PREDICTING FRACTION AND ALGEBRA ACHIEVEMENTS ONLINE

Predicting Fraction and Algebra Achievements Online: A Large-Scale Longitudinal Study Using Data from an Online Learning Environment

Markus Wolfgang Hermann Spitzer^{1*},

Korbinian Moeller^{2, 3, 4, 5}

¹Albert-Ludwigs-Universität Freiburg, 79085 Freiburg, Germany

² Centre for Mathematical Cognition, School of Science, Loughborough University, Loughborough, United Kingdom

³Leibniz-Institut fuer Wissensmedien, Tuebingen, Germany

⁴LEAD Graduate School and Research Network, University of Tuebingen, Germany

⁵Individual Development and Adaptive Education for Children at Risk Center, Frankfurt am Main, Germany

Statement:

This research did not receive funding. We thank Bettermarks for sharing their data with us. The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of Bettermarks. Bettermarks had no role in study design, and data analysis, decision to publish, or preparation of the manuscript. The data that support the findings of this study are available from the corresponding author upon reasonable request. In this study, we are reporting a retrospective study of archived data which was fully anonymized before data analysis. Thus, it is not possible to track the data back to any software user. When signing up with the software, each software user agrees that their data will be stored anonymously and used for data analyses.

*Correspondence:

E-Mail: Markus.spitzer@psychologie.uni-freiburg.de

Abstract

Background: Mastering fractions seems among the most critical mathematical skills for students to acquire in school as fraction understanding significantly predicts later mathematic achievements, but also broader academic and vocational prospects. As such, identifying longitudinal predictors of fraction understanding (e.g., mastery of numbers and operations) is highly relevant. However, almost all existing studies identifying more basic numerical skills as predictors of fraction understanding rest on data acquired in face-to-face testing—mostly in classrooms.

Objectives: In this article, we evaluated whether results obtained in these previous studies generalized to data from the curriculum-based online learning environment *Bettermarks* for mathematics used in schools in the Netherlands.

Methods: We considered data from more than 5,000 students who solved over 1 million mathematical problem sets on basic mathematical skills, fractions, but also algebra.

Results and Conclusions: In line with previous findings, we found that fraction understanding was predicted significantly by more basic mathematical skills. Our analyzes also indicated that algebra achievement was predicted significantly by fraction understanding beyond influences of more basic mathematical skills.

Implications: Together, these findings generalized previous results based on face-to-face testing to the context of data from online learning environments and thus, indicate that data from such large-scale online learning environments may well qualify to provide significant insights into the hierarchical development of mathematical skills.

Keywords: Fraction, mathematical education, academic achievement, e-learning, online learning environments, learning analytics, educational data mining.

Introduction

Fraction understanding is one of the most critical and difficult mathematical skills to learn for children in school (Lortie-Forgues et al., 2015; Siegler et al., 2011; Siegler & Lortie-Forgues, 2015, 2017; Siegler & Pyke, 2013). At the same time, however, mastery of fraction understanding was observed to predict future achievements on later mathematics topics (such as algebra) in high-school (Bailey et al., 2012; Booth & Newton, 2012; Siegler & Pyke, 2013). Moreover, mathematics achievement in general and achievements on algebra in particular, have been associated with better job prospects during adulthood and thus higher socioeconomic status later in life (Murnane et al., 1995; National Mathematics Advisory Panel, 2008; Parsons & Bynner, 2007; Ritchie & Bates, 2013; Rivera-Batiz, 1992). In other words, the ability to successfully operate with fractions plays an important role in children's mathematics education, paving the ground for later academic and life prospects. Therefore, identifying predictors of fraction understanding is highly relevant.

A growing body of longitudinal studies indicates that more basic mathematical skills like children's understanding of (whole) numbers and operations predict later fraction understanding significantly—even when controlling for important covariates like intelligence, SES, gender, or race. (e.g., Bailey et al., 2014; Hansen et al., 2015; Hecht & Vagi, 2010; Jordan et al., 2013; Vukovic et al., 2014, see below for a more detailed description). Importantly, however, almost all of these studies rest on data collected through face-to-face testing—mostly in classrooms—while data from online learning environments have largely been neglected. This is surprising as online learning environments have been continuously collecting large amounts of data over years on students' mastery of various mathematical topics. However, it remains to be evaluated whether the results observed from data acquired in face-to-face testing generalize to data from online learning

environments. If so, this would open a new avenue for research on the development of mathematical skills with datasets of considerable size. Accordingly, this research project sought to generalize findings from face-to-face testing to data collected with an online learning environment.

Therefore, we analyzed a large-scale and longitudinal dataset (5,000 students who calculated over 1 million mathematical problem sets) taken from the curriculum-based online learning environment *Bettermarks* which is used within the class-context in the Netherlands. In a first step, similar to previous longitudinal studies collecting data from face-to-face testing, but with data obtained from the online learning environment *Bettermarks*, we investigated whether prior skills on basic mathematical topics (i.e., *Geometry*, *Basic Arithmetic*, *Measurement Units*, and *Advanced Arithmetic*) implemented within *Bettermarks* are predictive of students' achievements on fractions (for an overview of how these topics appear in *Bettermarks* see Figure 1).

