Regularized Structural Equation Modeling with Stability Selection

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Abstract

Regularization methods such as the least absolute shrinkage and selection operator (LASSO) are commonly used in high dimensional data to achieve sparser solutions. They are also becoming increasingly popular in social and behavioral research. Recently methods such as regularized structural equation modeling (SEM) and penalized likelihood SEM have been proposed, trying to transfer the benefits of regularization to models with latent variables involved. However, some drawbacks of the LASSO such as high false positive rates (FPRs) and inconsistency in selection results persist at the same time. We propose the use of stability selection (Meinshausen & Bühlmann, 2010) as a mechanism to overcome these limitations, demonstrating simulation conditions in which it improves performance, and simulation conditions in which it does not. In this paper, we point out that there is no free lunch, and researchers should be aware of those problems when applying regularization to latent variable models, concluding with an empirical example and further discussion of the application of regularization to SEM.

Keywords: structural equation modeling, regularization, LASSO, stability selection, data mining, machine learning

Introduction

Stemming from statistical learning, regularized regression methods, such as the least absolute shrinkage and selection operator (LASSO; Tibshirani, 1996), have been shown to effectively produce sparse solutions by penalizing model complexity. Although regularization is a widely adopted method in regression and has been shown to possess many desirable properties, it wasn't until recently that regularization has been applied to the structural equation modeling (SEM) framework in regularized structural equation modeling (regularized SEM; Jacobucci, Grimm & McArdle, 2016) and the penalized likelihood (PL) method for structural equation models (Huang, Chen & Wen, 2017). Through penalizing specific parameters in an SEM, regularized SEM attempts to transfer the benefits of regularization to models with latent variables. The goals of this include creating simpler models, while still demonstrating good fit, or overcoming problems with maximum likelihood estimation (MLE), for instance, "Heywood cases" (Bentler & Chou, 1988), nonconvergence, underidentification, and many others. With a penalty term applied to different parameters, the regularization process could take many different forms, for example, selecting among multiple predictors of a latent variable (Jacobucci, Brandmaier & Kievit, 2019), simplifying factor structure by removing cross-loadings (Scharf & Nestler, 2019), and determining whether additional linear terms are necessary in longitudinal models, and many others.

The idea of applying regularization/penalization¹ to obtain sparser solutions has been applied broadly in psychometrics. For example, LASSO regularization has been applied in item response theory (IRT) to achieve latent variable selection (Sun, Chen & Liu, 2016). More recently, Bauer, Belzak, and Cole (2019) proposed a regularization approach to moderated nonlinear factor analysis estimation, penalizing the likelihood for differential item functioning (DIF) parameters, to simplify the assessment of measurement invariance. Liang & Jacobucci (2019) studied the

performance of several regularization methods for detecting measurement bias, with focus put on complex SEM with small sample sizes. In confirmatory factor analysis (CFA), Pan, Ip and Dubè (2017) proposed a post hoc model modification method using the Bayesian LASSO. In exploratory factor analysis (EFA), besides the traditional two-step approach of applying factor rotation to the least squares or maximum likelihood estimators to obtain sparser solutions, penalized least squares (Sitek, Gullberg & Huesman, 2002) and penalized maximum likelihood (Zou, Choi & Oehlert, 2010; Hirose & Yamamoto, 2013; Hirose & Yamamoto, 2015) techniques have been proposed. Furthermore, Scharf and Nestler (2019) showed that though both rotated and regularized EFA tended to underestimate cross-loadings and inflate factor correlations when the factor structures are complex, regularized EFA was able to recover loading patterns as long as some of the items followed a simple structure.

Current applications of regularized SEM, and regularization in psychometrics more broadly, typically utilize the LASSO penalty due to its ability to perform variable selection and its computational simplicity. However, two prominent problems with the LASSO have been noted in previous studies. The first problem is with the consistency of LASSO. A variable selection procedure is consistent if the probability of selecting the correct model tends to one as sample size goes to infinity. However, it has been shown that non-trivial conditions are needed for the LASSO to provide consistent selection results (Zhao & Yu, 2006; Zou, 2006; Meinshausen & Bühlmann, 2006; Yuan & Lin 2007). As a stricter property, a method is said to possess the oracle property if it is consistent in the variable selection procedure, and it estimates the non-zero parameters as efficiently as could be possible if we knew which variables were uninformative ahead of time. In short, an oracle estimator needs to be consistent in both variable selection and parameter estimation. Moreover, Fan and Li (2001) proved that the oracle

properties do not hold for LASSO by arguing that large coefficients are biased downward when estimated with LASSO, which could be suboptimal in terms of estimation rate. Besides, the LASSO depends upon a regularizing parameter λ , which controls the amount of shrinkage applied to the coefficients. This value must be chosen in some fashion, for instance, by cross-validation. However, it has been noted that the result could be sensitive to “fold assignment” (Bovelstad et al., 2007; Roberts & Nowak, 2014; Datta & Zou, 2019). On top of that, Meinshausen and Bühlmann (2006) showed the conflict of optimal prediction and consistent variable selection in the LASSO. When choosing the optimal tuning parameter λ , a criterion like prediction accuracy is needed, which, in general does not give consistent variable selection result (Leng, Lin, & Wahba, 2004).

