Cultural Consensus Theory for Two-Dimensional Location Judgments

Maren Mayer[1,2] & Daniel W. Heck[3]

[1] Leibniz-Institut für Wissensmedien (Knowledge Media Research Center) Tübingen

[2] Heidelberg Academy of Sciences and Humanities

[3] University of Marburg

Author Note

Maren Mayer, Leibniz-Institut für Wissensmedien (Knowledge Media Research Center), Tübingen, Germany. https://orcid.org/0000-0002-6830-7768

Daniel W. Heck, Department of Psychology, University of Marburg, Germany. https://orcid.org/0000-0002-6302-9252

Data and R scripts for the analyses are available at the Open Science Framework (https://osf.io/jbzk7/).

The present work was presented at the SJDM Annual Meeting 2020 (Virtual Conference) and at the 15th Conference of the Section 'Methods and Evaluation' (2021) of the German Psychological Society (DGPs). The present manuscript has not yet been peer reviewed. A preprint was uploaded to PsyArXiv and ResearchGate for timely dissemination (version: January 16, 2023).

This work was funded by the WIN programme of the Heidelberg Academy of Sciences and Humanities, financed by the Ministry of Science, Research and the Arts of the State of Baden-Württemberg and also supported by the Research Training Group "Statistical Modeling in Psychology" funded by the German Research Foundation (DFG grant GRK 2277).

The authors made the following contributions. Maren Mayer: Conceptualization, Investigation, Methodology, Writing - Original Draft, Writing - Review & Editing; Daniel W. Heck: Conceptualization, Methodology, Writing - Review & Editing.

Abstract

Cultural consensus theory is a model-based approach for analyzing responses of informants when correct answers are unknown. The model provides aggregate estimates of the latent consensus knowledge at the group level while accounting for heterogeneity in informant competence and item difficulty. We develop a new version of cultural consensus theory for two-dimensional continuous judgments which are obtained when asking informants to locate a set of unknown sites on a geographic map. The new model is fitted using hierarchical Bayesian modeling. A simulation study shows satisfactory parameter recovery for realistic numbers of informants and items. We also assess the accuracy of the aggregate location estimates by comparing the new model against simply computing the unweighted average of the informants' judgments. A simulation study shows that, due to weighing judgments by the inferred competence of the informants, cultural consensus theory provides more accurate location estimates than unweighted averaging. The new model also showed a higher accuracy in an empirical study in which individuals judged the location of 57 European cities on maps.

Keywords: wisdom of crowds, group decision making, Bayesian modeling, test theory, psychometrics

Cultural Consensus Theory for Two-Dimensional Location Judgments

1 Introduction

In many domains in the social sciences and particularly in psychology, participants provide responses to questions for which correct answers are not known or not defined. For instance, researchers may elicit probability judgments of future events (Anders et al., 2014) or ask whether one agrees or disagrees with a set of statements about a certain topic such as beliefs about the contagiousness of AIDS (Trotter et al., 1999). Cultural consensus theory (CCT, Romney et al., 1986) is a method for aggregating the responses from several informants to estimate the shared knowledge of a group. Essentially, the model infers the latent cultural consensus of a group while considering variance both in the competence of informants and in the difficulty of items. This is achieved by assuming that the experts in a domain are those informants who provide "correct answers' ' in the sense that their responses consistently reflect the shared cultural beliefs.

The fact that normatively correct answers are unknown complicates the aggregation of informants' responses because it is not clear which of the informants are most competent in the sense that they provide judgments close to the unknown cultural truth. As a remedy, CCT allows researchers to infer the latent cultural truth as well as the competence of each informant simultaneously. The main principle of CCT is that informants with more cultural knowledge are likely to show similar answer patterns across the set of questions asked because their judgments consistently reflect the shared cultural truth (Romney et al., 1986). Based on the correlation of the observed answer patterns, the method jointly estimates the cultural truth at the group level and the informants' competence at the individual level. This requires that multiple informants provide judgments to a set of items from the same knowledge domain (Weller, 2007).

1.1 Applications and Extensions of Cultural Consensus Theory

CCT was first developed in anthropological research for questionnaires about cultural topics with a dichotomous response format (Romney et al., 1986) and has also been described as "test theory without an answer key" (Batchelder & Romney, 1988). For instance, one of the first applications investigated the intracultural variability of beliefs about whether illnesses are contagious (Romney et al., 1986). The method has since been applied in various contexts such as aggregating eyewitness reports (Waubert de Puiseau et al., 2017; Waubert de Puiseau et al., 2012), obtaining forecasts for various events (Anders et al., 2014; Merkle et al., 2020), or estimating social networks where individuals provide information about social relations among different people (Batchelder et al., 1997; Batchelder, 2009).

The original version of CCT was applicable only to dichotomous data and assumed that all informants belong to a single shared cultural truth. As it may be possible that not all informants share a common consensus, Anders and Batchelder (2012) extended CCT to multiple cultural truths (see also Aßfalg & Klauer, 2020). Essentially, such extended models assume that informants belong to separate latent classes which differ with respect to the assumed cultural truth. For instance, medical professionals and lay people may differ with respect to medical beliefs resulting in two cultural truths which are latent if group membership is unknown.

CCT has also been extended to response formats other than binary answers. Extensions have been developed for continuous data (Anders et al., 2014; Batchelder & Anders, 2012), ordinal responses (Anders & Batchelder, 2015), and mixed response formats (Aßfalg, 2018) in order to aggregate ratings about the grammatical acceptability of English phrases or to measure shared beliefs about the importance of various health behaviors. Statistical inference for such extended CCT models has often relied on hierarchical Bayesian modeling in which parameter estimates are obtained via Markov chain Monte Carlo (MCMC) sampling (Anders et al., 2014; Anders & Batchelder, 2012; Aßfalg & Klauer, 2020). Overall, these extensions have enabled researchers to adapt the CCT approach to various types of data while assuming a certain structure of cultural truths underlying informants' answers.

CCT is also applicable to scenarios in which correct answers are not known during the time of data collection but may become available later. Such forecasting applications are especially interesting because the performance of different aggregation methods including CCT can be directly compared against each other once the correct answers become available. If factually correct answers are available, it is also possible to check whether the expertise estimates of CCT correlate with the accuracy scores of individuals. In judgment and decision making, it is well known that the aggregation of independent individual judgments (e.g., by computing an unweighted average) results in highly accurate group estimates, a phenomenon referred to as wisdom of crowds (Hueffer et al., 2013; Larrick & Soll, 2006; Steyvers et al., 2009; Surowiecki, 2005). This high level of accuracy across various tasks and contexts is surprising given that all judgments are weighted equally without considering informants' competence (or incompetence) regarding the domain of interest.

The accuracy of wisdom of crowds can be improved by weighing individual judgments by the expertise of informants. For instance, Budescu and Chen (2015) relied on prior judgments of participants to estimate the competence of each individual relative to the crowd. Using the estimated expertise as weights improved the accuracy of the aggregate estimates. CCT also weighs judgments by expertise, but it does not rely on informants' performance on previous items. Instead, CCT relies on a statistical model to simultaneously estimate individuals' expertise while using these estimates as weights for the aggregation of judgments. With respect to forecasting, Merkle et al. (2020) showed that such a CCT-inspired aggregation mechanism does indeed outperform unweighted averaging. Similarly, the accuracy of aggregated eyewitness testimonies increases when using a CCT model since it infers the witnesses' competence (Waubert de Puiseau et al., 2017). Overall, CCT is thus a useful tool for the aggregation of judgments when the ground truth becomes available only at a later time.

1.2 Geographical Data and Location Judgments

CCT has been adapted to several types of response formats and applications, but an extension for two- or higher-dimensional judgments has not been developed yet. One important type of two-dimensional continuous data are geographical judgments which arise whenever individuals locate sites on a map (e.g., Friedman, Kerkman, et al., 2002; Mayer & Heck, 2022). In such scenarios, the actual locations are usually unknown to the informants, either because of a lack of precise knowledge (e.g., for cities) or because there is no factually "correct' ' location (e.g., when asking for preferences or beliefs). Extending CCT to location judgments provides a principled method of inferring the common consensus of a group about the location of the sites of interest.

A CCT model for two-dimensional location judgments can be beneficial in various contexts and tasks, both in psychology and beyond. First, CCT is a useful tool for aggregating subjective judgments even when the actual locations of sites can in principle be known. For instance, Friedman and colleagues (Friedman, Brown, et al., 2002; Friedman et al., 2012, 2005; Friedman, Kerkman, et al., 2002) examined the role of individuals' place of residence on their geographical knowledge and representation. Several studies showed that there are considerable differences in location judgments for individuals living in Canada, Mexico, and the United States when it comes to locating cities in all three countries. However, participants in these studies only provided one-dimensional judgments of the latitude of cities as this facilitated the statistical analysis. An extended CCT model for two-dimensional continuous data would allow researchers to collect and aggregate location judgments with respect to both latitude and longitude. Moreover, a CCT model could be used to explain variance in judgments by informants' expertise, or to compare model-based location estimates between different manifest groups or cultures. Note that location judgments of cities have also been used to compare the performance of different approaches of judgment aggregation in online collaborative projects (Mayer & Heck, 2022).

Second, an extension of CCT for two-dimensional continuous data is especially useful for the aggregation of individual location judgments when the factually correct or optimal locations are unknown. For instance, Surowiecki (2005) describes how a lost submarine was found by aggregating the judgments of experts on its most likely location. Similar applications are in principle possible when selecting optimal locations for park-and-ride facilities (Faghri et al., 2002), suitable areas for ecotourism (Mahdavi et al., 2015), or uncovering ancient archaeological sites (Casana, 2014) and natural resources (e.g., water harvesting sites, Al-shabeeb, 2016). However, a statistical aggregation of location judgments based on CCT is only applicable in scenarios where several informants provide location judgments for multiple sites. If these requirements are met, CCT is ideally suited to infer the shared, common consensus about the unknown locations.

In the following, we develop a new CCT model for two-dimensional location judgments based on Anders et al.'s (2014) CCT model for one-dimensional continuous responses. We check the validity and performance of the proposed CCT model and its Bayesian implementation in JAGS (Plummer, 2003) by investigating parameter convergence and recovery in a Monte Carlo simulation. Moreover, we use simulations to examine under which conditions the weighting of judgments by individuals' competence improves the accuracy of location estimates at the group level. Empirically, we apply the new model to reanalyze location judgments of European cities on maps (Mayer & Heck, 2022) and compare the accuracy of the aggregate location estimates to those obtained with unweighted averaging. Thereby, our work contributes to prior research showing that wisdom of crowds can be improved by weighing judgments by expertise (Budescu & Chen, 2015; Merkle et al., 2020). Overall, the results of our simulation studies and the empirical reanalysis show that the weighting of individual location judgments by informants' competence improves estimation accuracy of CCT compared to weighting all judgments equally.

The proposed extension of CCT for two-dimensional continuous responses is specifically tailored to geographical data where informants provide location judgments. Of course, two-dimensional continuous data are also collected in other tasks and contexts in psychology. For instance, participants may have to rate items with respect to two features such as the valence and arousal of images (Funke & Reips, 2012; Reips & Funke, 2008) or facial images for their attractiveness and trustworthiness (Oosterhof & Todorov, 2008). The proposed CCT model may not be directly applicable to such data because of certain assumptions that are specific to location judgments (e.g., assumptions about the dimensionality of informants' competence or the correlation of errors across dimensions).

In the Discussion, we elaborate on how the model can be adapted to multidimensional, continuous judgments in other tasks and contexts besides location judgments.

