Naturalistic decision-making: continuous, open-world, and recursive

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ABSTRACT

Real-world decisions take place over time, in dynamics contexts, and with frequently shifting choice sets. In contrast, most studies of decision making have focused on single, abstracted choices among small numbers of discrete options. Here, we advocate for a shift in emphasis to what we call *continuous decisions*, which better capture the complexity of decision-making in the wild. Continuous decisions involve a continuum of possible responses and take place over an extended period of time during which the response is continuously subject to modification. The range of options available at any time can fluctuate and is affected by recent responses, making consideration of feedback between choices and the environment essential. The study of continuous decisions emphasizes distinct questions not easily captured by discrete decisions, including questions about how the brain integrates choices with movement, bridges representations through time, and maintains hierarchically organized information. While microeconomic theory has proven invaluable for discrete decisions, we propose that engineering control theory may serve as a better foundation for continuous ones. And while the concept of value has proven foundational for discrete decisions, goal states and policies may prove more useful for continuous ones.

MAIN TEXT

Introduction

Decisions as studied in the lab are typically *discrete*. Consider, for example, a paradigmatic example, the Allais Paradox (Allais, 1953, **Figure 1**). It is one of the most influential artificial choices of the 20th century, and one that, probably more than any other, inaugurated behavioral economics. The Allais Paradox involves a single choice between a pair of categorically distinct, fully-specified options. It is the prototype of the vast majority of decisions studied in microeconomics, behavioral economics, and neuroeconomics (Kahneman and Tversky, 1979, Glimcher and Fehr, 2013). The decision-maker has, in principle, as long as they want to decide. The result of the choice does not affect what options are made available in the future. The choice itself takes place either instantaneously or irrespective of time, and, once made, it is irrevocable. The decision-maker can express their preference verbally, with a button press, or with any other response modality. The decision-maker's choice is free of context; that is, it is *disembodied* (Lepora and Pezzulo, 2015; Pezzulo and Cisek, 2016).

The Allais paradox is the epitome of what one might call a *discrete decision*. Many of the decisions we make in our daily lives approximate such discrete decisions (Platt and Huettel, 2008). For example, selecting an item from a menu in a restaurant and selecting a life insurance plan can be conveniently thought of as discrete decisions. However, many other decisions we face in our daily lives have different characteristics. Even quotidian decisions may involve a continuum of options or a changing option set. They can take

place over time and are enacted through complex movements (i.e. they are embodied). They may take place with varying levels of conscious awareness and involve feedback between the decider and the external world. Here we use the term *continuous decisions* to describe them.

The major goal of the present article is to introduce and motivate the study of continuous decisions. As we will argue, such decisions capture aspects of real behavior often neglected in the fields that comprise decision science, but they offer potentially unique insights into natural behavior. For concreteness, consider - in contrast to the Allai paradox - a canonical continuous decision: the pursuit behavior of the window-skimmer dragonfly (*Libellula luctuosa*). As it pursues its prey (a mosquito), the dragonfly must continuously monitor the mosquito's position while both move rapidly through three-dimensional space (Gonzalo-Bellido et al., 2016; Gonzalo-Bellido, 2020). Here, there is no longer any obvious discrete decision among fixed options: rather, the dragonfly must carefully regulate its wings to stabilize its body, track and extrapolate the future position of the prey, and plot an intercept course. Actions at each moment come from a large continuum of possibilities, each flowing into the next. Moreover, the mosquito will change its behavior based on what the dragonfly does, and the dragonfly can exploit its knowledge of that fact to improve its foraging.

While such complex behavior may be conceptualized or modeled mathematically as a series of discrete economic choices, one at each moment, we argue that something in this formulation fails to do the dragonfly justice. Because while the overall behavior apparently involves a deliberate selection of flight plan and final strike time, as well as a

balancing of costs (time, energy, risk) and benefits (caloric intake), it seems clearer to view the notion of decision as encompassing the entire process, from the continuum of motor adjustments to the selection of flight control strategies. From this perspective, the choice first posed by Allais appears as a degenerate example, one in which the various aspects are discretized, disembodied, and collapsed into a single moment. Even though it is a special case, economics, as studied in psychology and neuroscience, has focused almost exclusively on the discrete case and ignored the much larger world of continuous decisions.