An additional problem with the LASSO is the high false positive rates (FPRs), that is, null features, or, noise variables being selected. Meinshausen (2007) pointed out that many noise variables are selected by LASSO regression if the estimators are chosen by cross-validation. Moreover, false discoveries may occur very early on the LASSO path (Fan & Song, 2010), and true features and null features are always interspersed on the LASSO path, no matter how strong the effect sizes are (Su, Bogdan & Candes, 2017), leading to a practical tradeoff between FPRs and FNRs. This pattern is also observed in regularized SEM, as demonstrated in Figure 1. It is observed that although regularized SEM generally produces very low averaged false negative rates (FNRs) on small samples, the averaged FPRs is usually relatively high (e.g., Jacobucci, Brandmaier, & Kievit, 2019). That is, when the LASSO penalty is used, regularized SEM does not tend to over-penalize signals (true features), but it may not give enough penalization, so that a lot of noise variables (null features) are often included. Nevertheless, this high averaged FPRs stays even with large enough sample size. For instance, in Jacobucci, Brandmaier and Kievit (2019), an

average of over 35% of noise variables are selected to the final set by regularized SEM with small sample sizes, and the average FPRs stays over 30% even with sample size of 2000. This is in line with the performance of LASSO in regression (Su, Bogdan & Candes, 2017; Drysdale, et al., 2019), tracing back to the problem of inconsistency. In many other applications in psychometrics, similar patterns are observed. For example, when LASSO is used in regularized DIF (Bauer, Belzak & Cole, 2019), with the tuning parameter determined by minimizing the Bayesian information criteria (BIC; Schwarz, 1978), the FPRs could be as high as 48% when DIF is pervasive (66%) and of large magnitude, at sample size of 2000, though this rate is lower than using the item response theory likelihood ratio DIF detection procedure (IRT-LR-DIF; Thissen et al., 1993).

In statistical learning, a large body of research has shown that inconsistency could be reduced by aggregating over a large set of competing models (e.g., bootstrap aggregating, Breiman, 1996; random forest, Breiman, 2001; boosting, Friedman, 2001). Moreover, Barber and Candès (2015) pointed out that effective FPR control helps in improving model consistency and interpretation. Stability selection, as proposed by Meinshausen and Bühlmann (2010), was designed to improve the performance of variable selection and structure estimation algorithms by aggregating the selection results from bootstrap samples. It provides finite sample control for FPRs and hence a transparent principle to choose a proper amount of regularization. It is a very general technique based on resampling that has a wide range of applicability, for instance, in neuroscience (Ryali, et al., 2012; Ye et al., 2012), genetics (Alexander & Lange, 2011), and bioinformatics (Hofner, Boccuto, & Göker, 2015).

In this study, we aim to draw researchers' attention to possible problems of applying LASSO regularization in latent variable models, and provide a potential solution for improving the consistency and controlling the high FPRs by incorporating stability selection in regularized

SEM. We first provide an overview of regularized SEM and stability selection, then present the proposed algorithms to utilize regularization and stability selection under the SEM framework. We then compare the performances of stability selection as well as a number of extensions based on the LASSO, in terms of FPRs and FNRs across two simulation studies, focusing on selecting predictors of latent variables, and selecting cross loadings, respectively. Lastly, we perform stability selection on an empirical example to further discuss the practical advantages and issues of the proposed method. Although we focus on the application of regularized SEM, we believe that our proposed procedure and findings would pertain to other types of psychometric models with regularization applied.

Regularized Structural Equation Modeling

Regularized SEM (Jacobucci, Grimm & McArdle, 2016) adds a penalty term to the traditional maximum likelihood estimation (MLE) fit function:

$$F_{regsem} = \underbrace{\log(|\Sigma|) + tr(C * \Sigma^{-1}) - \log(|C|) - p}_{MLE} + \underbrace{\lambda P(\cdot)}_{penalty}, \quad (1)$$

where Σ is the model implied covariance matrix, C is the observed covariance matrix, and p is the number of variables. The regularization parameter λ is a tuning parameter that cannot be determined jointly with the coefficients. It quantifies the influence of the penalty, with larger λ incurring larger penalty, and thus result in greater shrinkage of the coefficient sizes. Its optimal value are often determined through cross validation or based on information criteria, such as Akaike information criterion (AIC; Akaike, 1974)

$$AIC = 2k - 2 \ln(\hat{L}),$$

where $\hat{L}$ is the maximum value of the likelihood function for the model, and Bayesian information criteria (BIC; Schwarz, 1978)

$$BIC = \ln(N) k - 2 \ln(\hat{L}).$$

As can be seen in the formulas, the AIC does not depend directly on the sample size N , making it prone to overfitting, whereas BIC does depend on sample size, and is more conservative relative to AIC in most cases. In practice, the BIC generally performs well in finding the regularization parameter λ that results in reasonable parameter estimates for regularized factor analysis models (Hirose & Yamamoto, 2015; Jacobucci et al., 2016).

$P(\cdot)$ is a general penalty function which reflects the sum of all selected coefficients, and can take many different forms like the LASSO ($\|\cdot\|_1$), ridge ($\|\cdot\|_2$), and elastic net ($(1 - \alpha)\|\cdot\|_1 + \alpha\|\cdot\|_2$). Although the LASSO is widely used in regression, using only one value of λ could lead to predictable bias in the resulting estimates, particularly when the scale of the variables are dramatically² different (Fang & Li, 2001). For this reason, a two-stage procedure termed the “relaxed LASSO” (Meinshausen, 2007) is usually performed when the LASSO is chosen as the penalty type in regularized SEM. That is, we first perform variable/path selection using LASSO, and then we fit an SEM model with only the selected variables/paths to get their unbiased estimates. It has been shown that the relaxed LASSO produces unbiased parameter estimates when applied to mediation models estimated under the SEM framework (Serang et al., 2017).

