Cubic Response Surface Analysis: Investigating Asymmetric and Level-Dependent Congruence
Effects With Third-Order Polynomial Models

Sarah Humberg

Westfälische Wilhelms-University Münster

Felix D. Schönbrodt

Ludwig-Maximilians-Universität München

Mitja D. Back

Westfälische Wilhelms-University Münster

Steffen Nestler

Westfälische Wilhelms-University Münster

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Author Note:

Additional materials for this article can be found at osf.io/drv3a.

The OSF materials include the proofs of the central mathematical statements in this article, overview tables that show how to test variants of the hypotheses that are discussed in the manuscript, example R-code, which guides the user through the investigation of asymmetric or level-dependent congruence effects, and all data and R scripts necessary to reproduce the example analyses reported in this manuscript.

Correspondence concerning this article should be sent to Sarah Humberg, University of Münster, Department of Psychology, Fliednerstr. 21, 48149 Münster, Germany. E-Mail: sarah.humberg@wwu.de

Some contents of this article have been presented at scientific conferences (13th and 14th meeting of the section Methods and Evaluation of the German Psychological Society). The illustrative data used in Case Study 3 is publicly available and has been analyzed for other publications.

Abstract

Congruence hypotheses play a major role in many areas of psychology. They refer to, for example, the consequences of person-environment fit, similarity, or self-other agreement. For example, are people psychologically better adjusted when their self-view is in line with their reputation? A valid statistical approach that can be applied to investigate congruence hypotheses of this kind is quadratic Response Surface Analysis (RSA) in which a second-order polynomial model is fit to the data and appropriately interpreted. However, quadratic RSA does not allow researchers to investigate more precise expectations about a congruence effect. Do the data support an *asymmetric* congruence effect, in the sense that congruence leads to the highest (or lowest) outcome, but incongruence in one direction (e.g., self-view exceeds reputation) affects the outcome differently than incongruence in the other direction (e.g., self-view falls behind reputation)? Is there a *level-dependent* congruence effect, such that the amount of congruence is more strongly related to the outcome variable for some levels of the predictors (e.g., high self-view and reputation) than for others (e.g., low self-view and reputation)? Such complex congruence hypotheses have frequently been suggested in the literature, but they could not be investigated because an appropriate statistical approach has yet to be developed. Here, we present analytical strategies, based on third-order polynomial models, that enable users to investigate asymmetric and level-dependent congruence effects, respectively. To facilitate the correct application of the suggested approaches, we provide respective step-by-step guidelines, corresponding R syntax, and illustrative analyses using simulated and real data.

Keywords: similarity, agreement, polynomial regression, asymmetric congruence effect, level-dependent congruence effect

Cubic Response Surface Analysis: Investigating Asymmetric and Level-Dependent Congruence Effects With Third-Order Polynomial Models

Congruence hypotheses are key to many psychological theories and are prevalent across diverse psychological research domains. For example, are people psychologically better adjusted when they maintain intelligence self-views that are congruent with their real abilities (e.g., Higgins, 1996; Wilson & Dunn, 2004)? Are people more attracted to others who have similar values, interests, or attractiveness (Olderbak et al., 2017)? Do clients perceive psychotherapy sessions as deeper when they and their counselor have similar perceptions of the therapy alliance (Marmarosh & Kivlighan, 2012)? Do employees maintain more positive personal or work outcomes when they “fit” the organization they work for (Kristof, 1996, see Table 1 for further examples)?

A statistical approach that is gaining popularity for investigating congruence hypotheses is (quadratic) Response Surface Analysis (RSA; Edwards, 1994, 2002; Edwards & Parry, 1993; for further introductory descriptions of quadratic RSA, see also Edwards, 2007; Humberg, Nestler, et al., 2018; Nestler et al., 2015; Schönbrodt, 2016). RSA is an analytical tool that is based on the estimation of a polynomial regression model that predicts the potential outcome variable z (e.g., psychological adjustment) from the two predictor variables x and y (e.g., self-view and reality criterion), their squared terms, and their interaction term (thus making the model a *second-order* or *quadratic* polynomial model):

$z = b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + \varepsilon$. The graph of the model is then used as a guide to interpret the estimated regression coefficients with regard to the suggested effect.

When appropriately applied, quadratic RSA is well suited to investigate congruence hypotheses. However, it is not possible to use this approach to investigate more precise details about congruence effects that are often of key interest (see also Table 1). (a) Is there an *asymmetric* congruence effect in the sense that, for example, people are psychologically better adjusted when their self-view of intelligence is more congruent with reality, but is overestimation more detrimental for psychological adjustment than underestimation? (b) Does the strength of a congruence effect vary with the mean predictor level (*level-dependent congruence effect*), such that an accurate self-view of intelligence is especially relevant for people at low ability levels, whereas congruence plays a minor role at high levels? Neither the asymmetric nor the level-dependent congruence effect can be reflected in a quadratic polynomial model because the restriction to second-order terms (i.e., the most complex terms

are the squares and the interaction) do not allow enough flexibility (see also Humberg, Nestler, et al., 2018). Instead, complex congruence hypotheses of the sort just described can be investigated by extending the polynomial model underlying RSA so that it becomes a cubic model (i.e., a *third-order* model, including the terms x^3 , x^2y , xy^2 , y^3 , where, e.g., x = self-view, y = reality criterion).

However, previous attempts to investigate asymmetric or level-dependent congruence hypotheses have not relied on a third-order polynomial model (see Ilmarinen et al., 2016, for one exception). Instead, quadratic RSA has been applied to investigate these hypotheses (which is impossible and thus leads to misconclusions), or the respective hypothesis was simply not investigated. We can identify two potential reasons for this situation. First, researchers might not be fully informed about which hypotheses are too complex to be investigated with quadratic RSA and require a third-order model instead. Second, although it is straightforward to extend the regression model by including cubic terms so that it can theoretically reflect complex congruence effects, interpreting the coefficients to determine whether the estimated model *actually* indicates the suggested effect can be very complicated and prone to errors even for mathematically trained researchers.

Here, we aim to provide (a) an overview of the hypotheses that are too complex for quadratic RSA and (b) a valid and easy-to-apply method to investigate complex congruence hypotheses with *cubic* RSA. To these aims, we first present a brief introduction to the investigation of “simple” congruence hypotheses with second-order models. We then show how these models can be extended in a straightforward way to obtain third-order polynomial models that can be used to investigate asymmetric and level-dependent congruence hypotheses, respectively. These models are special cases of the full third-order polynomial model, with parameter constraints that correspond to the respective hypothesis. We provide guidelines for investigating whether one’s data are in line with the suggested effect by using these models, and we derive additional conditions that must be considered to avoid misinterpretation (i.e., so that model predictions will be interpreted for all realistic predictor combinations). We incorporated all suggested third-order models into the R package *RSA* (Schönbrodt & Humberg, 2020), we implemented additional functions to ease the application of the suggested strategy, and we demonstrate the application and interpretation of cubic RSA by presenting illustrative analyses with simulated and with real data.

Table 1

Definitions and Examples of Simple, Asymmetric, and Level-Dependent Congruence Hypotheses

Simple congruence effect (testable with quadratic RSA)

Definition: The closer the values of two predictor variables x and y are to one another, the higher (or lower) is the value of a respective outcome variable z .

Examples: People are better adjusted the more accurately they judge their own favorable attributes (Higgins, 1996; Wilson & Dunn, 2004).
 People are more attracted to individuals whose mate value is similar to their own mate value (Kavanagh et al., 2010; Walster et al., 1966; Wurst et al., 2017b).
 People are more satisfied with a job when their goals and values are congruent with the organizational goals or values of the work place (Hoffman & Woehr, 2006; Kristof, 1996; Kristof-Brown et al., 2005).

