Evaluating the Internal Consistency of Subtraction-Based and Residualized Difference Scores:
Considerations for Studies of Event-Related Potentials

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Abstract

In studies of event-related brain potentials (ERPs), difference scores between conditions in a task are frequently used to isolate neural activity for use as a dependent or independent variable.

Adequate score reliability is a prerequisite for studies examining relationships between ERPs and external correlates, but there is no extensive treatment on the suitability of the various available approaches to estimating difference score reliability that focus on ERP research. In the present study, we provide formulas from classical test theory and generalizability theory for estimating the internal consistency of subtraction-based and residualized difference scores. These formulas are then applied to error-related negativity (ERN) and reward positivity (RewP) difference scores from the same sample of 117 participants. Analyses demonstrate that ERN difference scores can be reliable, which supports their use in studies of individual differences. However, RewP difference scores yielded poor reliability due to the high correlation between the constituent reward and non-reward ERPs. Findings emphasize that difference score reliability largely depends on the internal consistency of constituent scores and the correlation between those scores. Furthermore, generalizability theory yields more suitable estimates of internal consistency for subtraction-based difference scores than classical test theory. We conclude that ERP difference scores can show adequate reliability and be useful for isolating neural activity in studies of individual differences.

Keywords: reliability, difference score, generalizability theory, event-related potentials (ERPs), ERP Reliability Analysis (ERA) Toolbox, psychometrics

The suitability of various approaches to estimating the internal consistency of difference scores has received a great deal of debate in the psychometric literature for decades (Cattell, 1982; Cronbach & Furby, 1970; Draheim, Mashburn, Martin, & Engle, 2019; Jennings & Cribbie, 2016; Lord, 1956; Rachor & Cizek, 1996; Trafimow, 2015; Yin & Brennan, 2002). In studies of event-related brain potentials (ERPs) or functional magnetic resonance imaging (fMRI), a difference score¹ between two conditions is commonly used to isolate activity for use as a dependent or independent variable (Luck, 2014; Picton et al., 2000; Urbach & Kutas, 2006). However, there is no extensive treatment on which approaches for estimating the internal consistency of difference scores are most appropriate for ERP scores, which undermines the efforts to encourage the routine evaluation of psychometric reliability. The verification of score reliability should be an early step in data analysis because unreliable data call into a question a study's conclusions. Therefore, in this paper we describe optimal approaches for estimating the internal consistency of ERP difference scores and clarify the measurement characteristics that improve difference-score reliability.

In many areas of neuroscience research, including ERP studies, a common approach for calculating difference scores is to subtract activity during one condition (X , the subtrahend) from activity during another condition (Y , the minuend; $Y - X$). Another approach increasing in popularity is a residualized difference score (e.g., Crane et al., 2021; Hanna et al., 2020; Klawohn, Meyer, Weinberg, & Hajcak, 2020; Macatee et al., 2020; Meyer, Lerner, De Los Reyes, Laird, & Hajcak, 2017; Muir, Hedges-Muncy, Clawson, Carbine, & Larson, 2020; Sandre

¹ For the purpose this manuscript, we only focus on difference scores used to isolate neural activity. However, the described approaches have application to other difference scores, such as differences between two related ERP components, differences between tasks, or pre- and post-intervention differences between the same ERP components.

et al., 2020; Schroder et al., 2020; Suor et al., in press). A residualized difference score is the difference between the actual score (Y) and predicted score ($\hat{Y}$) using a linear regression equation ($Y - \hat{Y}$), wherein Y scores are regressed on X . Hence, the residualized difference scores characterize participants with larger or smaller differences in Y scores than would be expected based on X scores for a group of participants.

If ERP difference scores are to be used in studies of individual differences, their internal consistency needs to be verified. Estimates of internal consistency characterize how well scores can distinguish between people. Poor internal consistency can lead to spurious relationships (Flegal, Kit, & Graubard, 2017; Gelman & Carlin, 2014) and low statistical power (Boudewyn, Luck, Farrens, & Kappenman, 2017; Clayson & Miller, 2017b; Fischer, Klein, & Ullsperger, 2017; Kolossa & Kopp, 2018; Luck & Gaspelin, 2017). As such, poor internal consistency renders any relationship between ERPs and external correlates tentative. What makes the internal consistency of difference scores potentially problematic is that difference scores contain measurement error from each constituent measure and is impacted by the correlation between the two constituent measures.

We cover approaches for estimating the internal consistency of subtraction-based difference scores and discuss issues relevant for ERP research. We review classical test theory and generalizability theory approaches to estimate the reliability of subtraction-based difference scores. Then, we cover issues related to the estimation of the reliability of residualized difference scores. After introducing the different formulas for calculating difference score reliability, we demonstrate the application of these formulas to characterizing difference score reliability of the error-related negativity (ERN) and reward positivity (RewP) components. We close with a discussion of how and when difference scores can be reliable in ERP research.

Subtraction-Based Difference Scores

Subtraction-based difference scores include instances when ERP activity recorded during one condition is subtracted from ERP activity during another condition. Two relevant measurement theories for assessing the internal consistency of ERP difference scores are classical test theory and generalizability theory.

Classical Test Theory

The internal consistency of difference scores was historically assumed to be much lower than the constituent scores (i.e., the subtrahend and minuend) and consequently inherently unreliable (e.g., Cronbach & Furby, 1970; Lord, 1956; Overall & Woodward, 1975). These early estimates of difference score reliability relied on mathematical formulas that assumed equal variance of the constituent scores. For example, consider the following equation:

$$\rho_{DD'} = \frac{\rho_{XX} + \rho_{YY} - 2\rho_{XY}}{2 - 2\rho_{XY}} \quad (\text{Eq. 1})$$

Here, the internal consistency of a subtraction-based difference score ($\rho_{DD'}$) is estimated from the separate internal consistencies of X (ρ_{XX}) and Y (ρ_{YY}) and the correlation between X and Y (ρ_{XY}). If the separate internal consistencies of X (ρ_{XX}) and Y (ρ_{YY}) scores are held constant, any increase in the correlation² between X and Y (ρ_{XY}) will decrease the reliability of subtraction-based difference scores (see also Thomas & Zumbo, 2011), and this is the case for all approaches to estimating the reliability of difference scores described in the present manuscript.

