Situational optimization function analysis:**An ideal performance analysis inspired on Lewin's Equation**

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Abstract

This study presents the situational optimization function analysis (SOFA) and has three aims. First, to develop a Bayesian implementation of SOFA. Second, to compare this implementation with three other maximum likelihood-based models in their accuracy to estimate true scores. The third aim is to show how joint modeling can be used for validity research. A simulation study was used to examine the second aim, while an empirical example was used to illustrate the third aim. The simulation study used three data generating processes, with varying degrees of deviation from linear models and with different sample sizes. Results of the simulation study showed that the Bayesian implementation supersedes the other models. In the empirical example, data collected from 66 participants using an iterated prisoner dilemma and a scale measuring cooperation-competition attitudes were used. Results showed that joint modeling is the best fitting model, also increasing the correlation between the true scores of both measures (deviations from the iterated prisoner dilemma and the scale). Finally, implications, limitations and future studies are discussed.

Keywords: Measurement; SOFA framework; Bayesian modeling; joint modeling.

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Psychometrics is the field in psychology dedicated to theory and practice of measuring psychological constructs (Borsboom, 2005). For most measurement models, responses are considered to be a function of the effect of a dispositional trait and particular characteristics of the items used as stimuli in a test or questionnaire (McDonald, 1999). For models developed in the item response theory framework (IRT; van der Linden & Hambleton, 2013; De Ayala, 2009), respondents' dispositional trait θ (i.e., intelligence, attitudes, beliefs, personality, and so on) has an additive interaction with items' difficulty parameter δ in score responses observed in questionnaires and tests.

On the other hand, researchers in psychology typically use many data types that are not always suitable to be analyzed with IRT-derived models, especially behavioral data coming from experimental settings (Eid & Diener, 2006). Psychometric models are generally fit to correlational data (van der Linden & Hambleton, 2013). A common justification for this is the fact that experimental manipulation of psychological variables is either very difficult, impossible or unethical (Meehl, 1967). However, several alternative measurement models, based on a more diverse theoretical background and on information gathered in experimental settings, have been proposed by cognitive psychologists (Farrell & Lewandowsky, 2018; Lee & Wagenmakers, 2014).

The overall aim of this study was to propose the methodological framework of situational optimization function analysis (SOFA), inspired by Lewin's equation, and ideal performance analysis (Geisler, 1989; Geisler, 2003; Thomson & Kristan, 2005), which can be used to develop psychometric-like measures derived from experimental studies. A further aim was to propose a Bayesian implementation of SOFA and compare it with three other maximum likelihood-based approaches in its efficiency to estimate dispositional traits from

simulated data. A final aim was to illustrate how construct validity analysis can be conducted in this framework by means of joint modeling (Turner et al, 2013) using real data.

Lewin's equation: Identifying Dispositional and Situational Influences

According to the field theory proposed by Lewin (1936), it is a somewhat agreed principle that a number of different and competing influences affect individuals' behavior (Furr & Funder, in press). Stemming from this principle, two areas of research were developed: social psychology and personality psychology (McAdams, 1997). While social psychology mainly studies situational causes of behavior, personality psychology seeks to identify dispositional traits that influence behavior. In more practical terms, traditionally, social psychology is rooted in experimental designs while personality psychology relies more on psychometrics (Furr & Funder, in press).

Despite constituting seemingly different approaches, personality and social psychology are combined in several studies, as researchers in these areas understand that both are necessary to better explain human behavior (Furr & Funder, in press). The complementarity of both approaches was stated long ago in Lewin's (1936) equation, where behavior (B) is defined as a function of the person (P ; the dispositional traits) and the environment (E ; the situational variables):

$$B = f(P, E). \quad (1)$$

Equation (1) is only presented as a heuristic formula for how researchers in psychology should approach the study of behavior. This means that the function relating the person and the environment to behavior may vary on a case-by-case basis (Furr & Funder, in press).

A methodological framework for Lewin's equation can be proposed by combining experimental research design and causal inference (Pearl, 2000) with latent variable mixture modeling (Muthén, 2001). Figure 1 represents the fundamental problem faced by a social

psychologist. In this case, one is interested in testing the effect of a situational variable E on the behavior B . The node Z represents all the other possible causes for both E and B , not directly measured in the study. Therefore, Z represents both dispositional variables as well as other situational variables, less relevant for the study.

Figure 1. Representation of the fundamental problem for social psychologists: how to identify the causal effect of the environment (E) on the behavior (B), given possible unobserved confounder (Z)?

This type of problem is a common, and somewhat simple, example studied in the causal inference literature (Pearl, 2000). It is a known fact that, to be able to measure the causal effect of E on B in this scenario, it is necessary to control Z or to experimentally act on E (Pearl et al., 2016, p. 55). The procedure of experimentally acting on E causes the arrow between Z and E to disappear, meaning that both nodes become independent (Pearl, 2000). This new scenario is represented in Figure 2, where E is grayed out to represent that this variable was experimentally acted on. To fully determine the SOFA framework, Z must then be decomposed into two mixture components: a stochastic error (v); and the dispositional trait (P). Mixture components are subpopulations within a sample (Muthén, 2001). In the present case, the mixture components represent the two subpopulations of effects of unobserved

variables on behavior: stochastic and systematic (due to individual differences in P). The SOFA framework can then be represented by the modeling of behavioral data from implementations of the following equation

$$B = f(E) + F(v, P), \quad (2)$$

where $F(\cdot)$ is the mixture model relating to both the effects of the stochastic error and those of the dispositional variables on the behavior.

