MODELING ATTENTION AND MAGNITUDE

Attention as a source of variability in decision-making: Accounting for overall-value effects with
diffusion models

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Running Head: MODELING ATTENTION AND MAGNITUDE

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Abstract

Research has demonstrated that value-based decisions depend not only on the relative value difference between options, but also on their overall value. In particular, response times tend to decrease as the overall value of the options increase. Standard sequential sampling models such as the diffusion model can account for this fact by assuming that decision thresholds or noise vary with overall value. Alternatively, attention-based models that incorporate eye-tracking data accommodate this overall-value effect directly as a consequence of the multiplicative relationship between attention and value magnitude. Using non-attentional diffusion models fit to data simulated with an attention-based model, we find that parameters related to decision thresholds or noise vary as a function of overall value, even though these were not features of the data generating process. We find additional evidence for misidentified parameters in a similar analysis using empirical data. Our results indicate that neglecting attentional effects can lead to mistaken conclusions about which decision parameters are sensitive to overall value.

Keywords: Attention, diffusion model, overall-value effect, response time models

In their daily lives, people are confronted by decisions with values that can vary substantially from one context to the next. In the very same hour, one may go from selecting a car to purchase to choosing a pen with which to sign the offer. For preference-based decisions, one obvious dimension along which they can vary is in the overall sum of the available options' subjective values. Decisions can range from very low value (e.g., which pen to use) to very high value (e.g., which car to purchase). An unresolved issue is how overall value (OV) affects the process of making decisions.

A common perspective is that OV should not affect the decision process. Instead, the importance of a decision should depend on the value difference between the available options. This is because the relative value difference is what the decision-maker stands to gain by selecting the best option, relative to the other options. The value difference between the options also determines the ease of making decisions because larger value differences make it easier to discriminate between the options and identify the best one. The value-based (i.e., preference-based) decision-making literature is replete with examples demonstrating decision-makers' proficiency at quickly discriminating between unlike options (Busemeyer & Townsend, 1993; Clithero, 2018; Dai et al., 2018; De Martino et al., 2013; Hunt et al., 2012; Konovalov & Krajbich, 2019; Krajbich et al., 2010; Mormann et al., 2010; Pirrone, Azab, et al., 2018; Pisauro et al., 2017; Polanía et al., 2014; Shenhav et al., 2014; Sullivan et al., 2015). The fact that response times (RT) are shorter for decisions involving options that are on opposite ends of the value scale seems almost lawlike in nature.

Over the years, researchers have noted that, in addition to the value-difference effect, OV also impacts the decision process. In particular, OV tends to correlate negatively with RT. In other words, high-value decisions tend to be faster than low-value decisions (Hunt et al., 2012;

Pirrone, Azab, et al., 2018; Polanía et al., 2014; S. Smith & Krajbich, 2018). In a particularly evocative examination of this effect, Pirrone et al. (2018) studied rhesus monkeys choosing between colored squares, each representing a different quantity of juice. The researchers included occasional trials with two identical options. They found that RTs were shorter for choices between two identically large quantities of juice than for two identically small quantities of juice.

From a normative perspective, making high-value decisions quickly can be beneficial if the decision-maker is satisficing rather than maximizing (e.g., Marshall et al., 2009; Pais et al., 2013). For example, a scavenging animal needs only a certain number of calories to survive the day. If the immediately available options are all rich enough, the animal's optimal strategy is to choose quickly and avoid competition or predation. For human decision-makers, if the goal is to accumulate resources up to a target amount, then the optimal strategy is similarly to choose quickly when all options are highly rewarding (Pirrone, Azab, et al., 2018; Tajima et al., 2016; Teodorescu et al., 2016).

Interestingly, these OV effects are not unique to value-based decisions. In perceptual decision-making, researchers have also noted that decisions involving high-magnitude stimuli tend to be fast (Bose et al., 2020; Kvam & Pleskac, 2016; Polanía et al., 2014; Ratcliff & McKoon, 2018; Ratcliff et al., 2018; Teodorescu et al., 2016). In the same article by Pirrone et al. (2018) discussed above, the authors also studied humans completing a brightness discrimination task in which participants were required to indicate which of two grayscale patches was brighter. Embedded in this task were trials where both alternatives were of equal luminance. The researchers found that participants responded faster when choosing between high-equal-luminance options than low-equal-luminance options. But unlike in value-based

domains, decision-makers do not derive additional benefits from quickly deciding between high-luminance options than from low-luminance options. Therefore, these magnitude effects in perceptual decision-making suggest that the satisficing account cannot be the complete explanation for the OV effect in value-based decision-making.

To try to understand the OV effect, we turn to sequential sampling models (SSM), which are canonical models for the decision process. In the SSM framework, the decision-maker evaluates the options, generating and accumulating noisy evidence for each, until one option accumulates enough evidence to be chosen.

The most well-known SSM, the diffusion model (DDM; e.g., Ratcliff & McKoon, 2008), does not account for the OV effect under its usual assumption that parameters are held constant across OV conditions. In the DDM, there is a single decision variable that represents the relative evidence in favor of one option over the other. This decision variable evolves with a *drift rate* that only depends on the relative value difference, while ignoring OV. However, Ratcliff et al. (2018) found that allowing some of the DDM's variability parameters to increase linearly with stimulus magnitude enabled it to better account for magnitude effects. Additionally, there are several other SSMs that share the assumption that within-trial variability increases with stimulus magnitude (Bose et al., 2020; Brunton et al., 2013; Kvam & Pleskac, 2016; Teodorescu et al., 2016).

Another important parameter in SSMs is boundary separation. Boundary separation indexes the decision criterion, the quantity of evidence a decision-maker requires before selecting an alternative. Typically, boundary separation is thought to be fixed before the onset of a trial (Ratcliff & McKoon, 2008; although, this assumption has been challenged by empirical work on cognitive control, see Botvinick et al., 2001; Vassena et al., 2020). Nonetheless, some

researchers have allowed boundary separation to decrease with higher OV, which reduces RT with only small decrements in accuracy (J. F. Cavanagh et al., 2014; Pirrone, Wen, et al., 2018). However, this requires a mechanism for using incoming stimulus information not only to drive the accumulation of evidence, but also to decide when to stop.