Another method proposed to overcome this biasedness of the LASSO is through using the adaptive LASSO (ALASSO; Zou, 2006). When the ALASSO is applied in SEM, each parameter to be penalized is first scaled by the MLE estimates before LASSO is performed ($\|\beta_{ML}^{-1} * \beta_{pen}\|_1$). With this scaling, smaller parameters would receive larger penalties, while limiting the bias in the estimation of larger parameters. However, since MLE parameter estimates are needed when

applied to SEM, the ALASSO may not be applicable when the model has a large number of variables in relation to sample size, or other estimation difficulties have occurred.

As we have discussed previously in the introduction, besides biasedness, the high FPRs and inconsistency of the selection results are two major concerns of LASSO. Other penalties have been proposed to deal with the above two problems, for example, the smoothly clipped absolute deviation (SCAD; Fan & Li, 2001), and minimax concave penalty (MCP; Zhang, 2010) have been shown to possess the oracle properties. However, some penalties may involve minimization of nonconvex function, which could be computationally complex (Meinshausen, 2007; Fan & Peng, 2004). Jacobucci (2017) and Huang et al. (2017) demonstrated the algorithmic details of both the MCP and SCAD, however, in practice they are difficult to estimate, particularly in complex SEMs.

Stability Selection

Stability selection utilizes resampling for the purpose of selecting variables. The method is very general and can be paired with multiple variable selection or structural estimation techniques. Before we introduce the idea of stability selection, we first formalize the variable selection (or, structure estimation) problem: Suppose we have a p -dimensional vector β , with s ($s < p$) components being non-zero. We denote the set of non-zero values by $S = \{k: \beta_k \neq 0\}$, and the set of zero coefficients by $V = \{k: \beta_k = 0\}$. The goal of variable selection (or, structure estimation) is then to infer the set S from the p possible predictors. For every tuning parameter $\lambda \in \Lambda \subseteq \mathbb{R}^+$ that determines a specific amount of regularization, we could get a structure estimate $\hat{S}^\lambda \subseteq \{1, \dots, p\}$. The goal is then to determine whether there exists a $\lambda \in \Lambda$ such that $\hat{S}^\lambda = S$ with high probability, and to achieve (or get a close approximation) of this λ .

Stability selection does not focus on finding a single λ , rather, it aggregates across a range Λ of regularization values. The procedure is as follows: Let I be a random sample of $\{1, \dots, n\}$ of size $\lfloor \frac{n}{2} \rfloor$ drawn without replacement ($\lfloor \frac{n}{2} \rfloor$ is chosen as it mostly closely resembles bootstrap, while allowing for a computationally efficient implementation). For every set $K \subseteq \{1, \dots, p\}$, the probability of being in the selected set is $\hat{\Pi}_K^\lambda = P^*(K \subseteq \hat{S}^\lambda(I))$. Then for each predictor $k = 1, \dots, p$, the stability path is given by the selection probabilities $\hat{\Pi}_k^\lambda, \lambda \in \Lambda$. For each cutoff $\pi_{thr} \in (0,1)$, and a set of regularization parameters Λ , the set of stable predictors is then defined as $\hat{S}^{stable} = \{k: \max_{\lambda \in \Lambda} \hat{\Pi}_k^\lambda \geq \pi_{thr}\}$. Predictors with high selection probabilities are kept (e.g., > 0.8 or 0.9) and those with low selection probabilities are discarded.

To see the advantage of stability selection, let Λ denote the set of regularization parameters. To perform variable selection, traditional techniques pick one model from the set $\{\hat{S}^\lambda: \lambda \in \Lambda\}$, which may lead to the two potential problems. First is that the correct model S may not be contained in the above set. Even in the absence of this problem, the second problem is that even if the correct model S is contained in the set, we may not be able to accurately determine a correct degree of regularization λ to select exactly S , or even a close approximation of S . Perturbing the data multiple times and choosing structures or variables that occur with high probability in the selection set allows for more flexible choices which are not limited to one single model from the set (Meinshausen & Bühlmann, 2010), and thus, potentially eliminating the two identified problems. It has also been shown that stability selection can control the familywise type I error rate (FPR) in multiple testing for finite sample size and thus markedly improve the structure estimation or selection methods (Meinshausen & Bühlmann, 2010).

Regularized Structural Equation Modeling with Stability Selection

We now provide the details on pairing regularized SEM with stability selection:

1. Standardize the variables to place them on the same scale.
2. Select a reasonable range for Λ (discussed below).
3. For each $\lambda \in \Lambda$, obtain M (e.g. $M=100$) bootstrap samples of $I=\{1, \dots, n\}$.
 - a. For each bootstrap sample T , perform regularized SEM with this specific amount of regularization λ , and obtain a set of selected predictors $\hat{S}^\lambda(T) \subseteq \{1, \dots, p\}$.
 - b. Consider the M sets of selected predictors altogether and obtain the selection probabilities $\hat{\Pi}_k^\lambda$ for each k in $1, \dots, p$, at this particular λ .
4. Select the final set $\hat{S}^{stable} = \{k: \max_{\lambda \in \Lambda} \hat{\Pi}_k^\lambda \geq \pi_{thr}\}$ based on the choice of probability cutoff π_{thr} .
 - a. Here, we select all predictors that have selection probability greater than or equal to π_{thr} at some $\lambda \in \Lambda$.
5. Refit the SEM model without any regularization, using only the subset of selected predictors.