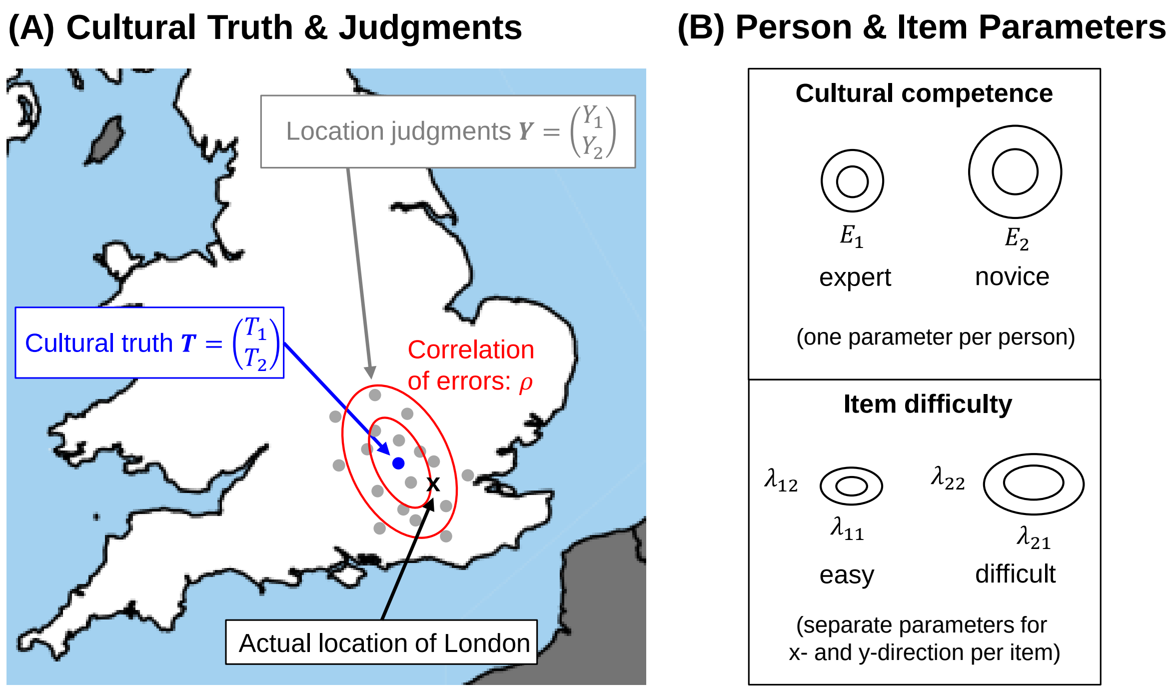
2 Model extension for two-dimensional continuous responses

2.1 Data structure

We extend the CCT model for one-dimensional continuous responses by Anders et al. (2014) to two-dimensional continuous judgments. Similar to all CCT models, the new model requires that multiple informants provide location judgments for a set of items from the same competence domain (Weller, 2007). For instance, as illustrated in Figure 1A, several informants could be asked to locate different European cities such as London on geographic maps (Mayer & Heck, 2022). Locations can be measured in different units depending on the application. For instance, one may use pixels of the presented image as in our empirical study below or geographical coordinates such as longitude and latitude, but other two-dimensional judgments are also feasible.

Figure 1

Data structure and CCT parameters for modeling location judgments of London.



Regarding notation, we assume that $i = 1, \ldots, N$ informants answer $k = 1, \ldots, M$ items by providing continuous, two-dimensional location judgments

$$\boldsymbol{Y}_{ik} = \begin{pmatrix} Y_{ik1} \\ Y_{ik2} \end{pmatrix}. \tag{1}$$

This means that each location judgment contains two components with Y_{ik1} referring to the first dimension (e.g., the x-axis or longitude on a map) and Y_{ik2} referring to the second dimension (e.g., the y-axis or latitude).

2.2 Model specification

The CCT model for two-dimensional judgments (CCT-2D) assumes that all respondents share a single latent cultural truth $\boldsymbol{T}_k$ for each item k. In our empirical example, the latent-truth parameters refer to the group's consensus knowledge about the location of London and other European cities on a map. Note that our example concerns a case where the true locations are in principle available, but of course, the model also applies to scenarios in which this is not the case.

As displayed in Figure 1A, we assume that the observed judgments $\boldsymbol{Y}_{ik}$ can be modeled by two additive components, the shared cultural truth and an unsystematic judgment error,

$$\boldsymbol{Y}_{ik} = \boldsymbol{T}_k + \boldsymbol{\varepsilon}_{ik}. \tag{2}$$

This additive structure of a true score and an error term is very common for CCT models (Anders et al., 2014; Anders & Batchelder, 2012) and can also be found in classical test theory (Lord et al., 1968). Similar to other CCT models (Anders et al., 2014) and item response theory in general (Embretson & Reise, 2000), we assume that the errors $\boldsymbol{\varepsilon}_{ik}$ are conditionally independent given the informant competence E_i and the item difficulty $\boldsymbol{\lambda}_k$. Conditional independence of errors means that judgment errors are assumed to be uncorrelated once competence and difficulty are accounted for by the model, whereas errors may still be correlated when ignoring the clustering by person and item. Moreover, since judgments are continuous, we assume a bivariate normal distribution of errors,

$$(\boldsymbol{\varepsilon}_{ik} \mid E_i, \boldsymbol{\lambda}_k) \overset{\text{iid}}{\sim} \text{MV-Normal}\left(\boldsymbol{\mu} = \boldsymbol{0}, \boldsymbol{\Sigma} = \boldsymbol{\Sigma}_{ik}\right). \tag{3}$$

The covariance matrix $\boldsymbol{\Sigma}_{ik}$ of judgment errors is modeled as a function of informant competence and item difficulty. The error variances in the x- and y-direction (i.e., the diagonal elements of $\boldsymbol{\Sigma}_{ik}$) are assumed to be smaller for persons with higher cultural competence and for items that are easier, meaning that in such cases the observed judgments are closer to the cultural truth. For instance, when asked to locate cities in the United Kingdom, informants with high competence will position these cities closer to the shared cultural knowledge about the location. Formally, this idea is implemented by defining the person competence E_i and the item difficulty λ_{kd} as multiplicative factors which jointly determine the standard deviation of informants' judgments around the cultural truth in the d-th dimension,

$$\sigma_{ikd} = E_i\,\lambda_{kd} \tag{4}$$

Essentially, smaller values of E_i reflect a higher competence since judgments are closer to the cultural truth.[1] Figure 1B illustrates how the parameter E_i affects the variance of the distribution of errors. Since cultural competence is modeled as a multiplicative factor affecting the standard deviation, the parameter E_i is restricted to be positive ($E_i > 0$).

Recent versions of CCT (e.g., Anders et al., 2014) also assume that items vary in difficulty such that more difficult items result in a larger variance of judgments around the cultural truth. For the present case of location judgments, we define a vector-valued item-difficulty parameter $\boldsymbol{\lambda}_k$ for each item with two components $\lambda_{k1} > 0$ and $\lambda_{k2} > 0$ for the x- and y-dimension, respectively. We model the difficulty of each item with two instead of only one value because the x- and y-dimension may differ in difficulty.

2.3 Model assumptions specific to location judgments

Two-dimensional location judgments have some unique features which require special consideration in model development. Imagine that informants are asked to locate London, Birmingham, Glasgow, Liverpool, and Dublin on a map of the United Kingdom and Ireland similar to Figure 1A. The CCT-2D model outlined above accounts for such two-dimensional continuous responses by assuming that all informants answer according to the same underlying cultural truth. Here, the latent truth $\boldsymbol{T}_k$ refers to the group's shared knowledge about the positions of city k on the map. The model assumes that the location judgments of an informant are closer or further away from the shared consensus knowledge depending on their competence level. Importantly, the parameter E_i refers to the *general* competence of an informant irrespective of the x- or y-direction. Hence, when an informant knows that London is located in the south of the United Kingdom, it is also likely that they know whether it is located more to the west or to the east. This restriction simplifies the interpretation of the competency parameter E_i as a one-dimensional trait or construct.

Whereas competence is modeled as a one-dimensional parameter, the model assumes that each item has separate and possibly different difficulties λ_{k1} and λ_{k2} in the

[1] Due to the reversed direction of the effect, one could alternatively refer to E_i as an *incompetence* parameter.

x- and y-direction, respectively. Due to geographical features of a map such as borders, lakes, coasts, or other anchor points, informants may be naturally restricted in positioning a site in the vertical direction but not in the horizontal direction or vice versa. For instance, when positioning Liverpool and Dublin, informants are limited by the coastline to the West and the East, respectively, which may in turn result in a reduced variance of judgments in the x-direction (longitude) compared to the y-direction (latitude).

More generally, certain features of geographic maps such as coastlines may also lead to spatially correlated errors of location judgments. For instance, a positive correlation may emerge when positioning cities on a map which are closely located to a "diagonal" coastline (e.g., Aberdeen which is located close to a coast going from South-West to North-East). In other cases, however, informants are not restricted by nearby coasts (e.g., Birmingham), meaning that judgment errors in x- and y- direction may be uncorrelated. Overall, these considerations lead us to allow for a stochastic dependence of the judgment errors ε_{ik1} and ε_{ik2} in the x- and y-direction, respectively. We thus assume that, for each item k, the normally-distributed errors may correlate between the two dimensions with correlation ρ_k (as illustrated by the tilted red ellipses in Figure 1A). This results in the following covariance matrix of the two-dimensional judgment errors in Equation 3:

$$\mathbf{\Sigma}_{ik} = \begin{pmatrix} (E_i \lambda_{k1})^2 & \rho_k E_i^2 \lambda_{k1} \lambda_{k2} \\ \rho_k E_i^2 \lambda_{k1} \lambda_{k2} & (E_i \lambda_{k2})^2 \end{pmatrix}. \tag{5}$$

Hence, the errors may be correlated between the two dimensions within each item for each informant, which does, however, not imply that the errors are correlated across items or informants. Therefore, the CCT-2D model still satisfies the conditional-independence assumption with respect to the two-dimensional vector of errors $\boldsymbol{\varepsilon}_{ik}$.

2.4 Model simplifications

Compared to the CCT model for one-dimensional continuous data developed by Anders et al. (2014), we simplified the CCT-2D model for two-dimensional judgments with respect to several aspects. First, we do not assume multiple cultural truths. In our example of positioning cities on a map of the United Kingdom and Ireland, multiple cultural truths would imply that there are two or more latent classes of informants with each group having a different consensus of where the cities are located (Anders & Batchelder, 2012). When inferring the position of unknown locations such as natural resources, missing victims, or ancient archaeological sites, we assume that informants often use similar information and background knowledge to form their judgment. Thus, a multimodal distribution of distinct patterns of location judgments is possible but rather unlikely. In other scenarios such as the city-location task, a single correct position on the map does exist but is not available to the informants. In such cases, CCT is most useful when it provides a single, competence-weighted group-level estimate for each item which can then be compared to the accuracy of other aggregation approaches such as unweighted averaging (Merkle et al., 2020; Waubert de Puiseau et al., 2017).

Second, we do not consider systematic biases of location judgments which were included in the model by Anders et al. (2014) for one-dimensional judgments. For rating judgments, a shifting response bias implies that each informant shifts all of their answers up or down on the response scale by a certain amount. Anders et al. modeled this by an idiosyncratic, additive component for each individual. When positioning cities on a map of the United Kingdom, a response bias would imply that each informant exhibits an individual bias shifting all location judgments in a certain direction by a fixed distance (e.g., horizontally, vertically, or diagonally). However, such a general shift of location judgments for all sites of interest seems to be unlikely given that certain cues provided by the map (e.g., borders, coasts, or other geographic features) constrain the possible responses for each item in a different way. For instance, when positioning cities on a map of the United Kingdom and Ireland, a bias to the east would simply result in slightly biased judgments for some cities (e.g. London, Birmingham, and Manchester) but to judgments located in the ocean for others (e.g., Glasgow and Dublin). Hence, the CCT-2D model does not assume a shifting response bias.