In a second step, using follow-up data from the same students considered in the first step, we then evaluated whether fraction understanding predicts achievements in algebra over and beyond the predictive value of prior skills on basic mathematical topics (which may predict fraction understanding). While these analyses aimed to generalize previous findings from face-to-face testing using large-scale data from an online learning environment, they also aimed to substantiate previous findings investigating a specific chain of predictions: whether achievements in basic mathematical skills predict fraction achievements and then fraction achievements predict algebra achievements over and beyond influences of achievements in basic mathematical skills.

In the following, we first introduce the integrated theory of numerical development (Siegler et al., 2011). We then review previous longitudinal studies, followed by a brief

introduction of the advantages large-scale data from online learning environments may have before describing the details of the current study.

PREDICTING FRACTION AND ALGEBRA ACHIEVEMENTS ONLINE

Figure 1. User Interface of *Bettermarks* for the Netherlands. A tutorial (0 Tutorial) and different topics can be selected. The language can be set to English or Dutch for the *Bettermarks* system in the Netherlands. TTO stands for “tweetalig onderwijs” and means bilingual education as the language in this example was set to English. Please note that the tutorial as well as the 7th topic (1TT07) was not part of this investigation.

Basic Mathematical Skills Predict Fraction and Algebra Achievements

The integrated theory of numerical development (Siegler et al., 2011; Siegler & Lortie-Forgues, 2014) proposes that the representation of number magnitude is at the core of numerical development. With age, children first increase the range of whole numbers they understand, then integrate negative numbers, and finally fraction magnitudes into their mental representation of numbers. The theory predicts that those with better understanding of earlier instantiations/representations of number magnitude (i.e., whole numbers) should be better off when learning new classes of numbers like fractions (Siegler, 2016; Siegler et al., 2011, 2013; Siegler & Braithwaite, 2017). Support for this theoretical approach comes from longitudinal studies showing that fraction understanding is indeed predicted by earlier whole number processing and at the same time is predictive of more advanced mathematical concepts such as algebra. We review evidence on this in the following two sections.

A growing number of longitudinal studies reported that early mathematical skills seem to serve as building blocks for later achievements in mathematics. On the one hand, several longitudinal studies observed that fraction understanding was predicted significantly by prior skills on whole number processing and arithmetic (Bailey et al., 2014; Hansen et al., 2015; Hecht & Vagi, 2010; Jordan et al., 2013; Vukovic et al., 2014). For instance, Hecht and Vagi (2010) followed 181 students for two years from fourth through fifth grade and found that their fraction understanding was predicted significantly by their earlier whole number arithmetic performance (Hecht & Vagi, 2010). In a similar longitudinal study, Jordan et al. (2013) assessed students' earlier whole number arithmetic performance at the beginning of third grade. When assessing their fraction understanding at the end of grade four ($n=357$) the authors found that it was predicted significantly by children's performance in whole number arithmetic over and above influences of

working memory and attention (Jordan et al., 2013). Another study by Vukovic et al. (2014) which included 163 students replicated this longitudinal association between fraction understanding (at age 10) and prior achievements in whole number arithmetic (at age 7). Additionally, analyzing data from 334 students collected over one school-year, Hansen et al., 2015 observed that earlier whole number arithmetic performance in fifth grade significantly predicted fraction understanding in sixth grade. Finally, a study which followed 162 students from first to eighth grade found that fraction understanding in seventh and eighth grade were predicted significantly by earlier arithmetic performance in first grade, controlling for gender, race, domain general cognitive abilities, parental income, and education (Bailey et al., 2014).

On the other hand, another set of longitudinal studies reported that fraction understanding significantly predicts achievement in algebra (Bailey et al., 2012; Barbieri et al., 2021; Siegler, Duncan, Davis-kean, et al., 2012). For instance, Bailey et al. (2012) examined 212 students in grades six and seven and reported that achievements in mathematics in seventh grade (which included algebra) were predicted significantly by children's fraction understanding in grade six (Bailey et al., 2012). Similarly, Siegler et al. (2012) examined 3,677 UK students and 599 students from the US. For both samples they observed that students' whole number arithmetic performance as well as fraction understanding when 10 years old uniquely predicted their achievement in algebra and overall mathematics achievement six years later (when aged 16)—even after controlling for whole number knowledge, reading achievement, working memory, family income, and intelligence. Furthermore, Booth and Newton (2012) collected data of 32 students (age range 12-14) and observed that fraction understanding was the strongest predictor for achievements in algebra. Finally, another recent study including 350 middle school students between 11-13 years

of age reported that fraction understanding at the beginning of a school year was the best predictor for algebra achievement at the end of the same school year (Barbieri et al., 2021).

Together, previous longitudinal studies consistently supported the hierarchical nature of acquiring mathematical skills suggesting that (a) achievements in fraction understanding can be predicted by prior skills on whole number processing and arithmetic (depending on age/grade level), and (b) achievements in algebra are predicted by fraction understanding. Yet, most of the reported studies share a few commonalities: (a) assessing data from US students (except: Siegler, Duncan, Davis-Kean, et al., 2012); (b) collecting data of a few hundred students (except: Siegler, Duncan, Davis-Kean, et al., 2012); (c) assembling data through non-digital assessments over long periods of time; and (d) collecting data at two time points to investigate the influence of variables at timepoint one on (outcome) variables on timepoint two. However, a longitudinal study on the effects of prior mathematical achievements on fraction achievements with a follow-up on the influence of prior skills *and* fraction achievements on algebra achievements has thus far, and to the best of our knowledge, not been conducted. Accordingly, the present article aimed to evaluate whether the predictive effects of basic mathematical skills on later achievements in mathematics generalize to data acquired in an online learning environment used in schools within the class-context.