² Notably, simply increasing the internal consistencies of X and Y does not have a straightforward impact on difference score reliability. If there is a “true” correlation between the two scores, their correlation will be attenuated when their internal consistencies are low. Although increasing the internal consistencies of each score would hopefully lead to higher reliability of the difference score, higher internal consistencies should also deattenuate the correlation between X and Y , subsequently reducing the overall difference score reliability (Bereiter, 1963).

An assumption of Equation 1 is that the variances of X and Y are constant, but this assumption is unlikely to be true in most cases (Yin & Brennan, 2002; Zimmerman & Williams, 1982a; Zumbo, 1999), particularly in ERP research. In ERP studies the assumption of equal variances is often violated (Jennings, 1987; Picton et al., 2000). An expression of subtraction-based difference score reliability was derived that relaxed the restriction of equal variances of the constituent scores (Zimmerman & Williams, 1982a, 1998). This later formula is more appropriate for ERPs than earlier formulas, because the variance of each ERP is accounted for in the estimation of reliability. The reliability of subtraction-based differences scores between two measures, X and Y , is shown here in Equation 2.

$$\rho_{DD'} = \frac{\rho_{XX}\lambda + \rho_{YY}\lambda^{-1} - 2\rho_{XY}}{\lambda + \lambda^{-1} - 2\rho_{XY}} \quad (\text{Eq. 2})$$

The internal consistency of a subtraction-based difference score ($\rho_{DD'}$) is estimated from the separate internal consistencies of X (ρ_{XX}) and Y (ρ_{YY}), the ratio of the standard deviations³ of X and Y ($\lambda = \sigma_X/\sigma_Y$), and the correlation between X and Y (ρ_{XY}). When the ratio of standard deviations (λ) is not 1, the estimated internal consistency of the difference score can be higher than when this ratio is ignored (Rachor & Cizek, 1996; Williams, Zimmerman, Rich, & Steed, 1984; Yin & Brennan, 2002; Zimmerman & Williams, 1982a, 1982c). Furthermore, as described above, as the correlation between X and Y (ρ_{XY}) approaches one, the difference score reliability approaches zero (Thomas & Zumbo, 2011).

In the psychometric literature, the approach to estimating the internal consistency of a difference score (Equation 2) commonly uses coefficient alphas (i.e., Cronbach's alpha) to

³ For a discussion of the conceptual differences and relationship between standard deviations and psychometric reliability, see Clayson, Brush, and Hajcak (2020) and Luck, Stewart, Simmons, and Rhemtulla (2020).

characterize the internal consistency of X and Y . However, the common formulation of coefficient alpha is not well suited for estimating the internal consistency of ERP scores due to the unbalanced nature of ERP scores across participants. Coefficient alpha requires participants to have the same number of observations. As a result, a split-half reliability estimate, such as two-part coefficient alpha (Cronbach, 1951; Rulon, 1939), is better suited to characterizing data from participants with unequal observations (see Appendix). However, the split-half reliability approach is susceptible to sampling variability due to splitting the data into two arbitrary halves, which is particularly problematic when few trials are retained for averaging (see Clayson, Brush, et al., 2020).

Generalizability Theory

Generalizability theory approaches circumvent the sampling variability endemic to split-half estimates by using all ERP trials from all participants in the estimation of reliability (Baldwin, Larson, & Clayson, 2015; Clayson, Carbine, Baldwin, Olsen, & Larson, 2020; Clayson & Miller, 2017a). Generalizability theory provides a multifaceted framework for estimating score reliability that simultaneously considers multiple sources of variance (Brennan, 2001; Cronbach, Gleser, Nanda, & Rajaratnum, 1972; Vispoel, Morris, & Kilinc, 2018). ERP difference score reliability can be estimated using a multivariate extension of the univariate generalizability theory formulas (see Baldwin et al., 2015; Clayson & Miller, 2017a, for an introduction to the univariate approach).

When estimating difference score reliability with generalizability theory, there are two different types of reliability coefficients to consider, and the choice between which coefficient to use is left up to the researcher and the questions of interest (Brennan, 2001; Yin & Brennan, 2002). The two reliability coefficients are a generalizability coefficient ($E\rho^2$) and a

dependability coefficient (ϕ). A generalizability coefficient is akin to classical test theory estimates, such as coefficient alpha or split-half reliability, and only considers the ranking of individuals in its estimation:

$$E\rho_D^2 = \frac{\sigma_X^2(p) + \sigma_Y^2(p) - 2\sigma_{XY}(p)}{\sigma_X^2(p) + \sigma_Y^2(p) - 2\sigma_{XY}(p) + \frac{\sigma_X^2(e)}{n'_{iX}} + \frac{\sigma_Y^2(e)}{n'_{iY}} - \frac{2\sigma_{XY}(e)}{\bar{n}'_i}} \quad (\text{Eq. 3})$$

The generalizability coefficient of a difference score ($E\rho_D^2$) involves estimating between-person variance (p) and error variance (e ; i.e., residual variance). Between-person parameters⁴ are the between-person variances of X ($\sigma_X^2(p)$) and Y ($\sigma_Y^2(p)$) and their covariance ($\sigma_{XY}(p)$). The within-person parameters include the residual variance for X ($\sigma_X^2(e)$), Y ($\sigma_Y^2(e)$) and the covariance between residuals ($\sigma_{XY}(e)$). The contribution of error variance to the internal consistency of the difference score is impacted by the average number of X items (i) or trials (n'_{iX}), the average number of Y trials (n'_{iY}), and the harmonic mean of X and Y ($\bar{n}'_i$). In the generalizability coefficient, it can be seen that a large between-person covariance of X and Y ($\sigma_{XY}(p)$) can reduce the reliability of the difference score due to its impact on the numerator, and the impact of error variance on reliability can be minimized by retaining many trials for analysis.