But considering decisions more broadly is valuable. From our perspective, the greatest of these is the benefits to neuroscience that come from studying the types of decisions our brains were evolved to handle (Brown and De Bivort, 2018; Calhoun and Hayden, 2015; Kacelnik et al., 2011; Krakauer et al., 2017; Pearson et al., 2014). Indeed, differences in sensory coding between artificial and natural stimuli are often stark (Brenner et al., 2000; Vinje and Gallant, 2000; David et al., 2004), raising the possibility that conclusions about choice coding derived from highly circumscribed and overtrained paradigms may be less general than hoped. In addition, continuous decisions require careful consideration of the sensorimotor context in which they are embedded. Rather than attempt to divorce decision as cleanly as possible from perception and action, studying continuous decisions requires us to confront the widespread observation that neural representations often intermingle information about decisions, actions, and outcomes, even in putative “decision” areas (Fusi et al., 2013; Yuste, 2015; Fusi et al., 2016). Finally, natural decisions focus on dynamics. Here again, while systems neuroscience has gained

tremendous insights from the study of steady-state coding and ballistic movement, natural actions are primarily characterized by transition and the absence of equilibrium. Indeed, several of the most exciting recent ideas in decision-making have looked to decision dynamics as key (e.g. Mante et al., 2013; Resulaj et al., 2009; Stoke et al., 2013; Kobak et al., 2016).

Figure 1. The Allais paradox (left) is a canonical example of what we call a discrete decision. It involves a single choice between two options; the choice is independent of action, context, and time. It has no influence on what options are available in the future. A dog moving along a bank preparing to jump and grab a fish (right) is a canonical example of a continuous decision.

Foraging decisions are continuous decisions

The study of naturalistic decisions is the central concern of foraging theory, an important branch of behavioral ecology (Stephens and Krebs, 1986; Stephens et al., 2008). The centrality of continuousness in foraging theory is demonstrated by its two foundational problems - *patch-leaving* and *diet selection* (Stephens and Krebs, 1986). In patch-leaving, the decision-maker chooses between leaving and staying; in diet selection, between accepting and rejecting an encountered prey item. And while both might appear at first

glance to be discrete decisions, they are best understood as part of the framework of continuous decisions as described above. Patch-leaving, for instance, is most properly understood not as a binary stay or go decision, but as a *when* to go decision, the selection of a moment to leave the current food patch in search of better options. Likewise, prey selection is a binary decision — pursue this item or not? — taken against the backdrop of *asynchronous* prey encounters. That is, both require a fuller picture of dynamic context and feedback than that typically used in laboratory studies of discrete decisions. (Stephens and Anderson, 2001; Blanchard et al., 2013; Pearson et al., 2010).

Similar arguments can be advanced for other, less well-known types of foraging decisions. For instance, the tradeoffs involved in behavioral scheduling — when to eat, when to sleep, when to mate — involve more than isolated value comparisons. Since each of these behaviors is necessary but they are (usually) mutually exclusive, current opportunities, physiological state, and recent history must therefore alter their respective values to produce the observed behavioral dynamics. Thus, a “meta-task” perspective, which fits naturally into the hierarchical framework for continuous decisions we describe below, is both natural and necessary for real-world behavior.

The same holds true even more strongly for social decisions, where the presence of other agents renders the choice space both larger and dynamic (Giraleau and Caraco, 2018). Indeed, behaviors such as swarming, flocking, and pack hunting integrate both movements and decisions across multiple agents, producing a synthesis that seems impossible to model with discrete decisions (Couzin and Krause, 2003, Camazine et al., 2003; Viswanathan et al., 2011). And while traditional game theory has worked well to

161 identify situations like sparse aggressive interactions that can be modeled as one-shot,
162 cases in which equilibrium behavior is informative, these methods do not naturally extend
163 to the short time-scale dynamics of social interactions. As we will argue below, extensions
164 like differential games can help address this concern, but these are best situated within the
165 broader perspective we have outlined here.