Learning Mathematics by Means of Online Learning Environments

During the last decade, the number of students who use online learning environments to study mathematics increased steadily until the outbreak of the COVID-19 pandemic. Then, the use of online learning environments increased steeply due to COVID-19 related school closures (Meeter, 2021; Spitzer et al., 2021; Spitzer & Musslick, 2021; Tomasik et al., 2020; Velde et al., 2021). For instance, the usage (in terms of students enrolled and problem sets computed per month)

of the curriculum-based online learning environment *Bettermarks* increased by about 300% within March 2020. Currently, *Bettermarks* is being used by over 300,000 students who solve over 5 million mathematical problem sets a month. *Bettermarks* is most prominently used in schools in Germany, the Netherlands, and Uruguay (Spitzer, 2021). *Bettermarks* is employed in the class-context with teachers assigning problem sets to their students which they have to work on in the classroom, or from home, while both students and teachers are able to track the learning progress online. Therefore, the software automatically and permanently records students' anonymous performance. Consequently, such an online learning environment provides researchers with rich data sets on students' performance on different mathematical topics assessed over long periods of time (Koedinger et al., 2007, 2015; Spitzer, 2021). As such, these data allow to distill factors that play a major role on students' learning trajectories (Baker & Inventado, 2016; Spitzer, 2021).

The Present Study

As the usage of online learning environments is constantly growing, one may ask whether the data obtained from these online learning environments reflects similar developmental trajectories to previous studies which collected data face-to-face inside classrooms. Accordingly, this study considered data collected with the online learning environment *Bettermarks* to evaluate the hierarchical nature of numerical development as suggested by the integrative theory of numerical development (Siegler et al., 2011) and corroborated by above-described longitudinal studies on the predictive power of basic mathematical skills on later fraction understanding and fraction understanding on more general mathematical achievements (including algebra). We specifically considered data of several hundred so called digital-learning classes in the Netherlands that have been studying purely with *Bettermarks*. That is, these classes consistently worked through *Bettermarks* topic by topic assigned by their teachers. Thus, these classes provide an ideal

use case for a longitudinal study on the effects of basic mathematical skills on later mathematical achievement in digital learning environments.

In particular, we evaluated whether students fraction understanding i) is predicted by basic mathematical skills and ii) predicts later algebra achievement over and beyond contributions of basic mathematical skills in a follow-up analysis. Therefore, we considered data of six topics (so called books within *Bettermarks*, labeled as (i) *Geometry*; (ii) *Basic Arithmetic*; (iii) *Measurement Units*; (iv) *Advanced Arithmetic*; (v) *Fractions*; and (vi) *Algebra*). Basic mathematical skills were operationalized by means of the first four topics which students completed before working on *Fractions* and *Algebra* (see Figure 1 for an overview of the topics).

We conducted two sets of analyses. In a first step, we investigated whether and if so, which earlier mathematical skills predicted fraction understanding. In a second step, we evaluated whether fraction understanding predicted achievements in algebra over and beyond contributions of the first four basic mathematical topics. Based on previous longitudinal studies, we expected students' performance on the two topics *Basic Arithmetic* and *Advanced Arithmetic* to significantly predict students' fraction understanding. In addition, we hypothesized that the ability to operate on different measurement units should also predict fraction understanding as it also reflects processing number magnitude. We had no specific expectation as to the influence of *Geometry* on students' fraction understanding. As regards the second step, we expected fraction understanding to significantly predict algebra achievement as observed previously. The hypothesized results pattern would indicate that longitudinal associations between the learning of basic and more advanced mathematical skills as previously observed for fraction understanding and algebra generalize to data from online learning environments. In turn, this would indicate that online

learning environments provide a valid source of data to investigate the development of mathematical skills

Methods

Online Learning Environment

The online learning environment *Bettermarks* is used by teachers and students from classes 4-10 (age range: 9-16; also see: Spitzer, 2021). It includes over 100 different “book topics”, comprising different mathematical topics such as geometry, algebra, number theory, and probability. These different book topics cover different chapters each of which contains several problem sets (nine on average per chapter). In total, the learning environment comprises over 2,000 curricular based problem sets each of which contains eight single mathematical problems on average. Teachers use *Bettermarks* within mathematics classes and assign students problem sets to solve either as homework or within the classroom. When students solve assigned problem sets, they receive automatic feedback after each problem. After they finished a problem set, they are feedbacked their average score on this set. Students may repeat a problem set, but problems within sets change on each new attempt and thus, memorizing the results of a problem set does not help on a new attempt. Teachers automatically receive feedback on how many assigned problem sets their students worked on as well as their accuracy. The data of solved problem sets is stored fully anonymized. Therefore, demographic information such as exact age or gender is not available within the system.

Inclusion Criteria

The current study included Dutch students who used *Bettermarks* together with their classmates and their teachers between January 1st, 2016 and December 31st, 2020. We carried out two sets of analyses which differed with respect to the dependent variable. In the first set of

analyses, we investigated the predictive power of earlier mathematical skills on fraction understanding. Therefore, we included problem sets of the five topics (i.e., 1TTO1 Lines and angles – labeled here as - *Geometry*; 1TTO2 Numbers – *Basic Arithmetic*; 1TTO3 Units of measurement – *Measurement Units*; 1TTO4 Order of operations, powers and negative numbers – *Advanced Arithmetic*; 1TTO5 Fractions – *Fractions*) and investigated the predictive power of the four topics completed before *Fractions* on achievements in *Fractions* (see Figure 1 for an example user interface of *Bettermarks*). All topics as well as the order of topics (i.e., *Geometry* before *Basic Arithmetic* and *Basic Arithmetic* before *Fractions* etc.) were designed by *Bettermarks* according to the mathematics curriculum of students in their respective country. Appendix A describes the chapters of each book topic in more detail.

