A Bayesian Approach to Estimating Reciprocal Effects with the Bivariate STARTS Model
using Markov Chain Monte Carlo

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Abstract

The bivariate Stable Trait, AutoRegressive Trait, and State (STARTS) model provides a general approach for estimating reciprocal effects between constructs over time. However, previous research has shown that this model is difficult to estimate using the maximum likelihood (ML) method (e.g., nonconvergence). In this article, we introduce a Bayesian approach for estimating the bivariate STARTS model and implement it in the software Stan. We discuss issues of model parameterization and show how appropriate prior distributions for model parameters can be selected. Specifically, we propose the four-parameter beta distribution as a flexible prior distribution for the autoregressive and cross-lagged effects. Using a simulation study, we show that the proposed Bayesian approach provides more accurate estimates than ML estimation in challenging data constellations. An example is presented to illustrate how the Bayesian approach can be used to stabilize the parameter estimates of the bivariate STARTS model.

Keywords: STARTS model, state-trait models, reciprocal effects, Bayesian methods, Markov chain Monte Carlo (MCMC) techniques

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Studying reciprocal effects between psychological constructs (e.g., self-esteem and depression) across time is a central topic of longitudinal research. In many areas of psychological research, scholars are interested in whether changes in one construct are related to changes in other constructs (Little, 2013; Marsh et al., 2005; Orth et al., 2020). A general modeling approach for estimating reciprocal effects was provided by Kenny and Zautra (1995, 2001), who proposed the STARTS (Stable Trait, AutoRegressive Trait, and State) model. In this model, individual differences across time in a construct are decomposed into three sources of variation: first, a time-invariant stable trait; second, a time-varying, partly stable autoregressive trait; and third, an occasion-specific state component. In its bivariate version, the STARTS model allows decomposing longitudinal associations between two constructs into stable between-person associations (i.e., a correlation between the time-invariant stable traits) and temporal within-person dynamics (i.e., autoregressive and cross-lagged effects between the autoregressive traits).

A further attractive feature of the bivariate STARTS model is the inclusion of occasion-specific state factors that control for transient measurement error (Campbell & Kenny, 1999; McCrae, 2015) when estimating the autoregressive and cross-lagged effects at the within-person level. This was also recently pointed out by Usami, Murayama, and Hamaker (2019) in their unified framework of various longitudinal models, in which they clarified that the bivariate STARTS model extends the random intercept cross-lagged panel model (RI-CLPM; Hamaker et al., 2015) to allow for measurement error. The RI-CLPM has been introduced as an extension of the traditional cross-lagged panel model (CLPM) that controls stable trait factors. It has received considerable attention in applied methodological literature (Bailey et al., 2020; Berry & Willoughby, 2017; Ehm et al., 2019; Ruzek &

Schenke, 2019; Orth et al., 2020) but, at least in its original version (see Mulder & Hamaker, 2020), assumes that the variables are measured without error.

Major obstacles to applying the bivariate STARTS model in research practice are the serious estimation problems, such as nonconvergence or inadmissible estimates (e.g., negative variances), that frequently occur with the use of maximum likelihood (ML) estimation (Kenny & Zautra, 2001). For example, Orth and colleagues (2020) were interested in estimating the reciprocal effects of low self-esteem and depression in 10 longitudinal samples with different statistical models. Using ML estimation, they were unable to fit the bivariate STARTS model for almost all data sets and encountered serious estimation problems. These estimation problems are in line with previous simulation research that showed that with ML estimation, relatively large sample sizes (at least 500) and a large number of measurement waves are needed to obtain valid estimates for the univariate and bivariate STARTS models (Cole et al., 2005; Geiser, 2021; Lüdtke et al., 2018; Usami, Todo, & Murayama, 2019).

Lüdtke et al. (2018) introduced a Bayesian approach to estimating the univariate STARTS model and showed that the Bayesian approach avoids estimation problems with appropriate prior distributions. Furthermore, the Bayesian approach allows for the stabilization of parameter estimates by specifying weakly-informative prior distributions (see Depaoli & Clifton, 2015; Smid et al., 2020; Zitzmann et al., 2020). This study proposes a Bayesian approach to estimating the bivariate STARTS model and implementing it in the software Stan (Stan Development Team, 2019). We discuss issues of model parameterization and show how appropriate prior distributions for model parameters can be selected. More specifically, we suggest the four-parameter beta distribution as a flexible prior distribution for the autoregressive and cross-lagged effects. In a simulation study, we evaluated the statistical properties of the proposed Bayesian approach and compared it with ML estimation. Finally, we apply the bivariate STARTS model to a data set from Orth et al. (2020) and demonstrate that the Bayesian approach provides interpretable estimates of the longitudinal association

between depression and self-esteem. We start by briefly describing the univariate STARTS model.

Univariate STARTS Model

Let us assume that a single variable y_{it} (e.g., self-esteem) was assessed for persons i ($i = 1, \dots, N$) across time t ($t = 1, \dots, W$) in a longitudinal study with W equally spaced measurement waves. In the STARTS model (Kenny & Zautra, 1995, 2001), the measurements at each time point are modeled as a function of three independent latent variables:

$$y_{it} = \mu_{yt} + T_{yi} + A_{yit} + S_{yit} \quad , \quad (1)$$

where μ_{yt} represents the mean at time t , T_{yi} is a time-invariant stable trait (e.g., a completely stable component of self-esteem), A_{yit} is a time-varying autoregressive trait (e.g., a changing component of self-esteem that is partly stable), and S_{yit} is an occasion-specific state component that does not differentiate between time-specific effects and random measurement error (e.g., a component of self-esteem that is completely occasion-specific). All three latent variables are assumed to be mean-centered and normally distributed with variances; ϕ_{Ty} , ϕ_{Ay} , and ϕ_{Sy} . The stability of the autoregressive trait across time is given by:

$$A_{yit} = \beta_{yy} A_{yi,t-1} + D_{yit} \quad , \quad (2)$$

where β_{yy} indicates the part of A_{yit} that depends on the previous time point and is assumed to be invariant across time (i.e., the first-order autoregressive process). The disturbance term D_{yit} denotes a random component that is unrelated to previous time points and is normally distributed with a zero mean and variance ϕ_{Dy} . Furthermore, the STARTS model assumes that the variance explained by the stable trait (T_{yi}), the autoregressive trait (A_{yit}), and the state (S_{yit}) is the same at each measurement point (stationarity assumption). Given the first-order autoregressive structure of the STARTS model, this implies that the disturbance variance is a function of the autoregressive stability and autoregressive trait variance: $\phi_{Dy} = \phi_{Ay}(1 -$

β_{yy}^2). Thus, the univariate STARTS model estimates four parameters $\theta =$

$(\phi_{Ty}, \phi_{Ay}, \phi_{Sy}, \beta_{yy})$ to describe interindividual differences in stability and change over time.

This has been applied to constructs such as life satisfaction (Lucas & Donellan, 2007;

Schimmack et al., 2010), academic self-concept (Jansen et al., 2020), self-esteem (Alessandri

et al., 2016; Donellan et al., 2012; Wagner et al., 2016), loneliness (Mund et al., 2020), and

personality traits (Anusic et al., 2012; Anusic & Schimmack, 2016).

Examining the covariance structure implied by the STARTS model (Bollen, 1989) helps to understand how the model is used to represent longitudinal data. For W time points, the model-implied covariances can be written as a (nonlinear) function of the four STARTS model parameters and the time lag h :

$$\phi_{t+h,t} = \text{Cov}(y_{i,t+h}, y_{it}) = \phi_{Ty} + \beta_{yy}^h \phi_{Ay} + \mathbf{1}_{\{h=0\}} \phi_{Sy} \quad , \quad (3)$$

where $\mathbf{1}_{\{h=0\}}$ denotes an indicator function that takes a value of 1 for the lag 0 covariance

(i.e., the variance of the variable) and 0 otherwise. Assuming that the variable is standardized

(i.e., the covariance at time lag $h = 0$ is one), the STARTS model captures three essential

features of retest correlations that are often observed in empirical longitudinal data (Campbell

& Kenny, 1999). First, even for very short retest intervals, the retest correlations are never

one, which indicates that a substantial part of the individual differences in psychological

constructs is due to transient factors that are occasion-specific (Gnambs, 2014; McCrae,

2015). Second, retest correlations usually decrease with increasing time lags between

measurements (Little, 2013). This feature of retest correlations is captured by the

autoregressive component, which reflects changing factors that are partly stable across time

(and are modeled by a first-order autoregressive process; see β_{yy} in Equation 3). Third, retest

correlations for psychological constructs often do not approach zero, even across very long

time intervals (Anusic & Schimmack, 2016; Roberts & DelVecchio, 2000). This is reflected

in the STARTS model by a component that is completely stable over time.

Multivariate STARTS Model

The STARTS model has been extended to two variables measured over time (Kenny & Zautra, 1995; see also Zautra et al., 1995). Let us assume a second variable x_{it} (e.g., depression) that is also represented as a function of the STARTS model parameters. Then, the bivariate STARTS model is given by:

$$\begin{aligned} x_{it} &= \mu_{xt} + T_{xi} + A_{xit} + S_{xit} \\ y_{it} &= \mu_{yt} + T_{yi} + A_{yit} + S_{yit} \end{aligned} \quad , \quad (4)$$

where μ_{xt} and μ_{yt} represent the two means at time t , T_{xi} and T_{yi} are the stable trait factors, A_{xit} and A_{yit} are the time-varying autoregressive trait factors, and S_{xit} and S_{yit} are the occasion-specific states. As in the univariate version, interindividual differences in both variables are decomposed into three orthogonal variance components: stable trait variances (ϕ_{Tx} and ϕ_{Ty}), autoregressive trait variances (ϕ_{Ax} and ϕ_{Ay}), and state variances (ϕ_{Sx} and ϕ_{Sy}). In addition, three cross-variable covariances are estimated in the bivariate STARTS model: the covariance between the stable trait factors ϕ_{Txy} (e.g., whether individuals with high self-esteem, in general, are also more depressive), the covariance between the autoregressive trait factors ϕ_{Axy} (e.g., whether changes in self-esteem are associated with changes in depression), and the covariance between the state factors ϕ_{Sxy} (e.g., whether the completely occasion-specific parts of self-esteem and depression are associated).

The effects among the autoregressive traits across time are modeled as:

$$\begin{aligned} A_{xit} &= \beta_{xx}A_{xi,t-1} + \beta_{xy}A_{yi,t-1} + D_{xit} \\ A_{yit} &= \beta_{yy}A_{yi,t-1} + \beta_{yx}A_{xi,t-1} + D_{yit} \end{aligned} \quad , \quad (5)$$

where the coefficients β_{xx} and β_{yy} represent the autoregressive effects from $A_{xi,t-1}$ to A_{xit} , and from $A_{yi,t-1}$ to A_{yit} . The coefficients β_{xy} and β_{yx} represent the cross-lagged effects from $A_{yi,t-1}$ to A_{xit} , and from $A_{xi,t-1}$ to A_{yit} . For example, a negative β_{xy} coefficient would indicate that low self-esteem is negatively related to subsequent depression. Again, the disturbance terms D_{xit}

and D_{yit} denote random components that are unrelated to previous time points and are normally distributed with zero means, variances ϕ_{Dx} and ϕ_{Dy} , and covariance ϕ_{Dxy} . However, given the stationarity assumption, these variances and covariances of the disturbances are not estimated but implied by the other STARTS model parameters (Kenny & Zautra, 1995). Overall, there are a total of 13 parameters that are estimated in the bivariate STARTS model, which are summarized in the vector $\boldsymbol{\theta}$: six variances (ϕ_{Tx} , ϕ_{Ax} , and ϕ_{Sx} ; ϕ_{Ty} , ϕ_{Ay} , and ϕ_{Sy}), three covariances (ϕ_{Txy} , ϕ_{Axy} , and ϕ_{Sxy}), and four regression coefficients (β_{xx} , β_{yy} , β_{xy} , and β_{yx}). The bivariate STARTS model is presented as a path diagram in Figure 1.