Still other SSMs, such as the leaky competing accumulator (LCA; Usher & McClelland, 2001) model and Wang model (Lo & Wang, 2006), assume separate evidence accumulators for each option. In these models, stronger inputs (i.e. higher values) lead to faster evidence accumulation and thus shorter RTs (Hunt et al., 2012; Lo & Wang, 2006; Usher & McClelland, 2001).

Finally, research incorporating eye-tracking suggests that the interaction between attention and value can explain the OV effect. The attentional DDM (aDDM; Krajbich et al., 2010) inherently predicts that RT is tied to OV. In the aDDM, drift rates are a function of the difference in the options' subjective values, which are scaled by attention-based weights. Attention has a magnifying effect on value, providing a greater boost to options with higher values. Smith and Krajbich (2019) tested this hypothesized interaction across six datasets, finding consistent support for a multiplicative relationship between attention (as measured by gaze duration) and subjective value. The advantage of this particular account is that it explains two patterns in the data (RT and gaze-weighted effects on choice) rather than just one (RT).

Since the standard DDM does not incorporate eye-tracking, it has to account for the OV effect with other parameters. For example, Ratcliff et al. (2018)'s conclusions are in line with the aDDM explanation, namely that noise in the decision process increases with value. The difference is that the noise in the aDDM is specifically due to shifts in attention, and is tied to

observable eye-tracking data. The noise in the standard DDM could be due to any number of stimulus features and cognitive processes.

Here, we investigate how standard DDM estimation may be distorted when the true data generating process is the aDDM. In other words, assuming that the OV effect is truly due to attention, how do diffusion models without attention parameters account for this missing mechanism? First, we simulate an aDDM with constant noise and boundary-separation parameters, and show how a standard DDM fit to those data yields varying noise or boundary separation parameters. Second, we analyze two empirical eye-tracking datasets that have aDDM-like properties and show how a standard DDM fit to those data can yield varying noise or boundary separation parameters. In sum, we show how not accounting for visual attention can lead to incorrect conclusions about the extent to which changing boundaries or noise levels can explain the OV effect. This highlights the importance of accounting for attention when analyzing decision-making data.

The DDM

In the DDM (Ratcliff, 1978; Ratcliff & McKoon, 2008), decisions are made when option information accumulated from a starting point z , reaches one of two boundaries, 0 or a . The drift rate of the accumulation process, v , indexes the rate at which evidence is acquired. In value-based two-alternative forced-choice tasks (2AFC), this parameter represents the relative difference in strength of preference between the two options. Processes outside of the evidence accumulation are represented by the non-decision time parameter T_{er} . The starting point z is assumed to vary from trial-to-trial, and is drawn from a uniform distribution with range sz . Non-decision time is also assumed to have a uniformly distributed trial-to-trial variability with range

st. Furthermore, the trial-to-trial standard deviation in drift rate is expressed by the parameter η . The model also assumes there is within-trial variability in the drift rate from one moment to another. This parameter, s , is typically fixed to 0.1 to serve as a scaling parameter, since the units of evidence in the model are arbitrary.

Usually, the DDM is parameterized such that the boundary separation, non-decision time, and starting point are determined before the beginning of the decision. In other words, these parameters do not depend on stimulus properties. This means that under usual assumptions, these parameters should not vary with OV. In some cases, relaxing this assumption has led to better model fits. Therefore, in the following analyses we bin the data into different conditions based on OV and estimate parameters separately across these conditions.

The aDDM

The aDDM (Krajbich et al., 2010) similarly views 2AFC as involving a noisy evidence accumulation process with two boundaries. The primary difference between this model and the standard DDM is that, in the aDDM, the drift rate changes depending on gaze location. In the standard DDM one can model the drift rate as a function of the two option values:

$$v = d \cdot (r_{left} - \theta r_{right})$$

where r_{left} and r_{right} represent the stimulus values of the options on the left- and right-hand side of the screen, and d is a scaling parameter to account for the units of r . In the aDDM, this function has an additional parameter θ that discounts the value of the non-fixated option.¹ More specifically, there are two drift rates, depending on gaze location:

¹ Unlike the standard DDM, the aDDM typically holds the boundary separation parameter constant, and instead allows within-trial variability in drift rate to vary.

$$\text{gaze left: } v = d \cdot (r_{\text{left}} - \theta r_{\text{right}})$$

$$\text{gaze right: } v = d \cdot (\theta r_{\text{left}} - r_{\text{right}})$$

Since θ multiplies the non-fixated item's value, the gaze-dependent shifts in drift rate depend on OV. To illustrate this effect, assume r_{Left} and r_{right} are both set to 1, $d = 0.5$, and $\theta = 0.3$:

$$v = d \cdot (r_{\text{left}} - \theta r_{\text{right}}) = 0.5(1 - (0.3 * 1)) = 0.35$$

$$v = d \cdot (\theta r_{\text{left}} - r_{\text{right}}) = 0.5((0.3 * 1) - 1) = -0.35$$

However, if r_{Left} and r_{right} are both set to 10 (using the same values for d and θ):

$$v = d \cdot (r_{\text{left}} - \theta r_{\text{right}}) = 0.5(10 - (0.3 * 10)) = 3.5$$

$$v = d \cdot (\theta r_{\text{left}} - r_{\text{right}}) = 0.5((0.3 * 10) - 10) = -3.5$$

On average, the decision process will terminate more quickly in the latter case, due to the increased variability in the drift rate (Fig. 1).

Figure 1. Illustration of the multiplicative effect of attention on the decision process. In this model, attention successively shifts between the left (L) and right (R) options. The magnitude of this shift is greater for higher OVs. Reprinted with permission from Smith & Krajbich, 2019.

