**Validation as Evaluating Desired and Undesired Effects:
Insights from Cross-Classified Mixed Effects Model**

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This study has two purposes. The first is to showcase how validation can be undertaken as an exercise of identifying and explaining desired and undesired effects (or score variations). The second is to elaborate on the fruitfulness of cross-classified mixed effect models (CCMEM) as a validation tool.

Measurement is crucial to quantitative research. For some, measurement is seen as “the process of linking abstract concepts [underlying unobservable phenomenon] to empirical indicants [observable response]” (Zeller & Carmines, 1980, p. 2). In a more straightforward view, measurement is a systematic operation for quantifying and classifying the qualities of a phenomenon¹. Conceptually, measurement error is the discrepancy between the true value and the measured quantity. It is inevitable and may result in both random errors and systematic errors in the measured scores which can discredit the trustworthiness of the measured scores and obfuscate the meaning of the measured scores (Cole & Preacher, 2014; Loken & Gelman, 2017). Research findings based on such scores are called into question.

Reliability and validity are two major properties for examining the impact of measurement errors. Reliability is the degree of score consistency among repeated measurements of the same phenomenon administered to the same individual. Validity refers to “the degree to which evidence and theory support the interpretation of test scores for the proposed uses of tests” (AERA et al., 2014, p. 11). They are viewed as distinct but related psychometric properties because reliability pertains to the quality of scores (Thompson & Vacha-Hasse, 2000), and validity is pertinent to the

¹ A phenomenon can refer to both construct and “non-construct” (e.g., criterion or universe of content). Cronbach and Meehl (1955) did not assert that construct validity is ubiquitously applicable to all types of tests. In lieu, they asserted that construct validity “. . . must be investigated whenever no criterion or universe of content is accepted as entirely adequate to define the quality to be measured” (p. 282).

quality of the inferences and decisions made based on the scores. Therefore, reliability is a necessary but insufficient condition for validity. In practice, however, reliability is often confused for validity. Also, it is not unusual that reliability is reported alone with no accompanying validity evidence. As concluded in Zumbo and Chan (2014), there is a very low rate and poor reporting of validity evidence in empirical studies.

Measurement as a System

Measurement can be viewed as a systematic operation for quantifying and classifying the characteristics of a phenomenon, with five sequentially related components: framework, operationalization, interpretation, utility, and evaluation. A *framework* provides the background, context, and structure of the qualities for the (latent) phenomenon to be measured. *Operationalization* concerns the players (e.g., respondents, raters) and mechanisms (e.g., test design and scoring rubrics) that transform the framework for the phenomenon into scores or labels. *Interpretation* assigns substantive or practical meaning to the measured scores (e.g., normed vs. criterion referenced interpretation). *Utility* concerns the decisions made or actions taken based on the measured scores (e.g., admitted to a college or not). *Evaluation* assesses the degree of coherence (i.e., validity) among the first four components, and, in essence, is an exercise of validation.

Undoubtedly, each component in the measurement system introduces both random and systematic errors into the measured scores (Cole & Preacher, 2014; Loken & Gelman, 2017). On the one hand, random errors are score variations caused by untraceable factors. They are unexplainable but quantifiable as reliability. On the other hand, factors for systematic errors are not only quantifiable but also trackable, thus explainable. Systematic errors threaten validity because they obscure the meaning of measured scores.

It is critical to highlight that not all identified systematic score variations should be considered errors. Only those score variations that are *not relevant* to the phenomenon of interest and/or have

the potential to jeopardize the intended score interpretation (or use) should be considered errors.

Relevance should be reasoned by the researchers based on the framework of the measurement system (e.g., prior knowledge, theory, empirical studies, or simply common sense about the phenomenon) to distinguish whether systematic score variation is an error or not. Using the phenomenon of reading comprehension as an example, Table 1 organizes four scenarios, cross-tabulated by relevance to the measurement framework and the two most common elements of operationalization: person and item.

Scenario 1 described an undesired situation due to the lack of measurement invariance. Measurement invariance was historically defined as the equivalent functioning of psychometric instruments when applied across different groups (Millsap, 2011). For example, if groups of test-takers carry the characteristics (e.g., language versions of the test²) irrelevant to reading comprehension, it will compromise the comparability of the scores between groups. Because test-takers' assessment scores should reflect their levels of reading comprehension rather than be confounded with the language versions of the tests, score variation due to such group difference is seen as error variance. Scenario 2 is also an undesired situation pertaining to irrelevant characteristics of items (e.g., item response formats). The performance of test-takers should be determined by their reading comprehension ability, not their familiarity with or preference for a certain response format.