The bivariate STARTS model can be generalized to a multivariate version. The resulting multivariate STARTS model can be written compactly in matrix notation as:

$$\mathbf{y}_{it} = \boldsymbol{\mu}_t + \mathbf{T}_i + \mathbf{A}_{it} + \mathbf{S}_{it} \quad , \quad (6)$$

where $\boldsymbol{\mu}_t$ is a vector of means, and $\mathbf{T}_i$, $\mathbf{A}_{it}$, and $\mathbf{S}_{it}$ are vectors of stable trait factors, autoregressive trait factors, and state factors, respectively. The effects of the autoregressive traits are modeled as:

$$\mathbf{A}_{it} = \mathbf{B}\mathbf{A}_{i,t-1} + \mathbf{D}_{it} \quad , \quad (7)$$

where $\mathbf{B}$ is the matrix of the regression coefficients that contains the autoregressive and cross-lagged coefficients. The model-implied covariance matrix can be expressed as a function of the model parameters and time lag h as:

$$\boldsymbol{\Sigma}_h = \text{Cov}(\mathbf{y}_{t+h}, \mathbf{y}_t) = \boldsymbol{\Sigma}_T + \mathbf{B}^h \boldsymbol{\Sigma}_A + \mathbf{1}_{\{h=0\}} \boldsymbol{\Sigma}_S \quad , \quad (8)$$

where $\boldsymbol{\Sigma}_T$ denotes the covariance matrix of the stable trait factors, and $\boldsymbol{\Sigma}_A$ and $\boldsymbol{\Sigma}_S$ are the covariance matrices of the autoregressive trait and state factors, respectively. The effects between the autoregressive factors at lag h are given in $\mathbf{B}^h$. For example, in a bivariate STARTS model with $\mathbf{y}_{it} = (x_{it}, y_{it})$, the implied lag- h covariance matrix $\boldsymbol{\Sigma}_h$ for $h \geq 0$ can be written as follows:

$$\begin{aligned}
\underbrace{\begin{pmatrix} \phi_{x_{t+h}x_t} & \phi_{x_{t+h}y_t} \\ \phi_{y_{t+h}x_t} & \phi_{y_{t+h}y_t} \end{pmatrix}}_{\Sigma_h} &= \underbrace{\begin{pmatrix} \phi_{Tx x} & \phi_{Ty x} \\ \phi_{Ty x} & \phi_{Ty y} \end{pmatrix}}_{\Sigma_T} + \underbrace{\begin{pmatrix} \beta_{xx} & \beta_{xy} \\ \beta_{yx} & \beta_{yy} \end{pmatrix}}_{\mathbf{B}^h} \underbrace{\begin{pmatrix} \phi_{Ax x} & \phi_{Ay x} \\ \phi_{Ay x} & \phi_{Ay y} \end{pmatrix}}_{\Sigma_A} \\
&\quad + \mathbf{1}_{\{h=0\}} \underbrace{\begin{pmatrix} \phi_{Sxx} & \phi_{Syx} \\ \phi_{Syx} & \phi_{Syy} \end{pmatrix}}_{\Sigma_S}.
\end{aligned} \tag{9}$$

In the case of $W = 4$ measurements, the model-implied covariance matrix is given by:

$$\mathbf{\Sigma}(\boldsymbol{\theta}) = \begin{pmatrix} \Sigma_0 & \Sigma_1 & \Sigma_2 & \Sigma_3 \\ \Sigma_1 & \Sigma_0 & \Sigma_1 & \Sigma_2 \\ \Sigma_2 & \Sigma_1 & \Sigma_0 & \Sigma_1 \\ \Sigma_3 & \Sigma_2 & \Sigma_1 & \Sigma_0 \end{pmatrix}, \tag{10}$$

where $\mathbf{\Sigma}(\boldsymbol{\theta})$ is a symmetric 8×8 block matrix that is partitioned into 2×2 submatrices, Σ_h . In Appendix A, we show that at least four measurement waves are needed to identify the parameters of the multivariate STARTS model. A sufficient precondition is that the covariance matrices Σ_T , Σ_A , and Σ_S are positive definite and that the coefficient matrix $\mathbf{B}$ is invertible.

Finally, it should be mentioned that in many longitudinal studies, the time intervals between the measurement occasions differ from wave to wave (e.g., one year between waves 1 and 2, and two years between waves 2 and 3). These nonequidistant time intervals can be considered in the STARTS model by recoding time to reflect a uniform metric across measurement waves. More specifically, let us assume that differences between measurement waves can be written as $t_2 - t_1 = (\delta_2 - \delta_1)\Delta$, where Δ is an integer. More generally, for $i > k$, we can write $t_i - t_k = (\delta_i - \delta_k)\Delta$, and the covariance is given as:

$$\text{Cov}(\mathbf{y}_{t_i}, \mathbf{y}_{t_k}) = \Sigma_T + \mathbf{B}_\Delta^{(\delta_i - \delta_k)} \Sigma_A. \tag{11}$$

The covariances for the other time lags can be obtained accordingly. Thus, the general estimation approach for the STARTS model does not change with nonequidistant time points. Only appropriate time differences need to be specified when raising powers of the matrix of

regression coefficients $\mathbf{B}_\Delta$ (Kuiper & Ryan, 2018; see also Jansen et al., 2020). Note that the estimated parameters have to be interpreted relative to the unit time interval Δ .

Maximum Likelihood Estimation of the STARTS Model

Maximum likelihood estimation is routinely used to obtain parameter estimates for the STARTS model (Orth et al., 2020; Usami, Murayama, & Hamaker, 2019). In the structural equation modeling (SEM) framework, the sample-based covariance matrix $\mathbf{S} =$

$\frac{1}{n} \sum_{i=1}^n (\mathbf{y}_i - \bar{\mathbf{y}})(\mathbf{y}_i - \bar{\mathbf{y}})'$ is used as the input, and the structure of the means is assumed to be saturated. The log-likelihood can then be written as (Bollen, 1989):

$$l(\boldsymbol{\theta}) = \log L(\boldsymbol{\theta}|\mathbf{S}) = -\frac{n}{2} [p \cdot \log(2\pi) + \log|\boldsymbol{\Sigma}(\boldsymbol{\theta})| + \text{tr}(\boldsymbol{\Sigma}^{-1}(\boldsymbol{\theta})\mathbf{S})] \quad ; \quad (12)$$

where tr is the trace operator, that is, the sum of the diagonal elements of a square matrix;

$|\boldsymbol{\Sigma}(\boldsymbol{\theta})|$ denotes the determinant of the matrix $\boldsymbol{\Sigma}(\boldsymbol{\theta})$; and the value $\hat{\boldsymbol{\theta}}_{\text{ML}} = \arg \max_{\boldsymbol{\theta}} l(\boldsymbol{\theta})$, that

maximizes $l(\boldsymbol{\theta})$, is the ML estimate. However, the desirable properties of ML (e.g., most efficient estimates) are based on asymptotic theory and are only guaranteed to hold with large sample sizes (Yuan & Bentler, 2007). It has repeatedly been shown that in small samples and with complex models, ML is prone to serious estimation problems such as failure to converge or inadmissible solutions (Anderson & Gerbing, 1984; Wothke, 1993).

To mitigate estimation problems, *constrained* ML estimation has been suggested for obtaining parameter estimates in SEM (Chen et al., 2001). In default, ML estimation parameter estimates are not constrained to any specific interval; for example, nothing prevents variance estimates from becoming negative (Held & Bové, 2014; Savalei & Kolenikov, 2008). In constrained estimation, the parameter space over which optimization is performed is restricted to admissible values (e.g., variances are constrained to be positive; Schoenberg, 1997). To this end, inequality constraints that restrict parameter estimates to lower and upper bounds must be specified (see Savalei & Kolenikov, 2008). More specifically, in the constrained estimation approach, a multivalued function, $\mathbf{h}$, is specified on the vector of

parameters, that is $\mathbf{h}(\boldsymbol{\theta}) > \mathbf{0}$. For example, if a parameter θ (e.g., a correlation) is supposed to be bounded by a lower bound l , and an upper bound u , that is $l < \theta < u$, the constraints are given as follows: $\mathbf{h}(\theta) = (\theta - l, u - \theta) > (0, 0)$. The constrained ML estimator is the parameter vector $\boldsymbol{\theta}$ that maximizes the log-likelihood in Equation 12 and fulfills the constraints that are imposed in $\mathbf{h}$. For the univariate STARTS model, Lüdtke et al. (2018) showed in simulation studies that constrained ML estimation clearly outperformed unconstrained ML estimation in terms of the frequency of estimation problems and the accuracy of the parameter estimates (see also Lüdtke et al., 2020). Therefore, we focus on constrained ML estimation in the present article and compare its performance to the Bayesian estimation approach discussed in the following section.

Bayesian Approach to Estimating the Bivariate STARTS Model

In the Bayesian approach, statistical inference is based on the posterior distribution, which is determined by the likelihood function and the prior distribution, $\pi(\boldsymbol{\theta})$, of model parameters (for a general introduction to the Bayesian approach, see Gelman et al., 2014; Jackman, 2009; Lynch, 2007). Using the observed data $\mathbf{S}$ and the prior distributions, the joint posterior distribution, $p(\boldsymbol{\theta}|\mathbf{S})$, of the parameters is determined by multiplying the likelihood with the prior:

$$p(\boldsymbol{\theta}|\mathbf{S}) = \frac{L(\boldsymbol{\theta}|\mathbf{S})\pi(\boldsymbol{\theta})}{\int L(\boldsymbol{\theta}|\mathbf{S})\pi(\boldsymbol{\theta})d\boldsymbol{\theta}} = L(\boldsymbol{\theta}|\mathbf{S})\pi(\boldsymbol{\theta})C \propto L(\boldsymbol{\theta}|\mathbf{S}) \cdot \pi(\boldsymbol{\theta}) \quad , \quad (13)$$

where $C = 1 / \int L(\boldsymbol{\theta}|\mathbf{S})\pi(\boldsymbol{\theta})d\boldsymbol{\theta}$ is a normalizing constant. As can be seen, the posterior distribution is proportional to the product of the likelihood and the prior. If a researcher is unwilling to make assumptions about a parameter, noninformative prior distributions that have only a minimal influence on the results are selected. Two methods for obtaining Bayesian estimates from the posterior distribution have been distinguished (Gelman et al., 2014): the penalized maximum likelihood (PML) and Markov Chain Monte Carlo (MCMC)

methods. In the following, we focus on the MCMC methods and discuss the PML method in the section “Concluding remarks”.

The MCMC methods apply simulation-based techniques and are motivated by the fact that the joint posterior distribution of the parameters is often difficult to evaluate in practice. This strategy is implemented in most general Bayesian software such as WinBUGS (Spiegelhalter et al., 2003), JAGS (Plummer, 2003), or Stan (Stan Development Team, 2019). They use MCMC algorithms to approximate the posterior distribution by iteratively sampling from conditional distributions. The most prominent MCMC methods are Gibbs sampling, Metropolis–Hastings sampling, and the no-U-turn sampler (Gelman et al., 2014; Junker et al., 2016).

In these methods, point and interval estimators for $\boldsymbol{\theta} = (\theta_1, \dots, \theta_q)$ are constructed from the posterior distribution by iteratively sampling from conditional distributions, constituting an MCMC chain ($m = 1, \dots, M$):

$$\boldsymbol{\theta}^{(m)} = \left(\theta_1^{(m)}, \dots, \theta_d^{(m)}, \dots, \theta_q^{(m)} \right) \sim p(\boldsymbol{\theta}|\mathbf{S}) \propto L(\boldsymbol{\theta}|\mathbf{S}) \cdot \pi(\boldsymbol{\theta}) \quad . \quad (14)$$

Usually, the initial samples are discarded (burn-in period) because the initial values are affected by the starting values (for a detailed discussion of assessing convergence in MCMC, see Gelman et al., 2014; Gill, 2007; Jackman, 2009). In MCMC, the mean and mode are often used as Bayesian estimates of the parameters. More specifically, the expected a posteriori (EAP) estimator for a single parameter θ_d is obtained by averaging across the M iterations, that is, $\hat{\theta}_{d,\text{EAP}} = M^{-1} \sum_{m=1}^M \theta_d^{(m)}$. The mode $\hat{\theta}_{d,\text{MAP}}$ can be defined as the univariate mode of the kernel density estimate (Silverman, 1998) of the univariate density for the sample of $\theta_d^{(m)}$ ($m = 1, \dots, M$).

Finally, it should be emphasized that in the presented MCMC approach, the parameter estimates are based on the marginal likelihood $L(\boldsymbol{\theta}|\mathbf{S})$ (see Equation 12) in which the latent variables $\boldsymbol{\eta}$, that is, individual factor scores of the stable trait, autoregressive trait, and

occasion-specific states, are integrated. Note that in many applications of MCMC-based latent variable modeling, a joint estimation approach is used that relies on the joint likelihood $L(\boldsymbol{\theta}, \boldsymbol{\eta}|\mathbf{X})$, where the latent variables $\boldsymbol{\eta}$ are included in the likelihood (Lee, 2007; Muthén & Asparouhov, 2012) and sampled at each draw from the posterior. However, when individual factor scores are not of interest, these become nuisance parameters that may deteriorate the computational efficiency of the MCMC algorithm (Hayashi & Arav, 2006; Hecht et al., 2020; Lüdtke et al., 2018).