Simulation Study

We first generated simulated data using the aDDM and then evaluated how fitted parameters of the standard DDM varied across OV conditions to accommodate the patterns in the simulated data. We simulated the data using the values of the aDDM's parameters that best fit the data from Krajbich et al. (2010). The parameters used in the simulation were $d = 0.0002$, $\theta = 0.3$, $s = 0.02$ per ms, and $T_{er} = 355 \text{ ms}^2$. Gaze durations were drawn from a uniform distribution, $U(\text{gaze}_{median} - 0.5 * \text{gaze}_{median}, \text{gaze}_{median} + 0.5 * \text{gaze}_{median})$. Two separate datasets were generated using median gaze durations of 250 ms and 500 ms, respectively.

The simulations consisted of nine conditions: three levels of value difference {1,2,3}, and for each of those, three levels of OV {low, medium, high}. The specific values for each condition can be found in Table 1. For each condition, we simulated 10,000 trials. For each trial, option values were randomly assigned to the left-right locations, and the first fixation location was determined by trial number: even-numbered trials started with a left fixation, and odd-numbered trials started with a right fixation.

Table 1. Values used to simulate data

OV Condition	Value-Difference (VD) Condition	Values
Low value	Easy (VD = 3)	{0, 3}
Medium value	Easy (VD = 3)	{3.5, 6.5}
High value	Easy (VD = 3)	{7, 10}
Low value	Moderate (VD = 2)	{0, 2}
Medium value	Moderate (VD = 2)	{4, 6}
High value	Moderate (VD = 2)	{8, 10}
Low value	Hard (VD = 1)	{0, 1}
Medium value	Hard (VD = 1)	{4.5, 6.5}
High value	Hard (VD = 1)	{9, 10}

² We generated the simulated data using the random walk approximation (Tuerlinckx et al., 2001).

We fit the simulated data using various non-attentional DDMs. The first 8 models had the starting-point parameter set to $z = a/2$ and the within-trial variability in drift rate parameter set to $s = 0.1$. The other variability parameters st and sz were assumed to be constant across all conditions. The drift rate v was assumed to be constant across OV conditions; it was only allowed to vary with value-difference. The other parameters (i.e., a , T_{er} , and η) were either held constant or allowed to vary across conditions (Table 2).

Table 2. Description of the models fit to simulated data

Model	Free parameters	Constant across OV	Constant across all conditions	Fixed parameters
Model 1	a, T_{er}, η	v	sz, st	z, s
Model 2	T_{er}, η	v	a, sz, st	z, s
Model 3	a, η	v	T_{er}, sz, st	z, s
Model 4	a, T_{er}	v	η, sz, st	z, s
Model 5	η	v	a, T_{er}, sz, st	z, s
Model 6	T_{er}	v	a, η, sz, st	z, s
Model 7	a	v	T_{er}, η, sz, st	z, s
Model 8	—	v	a, T_{er}, η, sz, st	z, s
Model 9	η	v	s, T_{er}, sz, st	z, a
Model 10	s	v	η, T_{er}, sz, st	z, a

Note: Free parameters were estimated independently per condition. Fixed parameters were not estimated from the data. “ a ” stands for boundary separation, “ T_{er} ” for non-decision time, “ η ” for across-trial noise in drift rate, “ sz ” for across-trial noise in starting point, and “ st ” for across-trial noise in non-decision time, “ v ” for drift rate, “ z ” for starting point, and “ s ” for within-trial noise in drift rate.

We fit two additional models, which were variations of the standard DDM. While the DDM typically keeps within-trial variability in drift rate constant at $s = 0.1$, we allowed it to vary while keeping boundary separation constant. These models held the boundary separation parameter constant at $a = 0.3$ and starting point at $z = 0.15$. Model 9 constrained s to be constant across conditions, while allowing η to vary. Model 10 constrained η to be constant across

conditions, while allowing s to vary. Models 9 and 10 closely resembles the η Model and σ Model presented by Ratcliff et al. (2018), respectively, where across-trial variability in drift and within-trial variability in drift were each estimated as a function of overall magnitude (see Supplementary Material for tests of these models)

Figure 2. Quantile-probability (Q-P) plots for exemplar DDM models fit to simulated data (500 ms fixations). In these figures, the numbers represent different difficulty and OV conditions in the simulated data: 1 (black) = .1 RT quantile, 2 (red) = .3 RT quantile, 3 (green) = .5 RT quantile, 4 (dark blue) = .7 RT quantile, and 5 (light blue) = .9 RT quantile. Within a given difficulty condition, the correct and error responses in the simulated data display a U-shaped pattern, where accuracy and RTs decrease with OV. Circle and lines represent model predictions. (a) Fits for Model 7, where boundary separation (a) alone varies by OV condition. (b) Fits for Model 5, where across-trial variability in drift rates (η) alone varies by OV condition. (c) Fits for Model 6, where non-decision time (T_{er}) alone varies by OV condition. (d) Fits for Model 10, where within-trial variability in drift rates (s) alone varies by OV condition.

We fit the models to choice outcomes (correct vs. error) and RT quantiles (0.1, 0.3, 0.5, 0.7, 0.9) within each condition. We used generic initial parameter values and then used a simplex function (Nelder & Mead, 1965) to minimize the difference between each model's predictions and the simulated data, as quantified by a chi-square (χ^2) statistic (Ratcliff & Tuerlinckx, 2002).

Results

In both the 250 ms and 500 ms median gaze duration data, the best fitting models were those in which a and T_{er} varied with OV (Table 3). For the standard DDM (with s fixed), only models which allowed a to vary (i.e., Models 1, 3, 4, and 7) were able to replicate the U-shaped pattern found in the simulated RTs. This qualitative pattern can be fit by varying just the a parameter (i.e., Model 7: Fig. 2A). In these models, the fitted a values were smaller for higher OV. Within Model 7, the average values of a were 0.324 ($SD = 0.001$) for low-value conditions, 0.309 ($SD = 0.005$) for medium-value conditions, and 0.275 ($SD = 0.024$) for high-value conditions (Supplementary Tables S13-14). In other words, these models tell us that boundary separation is decreasing with OV, even though we know that the generating model held boundaries constant.