An important distinction between generalizability and coefficients is that the dependability coefficient also considers the actual magnitude of change in differences (Brennan, 2001; Yin & Brennan, 2002). In the classical test theory estimates and generalizability coefficient, the average change in scores is essentially subtracted out of the difference score. For example, if there is a large average difference between two ERPs, the magnitude of this

⁴ The relationship between variance components estimated within a generalizability theory framework are not transparently related to conventional coefficients within a classical test theory framework. The interested reader is directed to an in-depth comparison of generalizability theory and classical test theory coefficients in Vispoel et al. (2018).

difference has no impact on the generalizability coefficient for the difference score (see also Yin & Brennan, 2002). When the goal of the measurement is to make inferences about the magnitude of differences between X and Y , then the dependability coefficient is the appropriate coefficient.

$$\phi_D = \frac{\sigma_X^2(p) + \sigma_Y^2(p) - 2\sigma_{XY}(p)}{\sigma_X^2(p) + \sigma_Y^2(p) - 2\sigma_{XY}(p) + \frac{\sigma_X^2(i)}{n'_{iX}} + \frac{\sigma_Y^2(i)}{n'_{iY}} - \frac{2\sigma_{XY}(i)}{n'_i} + \frac{\sigma_X^2(e)}{n'_{iX}} + \frac{\sigma_Y^2(e)}{n'_{iY}} - \frac{2\sigma_{XY}(e)}{n'_i}} \quad (\text{Eq. 4})$$

The estimation of the dependability coefficient differs from the generalizability coefficient by including estimates of between-trial or between-item (i) (co)variances. Between-trial variances are estimated for X ($\sigma_X^2(i)$), Y ($\sigma_Y^2(i)$), and their covariance ($\sigma_{XY}(i)$). Similar to error variances, the impact of between-trial variances is impacted by the number of ERP trials for X and Y .

Although Equation 3 and 4 include the residual covariance, not all ERP designs will have correlated residuals. In ERP studies, the estimation of error covariance is sensible when measuring multiple ERP features of the same trial. Some examples include when using a peak-to-peak amplitude difference score or when the ERP is defined as the difference between electrode sites (e.g., ipsilateral vs. contralateral activity), such as N2pc, the lateralized readiness potential (LRP), or ERPs related to the interhemispheric transfer of information. When ERP difference scores are measured for two separate conditions, these are not considered naturally cooccurring events, and the error covariances in Equations 3 and 4 are set to zero ($\sigma_{XY}(e) = 0$).

Residualized Difference Scores

A common criticism of subtraction-based difference scores is that they are negatively correlated with the subtrahend, X , in Y minus X . A residualized difference score has a zero correlation with X , when Y scores are regressed on X (Traub, 1967). It is argued in the psychometric literature that the spurious correlation between the subtraction-based difference score and the subtrahend is a major disadvantage of subtraction-based difference scores, and likely reason enough to abandon them (Cronbach & Furby, 1970; Linn & Slinde, 1977; Lord,

1963; O'Connor Jr, 1972). The reason being that a subtraction-based difference score might be unfair and potentially disadvantage those with certain scores (typically high scores), although this is a matter of considerable debate (see Unfairness section in Zumbo, 1999).

The correlation between the pretest score and subtraction-based difference score can occur somewhat artifactually simply due to the relationship between the variances of X and Y (Rachor & Cizek, 1996; Trafimow, 2015; Zimmerman & Williams, 1982a, 1982b), and the correlation can be due to several possible explanations (see Rogosa, Brandt, & Zimowski, 1982; Zimmerman & Williams, 1982b; Zumbo, 1999). Explanations include floor and ceiling effects in the data at pretest, regression toward the mean, or fanspread (i.e., when variances increase over time). With regard to ERP scores, voltage measurements are not likely restricted by floor and ceiling effects, and regression toward the mean and fanspread are more likely applicable to pre-test/post-test difference scores rather than between-condition difference scores—the kind commonly used in ERP research. Furthermore, the correlation between the subtrahend score and difference score does not invalidate the difference score or its interpretation (Allison, 1990; Kenny, 1975).

Another reason to possibly use subtraction-based difference scores is that they are an unbiased estimate of change, unlike residualized scores (Rogosa et al., 1982). Furthermore, data simulations illustrate that subtraction-based difference scores tend to show comparable or better Type 1 error rates, statistical power, and measurement bias to residualized scores in studies that use random group assignment (Jennings & Cribbie, 2016; Trafimow, 2015). When pre-existing group membership is related to ERP scores from the subtrahend condition (i.e., X , in Y minus X), type 1 error rates are higher, statistical power is lower, and measurement bias is higher for residualized difference scores than for subtraction-based difference scores (Jennings & Cribbie,

2016; see also Linn & Slinde, 1977; Trafimow, 2015). These findings are particularly relevant for ERP psychopathology researchers that often study pre-defined groups and suggest that a subtraction-based difference score is preferred in some instances. It suffices to say that there are many considerations when deciding between a subtraction-based difference score and a residualized difference score, but it is likely premature to disregard either.

The internal consistency of residualized difference scores can be estimated using the following equation (Williams & Zimmerman, 1982):

$$\rho_{RR'} = \frac{\rho_{YY} + \rho_{XY}^2 \rho_{XX} - 2\rho_{XY}^2}{1 - \rho_{XY}^2} \quad (\text{Eq. 5})$$

When Y scores are regressed on X scores, the residualized difference score ($Y - \hat{Y}$) is impacted by the internal consistencies of X (ρ_{XX}) and Y (ρ_{YY}) and their squared correlation (ρ_{XY}^2). Unlike subtraction-based difference scores, the residualized difference score is not directly impacted by the variability in X or Y (although the internal consistencies of X and Y are impacted by their variances). Similar to the previous reliability coefficients, a large correlation between X and Y generally reduces the internal consistency estimate of the residualized difference scores.