Specification of Prior Distributions

In the case of a bivariate STARTS model (see Equations 4 and 5), we need to select a prior distribution for the covariance matrices, $\boldsymbol{\Sigma}_T$, $\boldsymbol{\Sigma}_A$, and $\boldsymbol{\Sigma}_S$, as well as the matrix of regression coefficients $\mathbf{B}$. For the covariance matrices, we suggest using a separation strategy to specify a separate prior distribution for the two variances and the correlation (Schoorman et al., 2016). For example, in the case of the 2×2 covariance matrix of stable trait factors $\boldsymbol{\Sigma}_T$, separate prior distributions for the variances ϕ_{Txx} and ϕ_{Tyy} and the correlation ρ_{Txy} are specified. Prior distributions for $\boldsymbol{\Sigma}_A$ and $\boldsymbol{\Sigma}_S$ can be specified accordingly.

When researchers are unwilling to incorporate prior information to estimate the bivariate STARTS model, uniform prior distributions can be employed for all model parameters. For the variance parameters, an improper uniform prior can be chosen that only restricts variances to be non-negative (Gelman et al., 2014; Muthén & Asparouhov, 2012). Note that some Bayesian software allows the specification of improper priors (e.g., Stan), while it is not allowed in other implementations (e.g., WinBUGS). In the latter case, the improper prior can be approximated by a uniform distribution on the interval [0,1000] that practically does not provide prior information. For the three correlations between the stable trait factors (ρ_{Txy}), the autoregressive trait factors (ρ_{Axy}), and the state factors (ρ_{Sxy}), as well

as the autoregressive parameters (i.e., β_{xx} and β_{yy}), and the cross-lagged coefficients (i.e., β_{yx} and β_{xy}), uniform distributions on the interval $[-1, 1]$ can be specified.

For stabilizing parameter estimates, we suggest employing an inverse-gamma (IG) distribution, which is commonly used as a prior distribution for variances. There are several options for specifying the parameters of the IG distribution (Gelman et al., 2014). In the context of the univariate STARTS model, Lüdtke et al. (2018) used the following specification:

$$\phi \sim \text{IG}(v_\phi/2, v_\phi\mu_\phi/2) \quad , \quad (15)$$

where μ_ϕ can be interpreted as a prior guess for the variance component and v_ϕ as the sample size of prior observations on which the prior guess is based (see Gelman et al., 2014; Hoff, 2009). If only little information is available about a variance component from previous studies, a small value for v_ϕ would be chosen so that the prior distribution will be only weakly centered around the prior guess μ_ϕ . A reasonable prior guess would be to assume that the total variance of the observed measurements for a construct is partitioned into variance components of equal magnitude when there is no previous knowledge. Therefore, if $\tilde{\mu}_{\phi x}$ is the prior guess for the total variance of one of the two constructs in the bivariate STARTS model, it would be assumed that $\mu_{\phi Tx} = \mu_{\phi Ax} = \mu_{\phi Sx} = 1/3\tilde{\mu}_{\phi x} = .33\tilde{\mu}_{\phi x}$. Through transforming the manifest variables by applying an appropriate scaling constant (e.g., the average standard deviation across the measurement waves), the total variance can be assumed to be one for both constructs (i.e., $\tilde{\mu}_{\phi x} = 1$, and $\tilde{\mu}_{\phi y} = 1$), and the IG distributions, for example, for variable x are given as follows¹: $\phi_{Tx} \sim \text{IG}(v_{\phi Tx}/2, v_{\phi Tx} \cdot .33/2)$, $\phi_{Ax} \sim \text{IG}(v_{\phi Ax}/2, v_{\phi Ax} \cdot .33/2)$, and $\phi_{Sx} \sim \text{IG}(v_{\phi Sx}/2, v_{\phi Sx} \cdot .33/2)$. Figure 2 (left panel) illustrates the influence of

¹ It should be emphasized that we standardize the variables across waves. For example, in the case of $W = 4$ and four variables, X_1 , X_2 , X_3 , and X_4 , each of the four variables would be divided by the average standard deviation across the four variables.

the prior sample size on the shape of an IG distribution for a prior guess of $\mu_\phi = 1/3$. It is evident that with a larger prior sample size v_ϕ , the distribution puts more probability mass around the prior guess one third. Lüdtke et al. (2018) found that prior sample sizes of 1 or 3 had a stabilizing influence on the estimates of the variance parameters of the univariate STARTS model in a wide range of data constellations.

We suggest employing a four-parameter beta distribution (Lüdtke et al., 2020; Zeng, 1997) for the three correlations and the regression coefficients. The four-parameter beta distribution extends the beta distribution (which is restricted to the interval between 0 and 1) to arbitrary intervals and allows a flexible specification of prior distributions for bounded parameters. The four-parameter beta distribution $Y \sim \text{Beta4}(a, b, l, u)$ can be obtained by shifting a beta distributed random variable² $X \sim \text{Beta}(a, b)$ by lower (l) and upper (u) bounds: $Y = l + (u - l)X$. The density of Y is then given as:

$$f(x) = (u-l)^{-1} B_{a,b}^{-1} \left(\frac{x-l}{u-l} \right)^{a-1} \left(\frac{u-x}{u-l} \right)^{b-1}, \quad x \in [l, u] \quad . \quad (16)$$

The four-parameter beta distribution can be reparameterized as $Y \sim \text{Beta4}(\mu, v, l, u)$ with prior guess $\mu = a(a+b)^{-1}$, and prior sample size $v = a+b-2$. The prior sample size is motivated by the fact that the uniform distribution, which reflects complete ignorance about a parameter, is given by setting $a = b = 1$. Thus a prior sample size of $v = 1 + 1 - 2 = 0$ corresponds to the uniform prior on $[l, u]$. The parameters of the original specification $Y \sim \text{Beta4}(a, b, l, u)$ can be obtained as $a = (v+2)(\mu-l)(u-l)^{-1}$, and $b = (v+2)(u-\mu)(u-l)^{-1}$.

For instance, for a regression coefficient that can be assumed to be bounded between -1 and 1 , we suggest a $\text{Beta4}(\mu_\beta, v_\beta, -1, 1)$ and interpret μ_β as a prior guess for the regression coefficient and v_β as the sample size of prior observations on which the prior guess is based

² The density of the beta distribution $X \sim \text{Beta}(a, b)$ on the interval $[0, 1]$ is given as $f(x) = B_{a,b}^{-1} x^{a-1}$, where $B_{a,b}$ is a normalizing constant. The mean and the variance can be computed as $E(X) = a/(a+b)$, and $\text{Var}(X) = ab/[(a+b)^2(a+b+1)]$.

(Lüdtke et al., 2018). Figure 2 (right panel) shows for a prior guess of $\mu_\beta = 0.2$ (e.g., small positive cross-lagged effect) how increasing v_β (i.e., prior sample sizes of 1, 3, and 10) changes the shape of the four-parameter beta distribution. As can be seen, with a small value for v_β , the prior distribution is only weakly centered around the prior guess μ_β . Note that the four-parameter beta distribution with a prior guess $\mu_\beta = 0$ and a prior sample size $v_\beta = 0$ corresponds to a uniform distribution on $[-1, 1]$, which reflects complete ignorance about the size of the regression coefficient. Prior distributions for the correlations can be constructed accordingly. In Appendix B, we provide the Stan code for a bivariate STARTS model with four-parameter beta distributions as prior distributions for the correlations and regression coefficients.

Simulation Study

In the simulation study, we evaluated the performance of different approaches to estimating the bivariate STARTS model. The study had two main goals. First, we compared the Bayesian approach with noninformative (uniform) prior distributions to constrained ML estimation. We expected that constrained ML avoids the estimation problems of unconstrained ML, even for small sample sizes. Furthermore, we assumed that the Bayesian approach produces more stable estimates of reciprocal effects than constrained ML, particularly in conditions with smaller sample sizes and a minimum number of four measurement points. As we did not incorporate any prior information (i.e., select uniform distributions) in the Bayesian approach in this specification, the performance difference between the Bayesian approach and ML estimation can be explained by the fact that the MCMC method was used (see Lüdtke et al., 2020).

Second, we investigated the extent to which the parameter estimates of the Bayesian approach are sensitive to different specifications of the prior distributions. We expected that by selecting weakly informative and correctly specified prior distributions (i.e., prior guess matches the true values of the data-generating model), the estimates of the bivariate STARTS

model parameters can be further improved. In addition, we evaluated the performance of the Bayesian approach with prior distributions that incorporate the default prior guesses outlined above (i.e., 0.33 for the variances and 0 for the correlations and regression coefficients). Since, in practice, these prior guesses are likely to deviate from the true parameters, we contrasted the stabilizing effects of the Bayesian approach with mildly misspecified prior distributions against the Bayesian approach with uniform prior distributions.

Simulation Model and Conditions

The data-generating model was a bivariate STARTS model, as shown in Figure 1. We set the stable trait variances for both variables to 0.25, autoregressive trait variances to 0.55, and occasion-specific state variances to 0.2. These variance proportions are in line with findings from longitudinal research that applied the univariate STARTS model (Jansen et al., 2020; Mund et al., 2020; see also Usami, Todo, & Murayama, 2019). The total variance of the manifest variables was set to 1. The correlation between the stable trait factors was set to 0.7, the correlation between the autoregressive traits was fixed at 0.4, and the correlation between the state factors was set to 0.2. Finally, we set the autoregressive effect for both variables to $\beta_{xx} = \beta_{yy} = .5$, and fixed the cross-lagged effects to $\beta_{yx} = 0$ and $\beta_{xy} = .3$.

The number of observations was set to $N = 200, 500$, and 1000 . A sample size of 200 can be considered relatively small for longitudinal latent variable models (see Usami, Todo, & Murayama, 2019). The number of measurement points was set to $W = 4, 6$, and 9 . For each of the $3 \times 3 = 9$ conditions, we generated 500 simulated datasets.

Analysis Models and Outcomes

Each of the simulated datasets was analyzed with both unconstrained and constrained ML estimations and the different specifications of the Bayesian approach. The software lavaan (Rosseel, 2012) was used for ML estimation (see online supplemental material for the

lavaan code).³ Furthermore, in constrained estimation, the parameter space over which the optimization is performed was restricted to admissible values. To avoid estimates located at the boundaries, parameters can be coerced away from the boundaries by a small value ϵ (i.e., $\epsilon = .01$; see Lüdtke et al., 2020). That is, variances were constrained to values greater than $0 + \epsilon$, and correlations and regression coefficients were constrained to take values between $-1 + \epsilon$ and $1 - \epsilon$. The standard errors were calculated by employing delete- k Jackknifing (Efron, 1981) in which k subjects were removed from each jackknife sample. For a sample size of N , k was chosen as $k = N/50$. Hence, the computation of standard errors relied on 50 Jackknife replications.

The Bayesian approach was implemented using Stan software. We varied the prior distributions for the 13 parameters of the bivariate STARTS model. First, we specified a model in which we selected noninformative uniform prior distributions (i.e., constant across the admissible parameter space) for all model parameters. For the six variances, we specified improper uniform distributions with a constant value from 0 to infinity. As in constrained ML estimation, it can improve the numerical stability of parameter estimates to coerce parameters further away from the boundary. Thus, we selected $\text{Unif}(\epsilon, \text{Inf})$ as prior distributions for the variances (again with $\epsilon = .01$). For the three correlations and the regression coefficients, we set the prior distributions to $\text{Unif}(-1 + \epsilon, 1 - \epsilon)$. Second, we specified prior distributions with prior guesses that matched the true values of the data-generating model. For the variances, correlations and regression coefficients, we set the prior guesses in the IG distribution and four-parameter beta distributions to the true values and only varied the prior sample sizes (e.g., $v_\phi = 1$ and 3 for the variance parameters). Note that these prior

³ Note that for the estimation of the bivariate STARTS model in SEM software three nonlinear constraints for the variances and covariances of the disturbances need to be specified (Kenny & Zautra, 1995). For example, the variance of the disturbances for x is constrained as follows: $\phi_{Dxx} = \phi_{Axx} - \beta_{xx}^2 \phi_{Axx} - \beta_{xy}^2 \phi_{Ayy} - 2\beta_{xx}\beta_{xy}\phi_{Axy}$. The constraint for the covariance is given by: $\phi_{Dxy} = \phi_{Axy}(1 - \beta_{xx}\beta_{yy} - \beta_{xy}\beta_{yx}) - \beta_{xx}\beta_{yx}\phi_{Axx} - \beta_{yy}\beta_{xy}\phi_{Ayy}$. These constraints are not necessary in the Bayesian approach (see Appendix B).

distributions were correctly specified and only differed in the amount of information incorporated in the prior specification (i.e., prior sample sizes of 1 and 3). These two specifications were labeled True_1 and True_3. Third, we specified IG distributions as prior distributions for the six variances with the default prior guesses of 0.33 and prior sample sizes of 1 and 3, while employing uniform prior distributions for the remaining model parameters (labeled as IG_1 and IG_3, respectively). This allows investigating the impact of mildly misspecified prior distributions for the variance parameters on the estimates of correlations and regression coefficients. Finally, the weakly informative IG prior distributions were combined with four-parameter beta priors $\text{Beta4}(\mu_\rho, v_\rho, -1 + \varepsilon, 1 - \varepsilon)$ and $\text{Beta4}(\mu_\beta, v_\beta, -1 + \varepsilon, 1 - \varepsilon)$ for the correlations and regression coefficients, using the default prior guesses of $\mu_\rho = \mu_\beta = 0$ outlined above, and prior sample sizes of 1 and 3 (labeled as Beta4_1 and Beta4_3, respectively). Note that all prior distributions were mildly misspecified in these two specifications, except for the cross-lagged effect of x on y , which was set to 0 in the data-generating model (i.e., $\beta_{yx} = 0$).