Models that allowed η to vary and not a (i.e., Models 2, 5, and 9) over-estimated the prevalence of slow errors in the data (Fig. 2B). In these models, the fitted η values were larger for higher OVs. Within Model 5, the average values of η were 0.007 ($SD = 0.004$) for low-value conditions, 0.018 ($SD = 0.005$) for medium-value conditions, and 0.073 ($SD = 0.040$) for high-value conditions (Supplementary Table S9-10). These models tell us that across-trial variability in drift rate is increasing with OV. While this interpretation is not entirely inaccurate, the source of the variability is not explicitly attributed to attention.

Models that allowed T_{er} to vary and not a (i.e., Models 2 and 6) could not account for the differences in accuracy across OV conditions (see Fig. 2C). In these models, the fitted T_{er} values were smaller for higher OVs. Within Model 6, the average values of T_{er} were 0.500 ($SD = 0.073$) for low-value conditions, 0.413 ($SD = 0.052$) for medium-value conditions, and 0.278 ($SD = 0.030$) for high-value conditions (Supplementary Table S11-12). These models tell us that non-decision time is decreasing with OV, which we know is not the case.

The only other model that captured the patterns found in the simulated data was Model 10, which held a constant, while allowing s to vary (Fig. 2D). Within this model, the average values of s were 0.092 ($SD = 0.001$) for low-value conditions, 0.096 ($SD = 0.001$) for medium-value conditions, and 0.110 ($SD = 0.010$) for high-value conditions (Supplementary Table S19-20). The interpretation of this non-standard DDM is that OV increases the within-trial noise in the decision process. Again, this is not entirely inaccurate, but the source of the variability is not attributed to attention.

The results of this simulation study reveal how fits of the DDM can yield incorrect explanations for a behavioral phenomenon when relaxing common assumptions about boundary separation and non-decision time. If the DDM were fit holding variability in drift rate constant across conditions, a likely conclusion of such a study would be that OV affects boundary separation. If the researchers instead held boundary separation constant as a function of stimulus properties, as is often assumed, then the conclusion would be that OV affects non-decision time. If the generating process is the aDDM, then neither of these conclusions would reflect reality.

Table 3. Simulation study model comparison

Model	Free parameters	Dataset	# of parameters	χ^2
Model 1	a, T_{er}, η	250 ms fixations	32	87.729
		500 ms fixations	32	225.376
Model 2	t_{er}, η	250 ms fixations	24	100.672
		500 ms fixations	24	277.725
Model 3	a, η	250 ms fixations	24	94.377
		500 ms fixations	24	240.711
Model 4	a, T_{er}	250 ms fixations	24	89.119
		500 ms fixations	24	225.820
Model 5	η	250 ms fixations	16	139.546
		500 ms fixations	16	362.495
Model 6	T_{er}	250 ms fixations	16	114.947
		500 ms fixations	16	480.044
Model 7	a	250 ms fixations	16	96.634
		500 ms fixations	16	241.021
Model 8	—	250 ms fixations	8	169.635
		500 ms fixations	8	816.033
Model 9	η	250 ms fixations	16	139.281
		500 ms fixations	16	367.362
Model 10	s	250 ms fixations	16	99.72
		500 ms fixations	16	233.104

Note: “ a ” stands for boundary separation, “ T_{er} ” for non-decision time, “ η ” for across-trial noise in drift rate, “ sz ” for across-trial noise in starting point, and “ st ” for across-trial noise in non-decision time, “ v ” for drift rate, “ z ” for starting point, and “ s ” for within-trial noise in drift rate.

Empirical Study

The results of our stimulation study indicate that data generated with a gaze-weighted drift rate can lead to a variety of patterns in the parameter estimates of a standard DDM. As a result, researchers may come to inaccurate conclusions about the mechanisms responsible for behavioral phenomena, such as the OV effect. Without accounting for attention, researchers might, for example, falsely conclude that decision boundaries are sensitive to value.

In one relevant example, Cavanagh et al. (2014) proposed value-dependent decision boundaries in conjunction with gaze-weighted evidence accumulation. While their model did incorporate gaze time into the calculation of drift rates, they assumed that gaze provides a constant boost to the drift rate independent of option value. As demonstrated by Smith & Krajbich (2019), the aDDM’s multiplicative attentional weights are able to account for the OV effect, while additive attentional weights are not. It is therefore possible that the authors found evidence for value-dependent boundaries because they focused on an additive attentional model.

The goal of this section is to compare DDM and aDDM fits to real data, to verify that these two models can generate different conclusions about parameters such as boundary separation, as implied by our simulation study. Using the data from Cavanagh et al. (2014), as well as data from Smith and Krajbich, 2018, we study the OV effect using both models.

To preview the results, we partially confirm the conclusions of our simulation study. We find that each version of the standard DDM yields differences in some parameters across value conditions. We also find that in models without an attention-based mechanism, value-dependent boundary separation improves model fit. Finally, including attention in the drift rate calculations reduces, but does not eliminate, the need for value-dependent boundaries.

Method

Dataset 1

In this reinforcement learning study (Cavanagh et al., 2014), participants ($N = 20$) initially learned three stimulus pairings with the following probabilities of being the correct choice: A:B (80%:20%), C:D (70%:30%), and E:F (60%:40%). Each stimulus was a unique Japanese Hiragana character. Participants went through this training phase until they reached a

minimum criterion of choosing the better stimulus in each pair. Participants then proceeded to the testing phase where they chose between all possible pairings for 240 trials.

We classified each trial according to the OV of the stimuli in the trial: low value trials containing only low probability stimuli (20%, 30%, or 40%), high value trials containing only high probability stimuli (60%, 70%, or 80%), or mixed value trials containing one low value stimulus and one high value stimulus. We excluded trials with excessively fast (slow) decisions, less (more) than 2 standard deviations below (above) participant-level means, using $\log(RT)$. Such trials are likely to have been terminated due to accidental button presses, intentional skipping, or distraction. Using these criteria, we excluded 3.7% of trials.