The CCT-2D model does also not assume a scaling response bias. For one-dimensional continuous data, a scaling bias refers to a multiplicative bias (i.e., a "stretching factor") for each informant which is assumed to affect the judgments for all items (Anders et al., 2014). Thereby, the model accounts for the fact that individuals differ in their preference for providing extreme judgments at the lower or upper end of the response scale (e.g., Plieninger & Heck, 2018). For location judgments, a scaling bias would imply that informants' judgments are scaled by a multiplicative factor on each axis and for all items. The effect of such a bias depends on the location of the origin of the coordinate system since judgments closer to the origin are less affected by a multiplicative factor compared to judgments further away. However, whereas the midpoint corresponds to a natural origin for one-dimensional judgments on a continuous response scale, location judgments on a map do not necessarily have a natural origin. This is due to the fact that location judgments can simply be rescaled by choosing a different origin (i.e., by defining that the vector $(0, 0)$ corresponds to the lower left corner of the map, the center of the map, or the center of the geographical region of interest). Since we do not have any theoretical or empirical justification for a specific type of scaling bias, we did not implement such a bias in the CCT-2D model for location judgments.

2.5 Hierarchical Bayesian modeling

To fit the CCT-2D model to data and estimate its parameters, we adopt the hierarchical Bayesian modeling approach by Anders et al. (2014). Hierarchical modeling allows researchers to specify a population distribution for a set of model parameters such as person abilities or item difficulties (Lee & Wagenmakers, 2014). This provides many benefits such as a partial pooling of the information between the individual and the group level, which in turn results in shrinkage of the estimates (e.g., Heck, 2019; Singmann & Kellen, 2019). In our case, we assume separate population distributions of the competence parameters E_i across informants and of the item difficulty parameters $\boldsymbol{\lambda}_k$ across items

Besides specifying hierarchical distributions, the Bayesian framework also requires to define prior distributions. In the following, we adopt the common notation of distributions of the software JAGS (Plummer, 2003) which is used to fit the CCT-2D model below. The normal distribution is thus not parameterized by the mean μ and the standard deviation σ, but rather by the mean μ and the precision parameter $\tau = 1/\sigma^2$ (i.e., the inverse of the variance). Similarly, for the t distribution, the second parameter refers to the precision and not to the scale parameter.

Often, normal distributions are assumed as hierarchical group-level distributions. Concerning the latent truth for each item k, we assume that the cultural truth coordinates T_{kd} (with dimension index $d = 1, 2$) are located on the real line and are normally distributed across items,

$$T_{kd} \sim \text{Normal}(\mu = \mu_{T,d}, \tau = \tau_{T,d}). \tag{6}$$

In contrast, the parameters E_i and λ_{kd} are constrained to be positive. As a remedy, we first apply a log transformation to obtain parameters on the real line for which we can assume unbounded normal distributions (Anders et al., 2014). Taking the dimensionality of the parameters into account, the CCT-2D model assumes a one-dimensional hierarchical distribution of the informants' competence,

$$\log E_i \sim \text{Normal}(\mu = \mu_{\log E}, \tau = \tau_{\log E}), \tag{7}$$

and a two-dimensional distribution (with dimensions $d \in \{1, 2\}$) of the items' difficulty,

$$\log \boldsymbol{\lambda}_k \sim \text{MV-Normal}(\boldsymbol{\mu} = \boldsymbol{\mu}_{\log \lambda}, \boldsymbol{\Sigma} = \boldsymbol{\Sigma}^{-1}_{\log \lambda}). \tag{8}$$

For Bayesian inference, it is necessary to specify prior distributions for the hyperparameters of the hierarchical group-level distributions (e.g., for $\mu_{\log E}$ and $\boldsymbol{\mu}_{\log \lambda}$). Our main goal is to estimate the parameters for cultural truth, competence, and item difficulty. Since we are not interested in testing hypotheses with theoretically informed prior distributions (e.g., via Bayes factors, Heck et al., 2022), we rely on default prior distributions that are only weakly informative. Nevertheless, for some parameters such as the cultural-truth locations, it is important that the scaling of the priors aligns with that of the judgments $\boldsymbol{Y}_{ik}$. In our simulation, this was ensured by selecting data-generating parameters in the range of the default prior distributions. In our empirical example, we first standardized all judgments in order to align the data with the default priors.

For the correlation of judgment errors in the x- and y-direction for item k, we assume the following prior:

$$\rho_k \sim \text{Uniform}(\min = -1, \max = 1). \tag{9}$$

For the mean and precision of the latent truth coordinates, we assume

$$\mu_{T,d} \sim \text{Normal}(\mu = 0, \tau = 0.25) \tag{10}$$

$$\tau_{T,d} \sim \text{Half-}t_{\text{df}=1}(\mu = 0, \tau = 1). \tag{11}$$

The half-t-distribution is a t-distribution truncated to positive values. By defining a location parameter of $\mu = 0$, degrees of freedom $\text{df} = 1$, and a precision parameter of $\tau = 1$, the distribution is identical to a half-Cauchy distribution. For the mean and standard deviation of the (log) competence, the prior is

$$\mu_{\log E} = 0 \tag{12}$$

$$\sigma_{\log E} \sim \text{Half-}t_{\text{df}=1}(\mu = 0, \tau = 1). \tag{13}$$

For the mean and standard deviation of the (log) difficulty parameters, we assume

$$\mu_{\log \lambda, d} = 0 \tag{14}$$

$$\sigma_{\log \lambda, d} \sim \text{Half-}t_{\text{df}=1}(\mu = 0, \tau = 3). \tag{15}$$

Note that the hyperparameters for $\mu_{\log E}$ and $\mu_{\log \lambda, d}$ are fixed to constants to ensure the identifiability of the resulting model similar as in item response theory (Embretson & Reise, 2000). Item parameters are often modeled as fixed effects in item response theory, and thus, we reduced the amount of shrinkage for these parameters by assuming a less informative prior for item difficulty than for informant expertise. Finally, the prior for the correlation of the (log) difficulty in x- and y-direction across items is

$$\rho_{\log \lambda} \sim \text{Uniform}(\min = -1, \max = 1). \tag{16}$$

A positive correlation $\rho_{\log \lambda}$ means that, if locating a city is difficult with respect to one axis, it is also difficult with respect to the other axis. By changing the provided JAGS code of the model, researchers can easily adopt different prior distributions for other domains and applications.

3 Simulation study

We performed a simulation study to examine general properties of the CCT-2D model. First, we want to assess how well the model can recover the true, data-generating parameters in various, realistic scenarios. Second, we compare the accuracy of location estimates obtained with the CCT model for two-dimensional continuous data to location estimates obtained with the unweighted aggregation of judgments. Simulated data and R scripts are available at https://osf.io/jbzk7/.

3.1 Method

In the simulation study, the following factors were varied in a fully crossed design using 100 replications per cell:

- Number of informants: $N = 10, 20, 50, 100$
- Number of items: $M = 5, 10, 25, 50$
- Standard deviation of log informants' competence: $\sigma_{\log E} = 0, 0.25, 0.5, 1$
- Standard deviation of log item difficulty: $\sigma_{\log \lambda} = 0, 0.25, 0.5, 1$

We chose a wide range for the sample size N to illustrate the effect of having few or many informants on parameter recovery and on the relative performance of CCT-2D compared to unweighted averaging. However, informants' competence can only be estimated precisely if the number of items is sufficiently large. Hence, we also varied the number of items M on a large range. Overall, these settings reflect the fact that CCT is useful for a wide range of scenarios with both smaller and larger numbers of informants who answer more or less questions (e.g., Waubert de Puiseau et al., 2012).

Furthermore, we varied the standard deviation of the logarithm of informants' competence ($\sigma_{\log E}$) and the standard deviation of the logarithm of item difficulty ($\sigma_{\log \lambda}$) on a large range, including conditions with no variance at all. The standard deviations refer to the logarithm of these parameters since informants' competence and item difficulty must be positive, which also reflects the model's assumption that the log-transformed parameters follow unbounded normal distributions. While both types of variances can be expected to affect parameter recovery of their respective parameters, $\sigma_{\log E}$ is especially relevant for the comparison of the accuracy of estimates obtained with CCT-2D and unweighted averaging. Without any variance in informants' competence, CCT and unweighted averaging are expected to perform approximately equally well because equal weighting of judgments leads to optimal performance (Davis-Stober et al., 2014). However, if the variance in informants' location judgments partially emerges due to differences in informants' competence, CCT-2D is expected to result in more accurate estimates than unweighted averaging because it assigns larger weights to more competent informants (Merkle et al., 2020).

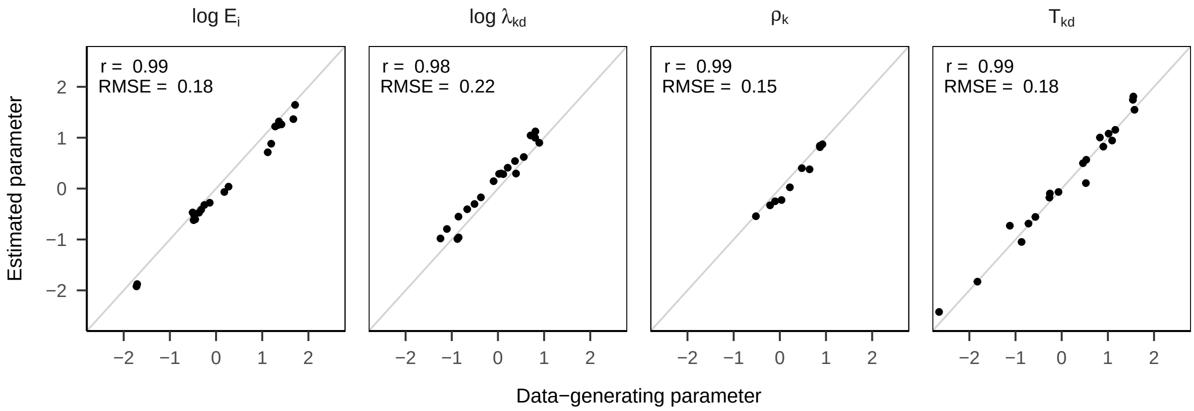
All simulations were conducted with the software JAGS (Plummer, 2003) in R using the packages `rjags` and `runjags` (Denwood, 2016; Plummer, 2021). For parameter estimation, we used six MCMC chains with 1,000 adaptions, 1,500 burn-in iterations, and 8,000 samples each (after thinning by a factor of three). These MCMC settings were selected to achieve a potential scale reduction factor of $\hat{R} < 1.1$ for all parameters. For this purpose, we first performed a small-scale simulation study with only few informants, few items, and a small variance in informants' competence and item difficulty to adjust the setting for JAGS. In the main simulation study, only 56 simulations (0.22%) did not converge with more than 10% of parameters having a potential scale reduction factor of $\hat{R} > 1.1$ and were, thus, excluded from the analysis. For the remaining simulations, the average potential scale reduction factor was $\hat{R} = 1.002$ (99% quantile = 1.02). The model code for JAGS can be found in Appendix A.

3.2 Parameter recovery

To examine parameter recovery in our extended CCT model, we first investigate parameter recovery using a single simulated data set. For this example, we chose a model with $N = 20$ informants, $M = 10$ items, a standard deviation of informants' competence of $\sigma_{\log E} = 1$, and a standard deviation of item difficulty of $\sigma_{\log \lambda} = 0.5$. Figure 2 shows the data-generating and estimated parameters for $\log E_i$, $\log \lambda_{kd}$, ρ_k, and T_{kd} including the correlation of data-generating and estimated parameters and the root-mean-square

Figure 2

Parameter recovery of the CCT-2D model for a single simulated data set.