On the contrary, Scenario 3 is a desired situation where person characteristics are relevant and desired. For example, languages spoken at home (i.e., home language) can have a significant impact on their reading comprehension. Students who speak the same language as the test is written in would perform better in reading comprehension than students who rarely speak the test language at

² Language version is the grouping variable. When examinees from different linguistic groups take tests in different languages (e.g., PIRLS, TIMSS), the comparability of results from different language versions is routinely examined. (e.g., Ercikan et al., 2010; Wu & Ercikan, 2006).

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home (Brochu et al., 2018). Hence, home language is relevant to the assessment of reading comprehension; score variation due to such person characteristics can be argued to be desired. Scenario 4 is also a desired situation where the score variation is due to an item characteristic relevant to reading comprehension. For example, it is desired if the score variation reflects the different cognitive levels of reading comprehension that the items are written to tap. Score variation due to such item characteristics reflects test-takers' true differences in reading comprehension.

[Insert Table 1 here]

It is important to note that the *justification* for the desired or undesired score variation serves as the backbone for the validation method demonstration in the current study. From this stance, validation is an exercise of evaluating whether the sources of score variation (desired and undesired) are coherent with the system of measurement (i.e., whether the components of framework, operationalization, interpretation, and utility are properly interconnected). In doing so, this validation method should entail a statistical model that can account for the identified desired and undesired sources of score variation, in addition to usual estimation of reliability, person ability and item difficulty. The cross-classified random effects model (CCREM; Goldstein, 1994; 2011; Raudenbush, 1993) is one of the available tools that can fulfill these needs. Of note, we referred to CCREM as CCMEM (*cross-classified mixed effects model*) because we fitted models with explanatory variables having a *mix* of fixed and random effects on the response outcomes.

CCMEM is not a new modeling framework in psychometrics. It was originally introduced as a method for evaluating dependability (reliability) through the lenses of generalizability theory (Brennan, 2001; Jiang et al., 2020). Despite its usefulness, practical guidance and real data examples are still scant, in particular using it as one tool for validation practice (see Choi & Wilson; 2018; LoPilato et al., 2015; Huebner & Lucht, 2019).

Purpose

This paper has two related objectives: (a) to showcase how validation practice can be undertaken as an exercise of evaluating the coherence between the components in the measurement system as described earlier; and (b) to elaborate on the fruitfulness of CCMEM as a validation tool for a validation exercise as stated in objective (a). Specifically, we will demonstrate the use of CCMEM to explain the score variation associated with the item and person characteristics that are relevant and irrelevant to reading comprehension as assessed by the Progress in International Reading Literacy Study (PIRLS, Mullis et al., 2015).

The rest of this paper is divided into three sections. First, to demonstrate how to implement our proposed validation approach, we will explicate the formulation of CCMEM and elaborate on its usefulness in evaluating reliability and validity evidence. Next, we will demonstrate the CCMEM-based validation method with the PIRLS dataset. Finally, we will provide our closing comments and discussions about validation as the exercise of evaluating coherence in the measurement system and the use of CCMEM for validation.

Formulation of CCMEM

In a typical educational test, each response can be seen as being nested within the clusters that are crossed by the two design factors of item and person. Cross-classified data are less recognized despite the fact that methods for modeling such data have long been available (Luo & Kwok, 2009). These designs can lead to observation dependence that violates the assumption of many basic statistical methods. The CCMEM was proposed specifically for modeling cross-classified data. In this paper, we will highlight, in addition to estimating reliability as it is mostly used for in existing practice, how CCMEM enables researchers to include item and person characteristics as explanatory variables to evaluate validity evidence.

In the following explanation of CCMEM models, we will use reading comprehension as the phenomenon to be measured when a concrete example is needed, where the measurement involves two design factors of person and item.

Evaluating Item Easiness, Person Ability and Reliability

CCMEM starts with specification at the cell level. Each cell $c(i, j)$ contains an observed response $Y_{c(i, j)}$ that is cross classified by item i and person j and is treated as the unit of analysis. The parenthesis indicates item and person are at the *same* cross-class (cluster) level, under which each cell response is observed (Rasbash & Browne, 2001).