Figure 2. Representation of the SOFA framework for the case where P (dispositions) and v (random noise) are both independent of E (environmental variable experimentally acted on).

It is important to note that, according to field theory, the person and the environment are considered as a “constellation” of interdependent factors (Lewin, 1946). This means that Figure 2 is not the only possible scenario regarding the effects of experimentally acting on the environment. The other possible scenario is represented in Figure 3, where a causal effect departs from the environment to the mixture of secondary environmental effects and the person. In this case, it is not possible to derive an identifiable model, as P is not directly observed and, therefore, any type of mediation model from partial to full mediation of the effect of E on B by P may fit the data equally well.

It is also worth adding that, for Lewin (1936; 1946), the person (P) can be understood as: (i) attributes of the individual (i.e., fixed dispositional traits or momentary dispositional states); (ii) the individual perception of the environment; and (iii) the individual perception of the relations between the environment with oneself. If Figure 2 is assumed as the most likely scenario for the experimental task, P is better interpreted as a dispositional trait (therefore, reasonably independent of the situation; Fridhandler, 1986). However, if Figure 3 is assumed as the most likely scenario for a given experimental task, further theoretical assumptions regarding both P and the underlying causal process (Pearl, 2000) must be taken into account before a measurement model can be derived. Because the present study is an introduction to the SOFA framework, and also because the scenario represented in Figure 3 is highly dependent on particular theories and research context, we focus in the rest of the paper only on implementing a model based on the context expressed in Figure 2.

Figure 3. Representation of the SOFA framework for the case where P (dispositions), v (random noise) or both are not independent of E (environmental variable experimentally acted on).

Assessing dispositions with Stochastic Frontier Analysis and ideal performance analysis

A similar reasoning as the one that originated Eq. 2 was previously employed in economic modeling (e.g., Aigner et al., 1977). One particular method, known as stochastic frontier analysis (SFA; Aigner et al, 1977), is already recognized as a robust analytical tool to assess the efficiency of business' firms. The SFA model can be seen as a conjoint measurement model (Luce & Tukey, 1964), as it simultaneously estimates a production function, as well as the efficiency of firms. A production function is usually interpreted as a threshold of maximal possible production of outputs, given certain levels of inputs. Efficiency is defined as the distance of particular firms from the production function. Firms on the production function are relatively efficient (i.e., produce the maximal possible output, considering the amount of inputs). Firms below the production function are relatively inefficient (i.e., produce less output than they should, considering the amount of inputs). SFA also takes into account that measurement may be imperfect and, therefore, with expectancy equal to zero, values vary randomly around the production function.

The SFA model, with output y and a set of inputs $\mathbf{x}$, can be represented as

$$y = f(\mathbf{x}; \boldsymbol{\beta}) + v - u, \quad (3)$$

where $\boldsymbol{\beta}$ is the set of regression coefficients for $\mathbf{x}$, v is a stochastic component, similar to the error term in a regression model, and u is the non-negative inefficiency component. Note, Eq. 3 is simply a generic regression representation, where the error term is decomposed into the additive effect $v - u$. For Eq. 3 to be identifiable, independence within $\mathbf{x}$ and between $\mathbf{x}$, v , and u is assumed. It is also necessary to specify distributional assumptions of the error components (Greene, 1990). For the stochastic component v , generally a normal distribution with a mean equaling 0 is used. For the inefficiency component, truncated normal, exponential, or gamma distributions are common choices.

In terms of interpretation, when SFA is extended to SOFA, both assumptions (independence and error terms' distributions) also lead to a reasonable model to be implemented. As illustrated in Figure 2, experimentally controlling the situational variable, which corresponds to the set $\mathbf{x}$ in Eq. 3, guarantees that, at least, the situational effects on behavior are identifiable. This means that experimentally controlling the situational variable is necessary for the interpretability of the model estimates. The second necessary condition is that u , paralleled with P in Eq. 2, is a psychological trait and, therefore, independent of the situation. The use of a mixture modeling procedure with a 0 mean normal distribution for the stochastic error and a non-negative component for the dispositional variable is the same as stating that the “optimal” behavior is determined by situational contingencies; hence, the name of the framework. “Situational optimization” means that the models developed in the SOFA framework should be based on analyses of functions that find the optimal effect of situational variables.

Another possible interpretation for Eq. 3, from a psychological perspective, is that it represents a type of ideal observer analysis (Geisler, 2003). In ideal observer analysis, a rational, or utility maximizing, behavior can be deduced from the experimental task. If no constraints are posed, then the observer achieves the ideal performance. The ideal performance is the upper limit of performance imposed by the design of the experimental task, and it is impossible for the observer to perform better than ideal. The production function in Eq. 3, therefore, is the ideal performance given the situation, and P is the trait of the observer that constrains the performance. For instance, imagine a psychophysical study where the independent variable is the ratio of noise over signal for a series of stimuli. The dependent variable is the average number of errors when identifying a signal. In this case, the dispositional variable, which could be defined as “visual acuity”, cannot improve the

behavior of signal identification beyond the ideal performance established by the situational contingencies.