Asymmetric congruence effect (*not* testable with quadratic RSA but requires cubic RSA)

Definition: The closer the values of two predictor variables x and y are to one another, the higher (or lower) is the value of a respective outcome variable z , and in addition, the outcome is higher (or lower) when x exceeds y than when y exceeds x to the same extent.

Examples: Women experience lower feelings of shame and guilt when their actual weight matches the weight they evaluate as ideal, and women feel more ashamed and guilty when they weigh “more than ideal” in comparison with weighing “less than ideal” (Castonguay et al., 2012).
 Romantic partners are intrapersonally better adjusted on days when they provide similarly strong social support for each other, and providing more support than one’s partner comes at a greater cost than providing less support than one’s partner (Bar-Kalifa et al., 2017).
 Managers whose explicit and implicit power motives are congruent have the highest well-being, and the detrimental effect of incongruence should be stronger when explicit motives exceed implicit motives (Kazén & Kuhl, 2011).

Level-dependent congruence effect (*not* testable with quadratic RSA but requires cubic RSA)

Definition: The closer the values of two predictor variables x and y are to one another, the higher (or lower) is the value of a respective outcome variable z , and this effect of congruence is stronger (or weaker) for higher levels of x and y than for lower levels.

Examples: Children who resemble their parents with respect to personality experience more externalizing problems in adulthood, and the impact of similarity differs in strength for low versus high levels of the personality trait (Franken et al., 2017).
 People who resemble each other in terms of dark personality traits like each other more, and this effect is stronger for people with low levels of these traits (Ilmarinen et al., 2016).

Investigating “Simple” Congruence Hypotheses with Quadratic RSA

Many branches of psychological research have viewed the comparison of two constructs x and y as a meaningful source of interpersonal variation. When x and y have been assessed on the same numerical scale, the directed discrepancy $x - y$ has often been taken as an operationalization of this comparison. The value $x - y = 0$ has usually played a special role because it represents people whose x and y variables match exactly, that is, people whose x and y values are perfectly congruent. For two people, we say that the person whose difference $x - y$ is closer to zero (i.e., the person with the lower value in the absolute difference $|x - y|$) is *more congruent*. A common hypothesis in congruence research posits that $x - y$ should explain variation in some outcome variable z such that z is higher (or lower, for a reversed version of the hypothesis) for people who are more congruent than for people who are less congruent. For example, one idea could be that for two people, the happier person is the person whose self-view with regard to some trait (e.g., intelligence, a personality trait) is closer to the person’s reputation for embodying this trait. To investigate this hypothesis, we would assess people’s self-view x and their reputation y regarding the trait of interest using the same scale (e.g., the same Likert scale). We would also assess their happiness z .

The verbal hypothesis “the closer x and y are to one another, the higher is z ” has been interpreted in different ways in the literature (see also Edwards, 2002). One common specification is the expectation that for two people, the person whose x and y values are closer to one another has the higher value of z , where the function that describes this association is assumed to have a smooth shape; we label this the *strict simple congruence hypothesis* (see also Table 2). The smoothness assumption has usually not been made explicit in theoretical or empirical work on congruence effects. We verbalize it here to differentiate between the hypotheses that we are addressing in this article from other common specifications of the verbal hypothesis. Stating that the functional form that relates $x - y$ to z is assumed to be “smooth” means that the association is expected to resemble an inverted U-shape rather than an inverted V-shape, where the latter would have a sharp change in direction at the point of congruence. The U-shape assumption mirrors a decision that has often (implicitly) been made in psychological research (e.g., in all studies that have applied quadratic RSA to investigate a congruence hypothesis) and is justified from a processual perspective for many research objectives in which congruence effects are of interest (e.g., because most psychological processes imply smooth changes in an outcome). Accordingly, all models that we describe in

this manuscript imply the smoothness assumption. By contrast, readers whose theory is instead in line with (inverted) V-shaped versions of the hypotheses need to apply a fundamentally different class of models (absolute difference or linear spline models); we refer these readers to the work by Edwards (2002) and by Edwards and Parry (2018).

Note that to clearly differentiate between the diverse hypotheses that we refer to in this article, we apply the following terminology (see also Table 2): We use the adjective “simple” for hypotheses and respective models that can be represented by second-order polynomials. We use “complex” or the specifications “asymmetric” and “level-dependent” for hypotheses that require third-order models. We use the adjective “strict” to refer to hypotheses and models that do not allow a main effect of the predictor levels in addition to a (simple or complex) congruence effect. Finally, we use “rising ridge” for hypotheses that allow such a main effect.

The strict simple congruence hypothesis can be represented by the following *strict simple congruence model* (Edwards, 2002; see also Schönbrodt, 2016, see OSF Material A at osf.io/drv3a for the proof that the model satisfies the hypothesis’ properties listed in Table 2):

Strict simple congruence model:

$$\begin{aligned} z &= c_0 + c_1(x - y)^2 + \varepsilon \\ &= c_0 + 0x + 0y + c_1x^2 - 2c_1xy + c_1y^2 + \varepsilon \end{aligned} \tag{1}$$

To investigate the strict simple congruence hypothesis following the RSA strategy, one would fit this model to the data¹ and implement two subsequent steps (the following strategy is adapted from Edwards, 1994, 2002; see also Schönbrodt, 2016).

Step 1: Are the constraints imposed by the strict simple congruence model justified? As can be seen in the second row of Equation 1, the strict simple congruence model is a restricted version of the *full* second-order polynomial model:

Full second-order model:

$$z = b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + \varepsilon \tag{2}$$

In comparison with the full second-order model (Equation 2), the strict simple congruence model (Equation 1) imposes the constraints $b_1 = 0$, $b_2 = 0$, $b_4 = -2b_3$, and $b_5 = b_3$ on the coefficients of the full model (see also Table 3). In a first analytical step, we must

¹ All regression models that are presented in this article are based on the classical assumptions of multiple regression (i.e., independent identically normally distributed errors with mean zero).

explore whether it is justified to impose these constraints, in comparison with estimating the coefficients freely as in the full model. That is, we must compare the fit of the strict simple congruence model against the fit of the full model, for example, by using a likelihood ratio test. If the full model does not increase the explanatory power of the model by a significant amount, it is justified to base inferences on the estimated strict simple congruence model by interpreting its coefficients in a second analytical step.

To get an intuition of why this technical-looking step of testing model constraints is necessary, we can view it from a modeling perspective (see also Rodgers, 2010): The strict simple congruence model is a mathematical representation of our theoretical hypothesis (strict simple congruence effect). While being aware that all models are simplifications of complex reality, in positing the hypothesis, we suggest that this model is nonetheless a good approximation of the true associations in empirical data. In comparing the fit of our model against the full second-order polynomial (which is more flexible in approaching the data), we put this expectation to a critical test. If the full model does fit the data significantly better than the strict simple congruence model, this is a sign that the patterns in the data are different from the assumptions reflected in our model; we would reject the strict simple congruence hypothesis. In such a situation, one way to proceed is to reflect on alternative patterns that are theoretically plausible. For example, one possible next (exploratory) step could be to test whether linear main effects occur in addition to a congruence effect (see the section on rising ridge models below) or whether the data reflect main effects without a congruence effect (e.g., by comparing the full model to a linear model with predictors x and y). One can also inspect the coefficients of the estimated full second-order model and interpret them by using response surface methodology (e.g., see Edwards, 2002; Humberg, Nestler, et al., 2018).