Application of Internal Consistency Estimates

The psychometric properties of ERN and RewP components are of considerable interest because they are often used in clinical research to investigate individual differences with clinical symptoms, such as depression (Clayson, Carbine, & Larson, 2020; Moran, Schroder, Kneip, & Moser, 2017), anxiety (Hajcak, Klawohn, & Meyer, 2019; Moser, Moran, Kneip, Schroder, & Larson, 2016), and psychotic features (Clayson, Wynn, Green, & Horan, 2018; Clayson et al., 2019). ERN represents activity related to performance monitoring that is sensitive to behavioral errors (Larson, Clayson, & Clawson, 2014) and is a negative deflection in the scalp-recorded ERP that occurs within 100ms of error commission (Falkenstein, Hohnsbein, Hoormann, &

Banke, 1991; Nieuwenhuis, Ridderinkhof, Blom, Band, & Kok, 2001). The correct-related negativity (CRN) is the correct-trial counterpart to ERN and reflects overall response monitoring (Vidal, Hasbroucq, Grapperon, & Bonnet, 2000). A difference score using ERN (Δ ERN) is commonly calculated by subtracting CRN amplitude from ERN amplitude (i.e., error minus correct activity) or by regressing ERN on CRN. Δ ERN is thought to isolate activity related to error monitoring from activity related to overall performance monitoring.

Whereas ERN is associated with the internal monitoring of errors, the monitoring of external feedback and reward is indexed by RewP (Proudfit, 2015). RewP is characterized by a positive deflection in the scalp-recorded ERP approximately 200 to 300ms following presentation of positive feedback and is more positive for reward than for neutral trials (Holroyd, Pakzad-Vaezi, & Krigolson, 2008; Kujawa, Smith, Luhmann, & Hajcak, 2013; Proudfit, 2015). The feedback negativity (FN) is the loss-trial counterpart to RewP. A difference score using RewP (Δ RewP) can be calculated by subtracting FN amplitude from RewP amplitude (i.e., reward minus loss activity) or by regressing RewP on FN. Δ RewP is thought to isolate activity related to reward processing from activity related to loss feedback processing.

We now estimate the internal consistency of subtraction-based and residualized difference scores using published ERN and RewP data (Muir et al., 2019). The purpose of the Muir et al. (2019) study was to examine the relationship between perfectionism, locus of control, and ERPs in healthy undergraduates. The interested reader is directed to Muir et al. (2019) for more information about those relationships and in-depth details on the sample and measures. The ERN and RewP waveforms are also shown there. Participants from the published study with at least two trials from each condition of ERN and of RewP were included in the present analyses. A minimum of two trials was necessary to use a random resampling procedure for calculating

difference score reliability (so at least one trial was available for all participants after splitting a condition in half, see Data Analysis section). The sample comprised 117 undergraduate student participants (64 women), and the average age of participants was 20.6 ($SD = 2.0$).

The present analyses used the previously reported data processing pipeline. Hence, for all relevant characteristics of data recording and reduction, see Muir et al. (2019). Briefly, ERN was recorded during a modified version of the flanker task that consisted of 600 total trials, and single-trial ERN was scored as the average activity from 0 to 100 ms following the participant's response across four fronto-central electrode sites (6 [FCz], 129 [Cz], 7, 106; for electrode configuration, see Clayson & Larson, 2013). The difference score was estimated between correct- and error-trial ERN scores (Δ ERN; subtraction-based difference score: error minus correct; residualized difference score: error regressed on correct). There was an average of 487 correct trials ($SD = 92$, range: 35 to 583) and an average of 25 error trials ($SD = 20$, range: 2 to 103) after artifact rejection.

Single-trial ERP scores were recorded during a doors task that consisted of 50 total trials (RewP: 25 gain trials; FN: 25 loss trials). These ERPs were scored as the average activity from 250 to 325 ms across the same four fronto-central electrode sites as CRN/ERN. There was an average of 20 gain trials ($SD = 5$; range: 3 to 25) and an average of 18 loss trials ($SD = 5$; range: 2 to 25) after artifact rejection. The difference score was estimated between RewP and FN (Δ RewP; subtraction-based difference score: RewP minus FN; residualized difference score: RewP regressed on FN).

Data Analysis

Several different coefficients of subtraction-based and residualized difference scores for Δ ERN and Δ RewP were estimated. The formulas used for the estimation of difference score

reliability are shown in Equation 2 through 5. To estimate the separate internal consistencies of each ERP condition (ρ_{XX} , ρ_{YY}) for Equation 2 and Equation 5, we used a resampling procedure (see Clayson, Brush, et al., 2020). For ERP scores in a condition, the data were randomly split in half, scores from each half were averaged, the correlation between the two halves was then calculated, and the two-part alpha coefficient was estimated using the formula shown in the Appendix. This random resampling procedure was performed 10,000 times, and the average point estimate of all random resampling procedures was used to characterize the split-half reliability for ρ_{XX} and ρ_{YY} . The 95% confidence interval characterized the point estimates across all iterations.

A similar random resampling procedure was used to estimate the split-half internal consistency of subtraction-based and residualized difference scores (see Appendix). In this procedure, ERP scores from the two conditions were separately and randomly split into halves. Then, each half was separately averaged, resulting in two halves for each separate condition. The subtraction-based difference scores and residualized difference scores were then calculated using one half of a condition and one half of the other condition and then separately using the remaining halves from each condition. This resulted in two separate difference scores. The reliability was then estimated using the two-part alpha coefficient (see Appendix). This random resampling procedure was performed 10,000 times, and the average point estimate of all random resampling procedures was used to characterize the split-half reliability. The 95% confidence interval characterized the middle 95% of estimates across all 10,000 iterations.