In the case of binary response outcomes, the observations will either be $Y_{c(i, j)} = 1$ (correct answer) or $Y_{c(i, j)} = 0$ (in correct answer). These categorical response outcomes are transformed into quantitative values of logits with the function given in M0,

$$\text{logit}(P_{c(i, j)}) = \log \frac{P_{c(i, j)}}{1 - P_{c(i, j)}} = \beta_{(i, j)} \quad \text{M0}$$

where $P_{c(i, j)}$ refers to the probability of $Y_{c(i, j)} = 1$. The transformed values of $\beta_{(i, j)}$ will be referred to as *cell scores* hereafter. Note that, although M0 does not include a residual term, the residual variance can be approximated via the threshold modeling framework in M0' (Long, 1997; Muthén, 1984; Snijders & Bosker, 2011) given as,

$$\check{Y}_{c(i, j)} = \beta_{(i, j)} + r_{c(i, j)} \quad \text{M0'}$$

such that the binary response variable Y is conceptualized as the results of dichotomizing the distribution of a continuous latent response variable $\check{Y}$, assumed to underlie Y . That is, $Y = 0$ if $\check{Y} \leq \check{t}$, and $Y = 1$ if $\check{Y} > \check{t}$, where $\check{t}$ is a threshold on the continuum of $\check{Y}$. When the residual term $r_{c(i, j)}$

of $\tilde{Y}$ is assumed to follow a standard logistic distribution³ with a mean of 0 and variance of $\pi^2/3$, M0' is equivalent to the logit model in M0 (Long, 1997; Long & Fresse, 2014; Snijders & Bosker, 2011). Hence, $\pi^2/3$ is taken as the residual variance for M0 and will be used later to compute reliability.

At the cluster level, the values of $\beta_{(i,j)}$ are modeled by three terms: i) a grand mean, μ , ii) the random effects of items u_i , and iii) the random effects of persons v_j . This model is given in M1 as,

$$\begin{aligned}\beta_{(i,j)} &= \mu + u_i + v_j \\ u_i &\sim N(0, \sigma_u^2) \\ v_j &\sim N(0, \sigma_v^2)\end{aligned}\tag{M1}$$

The u_i is formulated as random effects of item identifier and the v_j as random effects of person identifier. Random effects like u_i and v_j can be understood as parameters that are represented in a form of a normally distributed random variable with a mean (usually set to zeros) and a variance of σ_u^2 and σ_v^2 , respectively.⁴ Thus, u_i is a latent variable representing item differences (i.e., item easiness). A higher u_i means an item is more likely to be answered correctly. Likewise, v_j is a latent variable representing person differences (i.e., ability). A higher v_j value means a person has a higher level of ability.

M1 is a *descriptive* model because it only includes item and person identifier variables (as random effects) and no characteristic variables are included to *explain* the score variation. This model is referred to as the baseline model.

³ The residual error variance can also be set to be 2.89 for with a logit link, which is the squared of the scaling constant (1.7), see De Boeck (2008). When the residual is assumed to follow a standard normal distribution $N(0, 1)$, it will be a model with a probity link, and the error variance is set to 1.

⁴ The interaction between the two random variables u_i and v_j not be included in M1 because there is only one score per cell that is the response to of a person to an item (Raudenbush & Bryk, 2002).

The three variance components in the baseline model are: person variance σ_v^2 , item variance σ_u^2 , and error variance $\sigma_e^2 (= \pi^2/3)$ serves as the bases for computing the reliability of measuring the objects of measurement (i.e., persons) with k measures (i.e., items). Two kinds of reliability, relative consistency and absolute agreement, can be computed (McGraw & Wong, 1996) and are given as,

$$\text{relative consistency} = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2/k} \quad (1)$$

$$\text{absolute agreement} = \frac{\sigma_v^2}{\sigma_v^2 + (\sigma_u^2 + \sigma_e^2)/k} \quad (2)$$

The *relative consistency reliability* in (1) is the degree of consistency in measurement among averages of k independent items on randomly selected persons. It is appropriate when the score interpretation is norm-referenced and scores are used to make relative decisions (e.g., ranking individuals). The *absolute agreement reliability* in (2) is the degree of absolute agreement among the averages of k independent items on randomly selected persons. It is suitable when the score interpretation is criterion-referenced and scores are used to make absolute decisions (e.g., passing or failing a person). The only difference in computing the two kinds of reliability is that the item variance σ_u^2 is regarded as error variance when computing absolute agreement because the pass/fail decision could be different for the same test taker, depending on which set of k items is sampled for the test-taker.

Evaluating the Effects of Item and Person Characteristics

One of the advantages of CCMEM is that it can include observed item and person characteristic variables and their interactions with other variables, fixed or random effects, to explain the score variation. This makes CCMEM a very flexible model for modeling score variation, desired or undesired, depending on whether the item and person characteristics are relevant or irrelevant to

the phenomenon being measured according to the researchers' justification, as organized in Table 1

In what follows, we will discuss six *explanatory* models (M2 to M7) that include person and item characteristics to evaluate validity evidence. All of them are built on M1, the baseline descriptive model.