Note. Parameter recovery for a single simulated data set with $N = 20$ informants, $M = 10$ items, $\sigma_{\log E} = 1$, and $\sigma_{\log \lambda} = 0.5$. The first two panels show the logarithm of informants' competence ($\log E_i$) and item difficulty ($\log \lambda_{kd}$).

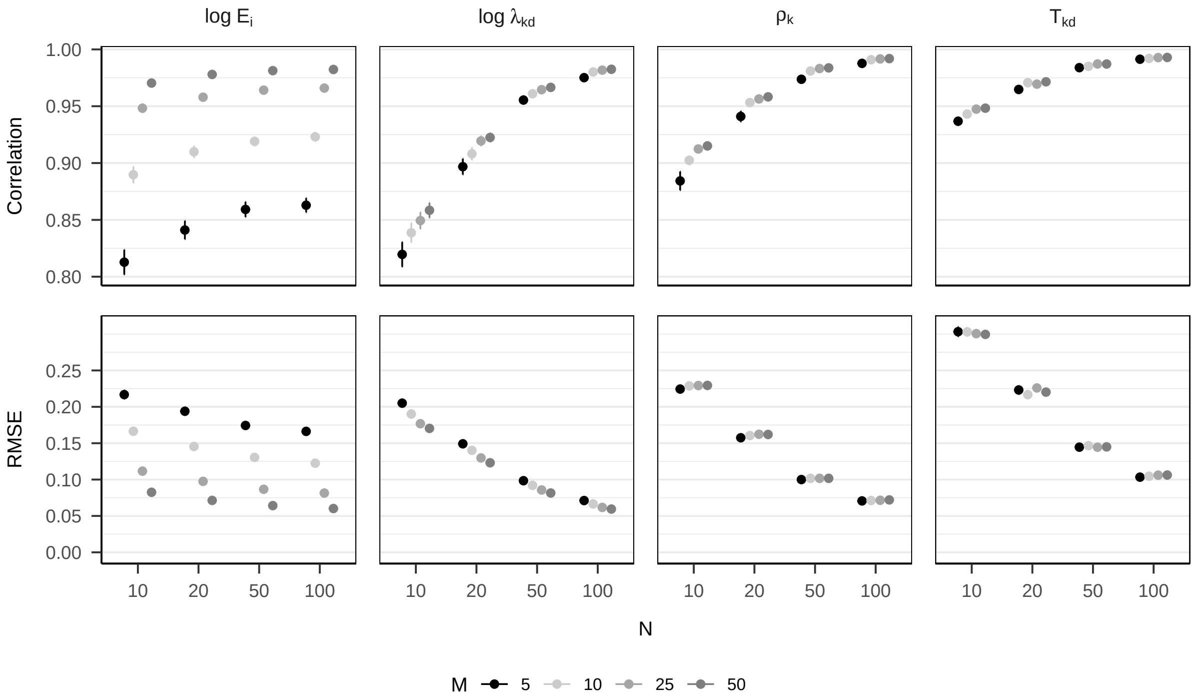
error (RMSE). For the vector-valued parameters $\boldsymbol{\lambda}_k$ and $\boldsymbol{T}_k$, the data-generating and estimated values for the x- and y-dimension are displayed jointly in the respective panels. All correlations are above .98 with the RMSE of the estimates ranging between 0.15 and 0.22. This indicates that the CCT-2D model performs quite well even with a moderate number of informants and items.

To judge the performance of the CCT-2D model for various scenarios, we assess the parameter recovery by computing the average correlation and RMSE of the data-generating and the estimated parameters across all 25,544 replications. Again, we display the correlation and RMSE for $\log \lambda_{kd}$ and T_{kd} for both dimensions in one panel. For all simulations with $\sigma_{\log E} = 0$ or $\sigma_{\log \lambda} = 0$, the correlation of generated and posterior values for $\log E_i$ and $\log \lambda_{kd}$, respectively, cannot be computed. This affected 11,188 replications for which either $\sigma_{\log E}$, $\sigma_{\log \lambda}$, or both were zero.

Figure 3 displays the average correlation and RMSE for all combinations of N and M. The item parameters $\log \lambda_{kd}$, ρ_k, and T_{kd} were clearly affected by the number of informants (N). This is due to the item parameters requiring a certain number of

Figure 3

Parameter recovery across 25,544 replications.



Note. Average correlations of data-generating and estimated parameters and RMSEs are displayed with 95% confidence intervals. For simulations with $\log \sigma_E = 0$ and $\log \sigma_\lambda = 0$, no correlations could be computed for the parameters $\log E_i$ and $\log \lambda_{kd}$, respectively.

informants who answer these items to yield reliable parameter estimates. In contrast, the person parameters E_i were more strongly affected by the number of items (M). This shows that the estimation of person parameters requires a certain number of items to be reliable. Of all parameters, RMSEs of the cultural truth T_{kd} were somewhat more affected by varying levels of N than those of all other parameters with RMSEs as high as 0.30. However, correlations of data-generating and estimated parameters of $\log \lambda_{kd}$ and $\log E_i$ were more strongly affected by varying levels of N and M respectively with correlations just above .80 for both parameters. Similar patterns emerged for the average width of the credible intervals of the parameters (see Appendix B). The precision of the item parameters ρ_k and T_{kd} was strongly affected by the number of informants whereas the precision of the person parameters E_i was more strongly affected by the number of items. In contrast to parameter recovery, the widths of the credible intervals for the item difficulties $\log \lambda_{kd}$ were affected by both N and M.

Furthermore, Figure 4 displays the parameter recovery of $\log E_i$ (Panel A) and $\log \lambda_{kd}$ (Panel B) for varying levels of $\sigma_{\log E}$ and $\sigma_{\log \lambda}$. While RMSEs are very small when there is no variance in either of the parameters, the recovery of E_i and λ_{kd} is worse for low levels of $\sigma_{\log E}$ and $\sigma_{\log \lambda}$, respectively, with correlations between data-generated parameters and estimated parameters as low as .64 for $\log E_i$ and .65 for $\log \lambda_{kd}$. However, as already observed in Figure 3, with increasing M, parameter recovery for $\log E_i$ improves, and with increasing N, parameter recovery for $\log \lambda_{kd}$ improves.

Overall, parameter recovery is acceptable for small N and M as well as low levels of $\sigma_{\log E}$ and $\sigma_{\log \lambda}$. As expected, all parameters show better recovery and smaller credible intervals the larger N and M are and the larger the variances in informants' competence and item difficulty are. Accordingly, if N and M are small while there is little variance in $\sigma_{\log E}$ and $\sigma_{\log \lambda}$, the parameters $\log E_i$ or $\log \lambda_{kd}$ cannot be estimated reliably.

3.3 Comparing the accuracy of CCT-2D and unweighted averaging

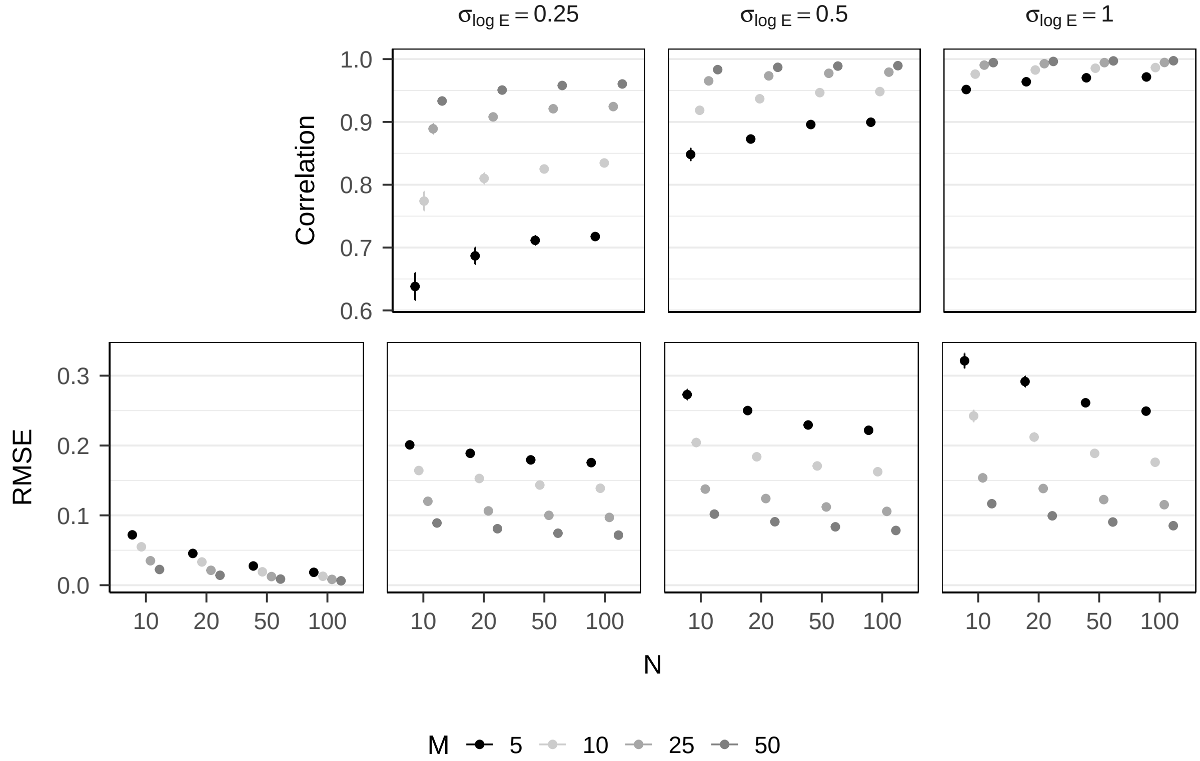
In the following, we compare the accuracy of aggregating two-dimensional location judgments either with the CCT-2D model or with unweighted averaging. To obtain unweighted group-level estimates, we simply computed the unweighted mean of all location judgments for each item (separately for the x- and the y-coordinate). As a measure of accuracy, we use the Euclidean distance to the correct position for each item. Figure 5 displays the mean Euclidean distances across all items between the correct values and the CCT-2D estimates (gray points) and between the correct values and the estimates obtained with unweighted averaging (black points). To facilitate interpretation of the results, we aggregated across replications with varying numbers of items.

As expected, Figure 5 shows that aggregating location judgments with CCT-2D yielded more accurate estimates than aggregating judgments with unweighted averaging. However, without any variance in informants' competence ($\sigma_{\log E} = 0$) or item difficulty ($\sigma_{\log \lambda} = 0$), both methods lead to equally accurate location estimates (upper left panel).