Table 2

Formal Definitions of the Hypotheses that can be Investigated with (Quadratic or Cubic) RSA

Hypotheses definitions
<p>Strict simple congruence hypothesis (see also Figure 1)</p> <p>For two people, the person whose x and y values are closer to one another has the higher (or lower) value of z. [<i>congruence effect</i>]</p> <p>Strict asymmetric congruence hypothesis(see also Figure 4)</p> <p>For two people who either both have $x > y$ or both have $x < y$, the person whose x and y values are closer to one another has the higher (or lower) value of z, [<i>congruence effect</i>]</p> <p>and for two people whose discrepancy values $x - y$ are equal in magnitude but opposite in sign, the person with $x < y$ has the higher (or lower) value of z. [<i>asymmetry effect</i>]</p> <p>Strict level-dependent congruence hypothesis (see also Figure 6)</p> <p>For two people with the same mean predictor level $(x + y)/2$, the person whose x and y values are closer to one another has the higher (or lower) value of z, [<i>congruence effect</i>]</p> <p>and for four people A, B, C, and D of which A and B, as well as C and D, respectively, have equal mean predictor levels $(x + y)/2$, where A and B have a lower mean predictor level than C and D, and of which A and C, as well as B and D, respectively, have equal discrepancy values $x - y$, the absolute difference between A's and B's z values is larger (or smaller) than the absolute difference between C's and D's z values. [<i>level-dependency effect</i>]</p> <p>Rising ridge simple congruence hypothesis (see also Figure 8a)</p> <p>For two people with the same mean predictor level $(x + y)/2$, the person whose x and y values are closer to one another has the higher (or lower) value of z, [<i>congruence effect</i>]</p> <p>and for two people with the same value on the discrepancy $x - y$, the person who has the higher mean predictor level $(x + y)/2$ has the higher (or lower) value of z. [<i>linear level effect</i>]</p> <p>Rising ridge asymmetric congruence hypothesis (see also Figure 8b)</p> <p>For two people who either both have $x > y$ or both have $x < y$ and who have the same mean predictor level $(x + y)/2$, the person whose x and y values are closer to one another has the higher (or lower) value of z, [<i>congruence effect</i>]</p> <p>and for two people with the same value on the discrepancy $x - y$, the person who has the higher mean predictor level $(x + y)/2$ has the higher (or lower) value of z, [<i>linear level effect</i>]</p> <p>and for two people with the same mean predictor level $(x + y)/2$ and whose discrepancy values $x - y$ are equal in magnitude but opposite in sign, the person with $x < y$ has the higher (or lower) value of z. [<i>asymmetry effect</i>]</p> <p>Rising ridge level-dependent congruence hypothesis (see also Figure 8c)</p> <p>For two people with the same mean predictor level $(x + y)/2$, the person whose x and y values are closer to one another has the higher (or lower) value of z, [<i>congruence effect</i>]</p> <p>and for two people with the same value on the discrepancy $x - y$, the person who has the higher mean predictor level $(x + y)/2$ has the higher (or lower) value of z, [<i>linear level effect</i>]</p> <p>and for four people A, B, C, and D of which A and B, as well as C and D, respectively, have equal mean predictor levels $(x + y)/2$, where A and B have a lower mean predictor level than C and D, and of which A and C, as well as B and D, respectively, have equal discrepancy values $x - y$, the absolute difference between A's and B's z values is larger (or smaller) than the absolute difference between C's and D's z values. [<i>level-dependency effect</i>]</p>

Note. The polynomial models that are introduced in this article as representations of the hypotheses additionally assume that the function that describes the association between (x, y) and z is differentiable (smoothness assumption). In the additional OSF materials (osf.io/drv3a), we provide mathematical proofs for the claims that the polynomial models are in line with the predictions of the respective hypotheses. In OSF Material I, we provide example hypotheses that correspond to the formal definitions in this table.

Table 3
Steps for Investigating a Strict Simple Congruence Hypothesis

Hypothesis	Example Figure	Step 1: Test the model constraints on the full second-order model	Step 2: Inspect the Strict Simple Congruence Model There should be a congruence effect in the expected direction
Congruence is associated with the highest outcome values	Figure 1a	$b_1 = 0, b_2 = 0,$ $b_4 = -2b_3, b_5 = b_3$	$\hat{c}_1 = \hat{b}_3 < 0$
Congruence is associated with the lowest outcome values	Figure 1b	$b_1 = 0, b_2 = 0,$ $b_4 = -2b_3, b_5 = b_3$	$\hat{c}_1 = \hat{b}_3 > 0$

Step 2: Does the estimated model reflect the suggested effect? If the first step reveals that the data support the constraints of the strict simple congruence model, the model coefficient $\hat{c}_1$ must be inspected in a second step to reveal whether it supports the suggested congruence effect (see Table 3). If the coefficient $\hat{c}_1$ were zero, the strict simple congruence model would equal an intercept only model $z = c_0$. The significance test $\hat{c}_1$ thus informs us of whether the null hypothesis positing that x and y are unrelated to z can be rejected in favor of the hypothesis that z is associated with the level of congruence between x and y .² The graph of the estimated model can serve as a guide for understanding how $\hat{c}_1$ is related to a congruence effect (see Figure 1 for example graphs). When $\hat{c}_1$ is significantly negative (as in Figure 1a), the constraints in the strict simple congruence model ensure that the respective surface is highest for predictor combinations that lie on the so-called *line of congruence* (LOC) defined by predictor combinations with $x = y$ (see the blue line ranging from the front corner to the back corner of the coordinate cube; Edwards, 2002). Analogously, when $\hat{c}_1$ is significantly positive, the surface is lowest for congruent predictor combinations (see Figure 1b and Table 3). When the strict simple congruence model is implemented as a constrained version of the full model, as in the RSA package that we will use for model

² Note that this test renders it unnecessary to test the overall model significance of the strict simple congruence model, although one can of course insert such an omnibus test as an additional step before inspecting the model coefficients.

estimation, then $\hat{c}_1$ equals the coefficient $\hat{b}_3$ of x^2 (see Equations 1 and 2).

Overall, to investigate our hypothesis in which we posit that congruence between a person’s self-view and reputation is beneficial, the data would support our hypothesis if we found that the constraints of the strict simple congruence model were justified (Step 1) and that $\hat{c}_1$ was significantly negative (Step 2, Figure 1a; see Table 3).

(a) Congruence associated with the highest outcome values.

(b) Congruence associated with the lowest outcome values.

Figure 1. Prototypical strict simple congruence models for the different variations of the hypothesis.

“Complex” Congruence Hypotheses that cannot be Investigated with Quadratic RSA: Asymmetric and Level-Dependent Congruence Effects

In diverse research domains, a number of more nuanced versions of congruence hypotheses have recently been formulated (see Table 1 for examples). The most frequent hypotheses can be divided into two classes that we will label asymmetric congruence hypotheses and level-dependent congruence hypotheses. To date, empirical investigations of these complex congruence hypotheses have been based on quadratic RSA. We will now outline why this approach is inappropriate because second-order models cannot reflect these effects (see also Humberg, Nestler, et al., 2018). In doing so, we also introduce mathematical concepts that will be needed in the remainder of this article.

First, the *strict asymmetric congruence* hypothesis states that the outcome is higher (or lower) the more congruent the predictor values x and y are but that a mismatch in one direction (e.g., $x > y$) leads to steeper decreases in the outcome than a mismatch in the other

direction (e.g., $x < y$; see Table 2 for a formal definition of the hypothesis). For example, one might hypothesize that congruence between self-view and reputation will lead to the highest happiness, but holding a self-view that exceeds one's reputation should be worse than an incongruence in the other direction (see also Table 1 for some real examples from the literature).

The strict asymmetric congruence hypothesis cannot be investigated with the strict simple congruence model presented in Equation 1. This can be seen by inspecting the shape of the surface above the lines that are perpendicular to the LOC, that is, when moving away from the LOC in the direction of incongruence (see Figure 2). For an arbitrary value $k \in \mathbb{R}$, consider the line g_k , which lies in the xy plane, is perpendicular to the LOC, and includes the point (k, k) (see the lines depicted at the bottom of the coordinate cube in Figure 2 for examples with $k = -1$, $k = 0$, and $k = 1$). By definition, g_k consists of all points (x, y) with $y = -x + 2k$.