Person (or Item) Characteristics Having “Fixed” Effect. One can add a person characteristic variable G_j as shown in M2,

$$\beta_{(i,j)} = \mu + \gamma_1 G_j + u_i + v_j \quad \text{M2}$$

Take language versions of test (English vs French), a person characteristic as an example for G_j . The coefficient γ_1 is the fixed effect – the difference in the logit, comparing the English group to the French group, where the subscript “1” denotes the slope coefficient of the “first” and only person characteristic variable in M2. If there are $G_1 \dots G_Q$ person characteristic variables, their fixed effects would be denoted as $\gamma_1, \gamma_2, \dots, \gamma_Q$. The γ_1 is assumed to be a fixed effect, hence is a constant value.

When being evaluated for validity, a non-zero γ_1 of test language is undesired because it affects score variation, after taking into account item easiness u_i and person ability v_j . This is an example of Scenario 1 in Table 1. In contrast, γ_1 could be a desired effect of Scenario 3 in Table 1. The languages spoken at home can have a tremendous influence on their reading comprehension. Thus, it is expected that those whose primary home language is the language that the reading comprehension test is written in will perform better than those whose home language is not the test language (Brochu et al., 2018). Thus, such an effect, if found, can be argued to be relevant and desired.

By the same token, one could include an item characteristic variable W_i to explain the variation in $\beta_{(i,j)}$, as shown in M3,

$$\beta_{(i,j)} = \mu + \lambda_1 W_i + u_i + v_j \quad \text{M3}$$

An example of an item characteristic is response format in multiple choice (MC) or constructed response (CR). The λ_1 coefficient for W_i indicates the difference in the logit between the two formats, where “1” in the subscript denotes slope coefficient of the “first” (and only) item characteristic variable in M3. If there are $W_1 \dots W_R$ item characteristic variables, their effects would be denoted as $\lambda_1 \dots \lambda_R$. The effect of the response format is assumed to be fixed, hence represented by a constant coefficient λ_1 . M3 is equivalent to the random effects linear logistic test model (random effects-LLTM or LLTM + error; Janssen et al., 2004).

When being evaluated for validity, a non-zero λ_1 of response format would be deemed an undesired effect as in Scenario 2 in Table 3.1 because response format affects scores after accounting for item difficulty and person ability. This undesired effect may occur when test-takers have different preferences for and familiarity with certain response formats.

In contrast, λ_1 in M3 can be a desired effect as in Scenario 4 in Table 1. For example, say that each item in the test is written to tap one of two levels of cognitive skills of information-retrieval and inference-making for reading comprehension. A non-zero λ_1 of item cognitive level is desired if it shows that the information retrieval items are, on average, easier to answer than the inference-making items. This finding would provide strong validity evidence for score variation due to item cognitive level.

Person (or Item) Characteristic Having “Random” Effects. One can model an *item-specific person characteristic effect* by introducing a random variable γ_{1i} to interact with the person characteristic variable G_j , expressed as the product term $\gamma_{1i}G_j$ given in M4 as,

$$\beta_{(i,j)} = \mu + \gamma_1 G_j + \gamma_{1i} G_j + u_i + v_j \quad \text{M4}$$

$$\beta_{(i,j)} = \mu + (\gamma_1 + \gamma_{1i})G_j + u_i + v_j \quad \text{M4'}$$

where γ_{1i} is the random effects of G_j that vary across items, with $\gamma_{1i} \sim N(\mathbf{0}, \sigma_{\gamma_1}^2)$. The product term of $\gamma_{1i}G_j$ can be understood as a moderation effect of γ_{1i} . This means that the effect of G_j on the cell scores is moderated by the values of γ_{1i} . Thus, γ_{1i} functions like a *latent item moderator*. This moderation effect can be more easily understood by M4', reorganized M4 with respect to G_j . The effect of person characteristic G_j is expressed by the two parts in the parentheses: the fixed effect of γ_1 that is the baseline (common to) all items, and the random effects of γ_{1i} that is specific to (varies across) items.

The presence of the γ_{1i} moderation means that the person characteristic effect is more pronounced for some items than for others, which helps to explain how the person characteristic, desired or undesired, interferes with different test questions to affect score variation. A non-zero variance of γ_{1i} (i.e., statistically significant $\sigma_{\gamma_1}^2$) indicates the presence of such an item-specific person characteristic effect. If G_j is test language, an undesired person characteristic, coefficient γ_1 is the overall amount of the undesired effect of the test language; on top of γ_1 , each item has its own amount of undesired test language effect of γ_{1i} .