Before running the main simulation study, we investigated the behavior of the MCMC sampler in preliminary simulations by inspecting two criteria: (a) the potential scale reduction factor (PSR; Gelman et al., 2014), and (b) the effective sample size. Applying these two criteria suggested that an average chain length of 10,000 iterations with a burn-period of 5,000 iterations was sufficient to provide a good approximation of the posterior distribution. We assessed both the mean and mode of the posterior distribution (EAP and MAP) as Bayesian point estimates. Furthermore, the Bayesian credible interval (BCI) was defined by the 2.5th and 97.5th percentiles of the posterior distribution (Gelman et al., 2014)

We used three criteria to evaluate the performance of the different approaches: bias, root mean square error (RMSE), and coverage rate. The bias was calculated as the difference between the mean parameter estimate and the true population parameter value from each design cell. We assessed the overall accuracy with the (empirical) RMSE, which combines

the squared empirical bias and variance of the parameter estimates to quantify the overall accuracy. Finally, we determined the coverage rate of the 95% confidence intervals. A coverage rate between 91% and 98% was considered acceptable (Muthén & Muthén, 2002).

Results

For unconstrained ML estimation, we encountered serious convergence issues across all conditions (see Table 1). These strongly depended on the sample size and the number of measurement waves. For example, in the conditions with $N = 200$, 30.80% of the replications converged for $W = 4$, and 53.8% of the replications converged for $W = 6$. By employing constrained ML estimation, all models converged. Thus, constrained ML helped mitigate the estimation problems that were frequently observed with unconstrained ML estimation. However, with small sample sizes and/or few measurement occasions, we often did not obtain standard error estimates with constrained ML estimation. For example, in the condition with $N = 200$ and $W = 4$, standard error estimates were computed for only 70% of the replications. This issue diminished with an increasing sample size as well as an increasing number of measurement occasions, such that under conditions with $N \geq 500$ and $T \geq 6$, standard errors were provided for at least 95% of the replications. Note that the following analyses were based on all solutions (including those without standard error estimates), as we did not rely on standard errors computed by lavaan but used Jackknifing. For Bayesian model specifications, in contrast, we did not encounter issues concerning the quality of the estimation procedure. After 10,000 iterations, the proportion of replications in which the PSR value was below 1.05, and the effective sample size was above 400 for all parameters was at least 99% across all conditions. In the following analyses, we focused on constrained ML estimation and compared it to the results from the Bayesian model specifications.

Table 2 shows the bias for the different estimation approaches. For simplicity, we focus on the results for conditions with a sample size of $N = 500$ and present the results for

the three variance components for only one of the two variables⁴ (see online supplemental material for information about the other conditions). With a larger number of measurement waves ($W \geq 6$), constrained ML estimation (CML), and the Bayesian approach with uniform distributions (Uniform) provided approximately unbiased estimates. However, when the number of measurement waves was $W = 4$, the estimates for some of the STARTS model parameters (especially the correlation between the state factors and the cross-lagged effect β_{yx}) were substantially biased. The bias of the Bayesian approach only slightly decreased when correctly specified prior information was added to the model (True_1 and True_3), suggesting that larger prior sample sizes would be needed in order to increase the precision of the Bayesian estimates.

The results for the RMSE (see Table 3) show that the Bayesian approach with uniform distributions clearly outperformed constrained ML across all conditions and STARTS model parameters (CML: $M = 0.12$, range = 0.02 to 0.70; Uniform: $M = 0.08$, range = 0.02 to 0.23). As expected, the differences were most pronounced in the condition with $W = 4$, and almost negligible with $W = 9$. The results also show that selecting a more informative correctly specified prior distribution (True_3 instead of True_1) slightly improved the accuracy of the parameter estimates. This was additionally true for the Bayesian approach with the weakly informative default prior distributions, indicating that the Bayesian approach produced more accurate estimates than constrained ML, even when the prior distribution was slightly misspecified. Furthermore, the results show that the two Bayesian models that included weakly informative distributions for the correlations and regression coefficients (Beta4_1 and Beta4_4) were more accurate than the two approaches that only added information to the estimation of the variance parameters (IG_1 and IG_3). The mode of the posterior distribution

⁴ Results for the variance components of the second variable were essentially equal to those of the first variable.

(MAP) as a Bayesian point estimate showed a slightly better performance than the EAP and further reduced the bias in the Bayesian estimates (see online supplemental material).

Finally, we assessed the coverage rates of the 95% CI for the ML approach and the 95% BCI for the Bayesian approach (see Table 4). The coverage rates produced by constrained ML were often too low in conditions with $W \leq 6$ ($M = 89.25\%$, range = 72.20% to 99.20%) but were almost acceptable with $W = 9$ ($M = 93.79\%$, range = 85.20% to 99.00%). For the Bayesian approach with uniform prior distributions (Uniform) and correctly specified prior distribution (True_1 and True_3), the coverage rates were often too high with $W = 4$ ($M = 99.11\%$, range = 94.60% to 100%), but were close to the nominal value with $W \geq 6$ ($M = 96.63\%$, range = 92.80% to 99.80%). The coverage rates provided by the Bayesian approach with slightly misspecified default priors were very similar but showed a tendency to be slightly too high ($M = 97.83\%$, range = 92.40% to 100%).

Summary and Discussion

The main findings of the simulation can be summarized as follows: First, constrained ML shows clear improvement over unconstrained ML estimation and produced converged solutions for all replications. This allowed for a fair comparison between the ML and Bayesian approaches because no replications had to be excluded from the analysis. Second, the Bayesian approaches outperformed the ML approach and produced more accurate parameter estimates, particularly in conditions with a minimal number of measurement waves. However, with more empirical information (i.e., large sample size and nine-wave designs), the differences between the two approaches vanished, and the Bayesian estimates became approximately equivalent to the ML estimates. Third, it should be emphasized that even the Bayesian approach with uniform prior distributions, which added no information to the likelihood, provided considerably more accurate estimates than the ML approach. Thus, some of the performance gains for the Bayesian approach should be ascribed to the fact that the MCMC method utilizes marginal posteriors to obtain Bayesian point estimates (Lüdtke et

al., 2020). Fourth, the effect of selecting different prior distributions for the bivariate STARTS model was more pronounced in more problematic longitudinal designs (e.g., small sample sizes and only four measurement waves). However, even for slightly misspecified prior distributions, the Bayesian approach performed at least as good as the constrained ML. It is an important topic for future simulation research to further investigate the impact of misspecified prior distributions with more extreme data-generating parameters (e.g., very small variance components or large autoregressive effects) and more strongly misspecified prior information.

Data Example

In this section, we provide a reanalysis of a dataset that was used by Orth et al. (2020) to compare different longitudinal models for estimating the reciprocal effects between self-esteem and depression (the original data set can be retrieved from <https://osf.io/5rjsm/>). The considered dataset stems from the My Work and I (MWI) study (Meier & Spector, 2013), comprising 663 individuals reporting work experiences and well-being. Data were collected in 2009 and 2010 in Switzerland at five measurement occasions, spaced two months apart. Self-esteem and depression were measured using the 10-item Rosenberg Self-Esteem Scale (Rosenberg, 1965) and the 20-item Center for Epidemiologic Studies Depression Scale (Radloff, 1977). Cronbach's alpha reliabilities of both scales were satisfactory (Orth et al., 2020, reported α s of .89 for both self-esteem and depression). As in Orth et al. (2020), we employed scale scores as observed indicators. For both constructs, we transformed the scale scores by dividing each score by the average standard deviation across the five waves. This allowed us to assume that the total variance was one for each construct.

The bivariate STARTS model was estimated with constrained ML estimation and the Bayesian approach with five different specifications of the prior distributions.⁵ First, we

⁵ We also tried analyzing the data with unconstrained ML. However, similarly to Orth et al. (2020), we were unable to retrieve results.

employed uniform priors for all model parameters (Uniform). Second, we specified weakly informative IG distributions for the variance components, employing a prior guess of 0.33 and prior sample sizes of 1 and 3, respectively. For correlation and regression coefficients, uniform priors were specified (IG_1 and IG_3). Third, the weakly informative IG prior distributions for the variance components were combined with four-parameter beta prior distributions for correlation and regression coefficients. For the four-parameter beta prior distributions, we specified a prior guess of zero with prior sample sizes of 1 and 3 (Beta4_1 and Beta4_3).

In the considered dataset, 27% of the observations were missing. The amount of missing data increased across measurement occasions, with 0.60% and 43% of the observations missing at the first and fifth measurement occasions, respectively. We used multiple imputations to deal with missing data, employing the R package mice (van Buuren & Groothuis-Oudshoorn, 2011). We used predictive mean matching by imputing every variable with missing values based on a regression model with all other variables as predictors and 20 imputed datasets. For constrained ML, Jackknife standard errors were pooled according to Rubin's rules (Rubin, 1987). In the Bayesian model specifications, we combined posterior draws from the analysis of each imputed dataset and used the combined draws to summarize the posterior distribution, that is, to obtain EAPs and posterior standard deviations (see Zhou & Reiter, 2010). Furthermore, for each model in the Bayesian approach, we specified a total of 10,000 iterations, from which 5,000 iterations served as a burn-in period. For each model parameter and in every imputed dataset, the PSR was smaller than 1.05, and the effective sample sizes were larger than 400.

We observed considerable differences in parameter estimates across the six model specifications, with the largest differences being observable between constrained ML and the Bayesian specifications (see Table 5). Concerning the variance components, there was considerable disagreement between estimates of stable trait and autoregressive trait

components, particularly for self-esteem. The ML approach yielded variance estimates for the stable trait and autoregressive components of 0.16 and 0.70, suggesting that the vast majority of differences in self-esteem across time can be attributed to time-varying partly stable differences. In contrast, the Bayesian specifications ascribed a much more considerable amount of variance (0.42 to 0.54) to the stable trait factor and a smaller amount (0.33 to 0.46) to the autoregressive trait factor. Nevertheless, the most pronounced differences occurred in the correlation between the stable trait components. While constrained ML yielded a very small positive estimate, the Bayesian specifications yielded a large negative association (correlations between -0.63 and -0.53), suggesting that individuals with generally higher self-esteem tend to have lower depression scores.

Overall, the different Bayesian model specifications led to comparable conclusions concerning the longitudinal associations between self-esteem and depression. One consistent finding for the Bayesian models was that smaller posterior standard deviations were obtained when more informative prior distributions were selected (e.g., Beta4_3 instead of Uniform). Across all Bayesian model specifications, both cross-lagged effects were negative, indicating that low self-esteem and high depression, was associated with an increase in subsequent depression and a decrease in self-esteem. We observed a larger effect of self-esteem on subsequent depression than vice versa. However, both cross-lagged effects were not statistically significantly different from zero.

Concluding Remarks

The bivariate STARTS model provides a general approach for modeling the longitudinal association between two constructs across time (Usami, Murayama, & Hamaker 2019). However, previous research has shown that relatively large sample sizes and a large number of measurement waves are needed to obtain valid parameter estimates. In this article, we introduced a Bayesian approach to estimating the bivariate STARTS model in the software Stan. In a simulation study, we evaluated the Bayesian approach and showed that it

outperforms the ML approach (unconstrained and constrained) that has been used to estimate the STARTS model, particularly in conditions with a small number of measurement waves.

The results of our study could be extended in several directions. First, constructs are often conceptualized as latent variables that are measured by a set of multiple indicators that include measurement error. The state factors in the STARTS models confound occasion-specific effects and measurement error (Donnellan et al., 2012; Geiser, 2021). It would be an important topic for future research to extend the proposed Bayesian approach to multiple indicators that could be ordered-categorical or continuous (Lee, 2007; Song & Lee, 2009). This would allow researchers to disentangle occasion-specific state effects from random measurement errors to estimate reciprocal effects (Usami, Murayama, & Hamaker, 2019). The bivariate STARTS model with multiple indicators would also be an extension of the recently introduced multiple indicator version of the random intercept cross-lagged panel model (Mulder & Hamaker, 2020), which does not include occasion-specific state effects.