Figure 2. Visual representation of the lines g_k defined by $y = -x + 2k$, and visual representation of the shape of the graph of the strict simple congruence model above these lines.

The shape of the surface above g_k for any arbitrary value of k can be computed by substituting $y = -x + 2k$ into the estimated regression model from Equation 1:

Strict simple congruence model above g_k :

$$\begin{aligned} z &= \hat{c}_0 + \hat{c}_1(x - (-x + 2k))^2 \\ &= (\hat{c}_0 + 4\hat{c}_1k^2) - 8\hat{c}_1kx + 4\hat{c}_1x^2 \end{aligned} \quad (3)$$

An asymmetric congruence effect would require that (a) the surface is highest above the LOC, meaning that the surface above each g_k has a maximum above the LOC, and (b) the surface falls more quickly to one side of the LOC than to the other side, meaning that the surface above each g_k falls asymmetrically to the left versus the right side of its vertex. However, the surface above each g_k is given by a quadratic equation (Equation 3), and its graph is thus symmetric around its vertex. For example, Figure 3a shows the graph of the strict simple congruence model from Figure 2 above the lines g_{-1} , g_0 , and g_1 ; all of these lines fall symmetrically to both sides of $x = 0$ (i.e., to both sides of $(x, y) = (k, k)$ in Figure 2). Consequently, the strict simple congruence model cannot reflect an asymmetric congruence effect.

More generally, an asymmetric congruence effect cannot be reflected by *any* second-order polynomial model. The full second-order polynomial model (Equation 2) above the line g_k for an arbitrary value of k is given by:

Full second-order model above g_k :

$$\begin{aligned} z &= \hat{b}_0 + \hat{b}_1x + \hat{b}_2(-x + 2k) + \hat{b}_3x^2 + \hat{b}_4x(-x + 2k) + \hat{b}_5(-x + 2k)^2 \\ &= (\hat{b}_0 + 2\hat{b}_2k + 4\hat{b}_5k^2) + (\hat{b}_1 - \hat{b}_2 + 2\hat{b}_4k - 4\hat{b}_5k)x + (\hat{b}_3 - \hat{b}_4 + \hat{b}_5)x^2 \end{aligned} \quad (4)$$

Again, the surface above each g_k is given by a quadratic equation that cannot reflect the suggested asymmetric effect.

Second, the *strict level-dependent congruence* hypothesis posits that the outcome will be higher (or lower) for more congruent predictor combinations but that the strength of this effect will vary in accordance with the average level of the predictors (see Table 2 for a formal definition of the hypothesis). For example, having a self-view that is in line with one's reputation might beneficially affect happiness for people with low self-view and reputation levels, whereas congruence might play a minor or no role for people with high levels (see Table 1 for some real examples from the literature).

Surface above g_{-1} . Surface above g_0 . Surface above g_1 .

(a) Graphs of the functions that describe the strict simple congruence model from Figure 2 above g_{-1} , g_0 , and g_1 (see Equation 3).

Surface above g_{-1} . Surface above g_0 . Surface above g_1 .

(b) Graphs of the functions that describe the strict asymmetric congruence model from Figure 4a above g_{-1} , g_0 , and g_1 (see Equation 6).

Surface above g_{-1} . Surface above g_0 . Surface above g_1 .

(c) Graphs of the functions that describe the strict level-dependent congruence model from Figure 6b above g_{-1} , g_0 , and g_1 (see Equation 14).

Figure 3. Graphs of the functions that describe a strict simple congruence model (Panel a), a strict asymmetric congruence model (Panel b), and a strict level-dependent congruence model (Panel c) above the lines g_{-1} , g_0 , and g_1 . The strict simple congruence model above g_k (Panel a) is symmetric around its vertex; this property is loosened in the strict asymmetric congruence model (Panel b). Moreover, the strict simple congruence model above g_k (Panel a) has the same curvature for all k ; this property is loosened in the strict level-dependent congruence model (Panel c).

Again, this expectation cannot be investigated with the strict simple congruence model or with any second-order polynomial model. This can be seen with the help of the lines g_k , which are directly related to the average predictor level: For any value of k , the line g_k consists exactly of the predictor combinations $(x, y) = (x, -x + 2k)$ with an average level of $(x + y)/2 = k$. The strict level-dependent congruence hypothesis thus posits that the surface above g_k on a low average predictor level k (e.g., g_0 in Figure 2) should have a steeper curvature than the surface above g_k on a higher level (e.g., g_1 in Figure 2). However, the curvature above g_k (i.e., the coefficient of x^2 in Equations 3 and 4) is independent of k , both in the strict simple congruence model (where the curvature is $4\hat{c}_1$) and in the full second-order model (curvature = $\hat{b}_3 - \hat{b}_4 + \hat{b}_5$). That is, the curvature of the surface is equally steep for all mean predictor levels k (see Figure 3a). Consequently, a second-order polynomial model cannot reflect a level-dependent congruence effect.

To sum up, second-order polynomial models can be used to investigate the strict simple congruence hypothesis. However, these models cannot be applied to study asymmetric or level-dependent congruence effects. As a valid approach to investigate such complex congruence hypotheses, one can apply cubic instead of quadratic RSA.

Investigating Complex Congruence Hypotheses with Cubic RSA

The strict simple congruence model (Equation 1) can be extended by adding third-order terms to provide greater flexibility, which is necessary to model more complex hypotheses. This approach allows us to investigate for whether there are congruence effects that are asymmetric or level-dependent.

Strict Asymmetric Congruence Hypothesis

Imagine that we want to investigate the *strict asymmetric congruence hypothesis* that posits that people are happier when there is greater congruence between their self-view and reputation, but holding a self-view that exceeds one's reputation is worse for happiness than holding a self-view that lags behind one's reputation by the same amount (e.g., self-viewed intelligence 10 IQ points more vs. less than one's reputation; see Table 2 for a formal definition of the hypothesis).

A model that reflects this hypothesis can be derived as a natural extension of the strict simple congruence model (Equation 1), which predicts the outcome z from $(x - y)^2$. To loosen the symmetry property of this model, we introduce the cubic term $(x - y)^3$ as an additional

predictor:³

Strict asymmetric congruence model:

$$\begin{aligned} z &= c_0 + c_1(x - y)^2 + c_2(x - y)^3 + \varepsilon \\ &= c_0 + 0x + 0y + c_1x^2 - 2c_1xy + c_1y^2 + c_2x^3 - 3c_2x^2y + 3c_2xy^2 - c_2y^3 + \varepsilon \end{aligned} \quad (5)$$

(a) Congruence is associated with the highest outcome, and incongruence $x > y$ is associated with a lower outcome than incongruence $x < y$.

(b) Congruence is associated with the highest outcome, and incongruence $x > y$ is associated with a higher outcome than incongruence $x < y$.

(c) Congruence is associated with the lowest outcome, and incongruence $x > y$ is associated with a lower outcome than incongruence $x < y$.

(d) Congruence is associated with the lowest outcome, and incongruence $x > y$ is associated with a higher outcome than incongruence $x < y$.

Figure 4. Prototypical strict asymmetric congruence models for the different variations of the hypothesis.

³ It might seem counterintuitive to include the cubic term $(x - y)^3$ but not the linear term $(x - y)$ in this model. However, if $(x - y)$ were included and had a nonzero coefficient, the model would systematically predict the highest z values for people with incongruent predictor combinations, which would contradict the congruence effect (see OSF Material B at osf.io/drv3a for the proof).