One issue with the traditional form of stability selection is that although it is said that the empirical results vary little for threshold values in the range (0.6, 0.9), and are not sensitive to choices of Λ for high dimension cases (Meinshausen & Bühlmann, 2010), it is not the case in low dimension settings (Huang & Marchetti-Bowick, 2014). Thus, when applied in psychological research where the dimension is usually not high, selecting a proper range Λ and a proper threshold value π_{thr} becomes crucial.

Choosing the probability cutoff π_{thr} is an extremely important component of this procedure as it determines the final selection result. This value could be pre-determined as a fixed

value by researchers (e.g., 0.8), or it could be tuned based on model fit. To be specific, we could refit the SEM model based on a different cutoff value π_{thr} , then compare the corresponding model fit, choosing the threshold that corresponds to the best fitted model.

Since we are aggregating the maximum selection probabilities $\max_{\lambda \in \Lambda} \hat{\pi}_k^\lambda$ of each variables across a range of regularization Λ , this range Λ needs to be carefully chosen. It could either be determined by theory or by pre-run results. For example, we could pre-run RegSEM on bootstrap samples of the data to get the empirical distribution (left panel of Figure 1) of the final λ values chosen by regularized SEM. Then we could use the inter-quartile range of the pre-run λ values as the range for stability selection. As demonstrated in the right panel of Figure 1, the region between the two dash-dotted lines corresponds to the inter-quartile range of the λ values for the example data. In this region, the FPR and FNR are relatively balanced, and the cross point of the two error rate curves falls in this region most of the time. This range is utilized throughout this study, while we leave it open for better choices.

Simulation Studies

Methods

In this section, we present the results of the two simulation studies on the performance of regularized SEM (RegSEM) with stability selection and a number of extensions to the LASSO, with a main focus put on FPRs and FNRs. In each study, eight methods are compared. Besides the traditional p -value based selection with MLE (using 0.05 as a cutoff) and RegSEM-LASSO procedure, we also tested the performance of RegSEM-ALASSO, in hope that it could get rid of some of the drawbacks of LASSO penalty. Five-fold cross validated RegSEM-LASSO and RegSEM-LASSO based on one hundred bootstrap samples are also tested, to see if they could

lower the FPR. For RegSEM with stability selection, we tried to vary the penalty types and the fit indices used for tuning the probability threshold π_{thr} . Four combinations are considered: stability selection-LASSO-BIC, stability selection-LASSO-AIC, stability selection-ALASSO-BIC, and stability selection-ALASSO-AIC. In each case, 50 RegSEM pre-runs are performed on bootstrap samples of the data to determine the range Λ for the regularization values. The range Λ is then divided to 10 equal distant values of λ , and the selection probability at each of these λ values are obtained from 100 replications of RegSEM with degree of penalty fixed at this λ on bootstrap samples. For all the methods stated above, whenever RegSEM is performed, its optimal λ value is chosen based on the BIC (Jacobucci, Grimm & McArdle, 2016).

The two studies consider different application models. The first study considers selecting among multiple predictors of a latent variable, following the general form of Jacobucci, Brandmaier, and Kievit (2019) to determine whether our proposed methods can improve upon the results of their simulation study. The data generating model is a Multiple Indicators Multiple Causes (MIMIC; Joreskog & Goldberger, 1975) model with one latent factor F_1 . There are in total six indicators ($Y_1, \dots, Y_6$), and thirty predictors of F_1 . The penalty is applied to all thirty regression paths. For identification purposes, the variance of the latent factor is fixed at 1, while all other factor loadings and regression paths are freely estimated. The mean structure is not estimated in this model. We tested two models with different effect sizes: the first model tested a wider range (0.2 to 0.8) of regression coefficients (Jacobucci, Brandmaier & Kievit, 2019), while the second model considers more realistic situations where all regression coefficients are relatively small in magnitude (0.1 to 0.3). The corresponding path diagrams are depicted in Figure 2. In both conditions, $X_1, \dots, X_{15}$ are simulated to be uninformative “noise” variables, with coefficients simulated as 0; $X_{16}, \dots, X_{20}$ are simulated to be predictors with relatively small effect sizes, having

coefficients 0.2 in the large effect size condition and 0.1 in the small effect size condition; $X_{21}, \dots, X_{25}$ are relatively medium effect size predictors, having coefficients 0.5 in the large effect size condition and 0.2 in the small effect size condition; $X_{26}, \dots, X_{30}$ are relatively large effect size predictors, with coefficient 0.8 and 0.3 in the large effect size condition and small effect size condition, respectively. We tested across sample sizes of 75, 150, 400, 800, and 2000, to see when the performance of MLE degrades, and when the use of regularization becomes beneficial.

The second study considers selecting cross loadings. A two-factor model is generated with simulated parameters as depicted in Figure 3. Each factor has five large loadings of 0.8, three loadings of 0.6, and two loadings of 0.4. While the loadings in solid lines are considered certain and are always included in the model, the paths in dashed lines are the cross loadings that we are uncertain about and are considered for removal to make the final model more parsimonious. The dash-dotted paths have a simulated loading of 0.3, while the dashed paths are the true “noise” which are simulated to have loadings of zero. Penalties are only put on those 14 paths. As we mentioned earlier, RegSEM when combined with LASSO penalty tends to give not enough penalization, so that the FPR are very high. Our goal here is to try to remove those true “noise” cross loadings, while keeping those non-zero loadings. In the data generating model, each factor is simulated to have a variance of 1 and the correlation of the two factors is simulated to be 0.3. For this study, sample sizes of 40, 50, 75, 150, 400, 800, and 2000 are considered. Across each of the simulation condition, each cell is replicated 100 times.