It is important to note that, while the ideal performance interpretation of the production frontier is necessary for SOFA, the theoretical assumption that it is impossible for the observer to perform better than ideal is not. The reason for that is because in several scenarios there is no evidently “ideal” or “rational” behavior for the observer to emit. On the other hand, on these scenarios, different challenges for model identification exist, as both P and v could have a location parameter equal to 0 but no identifiable scales (as they represent latent variables). Due to these complexities, in this study, we paralleled our implementation with the more traditional implementation of the SFA, where dispositional variables are assumed to follow a non-negative distribution (i.e., all values are below the regression function).

In terms of application, Eq. 3 can be extended to different types of SFA models. Traditional linear and log-linear models are likely to be the most commonly used SFA models (Griffin & Steel, 2007). However, semi- and nonparametric estimation techniques (Fan et al., 1996), as well as time-varying inefficiency techniques (Cornwell et al., 1990; Kumbhakar, 1990) have also been considered. In this study, we focused exclusively on semiparametric models for cross-sectional data. The reason for this is twofold. First, because linear models are the default in many analyses used in psychology and, therefore, non-linear and non-monotonic relations are usually overlooked (Beller & Baier, 2013). The second reason is that even in the traditional IRT literature, some authors have defended that semi- and nonparametric models should become the default, or at least initial diagnosing tools, instead of the more traditional parametric and linear, or linearized, models (e.g., Ramsay, 1991; Sijtsma & van der Ark, 2017).

Fitting a nonparametric SFA model

In the implementation process of a semiparametric SFA model, it is common to use a two-step approach, as first proposed by Fan et al. (1996). In the first step, a semiparametric or nonparametric regression technique is used to estimate the conditional expectation. Previous studies (Ferrara & Vidoli, 2017) applied general additive models (GAMs), kernel, and locally estimated scatterplot smoothing regression (Loess) techniques for estimating this first step. In the second step the stochastic error and the inefficiencies are estimated by maximizing a pseudo-likelihood function (Fan et al., 1996). Figure 4 represents this approach, where the graph on the left represents the result after applying the first step and the graph on the right represents the frontier (line) estimated after the second step.

Figure 4. Two-step approach to estimate semiparametric SFA models.

In this study, we developed a Bayesian semiparametric SFA model, following similar approaches as in Griffin and Steel (2004) and Tsionas and Mallick (2019). The model for predicting the behavioral response y_j from the j^{th} individual is defined as

$$y_j = \beta_0 + \sum_{k=1}^K \beta_k \psi_{jk} + u_j, \quad (4)$$

where β_0 is the intercept, and u_j is the estimate of the dispositional trait for individual j . The $\sum_{k=1}^K \beta_k \psi_{jk}$ expression represents the summation of all the basis transformation of the

manipulated independent variable for individual j , x_j , using a polynomial Gaussian radial basis function. The basis is calculated with

$$\psi_{jk} = \left(\frac{x_j - m_k}{\sigma_x} \right)^{\phi_k}, \quad (5)$$

where m_k is the value of the k^{th} knot, ϕ_k is the adjusted exponent of knot k , and σ_x is the standard deviation of x .

In terms of distributions required for the full Bayesian specification of the model, y_j is assumed to follow a normal distribution with mean μ_{yi} and standard deviation v , which is assumed a priori to come from a gamma distribution with shape and rate equal to .001. The dispositional trait u is assumed to follow an exponential distribution with rate parameter equal to λ , which a priori follows a gamma distribution with shape and rate equal to .001. The intercept of the regression model, β_0 , is assumed a priori to come from a normal distribution with mean 0 and standard deviation 1. The regression coefficients, β_k , are assumed a priori to come from a Laplace distribution with mean 0 and scale 1. This Laplace prior was used since it can be considered as a Bayesian implementation of the LASSO regression (Park & Casella, 2008), and, therefore, it can be used to prevent overfitting of the model by reducing the effect of less relevant knots. The knots' exponents, ϕ_{0k} , are drawn from a binomial distribution with bias equal to ζ and maximum possible degree equal to p . The bias is drawn from a beta 1-1 distribution, while p is set by the researcher. In our Bayesian approach, the model seeks to estimate what the best exponent for each knot should be. As we use both polynomial Gaussian radial basis functions and the Laplace prior for the regression coefficients, our Bayesian model can be considered to be a combined GAM/LASSO regression, with an adaptive step for the bases' exponents.

The Bayesian model in our SOFA approach is shown in the net representation in Figure 5, following Lee's (2008) graphical standards. The observed variables are represented by shaded nodes and the unobserved variables are represented by unshaded nodes. Discrete

variables are represented by square nodes, while continuous variables are represented by circular nodes. Stochastic variables are represented by single-bordered nodes, and deterministic variables are represented by double-bordered nodes. Finally, encompassing plates are used to denote independent replications of the graph structure within the model.

Figure 5. Bayesian implementation of situational optimization function analysis (SOFA).