Figure 4 shows graphs of the strict asymmetric congruence model for different combinations of the estimated coefficients $\hat{c}_1$ and $\hat{c}_2$. As can be seen in these example surfaces, the strict asymmetric congruence model is no longer restricted to be symmetric around the LOC. This is also reflected in the shape of this model above the lines g_k perpendicular to the LOC, that is, the model predictions for the (x, y) combinations with $y = -x + 2k$:

Strict asymmetric congruence model above g_k :

$$\begin{aligned}\hat{z} &= \hat{c}_0 + \hat{c}_1(x - (-x + 2k))^2 + \hat{c}_2(x - (-x + 2k))^3 \\ &= (\hat{c}_0 + 4\hat{c}_1k^2 - 8\hat{c}_2k^3) + (-8\hat{c}_1k + 24\hat{c}_2k^2)x + (4\hat{c}_1 - 24\hat{c}_2k)x^2 + 8\hat{c}_2x^3\end{aligned}\tag{6}$$

That is, the shape above g_k is a cubic instead of a quadratic equation owing to the inclusion of third-order terms. Because cubic equations are not restricted to be symmetric around their maximum (or minimum), the model allows an asymmetric shape of the surface to exist (see also Figure 3b). The procedure for investigating the strict asymmetric congruence hypothesis again consists of first testing the constraints of the corresponding model (Equation 5), this time compared with the full *third-order* model, and then inspecting the coefficient estimates to reveal whether the model supports the suggested effect. Table 4 contains all variants of the strict asymmetric congruence hypothesis (first column) and summarizes the respective analytical steps.

Step 1: Are the constraints posed by the strict asymmetric congruence model justified? As is evident from the second row of Equation 5, the strict asymmetric congruence model is a constrained version of the full third-order polynomial model:

$$\text{Full third-order model:}\tag{7}$$

$$z = b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + b_6x^3 + b_7x^2y + b_8xy^2 + b_9y^3 + \varepsilon.$$

Compared with the full model in Equation 7, the strict asymmetric congruence model poses the constraints $b_1 = 0$, $b_2 = 0$, $b_4 = -2b_3$, $b_5 = b_3$, $b_7 = -3b_6$, $b_8 = 3b_6$, and $b_9 = -b_6$ on the coefficients (see also Table 4). As in the quadratic RSA, a likelihood ratio test can reveal whether these constraints are justified. If they are justified, one would proceed with Step 2. If they are not, the strict asymmetric congruence hypothesis would be rejected. In the latter situation, as for the investigation of simple congruence effects, one possible next step could be to test whether linear main effects occur in addition, or instead, of a congruence effect. Note that, in contrast to the second-order case, we recommend *against* inspecting the coefficients of the full third-order model (see also the Discussion section).

Table 4

Steps for Investigating a Strict Asymmetric Congruence Hypothesis

Hypothesis	Example Figure	Step 1:	Step 2: Inspect the Strict Asymmetric Congruence Model
		Test the model constraints on the full third-order model	There should be a congruence effect in the expected direction
		There should be an asymmetry effect in the expected direction	There should be no data behind E_2 with significantly higher outcome predictions than points on E_2
Congruence is associated with the highest outcome values, and incongruence $x > y$ is associated with lower outcome values than incongruence $x < y$.	Figure 4a	$b_1 = 0, b_2 = 0,$ $b_4 = -2b_3, b_5 = b_3,$ $b_7 = -3b_6, b_8 = 3b_6,$ $b_9 = -b_6$	$\hat{c}_1 = \hat{b}_3 < 0$ $\hat{c}_2 = \hat{b}_6 < 0$ There should be no data point (x_i, y_i) with $y_i > x_i + \frac{2\hat{c}_1}{3\hat{c}_2}$ and $\hat{z}_i \notin (-\infty, \hat{z}_r + t_{n-p}(1 - \alpha)se(\hat{z}_r)]$
Congruence is associated with the highest outcome values, and incongruence $x > y$ is associated with higher outcome values than incongruence $x < y$.	Figure 4b	$b_1 = 0, b_2 = 0,$ $b_4 = -2b_3, b_5 = b_3,$ $b_7 = -3b_6, b_8 = 3b_6,$ $b_9 = -b_6$	$\hat{c}_1 = \hat{b}_3 < 0$ $\hat{c}_2 = \hat{b}_6 > 0$ There should be no data point (x_i, y_i) with $y_i < x_i + \frac{2\hat{c}_1}{3\hat{c}_2}$ and $\hat{z}_i \notin (-\infty, \hat{z}_r + t_{n-p}(1 - \alpha)se(\hat{z}_r)]$
Congruence is associated with the lowest outcome values, and incongruence $x > y$ is associated with lower outcome values than incongruence $x < y$.	Figure 4c	$b_1 = 0, b_2 = 0,$ $b_4 = -2b_3, b_5 = b_3,$ $b_7 = -3b_6, b_8 = 3b_6,$ $b_9 = -b_6$	$\hat{c}_1 = \hat{b}_3 > 0$ $\hat{c}_2 = \hat{b}_6 < 0$ There should be no data point (x_i, y_i) with $y_i < x_i + \frac{2\hat{c}_1}{3\hat{c}_2}$ and $\hat{z}_i \notin [\hat{z}_r - t_{n-p}(1 - \alpha)se(\hat{z}_r), \infty)$
Congruence is associated with the lowest outcome values, and incongruence $x > y$ is associated with higher outcome values than incongruence $x < y$.	Figure 4d	$b_1 = 0, b_2 = 0,$ $b_4 = -2b_3, b_5 = b_3,$ $b_7 = -3b_6, b_8 = 3b_6,$ $b_9 = -b_6$	$\hat{c}_1 = \hat{b}_3 > 0$ $\hat{c}_2 = \hat{b}_6 > 0$ There should be no data point (x_i, y_i) with $y_i > x_i + \frac{2\hat{c}_1}{3\hat{c}_2}$ and $\hat{z}_i \notin [\hat{z}_r - t_{n-p}(1 - \alpha)se(\hat{z}_r), \infty)$

Note. E_2 denotes the second extremum line of the surface, consisting of all predictor combinations (x, y) with $y = x + \frac{2\hat{c}_1}{3\hat{c}_2}$. $\hat{z}_r$ denotes the predicted outcome at the point of intersection (x_r, y_r) between E_2 and g_0 , that is, at $(x_r, y_r) = \left(\frac{-\hat{c}_1}{3\hat{c}_2}, \frac{\hat{c}_1}{3\hat{c}_2}\right)$.

Step 2: Does the estimated model reflect the suggested effect? When the constraints of the strict asymmetric congruence model are found to be justified, the coefficients of this model can be inspected to reveal whether they support the suggested hypothesis. Analogous to the strict simple congruence model, the coefficient $\hat{c}_1$ (which equals $\hat{b}_3$ when the model is notated as a constrained full model) indicates the congruence effect (see also Table 4; see OSF Material B at osf.io/drv3a for the proof). When $\hat{c}_1$ is significantly negative (Figures 4a and 4b), the respective surface falls to both sides of the line of congruent predictor combinations (LOC). When $\hat{c}_1$ is significantly positive (Figures 4c and 4d), the surface rises on both sides of the LOC.

The cubic term coefficient $\hat{c}_2$ ($= \hat{b}_6$) in the strict asymmetric congruence model reflects the strength and direction of the asymmetry effect (see also Table 4; see OSF Material B at osf.io/drv3a for the proof): When $\hat{c}_2$ is significantly negative (Figures 4a and 4c), then a person (e.g., Kim) whose self-view x exceeds her reputation y by a specific amount has a lower model-predicted happiness value than a person (e.g., Sam) whose self-view falls behind his reputation by the same amount. When $\hat{c}_2$ is significantly positive (Figures 4b and 4d), the situation is reversed such that incongruence in the direction of $x > y$ (Kim) is associated with higher model-predicted outcome values than incongruence $x < y$ (Sam).