166 But perhaps the strongest argument for foraging decisions as continuous decisions
167 comes from psychophysics, where small changes in temporal contingency can result in
168 remarkable differences in task performance. For example, foragers that show steep - and
169 highly suboptimal - temporal discounting in standard laboratory setups tend to show much
170 better behavior in the foraging contexts (Hayden, 2016; Blanchard and Hayden, 2015
171 Cash-Padgett and Hayden, 2020), even when the delay structures of the task are highly
172 similar. One theory holds that the foraging context motivates the contemplation of the
173 problem as a continuous problem and, because this form of problem resembles the evolved
174 problem, it elicits more optimal behavior (Stephens and Anderson, 2001). Along similar
175 lines, it has been conjectured that this difference is related to learning, with the temporal
176 structure of foraging much easier to learn than the post-reward delays typically imposed in
177 intertemporal choice tasks (Pearson et al., 2010; Blanchard et al., 2013). This again argues
178 for the importance of considering continuous decisions as a framework better aligned with
179 the behaviors that brains evolved to organize.

180

Figure 2. A. A traditional discrete choice task used to study time preferences is the intertemporal choice task (reviewed in Hayden, 2016). B. A patch-leaving task has an ostensibly isomorphic structure, except that the specific options available on the next trial are determined entirely by the choices made on the present trial (Stephens and Anderson, 2001). This feature has major effects on choice patterns and, although the element of cross-trial dependency is ostensibly more difficult, renders behavior nearly optimal (Blanchard and Hayden, 2015). C. The patch-leaving task, but not the intertemporal choice task, is ethologically valid. Here, monkeys at the Cayo Santiago field station travel between their morning foraging site on Large Cay to their afternoon foraging site on Small Cay.

Continuous decisions are open-world, and the world is large

Above, we have argued that foraging decisions, even when reduced to laboratory tasks, point in the direction of key aspects of real-world decisions that better align with natural behavior. Here, we will consider a related aspect of continuous decisions that sets them apart from discrete decisions: what might be called their “open world”-ness.

We take the term “open world” from the domain of video games. Open world games like Grand Theft Auto and Minecraft present players with virtually limitless territory to explore, and terrain filled with a multitude of possible actions. Like the

situations in which many real world decisions occur, they contain more unknowns than knowns, which reward curiosity and search, and a large number of other agents with their own goals. Open world environments pose a significant computational challenge, elided in nearly all laboratory decisions, but central to real decisions, of narrowing down a plenitude of possible actions to a small set of “live” options. To meet these challenges, decision-makers must employ cognitive abilities that make these computationally intensive problems tractable, abilities like mental maps, curiosity, and heuristics.

To sharpen this theoretically, we adopt the language of reinforcement learning (RL), which offers several advantages (Sutton and Barto, 2018; Dayan and Abbott, 2001): First, RL does not need to either assume a model of the world, a fixed set of choices, or even a static set of values. A learner that follows RL can learn online (it does not need to be pre-trained). Second, while RL has traditionally been most effective in the tabular setting — the case in which options and contexts are discrete and finite, so that the value of each action can be stored in a lookup table — its formalism is generic enough to extend to the continuous decisions we envision here. Finally, recent progress in deep reinforcement learning has opened a new dialogue between machine learning and cognitive neuroscience, producing artificial agents capable of solving challenging continuous tasks like video games while underperforming humans in others like imitation learning (Mnih et al., 2013; Vinyals et al., 2019; Finn et al., 2017; Duan et al., 2017; Ho and Ermon, 2016).

In this language, the first distinction we want to draw between discrete and continuous tasks is in the dimension of what is called the state space. For a given problem, RL always assumes a *state*, the collection of variables needed to fully define the current

context (Sutton and Barto, 2018; Wikenheiser and Schoenbaum, 2016; Schuck et al., 2016; Wilson et al., 2014). For discrete choices, the state space (the set of all possible states) usually consists of a small number of variables, perhaps attribute information for each of the choice options available and some information about current preferences. For the dragonfly pursuit described above, the current state might include position and velocity information for all joints, estimated position and velocity of both the dragonfly and its prey, and the relative locations of relevant obstacles. In addition, the state might also include internal variables of the organisms that might be relevant to predicting behavior (e.g. hunger, fatigue, etc). Consequently, the state space in open-world decisions tends to be enormous. In part, the proliferation of these states is related to the mathematical requirements of RL, which assume that the state contains all information on which behavior at the current time depends. But this explosion of states is also inherent in the complexity of the behavior itself and the granularity at which it is modeled.