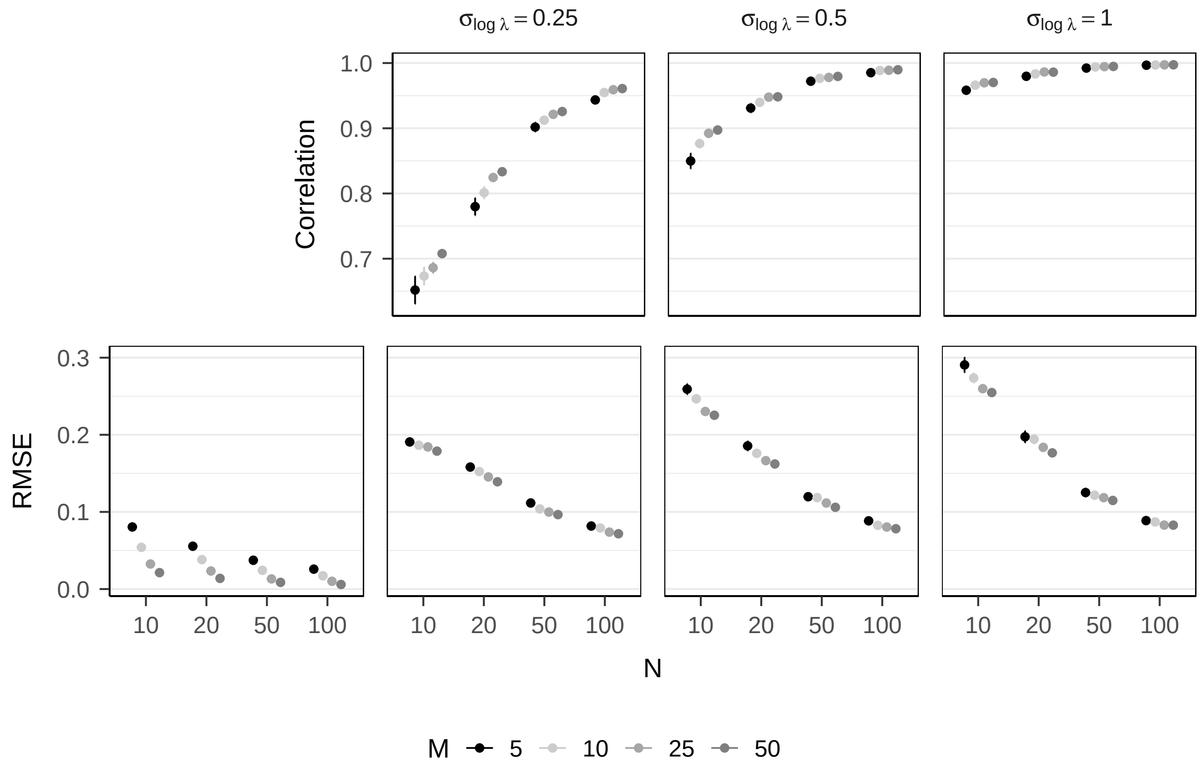
Figure 4

Average parameter recovery for different $\sigma_{\log E}$ or $\sigma_{\log \lambda}$.

(A) Parameter recovery of log E_i for varying levels of $\sigma_{\log E}$



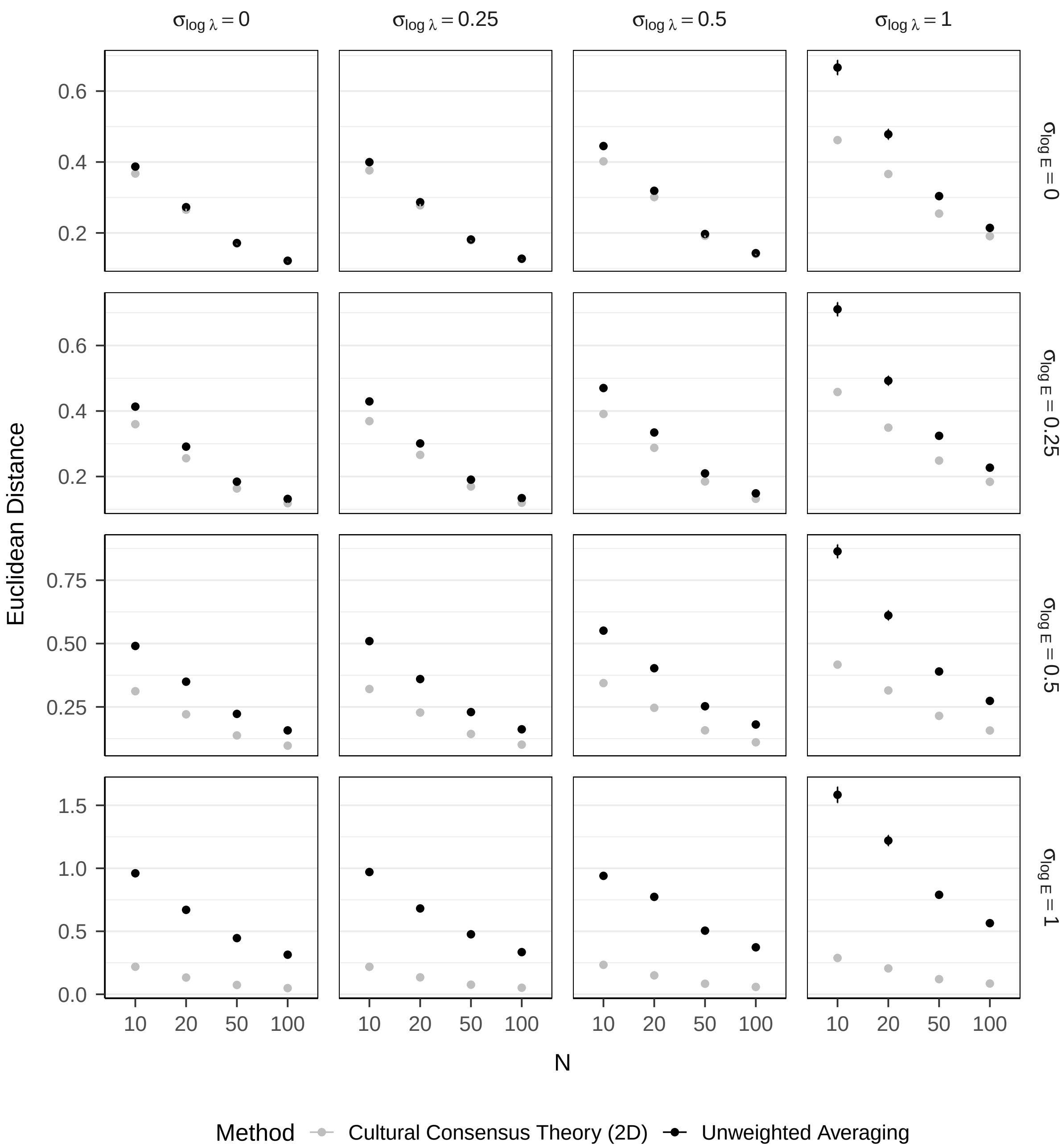
(B) Parameter recovery of log λ_{kd} for varying levels of $\sigma_{\log \lambda}$



Note. Mean correlations and RMSEs are displayed with 95% confidence intervals. For simulations with $\sigma_{\log E} = 0$ and $\sigma_{\log \lambda} = 0$ no correlations could be computed.

Figure 5

Marginal accuracy of aggregate location estimates.



Note. The scaling of the y-axis differs across rows to improve readability. Mean accuracy is displayed with 95% confidence intervals.

In line with the principles of averaging out individual errors, Figure 5 shows that both unweighted averaging and CCT generally provided more accurate estimates the larger the sample of informants was. However, increasing sample size was more beneficial for unweighted averaging than for CCT estimates. Furthermore, estimates obtained with unweighted averaging became worse the larger the variance in informants' competence became. This was expected since increasing the heterogeneity of informants' competence yields larger variation in judgments, which in turn results in larger Euclidean distances to the correct position. The CCT model accounts and corrects for this additional variance in the observed location judgments, thereby resulting in a better recovery of the latent truth.

Even in the absence of differences in competence (first row in Figure 5), CCT-2D resulted in more accurate location estimates than unweighted averaging. At first sight, this result might be surprising since there are no true differences in expertise which could be exploited by the weighting mechanism of the CCT model. However, the higher accuracy of CCT-2D can be explained by shrinkage of parameter estimates in hierarchical Bayesian models. The CCT-2D model assumes a hierarchical group-level distribution of the cultural-truth parameters $\boldsymbol{T}_k$ across items which is not the case for unweighted averaging. Shrinkage of the random-effect parameters generally results in estimates closer to the mean $\mu_{T,d}$ compared to estimates based on independent item parameters (i.e., fixed effects, Heck, 2019). As a consequence, extreme estimates are avoided especially when there are only few judgments for each item (i.e., if the sample size N is small). In our simulation study, the benefit of shrinkage was especially large since the CCT-2D perfectly represented the data-generating process with respect to all (hyper-)prior distributions of the parameters. However, even when the hierarchical distributions are not accurately specified, shrinkage generally results in increased overall accuracy (Efron & Morris, 1977; Stein, 1956). In Figure 5, shrinkage results in a higher accuracy of CCT-2D compared to unweighted averaging even in the absence of differences in competence. This benefit becomes larger as variance in item difficulties increases due to a stronger effect of shrinkage on the (random) item effects. However, with increasing numbers of judgments per item (i.e., for larger N), shrinkage is reduced as the item parameters can be estimated more precisely. In turn, this results in a similar accuracy for CCT-2D and unweighted averaging. Overall, our comparison shows that CCT-2D can increase the accuracy of aggregated location judgments by accounting for heterogeneity in competence and item difficulty.

4 Empirical study

In addition to the simulation study, we also apply the CCT-2D model to empirical data of participants who located various European cities on geographic maps (Mayer & Heck, 2022). Additionally, we compare the accuracy of aggregated location judgments of CCT-2D and unweighted averaging. Since multiple informants provided judgments for multiple items from the same knowledge domain (i.e., locations of European cities), the data fulfills the necessary requirements for an analysis with CCT-2D. All data and R scripts are available at https://osf.io/jbzk7/.

4.1 Methods

In the following, we reanalyze the data of a study by Mayer and Heck (2022) in which participants had to judge the location of 57 European cities on 7 different maps. We recruited 417 adult participants via a commercial German panel provider for an experiment on collaboration. 235 of these participants completed a condition in which they provided independent location judgments for all the presented items which makes their data suitable for an reanalysis with both CCT-2D and unweighted averaging. However, we excluded 7 participants who positioned more than 10% of the cities outside of the countries of interest (which were highlighted in white color), resulting in a total of 228 participants. In the remaining sample of participants, the mean age was 46.68 (SD = 15.23) and 46.9% of the participants were female. Most participants had a college degree (34.2%) or a high-school diploma (25.9%), while 24.1% had vocational education, and 15.8% had a lesser educational attainment.

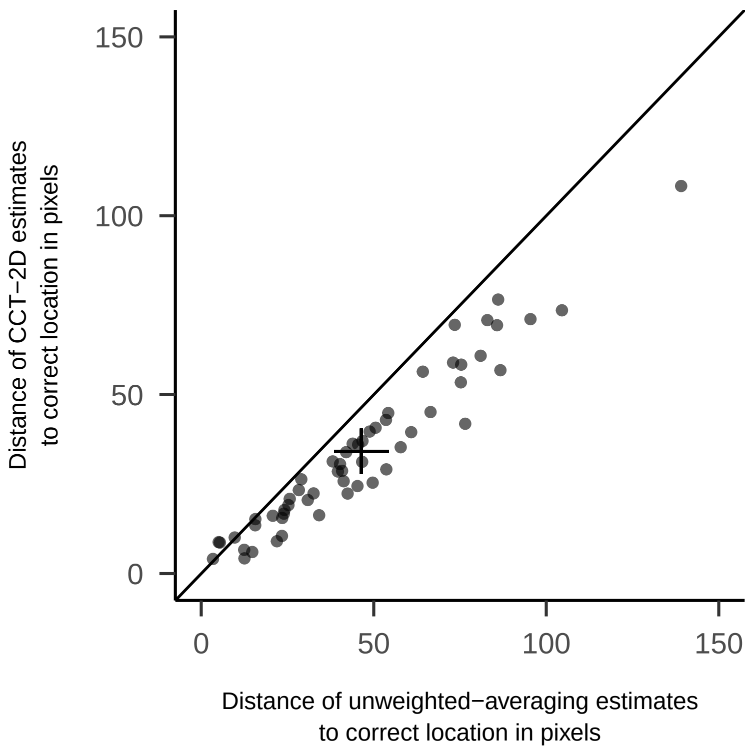
A comprehensive overview of all presented cities and maps can be found in Appendix C1. All maps were scaled to 1:5,000,000 and were presented as images with 800×500 pixels. At this scaling, the influence of earth's curvature is small and can be neglected in further analyses. The maps only showed oceans which were colored in blue, landmasses which were colored in white for countries of interest and in gray for all other countries, and national borders as black lines as shown in Figure 7.