In contrast to the investigation of the strict simple congruence model described above, a test of the estimated coefficients of the strict asymmetric congruence model is not sufficient for concluding that the data support the hypothesis. A further step is needed to ensure that the model indeed supports the suggested effect *for the whole range of realistic predictor values*. When considering the strict asymmetric congruence model for *all* possible combinations of the predictor variables, it does not infinitely fall (or rise, depending on $\hat{c}_1$) to both sides of the LOC. This is the case on one side of the LOC, but on the other side, the surface eventually turns and rises again, which is a consequence of the cubic term in the model (Equation 5). For example, consider Figure 5, which shows the graph of the same model as in Figure 4a for a larger range of predictor combinations. The line on the surface where the surface turns and rises again is depicted as a pink line. Its projection onto the xy plane is also depicted as a pink line, it is (by definition of the model) parallel to the LOC, and we refer to it as the *second extremum line* E_2 (in comparison with the LOC as the first extremum line).

Predictor combinations that lie behind E_2 , seen from the LOC, have higher model-predicted outcome values than combinations on E_2 , although the former combinations are more *incongruent*. The rising part of the surface for predictor combinations behind E_2 would thus contradict the hypothesis that more incongruence goes along with lower outcome values, *if* this part of the surface makes predictions for predictor combinations that are realistic (i.e., actually observed). To discover whether this is the case, we will now derive the formula for the position of E_2 and describe a strategy that can be applied to find out whether the range of realistic predictor combinations spreads into the part of the surface that (significantly) contradicts the hypothesis.⁴

Figure 5. The same strict asymmetric congruence model as in Figure 4a, plotted for a larger range of predictor values and with the model-predicted outcome values for the data points plotted on the surface. In this example, none of the predictor combinations in the data that lie behind the second extremum line E_2 have a significantly higher prediction of the outcome than predictor combinations on E_2 , so that this surface is in line with the suggested strict asymmetric congruence hypothesis (see also the first row of Table 4).

⁴ Theoretically, one could integrate this step into the model selection in Step 1 by constraining the strict asymmetric congruence model in such a way that the line E_2 is forced to lie outside of the area of observed predictor combinations. However, the model constraints would be highly sensitive to the influence of single data points, which would threaten the robustness of the results. We therefore prefer the strategy of inspecting the data points behind E_2 in a separate analytical step.

Position of the second extremum line E_2 . To derive the position of the second extremum line E_2 , consider again the surface above the lines g_k perpendicular to the LOC (Equation 6). When $\hat{c}_1$ and $\hat{c}_2$ are nonzero, the surface above g_k has two extrema. The first extremum has the x value $x_{k1} = k$ (i.e., the extremum is positioned at $(x_{k1}, y_{k1}) = (k, k)$ = above the LOC). The second extremum lies at $x_{k2} = k - \frac{\hat{c}_1}{3\hat{c}_2}$. The y value at x_{k2} is $y_{k2} = -x_{k2} + 2k$, and E_2 thus consists of all predictor combinations $(x_{k2}, y_{k2}) = (k - \frac{\hat{c}_1}{3\hat{c}_2}, -(k - \frac{\hat{c}_1}{3\hat{c}_2}) + 2k), k \in \mathbb{R}$. As a linear equation, E_2 is given by (see OSF Material B at osf.io/drv3a for the proof):

$$E_2 : \tag{8}$$

$$y = x + \frac{2\hat{c}_1}{3\hat{c}_2}$$

How many data points behind the second extremum line have significantly higher outcome predictions than more congruent predictor combinations? With the help of Equation 8, it is possible to determine how many predictor combinations in the data lie behind E_2 . When $\hat{c}_1$ and $\hat{c}_2$ are negative (as in Figure 5, and using Equation 8), the points behind the line E_2 are the points with $y_i > x_i + \frac{2\hat{c}_1}{3\hat{c}_2}$ (see the last column of Table 4 for other constellations of $\hat{c}_1$ and $\hat{c}_2$). For the simulated example data underlying Figure 5, this is the case for 4.5% of the data points. One could argue that the estimated model is in line with the strict asymmetric congruence hypothesis in the full range of realistic predictor combinations only if there are no data behind E_2 . This strategy would be rather conservative: Imagine that there are data points behind E_2 , but they are very close to the line, such that the outcome predictions for these points do not essentially differ from outcome predictions for more congruent combinations (e.g., for combinations on E_2). In this case, it might not be necessary to reject the hypothesis.

We therefore suggest a strategy that is based on the idea that a data point behind E_2 is considered to contradict the hypothesis only if the predicted outcome at this point is significantly higher than predictions for more congruent predictor combinations. To see how to identify such data points, consider, for example, Tom's predictor combination (x_{Tom}, y_{Tom}) , which lies behind E_2 (Figure 5), and his predicted outcome value $\hat{z}_{Tom}$. Is Tom's outcome prediction significantly higher than the prediction for any data point that is more congruent than (x_{Tom}, y_{Tom}) ? The "worst case" predictor combinations, namely, the more congruent ones (compared with Tom's combination) with the lowest possible outcome predictions, are

the points that lie on E_2 . Thus, it is sufficient to test whether $\hat{z}_{Tom}$ is significantly higher than the outcome prediction $\hat{z}$ of any predictor combination (x, y) on E_2 . Luckily, this significance test is exactly the same for all points on E_2 because the strict asymmetric congruence model makes the same outcome prediction $\hat{z}$ for all (x, y) on E_2 , and the standard error of $\hat{z}$ is also the same for all of these points (see OSF Material B at osf.io/drv3a for the proofs).

Consequently, without loss of generality, it is sufficient to compare $\hat{z}_{Tom}$ with the predicted outcome at an arbitrarily selected point on E_2 . A natural choice is the point (x_r, y_r) in which the line g_0 (defined by $y = -x$) and E_2 (Equation 8) intersect (see Figure 5), which has the following coordinates:

$$(x_r, y_r) = \left(\frac{-\hat{c}_1}{3\hat{c}_2}, \frac{\hat{c}_1}{3\hat{c}_2} \right) \quad (9)$$

To test whether Tom's predicted outcome $\hat{z}_{Tom}$ is significantly higher than the prediction $\hat{z}_r$ at (x_r, y_r) , the confidence interval of $\hat{z}_r$ can be computed and inspected for whether it includes $\hat{z}_{Tom}$. To this aim, we first need the value $\hat{z}_r$ at (x_r, y_r) , which can easily be computed with the model equation (Equation 5). Second, we need the standard error of $\hat{z}_r$. It can be computed as (e.g., see Casella & Berger, 2002):

$$se(\hat{z}_r) = \sqrt{\mathbf{x}_r^t \Sigma_{\hat{b}} \mathbf{x}_r} \quad (10)$$

where $\Sigma_{\hat{b}}$ is the covariance matrix of the vector $\hat{b} = (\hat{b}_0, \dots, \hat{b}_9)$ of the estimated regression coefficients, and $\mathbf{x}_r$ is the vector of predictor variables at the point (x_r, y_r) :

$$\mathbf{x}_r = (1, x_r, y_r, x_r^2, x_r y_r, y_r^2, x_r^3, x_r^2 y_r, x_r y_r^2, y_r^3). \quad (11)$$

The one-sided confidence interval of $\hat{z}_r$ to test for outcome predictions that are significantly higher than $\hat{z}_r$ is then given by:

$$(-\infty, \hat{z}_r + t_{n-p}(1 - \alpha)se(\hat{z}_r)] \quad (12)$$

where α is the respective significance level, n is the sample size, and p is the number of parameters in the model, that is, $p = 10$ (see Table 4 for the one-sided confidence intervals of $\hat{z}_r$ for models with other constellations of $\hat{c}_1$ and $\hat{c}_2$).