Moreover, it is not just the number of state variables that are increased in continuous decisions, but their ranges. In discrete RL settings, for S state variables with N values each, the size of the state space is N^S . That is, for a decision problem with five yes/no contextual variables (Am I hungry? For sweet or salty snacks? Is there food close by?), there are a total of $2^5=32$ possible states. . In the case of continuous states, where N becomes infinite, state spaces become so large that tabular approaches must fail. Moreover, in many cases of interest, states are only partially observed, requiring some form of state inference. For example, in games like poker, opponents' cards are key missing pieces of game state information. Finally, in continuous decisions, where possible actions include

246 continuous variables like forces and joint angles, the resulting space is also infinite
247 dimensional.

248 Fortunately, while practical and algorithmic challenges remain in these cases, they
249 are addressed by existing concepts like models, maps, trajectories, and policies that bridge
250 the discrete and continuous cases. The common insight underlying these extensions of
251 tabular RL is that, when states or actions are continuous, we can often make use of a
252 concept of distance, such that “nearby” states are similar, and quantities like value can be
253 assumed to change smoothly. That is, continuous spaces can often, paradoxically, have
254 *fewer* variables than discrete ones. For instance, if one is given a table of 100 numbers, this
255 data structure has, apparently, 100 free parameters. But if these numbers are samples from
256 a quadratic function at regularly spaced intervals along the x axis, they are encapsulated by
257 only three parameters. These sorts of geometric and smoothness assumptions often allow
258 one to replace tables with functions and an infinite number of individualized values with a
259 finite number of function parameters, rendering learning feasible once again.

260 Along related lines, much recent work on reinforcement learning has focused on
261 the existence and utility of mental models or maps (Schuck et al., 2016; Behrens et al.,
262 2018; Niv, 2019; Momennejad et al., 2019; Stachenfeld et al., 2017). What is important in
263 these cases is that not that such maps are necessary for learning — RL agents who lack
264 them can still eventually learn — but that these maps capture either regularities in state
265 sequences (like random walks on an unseen graph) or structure within the states (like
266 proportions of bird beaks and legs) that facilitate faster learning. In the language of
267 learning theory, they constitute *inductive biases* that allow learners to make the most of

each observation. Similarly, in a simulated pursuit task, we found evidence for such maps in the firing rates of single neurons in the dorsal anterior cingulate cortex (dACC). Specifically, dACC neurons have complex, non-regular multimodal firing rate place fields (Yoo et al., 2020), reminiscent of non-grid cells in the medial entorhinal cortex (Hardcastle et al., 2017). More importantly, such inductive biases, by making strong assumptions about the possible types of knowledge to be acquired, combat the curse of dimensionality. In the studies above, though numbers of states were large, their dimensionalities remained small. When dimensionality likewise increases, learning becomes an intractable search problem without some form of organizing assumption.

By the same logic, open-world decisions necessitate exploration, since large state spaces cannot be exhaustively visited. This means that continuous decision-makers are almost always highly information-starved. In typical studies of discrete decisions, it is assumed that decision-makers have access to all relevant information and that they have no cost to process it. When information is lacking, only one form of information is excluded, and its absence is typically made the focus of study. So, while discrete decisions can also be information-starved, that is not the standard case. Thus, the demand for information is typically a major influence on continuous decisions, whereas it is only a major influence on discrete ones when they are designed that way. This is likely one reason why humans and other foragers appear to have deeply evolved drives towards curiosity - willingness to pay for even ostensibly non-strategic information (Kidd and Hayden, 2016; Gottlieb et al., 2013; Wang and Hayden, 2019). They are trying to fill in the large gaps in their maps of the world as much as possible. Indeed, much recent work in RL is focused on efficient

methods of exploration in the absence of rewards — the search for an algorithmic curiosity. And such a focus is relevant in worlds like ours where decision-making is more often a problem of search for acceptable options than of choice between competing ones.

Finally, it is important to note that the goal of action is not necessarily to find the optimal course of action, but instead to find a course of action that is sufficiently good (Griffiths et al., 2015; Schwartz et al., 2002; Gigerenzer and Gaissmaier, 2011). Such approaches are valuable even in simple non-continuous decisions (e.g. Venkatraman et al., 2009; Heilbronner and Hayden, 2016). The additional complexity of natural decisions means that satisficing approaches, such as heuristics, are likely to be used in real-world decision-making. In continuous decisions, where the costs of computation are much greater and the time pressures more unrelenting, heuristics are likely to be even more important. Understanding the use of heuristics in continuous decisions, then, represents an important future direction for study.