It seems relevant to mention that the parameters p (order) and m_k (knots) are hyper, or tuning, parameters. This means that their values are not estimated, but rather chosen by the researcher before the data analysis. An automatic process for choosing the knots and order is with cross-validation (e.g., Ruppert, 2002). Cross-validation, however, usually increases the computational complexity of an analysis and does not scale well for large datasets (Magnusson et al., 2019), especially for nonparametric Bayesian models. Therefore, some simpler heuristics may be used. Regarding the knots, enough values should be chosen in order to capture essential features. For instance, if the predictor variable ranges from 0 to 1, an equal interval sequence of knots (e.g., 0, .2, .4, .6, .8, and 1) may be used. When it comes to order, it is typically chosen depending on how much nonmonotonicity may affect the model. Even orders will generate parabola-like relations. Odd orders will generate not decreasing or not increasing relations. Because 2 is the smallest possible even number to

generate nonmonotonic parabola-like relations, and 3 is the smallest possible odd number to generate nonmonotonic not decreasing/increasing relations, it is reasonable to set p as 2. This heuristic decision may help to prevent data overfitting by decreasing the error variance that may be captured by the model.

Another issue with nonparametric models is the fact that the model may be non-identifiable, i.e., the number of equally good solutions is infinite (Bamber & Van Santen, 2000). Even if the model is identifiable, the large number of parameters poses a challenge for many algorithms, as the algorithms may be stuck in local (i.e., sub-optimal) solutions for the values of the parameters. In the case of Bayesian estimation, the use of Markov Chain Monte Carlo (MCMC; Gamerman & Lopes, 2006) algorithms guarantees that the correct uncertainty distribution regarding the most likely values of the parameters (i.e., the posterior distribution) is found. Very flat distributions may indicate that the model is not identifiable, while multimodal solutions indicate the existence of many local or global solutions. Proper initial parameter values may help improve the performance of the MCMC algorithm, as long as the model is identifiable. Random initial parameter values followed by an increased number of simulation chains and longer simulation runs, despite being more computationally expensive, are an effective alternative for tuning initial parameters values.

Construct validity by joint modeling

A relevant matter in psychometric research is how to demonstrate that a given psychometric tool is really measuring what it aims to measure; this is known as validity research (Borsboom et al., 2004; Cronbach & Meehl, 1955). In traditional psychometrics, the content of test items and the correlations between tests with similar content are known as content validity and construct validity, respectively (Messick, 1989). In the SOFA framework, there will not be usually enough content to derive a general interpretation of what the latent

variable may be. For instance, if an experimental setting of a game favors competition over cooperation, then it is reasonable to assume that individuals who cooperate more have some dispositional trait (e.g., cooperative tendencies) that leads them to behave against the situational pressure (e.g., Goto, 1996). However, we cannot infer, from the behavior pattern alone, what exactly this dispositional trait is. For this reason, content validity in the SOFA framework is dependent on other evidences of construct validity.

One possible alternative for assessing validity in this context would be to use joint modeling (Turner et al., 2013). Joining different types of modeling in a single hierarchical framework allows for data with different sources to influence each other parameters, increasing interpretability of the parameters. Turner et al. (2013) used joint modeling for constraining parameters of a model from behavioral data using a neuroimaging model. In the present application, the idea is to combine both the SOFA approach with traditional psychometrics. For instance, the data from a questionnaire could be modeled by a two-parameter logistic item response model (2PLM; Swaminathan & Gifford, 1985) and the data from the experiment could be modeled by our Bayesian implementation. Then, estimates of the dispositional traits from both models could be correlated, as it is similarly done in traditional psychometric analysis of convergent validity (Carlson & Herdman, 2012).

In traditional psychometric analysis, convergent validity can be better explored using structural equation modeling (Raines-Eudy, 2000). This approach is normally preferred over using correction for attenuation of correlating scores estimated from two different tests or questionnaires (Osborn, 2003). The reason is that structural equation modeling has a natural correction for the measurement error, giving more reliable estimates of the correlations and factor loadings (Bagozzi, 1981). In this sense, joint modeling can be thought of as an extension of structural equation modeling where different measurement models are used for

each type of data, dependent on the data generating process of each type of data. This allows for much more complex types of models and theories to be estimated and tested.

Turner et al. (2013) proposed using the multivariate normal distribution to joint modeling parameters of interest. This approach needs no additional modifications if each parameter is indeed better described by a normal distribution. In the present application, if questionnaire/test data are to be used to be joint modeled with our Bayesian SOFA model, then the normal distribution fits well this type of data and model (i.e., the 2PLM). For the Bayesian SOFA model, however, the dispositional trait is better modeled using a non-negative distribution. Here we proposed to use exponential distribution. To model dependencies with different marginal distributions, one can use what is called copula dependence (Genest & MacKay, 1986). A copula is a multivariate cumulative distribution function (CDF) based on the fact that marginal CDFs are always uniformly distributed and, therefore, the marginals of a copula can be modeled using a uniform distribution (Embrechts & Hofert, 2013). From this fact, any type of dependence between continuous variables can be modeled using copulas and then be converted to the appropriate distribution using its marginals (Colonus, 2016). One easy way to fit copulas is to use the multivariate normal distribution to draw random values for the variables or parameters of interest, calculate the marginals' CDFs and then apply the quantile function to convert the values to the most appropriate targeted distribution for each variable or parameter (Meyer, 2013).