Generalizability theory estimates were calculated using version 0.5.2 of the ERP Reliability Analysis (ERA) Toolbox (Clayson, Carbine, Baldwin, et al., 2020; Clayson & Miller, 2017a), which is available for download at https://github.com/peclayson/ERA_Toolbox. The

ERA toolbox is wrapper around the open-source packages MatlabStan (Stan Development Team, 2016), CmdStan (Stan Development Team, 2019), and MatlabProcessManager (Lau, 2016) in order to implement Markov Chain Monte Carlo (MCMC) estimation procedures in Stan (Carpenter et al., 2017). (Co)variance components were estimated using MCMC procedures with 4 chains and 10,000 iterations. The convergence of chains was determined by verifying that the potential scale reduction for the scalar estimands ($\hat{R}$) were below 1.1, by verifying that the effective sample size for each scalar estimand was greater than 10 times the number of chains, and by visual inspection of trace plots. The estimated variance components were used to calculate the internal consistency within the ERA Toolbox.

Results

A summary of the internal consistency estimates for each ERP, the subtraction-based, and the residualized difference scores is shown in Table 1. Confidence intervals are shown for two-part coefficient alphas that were randomly resampled 10,000 times, and the estimates represent the upper and lower bounds of the 95% confidence interval of the point estimates. For generalizability theory estimates, 95% credible intervals are shown, and credible intervals are the Bayesian analog to confidence intervals.

The internal consistency of CRN scores was similar across classical test theory ($r_{2\alpha}$) and generalizability theory estimates (ϕ). Each estimate was above .80, which is the preferred threshold of well-developed ERP measures (Clayson & Miller, 2017b). Classical test theory and generalizability theory estimates diverged for ERN, FN, and RewP scores ($.61 < r_{2\alpha} < .78$; $.80 < \phi < .90$). The divergence is likely due to the inherent bias introduced by sampling variability that is endemic to split-half reliability, and this bias becomes more apparent when few trials are included in the estimation of internal consistency (Clayson, Brush, et al., 2020).

Subtraction-Based Difference Scores

The internal consistency of subtraction-based differences scores was numerically lower for classical test theory estimates ($\rho_{DD'}$, $r_{2\alpha}$) than for generalizability theory estimates ($E\rho_D^2$, ϕ_D). There was virtually no difference between the generalizability coefficient ($E\rho_D^2$) and dependability coefficient (ϕ_D). This is due to the very small between-item (co)variance components (see Table 2) that contribute to the denominator of Equation 5, which are made even smaller still by dividing by the number of retained trials. It is likely that larger between-item variances would be observed when using different stimuli within the same condition than when using identical stimuli repeatedly. Under such circumstances, the generalizability and dependability coefficients are more likely to diverge.

ΔRewP yielded numerically lower score internal consistency estimates than ΔERN. This is likely due to the higher correlation/covariance between FN and RewP than between CRN and ERN (see Table 2 and 3). To demonstrate the impact of the correlation of constituent scores and relative standard deviations of constituent scores on the internal consistency of difference scores, their impact is estimated for various levels of λ (σ_X/σ_Y) and correlations in Table 4. λ is the relationship between the standard deviations of the constituent scores. Across each level of λ (.5, 1, 1.5) difference score reliability decreased as the correlation between constituent scores increased, and there is a benefit to reliability of difference scores when $\lambda \neq 1$. When comparing across levels of λ , difference score reliability decreased more rapidly when λ is 1.5 than when λ is 0.5. This demonstrates that a smaller standard deviation for X (the subtrahend) than for Y (the minuend; i.e., when $\lambda < 1$) benefits the reliability of the difference score over the converse (see also Ranchor & Cizek, 1996).

Residualized Difference Scores

The residualized differences scores for ΔRewP are numerically smaller than for ΔERN, which is likely due to the higher correlation between FN and RewP than between CRN and ERN. Table 5 shows the impact of different correlation coefficients of the constituent scores on internal consistency of the difference score. Higher correlations lead to lower internal consistency. With regard to the estimation of reliability using Equation 5 ($\rho_{RR'}$) or the split-half resampling procedure ($r_{2\alpha}$), estimates of ΔERN were numerically similar for $\rho_{RR'}$ and for $r_{2\alpha}$; however, $r_{2\alpha}$ was numerically higher than $\rho_{RR'}$ for ΔRewP (Table 1). Given that the formula for $\rho_{RR'}$ explicitly corrects for the correlation between constituent scores, it likely provides a more valid estimate of the internal consistency than $r_{2\alpha}$ for residualized difference scores.

Comparison of Difference Scores

Generally speaking, generalizability theory estimates of subtraction-based difference scores were numerically larger than either classical test theory estimates of subtraction-based difference scores or residualized difference scores. With regard to classical test theory estimates, the reliability of subtraction-based difference scores was numerically similar to the reliability of residualized difference scores for ΔERN, but $r_{2\alpha}$ for residualized ΔRewP was numerically higher than other classical test theory estimates.

There are some notable differences between the estimation of $\rho_{DD'}$ in Equation 2 and $\rho_{RR'}$ in Equation 5. First, subtraction-based difference scores use $\lambda (\sigma_X/\sigma_Y)$ in their estimation. Subtraction-based difference scores should be more reliable than residualized difference scores when the correlation between the constituent parts is greater than λ , when $\lambda < 1$, and when $\rho_{YY} > \rho_{XX}$ (Williams & Zimmerman, 1982). For ΔRewP, the correlation between FN (X) and RewP (Y) was smaller than λ and the reliability of RewP scores was numerically greater than the reliability of FN scores ($\rho_{YY} > \rho_{XX}$), which resulted in the residualized difference score

demonstrating larger internal consistency. For ΔERN, the correlation between CRN (X) and ERN (Y) was smaller than λ but the internal consistency of ERN scores was numerically lower than the internal consistency of CRN scores ($\rho_{YY} < \rho_{XX}$), which resulted in the residualized difference score demonstrating smaller numerical internal consistency.