Dataset 2

Participants ($N = 44$) in this study (Smith & Krajbich, 2018) first rated 147 food items on a scale from -10 to 10 before making 200 choices between positively rated items (rating > 0). Although participants in this study completed three other choice tasks (200 trials each), we only focus on the food choice data³. Food items were selected such that no item was used in more than 7 trials and the maximum absolute rating difference was 5. Some participants ($n = 22$) did not have a sufficient number of positively rated food items to generate 200 valid trials, so these participants completed as many trials as could satisfy the constraints ($M = 171.3$ trials). Participants earned a \$5 show-up fee, additional money from the other tasks, and a food chosen from one random trial.

We classified each trial according to the OV of the food items in the trial. Low value trials were those where both food items were less than the median value (5.5), high value trials

³ The other tasks involved multiple attributes/stimuli per option, and thus require additional assumptions about how subjective value is calculated.

were those where both items were greater than the median value, and mixed value trials were those with one low and one high value item. We excluded participants ($n = 17$) with less than 20 trials in any of the three conditions. We also excluded one participant whose mean RT (3.39 sec) was 60% slower than the group mean excluding that participant (1.43 sec). Furthermore, we excluded trials with excessively fast (slow) decisions, less (more) than 2 standard deviations below (above) participant-level means, using $\log(\text{RT})$. Using these criteria, we excluded 4.3% of trials.

Analyses

Behavioral Analyses

We examined the effect of OV on accuracy and RT. We estimated two mixed-effects regression models with full random effects:

$$\log(\text{RT}) \sim \beta_0 + \beta_1 |r_{\text{left}} - r_{\text{right}}| + \beta_2 (r_{\text{left}} + r_{\text{right}}),$$

$$\text{logit}(\text{Accuracy}) \sim \beta_0 + \beta_1 |r_{\text{left}} - r_{\text{right}}| + \beta_2 (r_{\text{left}} + r_{\text{right}}).$$

In both models, β_2 is the primary variable of interest, as it relates to OV. At the same time, β_1 accounts for the value difference between the two options, since people tend to decide faster and more accurately with larger value differences.

DDM Analysis

We fit the data with three versions of the DDM. We examined the extent to which the boundary separation (a), across-trial variability in drift rate (η), and within-trial variability in drift rate (s) varied with stimulus magnitude in the two empirical datasets. For each model, we fixed one of the parameters (either a or s), constrained the second to be constant across

conditions, and estimated the third parameter separately for each condition. For the a Model and η Model, within-trial variability was fixed at $s = 0.1$. For the s Model, boundary separation was fixed at $a = 0.2$.

For this analysis, we excluded trials from the mixed-value condition. The specialized software we used to fit the s model cannot estimate drift rates using trial-level value difference. Instead, we would have needed to separately estimate drift rate parameters for each value-difference condition. Due to the limited number of trials per condition, we binned the data by OV instead. Therefore, we restricted our analyses to the low and high OV conditions because they have comparable levels of difficulty (i.e., value difference).

As in the simulation study, we fit all three models to each participant's choice outcomes and RT quantiles (0.1, 0.3, 0.5, 0.7, 0.9) for correct and error responses in each condition. We used generic initial parameter values and then used a simplex function (Nelder & Mead, 1965) to minimize the difference between the predictions and the empirical data, as quantified by a chi-square (χ^2) statistic (Ratcliff & Tuerlinckx, 2002).

aDDM Analysis

To show how an attention-based modeling approach can account for the OV effect, we also fit the data with the aDDM. We estimated the model using the HDDM Python package (Wiecki et al., 2013). In this model, the drift rate parameter was calculated as a regression of the relative value-weighted gaze-time for each option (Cavanagh et al. 2014; Smith, Krajbich, & Webb, 2019):

$$v \sim v_{Intercept} + v_1(g_{left}r_{left} - g_{right}r_{right}) + v_2(g_{right}r_{left} - g_{left}r_{right}),$$

where g_{left} and g_{right} refer to the proportion of gaze time on the left and right options, respectively.

We compared two versions of this model: one where boundary separation (a) was constant across OV conditions (aDDM1), and one where boundary separation varied based on condition (aDDM2). Here, we focus on boundary separation since this is the parameter most often discussed in other OV studies (J. F. Cavanagh et al., 2014; Pirrone, Wen, et al., 2018).

Furthermore, value-sensitive boundaries represent a truly different mechanism than attentional drift rate effects. Value-sensitive boundary separation implies that there is some mechanism that allows decision-makers to adjust their boundaries in response to trial-level information (Botvinick et al., 2001; J. F. Cavanagh et al., 2014; Vassena et al., 2020). In contrast, drift rate variability parameters make the less specific claim that stimulus strength increases noise in the evidence accumulation process (Ratcliff et al., 2018).

As a control, we also ran a model where the drift rate was calculated as a regression of the value difference between the stimuli:

$$v \sim v_{Intercept} + v_{Dif}(U_L - U_R).$$

We also ran two versions of this model: one where boundary separation was held constant across OV conditions (DDM1), and one where it varied based on condition (DDM2).

For both datasets, we included mixed OV trials. This allowed us to include more data when estimating the model, and gave us an additional condition to compare to. Instead of only comparing the low and high OV conditions directly, we could also compare each to the mixed value condition. Since we calculated drift rates as a function of trial-level value differences, this analysis controls for the lower difficulty (i.e., larger value differences) in the mixed trials. For all models, we used 5000 samples, discarding 1000 as burn-in, and fit the non-decision-time parameter at the subject level.