Simulation study

Method

Simulated data with three different sample sizes (100, 250 or 500 data points) for 100 iterations were generated. These data points were obtained using one of these three functions

(quadratic function, power function and a trigonometric function, respectively) or data generating processes (DGPs):

$$y = -x^2 + x + v + u; \quad (6)$$

$$y = 1.5^x + v + u; \quad (7)$$

$$y = x\sin(\pi x) + x\cos(\pi x) + v + u. \quad (8)$$

The variables x , v and u were drawn from a multivariate normal distribution with mean zero, variances equal to 1 and covariances equal to 0. This was done to guarantee that these values would be uncorrelated, as assumed by the SFA and SOFA models. Copula CDF transformation was used to convert the distribution of the dispositional trait, u , into a standardized truncated normal distribution, with lower bound equal to 0. Equations 6 (Quad), 7 (Power) and 8 (Sin/Cos), with v and u excluded, are represented from left to right in Figure 6.

Figure 6. DGPs' functions used to test the models' performance.

The simulated data were analyzed with four models: GAM-SFA; Kernel-SFA; Loess-SFA; and Bayesian-SOFA. For the GAM-SFA, smooth terms were calculated using penalized regression splines with dimension equal to -1 (Wood, 2003). The weight functions of the Kernel-SFA were computed using expected Kullback-Leibler cross-validation (Li & Racine, 2004). For the Loess-SFA, (Cleveland et al., 1992) the fit was made using weighted

points in relation to each value of the predictor variable and its neighborhood. The neighborhood was calculated by the ‘maximum distance’ fixed to 1.7 (the default value in the R package used). Finally, for the Bayesian-SOFA, p was fixed as 2 and the knots were calculated as the equidistant quantiles (from 0 to 1) of the observed values of the predictor variable, with length equal to $K = \lceil \sqrt{n - 2} + 1 \rceil$, where n is the sample size. The model was run for 65000 iterations, with a 5000 burn-in period and 5000 adaptation samples for each of the 4 chains, with no thinning.

To compare the performance of the models, three measures of accuracy were used. The first measure was the Spearman correlation between the true values of u and its estimated values. This measure was used since the latent variables and their estimates are expected to be monotonically related (Junker & Sijtsma, 2000). The next two measures used were the mean absolute error (MAE) and the root-mean squared error (RMSE) between the true values of u and their estimated values. When the values of MAE and RMSE are larger than zero and equal, it means that the bias is equal in all u ’s of the scale. If the bias is larger for some values of u in comparison to others, then RMSE will be larger than MAE. The most effective model will be the one with correlations closest to 1, and MAE and RMSE the most similar and closest to 0.

All the simulations and data analyses were conducted using the R software (R Core Team, 2020) and are available from the corresponding author upon request. To fit the GAM-SFA, the Kernel-SFA and the Loess-SFA, the `semsfa` package was used (Ferrara & Vidoli, 2018). To fit the Bayesian-SOFA, the model was implemented using the JAGS language and software (Plummer, 2003), interfacing with R by means of the `jagsUI` package (Kellner, 2019). We used the `gam` function from the `mgcv` package (Wood, 2012) to estimate the residuals of the additive regressions used to measure the error to calculate the bias.

Results

We first compared the overall performance of all the methods as displayed in Table 1. Per column, the best performances are in bold. Table 1 shows that Bayesian-SOFA and Kernel-SFA models have the same level of correlations with the true scores, followed by the GAM-SFA and the Loess-SFA model. In terms of the mean absolute error (MAE), the Bayesian-SOFA model has the smallest value, followed by the Kernel-SFA model. The GAM-SFA and the Loess-SFA model have very similar values of MAE. In terms of the root-mean squared error (RMSE), the same pattern is observed that was found with MAE.

Table 1

Overall performance of each method, measured by Spearman correlations, MAE and RMSE.

Method	Correlation	MAE	RMSE
General Additive Models (GAM-SFA)	.915	.875	1.107
Kernel smoothing (Kernel-SFA)	.924	.843	1.063
Locally estimated scatterplot smoothing (Loess-SFA)	.911	.876	1.109
Bayesian Implementation (Bayesian-SOFA)	.924	.474	.647

Note. MAE = mean absolute error. RMSE = root mean-squared error.

The effect of sample size on the accuracy of the models can be observed in Table 2. This time, numbers in bold are used to indicate which sample size, within each model, provided the best results. For the three frequentist methods (GAM, Kernel and Loess), the same pattern was found: Spearman correlations with the true scores increase as the sample size increases. MAE and RMSE, however, are better with a medium sample size (250 data points) than when compared with the large sample size (500 data points). For the Bayesian model, it is possible to perceive that increased sample size yields improvements over all the accuracy measures.

Table 2

Performance of each method, measured by Spearman correlations, MAE and RMSE, compared by sample size.

Method	Sample size	Correlation	MAE	RMSE
General Additive Models (GAM-SFA)	100	.857	1.036	1.299
	250	.933	.786	.994
	500	.954	.804	1.030
Kernel smoothing (Kernel-SFA)	100	.884	.931	1.147
	250	.933	.785	.992
	500	.954	.813	1.050
Locally estimated scatterplot smoothing (Loess-SFA)	100	.854	1.000	1.240
	250	.929	.805	1.023
	500	.952	.824	1.064
Bayesian Implementation (Bayesian-SOFA)	100	.892	.451	.594
	250	.929	.513	.705
	500	.950	.457	.643

Note. MAE = mean absolute error. RMSE = root mean-squared error.

The effect of the data generating process (DGP) on the accuracy of the models can be observed in Table 3. As was the case in Table 2, numbers in bold are used to evaluate the effect of the conditions (in this case, DGP) within each method. This time, the same pattern was observed for all the methods: Spearman correlations, MAE and RMSE, were better for the monotonic condition (Eq 7), followed by the quadratic condition (Eq 6), and worse for the trigonometric function (Eq 8).