In the example depicted in Figure 5, the one-sided confidence interval of $\hat{z}_r$ is $(-\infty, 12.8]$, and Tom's predicted outcome is included in this confidence interval. As noted above, this implies that $\hat{z}_{Tom}$ is not significantly higher than *any* predicted outcome for predictor combinations on E_2 and, even more generally, it does not differ significantly from the prediction of any more congruent point. That is, although Tom's predictor combination is

positioned behind E_2 , it does not “significantly” contradict the hypothesis that congruence maximizes the outcome. By applying this procedure to each data point that lies behind E_2 , we can determine whether there are realistic predictor combinations whose outcome predictions significantly contradict the hypothesis. In the R package *RSA*, we provide the R function `caRange` that can be used to identify and test the data points behind E_2 and to identify how many points significantly contradict the hypothesis (see the case studies below).

Overall, in an investigation of our asymmetric congruence hypothesis about the congruence between self-view and reputation (Figure 4a), the data support our hypothesis when (a) $\hat{c}_1$ is significantly negative, (b) $\hat{c}_2$ is also significantly negative, and (c) there are no data points behind E_2 whose predicted outcomes are significantly higher than the predictions on E_2 (see the Discussion section for considerations about loosening the latter condition). The respective conditions for all other variants of the strict asymmetric congruence hypothesis can be found in Table 4.

Strict Level-Dependent Congruence Hypothesis

A second complex congruence hypothesis that was suggested in the literature is the *strict level-dependent congruence hypothesis*. As an example, imagine that we expect that for low levels of self-view and reputation, those people are happier whose self-views and reputations are more congruent, whereas congruence plays a minor or even no role when predictor levels are moderate or high (see Table 2 for a formal definition of the hypothesis). In other words, according to this hypothesis, the mean predictor level $(x + y)/2$ is expected to moderate the congruence effect.

A model that represents the strict level-dependent congruence hypothesis can be obtained by extending the strict simple congruence model by including a respective moderation term. To streamline the definition of the model and without loss of generality, we include the sum of the predictors $(x + y)$ instead of their average $(x + y)/2$ as a moderator of the congruence effect:

Strict level-dependent congruence model:

$$\begin{aligned} z &= c_0 + c_1(x - y)^2 + c_3(x + y)(x - y)^2 + \varepsilon \\ &= c_0 + 0x + 0y + c_1x^2 - 2c_1xy + c_1y^2 + c_3x^3 - c_3x^2y - c_3xy^2 + c_3y^3 + \varepsilon \end{aligned} \tag{13}$$

(a) Congruence is associated with the highest outcome, and this effect is weaker or nonexistent for lower levels of the predictors than for higher levels.

(b) Congruence is associated with the highest outcome, and this effect is weaker or nonexistent for higher levels of the predictors than for lower levels.

(c) Congruence is associated with the lowest outcome, and this effect is weaker or nonexistent for higher levels of the predictors than for lower levels.

(d) Congruence is associated with the lowest outcome, and this effect is weaker or nonexistent for lower levels of the predictors than for higher levels.

Figure 6. Prototypical strict level-dependent congruence models for the different variations of the hypothesis.

Figure 6 shows graphs of the strict level-dependent congruence model for different combinations of the coefficients. As can be seen in these plots, this model allows changes in the strength of the congruence effect, depending on the mean predictor level. This is also reflected in the shape of the surface above the lines g_k :

Strict level-dependent congruence model above g_k :

$$\begin{aligned}\hat{z} &= \hat{c}_0 + \hat{c}_1(x - (-x + 2k))^2 + \hat{c}_3(x + (-x + 2k))(x - (-x + 2k))^2 \\ &= (\hat{c}_0 + 4\hat{c}_1k^2 + 8\hat{c}_3k^3) + (-8\hat{c}_1k - 16\hat{c}_3k^2)x + (4\hat{c}_1 + 8\hat{c}_3k)x^2\end{aligned}\quad (14)$$

That is, the quadratic term coefficient $4\hat{c}_1 + 8\hat{c}_3k$ of the surface above g_k (which directly affects the steepness of the congruence effect) can change as a function of the mean predictor level k (see also Figure 3c). To investigate the strict level-dependent congruence hypothesis,

the analytical procedure is again divided into two steps.

Step 1: Are the constraints imposed by the strict level-dependent congruence model justified? The strict level-dependent congruence model is a constrained version of the third-order full polynomial model (Equation 7), imposing the constraints $b_1 = 0$, $b_2 = 0$, $b_4 = -2b_3$, $b_5 = b_3$, $b_7 = -b_6$, $b_8 = -b_6$, and $b_9 = b_6$ on the coefficients (see also Table 5). Again, these constraints can be tested by comparing the strict level-dependent congruence model with the full third-order model with a likelihood ratio test. If the test is nonsignificant, one would proceed with Step 2. If they are not, the hypothesis must be rejected, and one can, for example, conduct the exploratory next steps that were described above for the strict asymmetric congruence hypothesis.

Step 2: Does the estimated model reflect the suggested effect? The coefficient $\hat{c}_3$ ($= \hat{b}_6$ in the notation of a constrained full model) reflects the level-dependency effect (see also Table 5). This can be seen by considering the surface above g_k (Equation 14). When $\hat{c}_3$ is negative (Figures 6a and 6c), then an increase in the mean predictor level k causes a decrease in the curvature⁵ of the surface above g_k (i.e., the curvature becomes more negative or less positive; see OSF Material C at osf.io/drv3a for the proof). When $\hat{c}_3$ is positive (Figures 6b and 6d), the curvature of the surface above g_k increases (i.e., becomes less negative or more positive) as the predictor level k increases.

In contrast to the preceding models, the coefficient $\hat{c}_1$ cannot be directly interpreted with respect to a congruence effect. Instead, before it is possible to determine whether the estimated model supports the suggested hypothesis, the range of the data must be taken into account. When there is a level-dependency effect (i.e., $\hat{c}_3 \neq 0$), the model can indicate different directions for the congruence effect, depending on the levels of the predictors. For example, Figure 7 shows the same model as in Figure 6b but for a larger range of predictor combinations. The curvature of the surface is significantly negative (i.e., congruence is associated with the highest outcome values) in the front part of the surface, nonsignificant (congruence is not significantly related to the outcome) in the middle part, and significantly positive (congruence is associated with the lowest outcome values) in the far back corner of the coordinate cube.

⁵ Formally, the *curvature* of the surface above g_k is defined as the second derivative of the function in Equation 14 (see Equation 15 for its formula). Intuitively, the curvature reflects the "steepness" of the parabola above the line g_k (i.e., the steepness of the red lines on the surface in Figure 2).

Table 5

Steps for Investigating a Strict Level-Dependent Congruence Hypothesis

Hypothesis	Example Figure	Step 1:	Step 2: Inspect the Strict Level-Dependent Congruence Model
At high predictor levels, congruence is associated with the highest outcome values, and this effect is weaker or nonexistent at lower predictor levels.	Figure 6a	<p>Test the model constraints on the full third-order model</p> $b_1 = 0, b_2 = 0,$ $b_4 = -2b_3, b_5 = b_3,$ $b_7 = -b_6, b_8 = -b_6,$ $b_9 = b_6$	<p>There should be a level-dependency effect in the expected direction</p> <p>The intersection areas of the data with the regions of (negative, non-, positive) significance should be in line with the suggested hypothesis</p>
At low predictor levels, congruence is associated with the highest outcome values, and this effect is weaker or nonexistent at higher predictor levels.	Figure 6b	$b_1 = 0, b_2 = 0,$ $b_4 = -2b_3, b_5 = b_3,$ $b_7 = -b_6, b_8 = -b_6,$ $b_9 = b_6$	$\hat{c}_3 = \hat{b}_6 < 0$ <p>There should be a region of negative significance that intersects with the data, and if a region of positive significance exists, there should be no data in it.</p>
At low predictor levels, congruence is associated with the lowest outcome values, and this effect is weaker or nonexistent at higher predictor levels.	Figure 6c	$b_1 = 0, b_2 = 0,$ $b_4 = -2b_3, b_5 = b_3,$ $b_7 = -b_6, b_8 = -b_6,$ $b_9 = b_6$	$\hat{c}_3 = \hat{b}_6 > 0$ <p>There should be a region of negative significance that intersects with the data, and if a region of positive significance exists, there should be no data in it.</p>
At high predictor levels, congruence is associated with the lowest outcome values, and this effect is weaker or nonexistent at lower predictor levels.	Figure 6d	$b_1 = 0, b_2 = 0,$ $b_4 = -2b_3, b_5 = b_3,$ $b_7 = -b_6, b_8 = -b_6,$ $b_9 = b_6$	$\hat{c}_3 = \hat{b}_6 < 0$ <p>There should be a region of positive significance that intersects with the data, and if a region of negative significance exists, there should be no data in it.</p>

Figure 7. The same strict level-dependent congruence model as in Figure 6b, plotted for a larger range of predictors and with the model-predicted outcome values for the data points plotted on the surface.