Behavioral Results

We replicated past analyses of these two datasets, confirming the OV effect on RT (and minimal effects on accuracy). We found a negative relationship between RT and OV in both datasets (Dataset 1: $\beta_2 = -0.029 \log(\text{ms})$ per rating, $SE = 0.005$, $p < .001$; $\beta_0 = 0.346$, $SE = 0.065$, $p < .001$; Dataset 2: $\beta_2 = -0.011 \log(\text{ms})$ per rating, $SE = 0.002$, $p < .001$; $\beta_0 = 0.459$, $SE = 0.056$, $p < .001$). Additionally, there was small negative relationship between choice accuracy (i.e., choosing the probabilistically better option) and OV in Dataset 1 ($\beta_2 = -0.012$, $SE = 0.012$, $p = .031$; $\beta_0 = 0.047$, $SE = 0.142$, $p = .737$), but a small positive relationship in Dataset 2 ($\beta_2 = 0.022$, $SE = 0.08$, $p = .005$; $\beta_0 = -1.298$, $SE = 0.105$, $p < .001$). These results mostly confirm the aDDM's behavioral predictions: higher OV induces shorter RT while minimally affecting accuracy (Figure 2).

Figure 2. Choice behavior by OV. Panels (A) and (B) are based on choice behavior from Data Set 1 (Cavanagh et al., 2014), where value represents probability of winning. Panels (C) and (D) are from Data Set 2 (Smith & Krajbich, 2019), where value represents subjective value ratings on a scale from 1 to 10. Panels (A) and (C) show the relationship between choice accuracy (i.e., the probability of choosing the better item) and OV. In (A) accuracy is marginally higher in the Low Value condition than in the High Value condition, but in (C) accuracy is higher in the High Value condition. Panels (B) and (D) show the negative relationship between average response times (RT) and OV. In all panels, circles are data points and the error bars represent the standard error of the mean across participants. The colored-lines in (A) and (C) represent logistic regressions fitted to the data, by OV condition. The green lines in (B) and (D) are simple linear regressions fitted to the data.

Model Fitting Results: Standard DDM

We used participant-level χ^2 values to assess how well the best-fitting models compared to the empirical data. On average, the η Model had the lowest χ^2 values for both datasets (Table 3). Similar to the simulation study, these models indicate that across-trial variability in drift rate was increasing with OV, consistent with, but not specific to, increasing attentional effects with OV.

For each DDM variation, we ran t-tests on the parameters that were estimated separately for the low and high OV conditions. As noted in the Analyses section, we restricted our analyses to the low and high OV conditions because they have comparable levels of difficulty (i.e., value difference). For the a Model, we found that boundary separation decreased in the high OV condition ($M = 0.170$, $SD = 0.046$) relative to the low OV condition in Dataset 1 ($M = 0.222$, $SD = 0.080$), $t(30) = -2.525$, $p = .017$ (95% CI: -0.094, -0.010). In contrast, in Dataset 2, there was no significant difference in boundary separation between conditions (low: $M = 0.178$, $SD = 0.047$; high: $M = 0.177$, $SD = 0.057$; $t(46) = 0.070$, $p = .945$, 95% CI: -0.029, 0.031).

For the s Model, in Dataset 1 we found that within-trial variability increased in the high OV condition ($M = 0.123$, $SD = 0.023$) relative to the low OV condition ($M = 0.094$, $SD = 0.027$), $t(37) = 3.616$, $p = .001$ (95% CI: 0.013, 0.045). In contrast, in Dataset 2, there was no significant difference in within-trial variability between conditions, although the pattern was in the predicted direction (low: $M = 0.112$, $SD = 0.023$; high: $M = 0.119$, $SD = 0.028$; $t(44) = -0.993$, $p = .326$, 95% CI: -0.023, 0.008).

For the η Model, in Dataset 1 we found that across-trial drift variability increased in the high OV condition ($M = 0.257$, $SD = 0.137$) relative to the low OV condition ($M = 0.063$, $SD = 0.091$), $t(33) = 5.286$, $p < .001$ (95% CI: 0.120, 0.269). We found a similar effect in Dataset 2,

where η was significantly larger in the high OV condition ($M = 0.801$, $SD = 0.134$) than the low OV condition ($M = 0.185$, $SD = 0.139$), $t(31) = 19.146$, $p < .001$ (95% CI: 0.550, 0.682).

Table 4. Mean parameter estimates of the DDM fits

<i>a</i> Model	a_{Low}	a_{High}	ter	η	sz	st	v	χ^2
Data Set 1	.222	.170	.598	.094	.081	.365	.032	68
Data Set 2	.178	.177	.776	.130	.036	.423	-.008	24
<i>s</i> Model	s_{Low}	s_{High}	ter	η	sz	st	v	χ^2
Data Set 1	.094	.123	.590	.093	.087	.381	.033	68
Data Set 2	.112	.119	.755	.148	.046	.299	-.007	26
η Model	a	ter	η_{Low}	η_{High}	sz	st	v	χ^2
Data Set 1	.204	.589	.063	.257	.063	.394	.032	64
Data Set 2	.185	.801	.171	.212	.037	.476	-.008	23

Note. In the *a* Model and η Model, *s* was held constant at 0.1. In the *s* Model, *a* was held constant at 0.2. Here, “*a*” stands for boundary separation, “*T_{er}*” for non-decision time, “ η ” for across-trial variability in drift, “*sz*” for across-trial variability in starting point, “*st*” for across-trial variability in non-decision time, and “*v*” for drift rate.

Overall, we found that several different model parameters could account for the OV effect in empirical data. Similar to the simulation study, boundary separation appeared to decrease in the high OV condition. However, the OV effect could also be accounted for with within-trial drift variability or across-trial drift variability. Unlike in the simulation study, the across-trial variability model provided the best fit to the data, though only slightly.

Model Fitting Results: aDDM

Replicating the simulation study, boundaries decreased as OV increased in the two models that separately estimated boundary separation for each value condition (Table 4). To statistically test the difference in parameter values, we used Bayesian estimation for two groups implemented within the HDDM package (Kruschke, 2012; Wiecki et al., 2013). We report a P statistic that corresponds to the proportion of the sampled posterior probability distribution of mean differences that was in line with each hypothesis.

In Dataset 1, we found strong evidence that boundary separation was smaller in the high OV condition than in the mixed OV condition for both the value-difference model (DDM2: $P_{high-high < high-low} = 0.951$) and the attention-based model (aDDM2: $P_{high-high < high-low} = 0.945$). We also found moderate evidence that boundary separation was larger in the low OV condition compared to the mixed condition in both models (DDM2: $P_{low-low > high-low} = 0.858$; aDDM2: $P_{low-low > high-low} = 0.866$).