Results

The results of the two simulation studies are summarized in Tables 1 and 2 respectively³. Since the overall patterns of the results are very similar across the two studies, we give an overview of the results together. As expected, the FPRs of MLE models falls around 0.05 in most cases.

However the high FNRs (for small effect size predictors in simulation 1, and for cross loading detections in simulation 2) shows that the power could be an issue. That is, paths with true signals are often discarded, especially for cases with small sample sizes.

Similar to what was found in previous studies (Jacobucci, Grimm & McArdle, 2016), RegSEM-LASSO displays high FPRs, while having relatively low FNRs. This means that RegSEM-LASSO is able to include most signal variables/paths, however a large number of noise variables/paths are included at the same time. With respect to consistency, although the FNRs could be reduced to nearly 0 when sample size gets large enough, the FPRs stay high.

When ALASSO is used instead, parameters with smaller MLE estimates are given larger penalties, thus the FPRs becomes lower than RegSEM-LASSO. However, similar to using MLE, the FNRs becomes high for small effect paths when sample size is small. One nice feature of RegSEM-ALASSO is that the FPRs and FNRs continues to drop as sample size gets larger. When sample size is large, the FPRs could get to lower than 0.05 (i.e. better than MLE), with FNR close to 0. This shows that, ALASSO gives consistent selection results under the two simulation conditions.

Five-fold cross validation and bootstrapping did help decrease the FPRs, slightly sacrificing the FNRs, but again, the selection results are inconsistent with the non-decreasing FPRs as sample size gets larger. Moreover, since sample splitting is required in the cross-validation process, it may not be applicable when sample size is small, hence high rates of non-convergence may occur in small sample conditions.

Stability selection-LASSO-BIC and stability selection-ALASSO-BIC both give similar results as MLE when sample size is small, and they perform better than MLE in terms of FPRs

when sample size is large. But again, like MLE, the FNRs is very high for small samples, indicating that the selection is too conservative. When AIC is used for determining π_{thr} instead, a smaller penalty is incurred for model complexity, and thus gives more balanced FPRs and FNRs, as what we are targeting at (e.g. FPRs and FNRs as within the dash-dotted region in right panel of Figure 1). The use of stability selection-ALASSO-AIC affords the nice property of decreasing both FPRs and FNRs as sample size gets larger, showing consistency in selection results.

To summarize, our simulation results evident that there is a practical tradeoff between FPRs and FNRs. Based on the simulation results of the current two studies, stability selection-ALASSO-AIC shows benefit over both MLE and regularized SEM and is preferred when computation resources permits. With limited computational resources, RegSEM-ALASSO is recommended over RegSEM-LASSO. However, it is possible in real application that the full model is unidentified, and thus MLE estimates are not available. In such scenarios, ALASSO is not applicable, and researchers could consider using stability selection-LASSO-BIC.

Empirical Example

Given that the application of regularized SEM has predominantly been applied to regression parameters in a MIMIC model or to factor loadings in a CFA model, our goal in the empirical example was to apply penalties in a unique way. For this, we applied penalties to residual covariances as a mechanism to overcome poor fit and identify possible latent variables that explain a facet of manifest variables not captured by a general factor. We tested this using personality data collected online and freely available at https://openpsychometrics.org/_rawdata/. Specifically, we analyzed responses to the 50-item International Personality Item Pool-Five-Factor Model measure (Goldberg, 1999). The dataset consists of 4,270 observations, but for the purposes of this analysis we randomly sampled 1,000 cases. Of the five scales, we focused on

Conscientiousness, fitting a one factor model to the ten observed variables. The item content is detailed in Appendix A. The one-factor model did not fit well, with a CFI of 0.907, TLI of 0.880, and RMSEA of 0.080 (90% CI of 0.071, 0.089). The unstandardized and standardized factor loadings are displayed in Table 3.

In contrast to the use of modification indices, we attempted to use a more global approach to identify specific components of misfit. As a first step, we fit a model with all possible residual covariances. Given that this model is not identified (-1 degrees of freedom), we do not report parameter estimates. As a second step, we added LASSO penalties to each of the residual covariances as a mechanism to identify which may be non-zero. This procedure is in line with the Bayesian Lasso approach of Pan, Ip, and Dubé (2017). Note that we do not use the ALASSO, even though it had superior performance in the simulation study, as the MLE parameter estimates are unstable given the non-identification. Therefore, we paired the stability selection-LASSO with BIC to select residual covariances, as this resulted in the best performance for LASSO penalties. The full analysis code and data will be put into an open science framework folder upon publication.

Results

Table 3 shows the selected residual covariances after running the stability procedure. After modifying the initial model to include those residual covariances, the new model evidenced far superior fit, with a CFI of 0.976, TLI of 0.963, and RMSEA of 0.044 (90% CI 0.034, 0.055). Given that we had additional data at our disposal, we tested the new model on a second sample of 1,000 observations to assess the replication of this new factor structure. Although we could just estimate the model in the holdout data, as is a common form of replication in factor analysis research, a stricter test is to use the expected covariance matrix from the first sample, then treat this as fixed

for the assessment of fit in the holdout. Re-estimating the model in the holdout sample resulted in an almost identical fit (unsurprisingly given that the sample characteristics are identical). This stricter form of replication resulted in only a slight decrement in fit, with a CFI of 0.966, TLI of 0.948, and RMSEA of 0.053⁴.

In examining the residual covariances in Table 3, we can see that C2, “I leave my belongings around”, accounts for the majority of relationships. In examining the other three items, we can see that they all generally assess an additional factor which could be described as messiness, a subfacet of Conscientiousness. These results are further confirmed in the use of exploratory factor analysis, where a two-factor solution (using oblimin rotation) has one factor that is mainly indicated by those same four variables.