Second, we restricted ourselves to settings that are typical for panel data. However, we believe that the bivariate STARTS model offers outstanding potential for analyzing intensive longitudinal designs that are applied, for example, in diary and experience sampling research. These designs are usually characterized by many measurement waves (Bolger & Laurenceau, 2013), and future studies should investigate if a large number of measurement waves might compensate for small sample sizes (Hecht & Zitzmann, 2020; Lüdtke et al., 2018). It would also be interesting to allow the STARTS model parameters (e.g., variance components and regression coefficients) to vary across persons in this context. This extension would integrate the bivariate STARTS model with the recently introduced dynamic SEM framework (Asparouhov et al., 2018; McNeish & Hamaker, 2020) and enable researchers to accommodate designs in which each subject is observed at potentially different time points (i.e., unbalanced design).

Third, we applied simulation-based MCMC techniques to calculate Bayesian estimates from the posterior distribution. An alternative method for obtaining Bayesian estimates is to maximize the log-posterior function directly (i.e., the sum of the log-likelihood and the prior density; see Equation 13) and calculate the multivariate mode of the posterior distribution (Gelman et al., 2014). This is also referred to as penalized ML because the logarithm of the prior distribution can be interpreted as a penalty term that is added to the log-likelihood function (Cole et al., 2014; Cousineau & Helle, 2013). Penalized ML shares the advantage of traditional ML in that a deterministic optimization of the log-posterior is performed. The constrained ML estimation approach used in the present study can be regarded as a variant of penalized ML when uniform prior distributions are imposed on the admissible parameter space (Rindskopf, 2012; Lüdtke et al., 2020). It would be interesting to further explore the potential of penalized ML to estimate the bivariate STARTS model in challenging data constellations. However, given the superior performance of MCMC with uniform priors in comparison with constrained ML estimation, we do not expect that penalized ML will outperform MCMC estimation in small sample sizes or few waves.

Finally, at a more conceptual level, it should be emphasized that with non-experimental panel data, very strong assumptions are needed to give the reciprocal effects a causal interpretation in the bivariate STARTS model. The main challenge in estimating causal effects is to control for all potential covariates that affect the treatment variable as well as the outcome (Hernán & Robins, 2020; Morgan & Winship, 2015). For the reciprocal effects in the bivariate STARTS model, this would imply that the adjustment for the stable trait and autoregressive trait factors provides an appropriate control for all confounding variables. In many applications, this assumption does not seem plausible, and researchers would be well advised to include further covariates in the analyses when they are interested in drawing causal conclusions (Usami, 2020; VanderWeele et al., 2020). Nevertheless, we believe that

the bivariate STARTS model is a powerful tool for understanding the longitudinal association between constructs across time.

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Table 1

Convergence Rates for Unconstrained and Constrained ML

<i>N</i>	<i>Unconstrained ML</i>			<i>Constrained ML</i>		
	<i>W</i> = 4	<i>W</i> = 6	<i>W</i> = 9	<i>W</i> = 4	<i>W</i> = 6	<i>W</i> = 9
200	0.31	0.54	0.72	1.00	1.00	1.00
500	0.32	0.52	0.57	1.00	1.00	1.00
1000	0.42	0.43	0.54	1.00	1.00	1.00

Note. *N*: sample size; *W*: time points.

Table 2

Bias for Constrained Maximum Likelihood and Bayesian Approaches (With Different Priors) for N=500

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	CML	-0.02	-0.03	0.05	0.01	-0.04	-0.01	-0.03	0.03	-0.02	-0.04
4	Uniform	0.01	-0.05	0.02	0.04	-0.02	-0.09	-0.05	0.06	0.01	-0.07
4	True_1	0.00	0.07	0.03	0.03	-0.02	-0.09	-0.03	0.07	-0.01	-0.07
4	True_3	0.01	0.06	0.01	0.02	-0.01	-0.09	-0.02	0.05	-0.01	-0.07
4	IG_1	0.01	-0.02	0.01	0.04	-0.01	-0.09	-0.05	0.06	0.01	-0.07
4	IG_3	0.04	0.03	-0.04	0.05	0.01	-0.16	-0.05	0.08	0.01	-0.12
4	Beta4_1	0.00	-0.05	0.01	0.04	-0.01	-0.06	-0.04	0.04	0.01	-0.05
4	Beta4_3	0.03	-0.03	-0.04	0.03	0.01	-0.09	-0.05	0.03	0.01	-0.06
6	CML	-0.00	-0.01	0.02	-0.00	-0.02	0.06	-0.01	0.00	-0.01	-0.01
6	Uniform	-0.01	-0.03	0.04	0.02	-0.02	-0.01	-0.01	0.02	-0.01	-0.02
6	True_1	-0.02	0.02	0.04	0.01	-0.02	-0.01	-0.01	0.02	-0.01	-0.02
6	True_3	-0.01	0.02	0.02	0.01	-0.01	-0.03	-0.01	0.03	-0.01	-0.03
6	IG_1	-0.01	-0.02	0.03	0.02	-0.02	-0.01	-0.01	0.02	-0.01	-0.02
6	IG_3	0.01	0.01	0.00	0.03	-0.00	-0.07	-0.02	0.04	-0.01	-0.05
6	Beta4_1	-0.01	-0.03	0.03	0.02	-0.02	-0.01	-0.01	0.02	-0.00	-0.01
6	Beta4_3	0.00	-0.02	0.00	0.02	-0.00	-0.05	-0.02	0.02	-0.00	-0.03
9	CML	-0.00	0.00	0.00	-0.00	-0.00	0.04	0.00	0.00	-0.00	-0.01
9	Uniform	-0.00	-0.00	0.02	-0.00	-0.01	0.05	0.00	-0.00	-0.01	-0.00
9	True_1	-0.01	-0.00	0.02	-0.00	-0.01	0.06	0.00	-0.00	-0.01	0.00
9	True_3	-0.00	0.00	0.02	0.00	-0.01	0.02	-0.00	0.01	-0.00	-0.01
9	IG_1	-0.00	-0.01	0.02	-0.00	-0.01	0.04	-0.00	0.00	-0.00	-0.00
9	IG_3	0.00	-0.00	0.01	0.01	-0.00	-0.00	-0.00	0.02	-0.00	-0.01
9	Beta4_1	-0.01	-0.01	0.02	0.00	-0.01	0.03	-0.00	0.00	-0.00	-0.00
9	Beta4_3	-0.00	-0.01	0.01	0.01	-0.00	-0.01	-0.00	0.01	-0.00	-0.01

Note. W: time points; CML: constrained ML; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.

Table 3

RMSE for Constrained Maximum Likelihood and Bayesian Approaches (With Different Priors) for N=500

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{yx}	β_{xy}
4	CML	0.12	0.44	0.12	0.11	0.09	0.61	0.15	0.16	0.13	0.16
4	Uniform	0.06	0.13	0.07	0.10	0.05	0.21	0.09	0.10	0.07	0.10
4	True_1	0.06	0.12	0.07	0.09	0.05	0.19	0.09	0.11	0.07	0.10
4	True_3	0.05	0.10	0.06	0.08	0.04	0.16	0.07	0.09	0.06	0.09
4	IG_1	0.06	0.11	0.07	0.10	0.04	0.21	0.08	0.09	0.07	0.11
4	IG_3	0.06	0.09	0.08	0.10	0.04	0.23	0.09	0.10	0.07	0.15
4	Beta4_1	0.06	0.12	0.07	0.09	0.04	0.17	0.08	0.08	0.06	0.08
4	Beta4_3	0.06	0.08	0.07	0.09	0.03	0.15	0.07	0.06	0.05	0.09
6	CML	0.07	0.19	0.07	0.09	0.05	0.39	0.08	0.10	0.08	0.10
6	Uniform	0.06	0.13	0.07	0.07	0.05	0.23	0.07	0.08	0.07	0.08
6	True_1	0.06	0.11	0.07	0.06	0.05	0.21	0.07	0.09	0.07	0.08
6	True_3	0.05	0.09	0.05	0.06	0.04	0.17	0.06	0.07	0.06	0.07
6	IG_1	0.05	0.11	0.06	0.07	0.04	0.22	0.06	0.07	0.06	0.08
6	IG_3	0.04	0.08	0.05	0.07	0.03	0.20	0.06	0.07	0.06	0.08
6	Beta4_1	0.05	0.11	0.06	0.06	0.04	0.19	0.06	0.07	0.06	0.07
6	Beta4_3	0.04	0.08	0.05	0.06	0.03	0.15	0.05	0.06	0.05	0.06
9	CML	0.03	0.05	0.04	0.07	0.03	0.26	0.05	0.07	0.05	0.06
9	Uniform	0.04	0.06	0.04	0.06	0.03	0.19	0.05	0.06	0.05	0.05
9	True_1	0.04	0.06	0.04	0.06	0.03	0.19	0.05	0.06	0.05	0.05
9	True_3	0.03	0.05	0.04	0.05	0.03	0.15	0.04	0.06	0.05	0.05
9	IG_1	0.04	0.06	0.04	0.06	0.03	0.19	0.05	0.06	0.05	0.05
9	IG_3	0.03	0.05	0.04	0.06	0.03	0.16	0.04	0.06	0.05	0.05
9	Beta4_1	0.04	0.06	0.04	0.05	0.03	0.17	0.04	0.06	0.05	0.05
9	Beta4_3	0.03	0.05	0.04	0.05	0.03	0.13	0.04	0.05	0.04	0.04

Note. W: time points; CML: constrained ML; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.

Table 4

Coverage Rates for Constrained Maximum Likelihood and Bayesian Approaches (With Different Priors) for N=500

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	CML	0.82	0.78	0.90	0.94	0.87	0.78	0.86	0.82	0.91	0.90
4	Uniform	0.99	0.99	0.99	0.98	0.99	0.99	0.99	1.00	1.00	0.99
4	True_1	1.00	0.98	0.99	0.99	0.99	0.99	0.99	0.99	1.00	0.99
4	True_3	1.00	0.99	0.99	0.99	1.00	1.00	0.99	0.99	1.00	0.99
4	IG_1	1.00	0.99	0.99	0.98	1.00	0.99	1.00	1.00	1.00	0.99
4	IG_3	0.99	0.99	0.99	0.96	0.99	0.98	0.99	0.99	1.00	0.98
4	Beta4_1	0.99	0.99	0.99	0.98	1.00	1.00	1.00	1.00	1.00	0.99
4	Beta4_3	0.99	0.99	0.99	0.98	1.00	1.00	1.00	1.00	1.00	0.99
6	CML	0.94	0.93	0.96	0.89	0.94	0.88	0.93	0.89	0.94	0.89
6	Uniform	0.97	0.94	0.95	0.97	0.94	0.97	0.97	0.98	0.98	0.97
6	True_1	0.97	0.93	0.95	0.97	0.94	0.98	0.96	0.97	0.97	0.97
6	True_3	0.98	0.94	0.97	0.97	0.96	0.98	0.98	0.99	0.98	0.98
6	IG_1	0.97	0.95	0.96	0.97	0.96	0.98	0.98	0.99	0.98	0.97
6	IG_3	0.97	0.94	0.98	0.95	0.97	0.95	0.99	0.98	0.99	0.96
6	Beta4_1	0.97	0.95	0.96	0.97	0.96	0.99	0.98	0.99	0.99	0.98
6	Beta4_3	0.98	0.96	0.98	0.97	0.97	0.98	0.99	1.00	0.99	0.98
9	CML	0.97	0.96	0.96	0.91	0.97	0.90	0.96	0.92	0.95	0.92
9	Uniform	0.95	0.96	0.95	0.96	0.97	0.96	0.95	0.96	0.95	0.96
9	True_1	0.96	0.96	0.95	0.96	0.96	0.96	0.95	0.95	0.95	0.96
9	True_3	0.96	0.96	0.95	0.96	0.97	0.97	0.96	0.98	0.96	0.97
9	IG_1	0.96	0.96	0.95	0.96	0.97	0.96	0.96	0.98	0.96	0.96
9	IG_3	0.97	0.96	0.97	0.95	0.98	0.96	0.97	0.98	0.96	0.97
9	Beta4_1	0.96	0.96	0.95	0.96	0.97	0.97	0.96	0.98	0.96	0.97
9	Beta4_3	0.96	0.96	0.97	0.96	0.98	0.97	0.97	0.99	0.97	0.97

Note. W: time points; CML: constrained ML; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.

