not involve processing the holistic distance between fractions. However, these strategies cannot be used for comparing between a symbolic fraction and a nonsymbolic ratio, because there is no metric for comparing their respective gaps. Unlike symbols, comparing nonsymbolic ratios is less susceptible to other strategies because the overall sizes of nonsymbolic ratio components are not important (ex. the size and the gap of ratio stimuli can be large, but it can still indicate a larger ratio). Therefore, it is possible that mixed notation may elicit stronger neural distance effects by forcing children to use distance-dependent strategies more consistently.

5. Conclusion

The present study offers the first evidence demonstrating the existence of a primitive cognitive architecture for ratio processing in young children at both behavioral and neural levels. This stands in stark contrast to theories positing that the human cognitive architecture is ill-equipped to process fractions magnitudes (e.g., Dehaene, 2011; Feigenson et al., 2004). More broadly, our findings suggest that the neurocognitive architecture initially tuned only to nonsymbolic ratio magnitudes becomes tuned to symbolic fractions as children receive formal instruction about fractions. Altogether, our findings align well with the RPS hypothesis suggesting a *neuronal recycling* of the RPS to acquire symbolic fraction knowledge. Future training studies would be necessary to verify this account of the *neuronal recycling* of the RPS, as they could directly test the effects of different types of instruction for the neurodevelopment of symbolic fraction processing. Our study features a promising cognitive architecture that may aid children's cognitive development and learning.

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