Second, the residualized difference score estimation uses the squared correlation (ρ_{XY}^2) rather than the simple correlation (ρ_{XY}), as in the subtraction-based difference score. The numerator weights the reliability for a residualized difference score toward the reliability of the minuend (ρ_{YY}), because the reliability of the subtrahend (ρ_{XX}) is multiplied by the squared correlation coefficient. Furthermore, the squared correlation is numerically smaller than the simple correlation, and this results in the correlation between X and Y having a smaller impact on reliability for a residualized difference score than for a subtraction-based difference score (e.g., compare Tables 4 and 5).

Discussion

The present analyses demonstrated that ERP difference scores, under some circumstances, can yield reliable estimates and should not be categorically dismissed based on the assumption that they are inherently unreliable. Generalizability theory estimates of subtraction-based ΔERN scores yielded adequate reliability (.80). The remaining subtraction-based and residualized difference scores yielded internal consistency estimates for ΔERN that were numerically lower (.57 to .62) than the generalizability-theory estimates. The ΔERN score reliability was much lower than it was for CRN (.96 to .98) and similar to or lower than ERN (.61 to .80). For the reliability of ΔRewP scores, internal consistency estimates were low (-.07 to .49) and were far lower than the internal consistency of FN and RewP (.75 to .90). Taken together, difference score reliability is impacted by the internal consistency of its constituent

scores and the correlation between them. As long as ERP difference score reliability is reported and verified in any given study, these difference scores can be useful in research.

For subtraction-based difference scores, generalizability theory estimates were numerically higher than classical test theory estimates. Multivariate generalizability theory estimates have considerable advantages over classical test theory estimates for difference score reliability that should lead to more advantageous estimates of reliability—though not necessarily higher ones. More specifically, the generalizability theory approach explicitly separates between-trial variances from error variances (see Eq. 4). This is an important consideration when a single ERP condition comprises different types of stimuli, such as different unpleasant images that vary on arousal ratings. When a condition comprises different stimuli, between-trial variance will likely be higher than when a condition comprises identical stimuli. (Co)variance component estimation within the generalizability theory framework also partially pools data across participants to offset statistically abnormal results for participants with few trials (Brennan, 2001; Yin & Brennan, 2002). In the present dataset, this was apparent in the estimation of ERN and Δ ERN score reliability, because the variability in the number of trials retained for averaging varied widely with as few as 2 trials (see also Appendix and Clayson, Brush, et al., 2020).

Another significant advantage of generalizability theory for the estimation of reliability is its flexibility. Based on the formulas provided here (Eq. 3 and 4), it is straightforward to extend the formulas to simultaneously estimate the internal consistency and test-retest reliability of difference scores (see Carbine, Clayson, Baldwin, LeCheminant, & Larson, 2020; Clayson, Carbine, Baldwin, et al., 2020), to estimate subject-level difference score reliability (see Clayson, Brush, et al., 2020), or to estimate the impact of any other factor(s) or combination of factors on difference score reliability. For the reasons highlighted above and elsewhere (Baldwin et al.,

2015; Clayson, Carbine, Baldwin, et al., 2020; Clayson & Miller, 2017a, 2017b), generalizability theory estimates of reliability are particularly well suited for ERP research.

For classical test theory estimates, subtraction-based difference scores and residualized difference scores yielded similar difference score reliability for Δ ERN and for Δ RewP. Residualized scores tend to demonstrate higher reliability when the ratio of standard deviations between constituent scores (i.e., λ) is close to 1 (compare $\lambda = 1$ analyses in Table 4 and reliabilities in Table 5). However, within the context of classical test theory, subtraction-based difference score reliability tends to be higher when the ratio of standard deviations between constituent scores (λ) is not around 1. Given these aspects of the internal consistency of difference scores, subtraction-based difference scores might be preferable over residualized difference scores in most cases on the grounds of parsimony and interpretability (see also Rachor & Cizek, 1996; Williams & Zimmerman, 1982). Subtraction-based scores might also be preferred in studies of psychopathology when groups differ on ERP scores from the subtrahend, because residualized difference scores show higher type 1 error rates, lower statistical power, and larger measurement bias than subtraction-based difference scores in such instances (e.g., Jennings & Cribbie, 2016). Generally speaking, a subtraction-based or residualized difference score is suitable when constituent scores show adequate reliability, when the appropriate difference score is used, and when a difference score is properly interpreted.

Although the internal consistency of each ERP alone was adequate, difference score reliability for Δ RewP was far lower than Δ ERN likely due to the association between FN and RewP. The correlation between FN and RewP (.79, see Table 2) was numerically higher than the

correlation between CRN and ERN (.28). Hence, when it is possible to do so⁵, efforts to reduce the correlation between the ERPs that comprise a difference score will lead to higher score reliability. With regard to the comparison between subtraction-based difference scores and residualized difference scores, there are mathematical formulas for deriving when the reliability of one difference score will be higher than another (see Williams & Zimmerman, 1982). However, with modern computing capabilities it is likely more straightforward to simply estimate the reliability of both approaches for the data in hand than it is to estimate and compare the components that describe the relationship between the two.