Table 5

Bivariate STARTS Model Results for Self-Esteem and Depression in Data Example

Specification	ϕ_{Tsese}	ϕ_{Tdede}	ρ_{Tsede}	ϕ_{Asese}	ϕ_{Adede}	ρ_{Asede}	ϕ_{Ssese}	ϕ_{Sdede}	ρ_{Ssede}	β_{sese}	β_{desede}	β_{sede}	β_{dese}
CML	0.16 (0.44)	0.28 (0.29)	0.03 (1.95)	0.70 (0.43)	0.57 (0.30)	-0.75 (0.19)	0.14 (0.02)	0.14 (0.12)	-0.33 (0.47)	0.93 (0.14)	0.44 (0.28)	0.01 (0.13)	-0.31 (0.26)
Uniform	0.42 (0.19)	0.37 (0.15)	-0.53 (0.35)	0.46 (0.19)	0.49 (0.15)	-0.68 (0.12)	0.13 (0.02)	0.14 (0.06)	-0.34 (0.22)	0.85 (0.12)	0.46 (0.17)	-0.02 (0.10)	-0.26 (0.18)
IG_1	0.42 (0.19)	0.37 (0.14)	-0.53 (0.34)	0.45 (0.19)	0.48 (0.15)	-0.68 (0.13)	0.13 (0.02)	0.15 (0.05)	-0.34 (0.18)	0.84 (0.12)	0.48 (0.15)	-0.02 (0.10)	-0.24 (0.18)
IG_3	0.45 (0.16)	0.39 (0.12)	-0.62 (0.21)	0.41 (0.16)	0.43 (0.12)	-0.68 (0.12)	0.13 (0.01)	0.17 (0.04)	-0.32 (0.14)	0.84 (0.11)	0.52 (0.14)	-0.03 (0.10)	-0.20 (0.16)
Beta4_1	0.47 (0.19)	0.39 (0.14)	-0.56 (0.29)	0.40 (0.18)	0.46 (0.14)	-0.67 (0.12)	0.13 (0.02)	0.15 (0.05)	-0.34 (0.18)	0.81 (0.12)	0.48 (0.15)	-0.04 (0.10)	-0.23 (0.17)
Beta4_3	0.54 (0.15)	0.42 (0.12)	-0.63 (0.16)	0.33 (0.14)	0.41 (0.11)	-0.66 (0.11)	0.13 (0.02)	0.16 (0.04)	-0.34 (0.14)	0.77 (0.12)	0.51 (0.12)	-0.06 (0.09)	-0.20 (0.16)

Note. CML: constrained ML; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regressions coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3; ϕ_{Tsese} , ϕ_{Tdede} , and ρ_{Tsede} denote the variances of the stable trait factors for self-esteem and depression, respectively, as well as their correlation; ϕ_{Asese} , ϕ_{Adede} , and ρ_{Asede} denote the variances of the autoregressive trait factors for self-esteem and depression as well as their correlation; ϕ_{Ssese} , ϕ_{Sdede} , and ρ_{Ssede} denote the variances of the state factors for self-esteem and depression as well as their correlation; β_{sese} and β_{desede} denote the autoregressive effects of self-esteem and depression; β_{sede} and β_{dese} denote the cross-lagged effects of self-esteem and depression, respectively. Posterior standard deviations, and standard errors constructed using Jackknifing are given in brackets. Parameters that are statistically significant at the $\alpha = .05$ level are printed in bold type.

Figure 1. Path diagram of the bivariate STARTS model for four measurement waves (see Kenny & Zautra, 1995). T : stable trait; A : autoregressive trait; S : state; and D : disturbance. X_1 , X_2 , X_3 , and X_4 as well as Y_1 , Y_2 , Y_3 , and Y_4 are the manifest indicator variables. Loadings of the manifest indicators on the stable trait factors, autoregressive trait factors, and state factors are set to 1.0.

Figure 2. Inverse-gamma distributions as prior distributions for a variance component ϕ (left panel), and four-parameter beta distributions as prior distributions for a regression coefficient β (right panel). With a larger prior sample size (v_ϕ or v_β), the distribution puts more probability mass around the prior guess (μ_ϕ or μ_β). Note that for β , the four-parameter beta distribution with $\mu_\beta = 0$ and $v_\beta = 0$ corresponds to a uniform distribution $[-1, 1]$.

Appendix A: Identification of Multivariate STARTS Model

In this Appendix, we show that the multivariate STARTS model is identified with at least four measurement waves ($W \geq 4$). Assuming a multivariate STARTS model, the covariance matrix $\Sigma_h = \text{Cov}(\mathbf{y}_{t+h}, \mathbf{y}_t)$ for lag h is given as:

$$\Sigma_h = \Sigma_T + \mathbf{B}^h \Sigma_A + \mathbf{1}_{\{h=0\}} \Sigma_S. \quad (\text{A1})$$

Let $\mathbf{C}_1 = \Sigma_2 - \Sigma_1$, and $\mathbf{C}_2 = \Sigma_3 - \Sigma_2$. We obtain:

$$\mathbf{C}_1 = \mathbf{B}^2 \Sigma_A - \mathbf{B} \Sigma_A = [\mathbf{B}^2 - \mathbf{B}] \Sigma_A \quad (\text{A2})$$

$$\mathbf{C}_2 = \mathbf{B}^3 \Sigma_A - \mathbf{B}^2 \Sigma_A = \mathbf{B}[\mathbf{B}^2 - \mathbf{B}] \Sigma_A. \quad (\text{A3})$$

From Equation A3, we also receive $\mathbf{C}_2 = \mathbf{B} \mathbf{C}_1$. Under the assumption that $\mathbf{C}_1$ is invertible, we can determine the matrix of the regression coefficients $\mathbf{B}$ as follows:

$$\mathbf{B} = \mathbf{C}_2 \mathbf{C}_1^{-1} = f_{\mathbf{B}}(\mathbf{C}_1, \mathbf{C}_2). \quad (\text{A4})$$

Furthermore, we compute for the matrix of autoregressive trait factors Σ_A

$$\Sigma_A = [\mathbf{B}^2 - \mathbf{B}]^{-1} \mathbf{C}_1 = f_{\Sigma_A}(\mathbf{C}_1, \mathbf{C}_2). \quad (\text{A5})$$

It also holds that

$$\Sigma_3 = \Sigma_T + \mathbf{B}^3 \Sigma_A. \quad (\text{A6})$$

Equation A6 can be used to determine Σ_T :

$$\Sigma_T = \Sigma_3 - \mathbf{B}^3 \Sigma_A = f_{\Sigma_T}(\mathbf{C}_1, \mathbf{C}_2, \Sigma_3). \quad (\text{A7})$$

Finally, we obtain Σ_S :

$$\Sigma_S = \Sigma_0 - \Sigma_T - \Sigma_A = f_{\Sigma_S}(\mathbf{C}_1, \mathbf{C}_2, \Sigma_3, \Sigma_0). \quad (\text{A8})$$

Thus, the model parameters can be determined as a function of the observed variances and covariances, and the multivariate STARTS model is identified.

Appendix B: Stan Code for the Bivariate STARTS Model

```
data {
  int<lower = 1> N; // number of persons
  int<lower = 1> W; // number of time points
  matrix[W*2, W*2] S; // scatter matrix (order of variables: se1, de1, se2,
  de2, ... seW, deW)
  real n0V; // prior sample size variances
  real n0C; // prior sample size correlations
  real n0A; // prior sample size regression coefficients
```

```

vector[3] V0; // prior guess variances
vector[3] C0; // prior guess correlation
vector[2] A0; // prior guess autoregressive effects
vector[2] B0; // prior guess cross-lagged effects
real eps; // epsilon value, using eps=.01 in simulation and empirical example
}
transformed data{
  real epsln = -1+eps;
  real epslp = 1-eps;
  real eps2 = 2*(1-eps);
  real eps2i = 1/eps2;
}
parameters {
  real<lower=0,upper=1> corTStar; // correlation traits
  vector<lower=0>[2] sigmaT; // var traits
  real<lower=0,upper=1> corSStar; // correlation states
  vector<lower=eps>[2] sigmaS; // var states
  real<lower=0,upper=1> corAStar; // correlation autoregressive traits
  vector<lower=eps>[2] sigmaA; // var autoregressive traits
  vector<lower=0, upper=1>[2] aStar; // autoregressive effects
  vector<lower=0, upper=1>[2] bStar; // crosslagged effects
}
transformed parameters{
  cov_matrix[2*W] Sigma; // model-implied covariance matrix
  cov_matrix[2] SigmaT; // trait covariance matrix
  matrix[2,2] SigmaA; // autoregressive trait covariance matrix
  cov_matrix[2] SigmaS; // state covariance matrix
  matrix[2,2] B[W-1]; // regression coefficients
  vector[2] a;
  vector[2] b;
  real corS = epsln + eps2*corSStar;
  real corA = epsln + eps2*corAStar;
  real corT = epsln + eps2*corTStar;
  a[1] = epsln + eps2*aStar[1];
  a[2] = epsln + eps2*aStar[2];
  b[1] = epsln + eps2*bStar[1];
  b[2] = epsln + eps2*bStar[2];
  SigmaT=quad_form_diag([1,corT],[corT,1], sqrt(sigmaT)); // trait covariance
matrix
  SigmaA=quad_form_diag([1,corA],[corA,1], sqrt(sigmaA)); // autoregressive trait
covariance matrix
  SigmaS=quad_form_diag([1,corS],[corS,1], sqrt(sigmaS)); // state covariance
matrix
  B[1] = [[a[1],b[1]], [b[2],a[2]]];
  for(t in 2:(W-1)){
    B[t]=B[1]*B[t-1];
  }
  for(t in 1:W){
    Sigma[(t*2-1):(t*2),(t*2-1):(t*2)] = SigmaT+SigmaA+SigmaS; // diagonal
  }
  for(t in 2:W){
    for (j in 1:(t-1)){
      Sigma[(t*2-1):(t*2),(j*2-1):(j*2)]= SigmaT+B[t-j]*SigmaA; // off-diagonal
    }
  }
  for(t in 1:(2*W)){
    for (j in 1:(t-1)){
      Sigma[j,t] = Sigma[t,j];
    }
  }
}
model {
  sigmaT ~ inv_gamma(n0V/2,n0V*V0[1]/2);
  sigmaA ~ inv_gamma(n0V/2,n0V*V0[2]/2);
  sigmaS ~ inv_gamma(n0V/2,n0V*V0[3]/2);
  corTStar ~ beta((n0C+2)*(C0[1]+epslp)*eps2i, (n0C+2)*(epslp -C0[1])*eps2i);
  corAStar ~ beta((n0C+2)*(C0[2]+ epslp)*eps2i, (n0C+2)*(epslp -C0[2])*eps2i );
  corSStar ~ beta((n0C+2)*(C0[3]+ epslp)*eps2i, (n0C+2)*(epslp -C0[3])*eps2i);
  aStar[1] ~ beta((n0A+2)*(A0[1]+ epslp)*eps2i, (n0A+2)*(epslp -A0[1])*eps2i);
  aStar[2] ~ beta((n0A+2)*(A0[2]+ epslp)*eps2i, (n0A+2)*(epslp -A0[2])*eps2i);

```

```
bStar[1] ~ beta((n0A+2)*(B0[1]+ eps1p)*eps2i, (n0A+2)*(eps1p -B0[1])*eps2i);  
bStar[2] ~ beta((n0A+2)*(B0[2]+ eps1p)*eps2i, (n0A+2)*(eps1p -B0[2])*eps2i);  
target += wishart_lpdf(S|N-1,Sigma);  
}
```

Supplementary Material

A Bayesian Approach to Estimating Reciprocal Effects with the Multivariate STARTS Model using Markov Chain Monte Carlo

Table 1

Bias for Constrained Maximum Likelihood and Bayesian Approaches (With Different Priors) for N=200

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	CML	-0.00	-0.03	0.03	0.01	-0.04	-0.05	-0.05	0.07	0.02	-0.11
4	Uniform	0.04	-0.06	-0.02	0.06	0.01	-0.15	-0.07	0.08	0.05	-0.12
4	True_1	0.02	0.08	0.01	0.04	-0.00	-0.12	-0.04	0.10	0.01	-0.11
4	True_3	0.02	0.07	-0.01	0.03	0.01	-0.10	-0.02	0.07	0.00	-0.09
4	IG_1	0.04	-0.03	-0.04	0.06	0.02	-0.15	-0.06	0.08	0.05	-0.14
4	IG_3	0.07	0.02	-0.10	0.07	0.04	-0.19	-0.07	0.10	0.06	-0.19
4	Beta4_1	0.03	-0.07	-0.03	0.05	0.01	-0.12	-0.06	0.04	0.04	-0.09
4	Beta4_3	0.06	-0.06	-0.09	0.03	0.03	-0.11	-0.06	0.01	0.04	-0.10
6	CML	-0.02	-0.04	0.04	0.02	-0.02	0.04	-0.00	0.03	-0.01	-0.02
6	Uniform	-0.01	-0.07	0.04	0.05	-0.01	-0.08	-0.02	0.05	0.01	-0.04
6	True_1	-0.02	0.04	0.05	0.04	-0.01	-0.07	-0.01	0.06	-0.01	-0.05
6	True_3	-0.00	0.04	0.02	0.03	-0.00	-0.08	-0.00	0.06	-0.01	-0.05
6	IG_1	-0.01	-0.04	0.03	0.05	-0.01	-0.08	-0.02	0.06	0.00	-0.04
6	IG_3	0.02	0.01	-0.02	0.06	0.01	-0.14	-0.02	0.08	0.01	-0.08
6	Beta4_1	-0.01	-0.07	0.03	0.04	-0.01	-0.06	-0.02	0.04	0.01	-0.03
6	Beta4_3	0.01	-0.05	-0.01	0.04	0.01	-0.08	-0.02	0.03	0.01	-0.04
9	CML	-0.01	-0.00	0.02	-0.00	-0.01	0.09	-0.00	0.01	-0.00	-0.01
9	Uniform	-0.01	-0.03	0.04	0.02	-0.01	-0.01	-0.01	0.02	-0.00	-0.02
9	True_1	-0.02	0.00	0.04	0.02	-0.01	0.01	-0.00	0.02	-0.00	-0.02
9	True_3	-0.01	0.01	0.03	0.02	-0.01	-0.02	-0.01	0.03	-0.00	-0.02
9	IG_1	-0.01	-0.03	0.03	0.02	-0.01	-0.01	-0.01	0.02	0.00	-0.02
9	IG_3	0.00	-0.01	0.01	0.03	0.00	-0.06	-0.01	0.05	0.00	-0.04
9	Beta4_1	-0.01	-0.04	0.03	0.02	-0.01	-0.01	-0.01	0.02	0.00	-0.01
9	Beta4_3	0.00	-0.03	0.01	0.03	0.00	-0.05	-0.01	0.03	0.00	-0.03

Note. W: time points; CML: constrained ML; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.