We only included students who computed at least 10 problem sets in each of the five topics. To assure that the topics used as predictors were indeed computed before the dependent topic (*Fractions* in this case), only problem sets of topics which served as independent variables (*Geometry*, *Measurement Units*, *Basic Arithmetic*, and *Advanced Arithmetic*) computed prior to working on the first problem set from the *Fractions* topic were included in the analyses. To estimate students' best achievements, only the best result of a problem set was included in the analyses. All other repeated problem sets were excluded from the first set of analyses.

We considered the best result of students on a given problems set as dependent variable, as students sometimes repeat problem sets several times, even after achieving high scores, but then eventually decide to “click through” some of the remaining problems after having restarted a problem set. By considering the best result, we aimed to circumvent potential biases which may occur from such behavior. However, to validate the use of this measure as dependent variable we conducted two control analyses to substantiate the robustness of the results described in the results

section. The first analysis only considered problem sets which were completed once (i.e., without repetition, see Appendix B). In a second control analysis, we only considered the last result of students on the respective problem sets (see Appendix B). Importantly, the result pattern of both control analyses was virtually identical to the results of our main analyses described in the results section.

With these inclusion criteria, the dataset of the first set of analyses encompassed a total of 5,294 Dutch students who computed 1,050,897 problem sets.

In a second step of analyses, we investigated the predictive power of prior mathematical topics on *Algebra* (labeled as 1TTO6 Formulas in *Bettermarks*) and included all students of the first analyses, who computed at least 10 problem sets in the *Algebra* topic. Similar to the first set of analyses, only problem sets of non- *Algebra* topics computed before the calculation of the first the *Algebra* problem set were incorporated within this analysis. Some of the students considered in the first set of analyses did not meet these inclusion criteria, as they i) did not compute enough problem sets (i.e., $n_{\text{problem sets}} < 10$) from the algebra topic ($n_{\text{students}} = 217$), ii) stopped using this online learning environment before computing algebra problem sets ($n_{\text{students}} = 400$), or iii) worked on algebra problem sets before they worked on fractions ($n_{\text{students}} = 277$). Thus, the data set of second set of analyses was reduced to 4,400 students who solved 892,285 problem sets.

Independent and Dependent Variables

With respect to the first set of analyses, the four topics (1) *Geometry*, (2) *Measurement Units*, (3), *Basic Arithmetic*, and (4) *Advanced Arithmetic* were used as predictor variables. Achievement in these four topics were operationalized by means of students' average accuracy on each of the topics. In addition, we aimed at controlling for differences in the time elapsed between working on the predictive topics and the *Fractions* or the *Algebra* topic in the linear regression

models. Therefore, we computed the average number of days between the first attempt on a problem set in the *Fractions* topic and the completion dates of problem sets in each of the other four topics, respectively (labeled as $Days_{topic}$; e.g. $Days_{Geometry}$) and later compared models with and without this covariate on their goodness of fit. The average accuracy for each student on the *Fractions* topic served as the criterion variable for the first set of analyses.

Regarding the second set of analyses, achievement on the *Fractions* topic was included as an independent predictor variable in addition to the four independent variables indicated above, whereas the *Algebra* topic functioned as the depended variable.

Data Analysis

The predictive power of topics in *Bettermarks* on achievements in the *Fractions* and *Algebra* topic was evaluated using multiple linear regression models run within the R environment (R Core Team, 2013; RStudio Team, 2015). Plots were generated with the *sjPlot* package directly from the model fits (Lüdtke, 2020). In addition, we analyzed the correlations between all topics of the first and second set of analyses. To control for differences in the time elapsed between predictive topics and the *Fractions* or the *Algebra* topic, we compared the model fit of two regression models, one including and one not including the control variable $Days_{topic}$. We then report the fit of the winning model for each set of analyses in the results section labeled as Step 1 (for the predictive power on achievements in the *Fractions* topic) and Step 2 (for the predictive power on achievements in the *Algebra* topic). We additionally correlated accuracy scores of all topics with each other and report these correlational results in the following section.

Results

We first computed the correlations between all variables and depict these correlations matrices for all topics considered in Step 1 and Step 2 in Figure 2. The model comparison for the

first step of analyses revealed that controlling for differences in time elapsed between predictive topics and the criterion did not increase model fits with a BIC of -13947 for the less complex model which was smaller compared to the BIC of -13925 for the model including the control variables. This was also the case for the second set of analyses with a smaller BIC score of -10521 for the simpler model compared to a BIC score of -10473. Please note that a difference in BIC of 10 is considered as strong difference between models (Raftery, 1995). We thus report the results of the less complex models which included accuracy scores of the four topics as predictors in Step 1 and accuracy scores of five topics as predictors in Step 2. Figure 3 plots the results of Step 1 whereas Figure 4 depicts the results of Step 2. On average students solved 155 (SD= 33.2) problem sets before approaching the *Fractions* topic and 203 (SD= 44.1) problem sets before approaching the *Algebra* topic.