By the same token, M5 given below allows *person-specific item characteristic effects* by introducing a random variable λ_{1j} to interact with W_i as the product term $\lambda_{1j}W_i$ given in M5,

$$\beta_{(i,j)} = \mu + \lambda_1 W_i + \lambda_{1j} W_i + u_i + v_j \quad \text{M5}$$

$$\beta_{(i,j)} = \mu + (\lambda_1 + \lambda_{1j})W_i + u_i + v_j \quad \text{M5'}$$

where λ_{1j} is the random effects of W_i that vary across persons with $\lambda_{1j} \sim N(0, \sigma_{\lambda_{1j}}^2)$. The product term of $\lambda_{1j}W_i$ in M5 be understood as a moderation effect of λ_{1j} . This means that the effect of W_i

on the cell scores is moderated by the values of λ_{1j} . Thus, λ_{1j} functions like a *latent person moderator*. This moderation effect can be more easily understood by M5', which reorganized M5 with respect to W_i . In M5', the item effect W_i is expressed by two parts in the parentheses: the fixed effect of λ_1 that is the baseline (common to) all items, and random effects of λ_{1j} that is specific to (varies) across persons.

The presence of the λ_{1j} moderation means that the item characteristic effect is more pronounced for some persons than for others, which helps to explain how the item characteristic, desired or undesired, interacts with different individuals to affect score variation. A non-zero variance of λ_{1j} (i.e., statistically significant $\sigma_{\lambda_1}^2$) suggests the presence of such person-specific item. If G_j is response format, an undesired item characteristic, coefficient λ_1 is the overall amount of the undesired effect of response format; On top of λ_1 , each person has their own amount of undesired response format effect of λ_{1j} .

Modeling Interaction between Person and Item Characteristics. One can also evaluate how the observed person and item characteristics interact to affect cell scores by adding the product term $G_j W_i$ to M4 or M5 as shown in M6 or M7,

$$\beta_{(i,j)} = \mu + \gamma_1 G_j + \lambda_1 W_i + \gamma_{1i} G_j + \kappa_1 W_i G_j + u_i + v_j \quad \text{M6}$$

$$\beta_{(i,j)} = \mu + \lambda_1 W_i + (\gamma_1 + \gamma_{1i} + \kappa_1 W_i) G_j + u_i + v_j \quad \text{M6'}$$

$$\beta_{(i,j)} = \mu + \gamma_1 G_j + \lambda_1 W_i + \lambda_{1j} W_i + \kappa_1 G_j W_i + u_i + v_j \quad \text{M7}$$

$$\beta_{(i,j)} = \mu + \gamma_1 G_j + (\lambda_1 + \lambda_{1j} + \kappa_1 G_j) W_i + u_i + v_j \quad \text{M7'}$$

For example, the test language can be specified to interact with item format to affect reading comprehension. That is, the undesired effect of test language may be more pronounced for CR items than for MC items. Alternatively, the undesired effect of item format may be more pronounced for the French test-language group than for the English test-language group. Namely, one of the characteristic variable functions as an *observed* moderator for the other characteristic variable. Note that M6 and M7 are almost identical, except that the $\gamma_{1i}G_j$ in M6 is the latent item moderation in M4 but the $\lambda_{1j}W_i$ in M7 is the latent person moderation in M5.

M6', reorganized M6 with respect to G_j , shows that $\kappa_1 W_i$ is added to the effect of G_j , on top of the two effects of γ_1 and γ_{1i} in M4'. A statically significant κ_1 indicates the presence of the additional effect of $\kappa_1 W_i$. In principle⁵, the presence of $\kappa_1 W_i$ will partial out some effect from γ_1 for G_j , leading to a smaller value of γ_1 as well as partial out some effect from γ_{1i} , leading to a smaller variance of γ_{1i} (i.e., smaller $\sigma_{\gamma_1}^2$), compared to those in M4'. Similarly, the value of λ_1 and the variance of λ_{1j} (i.e., $\sigma_{\lambda_1}^2$) in M7' will be, in principle, smaller compared to those in M5'.

Regarding the evaluation of validity, as long as either G_j or W_i is undesired, the interaction effect of $G_j W_i$ will be undesired. For example, a statistically significant κ_1 for the interaction between test language and the item cognitive level would be considered as undesired. This is because, although item cognitive level is a desired characteristic, its interaction with the undesired test language will be undesired. Such an interaction is simply a further step to understand how the undesired effect of test language will be moderated by item cognitive level, albeit a desired characteristic.

Method

⁵ In some cases, there could be suppression effects, leading to exceptional results.

Data Source. The sample includes 3,006 fourth grade Canadian students⁶ who participated in the ePIRLS 2016 (Mullis et al., 2015). The ePIRLS is a digitalized version of PIRLS. There are five text passages in the assessment along with 17 to 20 questions per passage. The ePIRLS 2016 assesses reading comprehension with the rotation test design. Each text passage would be assigned to 4 out of 20 possible rotations (test forms). We only selected the fifth testlet with the passage “*The Legend of Troy*” because it has an equal number of MC and CR items (9) and the greatest number of binarily scored items (17 out of 18 items⁷) for the current demonstration purpose.