Table 3

Performance of each method, measured by Spearman correlations, MAE and RMSE, compared by DGP.

Method	DGP	Correlation	MAE	RMSE
General Additive Models (GAM-SFA)	Equation 6	.914	.878	1.113
	Equation 7	.927	.809	1.005
	Equation 8	.902	.939	1.205

Kernel smoothing (Kernel-SFA)	Equation 6	.927	.826	1.041
	Equation 7	.932	.791	.985
	Equation 8	.912	.913	1.163
Locally estimated scatterplot smoothing (Loess-SFA)	Equation 6	.919	.842	1.061
	Equation 7	.920	.840	1.052
	Equation 8	.894	.949	1.223
Bayesian Implementation (Bayesian-SOFA)	Equation 6	.927	.500	.695
	Equation 7	.941	.298	.399
	Equation 8	.903	.623	.847

Note. MAE = mean absolute error. RMSE = root mean-squared error.

Empirical example

For illustration purposes, the present empirical example uses the data of a study that was carried out to measure cooperation/competition dispositions. In experimental social psychology, social dilemmas are games used to evaluate how people choose and are influenced by the context to behave in a particular manner (Van Lange et al., 2013). One of the most famous games is the iterated prisoner dilemma. This game was chosen as the optimal behavior is always to compete, especially when the other player is a very competitive person (or, in our case, when the simulated player acts more competitively). Therefore, there is a clear expected effect of the situation that can be modeled by the SOFA approach.

Method

Participants. Data from 66 participants, with mean age of 22.7 years and 50.8% women, were collected. No other sociodemographic data were collected.

Instruments. Two instruments were used: the cooperation-competition scale (Coop-Comp Scale; Teixeira et al., 2010) and an iterated prisoner dilemma game. The Coop-Comp Scale is composed by 11 sentences related to how much the respondent values shared rather than individual gains (e.g., “I think that group work is more important than winning”).

Participants had to indicate to what extent they agree to each sentence, ranging from 1 (totally disagree) to 5 (totally agree). Competition-related items (e.g., “I worry when other people perform better than me”) had their keys inverted so higher scores in all items represented a more cooperative attitude.

The prisoner dilemma is a “roleplay” game used in game theory and social dilemmas to study how rational behavior may lead to competition. In this game, two individuals have to decide to compete (betray the other player) or to cooperate (remain silent about their crime) as if they were prisoners being independently enquired by police detectives. If both decide to compete, each of them serves two years in prison. If one of them decides to compete but the other decides to cooperate, the one who competed will be set free and the one who cooperated will serve three years in prison. Finally, if both decide to cooperate, both of them will serve only one year in prison. In the iterated prisoner dilemma, players have to decide how to behave at each iteration (or more informally, each round). In our implementation, there were 10 rounds and the results of each round were summed over to return the total years in prison the participant would have to serve in case the scenario of the game was real. Also, the second player was a simulated participant. The probability of choosing to cooperate for each simulated participant was drawn from a standard uniform distribution. This probability was the predictor variable in the SOFA model.

Procedures. All the participants were invited from mailing lists and social network groups of several large universities. The study started with a brief explanation on its objectives, as well as a declaration of the intention of the participant to take part in the study. Next, the participant had to read a brief description of the iterated prisoner dilemma and start the game. The game had a duration of 10 rounds: at each round the participant had to choose between cooperating or competing. After finishing the game, the participant responded to the Coop-Comp Scale and to the sociodemographic (i.e., sex and age) questions.

Data analysis. We first reversed all negatively keyed items and applied item factor analysis to check for consistency of the scale (Wirth & Edwards, 2007). This analysis showed that item 11 from the Coop-Comp Scale was the only item with a factor loading below .30 and, therefore, it was deleted from the rest of the analyses. After that, we used the joint modeling approach. Data from the cooperation-competition scale was modeled using a Bayesian 2PLM, with discriminations constrained to be positive (as all items were reversed). Data from the iterated prisoner dilemma game was modeled with the Bayesian-SOFA approach, with higher values of dispositional trait representing the tendency to compete. We used four different analyses to assess convergent validity. First, we estimated the parameters of the Bayesian-SOFA model and of the 2PLM separately. Then, the attitudes' estimate of the 2PLM was correlated with the dispositional estimate of the Bayesian-SOFA model. For the second model, both models were modeled jointly, with a multivariate normal copula used to couple the attitudes' estimates of both the 2PLM and the Bayesian-SOFA model. The last two analyses were conducted forcing the estimates to be completely uncorrelated and perfectly negatively correlated, respectively. The perfect negative correlation was used because a maximum score at the Coop-Comp scale indicates someone that is very cooperative, while a high score in the Bayesian-SOFA model indicates someone that is very competitive.

Five fit indices were used to compare the models. The first fit index was the correlation, and the corresponding highest density intervals, between the estimate made with the 2PLM and the estimate made with the SOFA Bayesian model. The second index was the deviance information criterion (DIC; Spiegelhalter et al., 2014), which was used to measure the quality of the fit of the joint models; in which lower values are better. The next three fit indices were calculate using a similar procedure as the one proposed by Lewandowsky and Farrell (2010) to calculate the DIC difference (the third fit index), the DIC likelihood ratio

(LR; the fourth fit index) and the DICs weights (w ; the fifth fit index) of the models. The DIC difference represents how many units the compared model is larger than the best model; a value closer to 0 is better. The LR represents by what magnitude the compared model is as good as the best model; a value closer to 1 is better. Finally, w is the posterior probability of the models; values closer to 1 are better than values closer to 0.

