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Perceived number is not abstract

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Abstract

To support the claim that the ANS represents rational numbers, C&B argue that number perception is abstract and characterized by a second-order character. However, converging evidence from visual illusions and psychophysics suggests that perceived number is not abstract, but rather, is perceptually interdependent with other magnitudes. Moreover, number, as a concept, is second-order, but number, as a percept, is not.

Perceived number is not abstract

The concept “seven” applies just as easily to seven *elephants*, as seven *mice*, as seven *apples*. Numbers, as concepts, are abstract entities. “Seven” can be used to describe sets that differ vastly in their perceptual makeup because the symbol “seven” is dissociated from the sensory input (Dehaene, 1992). In making the case for non-symbolic number as a genuine dimension, C&B argue that the ANS, a perceptual system, is similarly abstract. Thus, whether elephants, mice, or apples, the ANS represents number, irrespective of their physical differences. In support of this perspective, C&B argue that (1) number is independent of other magnitudes; and (2) number, unlike other magnitudes, exhibits a ‘second-order’ property. Together, these arguments form the prerequisites for rational number.

Here, we argue that number is not (perceptually) independent of other magnitudes, nor is it unique by comparison. Moreover, we suggest that the appeal to a second-order character for number fundamentally confuses the distinction between percept and concept, such that number, as a percept (and like other magnitudes) is not second-order.

C&B argue that congruency effects do not dispute number as an abstract entity because the interactions between number and other magnitudes reflect post-perceptual processes such as a response-stage conflict. However, other evidence, not cited by C&B, provides more definitive evidence for the non-independence of number from other magnitudes. For example, by making clever use of Müller-Lyer (Dormal et al., 2018) and Ebbinghaus (Picon et al., 2019) illusions, researchers have demonstrated that number perception is influenced by illusory changes in non-numerical magnitudes such as length and density. Importantly, such effects were observed with estimation tasks in which participants estimated number, ruling out a response-stage conflict typical of magnitude comparison tasks, in which the response applies to both task-relevant and task-irrelevant magnitudes.

Our own research suggests that number and area are perceived holistically as integral dimensions (Aulet & Lourenco, 2021a). In Aulet and Lourenco, we found that perceived similarity for dot arrays, which varied parametrically in number and cumulative area, was best modeled by Euclidean, as opposed to city-block, distance within the stimulus space (Garner, 1974; Shepard, 1964), comparable to classically integral dimensions (e.g., brightness and saturation) but different from separable dimensions (e.g., shape and color). Importantly, results replicated across tasks and could not be explained by effects of confounding magnitudes or non-magnitude image similarity. In other words, perceived number may not be fully abstracted from co-occurring area but, instead, appears to be perceptually interdependent with it.

Relatedly, C&B claim that number is unique compared to other magnitudes—in terms of ratio and the second-order character. Others have similarly argued that number is uniquely salient (Cicchini et al., 2016; Ferrigno et al., 2017). Recent evidence from our lab, however, goes against the uniqueness claim. For example, we found that when perceptual discriminability between number and cumulative area was matched, area biased children’s number judgments more than the reverse (Aulet & Lourenco, 2021b) and children sorted visual stimuli according to area, not number (Aulet & Lourenco, 2021c), suggesting greater intrinsic salience for non-numerical magnitude, and consistent with others who have argued against the uniqueness of number (Leibovich et al., 2017; Newcombe, Levine, & Mix, 2015). Similarly, Testolin et al.

(2020) found that the internal encoding of ‘mature’ computational networks, trained to discriminate stimuli according to number, treated total perimeter and convex hull as comparable to number. These effects were even more striking in ‘young’ networks where the internal encoding was primarily driven by convex hull, not number.

C&B also posit that number, unlike other magnitudes, has a second-order character, such that the estimation of number requires stipulating what is being enumerated. For example, among a collection of shoes, number could apply to individual shoes or, alternatively, pairs of shoes. According to C&B, the representation of number is not set unless a relevant unit is specified (Burge, 2010; Frege, 1884). That is, a group of objects has no inherent number absent this stipulation. For the shoe example, the numerical value is n if considering individual shoes, but $n/2$ if considering pairs of shoes.

We agree that, when reasoning in this way, number exhibits a second-order character. C&B, however, apply this logic to the *perception* of number, which we would argue conflates the percept with the concept (Halberda, 2019). They describe ‘dumbbell’ studies in which participants underestimate individual dots that are connected by lines to form dumbbells (e.g., Franconeri et al., 2009). According to C&B, this effect suggests a second-order character for perceived number because participants’ number perception changes in the absence of changes to non-numerical properties (besides connectedness). However, if number perception was genuinely second-order, then it should be just as easy to continue perceiving the number of dots, instead of being biased towards the number of dumbbells. But this is not the case! Number percepts are not as flexible as number concepts. Number perception is constrained by physical (e.g., spatial individuation) and Gestalt principles (e.g., common motion; Wynn et al., 2002). Similar constraints (e.g., color grouping) apply to the ‘ratio’ experiments with infants (e.g., McCrink & Wynn, 2007) described by C&B. Moreover, number is perceived in accordance with these principles when arrays are passively, or even unconsciously, viewed (DeWind et al., 2019; Lucero et al., 2020; Fornaciai & Park, 2021; Van Rinsveld et al., 2020). Accordingly, we suggest that number perception, like the perception of other magnitudes, is a first-order property. We can *conceive* and count individual shoes, or the pairs of shoes they make up, but we *perceive* individual shoes. We can *conceive* and count dumbbells, or their individual component dots, but we *perceive* dumbbells.