PREDICTING FRACTION AND ALGEBRA ACHIEVEMENTS ONLINE

Figure 2. Correlation Matrices for Each Analysis. The correlation matrix depicts the correlations of each topic with each other included in the first analysis (Step 1: upper panel) and second analysis (Step 2: bottom panel). In both correlation analyses, we observed the strongest correlation between *Basic Arithmetic* and *Measurement Units*. All correlations were significant with a *p-value* below $<.05$.

Figure 3. Accuracy in *Fractions* as a Function of Accuracy in *Geometry*, *Measurement Units*, *Basic Arithmetic*, and *Advanced Arithmetic*. Each data point reflects the average score of a single student for problem sets in *Geometry* (upper left panel), *Basic Arithmetic* (upper right panel), *Units* (lower left panel), and *Advanced Arithmetic* (lower right panel) topics. Black lines indicate the linear regression line with the respective slope β . Grey shaded areas indicate one standard error of the mean.

PREDICTING FRACTION AND ALGEBRA ACHIEVEMENTS ONLINE

Figure 4. Accuracy in *Algebra* as a Function of Accuracy in *Geometry*, *Basic Arithmetic*, *Measurement Units*, *Advanced Arithmetic*, and *Fractions*. Each data point reflects the average score of a single student for problem sets in *Geometry* (upper left panel), *Basic Arithmetic* (upper middle panel), *Units* (upper right panel), *Advanced Arithmetic* (lower left panel), and *Fractions* (lower middle panel) topics. Black lines indicate the linear regression of the slope β . Grey shaded areas depict one standard error of the mean.

Step 1: Fraction Accuracy as a Function of Accuracy in *Geometry*, *Basic Arithmetic*, *Measurement Units*, and *Advanced Arithmetic*.

The linear regression model indicated significant prediction of *Fractions* accuracy ($F(4, 5289) = 855.5; p < .001$; multiple $R^2 = 0.393$; adjusted $R^2 = 0.392$) by the incorporated predictor variables. Inspection of beta weights indicated that the influence of *Geometry* was significant ($b = .077; t = 4.45; p < .001$), with higher accuracy on the *Geometry* topic being associated with higher accuracy on the *Fractions* topic. Additionally, there were significant influences of *Measurement Units* ($b = .152; t = 8.97; p < .001$) and *Advanced Arithmetic* ($b = .535; t = 34.84; p < .001$). In both cases higher accuracy scores on the respective topic were predictive of better performance on the *Fractions* topic. However, and different from our expectation the contribution of *Basic Arithmetic* was not significant ($b = -.001; t = -0.05; p = .955$).

The integrated theory of numerical development suggests a hierarchical nature of numerical development. Accordingly, more advanced topics should subsume the predictive power of more basic topics in mathematics for fraction achievements or algebra achievements. To test this, we conducted further linear regression models—dropping the respectively best predictor variable on each new regression model—to evaluate whether more advanced predictor topics indeed subsumed the predictive power of the predictor *Basic Arithmetic*. The results of these additional analyses revealed a significant predictive effect of the *Basic Arithmetic* topic on the *Fractions* topic when not including any one of the more advanced topics (i.e., *Measurement Units*, and *Advanced Arithmetic*; see Appendix D for a detailed description of these analyses).

In sum, these results suggest that performance on the three topics (1) *Geometry*, (2) *Measurement Units*, and (3) *Advanced Arithmetic*, uniquely predicted performance on the

Fractions topic. The results also indicate that the positive effect of the *Basic Arithmetic* topic on the *Fractions* topic was subsumed by more advanced topics.

Step 2: Algebra Accuracy as a Function of Accuracy in *Geometry*, *Basic Arithmetic*, *Measurement Units*, *Advanced Arithmetic*, and *Fractions*.

Comparable to the first step, the linear regression model indicated that *Algebra* accuracy was predicted significantly by the considered variables ($F(5, 4394) = 661.9; p < .001$; multiple $R^2 = 0.430$; adjusted $R^2 = 0.429$). Inspection of the beta weights showed that *Geometry* was a significant predictor of algebra ($b = .096; t = 4.57; p < .001$), with higher accuracy for *Geometry* associated with better scores on the *Algebra* topic. The influences of *Measurement Units* ($b = .093; t = 4.40; p < .001$), and *Advanced Arithmetic* were also significant ($b = .287; t = 13.37; p < .001$). Again, higher accuracy on the respective topics predicted higher accuracy for *Algebra*. Finally, also the contribution of *Fractions* was significant ($b = .497; t = 27.76; p < .001$), indicating that higher accuracy on the *Fractions* topic was associated with higher accuracy in *Algebra*. Comparable to the results of the analyses in Step 1, the influence of *Basic Arithmetic* was not significant ($b = -.039; t = -1.64; p = .099$).

In line with the previous analysis, we carried out additional analyses to investigate whether the predictive effect of *Basic Arithmetic* was subsumed by more advanced topics (see Appendix D). These analyses revealed that a significant predictive effect of *Basic Arithmetic* on *Algebra* was observed after dropping any one of the more advanced topics (i.e., *Fractions*, *Advanced Arithmetic*, and *Measurement Units*).

Together, these results suggest that performance on the four topics (1) *Geometry*, (2) *Measurement Units*, (3) *Advanced Arithmetic*, and (4) *Fractions* uniquely predicted performance

on the *Algebra* topic. Further analyses suggested that the effect of the topic *Basic Arithmetic* was subsumed by more advanced topics.