Explanatory Variables. Four characteristic variables were used as explanatory variables. The undesired person characteristic was the test language (English = 1 or French = 0). The desired person characteristic was home language (always or almost always speaking the test language at home = 1 vs. sometimes or never speaking the test language at home = 0). The undesired item characteristic was response format (MC = 1 or CR = 0). The desired item characteristic was four increasingly demanding cognitive levels of information retrieval (Retrieval), straightforward inferences making (Inferences), ideas and information integration (Integration), and evaluation and critique (Evaluation). Forward difference coding was used to make meaningful comparisons for *levels of comprehension* maintained by Lapp & Flood (1983), i.e., Retrieval vs. Inferences, Inferences vs. Integration, and Integration vs. Evaluation.

Analysis. The relative consistency reliability and absolute agreement reliability were estimated following equations (1) and (2), respectively. The seven CCMEM models (M2 to M7) were analyzed in the R package *glmmTMB* (Brooks et al., 2017) using maximum likelihood estimation. The *glmmTMB* was chosen due to its high estimation speed and flexibility in specifying the variance-covariance structure of the random effects.

⁶ Students with less than a year of instruction in the language of the test (English or French) were typically excluded in the PIRLS 2016 Canadian sample (Brochu et al., 2018)

⁷ Item 5 was removed because the scoring is not binary.

Results

Score Reliability

The estimates of person variance σ_v^2 and item σ_u^2 variance components for the baseline model were 1.363 and 1.251, respectively. Based on them, the estimates of relative consistency and absolute agreement reliability were .88 and .84.

Validity Evidence

For heuristic purpose⁸, we assessed validity evidence by examining undesired (Submodel a see Table 2) and desired (Submodel b see Table 3) score variations attributable to both item (M2, M4 and M6) and person characteristics (M3, M5 and M7).

[insert Table 2 and Table 3 here]

Undesired Score Variation. The “undesired” effects of *language of test* were examined via M2a, M4a, and M6a. The fixed slope of 0.75 in M2a (odd ratio (OR) = 2.13, 95%CI = [1.91, 2.36]) indicated that the odds of correct answer were 2.13 times higher for students who took the test in English (vs. French) after controlling for item easiness and person ability. This suggests that reading comprehension was invalidly assessed by the testlet due to test language issue. Furthermore, the significant random effect found in M4a revealed that the size of the undesired effect was item specific. Thus, it necessitated a further explanation. The results of M6a showed that response format explained about 20% of random effects (0.20 to 0.16 variance reduction) and that the fixed effect magnitude of test language varied depending upon the response format, -0.40, 95%CIs [-0.80, -0.01]⁹ (for multiple-choice item, OR_{English vs. French} = 1.71, 95%CIs [1.28, 2.27]; for constructed response, OR_{English vs. French} = 2.55, 95%CIs [1.89, 3.45]). The variance reduction and significant interaction effect suggested that response format could be one source of the undesired effect of test language. In

⁸ Note that results for this demonstrative study were meant to be interpreted for heuristic purpose and did not imply any substantive meaning.

⁹ $\exp(-0.40) = 0.67$ which is the ratio of the odds ratios. Namely, $1.71/2.55 = 0.67$.

contrast, the undesired effects of response format were not discovered, as evidenced by the non-significant of fixed effects and the zero variance for random effects in M3a and M5a. Therefore, M7a was not specified.

Desired Score Variation. The findings in M2b showed that students speaking the language of the test (English or French) *always or almost always* at home outperformed students who speak it *sometimes or never* at home, log-odds = 0.31, 95%CIs [0.20, 0.41] (OR = 1.36, 95%CIs [1.22, 1.15]). Moreover, the significant random effect in M4b revealed that desired effect was item specific. As shown in M6b, we employed *retrieval vs. inferences* to explain the random effect¹⁰. In contrast to the findings of M6a, the findings of M6b demonstrated a non-significant interaction effect and a smaller variance decrease of just 8%. In other words, it demanded additional investigation into other item characteristics.