Adequate internal consistency is essential in studies of individual differences, such as those describing associations between ERP scores and external correlates. For example, ERP studies might be interested in the use of auditory ERPs to predict response to treatment interventions (e.g., Clayson, Joshi, et al., 2020; Hochberger et al., 2019) or in determining relationships between ERPs and clinical symptoms, such as depression or anxiety (Hajcak et al., 2019; Moran et al., 2017; Moser et al., 2019). The viability of ERPs for use in predicting treatment differences between persons is limited by internal consistency. The internal consistency of scores must be verified prior to examining relationships between difference measures, because score reliability characterizes whether between-person variance is large relative to within-person/error variance (Hedge, Powell, & Sumner, 2017). That is, whether a between-person signal can be pulled out of the background between-trial noise. If adequate score

⁵ Consider an example of the late positive potential (LPP), which is more positive to arousing images than to neutral images (Cuthbert, Schupp, Bradley, Birbaumer, & Lang, 2000; De Cesarei & Codispoti, 2011; Weinberg & Hajcak, 2010). LPP amplitudes to high-arousing images and medium-arousing images will likely show a stronger correlation than amplitudes to high-arousing and neutral images. Hence, if an LPP difference score is of interest, attempting to use stimuli that have large differences in terms of arousal ratings would potentially yield difference scores with higher internal consistency estimates than images with similar arousal ratings.

reliability is not obtained, then any observed significant relationship between two variables could be spurious (Baldwin, 2017; Clayson, Brush, et al., 2020; Gelman & Carlin, 2014; Loken & Gelman, 2017; Schönbrodt & Perugini, 2013). For ERP difference scores to be useful as a dependent or independent variable, adequate score reliability needs to first be demonstrated, and we have provided several approaches for doing so.

Score reliability in studies of between-condition or between-group differences is not typically of primary concern, as it is in studies of between-person differences (Hedge et al., 2017). As an example, consider a hypothetical ERP obtained from two groups of participants: a healthy control group and a psychopathology group. This hypothetical ERP is universally intact in healthy controls, shows no variation between controls, and is measured with excellent precision (i.e., low between-trial variance). Despite yielding precise estimates, the ERP would show very poor internal consistency due to near zero between-person variance. Now, the same hypothetical ERP is very sensitive to presence of psychopathology (a large difference in amplitude) and is also measured with excellent precision. Such an ERP would similarly demonstrate poor internal consistency. Although this ERP would show robust group differences and would potentially be an excellent measure for doing so, it would not be well suited to studying individual differences because there is virtually no between-person difference variance to account for. To support the comparison of group differences in this scenario, estimates of ERP precision or data quality relative to the magnitude of group differences would be required (see Luck et al., 2020). In fact, repeated measures analyses of variance (ANOVA) or paired samples *t* tests can achieve very high statistical power for detecting condition differences despite zero classical score reliability (Overall & Woodward, 1975; Thomas & Zumbo, 2011). ERP precision

or data quality also contributes to score reliability, and the data quality metrics and score reliability can provide complementary information (Clayson, Brush, et al., 2020).

The focus of the present study was on the elaboration of approaches for estimating the internal consistency of ERP difference scores. Whether ERP difference scores are useful in research is also a theoretical question. The utility of difference scores for isolating neural activity depends on whether specific ERP features vary across the two conditions (see Kappenman & Luck, 2012; Luck, 2014, for detailed discussions). For example, between-condition differences in ERP component peak onsets and latencies or in the morphology of overlapping ERP components can contaminate difference scores. Despite such contamination, it is theoretically possible that high internal consistency of difference scores could be observed. However, the construct validity would be severely compromised, because the difference scores would be primarily due to non-neural differences. Hence, any statistical associations based on those difference scores would be flawed. Therefore, careful consideration of the validity and internal consistency of ERP difference scores is warranted.

In summary, subtraction-based and residualized ERP difference scores can demonstrate adequate internal consistency, which supports their use in research on individual differences. The internal consistency of difference scores must be demonstrated on a study-by-study basis to support their use as a dependent or independent variable. Generalizability theory approaches are better suited for estimating the reliability of subtraction-based ERP difference scores than classical test theory approaches.

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Table 1

Internal Consistency Estimates for Event-Related Potentials

Estimate	CRN		ERN		ΔERN	
	<u>Point</u>	<u>95% CI</u>	<u>Point</u>	<u>95% CI</u>	<u>Point</u>	<u>95% CI</u>
$r_{2\alpha}$	.96	.94 - .97	.61	.52 - .69		
ϕ	.98	.98 - .99	.80	.74 - .85		
<i>Subtraction-Based Scores</i>						
$\rho_{DD'}$					.61	--
$E\rho_D^2$					.80	.74 - .85
ϕ_D					.80	.74 - .85
$r_{2\alpha}$					.62	.53 - .70
<i>Residualized Scores</i>						
$\rho_{RR'}$					.57	--
$r_{2\alpha}$					.58	.49 - .67

Estimate	FN		RewP		ΔRewP	
	<u>Point</u>	<u>95% CI</u>	<u>Point</u>	<u>95% CI</u>	<u>Point</u>	<u>95% CI</u>
$r_{2\alpha}$	.75	.69 - .81	.78	.72 - .84		
ϕ	.87	.83 - .90	.90	.87 - .92		
<i>Subtraction-Based Scores</i>						
$\rho_{DD'}$					-.07	--
$E\rho_D^2$					.49	.35 - .62
ϕ_D					.49	.35 - .62
$r_{2\alpha}$					.25	.11 - .40
<i>Residualized Scores</i>						
$\rho_{RR'}$					.01	--
$r_{2\alpha}$					.35	.23 - .47

Note: For two-part coefficient alpha (also referred to as the Flanagan-Rulon formula; $r_{2\alpha}$) the

95% confidence intervals (CIs) represent the confidence interval of the point estimates across the

10,000 re-sampling iterations. For generalizability ($E\rho_D^2$) and dependability coefficients (ϕ, ϕ_D) the 95% credible intervals (CIs) are shown, which are the Bayesian analog to confidence intervals. CRN = correct-related negativity; ERN = error-related negativity; Δ ERN = ERN/CRN difference score; FN = feedback-related negativity; RewP = reward-related positivity; Δ RewP = RewP/FN difference score

Table 2

Summary Variance Components for Generalizability Theory Equations

		σ^2			σ^2
Between-Person (<i>p</i>)	Correct	2.40	Loss		7.56
	Error	3.89	Gain		10.76
	Covariance	1.09	Covariance		7.99
Between-Item (<i>i</i>)	Correct	0.03	Loss		0.09
	Error	0.22	Gain		0.05
	Covariance	-.01	Covariance		0.004
Error (<i>e</i>)	Correct	21.79	Loss		20.82
	Error	23.65	Gain		23.70
	Covariance	0	Covariance		0

Note: Covariance was set to be zero, because observations from the two conditions were never jointly observed.