Discussion

The purpose of this paper was to highlight two of the main drawbacks to regularization in psychometric models, namely inconsistency and high FPRs, while detailing and arguing for the application of stability selection. We demonstrated this in two simulation studies, which considered selecting predictors of latent variables and cross loadings in SEM. Stability selection with the ALASSO penalty and using the AIC to choose the probability threshold gave superior performance to other regularization methods. An empirical example using the proposed method to select among residual covariances in a CFA model that had poor initial fit was also demonstrated.

As with most things in statistics (and machine learning, e.g., the no free lunch theorem; Wolpert & Macready, 1997), no one method performed optimally across the simulation conditions. Given this, we put the onus on the researcher for justifying their choice of methodology. In many situations, the use of MLE works extremely well, particularly if the FPR is of utmost concern. If

parameters need to be added based on poor fit, the use of modification indices can work well as long as the number added is small. However, when the sample size is small relative to the number of parameters, estimation problems occur, or the number of parameters to be removed (or added and tested to see if they make a meaningful contribution) is large, a number of the proposed methods are recommended. When MLE parameter estimates can be reliably estimated, the stability selection-ALASSO-AIC demonstrated the best performance. When MLE exhibits problems, or the parameter estimates may be inflated due to other data characteristics, we recommend the stability selection-LASSO-BIC.

An additional consideration when choosing among various regularization procedures is the computational complexity of each. Stability selection methods require a number of bootstrap samples, greatly increasing the amount of time it takes to produce results. Particularly for large models, the computational complexity may be prohibitive, thus necessitating the use of MLE or RegSEM-LASSO depending on the research goals. As one mechanism for decreasing the computation time, parallel processing has been built into the R function that performs stability selection.

The use of these procedures does not negate the use or influence of theory into the modeling setting. This aspect of using regularized SEM is amplified by the characterization of regularization as falling under the umbrella of machine learning, typically conceptualized as a set of atheoretical algorithms. Instead, we see the utility of applying regularized SEM being amplified when theory fails as a mechanism to identify what specific aspects (of a potentially complex theoretical formulation) were missed or erroneously included. Additionally, in the nascent stages of theory generation, data may be available for the purposes of hypothesis formulation, to then be followed by a more rigorous testing in a follow up study.

Limitations and Future Directions

This study has a number of limitations which can be considered as future research directions. The first one is with respect to the choice of the initial range Λ and the probability threshold π_{thr} . Although Meinshausen and Bühlmann (2010) stated that the final results are insensitive to the choice of Λ and vary little for π_{thr} values in (0.6, 0.9), it is not always the case in real applications. Throughout the paper, we determined the range of Λ using the inter-quartile range of λ values chosen by regularized SEM on 50 bootstrap samples. However, there is no theoretical justification that this application is the best, and we believe there could be more sophisticated choices. As for π_{thr} , we tried treating it as a tuning parameter and selected it based on fit indices. The results, indeed, vary across different choice of fit indices. Although it sounds as if this method yields uncertain results, stability selection does provide a fixed maximal selection probability on Λ for each parameter being penalized, thus the sequence of variable selection is fixed, leading to another possible way of choosing π_{thr} based on theoretical justification. Again, we leave it open for better ways of determining this probability threshold π_{thr} .

Another limitation is that we are only focusing on FPRs and consistency in this study, ignoring other possible problems of LASSO, for example, correlation among variables, and parameter estimation bias. Other penalties such as elastic net have been demonstrated to work well when there is correlation among variables (Zou & Hastie, 2005), however, testing its performance would add another dimension to our simulation study and is out of the scope of the problems concerned in this paper. Investigating these further analysis conditions warrants future work however.

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Footnotes

¹ Throughout the paper, we use the term regularization and penalization interchangeably.

² It is typically recommended to standardize variables prior to analysis, however, there are some latent variable models where this is inappropriate.

³ Corresponding plots are available in supplementary materials online.

⁴ A problem occurred in estimating the 90% CI for the RMSEA.

Figure 1

Left panel: Histogram of possible optimal λ values chosen by regularized SEM on bootstrap samples of a simulated model with 1 factor and 10 causes.

Right panel: FPRs and FNRs on the full sample based on different λ values. Dashed lines represent the minimum and maximum of the chosen λ values based on bootstrap samples. Dash-dotted lines are the 25th and 75th quantiles of the chosen λ values based on bootstrap samples. Within this inter-quartile region, the FPRs and FNRs are relatively balanced.

Figure 2

Path diagram of the data generating model for simulation study 1. A MIMIC model with 30 causes of one latent variable is simulated. Among those 30 causes, the first 15 are simulated noise variables, the other 15 variables are simulated to have relatively small, medium, and large (5 each) effect on the latent variable. Two settings are considered: the left panel follows from the design used in Jacobucci, Brandmaier, and Kievit (2019), and has larger effect sizes than the right panel; the right panel considers more realistic conditions, where all the regression coefficients are smaller than 0.3.

Figure 3

Path diagram of the data generating model for simulation study 2. A two-factor model is simulated, with factor correlation of 0.3. Each factor has five large loadings of 0.8, three loadings of 0.6, and two loadings of 0.4, marked by solid line. Paths in dashed lines are the cross loadings that are considered for removal. The dashed paths are simulated to have zero loadings, and the dash-dotted paths have simulated loading of 0.3.

Table 1a

Results of simulation study 1, when the effect sizes of causes in the MIMIC model is large. NAs refer to conditions that no result was obtained.