Figure 3. Task and data from Yoo et al., 2020. **(A)** (Left) Cartoon of *virtual pursuit task*. Subject uses a joystick to control an avatar (circle) and pursue prey (square) on a computer screen. (Right) Avatar and prey trajectories on example trials. Grey: path of avatar; red/blue: path of prey. Color gradient indicates the time progression through the trial. **(B)** (Left) The generative model to explain the trajectory of nonhuman primates. (Right) The model result exhibiting that nonhuman primates are predicting the future position of the prey. **(C)** Example filters from neurons that are significantly tuned for the position of the prey within the monitor.

Continuous decisions involve dynamics and feedback

Just as large numbers and dimensions of states advantages agents who are able to make use of abstractions like maps and world models, so tasks that involve actions throughout time give rise to considerations of dynamics. Indeed, the importance of both dynamics (sometimes high-dimensional, often nonlinear) and feedback (from both other agents and the environment) is the most obvious feature setting off what we have called continuous decisions from their more conventional discrete counterparts. The problems imposed by dynamics encompass questions of control — how to move an arm to snatch a passing prey — and questions of timing — precisely *when* to make this move, adding

additional complexities often excluded in laboratory decision paradigms. Once again, we will argue that reinforcement learning, particularly its intersection with control theory, offers a conceptual framework for addressing these challenges, even in cases where learning is not the primary dynamic.

Before stepping further, it is helpful to draw the distinction between modeling a set of actions as sequential decisions and the cognitive experience of sequential decision making. For instance, while it is intuitive that the iterated Prisoner's Dilemma involves a sequence of cooperation and defection decisions by each player, it is less intuitive that a short walk down a straight hallway also comprises a series of decisions, one for each step. Yet both can be modeled mathematically as probabilistic sequences of "decisions," perhaps, in the case of walking, with rewards for staying upright and forward progress. Yet our subjective experience is one of long periods of automatic behavior punctuated by changes of mind, phenomena that are only imperfectly captured by the framework of sequential decisions. Instead, we might prefer to mirror mathematically the dissociation between rare goal changes and continuous execution of unconscious behaviors in time.

The first ingredient in such a recipe comes from control theory, which has a long and fruitful history in motor neuroscience (e.g. Todorov and Jordan, 2002; Wolpert et al., 2011; Gallivan et al., 2018; Diedreixsen et al., 2010). In the control framework, a system is endowed with a set of possible controls to apply (forces, torques, electronic commands), a system to which these controls may be applied (typically modeled as a set of differential equations), a goal or set point, and a cost for applying control (energy, price, regret). The aim of control theory is to solve for a *policy*, an algorithm that sets control as a function of

the system's state at a given time. The optimal policy is one that best balances control error (how close the system remains to its goal) against control costs. Such a framework automatically addresses two key problems identified above: First, control systems operate in time. Control policies are *functions*, not discrete actions, and specify how the controller co-evolves with the system. They are thus fundamentally dynamic quantities. Second, control policies do not require the semantics of decisions at all. They are conceptualized as automatic processes that operate adaptively, often deterministically, once a goal has been set. Thus, they have intuitive appeal for a framework that combines sparse, conscious decisions with automatic ongoing processes.

Again, this is complementary to the reinforcement learning perspective reviewed above (RL; Sutton and Barto, 2018), which likewise attempts to learn a policy that maximizes some set of rewards. In fact, many classic RL problems like cart-pole or learning to walk are control problems (Todorov and Jordan, 2002; Todorov, 2004; Todorov, 2009). Such links have often been overlooked in decision neuroscience, where RL actions are most frequently associated with decisions (Daw and Tobler, 2014), but there is nothing in the formalism to force this view. Before the advent of deep reinforcement learning, RL methods were often limited to low-dimensional, discrete state and action spaces, but modern applications now routinely feature complex, continuous policies parameterized by deep neural networks (Mnih et al., 2015; Van Hasselt et al., 2016;). Thus, RL likewise offers up the notion of a full policy in place of a discrete action as a building block of continuous decisions.