Discussion

In the present study, we evaluated whether longitudinal results obtained from face-to-face testing on predicting fraction outcomes generalize to data collected with an online learning environment. Therefore, we used a large dataset comprising over 5,000 students from the Netherlands who completed over 1 million problem sets in the online learning environment *Bettermarks* to evaluate whether prior skills on four basic mathematical topics (i.e., *Geometry*, *Basic Arithmetic*, *Measurement Units*, *Advanced Arithmetic*) predicted *Fractions* understanding. We then further investigated whether achievement in the *Algebra* topic was predicted by prior skills on the same four topics as well as prior skills on the *Fractions* topic. In a two-step within-student analyses approach, we first observed that students' achievement on the *Fractions* topic was predicted significantly by their prior achievement on the topics (1) *Geometry*, (2) *Measurement Units*, and (3) *Advanced Arithmetic*. The latter had the strongest effect with *Advanced Arithmetic* including operations of powers and negative numbers, followed by *Measurement Units*, and *Geometry*. Results of the second step indicated that students' achievements on the *Algebra* topic was predicted significantly by prior achievements on the topics of (1) *Geometry*, (2) *Measurement Units*, (3) *Advanced Arithmetic*, and (4) *Fractions*, with the strongest contribution made by the *Fractions* topic, followed by *Advanced Arithmetic*, *Geometry*, and *Measurement Units*. In contrast to our expectations, accuracy scores on the *Geometry* topic predicted students' accuracy on *Fractions* and *Algebra*, with a small but significant positive effect. In sum, the results of both analyses revealed similar results compared to previous longitudinal

studies from face-to-face testing and thus generalize these findings to a new context—learning within an online learning environment.

In addition, a general observation in our results was that more advanced topics were stronger predictors of later fraction and/or algebra achievements than more basic topics, with *Advanced Arithmetic* [including aspects such as order of operations (e.g., $3 * 2 + 5 * 3 = x$), calculating with powers (e.g., $4x = 16$), and negative numbers (e.g., $-34 - (-21) = x$)] as the strongest predictor for *Fractions* and *Fractions* as the strongest predictor for *Algebra*. Additional analyses suggested that the effect of prior skills in *Basic Arithmetic* [including calculations on positive decimal numbers (e.g., $6.45 + 8 = x$) and estimating the spatial position of numbers on a number line] was subsumed by more advanced topics such as *Measurement Units* and *Advanced Arithmetic* (in analysis Step 1) and *Measurement Units*, *Advanced Arithmetic*, and *Fractions* (in analysis Step 2). As such, these findings are in line and thus lend further support to the idea of a hierarchical nature of numerical development in terms of successively expanding the representation of number magnitude as argued by Siegler in his integrated theory of numerical development (Siegler et al., 2011; Siegler & Lortie-Forgues, 2014).

The present results extend previous longitudinal findings providing evidence *within one* and the same longitudinal sample of more than 5,000 students on the robust chain of predictions underlying children’s numerical development from whole number processing (i.e., *Basic Arithmetic* including estimating the spatial position of numbers on a number line; and *Advanced Arithmetic*: including aspects such as order of operations, e.g., $3 * 2 + 5 * 3 = x$) over fraction processing to algebra. Mastery of these numerical skills is crucial to children’s later academic achievements and life prospects (Murnane et al., 1995; National Mathematics Advisory Panel, 2008; Parsons & Bynner, 2007; Ritchie & Bates, 2013; Rivera-Batiz, 1992). As such, the results

of our two step analyses procedure, using the same cohort of students, substantiated previous findings suggesting that mastery of basic mathematical skills early on provides a building block for learning more advanced mathematical.

Additionally, it is worth noting that in our analyses we controlled for the time interval between predictor topics and dependent variable, as more advanced predictor topics may have had a shorter time interval to either *Fractions* or *Algebra*. However, model comparisons suggested that controlling for time differences between predictor topics and dependent variables did not improve model fits significantly. For reasons of parsimony, these model comparisons seem to suggest that the time between topics did not influence the results significantly.

Compared to previous longitudinal studies, this study differed in several respects. First, this study generalized previous results on predictive effects of more basic skills in mathematics on fraction understanding and algebra achievement in a European sample, whereas previous longitudinal studies were mostly conducted in the US (one single study included students from the UK: Siegler, Duncan, Davis-Kean, et al., 2012). Second, previous longitudinal studies (except the one of Siegler, Duncan, Davis-Kean, et al., 2012) considered samples of only a few hundred students, while this study is based on more students' data than any of the previous longitudinal studies.

Third, this study extends previous results to the predictive influence of curricular based problem sets teachers assigned to their students, who on average calculated 155 problem sets on other mathematical topics before approaching the fraction topic, while previous studies used standard tests or test batteries for the assessment of earlier (numerical) skills. Fourth, and similarly, this study extends previous longitudinal studies which examined predictive effects over a longer time period (at least half an academic year between the assessment of basic numerical skills and

that of the criterion variables, e.g., Fraction understanding), to data that reflect students' continuous use of the online learning environment *Bettermarks*.

Fifth, this study investigated whether basic mathematical skills beyond whole number processing and arithmetic predicted fraction understanding and algebra achievement. In particular, we also assessed and observed significant influences of children's understanding of measurement units and their achievement in geometry on fraction understanding and algebra achievement. However, the predictive effect of measurement units and geometry on fraction and algebra achievements was low, compared to the effect of more advanced topics.