In terms of cognitive level demand (see M3b), the positive estimates for the three contrasts (OR_{retrieval vs. inferences} = 2.85, OR_{inferences vs. integration} = 1.47, OR_{integration vs. evaluation} = 1.12) showed that the order in the easiness level of the four item types was: *retrieval, inferences, integration, and evaluation*, which was consistent with the levels of comprehension theory (Lapp & Flood, 1983) and the framework of PIRLS. Despite the non-significant estimates, these findings at least gave some evidence to the substantive meaning of the scores and can be considered a desired case. Following that, the results of M5b only revealed the variance for the random effects of retrieval vs. inferences was nonzero. However, the variance is not significantly different from zero¹¹. For heuristic purposes only, we further included *home language* to account for the person-specific item characteristic effect in M7b. Along with the marginally significant interaction effect, -0.15 (95%CI [-0.27, -0.02]), the inclusion of

¹⁰ The model with three person $\times$ item interaction terms did not reveal any significant main effects for item characteristics and interaction effects. For heuristic purpose, we chose the more parsimonious model with the interaction effect only between home language use and retrieval vs. inferences.

¹¹ Likelihood ratio (LR) test on mixture χ^2 distribution is used for statistical testing on random effects, especially for nonsignificant results based on -2 Δ LL LR test to avoid a Type II error (Hoffman, 2015).

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home language rendered a variance decrease of 15% for person-specific item effects of *retrieval vs. inferences*. This suggests that *home language* could be one source of desired score variation for *retrieval vs. inferences*.

Of note, the sequential patterns of the four cognitive demands became more pronounced as the model complexity increased from M3b, M5b to M7b (see Table 4). By including the other explanatory factors, such patterns were amplified. When relevant explanatory variables were introduced and the pattern of these intended effects became strikingly consistent with the theory, it would provide a strong form of validity evidence for the score meaning.

[Insert Table 4 here]

Discussion

In this study, validation practice is the process of identifying and explaining the desired and undesired score variation. First and foremost, evaluation of reliability is part of the validation exercise because, as a well-accepted belief, reliability is a necessary condition for validity. Reliability can be quantified by the unsystematic random score variation. Furthermore, validity evidence can be evaluated substantively as an evaluation of how well the measured scores vary coherently according to the design of the measurement system. CCMEM was demonstrated as a tool for this exercise. The degree of validity can be quantified by their fixed effects, random effects, or both.

To iterate, CCMEM has several advantages as a psychometric tool for validation practice. First, it can estimate the dependability for making relative and absolute decisions as in the generalizability theory (Jiang et al., 2020); Second, different from generalizability theory (i.e., a descriptive model by nature), CCMEM can include explanatory variables to account for the random effects of persons and the design factors (e.g., items). Therefore, CCMEM can be used for evaluating both reliability and validity evidence. Reliability can be estimated based on the variance

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components from the baseline model. As for validity, researchers can tailor their study by examining the effects that may support or jeopardize the intended score interpretation (or use).

It is noteworthy that the *justification* for the desiderata of systematic score variation forms the substantive foundation of this validation tool. The identified desired or undesired effects should be justified, as guided by the framework of the measurement system as well as prior knowledge, substantive theory, empirical studies, or simply common sense. For instance, we argued that the effects of *home language* (i.e., language of test spoken at home) on reading comprehension in Scenario -3 are desired because such findings are consistent with the results from empirical studies (e.g., Brochu et al., 2018, Dickinson & Porche, 2011; Lai et al., 2022). In other words, test scores accurately reflect the advantage of rich early language experience. Therefore, the effect of home language is desired and considered as validity evidence.

However, in other contexts, *home language* could be rendered as the source of bias or unfairness. As pointed out by Cizek (2020, p127), “the use of even psychometrically valid scores with strong support for their intended interpretations may nonetheless be considered to be unfair and a proposed use deemed unjustified”. As such, in the above example, if test scores were used to rank individual test-takers’ reading comprehension, it might produce unfair or biased ranking results due to the advantage of rich early language experience (or linguistic diversity of test-takers, see Solano-Flores, 2008).

Fairness (or bias) is closely related to validity. However, when evaluating whether a particular intended test use is justified, fairness (or bias) issues can arise beyond the scope of validity as well as just statistical considerations (Cizek, 2020). Moreover, the evaluations of fairness (or bias) are “inevitably shaped by the particular social context in which they are embedded” (Camilli, 2006, p. 221). Therefore, the evaluation of fairness (bias) is outside the scope of this demonstrative study. (see Oliveri et al. 2023 for a detailed discussion).

We close the discussion with an important emphasis. First, the proposed CCMEM validation method does not intend to discredit the popular validation methods (e.g., factor analysis, item response theory,¹² and structural equation modeling). Rather, we see it as an additional apparatus in the validation toolbox to use when the researchers see it suitable for their specific needs or when they need a different way of searching for validity evidence. Second, validation practice is beyond just reporting of statistical results (e.g., correlation, factor loadings, Cronbach's alpha). It requires a thoughtful justification and reasoning for evidence (or counterevidence). Our proposed method highlights such process. Finally, validation study based on this method only provides part of validity evidence and should not be the sole evidence. We hope this tool can help practitioners and applied researchers develop more thoughtful and tailored validation practices (cf. simple correlation-based evidence of validity) and gain more refined insights into the psychometrics and substantive literature.