Results

The results from the empirical example are presented in Table 4. The negative correlations found in the first two models, which allowed the correlation to be freely estimated, show that both measurements give similar results. Higher scores in the Coop-Comp scale characterize a person with more cooperative attitudes, while scores closer to 1 in the Bayesian SOFA model characterize someone with a competitive disposition. However, the results show that the joint model has a stronger correlation estimate when compared to the correlations made with separately estimated parameters. The strong negative correlation in the joint model ($-.83 [-.99, -.52]$) indicates that the coop-comp attitudes are what make people choose not to compete in a more competitive inducing situation, in a similar fashion as traditional convergence validity had shown before.

It is also possible to see that the model that best fits the data, according to the deviance information criterion (DIC), is the joint modeling of coop-comp scale and the SOFA approach (1,685.59) which allows for correlations to be freely estimated. Evaluating the LR, it is possible to see that even the second-best model is considerably worse than the joint model. The w shows that the joint model is almost the exclusive model in the capacity of explaining the data at hand, with a posterior probability equal to 1. Therefore, the models that force no correlation and perfect negative correlation between the dispositional estimates of

the coop-comp scale and the iterated prisoner dilemma do not provide a relatively good fit to data, when compared with the joint model.

Table 4

Different procedures to estimate construct validity.

	Correlation	DIC	ΔDIC	LR	w
Separately estimated	-.44 [-.62, -.22]	-	-	-	-
Joint modeling	-.83 [-.99, -.52]	1,685.59	0	1	1
No correlation	0	1,730.76	45.17	0	0
Perfect correlation	-1	1,720.64	35.05	0	0

Notes. DIC = deviance information criterion. Δ DIC = variation of DIC. LR = likelihood ratio. w = posterior probability of the models.

Discussion

This study had three aims: (1) to propose an analytical framework for psychometric modeling data dependent on a contextual variable, which we named situational optimization function analysis (SOFA); (2) to propose and compare a Bayesian implementation of SOFA with three other maximum likelihood-based approaches; and (3) to illustrate how construct validity can be conducted in the SOFA framework with joint modelling using real data. We showed that the SOFA framework, at least in the current Bayesian implementation, is ideal for situations where there is an expected optimal or ideal behavior, given situational contingencies.

Research paradigms such as the ideal observer (Kuss et al., 2005) or games with Nash equilibrium (Kalai & Lehrer, 1993) are good examples of possible designs where SOFA can be used to test hypotheses.

We believe that the SOFA framework is one of the first efforts towards an “experimental psychometrics” perspective. Other approaches, such as cognitive psychometrics (Batchelder, 2010) and explanatory item response theory (De Boeck & Wilson, 2004), have similar motivations of deriving measurement models that are more interpretable and theoretically meaningful. However, the SOFA framework is probably the

first to require experimental manipulations to be able to generate valid measures of psychological phenomena, attenuating consequences of not taking possible differences in measurement by experimental condition into account. For instance, it is known that even completely random data, like the flip of coins, can be fitted to a Rasch model (Wood, 1978). This happens because item response modeling, in general, is only decomposing error in arbitrary identifiable instances of the models' parameters (Engelhard, 1984). This is a problem because, even if the model fits the data well, the achieved measurement may not be an accurate representation of the real unobserved process that originates the observations/behavior. Therefore, when SOFA derived (or any experimentally derived) models are used, variance is decomposed in more meaningful components, generating more reliable and valid measures of the intended constructs.

The results of our simulation study indicate that the Bayesian implementation of the SOFA approach can outperform more traditional maximum-likelihood based SFA models in terms of mean absolute error (MAE) and root-mean squared error (RMSE) between the true scores and their estimated values. It has to be acknowledged, however, that the correlation between the true scores and their estimated values is quite similar, regardless of the model used. Because MAE does not increase with the variance of bias but RMSE does (Willmott & Matsuura, 2005), our results lead to the conclusion that more extreme values of the dispositional trait are especially biased when estimated with the maximum-likelihood based models. The better performance by the Bayesian implementation of the SOFA framework is probably due to the adaptive GAM/LASSO regression characteristic of the non-parametric regression in the model. However, caution should be taken in the sense that the great degree of flexibility of the adaptive GAM/LASSO regression can lead to overfitting (Wood, 2004) and further investigation should be carried out to better understand the optimal setting to assure that the LASSO part of our Bayesian model can decrease overfitting.

From our empirical example, we have three major outcomes. First, in traditional structural equation modeling, the joint modeling of two measurement models increases the correlation between the latent variables, due to the natural correction for attenuation of the correlations for the latent scores estimated by the model (Bagozzi, 1981; Osborne, 2003). The second outcome may be of special interest to researchers in social dilemmas, as our results present evidence for a strong effect of cooperation-competition attitudes on behavior. Of course, our study was only an empirical example and did not test possible correlations with time perspective and values (Milfont & Gouveia, 2006), social norms (Thøgersen, 2008), conversation (Sally, 1995), or other factors important for the study of cooperation-competition behaviors in social dilemmas. However, our model and results suggest that cooperation-competition attitudes may have more importance than previously thought. Future studies on social dilemmas should investigate this matter further in more complex experimental designs and using other subjective variables to improve the evidence of validity regarding the estimates of a SOFA-derived measurement model.