In Dataset 2, we found that boundary separation was smaller in the high OV condition than in the mixed condition, with $P_{high-high < high-low} = 0.849$ and $P_{high-high < high-low} = 0.810$ for the value-difference model (DDM2) and attention-based model (aDDM2), respectively. There was moderate evidence that boundary separation was larger in the low OV condition relative to the mixed OV condition (DDM2: $P_{low-low > high-low} = 0.762$; aDDM2: $P_{low-low > high-low} = 0.728$).

Table 4. Mean parameter estimates of the HDDM fits (95% HDI)

DDM1	ter	$v_{Intercept}$	v_{Dif}	a		DIC	BPIC		
Data Set 1	0.41 [0.38, 0.44]	0.02 [-0.04, 0.02]	0.28 [0.27, 0.30]	1.92 [1.76, 2.10]		5281	5325		
Data Set 2	0.57 [0.54, 0.60]	-0.02 [-0.06, 0.02]	0.21 [0.20, 0.22]	2.00 [1.82, 2.21]		11257	11315		
DDM2	ter	$v_{Intercept}$	v_{Dif}	$a_{Low-Low}$	$a_{High-Low}$	$a_{High-High}$	DIC	BPIC	
Data Set 1	0.42 [0.39, 0.45]	0.02 [-0.41, 0.08]	0.29 [0.27, 0.31]	2.06 [1.90, 2.22]	1.91 [1.76, 2.07]	1.75 [1.59, 1.91]	5266	5342	
Data Set 2	0.57 [0.54, 0.61]	-0.02 [-0.06, 0.03]	0.21 [0.20, 0.22]	2.07 [1.91, 2.24]	1.99 [1.82, 2.16]	1.87 [1.71, 2.04]	11258	11361	
aDDM1	ter	$v_{Intercept}$	v_1	v_2	a		DIC	BPIC	
Data Set 1	0.41 [0.38, 0.45]	-0.05 [-0.12, 0.02]	2.93 [2.28, 3.13]	1.58 [1.37, 1.79]	1.94 [1.78, 2.11]		5185	5233	
Data Set 2	0.57 [0.53, 0.60]	-0.01 [-0.06, 0.04]	0.25 [0.24, 0.26]	0.141 [0.13, 0.16]	2.04 [1.85, 2.24]		10923	10987	
aDDM2	ter	$v_{Intercept}$	v_1	v_2	$a_{Low-Low}$	$a_{High-Low}$	$a_{High-High}$	DIC	BPIC
Data Set 1	0.42 [0.38, 0.45]	-0.05 [-0.12, 0.02]	2.95 [2.75, 3.15]	1.62 [1.42, 1.84]	2.06 [1.90, 2.22]	1.94 [1.78, 2.10]	1.77 [1.61, 1.93]	5174	5253
Data Set 2	0.57 [0.54, 0.61]	-0.01 [-0.06, 0.05]	0.25 [0.23, 0.26]	0.14 [0.13, 0.16]	2.10 [1.92, 2.27]	2.02 [1.85, 2.20]	1.92 [1.74, 2.10]	10939	11048

Note: In these analyses, DDM1 refers to the model with value-difference drift rate (v) and constant boundaries (a), DDM2 to the model with value-difference drift rate and boundaries conditioned on OV, aDDM1 to the model with attention-based drift rate and constant boundaries, and aDDM2 to the model with attention-based drift rate and boundaries conditioned on OV.

When evaluating more complex models (e.g., models with variable boundaries) it is important to consider not only the parameter values, but also whether the extra flexibility significantly improves the model fits. Otherwise, the more complex models may simply be overfitting the data. We should be conservative in this regard, especially when deviating from standard modeling assumptions.

To evaluate whether variable boundaries significantly improved the models, we compared model fits using the deviance information criterion (DIC; Spiegelhalter et al., 2002) and the Bayesian predictive information criterion (BPIC; Ando, 2007). For both datasets, the models with attention-based drift rates outperformed the models that only incorporated value differences (Table 4; aDDM1 vs. DDM1 and aDDM2 vs. DDM2). However, the version of the attention-based model that best fit the data depended on the model diagnostic test. According to

the BPIC diagnostic, the aDDM with constant boundaries was the best fitting model for both datasets (Dataset 1: $\text{BPIC}_{\text{aDDM1}} = 5233 < \text{BPIC}_{\text{aDDM2}} = 5253$; Dataset 2: $\text{BPIC}_{\text{aDDM1}} = 10987 < \text{BPIC}_{\text{aDDM2}} = 11048$), though this was also true for the DDM models. With DIC, the aDDM with constant boundaries was favored in Dataset 2 ($\text{DIC}_{\text{aDDM1}} = 10923 < \text{DIC}_{\text{aDDM2}} = 10939$), while the aDDM with value-dependent boundaries was favored in Dataset 1 ($\text{DIC}_{\text{aDDM2}} = 5174 < \text{DIC}_{\text{aDDM1}} = 5185$). It is worth noting that the improvement in DIC for including value-dependent boundaries was smaller for the models with attention-based drift rates ($\Delta 9$) than those without attention ($\Delta 15$).

We conclude that adding value-dependent boundaries to the DDM/aDDM does not improve model fit enough to warrant their inclusion. The BPIC was developed to account for the DIC's tendency to favor models that overfit data (Ando, 2007). In the model comparison, the BPIC consistently favored the models with constant boundaries, while the DIC results were mixed (see Supplementary Materials). This suggests that value-dependent boundaries may be an overly-flexible and unnecessary mechanism, especially after accounting for the effects of attention.