Table 2

Bias for Constrained Maximum Likelihood and Bayesian Approaches (With Different Priors) for N=1000

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	CML	-0.04	-0.09	0.07	0.01	-0.03	0.09	-0.01	0.02	-0.00	-0.00
4	Uniform	-0.02	-0.07	0.05	0.03	-0.03	-0.02	-0.03	0.05	0.00	-0.03
4	True_1	-0.02	0.05	0.05	0.02	-0.03	-0.04	-0.02	0.05	-0.01	-0.04
4	True_3	-0.01	0.04	0.03	0.02	-0.02	-0.06	-0.02	0.05	-0.01	-0.04
4	IG_1	-0.02	-0.04	0.04	0.02	-0.02	-0.02	-0.03	0.04	0.00	-0.03
4	IG_3	0.01	0.03	-0.00	0.03	-0.01	-0.10	-0.03	0.06	-0.01	-0.07
4	Beta4_1	-0.02	-0.06	0.04	0.03	-0.02	-0.00	-0.03	0.04	0.01	-0.02
4	Beta4_3	0.01	-0.03	0.00	0.03	-0.01	-0.05	-0.03	0.03	0.01	-0.04
6	CML	-0.01	-0.00	0.01	-0.01	-0.01	0.08	0.00	-0.00	-0.01	0.00
6	Uniform	-0.01	-0.02	0.03	-0.00	-0.01	0.07	0.00	-0.00	-0.01	0.00
6	True_1	-0.02	-0.00	0.03	-0.00	-0.01	0.07	0.00	-0.00	-0.01	0.01
6	True_3	-0.01	0.00	0.02	0.00	-0.01	0.03	-0.00	0.01	-0.01	-0.00
6	IG_1	-0.01	-0.02	0.03	-0.00	-0.01	0.06	-0.00	0.00	-0.00	0.00
6	IG_3	-0.00	-0.00	0.01	0.01	-0.01	0.00	-0.01	0.02	-0.00	-0.02
6	Beta4_1	-0.01	-0.02	0.03	-0.00	-0.01	0.05	-0.00	0.00	-0.00	0.00
6	Beta4_3	-0.00	-0.01	0.01	0.01	-0.01	0.00	-0.01	0.01	-0.00	-0.01
9	CML	-0.00	-0.00	0.01	-0.01	-0.00	0.04	-0.00	-0.00	0.00	0.00
9	Uniform	-0.00	-0.01	0.02	-0.01	-0.01	0.07	-0.00	-0.01	-0.00	0.01
9	True_1	-0.01	-0.01	0.02	-0.01	-0.01	0.08	-0.00	-0.01	-0.00	0.01
9	True_3	-0.00	-0.01	0.01	-0.01	-0.01	0.05	-0.00	-0.00	-0.00	0.00
9	IG_1	-0.01	-0.01	0.02	-0.01	-0.01	0.07	-0.00	-0.01	0.00	0.01
9	IG_3	-0.00	-0.01	0.01	-0.00	-0.00	0.03	-0.00	0.00	0.00	-0.00
9	Beta4_1	-0.01	-0.01	0.01	-0.01	-0.01	0.06	-0.00	-0.01	-0.00	0.01
9	Beta4_3	-0.00	-0.01	0.01	-0.00	-0.00	0.02	-0.00	0.00	0.00	-0.00

Note. W: time points; CML: constrained ML; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.

Table 3

Bias for the Posterior Mode for Bayesian Approaches (With Different Priors) for N=200

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	Uniform	0.06	0.07	-0.02	0.07	0.01	-0.13	-0.05	0.06	0.00	-0.06
4	True_1	0.01	0.23	0.01	0.04	-0.01	-0.14	-0.01	0.09	-0.03	-0.06
4	True_3	0.01	0.16	-0.01	0.03	0.00	-0.12	-0.00	0.05	-0.02	-0.06
4	IG_1	0.05	0.06	-0.04	0.07	0.01	-0.14	-0.04	0.05	0.00	-0.07
4	IG_3	0.08	0.05	-0.10	0.09	0.04	-0.16	-0.04	0.08	0.01	-0.13
4	Beta4_1	0.03	0.03	-0.03	0.06	0.01	-0.12	-0.04	0.02	0.00	-0.04
4	Beta4_3	0.06	-0.01	-0.09	0.05	0.03	-0.11	-0.05	-0.01	0.01	-0.05
6	Uniform	-0.00	0.04	0.03	0.05	-0.00	-0.11	-0.01	0.03	-0.01	-0.02
6	True_1	-0.02	0.13	0.04	0.04	-0.00	-0.12	0.00	0.04	-0.02	-0.02
6	True_3	-0.01	0.07	0.02	0.04	0.00	-0.11	0.00	0.03	-0.02	-0.03
6	IG_1	-0.01	0.03	0.02	0.05	-0.00	-0.11	-0.01	0.03	-0.00	-0.02
6	IG_3	0.02	0.04	-0.03	0.07	0.01	-0.15	-0.01	0.05	-0.00	-0.06
6	Beta4_1	-0.01	0.01	0.02	0.05	0.00	-0.09	-0.01	0.01	-0.00	-0.01
6	Beta4_3	0.01	-0.01	-0.02	0.05	0.01	-0.10	-0.02	0.02	0.00	-0.02
9	Uniform	-0.00	0.02	0.03	0.02	-0.00	-0.06	-0.01	0.00	-0.01	-0.01
9	True_1	-0.02	0.03	0.04	0.02	-0.01	-0.05	-0.00	0.00	-0.01	-0.00
9	True_3	-0.01	0.03	0.02	0.02	-0.00	-0.06	-0.00	0.01	-0.01	-0.01
9	IG_1	-0.01	0.01	0.03	0.02	-0.00	-0.06	-0.01	0.00	-0.00	-0.01
9	IG_3	-0.00	0.02	0.00	0.04	0.01	-0.10	-0.01	0.03	0.00	-0.03
9	Beta4_1	-0.01	-0.00	0.02	0.02	-0.00	-0.05	-0.01	0.00	-0.00	-0.00
9	Beta4_3	-0.00	-0.01	0.00	0.03	0.01	-0.07	-0.01	0.01	0.00	-0.02

Note. W: time points; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.

Table 4

Bias for the Posterior Mode for Bayesian Approaches (With Different Priors) for N=500

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	Uniform	0.03	0.07	0.02	0.05	-0.01	-0.10	-0.04	0.04	-0.00	-0.03
4	True_1	0.01	0.19	0.03	0.03	-0.01	-0.12	-0.02	0.04	-0.02	-0.04
4	True_3	0.02	0.12	0.00	0.02	-0.00	-0.11	-0.01	0.03	-0.02	-0.04
4	IG_1	0.03	0.06	0.00	0.04	-0.01	-0.11	-0.04	0.03	-0.00	-0.03
4	IG_3	0.05	0.06	-0.05	0.05	0.01	-0.16	-0.04	0.05	-0.01	-0.09
4	Beta4_1	0.02	0.04	0.01	0.04	-0.00	-0.09	-0.03	0.01	0.00	-0.01
4	Beta4_3	0.04	0.01	-0.04	0.04	0.01	-0.10	-0.04	0.01	0.01	-0.03
6	Uniform	0.00	0.03	0.03	0.02	-0.01	-0.06	-0.01	0.00	-0.01	-0.01
6	True_1	-0.01	0.05	0.03	0.02	-0.01	-0.06	-0.00	-0.00	-0.02	-0.01
6	True_3	0.00	0.04	0.02	0.02	-0.01	-0.07	-0.01	0.01	-0.01	-0.01
6	IG_1	-0.00	0.02	0.02	0.02	-0.01	-0.06	-0.01	0.00	-0.01	-0.01
6	IG_3	0.01	0.03	-0.00	0.03	-0.00	-0.11	-0.02	0.02	-0.01	-0.04
6	Beta4_1	-0.00	0.01	0.02	0.02	-0.01	-0.05	-0.01	0.00	-0.00	-0.00
6	Beta4_3	0.01	0.01	-0.00	0.02	-0.00	-0.07	-0.02	0.01	-0.00	-0.02
9	Uniform	0.00	0.01	0.01	0.00	-0.00	-0.02	0.00	-0.01	-0.01	0.00
9	True_1	-0.00	0.01	0.02	0.00	-0.00	-0.01	0.00	-0.01	-0.01	0.01
9	True_3	-0.00	0.01	0.01	0.01	-0.00	-0.03	0.00	-0.00	-0.00	-0.00
9	IG_1	-0.00	0.00	0.01	0.00	-0.00	-0.02	-0.00	-0.01	-0.00	0.00
9	IG_3	0.00	0.01	0.00	0.02	0.00	-0.06	-0.00	0.01	-0.00	-0.01
9	Beta4_1	-0.00	0.00	0.01	0.00	-0.00	-0.02	-0.00	-0.01	-0.01	0.00
9	Beta4_3	-0.00	-0.00	0.00	0.01	0.00	-0.04	-0.00	0.00	-0.00	-0.01

Note. W : time points; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.

Table 5

Bias for the Posterior Mode for Bayesian Approaches (With Different Priors) for N=1000

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	Uniform	0.01	0.01	-0.00	0.03	-0.00	-0.07	-0.03	0.02	0.01	-0.02
4	True_1	-0.00	0.01	0.02	0.00	-0.00	-0.00	0.00	-0.01	-0.01	0.01
4	True_3	-0.01	0.02	0.03	0.00	-0.00	0.01	0.00	-0.02	-0.01	0.01
4	IG_1	-0.00	0.02	0.02	0.01	-0.00	-0.02	0.00	-0.00	-0.01	0.00
4	IG_3	-0.01	0.01	0.02	0.00	-0.01	-0.00	-0.00	-0.01	-0.00	0.01
4	Beta4_1	0.00	0.01	0.01	0.01	-0.00	-0.05	-0.01	0.01	-0.00	-0.01
4	Beta4_3	-0.01	0.00	0.02	0.00	-0.00	-0.00	-0.00	-0.01	-0.00	0.01
6	Uniform	0.00	0.00	0.01	0.01	-0.00	-0.03	-0.01	0.00	0.00	-0.00
6	True_1	-0.00	-0.00	0.01	-0.01	-0.00	0.00	-0.00	-0.01	-0.00	0.01
6	True_3	-0.01	-0.00	0.01	-0.01	-0.00	0.01	-0.00	-0.01	-0.00	0.01
6	IG_1	-0.00	-0.00	0.01	-0.00	-0.00	0.00	-0.00	-0.01	-0.00	0.01
6	IG_3	-0.00	-0.00	0.01	-0.01	-0.00	0.01	-0.00	-0.01	-0.00	0.01
6	Beta4_1	-0.00	-0.00	0.01	0.00	-0.00	-0.02	-0.00	0.00	0.00	-0.00
6	Beta4_3	-0.00	-0.01	0.01	-0.01	-0.00	0.00	-0.00	-0.01	-0.00	0.01
9	Uniform	-0.00	-0.01	0.01	0.00	-0.00	-0.01	-0.00	-0.00	0.00	0.00
9	True_1	0.01	0.01	-0.00	0.03	-0.00	-0.07	-0.03	0.02	0.01	-0.02
9	True_3	-0.00	0.01	0.02	0.00	-0.00	-0.00	0.00	-0.01	-0.01	0.01
9	IG_1	-0.01	0.02	0.03	0.00	-0.00	0.01	0.00	-0.02	-0.01	0.01
9	IG_3	-0.00	0.02	0.02	0.01	-0.00	-0.02	0.00	-0.00	-0.01	0.00
9	Beta4_1	-0.01	0.01	0.02	0.00	-0.01	-0.00	-0.00	-0.01	-0.00	0.01
9	Beta4_3	0.00	0.01	0.01	0.01	-0.00	-0.05	-0.01	0.01	-0.00	-0.01

Note. W: time points; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.