Sixth, finally and probably most critically, this study is, to the best of our knowledge, the first one to generalize the results of previous longitudinal studies, involving face-to-face data collection in classrooms to data from an online learning environment. This finding provides a relevant benchmark for future studies aiming at investigating the development of numerical and mathematical skills and wish to consider data from online learning environments. Future research is needed to provide further evidence on the replicability of exhaustively conducted face-to-face studies within online learning environments. Additionally, future studies may employ more sophisticated analyses looking into learning trajectories to better understand students' development of complex mathematical skills.

While our results suggest that prior achievements predict later achievements in mathematics, it remains to be investigated which order of learning topics may be optimal when learning mathematics. For instance, we investigated a specific learning curricular here (i.e., more basic skills before fractions, and fractions before algebra). However, it may be that another learning curricular (i.e., a different order of learning these contents) leads to better learning and thus higher achievements on the last topic learnt. Identifying the optimal curriculum for learning

fractions, algebra, or any other mathematical topic may thus be an important avenue for future research.

Thus far little is known about which specific learning curricular (i.e., which order) is optimal when learning rational numbers such as fractions, decimals, and percentages (Tian & Siegler, 2018). Nevertheless, the insights gained from this study show that mathematical achievements were predicted significantly by students' previous achievements on more basic math. Moreover, adaptive learning environments allow to assign specific earlier problem sets or topics *automatically* as recommended revisions for students who struggle with more advanced contents. In particular, our result that performance of some mathematical topics (e.g., *Fractions*) had a stronger predictive power for later topics (such as *Algebra*), points to the relevance of assuring that students have a sufficient understanding of prior topics before moving on to more advanced topics. A next step would be to investigate which specific curricular design would lead to optimal learning outcomes.

Conclusion

We investigated whether previous longitudinal findings from face-to-face testing, indicating that fraction understanding as well as achievements in algebra can robustly be predicted by prior mathematics skills, generalize to mathematics achievements within an online learning environment. In addition, we investigated a specific chain of predictions with the same cohort of students. Substantiating the expected chain of predictions, we observed that basic mathematics skills (*Geometry*, *Measurement Units*, and *Advanced Arithmetic*) significantly predicted achievements in *Fractions*. We also found that achievements in *Fractions* predicted achievements in later *Algebra*. As such, our results generalize previous findings on the predictors of *Fraction* and *Algebra* achievements to the context of large-scale data from online learning environments.

Moreover, our results substantiated the idea of a hierarchical nature of numerical development, as more advanced topics were stronger predictors for *Fractions* and *Algebra*, respectively, compared to more basic topics. This finding suggests that data collected with an online learning environment may well qualify for the investigation of children's numerical development. We hope that our findings are a first step towards building a strong foundation for future research considering data from online learning environments to better understand mathematical learning and development.

References

- Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology*, 113(3), 447–455. <https://doi.org/10.1016/j.jecp.2012.06.004>
- Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*, 17(5), 775–785. <https://doi.org/10.1111/desc.12155>
- Baker, R. S., & Inventado, P. S. (2016). Educational data mining and learning analytics: Potentials and possibilities for online education. In *Emergence and Innovation in Digital Learning: Foundations and Applications*. <https://doi.org/10.15215/aupress/9781771991490.01>
- Barbieri, C. A., Young, L. K., Newton, K. J., & Booth, J. L. (2021). *Predicting Middle School Profiles of Algebra Performance Using Fraction Knowledge*. 00(0), 1–22. <https://doi.org/10.1111/cdev.13568>
- Booth, J. L., & Newton, K. J. (2012). Fractions : Could they really be the gatekeeper ' s doorman ? *CONTEMPORARY EDUCATIONAL EDUCATIONAL PSYCHOLOGY*, 37(4), 247–253. <https://doi.org/10.1016/j.cedpsych.2012.07.001>
- Hansen, N., Jordan, N. C., Fernandez, E., Siegler, R. S., Fuchs, L., Gersten, R., & Micklos, D. (2015). General and math-specific predictors of sixth-graders' knowledge of fractions. *Cognitive Development*, 35, 34–49. <https://doi.org/10.1016/j.cogdev.2015.02.001>
- Hecht, S. A., & Vagi, K. J. (2010). Sources of Group and Individual Differences in Emerging Fraction Skills. *Journal of Educational Psychology*, 102(4), 843–859. <https://doi.org/10.1037/a0019824>
- Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013).

- Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology*, 116(1), 45–58. <https://doi.org/10.1016/j.jecp.2013.02.001>
- Koedinger, K. R., Anderson, J. R., Hadley, W. H., Mark, M. A., Koedinger, K. R., Anderson, J. R., Hadley, W. H., Mark, M. A., Tutoring, I., Koedinger, K. R., Anderson, J. R., Hadley, W. H., & Mark, M. A. (2007). *Intelligent Tutoring Goes To School in the Big City To cite this version : HAL Id : hal-00197383 Intelligent Tutoring Goes To School in the Big City*.
- Koedinger, K. R., Mello, S. D., Mclaughlin, E. A., Pardos, Z. A., & Rosé, C. P. (2015). Data mining and education. *WIREs Cognitive Science*, 6(August), 333–353. <https://doi.org/10.1002/wcs.1350>
- Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Why is learning fraction and decimal arithmetic so difficult? *Developmental Review*, 38, 201–221. <https://doi.org/10.1016/j.dr.2015.07.008>
- Lüdecke, D. (2020). *sjPlot: Data Visualization for Statistics in Social Science*. <https://doi.org/10.5281/zenodo.1308157>
- Meeter, M. (2021). Primary school mathematics during Covid-19: No evidence of learning gaps in adaptive practicing results. *Trends in Neuroscience and Education*, 25(October), 100163. <https://doi.org/10.1016/j.tine.2021.100163>
- Murnane, R. J., Willett, J. B., & Levy, F. (1995). The Growing Importance of cognitive skills in Wage Determination. *Review of Economics and Statistics*, 77(2), 251–266.
- National Mathematics Advisory Panel. (2008). Foundations for success: The final report of the National Mathematics Advisory Panel. In *U.S. Department of Education*.
- Parsons, S., & Bynner, J. (2007). *Research report over the lifecourse* (Issue November). <https://discovery.ucl.ac.uk/id/eprint/1566242/1/Parsons2007Illuminating.pdf>