Table 3

Summary of Estimates Used in Classical Theory Equations

	CRN	ERN	Δ ERN	FN	RewP	Δ RewP
σ	1.54	2.36		3.00	3.43	
λ			0.65			0.87
λ^{-1}			1.53			1.14
ρ_{XY}			.28			.79
ρ_{XY}^2			.08			.62

Note: σ = standard deviation; λ = ratio of the two standard deviations; ρ_{XY} = zero-order correlation between the two ERPs; CRN = correct-related negativity; ERN = error-related negativity; Δ ERN = ERN/CRN difference score; FN = feedback-related negativity; RewP = reward-related positivity; Δ RewP = RewP/FN difference score

Table 4

Subtraction-Based Score Reliability as a Function of Internal Consistency, Correlations, and the Ratio of Standard Deviations

Internal Consistency X and Y						
$\lambda = .5$	ρ_{XY}	.50	.60	.70	.80	.90
	0	.50	.60	.70	.80	.90
	.2	.40	.52	.64	.76	.88
	.4	.26	.41	.56	.71	.85
	.6	.04	.23	.42	.62	.81
	.8	-	-	.17	.44	.72
$\lambda = 1$	ρ_{XY}	.50	.60	.70	.80	.90
	0	.50	.60	.70	.80	.90
	.2	.38	.50	.63	.75	.88
	.4	.17	.33	.50	.67	.83
	.6	-	.00	.25	.50	.75
	.8	-	-	-	.00	.50
$\lambda = 1.5$	ρ_{XY}	.50	.60	.70	.80	.90
	0	.50	.60	.70	.80	.90
	.2	.39	.51	.63	.75	.88
	.4	.21	.37	.52	.68	.84
	.6	-	.10	.33	.55	.78
	.8	-	-	-	.24	.62

Note: λ represents the ratio of the standard deviation of X (σ_X) to the standard deviation of Y (σ_Y). ρ_{XY} characterizes the correlation between X and Y. Correlations below 0 are suppressed with dash (-).

Table 5

Residualized Difference Score Reliability as a Function of Internal Consistency and Correlations

ρ_{XY}	Internal Consistency X and Y				
	.50	.60	.70	.80	.90
0	.50	.60	.70	.80	.90
.2	.46	.57	.68	.78	.89
.4	.31	.45	.59	.72	.86
.6	-	.15	.36	.58	.79
.8	-	-	-	.09	.54

Note: ρ_{XY} characterizes the correlation between X and Y. Correlations below 0 are suppressed

with dash (-). The ratio (λ) of the standard deviation of X (σ_X) and standard deviation of Y (σ_Y)

does not directly impact the reliability of residual difference scores.

Appendix

An estimate of split-half reliability that is suitable for ERP scores is the two-part coefficient alpha (also referred to as the Flanagan-Rulon estimate; Cronbach, 1951). Two-part coefficient alpha is the preferred approach for estimating the internal consistency of scores when test halves are not strictly parallel (i.e., equal means, standard deviations, error variances, equal true scores for each half). Two-part coefficient alpha assumes tau-equivalence, which means that the test items are homogeneous and test halves have homogenous true score variance, and the assumption of tau-equivalence is less restrictive than parallel forms (Cho, 2016). The two-part coefficient alpha is also equivalent to the generic coefficient alpha (i.e., Cronbach's alpha) when a test only has two parts (Cronbach, 1951; Warrens, 2015).

Two-part coefficient is also preferred over Spearman-Brown adjusted split-half reliability, because the Spearman-Brown adjustment assumes parallel tests and overestimates the internal consistency of the full-length test when the variance from each half is unequal (Charter, 2001; Cho, 2016; Cronbach, 1951; Feldt & Charter, 2003; Warrens, 2015). For split-half estimates of ERP scores, there is likely to be unequal variance between any arbitrary halves when there are few trials retained for averaging (e.g., ERN). When the two test halves yield nearly identical standard deviations, the two-part coefficient alpha produces reliability estimates that are nearly identical to Spearman-Brown adjustment correlation coefficients, which has led to recommendations that two-part coefficient alpha should be routinely used as standard practice (Cronbach, 1951; Feldt & Charter, 2003; Osburn, 2000).

The two-part coefficient alpha is estimated using the following formula that includes an adjustment for estimating full-length test reliability from two halves (Cronbach, 1951; Rulon, 1939).

$$r_{2\alpha} = \frac{4\rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2}$$

Two-part coefficient alpha ($r_{2\alpha}$) is calculated using the correlation between test halves 1 and 2 (ρ_{12}), the standard deviations of scores from test halves 1 (σ_1) and 2 (σ_2), and the variances of score from each half (σ_1^2, σ_2^2). When an odd number of trials is recorded from a participant, the split-half procedure yields two “halves” with unequal lengths. One of the halves will have an additional trial over the other. Two-part coefficient alpha can be estimated in freely available Microsoft Excel-based package REL_{EX} (Steinke & Kopp, 2020) or in the *R* package *splithalf* (Parsons, 2020).

Although split-half reliability estimates are appropriate for data with unequal observations, they suffer from variability endemic to splitting a participant’s data in half, particularly when few trials are retained for averaging (Baldwin et al., 2015; Clayson, 2020; Clayson & Miller, 2017a, 2017b). An approach that appears more robust against splitting the data into any one particular half is to randomly re-split the data into numerous random halves (Clayson, Brush, et al., 2020). The split-half reliability is then estimated based on the central tendency of split-half estimates from the resampled iterations. This random resampling procedure can also be applied to estimating the reliability of both types of difference scores by randomly dividing both conditions into halves, calculating difference scores on separate halves, then computing correlations between two the difference scores.

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