Discussion

Empirically, increases in overall value (OV) result in shorter decision times. Recent research has argued that this OV effect is due to attentional amplification of value during the decision process, as described by the attentional diffusion model (aDDM; Smith & Krajbich 2019), but there are other explanations that involve drift rate variability or boundary separation. Here, we demonstrated how the standard DDM can misattribute the OV effect to boundary separations or drift rate variabilities if the true process is the aDDM. We have shown that aDDM-generated simulations can be fit with a non-attentional DDM that includes drift-rate

variability that increases with OV, or erroneously with decision boundaries that decrease with OV. Turning to empirical data, we have also shown that accounting for attention eliminates, or at least reduces, the model-fit advantage of allowing boundary separation to vary with OV.

In the empirical analysis, we focused on boundary separation because this is a parameter that other studies have allowed to vary with OV (e.g., Cavanagh et al., 2014; Pirrone et al., 2018). Alternatively, one can allow within-trial drift-rate-variability or across-trial drift-rate-variability parameters to account for OV (Bose et al., 2020; Brunton et al., 2013; Kvam & Pleskac, 2016; Ratcliff et al., 2018; Smith & Ratcliff, 2009; Teodorescu et al., 2016). We see the variability models as less conceptually different because they are simply less specific than the aDDM about the source of noise in the decision process. Moreover, there may likely be additional sources of drift-rate variability beyond attention. In contrast, diffusion models that condition boundary separation on OV offer a very different interpretation.

We used the aDDM as our default mechanism for the OV effect for the simple reason that it also explains the value-dependent effect of attention on choice (Smith & Krajbich, 2019). Over the past decade there have been a number of papers documenting the relationship between attention and choice (see Krajbich, 2019), including domains such as consumer/food choice (Fisher, 2017; Folke et al., 2016; Gluth et al., 2020; Krajbich et al., 2010; Sullivan et al., 2015; Towal et al., 2013), risky choice (Glickman et al., 2019; Gluth et al., 2018; Johnson & Busemeyer, 2016; Stewart, Hermens, et al., 2016), intertemporal choice (Amasino et al., 2019; Stewart et al., 2006), social preferences (Ashby et al., 2016; Smith & Krajbich, 2019; Stewart, Gächter, et al., 2016), aesthetic choice (Vaidya & Fellows, 2015), moral preferences (Fiedler & Glöckner, 2015; Pärnamets et al., 2015), and reinforcement learning (Cavanagh et al., 2014;

Cavanagh et al., 2019; Konovalov & Krajbich, 2020). Thus, we believe that it is vital to consider attention when trying to account for the OV effect.

One unanswered question is whether it would also be problematic to fit the aDDM to data generated from a non-attentional DDM with varying noise or boundaries. Even without eye-tracking data, the aDDM could be fit to the choice-conditioned RT distributions. If there were OV effects in the RTs (e.g., due to value dependent boundaries), this would presumably lead to an aDDM fit with significant attentional effects. We did not address this possibility here because every study using the aDDM has established a link between gaze and choice before fitting the model. Indeed, the aDDM's attentional discounting parameter (θ) can be estimated directly from the relationship between gaze proportions, option value, and choice probability (Smith, Krajbich, Webb, 2019), providing an alternative validation for the RT-based estimates. Therefore, any fits of the aDDM without eye-tracking data would need to be interpreted with caution.

Interestingly, the work by Cavanagh et al. (2014) did incorporate eye-tracking data. However, the researchers used an alternative formulation for how attention influences drift rate, namely, an additive effect that is constant regardless of the option values. This model does not generate the OV effect. In this study, they found that the additive model provided a better fit to their data than the multiplicative aDDM. (Smith & Krajbich, 2018) re-analyzed their dataset using models with stationary boundaries and found more equivocal results. Here, we confirmed Cavanagh et al.'s claim that boundary separation is decreasing with OV, even when using the aDDM. However, the advantage of doing so decreased substantially relative to the non-attentional model. Moreover, giving the models this added flexibility did not significantly improve their fits to the data.

Cavanagh et al. (2014) also linked changing boundary separation to changes in pupil diameter. While this provides an interesting new link between the DDM and biological measures, it does not by itself confirm that boundaries are changing with OV. It could be the case that pupil dilation simply reflects OV. Neuroimaging studies have found that brain activity does reflect OV, suggesting that this statistic is either represented or at least correlated with some computation that is represented in the brain (Hunt et al., 2012).

Our analyses in the simulation study found that any model that estimated boundary separation separately for each condition was able to qualitatively capture the patterns found in the simulated data. Additionally, the best fitting model was one where both boundary separation and non-decision time were estimated separately for different OV conditions. However, the data generating process included attention-weighted drift rates and not boundary separation. We therefore advise caution in interpreting decision boundaries that vary with OV without also considering attention during the value comparison process. More generally, we caution against letting boundary separation vary based on trial-level features, because what looks like evidence for varying boundaries may simply be unaccounted-for drift rate effects.

Looking outside the narrow scope of model fits, researchers should also consider the theoretical implications of their models' parameters. In the case of the DDM, magnitude-sensitive decision boundaries suggest that decision-makers are somehow adjusting their decision threshold each trial based on stimulus magnitudes. Such a strategy seems very cognitively costly relative to the small, debatable advantage that it conveys in terms of accuracy and RT. It also challenges the norms of diffusion modeling, where the decision threshold is typically set *a priori* to stimulus presentation and is constant within an experimental condition (Ratcliff & McKoon, 2008).

In contrast, magnitude-sensitive noise parameters have a more established pedigree in the DDM literature. Researchers have developed diffusion models where stimulus magnitudes are used to calculate the noise in the decision process in auditory discrimination tasks (Brunton et al., 2013), brightness discrimination tasks (Ratcliff et al., 2018; Teodorescu et al., 2016), numerosity magnitude estimation tasks (Ratcliff & McKoon, 2018), and motion discrimination tasks (Ratcliff et al., 2018). Additionally, the notion that perceptual noise increases in relation to magnitude is well established in psychophysics, going back to Fechner in 1860 (see Ratcliff et al., 2018 for a more thorough discussion of the history of magnitude-sensitive noise in the psychophysics literature). However, despite the theoretical appeal of DDMs with noise that increase with stimulus magnitude, our paper shows that such assumptions may not be necessary once one accounts for the role of attention in choice.

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