While completing the study, participants indicated the position of each of the 57 cities independently in separate trials. The seven maps were presented in random order such that all cities for a map were presented before a new map appeared. Within each map, the order of cities was also randomized. Since the study was conducted online, we implemented a maximum time limit of 40 seconds for each item to prevent looking up the correct locations of the cities (for details, see Mayer & Heck, 2022).

4.2 Results

Figure 6

Accuracy of location estimates for 57 European cities.



Note. Each point refers to one city. The black cross indicates the 95% confidence intervals for the average distance for each of the two methods. The reanalysis is based on $N = 228$ participants from the data set by Mayer and Heck (2022).

To compare the accuracy of CCT-2D and unweighted averaging, we first computed the group-level estimates for all locations of the 57 cities. For unweighted averaging, we simply aggregated the independent location judgments for each city by taking the mean in the x- and the y-direction. For the CCT-2D model, we extracted the posterior-mean estimates of the two-dimensional cultural-truth parameters $\boldsymbol{T}_k$. We then computed the accuracy of the estimated locations by the Euclidean distance to the actual location of the presented cities.

The CCT-2D model implemented in JAGS assumes prior distributions that are tailored to standardized judgments. For instance, the mean $\mu_{T,d}$ of the distribution of truth-parameters $\boldsymbol{T}_k$ follows a normal distribution with a mean of zero and a variance of four. However, the x- and y-coordinates of participants' judgments are measured in pixels with values between 0 px and 800 px. Besides distorting parameter estimates, such a misalignment between the scale of the priors and that of the data results in very long computation times of the JAGS sampler. As a remedy, we standardized the judgments before fitting the model based on the mean and SD of the x- and y-coordinates across all participants and items. In practice, it is easier to standardize the responses as opposed to aligning all prior distributions of the CCT-2D model to the measurement scale of the raw data. For the presentation of our results, the posterior-mean estimates of $\boldsymbol{T}_k$ are transformed back to pixels, thereby allowing for a comparison to unweighted averaging.

Figure 6 displays the Euclidean distance of the location estimates for the 57 cities obtained with CCT-2D and unweighted averaging, as well as 95% confidence intervals for the mean distances (indicated by a black cross). The results show that the aggregation of location judgments with CCT-2D resulted in more accurate estimates than unweighted averaging for almost all cities. To illustrate the improvement in accuracy provided by CCT-2D, Figure 7 displays the estimated locations of both methods as well as the actual locations of five cities on the map of the United Kingdom and Ireland. Moreover, the plot shows 50% confidence ellipses for the empirical distribution of individual judgments (solid lines) and for the distribution of judgments implied by the variance-covariance matrix of CCT-2D for an informant with average expertise (dashed lines). CCT-2D shows more accurate estimates than unweighted averaging for four of the five cities (i.e., Birmingham, Dublin, Glasgow, and London) and an equally accurate estimate for one city (Liverpool). Notably, for some cities such as London, the distance between the true and the estimated location is approximately half as large for CCT-2D compared to unweighted averaging. The supplementary material provides plots of all seven European maps used in the study, each displaying the location estimates obtained with unweighted averaging and CCT-2D as well as the cities' actual positions (see https://osf.io/jbzk7/).

The descriptive patterns shown in Figures 6 and 7 were also supported by a statistical analysis. A paired-sample t-test showed that the accuracy of the CCT-2D estimates was significantly higher than that of estimates obtained with unweighted averaging ($t(56) = 10.51, p < .001$). Notably, Cohen's d indicated a large effect size of $d = 1.39$. Across all cities, estimates were on average 12.24 pixels closer to the correct position, resembling the improvement for Glasgow in Figure 7 which was 15.63 pixels.

To further examine the validity of the CCT-2D model, we computed the correlation between the estimated competence parameters $\log E_i$ and the education level of the participants. Individuals with a higher education level should have more geographic knowledge and thus provide more accurate judgments which are closer to the cultural truth. Since smaller values of the competence parameter indicate higher individual competence (i.e., reflecting a smaller variance of judgments around the cultural truth), we expect a negative correlation between the estimated competence and education level. When encoding the education level as an ordinal variable, a Spearman rank correlation indeed showed a medium negative correlation of $-.35$ ($p < .001$), thus strengthening the validity of the CCT-2D model and the $\log E_i$ parameters.

We also fitted two alternative versions of CCT-2D to test whether certain assumptions for two-dimensional continuous data hold. The models differed in details of the specification of the parameters. The first alternative model assumed that the correlation ρ_k of judgment errors between the x- and y-dimension is zero for all items. This model resembles the CCT model for one-dimensional continuous data (Anders et al.,

2014) because the location parameters $\boldsymbol{T}_k$ are estimated independently for the x- and y-coordinates. However, this simplified model (DIC $= 31,999$) showed a worse performance than the original model (DIC $= 30,187$) with respect to the deviance information criterion (DIC, Plummer, 2008), for which smaller values indicate better model performance. This result is in line with our finding that for 35 of the 57 cities used in the empirical study (61.4%), the 95% credible interval of the correlation of errors between the two dimensions did not include the value zero. We also examined another CCT model version assuming that the item difficulty $\boldsymbol{\lambda}_k$ does not differ between x- and y-direction. Instead, the model assumes a single, one-dimensional difficulty parameter for each city. Even though the item difficulties for the x- and y-direction showed a substantial correlation in the original CCT-2D model ($r_\lambda = 0.85$; 95% credible interval $= [0.77, 0.92]$), the simplified model version assuming a one-dimensional item difficulty showed a worse performance (DIC $= 32,881$) than the original model (DIC $= 30,187$). This indicates that item difficulty indeed varies between both dimensions in our empirical study.

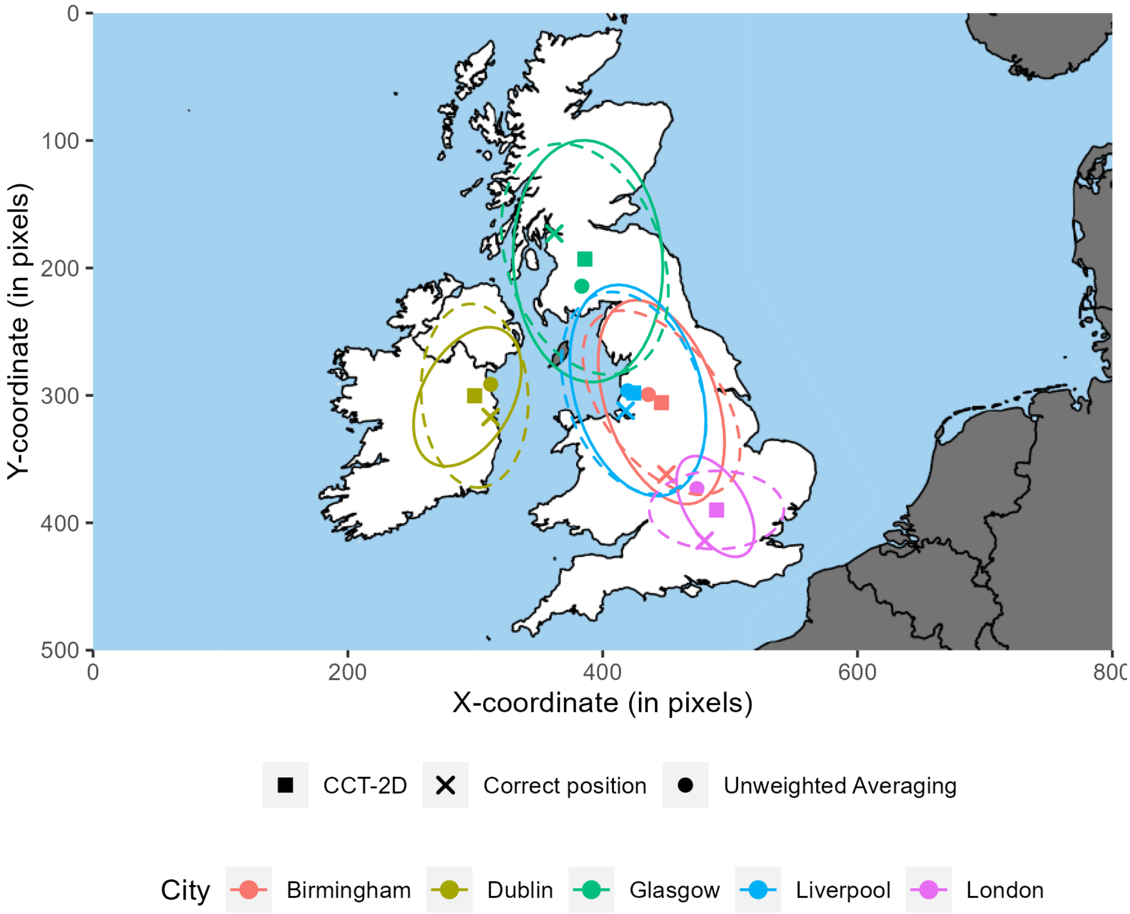
5 Discussion

We proposed a novel model of Cultural Consensus Theory for two-dimensional location judgments (CCT-2D). The model is based on the hierarchical Bayesian CCT model by Anders et al. (2014) for one-dimensional data. The CCT-2D model estimates the latent cultural truths of the presented items, that is, the group's consensus knowledge concerning the (unknown) positions of the items. To do so, the model infers the informants' competence based on the distance of their response patterns to the shared consensus, as well as the difficulty of the items. To account for the spatial structure of the two-dimensional data, the model assumes that judgment errors are correlated between the two dimensions for each item.

We successfully applied the new model both to simulated and empirical data. Using simulations, we showed that the CCT-2D model has a very good parameter recovery for a large range of numbers of informants and numbers of items. Moreover, the simulations showed that the CCT-2D group-level estimates for the latent truths of the

Figure 7

Estimated versus actual locations of five cities.



Note. Solid lines show 50% confidence ellipses based on the empirical distribution of judgments. Dashed lines visualize 50% posterior ellipses based on the estimated variance-covariance matrix of judgments in the CCT-2D model (for an informant with average competence). The variance-covariance matrix thereby depends on item difficulty and informant's expertise (see Equation 5).

locations were more accurate in terms of the Euclidean distance to the true locations than the estimates obtained with unweighted averaging of individual judgments. This is due to the fact that the CCT-2D model considers additional information obtained by inferring differences in the items' difficulty and the informants' competence. Furthermore, a reanalysis of an empirical study in which informants located 57 European cities on seven different maps showed a large effect concerning an increase in accuracy of CCT-2D compared to unweighted averaging. These findings conceptually replicate the results of Merkle et al. (2020) who found that a CCT-inspired mechanism of weighting informants' judgments by their expertise outperformed unweighted averaging for one-dimensional forecasting judgments (i.e., for point spread forecasts of the Australian Football League).

5.1 Limitations and future model developments

While our results provide preliminary evidence for the usefulness of the proposed CCT-2D model, the model has several limitations that should be addressed in the future. First, it is possible that response biases may lead to a general shift of location judgments away from the borders into the interior regions of the presented maps. A similar effect may also occur due to certain geographic features such as coastlines or national borders (Friedman, Brown, et al., 2002; Friedman et al., 2005). Note that a simple, additive shift of all location judgments into a certain direction by a certain distance similar as in the one-dimensional CCT model by Anders et al. (2014) cannot describe such a complex, nonlinear bias towards inner regions. However, it may generally be difficult to disentangle complex, item-independent response biases from distortions of the latent consensus knowledge about the locations of specific items both empirically and conceptually.