Just as importantly, while control systems are often assumed to be autonomous, with fixed, pre-specified set points, RL can be extended to the case of *hierarchical* learning and control (Doya et al., 2002; Precup & Sutton, 2000; Pearson & Platt, 2013; Merel et al., 2019; Botvinick et al., 2009). That is, the RL framework can easily accommodate the idea of *switching between policies* or changing goals within the same policy (McDonald et al., 2019; Iqbal et al., 2019). These high-level changes are most often slower (in the case of goals) or sparser (for policy switches) and map neatly onto the experience of rarer deliberative decisions setting in motion automatic behaviors. Moreover, hierarchical formulations of RL allow changes in state to drive high-level actions, facilitating “bottom-up” feedback that renders policies more flexible than purely “fire and forget” processes. And while training such hierarchical RL systems has continued to prove challenging (Precup and Sutton, 1998; Nachum et al., 2019), there is ample evidence that brains have evolved by solving exactly this problem (Niv, 2019).

Two recent paradigms from our groups illustrate precisely this sort of integration of continuous control with sparse strategy changes. In the first, monkeys and humans played a competitive video game against conspecifics that required continuous joystick input (McDonald et al., 2019; Iqbal et al., 2019, **Figure 4**). Based on the idea of a penalty shot in hockey, one player (the shooter) controlled the motion of a small circle (the puck), while the other (the goalie) controlled the vertical motion of a bar placed at the opposite side of the screen. Goalies were rewarded for blocking the puck with the bar, while shooters received rewards for reaching a goal line behind the bar. Though the game was highly

constrained, players' trajectories proved highly variable, revealing a variety of both individual strategies and inter-player dynamics.

These phenomena can be modeled as the result of a control model applied to an evolving value function incorporating each player's trajectory (Iqbal et al., 2019; McDonald et al., 2020), while directly modeling players' policies allows for the calculation of a measure of instantaneous coupling between the players (McDonald et al., 2019). However, in both human and monkey participants, it was found that the key strategic variable was not so much shooters' ability to adjust to their opponents as their ability to advantageously time their final movements on each trial (McDonald et al., 2019;). Such paradigms as this bear some relation to continuous games such as duels and are closely related to pursuit games in differential game theory, but in the latter, focus is typically on computing optimal solutions rather than modeling players' real behavior (Basar and Olsder, 1999; Braun et al., 2009).

Figure 4. Task from Iqbal et al., 2019. **(A)** Pairs of monkeys competed in a zero-sum, real-time game with continuous joystick inputs. One player controlled the position of a blue ball, the other the vertical position of a red bar. The goal of the blue player was to guide the ball past the dotted line at the right of the screen, while the red player attempted to block the ball with the bar. **(B)** Example of a single pair of player trajectories on a single trial. For display purposes, the trajectory of the bar has been extended in time along the x axis, though the bar could only move vertically. **(C)** Continuous control resulted in highly varied trajectories, but these could be captured by a model of players' coevolving goal states that generated realistic trials **(D)**. **(E)** In a version of the same task in human subjects, players' trajectories were characterized by sparse changes in trajectory (black dots), and these changepoints could be predicted by modeling players' policies as a function of continuous game state **(F)**. Black patches represent regions of increased probability of direction change. Blue traces indicate trials played against a human bar

opponent, green traces those played against a computer. Adapted from Iqbal, Yin et al., 2019; McDonald et al., 2019.

Implications for neuroscience: a preliminary sketch

As neuroscientists, our central concerns are on the neural mechanisms that drive choices. Our central argument here is for the importance of continuous decisions; our motivator is the overall lack of consideration for them in the field. As such, it may be too soon to draw any conclusions on the likely implications for neuroscience. We therefore choose to draw a narrow focus and consider one major question, that of modularity vs. distributedness.

Traditional “box-and-line” approaches to cognitive neuroscience presume the existence of discrete cognitive functions with intuitive roles. These functions, then, are assumed to be reified in neuroanatomy. For example, if choice consists of evaluation, comparison, and selection, then these three conceptually discrete functions ought to correspond to discrete anatomical substrates (Rangel et al., 2008; Padoa-Schioppa, 2011). An alternative viewpoint is distributed; it imagines that choice reflects an emergent process arising from multiple brain regions whose functions may not correspond to nameable processes, and/or that may largely overlap (Yoo et al., 2018; Hunt and Hayden, 2017; Cisek and Kalaska, 2010; Cisek, 2012).