	Sample Size	FPR	FNR		
			Small	Moderate	Large
MLE	75	0.127	0.634	0.076	0
	150	0.075	0.426	0.002	0
	250	0.068	0.204	0	0
	400	0.063	0.072	0	0
	800	0.051	0.002	0	0
	2000	0.045	0	0	0
RegSEM (LASSO)	75	0.459	0.306	0.008	0
	150	0.391	0.194	0	0
	250	0.432	0.050	0	0
	400	0.396	0.012	0	0
	800	0.405	0	0	0
	2000	0.393	0	0	0
RegSEM (Adaptive LASSO)	75	0.174	0.552	0.056	0
	150	0.117	0.380	0.004	0
	250	0.110	0.204	0	0
	400	0.099	0.056	0	0
	800	0.047	0.004	0	0
	2000	0.001	0	0	0
Cross Validated RegSEM (LASSO)	75	NA	NA	NA	NA
	150	NA	NA	NA	NA
	250	0.125	0.263	0	0
	400	0.142	0.066	0	0
	800	0.193	0	0	0
	2000	0.254	0	0	0
Bootstrap RegSEM (LASSO)	75	0.107	0.640	0.120	0
	150	0.193	0.274	0	0
	250	0.190	0.098	0	0
	400	0.164	0.026	0	0
	800	0.161	0	0	0
	2000	0.151	0	0	0
Stability Selection (LASSO) -BIC	75	0.107	0.676	0.092	0
	150	0.056	0.532	0.002	0
	250	0.039	0.328	0	0
	400	0.027	0.124	0	0
	800	0.017	0.006	0	0
	2000	0.023	0	0	0
Stability Selection (Adaptive LASSO) -BIC	75	0.106	0.667	0.079	0
	150	0.037	0.544	0.004	0
	250	0.030	0.372	0	0
	400	0.018	0.142	0	0
	800	0.009	0.008	0	0
	2000	0.003	0	0	0
Stability Selection (LASSO) -AIC	75	0.277	0.508	0.036	0
	150	0.203	0.262	0.002	0
	250	0.191	0.092	0	0
	400	0.169	0.020	0	0
	800	0.160	0	0	0

	2000	0.144	0	0	0
Stability	75	0.267	0.464	0.028	0
Selection	150	0.197	0.268	0.002	0
(Adaptive	250	0.189	0.104	0	0
LASSO)	400	0.162	0.024	0	0
-AIC	800	0.078	0.002	0	0
	2000	0.003	0	0	0

Table 1b

Results of simulation study 1, when the effect sizes of causes in the MIMIC model is small. NAs refer to conditions that no result was obtained.

Sample Size		FPR	FNR		
			Small	Moderate	Large
MLE	75	0.133	0.780	0.684	0.390
	150	0.095	0.798	0.456	0.148
	250	0.057	0.704	0.222	0.024
	400	0.053	0.562	0.074	0.002
	800	0.050	0.338	0.004	0
	2000	0.050	0.030	0	0
RegSEM (LASSO)	75	0.347	0.568	0.404	0.146
	150	0.207	0.620	0.268	0.080
	250	0.162	0.558	0.152	0.008
	400	0.172	0.404	0.044	0.004
	800	0.216	0.174	0.002	0
	2000	0.266	0.012	0	0
RegSEM (Adaptive LASSO)	75	0.094	0.840	0.730	0.490
	150	0.064	0.838	0.586	0.256
	250	0.073	0.690	0.226	0.020
	400	0.075	0.548	0.092	0
	800	0.041	0.368	0.010	0
	2000	0.002	0.260	0	0
Cross Validated RegSEM (LASSO)	75	NA	NA	NA	NA
	150	NA	NA	NA	NA
	250	0.169	0.596	0.190	0.014
	400	0.043	0.690	0.140	0.004
	800	0.055	0.472	0.020	0
	2000	0.129	0.030	0	0
Bootstrap RegSEM (LASSO)	75	0.167	0.800	0.400	0.200
	150	0.177	0.670	0.306	0.096
	250	0.151	0.554	0.136	0.012
	400	0.146	0.404	0.024	0.002
	800	0.158	0.180	0	0
	2000	0.156	0.010	0	0
Stability Selection (LASSO) -BIC	150	0.070	0.872	0.762	0.506
	250	0.043	0.884	0.608	0.236
	400	0.024	0.826	0.358	0.050
	800	0.013	0.764	0.168	0.002
	2000	0.007	0.532	0.012	0
	2000	0.007	0.122	0	0
Stability Selection (Adaptive LASSO) -BIC	75	0.073	0.870	0.770	0.500
	150	0.044	0.878	0.602	0.226
	250	0.018	0.820	0.358	0.042
	400	0.014	0.770	0.160	0.004
	800	0.008	0.560	0.016	0
	2000	0.003	0.184	0	0
Stability Selection (LASSO) -AIC	75	0.259	0.672	0.514	0.242
	150	0.197	0.642	0.278	0.084
	250	0.178	0.506	0.118	0.010
	400	0.172	0.368	0.016	0
	800	0.157	0.178	0	0

	2000	0.153	0.01	0	0
Stability	75	0.294	0.630	0.488	0.246
Selection	150	0.174	0.648	0.318	0.100
(Adaptive	250	0.157	0.528	0.116	0.008
LASSO)	400	0.150	0.390	0.020	0.002
-AIC	800	0.081	0.282	0.002	0
	2000	0.003	0.184	0	0

Table 2

Results of simulation study 2. NAs refer to conditions that no result was obtained.