Second, our model assumes that all informants share a single latent cultural truth across all items. This assumption is appropriate for applications similar to our example where we used CCT-2D to aggregate location judgments of a homogeneous group of informants when all items are from a single domain. However, the assumption may be violated in other applications. For instance, Friedman and colleagues (Friedman, Brown, et al., 2002; Friedman et al., 2012, 2005; Friedman, Kerkman, et al., 2002) demonstrated that geographical location judgments vary depending on the geographical region participants live in. In this example, different cultural truths can be modeled separately for each group. However, an informant's group membership may be latent, for instance, when their cultural background is unknown or when a person moved from one geographical region to another. In such cases, CCT-2D can be extended to multiple latent cultural truths by assuming different consensus locations and item difficulties for each latent group (Anders et al., 2014). If the presented items are from qualitatively different domains (e.g., informants have to locate cities in their home country and in a foreign country), the assumption that informants have the same competence for all items may be violated. As a remedy, one can fit separate CCT-2D models for different domains of items or extend the CCT-2D model while estimating multiple competence parameters per person.

Third, the proposed CCT-2D model assumes bivariate normal distributions of the observed location judgments and of the latent truths concerning the positions of the presented items. However, locations on maps are naturally constrained by the borders of the map and by geographic features such as coasts or national borders (Friedman et al., 2005). It is thus likely that our assumption that location judgments and latent truths follow bivariate normal distributions with unbounded support is violated. As a remedy, the CCT-2D model of location judgments may be improved by implementing a truncation of the support in the two-dimensional space by respecting geographic features of the map. For instance, when estimating the location of Dublin, one may exclude observed judgments that position the city in the Atlantic Ocean, while also implementing a corresponding truncation for the support of the bivariate normal distribution of observed judgments (Gelfand et al., 1992). For the application of our model to empirical data, we simply excluded participants who positioned more than 10% of their judgments outside the highlighted countries of interest to more adequately fulfill this assumption.

In principle, it is also possible to truncate the support of the bivariate distribution of latent truths to landmasses only. Thereby, one ensures that all posterior samples of the inferred locations in MCMC sampling are actually located on land and away from the sea. However, implementing complex, nonlinear, two-dimensional truncations in JAGS or other software is not straightforward. Even when considering only a set of simple, linear order constraints, tailored MCMC algorithms are usually required to ensure that all posterior samples satisfy the constraints (Heck & Davis-Stober, 2019). Moreover, these methods often assume that the truncated parameter space is convex which is not the case for landmasses on geographic maps. Thus, we leave it to future research to implement the truncation of distributions in the CCT-2D model.

Besides aggregating location judgments on geographic maps, our extension of CCT to two-dimensional continuous data can also be applied to other types of judgments such as continuous ratings of both the emotional arousal and valence of pictures on two visual analogue scales (Funke & Reips, 2012; Reips & Funke, 2008). Moreover, the CCT-2D model can also easily be extended to d-multivariate responses on an arbitrary number of judgment dimensions for continuous responses. Such an approach could be useful, for instance, when rating facial images with respect to several dimensions such as trustworthiness, attractiveness, and symmetry on continuous scales (Oosterhof & Todorov, 2008). However, depending on the application, it is necessary to adjust the model with respect to the dimensionality and dependence of the parameters. For instance, when eliciting judgments on several dimensions (e.g., subjective arousal and valence of pictures), it is more suitable to assume separate competence dimensions $E_{i,1}$ and $E_{i,2}$ as opposed to a single dimension as for geographical expertise. Moreover, response errors could be assumed to be uncorrelated across dimensions since our empirical example only provided evidence that correlated errors are required for modeling location judgments. When using visual analogue scales or sliders for eliciting judgments, it may also be necessary to include a shifting and a scaling response bias similar as in Anders et al. (2014). Before relying on the estimates of such alternative CCT-2D versions in other domains, it is of course necessary to test the fit and the validity of the model. In doing so, one should consider that averaging the judgments of all group members is only one among many possible aggregation mechanisms (for alternatives such as selecting a smaller crowd, see Galesic et al., 2018; Goldstein et al., 2014).

5.2 Conclusions

The proposed CCT-2D model extends the scope of applications of cultural consensus theory to two-dimensional continuous data. Researchers can now analyze and aggregate geographical location judgments consisting of x- and y-coordinates or longitude and latitude to infer the group's cultural knowledge about the unknown locations. In doing so, the model weighs the observed judgments both by the informants' competence and by the items' difficulty. Concerning the study design, it is necessary to recruit multiple informants who provide judgments for multiple items from the same knowledge domain. We showed that the CCT-2D model provides good parameter recovery and, in cases where the factual truth is known, provides aggregate group-level estimates that are more accurate than those obtained by the unweighted averaging of location judgments.

6 References

Al-shabeeb, A. R. (2016). The use of AHP within GIS in selecting potential sites for water harvesting sites in the Azraq Basin—Jordan. *Journal of Geographic Information System*, *8*(1), 73–88. https://doi.org/10.4236/jgis.2016.81008

Anders, R., & Batchelder, W. H. (2015). Cultural consensus theory for the ordinal data case. *Psychometrika*, *80*, 151–181. https://doi.org/10.1007/s11336-013-9382-9

Anders, R., & Batchelder, W. H. (2012). Cultural consensus theory for multiple consensus truths. *Journal of Mathematical Psychology*, *56*, 452–469. https://doi.org/10.1016/j.jmp.2013.01.004

Anders, R., Oravecz, Z., & Batchelder, W. H. (2014). Cultural consensus theory for continuous responses: A latent appraisal model for information pooling. *Journal of Mathematical Psychology*, *61*, 1–13. https://doi.org/10.1016/j.jmp.2014.06.001

Aßfalg, A. (2018). Consensus theory for mixed response formats. *Journal of Mathematical Psychology*, *86*, 51–63. https://doi.org/10.1016/j.jmp.2018.08.005

Aßfalg, A., & Klauer, K. C. (2020). Consensus theory for multiple latent traits and consensus groups. *Journal of Mathematical Psychology*, *97*, 102374. https://doi.org/10.1016/j.jmp.2020.102374

Batchelder, W. H. (2009). Cultural consensus theory: Aggregating expert judgments about ties in a social network. *Social Computing and Behavioral Modeling*, 1–9. https://doi.org/10.1007/978-1-4419-0056-2_5

Batchelder, W. H., & Anders, R. (2012). Cultural consensus theory: Comparing different concepts of cultural truth. *Journal of Mathematical Psychology*, *56*, 316–332. https://doi.org/10.1016/j.jmp.2012.06.002

Batchelder, W. H., Kumbasar, E., & Boyd, J. P. (1997). Consensus analysis of three-way social network data. *The Journal of Mathematical Sociology*, *22*, 29–58. https://doi.org/10.1080/0022250X.1997.9990193

Batchelder, W. H., & Romney, A. K. (1988). Test theory without an answer key. *Psychometrika*, *53*, 71–92. https://doi.org/10.1007/BF02294195

Budescu, D. V., & Chen, E. (2015). Identifying expertise to extract the wisdom of crowds. *Management Science*, *61*(2), 267–280. https://doi.org/10.1287/mnsc.2014.1909

Casana, J. (2014). Regional-scale archaeological remote sensing in the age of big data: Automated site discovery vs. Brute force methods. *Advances in Archaeological Practice*, *2*(3), 222–233. https://doi.org/10.7183/2326-3768.2.3.222

Davis-Stober, C. P., Budescu, D. V., Dana, J., & Broomell, S. B. (2014). When is a crowd wise? *Decision*, *1*, 79–101. https://doi.org/10.1037/dec0000004

Denwood, M. J. (2016). runjags: An R package providing interface utilities, model templates, parallel computing methods and additional distributions for MCMC models in JAGS. *Journal of Statistical Software*, *71*(9), 1–25. https://doi.org/10.18637/jss.v071.i09

Efron, B., & Morris, C. (1977). Stein's paradox in statistics. *Scientific American*, *236*, 119–127. https://www.jstor.org/stable/24954030

Embretson, S. E., & Reise, S. P. (2000). *Item response theory*. Psychology Press. https://doi.org/10.4324/9781410605269

Faghri, A., Lang, A., Hamad, K., & Henck, H. (2002). Integrated knowledge-based geographic information system for determining optimal location of park-and-ride facilities. *Journal of Urban Planning and Development*, *128*, 18–41. https://doi.org/10.1061/(ASCE)0733-9488(2002)128:1(18)

Friedman, A., Brown, N. R., & Mcgaffey, A. P. (2002). A basis for bias in geographical judgments. *Psychonomic Bulletin & Review*, *9*, 151–159. https://doi.org/10.3758/BF03196272

Friedman, A., Kerkman, D. D., & Brown, N. R. (2002). Spatial location judgments: A cross-national comparison of estimation bias in subjective North American geography. *Psychonomic Bulletin & Review*, *9*, 615–623. https://doi.org/10.3758/BF03196321

Friedman, A., Kerkman, D. D., Brown, N. R., Stea, D., & Cappello, H. M. (2005).

Cross-cultural similarities and differences in North Americans' geographic location judgments. *Psychonomic Bulletin & Review*, *12*, 1054–1060. https://doi.org/10.3758/BF03206443

Friedman, A., Mohr, C., & Brugger, P. (2012). Representational pseudoneglect and reference points both influence geographic location estimates. *Psychonomic Bulletin & Review*, *19*, 277–284. https://doi.org/10.3758/s13423-011-0202-x

Funke, F., & Reips, U.-D. (2012). Why semantic differentials in web-based research should be made from visual analogue scales and not from 5-point scales. *Field Methods*, *24*, 310–327. https://doi.org/10.1177/1525822X12444061

Galesic, M., Barkoczi, D., & Katsikopoulos, K. (2018). Smaller crowds outperform larger crowds and individuals in realistic task conditions. *Decision*, *5*(1), 1–15. https://doi.org/10.1037/dec0000059

Gelfand, A. E., Smith, A. F. M., & Lee, T.-M. (1992). Bayesian analysis of constrained parameter and truncated data problems using Gibbs sampling. *Journal of the American Statistical Association*, *87*, 523–532. https://doi.org/10.2307/2290286

Goldstein, D. G., McAfee, R. P., & Suri, S. (2014). The wisdom of smaller, smarter crowds. *Proceedings of the Fifteenth ACM Conference on Economics and Computation*, 471–488. https://doi.org/10.1145/2600057.2602886

Heck, D. W. (2019). Accounting for estimation uncertainty and shrinkage in Bayesian within-subject intervals: A comment on Nathoo, Kilshaw, and Masson (2018). *Journal of Mathematical Psychology*, *88*, 27–31. https://doi.org/10.1016/j.jmp.2018.11.002

Heck, D. W., Boehm, U., Böing-Messing, F., Bürkner, P.-C., Derks, K., Dienes, Z., Fu, Q., Gu, X., Karimova, D., Kiers, H., Klugkist, I., Kuiper, R. M., Lee, M. D., Leenders, R., Leplaa, H. J., Linde, M., Ly, A., Meijerink-Bosman, M., Moerbeek, M., … Hoijtink, H. (2022). A review of applications of the Bayes factor in psychological research. *Psychological Methods*. In press. https://doi.org/10.1037/met0000454

Heck, D. W., & Davis-Stober, C. P. (2019). Multinomial models with linear inequality constraints: Overview and improvements of computational methods for Bayesian inference. *Journal of Mathematical Psychology*, *91*, 70–87. https://doi.org/10.1016/j.jmp.2019.03.004

Hueffer, K., Fonseca, M. A., Leiserowitz, A., & Taylor, K. M. (2013). The wisdom of crowds: Predicting a weather and climate-related event. *Judgment and Decision Making*, *8*, 91–105.