The third outcome derived from our empirical study is one of the main contributions of the SOFA framework: subjective variables can be measured indirectly, with virtually no impact of response biases. The u parameter in the Bayes-SOFA model reflects a behavioral pattern that is not easily identified by either the participant or the researcher, as it is dependent on the expected response function. This means that it is not easy (or maybe not even possible) for participants to fake their behavior to express a specific value of u . In our empirical study, for instance, u is the deviation of the percentage of decisions to cooperate, given the expected response for the value of the predictor variable. Our results show that this value is strongly correlated to a psychometric measure of cooperation-competition attitudes and, therefore, it is reasonable to infer that both measurements are measuring the same construct. However, u is probably a less biased measure of this construct, as participants

could not know beforehand what the expected response function would be. Therefore, SOFA may be an important alternative to measure subjective variables in contexts where response biases may be a major concern (Furnham, 1986).

Despite the positive results of our study and the positive prospective of the SOFA framework, some limitations should also be noted. The first limitation is that our implementations of SOFA are univariate. Extending the univariate models to multivariable (i.e., with many predictor variables), or even multivariate (i.e., with many criterion variables) case is quite straightforward, as it is only necessary to duplicate the basis (in the multivariable case) or to model the covariance of the criterion variables (in the multivariate case). However, it may also be interesting to extend the models for cases of longitudinal, spatial and multilevel data, as well as when any or both the predictor and criterion variables are categorical or ordinal. In fact, in our implementations, both the predictor and criterion variables were assumed to be continuous, which is not very common in psychology. Therefore, the main priority for future methodological studies on the SOFA framework should be to develop a model which combines our Bayesian implementation with logistic regression and ANOVA models to better model variables found in the studies.

The second limitation is the assumption that minor situational variables (or random noise) as well as dispositions are not affected by the main situational variable. This assumption was contrasted with another possible scenario (which is also more likely in psychological research) in Figure 3. This scenario can be modeled by means of vector generalized linear and additive models (Yee, 2015), but identifiability issues must be evaluated case by case. It can also be the case that more than a single dispositional trait or state influences the behavior in a given context, which would require the disposition P to be decomposed into $P_1, P_2, \dots, P_l$, which, of course, would result in a non-identifiable model. Using joint-modeling, however, the P_l dispositional variables can be fixed to be perfectly

correlated with other estimates of the constructs and, therefore, allow for a more detailed validation of the processes underlying the behavior.

Another limitation, which is probably the main limitation of our Bayesian implementation, is the fact that, for identifiability reasons, all models tested in this study assume that the dispositional trait can only have positive values. In other words, our models assume that an optimal behavior, given the context, exist. This was expressed in Figure 2 and contrasted with a more reasonable scenario in psychological research in Figure 3. It is safe to say that researchers in psychology are not only interested in studies where an optimal behavior can be clearly defined. In fact, most research in psychology, and more specifically social psychology, does not define (or it does not even make sense to define) what would be the “expected” or “ideal” behavior given a situation. From a computational/statistical perspective, using combined moving average and Gaussian mixture models may be a way of surmounting this limitation (e.g., Yu et al., 2013), which may be of uttermost necessity to improve the applicability of the SOFA framework in psychological research. Therefore, future studies should also focus on identifying the conditions in which the model assuming an “optimal behavior” does not hold or does not make sense, as well as to what practical impacts this assumption has in the estimation of the dispositions.

In general, we believe that the SOFA framework has a great potential to improve measurement in research where response biases or situational influences are expected to affect behavior. For instance, it could be used in clinical research on the effects of antidepressants doses, and psychotherapy and stressor events’ frequency on self-injuring behavior. A longitudinal extension of the Bayes-SOFA could be used to measure how a construct (e.g., depression) changes over time, given the situational influences. The conventional psychometric approach would be to repeatedly apply the same instrument on depression, which would probably already be confounded with for example self-injuring

behavior. Another example could be a study where the participants were randomly assigned to do retrieval practice of different lengths and with different frequencies and were later evaluated in their performances in four standardized tests. In this case, a longitudinal/multilevel extension may also be used. The conventional psychometric approach would be to fit a growth curve model (Muthén, 2002), therefore not considering how the experimental conditions affect the performance in the standardized tests.

As a final note, it is important to explicitly locate the SOFA approach in the psychometric literature. The success of the factor analytical and item response paradigms (Rust & Golombok, 2014) is related to their easiness of understanding and generality of application. For instance, exploratory factor analysis is widely used in validation research on any type of data form, a questionnaire or test, no matter if it is an intelligence or a personality test, for instance (Thompson & Daniel, 1996). This can be seen as both a weakness and a strength of the factor analytical procedure, as it is a consequence of lack of theoretical assumptions on the underlying psychological process (Sijtsma, 2012). The SOFA approach is also basically atheoretical (with the exception of the Lewin's equation and optimal observer inspirations). However, because models developed from it need to be applied to experimental data, more objective interpretations and straightforward tests of theories are easier to make, but applications in non-experimental contexts may be limited. Future studies should try to identify when it is possible to apply SOFA to non-, or quasi-, experimental settings.

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