Sample Size		FPR			FNR	
		Y4, Y5, Y14, Y15	Y8, Y18	Y10, Y20	Y6, Y7, Y16, Y17	Y9, Y19
MLE	40	0.068	0.075	0.045	0.438	0.500
	50	0.075	0.090	0.035	0.358	0.440
	75	0.048	0.075	0.055	0.220	0.300
	150	0.048	0.035	0.055	0.020	0.035
	250	0.040	0.060	0.055	0	0
	400	0.055	0.050	0.055	0	0
	800	0.040	0.030	0.060	0	0
	2000	0.045	0.055	0.065	0	0
RegSEM (LASSO)	40	0.462	0.365	0.295	0.188	0.265
	50	0.482	0.295	0.315	0.123	0.180
	75	0.348	0.285	0.205	0.073	0.175
	150	0.392	0.220	0.210	0.005	0.010
	250	0.342	0.285	0.230	0	0
	400	0.365	0.215	0.210	0	0
	800	0.285	0.165	0.120	0	0
	2000	0.298	0.145	0.160	0	0
RegSEM (Adaptive LASSO)	40	0.110	0.125	0.085	0.422	0.495
	50	0.110	0.120	0.070	0.360	0.455
	75	0.100	0.105	0.090	0.202	0.320
	150	0.100	0.090	0.120	0.020	0.030
	250	0.068	0.080	0.060	0	0
	400	0.052	0.050	0.075	0	0
	800	0.002	0.010	0.010	0	0
	2000	0.008	0.025	0.025	0	0
Cross Validated RegSEM (LASSO)	40	NA	NA	NA	NA	NA
	50	NA	NA	NA	NA	NA
	75	NA	NA	NA	NA	NA
	150	0.240	0.080	0.075	0.022	0.060
	250	0.222	0.175	0.090	0.005	0.040
	400	0.288	0.145	0.150	0	0
	800	0.252	0.160	0.140	0	0
	2000	0.250	0.130	0.145	0	0
Bootstrap RegSEM (LASSO)	40	0.308	0.250	0.190	0.277	0.380
	50	0.247	0.182	0.096	0.237	0.338
	75	0.172	0.125	0.100	0.162	0.315
	150	0.200	0.070	0.070	0.017	0.050
	250	0.182	0.150	0.085	0	0.015
	400	0.170	0.085	0.100	0	0
	800	0.128	0.070	0.045	0	0
	2000	0.125	0.065	0.045	0	0
Stability selection (LASSO) -BIC	40	0.118	0.080	0.075	0.425	0.510
	50	0.088	0.070	0.050	0.368	0.505
	75	0.058	0.065	0.040	0.235	0.360
	150	0.035	0.015	0.035	0.027	0.100
	250	0.025	0.040	0.020	0	0.015
	400	0.020	0.005	0.015	0	0
	800	0.008	0.010	0.015	0	0
	2000	0.010	0.005	0.005	0	0
Stability	40	0.080	0.085	0.070	0.448	0.525

Selection	50	0.080	0.080	0.040	0.397	0.495
(Adaptive	75	0.043	0.060	0.040	0.235	0.345
LASSO)	150	0.023	0.015	0.030	0.027	0.095
-BIC	250	0.018	0.040	0.020	0	0.015
	400	0.018	0.005	0.015	0	0
	800	0.000	0.010	0.015	0	0
	2000	0.003	0	0	0	0
Stability	40	0.192	0.192	0.154	0.359	0.410
Selection	50	0.237	0.132	0.158	0.178	0.329
(LASSO)	75	0.184	0.105	0.132	0.099	0.158
-AIC	150	0.211	0.145	0.184	0.013	0
	250	0.158	0.171	0.158	0	0
	400	0.184	0.132	0.158	0	0
	800	0.151	0.158	0.105	0	0
	2000	0.171	0.118	0.237	0	0
Stability	40	0.203	0.175	0.185	0.273	0.320
Selection	50	0.193	0.170	0.170	0.195	0.250
(Adaptive	75	0.158	0.170	0.185	0.090	0.140
LASSO)	150	0.158	0.150	0.175	0.008	0.005
-AIC	250	0.123	0.170	0.105	0	0
	400	0.160	0.140	0.170	0	0
	800	0.055	0.035	0.060	0	0
	2000	0.003	0	0	0	0

Table 3

Results of the empirical example.

Factor Loadings	Original Model	New Model
C1	0.767	0.762
C2	0.749	0.627
C3	0.395	0.375
C4	0.781	0.672
C5	0.780	0.785
C6	0.890	0.791
C7	0.605	0.568
C8	0.598	0.533
C9	0.747	0.797
C10	0.481	0.537
Residual Variances		
C1	0.724	0.667
C2	1.282	1.432
C3	0.797	0.812
C4	1.014	1.108
C5	0.928	0.923
C6	1.194	1.325
C7	0.905	0.920
C8	0.894	0.922
C9	0.989	0.874
C10	0.749	0.688
Residual Covariances		
C2 WITH C4		0.257
C2 WITH C5		0.132
C2 WITH C6		0.429
C4 WITH C6		0.231
C4 WITH C8		0.176
C7 WITH C9		0.151

Note: All parameters were significant at $p < 0.05$.

Appendix A

- C1 I am always prepared.
- C2 I leave my belongings around.
- C3 I pay attention to details.
- C4 I make a mess of things.
- C5 I get chores done right away.
- C6 I often forget to put things back in their proper place.
- C7 I like order.
- C8 I shirk my duties.
- C9 I follow a schedule.
- C10 I am exacting in my work.