¹² IRT can be seen and estimated under the CCMEM framework (see De Boeck & Wilson, 2014)

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Table 1

Four Scenarios of Systematic Score Variation- Organized by Relevance to the Framework and Operationalization for Assessing Reading Comprehension

		Relevance to Framework	
		Irrelevant (Error) variation	Relevant (Desired) variation
Operationalization	Person	Scenario-1. Undesired systematic variation caused by irrelevant person characteristics (e.g., language versions of test)	Scenario-3. Desired systematic variation caused by person characteristics (e.g., whether the language examinees speak at home [home language] is the same language the test is written in)
	Item	Scenario-2. Undesired systematic variation caused by items (e.g., item response format of multiple choice or constructed response)	Scenario-4. Desired systematic variation caused by items (e.g., the different cognitive levels of reading comprehension the items were written to tap)

Table 2

Validity Evidence (Submodel a: Undesired effects in logit scale)

		Person			Item	
		M2a	M4a	M6a	M3a	M5a
Fixed Effects						
γ_1	Person:					
	Test language (English vs. French)	0.75 [0.65, 0.86]	0.72 [0.49, 0.96]	0.94 [0.64, 1.24]	--	--
λ_1	Item:					
	Responsa format (MC vs. CR)	--	--	0.92 [-0.12, 1.97]	0.62 [-0.41, 1.64]	0.62 [-0.41, 1.64]
κ_1	Person $\times$ Item Interaction	--	--	-0.40 [-0.80, -0.01]		
Random Effects						
$\sigma_{\gamma_1}^2$	Variance of γ_{1i}	--	0.2007 $p < .001$	0.1608 $p < .001$	--	--
$\sigma_{\lambda_1}^2$	Variance of λ_{1j}	--	--	--	--	0.00 ^a

Table 3

Validity Evidence (Submodel b: Desired effects in logit scale)

		Person			Item		
		M2b	M4b	M6b	M3b	M5b	M7b
Fixed Effects							
Person:							
γ_1	<i>home language use</i> (<i>always vs. never</i>)	0.31 [0.20, 0.41]	0.30 [0.17, 0.43]	0.29 [0.16, 0.42]	--	--	0.29 [0.19, 0.40]
Item:							
λ_1	<i>retrieval vs. inferences</i>	--	--	1.16 [-0.18, 2.50]	1.05 [-0.32, 2.41]	1.05 [-0.32, 2.42]	1.16 [-0.21, 2.53]
λ_2	<i>inferences vs. integration</i>	--	--	0.33 [-0.82, 1.48]	0.38 [-0.78, 1.55]	0.38 [-0.79, 1.55]	0.38 [-0.79, 1.55]
λ_3	<i>integration vs. evaluation</i>	--	--	0.16 [-1.22, 1.54]	0.12 [-1.29, 1.52]	0.12 [-1.30, 1.53]	0.12 [-1.29, 1.53]
Person $\times$ Item Interaction:							
κ_1	<i>home language use $\times$</i> <i>retrieval vs.</i> <i>inferences</i>	--	--	-0.13 [-0.36, 0.10]	--	--	-0.15 [-0.27, -0.02]
Random Effects							
$\sigma_{\gamma_1}^2$	Variance of γ_{1i}	--	0.02452 $p < .001$	0.02237 $p < .001$	--	--	--
$\sigma_{\lambda_1}^2$	Variance of λ_{1j}	--	--	--	--	0.04501 $p = .48$	0.03818 $p = .54$

Table 4

Odds Ratio of Pairwise Comparison by Forward Difference Coding (Row vs. Column)

Models		Odds Ratio (Row vs. Colum)		
M3b	Retrieval	Inference	Integration	Evaluation
Retrieval		<i>2.846</i>	<i>4.175</i>	4.693
Inference			<i>1.467</i>	1.649
Integration				<i>1.124</i>
Evaluation				
M5b	Retrieval	Inference	Integration	Evaluation
Retrieval		<i>2.862</i>	<i>4.201</i>	4.720
Inference			<i>1.467</i>	<i>1.649</i>
Integration				<i>1.124</i>
Evaluation				
M7b	Retrieval	Inference	Integration	Evaluation
Retrieval		<i>2.950</i>	<i>4.350</i>	4.910
Inference			<i>1.463</i>	1.654
Integration				<i>1.130</i>
Evaluation				