- R Core Team. (2013). *R: A Language and Environment for Statistical Computing*. <http://www.r-project.org/>
- Raftery, A. E. (1995). Bayesian Model Selection in Social Research. *Sociological Methodology*, 25(1995), 111–163.
- Ritchie, S. J., & Bates, T. C. (2013). Enduring Links From Childhood Mathematics and Reading Achievement to Adult Socioeconomic Status. *Psychological Science*, 24(7), 1301–1308. <https://doi.org/10.1177/0956797612466268>
- Rivera-Batiz, F. L. (1992). Quantitative Literacy and the Likelihood of Employment among Young Adults in the United States. *The Journal of Human Resources*, 27(2), 313–328.
- RStudio Team. (2015). *RStudio: Integrated Development Environment for R*.
- Siegler, R. S. (2016). Magnitude knowledge: The common core of numerical development. *Developmental Science*, 19(3), 341–361. <https://doi.org/10.1111/desc.12395>
- Siegler, R. S., & Braithwaite, D. W. (2017). Numerical Development. *Annual Review of Psychology*, 68, 187–213. <https://doi.org/10.1146/annurev-psych-010416-044101>
- Siegler, R. S., Duncan, G. J., Davis-kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). *Early Predictors of High School Mathematics Achievement*. <https://doi.org/10.1177/0956797612440101>
- Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early Predictors of High School Mathematics Achievement. *Psychological Science*, 23(7), 691–697. <https://doi.org/10.1177/0956797612440101>
- Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions : the new frontier for theories of numerical development. *Trends in Cognitive Sciences*, 17(1), 13–19.

<https://doi.org/10.1016/j.tics.2012.11.004>

Siegler, R. S., & Lortie-Forgues, H. (2014). An Integrative Theory of Numerical Development. *Child Development Perspectives*, 8, 144–150.

Siegler, R. S., & Lortie-Forgues, H. (2015). Conceptual knowledge of fraction arithmetic. *Journal of Educational Psychology*, 107(3), 909–918. <https://doi.org/10.1037/edu0000025>

Siegler, R. S., & Lortie-Forgues, H. (2017). Hard Lessons: Why Rational Number Arithmetic Is So Difficult for So Many People. *Current Directions in Psychological Science*, 26(4), 346–351. <https://doi.org/10.1177/0963721417700129>

Siegler, R. S., & Pyke, A. A. (2013). Developmental and individual differences in understanding of fractions. *Developmental Psychology*, 49(10), 1994–2004. <https://doi.org/10.1037/a0031200>

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, 62(4), 273–296. <https://doi.org/10.1016/j.cogpsych.2011.03.001>

Spitzer, M. W. H. (2021). Just do it! Study time increases mathematical achievement scores for grade 4-10 students in a large longitudinal cross-country study. *European Journal of Psychology of Education*. <https://doi.org/10.1007/s10212-021-00546-0>

Spitzer, M. W. H., Gutsfeld, R., Wirzberger, M., & Moeller, K. (2021). Evaluating students' engagement with an online learning environment during and after COVID-19 related school closures : A survival analysis approach. *Trends in Neuroscience and Education*, 25, 100168.

Spitzer, M. W. H., & Musslick, S. (2021). Academic performance of K-12 students in an online-learning environment for mathematics increased during the shutdown of schools in wake of the Covid-19. *PLoS ONE*, 16(8), 1–16. <https://doi.org/10.1371/journal.pone.0255629>

Tian, J., & Siegler, R. S. (2018). Which Type of Rational Numbers Should Students Learn First? *Educational Psychology Review*, 30(2), 351–372. [https://doi.org/10.1007/s10648-017-9417-](https://doi.org/10.1007/s10648-017-9417-3)

3

Tomasik, M. J., Helbling, L. A., & Moser, U. (2020). Educational gains of in-person vs. distance learning in primary and secondary schools: A natural experiment during the COVID-19 pandemic school closures in Switzerland. *International Journal of Psychology*, 56(4), 566–576. <https://doi.org/10.1002/ijop.12728>

Velde, M. van der, Sense, F., Spijkers, R., Meeter, M., & ... (2021). *Lockdown Learning: Changes in Online Study Activity and Performance of Dutch Secondary School Students during the COVID-19 Pandemic*. <https://psyarxiv.com/fr2v8/>

Vukovic, R. K., Fuchs, L. S., Geary, D. C., Jordan, N. C., Gersten, R., & Siegler, R. S. (2014). Sources of individual differences in children's understanding of fractions. *Child Development*, 85(4), 1461–1476. <https://doi.org/10.1111/cdev.12218>