Table 6

RMSE for Constrained Maximum Likelihood and Bayesian Approaches (With Different Priors) for N=200

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	CML	0.14	0.52	0.17	0.15	0.11	0.70	0.18	0.21	0.21	0.27
4	Uniform	0.08	0.14	0.11	0.14	0.06	0.23	0.10	0.11	0.11	0.16
4	True_1	0.07	0.12	0.09	0.12	0.05	0.20	0.08	0.13	0.09	0.14
4	True_3	0.06	0.11	0.08	0.10	0.04	0.17	0.07	0.10	0.07	0.11
4	IG_1	0.08	0.12	0.11	0.14	0.06	0.24	0.10	0.11	0.10	0.17
4	IG_3	0.09	0.10	0.13	0.16	0.06	0.26	0.11	0.12	0.11	0.22
4	Beta4_1	0.07	0.14	0.10	0.12	0.05	0.20	0.09	0.08	0.09	0.13
4	Beta4_3	0.08	0.11	0.12	0.11	0.05	0.17	0.09	0.06	0.08	0.12
6	CML	0.10	0.34	0.11	0.12	0.08	0.55	0.11	0.15	0.12	0.13
6	Uniform	0.07	0.16	0.09	0.09	0.05	0.22	0.08	0.10	0.08	0.09
6	True_1	0.06	0.12	0.08	0.09	0.05	0.20	0.08	0.12	0.08	0.09
6	True_3	0.05	0.10	0.07	0.08	0.04	0.17	0.06	0.10	0.07	0.08
6	IG_1	0.06	0.14	0.08	0.09	0.05	0.23	0.07	0.10	0.08	0.09
6	IG_3	0.05	0.10	0.07	0.10	0.04	0.23	0.07	0.11	0.08	0.11
6	Beta4_1	0.06	0.14	0.08	0.09	0.04	0.19	0.07	0.08	0.07	0.08
6	Beta4_3	0.05	0.10	0.06	0.08	0.04	0.16	0.06	0.07	0.06	0.07
9	CML	0.06	0.12	0.07	0.10	0.06	0.40	0.08	0.11	0.08	0.09
9	Uniform	0.06	0.12	0.07	0.07	0.05	0.21	0.07	0.08	0.07	0.07
9	True_1	0.06	0.10	0.07	0.07	0.05	0.20	0.08	0.09	0.07	0.07
9	True_3	0.05	0.08	0.06	0.07	0.04	0.16	0.06	0.08	0.06	0.06
9	IG_1	0.05	0.11	0.07	0.07	0.04	0.21	0.07	0.08	0.06	0.07
9	IG_3	0.05	0.09	0.05	0.08	0.04	0.19	0.06	0.08	0.06	0.08
9	Beta4_1	0.05	0.11	0.06	0.07	0.04	0.18	0.06	0.07	0.06	0.06
9	Beta4_3	0.04	0.09	0.05	0.07	0.03	0.15	0.05	0.06	0.05	0.06

Note. W: time points; CML: constrained ML; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.

Table 7

RMSE for Constrained Maximum Likelihood and Bayesian Approaches (With Different Priors) for N=1000

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	CML	0.11	0.46	0.12	0.09	0.07	0.50	0.12	0.13	0.10	0.12
4	Uniform	0.07	0.16	0.08	0.07	0.05	0.20	0.08	0.09	0.06	0.07
4	True_1	0.06	0.11	0.07	0.07	0.05	0.19	0.08	0.10	0.06	0.08
4	True_3	0.05	0.09	0.06	0.06	0.04	0.15	0.06	0.08	0.05	0.07
4	IG_1	0.06	0.12	0.07	0.07	0.04	0.20	0.07	0.08	0.06	0.07
4	IG_3	0.05	0.08	0.05	0.07	0.03	0.20	0.06	0.09	0.06	0.10
4	Beta4_1	0.06	0.13	0.07	0.07	0.04	0.16	0.06	0.07	0.06	0.06
4	Beta4_3	0.04	0.08	0.05	0.07	0.03	0.13	0.05	0.06	0.05	0.06
6	CML	0.05	0.08	0.05	0.07	0.04	0.31	0.06	0.08	0.06	0.07
6	Uniform	0.05	0.09	0.05	0.05	0.04	0.21	0.06	0.07	0.06	0.06
6	True_1	0.05	0.08	0.06	0.05	0.04	0.20	0.06	0.07	0.06	0.06
6	True_3	0.04	0.06	0.04	0.05	0.03	0.16	0.05	0.06	0.05	0.05
6	IG_1	0.05	0.08	0.05	0.05	0.03	0.20	0.06	0.06	0.05	0.06
6	IG_3	0.04	0.06	0.04	0.05	0.03	0.17	0.05	0.06	0.05	0.05
6	Beta4_1	0.05	0.08	0.05	0.05	0.03	0.18	0.05	0.06	0.05	0.05
6	Beta4_3	0.04	0.06	0.04	0.05	0.03	0.13	0.04	0.05	0.04	0.05
9	CML	0.02	0.04	0.03	0.05	0.03	0.18	0.04	0.05	0.04	0.04
9	Uniform	0.03	0.04	0.03	0.05	0.03	0.17	0.04	0.05	0.04	0.04
9	True_1	0.03	0.04	0.04	0.05	0.03	0.17	0.04	0.05	0.04	0.04
9	True_3	0.02	0.04	0.03	0.04	0.03	0.14	0.03	0.04	0.04	0.04
9	IG_1	0.03	0.04	0.04	0.05	0.03	0.16	0.03	0.05	0.04	0.04
9	IG_3	0.02	0.04	0.03	0.04	0.02	0.13	0.03	0.04	0.03	0.04
9	Beta4_1	0.03	0.04	0.03	0.04	0.03	0.15	0.03	0.05	0.04	0.04
9	Beta4_3	0.02	0.04	0.03	0.04	0.02	0.11	0.03	0.04	0.03	0.03

Note. W: time points; CML: constrained ML; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively. .

Table 8

Coverage Rates for Constrained Maximum Likelihood and Bayesian Approaches (With Different Priors) for N=200

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	CML	0.85	0.72	0.93	0.95	0.83	0.75	0.86	0.86	0.93	0.92
4	Uniform	0.99	1.00	0.99	0.97	0.99	1.00	1.00	1.00	1.00	1.00
4	True_1	1.00	0.99	0.99	0.98	1.00	0.99	1.00	1.00	1.00	0.99
4	True_3	1.00	0.99	1.00	0.99	1.00	1.00	1.00	1.00	1.00	0.99
4	IG_1	0.99	1.00	0.99	0.97	1.00	0.99	1.00	1.00	1.00	0.99
4	IG_3	0.98	0.99	0.94	0.93	0.99	0.96	1.00	0.99	1.00	0.97
4	Beta4_1	0.99	1.00	0.99	0.98	0.99	1.00	1.00	1.00	1.00	1.00
4	Beta4_3	0.98	0.99	0.95	0.99	0.99	1.00	1.00	1.00	1.00	0.99
6	CML	0.92	0.90	0.94	0.92	0.91	0.80	0.92	0.87	0.92	0.94
6	Uniform	0.98	0.98	0.97	0.97	0.98	0.99	1.00	0.98	0.98	0.99
6	True_1	0.99	0.98	0.97	0.98	0.98	0.99	1.00	0.98	0.97	0.99
6	True_3	1.00	0.97	0.99	0.98	0.99	0.99	1.00	0.98	0.98	0.99
6	IG_1	1.00	0.98	0.98	0.98	0.99	0.99	1.00	0.98	0.98	0.99
6	IG_3	0.99	0.97	0.98	0.96	0.99	0.97	1.00	0.97	0.98	0.98
6	Beta4_1	0.99	0.99	0.98	0.98	0.98	1.00	1.00	0.99	0.98	0.99
6	Beta4_3	0.99	0.98	0.99	0.99	0.99	1.00	1.00	0.99	0.99	0.99
9	CML	0.97	0.95	0.95	0.90	0.91	0.85	0.94	0.88	0.96	0.92
9	Uniform	0.97	0.95	0.96	0.97	0.96	0.99	0.96	0.98	0.98	0.98
9	True_1	0.96	0.95	0.94	0.97	0.95	0.99	0.96	0.98	0.98	0.97
9	True_3	0.99	0.96	0.97	0.97	0.97	0.99	0.98	0.98	0.99	0.98
9	IG_1	0.97	0.96	0.96	0.97	0.97	0.99	0.97	0.98	0.98	0.98
9	IG_3	0.98	0.95	0.98	0.96	0.98	0.96	0.98	0.97	0.98	0.96
9	Beta4_1	0.98	0.96	0.97	0.97	0.97	1.00	0.97	0.99	0.99	0.99
9	Beta4_3	0.98	0.96	0.98	0.97	0.97	0.99	0.98	0.99	0.99	0.98

Note. W: time points; CML: constrained ML; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.

Table 9

Coverage Rates for Constrained Maximum Likelihood and Bayesian Approaches (With Different Priors) for N=1000

W	Specification	Stable traits		Autoregressive traits		States		Regression coefficients			
		ϕ_{Txx}	ρ_{Txy}	ϕ_{Axx}	ρ_{Axy}	ϕ_{Sxx}	ρ_{Sxy}	β_{xx}	β_{yy}	β_{xy}	β_{yx}
4	CML	0.85	0.89	0.90	0.93	0.89	0.83	0.86	0.88	0.91	0.88
4	Uniform	0.98	0.99	0.96	0.97	0.98	1.00	0.98	0.99	1.00	1.00
4	True_1	0.99	0.99	0.95	0.97	0.98	0.99	0.98	0.98	1.00	0.99
4	True_3	0.99	0.99	0.97	0.97	0.99	0.99	0.99	0.99	1.00	1.00
4	IG_1	0.99	0.99	0.96	0.98	0.99	1.00	0.99	0.99	1.00	1.00
4	IG_3	0.99	0.99	0.98	0.96	1.00	0.98	0.99	0.99	1.00	0.99
4	Beta4_1	0.99	0.99	0.97	0.97	0.99	1.00	0.99	1.00	1.00	1.00
4	Beta4_3	0.99	0.99	0.99	0.97	1.00	1.00	1.00	1.00	1.00	1.00
6	CML	0.97	0.95	0.97	0.91	0.95	0.91	0.95	0.92	0.95	0.91
6	Uniform	0.96	0.95	0.95	0.97	0.96	0.97	0.95	0.97	0.96	0.97
6	True_1	0.94	0.95	0.95	0.97	0.95	0.97	0.95	0.97	0.95	0.97
6	True_3	0.97	0.95	0.96	0.97	0.96	0.97	0.96	0.98	0.96	0.98
6	IG_1	0.97	0.96	0.95	0.97	0.96	0.97	0.96	0.97	0.96	0.98
6	IG_3	0.98	0.95	0.98	0.96	0.98	0.96	0.98	0.98	0.97	0.97
6	Beta4_1	0.96	0.96	0.95	0.97	0.96	0.98	0.96	0.98	0.96	0.98
6	Beta4_3	0.98	0.96	0.98	0.97	0.99	0.98	0.98	0.99	0.97	0.98
9	CML	0.97	0.96	0.96	0.94	0.94	0.96	0.95	0.95	0.94	0.95
9	Uniform	0.96	0.94	0.93	0.96	0.93	0.96	0.95	0.95	0.94	0.95
9	True_1	0.96	0.95	0.93	0.96	0.93	0.95	0.95	0.95	0.94	0.95
9	True_3	0.97	0.94	0.94	0.97	0.94	0.97	0.95	0.97	0.95	0.96
9	IG_1	0.96	0.94	0.94	0.96	0.94	0.96	0.95	0.95	0.95	0.95
9	IG_3	0.97	0.95	0.95	0.97	0.94	0.96	0.96	0.98	0.96	0.97
9	Beta4_1	0.97	0.94	0.94	0.96	0.94	0.97	0.95	0.96	0.95	0.96
9	Beta4_3	0.97	0.94	0.95	0.97	0.95	0.98	0.97	0.98	0.96	0.97

Note. W: time points; CML: constrained ML; Uniform: uniform prior distributions for all parameters; True_1 and True_3: inverse gamma and four-parameter beta prior distributions for the variance components, correlations, and regression coefficients with the true parameters as prior guesses and prior sample sizes of 1 and 3, respectively; IG_1 and IG_3: inverse gamma prior distributions for the variance components with a prior guess of 0.33 and prior sample sizes of 1 and 3; Beta4_1 and Beta4_3: four-parameter beta prior distributions for the correlations and regression coefficients with a prior guess of 0 and prior sample sizes of 1 and 3 in addition to IG_1, IG_3. ϕ , ρ , and β denote variances, correlations, and regression coefficients, respectively.