The greatest debates about modularity occur in the prefrontal cortex, which can be readily divided into a few dozen areas based on cytoarchitecture; however, the functions of these areas remain difficult to definitively identify. In fact, this architectural diversity raises the question of why we have so many prefrontal regions to begin with. One possibility is that each region integrates new information from new sources and

participates in transforming that information via an untangling process (Yoo and Hayden, 2018). In this view, each area in sequence provides a partial transformation from one representation format to another (Azab and Hayden, 2017 and 2018). Indeed, it is this view, and not that of a modular system, that best accords with recent results in artificial neural networks (DiCarlo et al., 2012). Of course, some cognitive functions are localized at the coarsest levels. Our claim is simply that, once one embraces the framework of continuous decisions, with its tight coupling of action and decision, dynamics and feedback, the tight conceptual link between modularity in space and serial processing in time is no longer so natural as it has traditionally appeared.

The modular assumption becomes less tenable when moving from discrete to continuous decisions (Cisek, 2012). Consider choosing between two options. In this stereotypical situation, the motor action may not begin until comparison and choice of option are complete, and the action does not influence the decision. The serial model then neglects the physicality of the decision-making agent so action is a slave of choice. There are studies in which action cost is considered (refs; Hosokawa), but these are the rare exceptions. In continuous decisions, action is front and center; it is difficult to even think about how choice may occur without consideration of action. That includes not just action costs, but also forward models of the consequences of possible strategies. In other words, the obligatory embodiedness of continuous decisions means that action selection cannot be separated from choice.

In practice, subjects appear to initiate their action before they have fully decided on an option. That is, they hedge their bets and choose an intermediate path between the two

options (Haith et al., 2016; Christopoulos and Schrater, 2015). Then, as their decision process unfolds in their heads, their arm changes its pattern and begins to divert its path towards the chosen option. Critically, the initial hedging path lasts longer if the choice takes more time to unfold. In other words, the beginning of selection takes place before the choice is complete. Taking this idea literally, the choice process occurs simultaneously with the motor action. They are not discrete and the action would provide a feedback signal to the choice, which is opposed to the idea that movement does not influence the choice at all. Ultimately, then, resolution of the question of modularity requires testing under more naturalistic contexts. That is, natural tasks provide a valuable “stress test” for modular models.

Conclusion

Here, we have argued in favor of a decision-making framework that generalizes beyond discrete decisions among small, fixed option sets to continuous, open-world, dynamic processes that blend value and decision with control and movement. Of course, the greatest obstacle to the adoption of this framework, pieces of which go back decades, is the additional complexity involved in such unconstrained behaviors. As a result, we propose that the development of new experimental analysis techniques ought to be a major focus moving forward. For example, most traditional analyses in systems neuroscience have relied on averages, both in time and across trials, to reduce the complexity in data and achieve statistical power. But when trial-to-trial behavioral variability is large, these repetitions may no longer be comparable to one another. As a result, single trial analyses will be crucial (e.g. Sauerbrei et al., 2020; Pandarinath et al., 2018, also Iqbal et al., 2019;

McDonald et al., 2019, 2020). And these methods often benefit from denser recording techniques such as calcium imaging (Stringer et al., 2019;) and multi-contact multi-electrode recording (Gallego et al., 2017, 2018; Golub et al., 2018).

Along a different axis, there is much promise in recent studies that eschew single brain areas in favor of integrative analyses of simple, natural behaviors. Recent technical advances are making such studies possible, and this trend is likely to continue. A final example comes from problems introduced by distributed decision and control models - we need theoretical approaches that explain how multiple brain regions interact constructively without interfering (Hunt and Hayden, 2017; Yoo and Hayden 19; Eisenreich et al., 2017).

But we reiterate: to the degree that some complexity in behavior is irreducible, the study of continuous decisions is both necessary and inevitable. Not only are these decisions closer to natural behavior (and so easier to train in model species), they begin to tap the complexity we want to study, offering new puzzles that force us to integrate our knowledge across cognitive levels, brain regions, and time scales. If the behaviors we are interested in--our own included--take place in a changing, dynamic world, then our models of that world must grow to embrace these essential features.

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