In summary, although we agree with C&B’s description of the ANS as a perceptual system, we would argue that perceived number is not abstract, as it is to a conceptual system with access to symbolic representations such as number words. We have argued that perceived number may not be independent of other magnitudes and it does not appear to exhibit a unique status, including second-order character—calling into question the existence of an ANS that represents rational number.

Conflict of Interest

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References

- Aulet, L. S., & Lourenco, S. F. (2021a). Number and cumulative surface area are perceived holistically as integral dimensions. *Journal of Experimental Psychology: General*, *150*, 145-156.
- Aulet, L. S., & Lourenco, S. F. (2021b). The relative salience of numerical and non-numerical dimensions shifts over development: A re-analysis of Tomlinson, DeWind, and Brannon (2020). *Cognition*, *210*, 104610.
- Aulet, L. S., & Lourenco, S. F. (2021c). No intrinsic number bias: evaluating the role of perceptual discriminability in magnitude categorization. *PsyArXiv*. <https://doi.org/10.31234/osf.io/eh5pb>
- Burge, T. (2010). *The Origins of Objectivity*. Oxford: Oxford University Press.
- Cicchini, G. M., Anobile, G., & Burr, D. C. (2016). Spontaneous perception of numerosity in humans. *Nature Communications*, *7*, 1-7.
- Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*, 1-42.
- DeWind, N. K., Park, J., Woldorff, M. G., & Brannon, E. M. (2019). Numerical encoding in early visual cortex. *Cortex*, *114*, 76-89.
- Dormal, V., Larigaldie, N., Lefèvre, N., Pesenti, M., & Andres, M. (2018). Effect of perceived length on numerosity estimation: Evidence from the Müller-Lyer illusion. *Quarterly Journal of Experimental Psychology*, *71*, 2142-2151.
- Ferrigno, S., Jara-Ettinger, J., Piantadosi, S. T., & Cantlon, J. F. (2017). Universal and uniquely human factors in spontaneous number perception. *Nature Communications*, *8*, 1-10.
- Fornaciai, M., & Park, J. (2021). Disentangling feedforward versus feedback processing in numerosity representation. *Cortex*, *135*, 255-267.
- Franconeri, S. L., Bemis, D. K., & Alvarez, G. A. (2009). Number estimation relies on a set of segmented objects. *Cognition*, *113*, 1-13.
- Frege, G. (1884). *Die Grundlagen der Arithmetik: eine logisch mathematische Untersuchung über den Begriff der Zahl*, Breslau: W. Koebner; translated as *The Foundations of Arithmetic: A logico-mathematical enquiry into the concept of number*, by J.L. Austin, Oxford: Blackwell, second revised edition, 1974.
- Garner, W. R. (1974). The Stimulus in Information Processing. In H. R. Moskowitz, B. Scharf, & J. C. Stevens (Eds.), *Sensation and Measurement: Papers in Honor of S. S. Stevens* (pp. 77-90). Dordrecht: Springer Netherlands.
- Halberda, J. (2019). Perceptual input is not conceptual content. *Trends in Cognitive Sciences*, *23*, 636-638.
- Leibovich, T., Katzin, N., Harel, M., & Henik, A. (2017). From “sense of number” to “sense of magnitude”: The role of continuous magnitudes in numerical cognition. *Behavioral and Brain Sciences*, *40*.
- Lucero, C., Brookshire, G., Sava-Segal, C., Bottini, R., Goldin-Meadow, S., Vogel, E. K., & Casasanto, D. (2020). Unconscious number discrimination in the human visual system. *Cerebral Cortex*, *30*, 5821-5829.
- McCrink, K., & Wynn, K. (2007). Ratio abstraction by 6-month-old infants. *Psychological Science*, *18*, 740-745.
- Newcombe, N. S., Levine, S. C., & Mix, K. S. (2015). Thinking about quantity: The intertwined development of spatial and numerical cognition. *Wiley Interdisciplinary Reviews: Cognitive Science*, *6*, 491-505.
- Picon, E., Dramkin, D., & Odic, D. (2019). Visual illusions help reveal the primitives of number perception. *Journal of Experimental Psychology: General*, *148*, 1675-1687.
- Shepard, R. N. (1964). Attention and the metric structure of the stimulus space. *Journal of Mathematical Psychology*, *1*, 54-87.
- Testolin, A., Dolfi, S., Rochus, M., & Zorzi, M. (2020). Visual sense of number vs. sense of magnitude in humans and machines. *Scientific Reports*, *10*, 1-13.

- Van Rinsveld, A., Guillaume, M., Kohler, P. J., Schiltz, C., Gevers, W., & Content, A. (2020). The neural signature of numerosity by separating numerical and continuous magnitude extraction in visual cortex with frequency-tagged EEG. *Proceedings of the National Academy of Sciences*, 117, 5726-5732.
- Wynn, K., Bloom, P., & Chiang, W. C. (2002). Enumeration of collective entities by 5-month-old infants. *Cognition*, 83, B55-B62.