Larrick, R. P., & Soll, J. B. (2006). Intuitions about combining opinions: Misappreciation of the averaging principle. *Management Science*, *52*, 111–127. https://doi.org/10.1287/mnsc.1050.0459

Lee, M. D., & Wagenmakers, E.-J. (2014). *Bayesian cognitive modeling: A practical course*. Cambridge University Press.

Lord, F. M., Novick, M. R., & Birnbaum, A. (1968). *Statistical theories of mental test scores*. Addison-Wesley.

Mahdavi, A., Niknejad, M., & Karami, O. (2015). A fuzzy multi-criteria decision method for ecotourism development locating. *Caspian Journal of Environmental Sciences*, *13*(3), 221–236. https://cjes.guilan.ac.ir/article_1373.html

Mayer, M., & Heck, D. W. (2022). Sequential collaboration: The accuracy of dependent, incremental judgments. *Decision*, Advance online publication. https://doi.org/10.1037/dec0000193

Merkle, E. C., Saw, G., & Davis-Stober, C. (2020). Beating the average forecast: Regularization based on forecaster attributes. *Journal of Mathematical Psychology*, *98*, 102419. https://doi.org/10.1016/j.jmp.2020.102419

Oosterhof, N. N., & Todorov, A. (2008). The functional basis of face evaluation. *Proceedings of the National Academy of Sciences*, *105*, 11087–11092. https://doi.org/10.1073/pnas.0805664105

Plieninger, H., & Heck, D. W. (2018). A new model for acquiescence at the interface of psychometrics and cognitive psychology. *Multivariate Behavioral Research*, *53*, 633–654. https://doi.org/10.1080/00273171.2018.1469966

Plummer, M. (2008). Penalized loss functions for Bayesian model comparison. *Biostatistics*, *9*, 523–539. https://doi.org/10.1093/biostatistics/kxm049

Plummer, M. (2003). *JAGS: A program for analysis of bayesian graphical models using Gibbs sampling.*

Plummer, M. (2021). *rjags: Bayesian graphical models using MCMC.* https://CRAN.R-project.org/package=rjags

Reips, U.-D., & Funke, F. (2008). Interval-level measurement with visual analogue scales in internet-based research: VAS generator. *Behavior Research Methods*, *40*, 699–704. https://doi.org/10.3758/BRM.40.3.699

Romney, A. K., Weller, S. C., & Batchelder, W. H. (1986). Culture as consensus: A theory of culture and informant accuracy. *American Anthropologist*, *88*, 313–338. https://www.jstor.org/stable/677564

Singmann, H., & Kellen, D. (2019). An introduction to mixed models for experimental psychology. In D. H. Spieler & E. Schumacher (Eds.), *New Methods in Cognitive Psychology.* Psychology Press.

Stein, C. (1956). *Inadmissibility of the usual estimator for the mean of a multivariate normal distribution.* *1*, 197–206. https://projecteuclid.org/euclid.bsmsp/1200501656

Steyvers, M., Miller, B., Hemmer, P., & Lee, M. (2009). The wisdom of crowds in the recollection of order information. In Y. Bengio, D. Schuurmans, J. Lafferty, C. Williams, & A. Culotta (Eds.), *Advances in neural information processing systems* (Vol. 22). Curran Associates, Inc.

Surowiecki, J. (2005). *The wisdom of crowds.* Anchor Books.

Trotter, R., Weller, S., Baer, R., Pachter, L., Glazer, M., Alba-García, J., & Klein, R. (1999). Consensus theory model of AIDS/SIDA beliefs in four latino populations. *AIDS Education and Prevention*, *11*, 414–426.

Waubert de Puiseau, B., Aßfalg, A., Erdfelder, E., & Bernstein, D. M. (2012). Extracting the truth from conflicting eyewitness reports: A formal modeling approach. *Journal of Experimental Psychology: Applied*, *18*, 390–403. https://doi.org/10.1037/a0029801

Waubert de Puiseau, B., Greving, S., Aßfalg, A., & Musch, J. (2017). On the importance of considering heterogeneity in witnesses' competence levels when reconstructing crimes from multiple witness testimonies. *Psychological Research*, *81*, 947–960. https://doi.org/10.1007/s00426-016-0802-1

Weller, S. C. (2007). Cultural consensus theory: Applications and frequently asked questions. *Field Methods*, *19*, 339–368. https://doi.org/10.1177/1525822X07303502

Appendix A

JAGS code for the CCT-2D model of two-dimensional location judgments

```
model{

  for(i in 1:n){

    for(k in 1:m){

      sigma[i,k,1] <- E[i]*lam[k,1]

      sigma[i,k,2] <- E[i]*lam[k,2]

      Sigma[i,k,1,1] <- pow(sigma[i,k,1], 2)

      Sigma[i,k,2,2] <- pow(sigma[i,k,2], 2)

      Sigma[i,k,1,2] <- rho[k] * sigma[i,k,1] * sigma[i,k,2]

      Sigma[i,k,2,1] <- rho[k] * sigma[i,k,1] * sigma[i,k,2]

      Tau[i,k,1:2,1:2] <- inverse(Sigma[i,k,1:2,1:2])

      Y[i,k,1:2] ~ dmnorm(T[k,1:2], Tau[i,k,1:2,1:2])

    }

  }


# Parameters

  for (i in 1:n){

    Elog[i] ~ dnorm(Emu,Etau)

    E[i] <- exp(Elog[i])

  }

  lamSigma[1,1] <- pow(lamsigmax, 2)

  lamSigma[2,2] <- pow(lamsigmay, 2)

  lamSigma[1,2] <- lamrho * lamsigmax * lamsigmay

  lamSigma[2,1] <- lamSigma[1,2]

  for (k in 1:m){

    T[k,1] ~ dnorm(Tmu,Ttau)

    T[k,2] ~ dnorm(Tmu,Ttau)

    lamlog[k,1:2] ~ dmnorm.vcov(lammu[1:2], lamSigma[1:2,1:2])
```

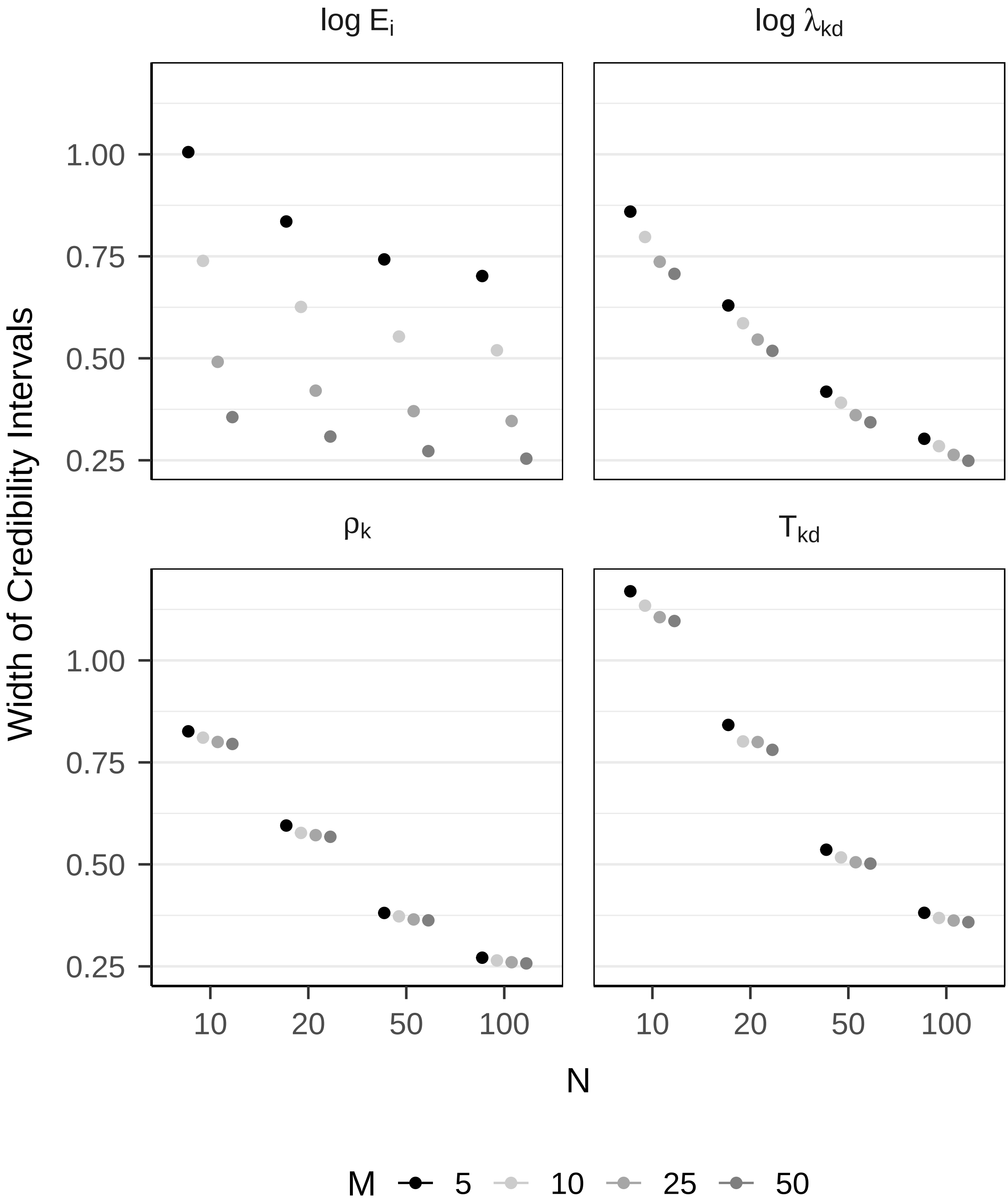
```
    lam[k,1] <- exp(lamlog[k,1])
    lam[k,2] <- exp(lamlog[k,2])
  }

# Hyperparameters
  Tmu ~ dnorm(0,0.25)
  Ttau ~ dt(0,1,1)T(0,)
  lammu[1] <- 0
  lammu[2] <- 0
  lamsigmax ~ dt(0,3,1)T(0,)
  lamsigmay ~ dt(0,3,1)T(0,)
  lamrho ~ dunif(-1, 1)
  Emu <- 0
  Etau <- pow(Esigma, -2)
  Esigma ~ dt(0,1,1)T(0,)

  for(k in 1:m){
    rho[k] ~ dunif(-1, 1)
  }
}
```

Appendix B

Width of Credibility Intervals

Figure B1

Width of Credibility Intervals Depending on Number of Informants N and Items M.



Note. Mean width of credibility intervals are displayed with 95% confidence intervals.

Appendix C

European cities used in the reanalysis

Table C1

European cities and maps from the study by Mayer and Heck (2021).

Item	Map	Cities
1	Austria and Switzerland	Zurich, Geneva, Basel, Bern, Vienna, Graz, Linz, Salzburg
2	France	Paris, Marseille, Lyon, Toulouse, Nice
3	Italy	Rome, Milan, Naples, Florence, Venice
4	Spain and Portugal	Madrid, Barcelona, Seville, Lisbon, Porto
5	United Kingdom and Ireland	London, Birmingham, Glasgow, Liverpool, Dublin
6	Poland, Czech, Hungary and Slovenia	Warsaw, Prague, Bratislava, Budapest
7	Germany	Berlin, Hamburg, Cologne, Frankfurt, Stuttgart, Düsseldorf, Leipzig, Dortmund, Essen, Bremen, Dresden, Hannover, Nuremberg, Duisburg, Wuppertal, Bielefeld, Bonn, Münster, Karlsruhe, Mannheim, Augsburg, Wiesbaden, Braunschweig, Kiel, Munich