The decision about whether the data support a respective hypothesis must be based on the parts of the surface that make predictions for realistic predictor combinations, where the question of which parts are “allowed” depends on the hypothesis at hand. For example, for our hypothesis that congruence is negatively related to happiness at low predictor levels and weaker or unrelated at higher levels, support for it would be found if realistic predictor combinations were situated only in the parts of the surface that are inverted U-shaped or nearly constant (as in Figure 7; see also Ilmarinen et al., 2016, for a related idea). By contrast, if the surface has parts that have a statistically significant U-shape in the range of realistic predictor combinations, we would reject the hypothesis.

We suggest that two steps be used to explore this: First, identify the predictor levels k for which the curvature of the surface above g_k is significantly negative, nonsignificant, or significantly positive (Bauer & Curran, 2005; Miller et al., 2013). Second, figure out which of these regions intersect with the ranges of realistic (i.e., actually observed) predictor combinations.⁶

⁶ As for the strict asymmetric congruence model (see Footnote 4), one could theoretically constrain the strict level-dependent congruence model in such a way that these two steps could be omitted. However, such a strategy would again threaten the robustness of the model estimation, and we therefore prefer the strategy of inspecting the regions of significance and their intersection with the data in separate steps.

Regions of (negative, non-, positive) significance for the curvature above g_k .

Following the strategy suggested by Bauer and Curran (2005), we will aim to identify the levels k_1 and k_2 at which the significance status of the curvature of the surface above g_k changes (see Figure 7). The significance status of each of the resulting intervals of k can then easily be determined by testing the curvature above g_k for an arbitrary value k within the respective interval.

The estimated curvature of the surface above g_k is the second derivative of the function in Equation 14. We denote it as $\hat{\eta}_k$:

$$\hat{\eta}_k = 8\hat{c}_1 + 16\hat{c}_3k \quad (15)$$

To obtain information about the significance status of $\hat{\eta}_k$ at some value k , we need to compute its test statistic $\hat{\eta}_k/\sqrt{\text{Var}(\hat{\eta}_k)}$ at the respective value of k . To this aim, the variance of $\hat{\eta}_k$ can be computed by applying the delta theorem (e.g., see Casella & Berger, 2002), which yields:

$$\text{Var}(\hat{\eta}_k) = 64 \text{Var}(\hat{c}_1) + 256 \text{Cov}(\hat{c}_1, \hat{c}_3)k + 256 \text{Var}(\hat{c}_3)k^2 \quad (16)$$

The values k_1 and k_2 where the significance status of $\hat{\eta}_k$ changes are the values of k where the test statistic $\hat{\eta}_k/\sqrt{\text{Var}(\hat{\eta}_k)}$ takes the critical values $\pm z_{\alpha/2}$ (e.g., ± 1.96 for $\alpha = .05$):

$$\begin{aligned} \frac{\hat{\eta}_k}{\sqrt{\text{Var}(\hat{\eta}_k)}} &= \pm z_{\alpha/2} \\ \Leftrightarrow \left(\frac{8\hat{c}_1 + 16\hat{c}_3k}{\sqrt{64 \text{Var}(\hat{c}_1) + 256 \text{Cov}(\hat{c}_1, \hat{c}_3)k + 256 \text{Var}(\hat{c}_3)k^2}} \right)^2 &= z_{\alpha/2}^2 \\ \Leftrightarrow ak^2 + bk + c &= 0 \end{aligned} \quad (17)$$

with

$$\begin{aligned} a &= 4\hat{c}_3^2 - 4 \text{Var}(\hat{c}_3)z_{\alpha/2}^2 \\ b &= 4\hat{c}_1\hat{c}_3 - 4 \text{Cov}(\hat{c}_1, \hat{c}_3)z_{\alpha/2}^2 \\ c &= \hat{c}_1^2 - \text{Var}(\hat{c}_1)z_{\alpha/2}^2 \end{aligned} \quad (18)$$

Thus, k_1 and k_2 can be computed via the quadratic formula:

$$k_{1,2} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}. \quad (19)$$

Let $k_1 < k_2$ without loss of generality. To reveal the significance status of $\hat{\eta}_k$ for each of the intervals $(-\infty, k_1]$, $[k_1, k_2]$, and $[k_2, \infty)$, an arbitrary value k_{test} can be selected from within the respective interval. Using Equations 15 and 16 to compute the test statistic $\hat{\eta}_{k_{test}}/\sqrt{\text{Var}(\hat{\eta}_{k_{test}})}$, it is possible to determine whether the curvature $\hat{\eta}_{k_{test}}$ of the surface above

$g_{k_{test}}$ is significantly negative, nonsignificant, or significantly positive, and this significance status will be the same for the entire respective interval.

Identify the regions that intersect with the data. In a second step, we need to determine which of the regions must be interpreted, that is, which regions intersect with the predictor combinations that are actually in the data. An arbitrary data point (x_i, y_i) lies on the line g_k with $k = (x_i + y_i)/2$. By using this observation to compute the respective mean level k for each combination of predictors in the data, it is easy to determine how many of the k values of the data points lie in each of the regions computed above.

Depending on our hypothesis, we must determine whether the data intersect with the “correct” regions of (negative, non-, or positive) significance (see also Table 5). For example, when we expect that congruence will lead to higher outcome values on some predictor levels and will be weaker or unrelated to the outcome on other levels (Figures 6a and 6b), then at least in some ranges of realistic predictor combinations, the curvature should be significantly negative (i.e., there should be a region of negative significance that contains a meaningful number of the data points), and there should be no data that lie on a line g_k above which the surface has a significantly positive curvature (i.e., if a region of positive significance exists, there should be no data in it).⁷ For the reverse effect, we must expect the opposite pattern (Figures 6c and 6d). These conditions can also be easily adapted to a situation in which we would expect a significant congruence effect for *all* realistic predictor levels, whereas we would expect the strength of the effect to vary. In this case, all data points should lie within a region of negative (or positive) significance. Analogously, the conditions can be adapted to investigate the expectation that the congruence effect changes direction (e.g., congruence leads to the highest outcome at low predictor levels and to the lowest outcome at high levels, or vice versa). In the R package *RSA*, we provide the function `clRange`, which computes the regions of significance and determines how many data points lie in each of them (see the case studies below).

Overall, to investigate our hypothesis that the benefits of congruent self-view and reputation levels should be more pronounced at lower predictor levels, the data support our hypothesis when (a) $\hat{c}_3$ is significantly positive and (b) an essential amount of the data lie in a

⁷ This strategy implies that there might be data points in U-shaped parts of the surface, as long as these parts are not significantly U-shaped (i.e., they lie in the U-shaped part of the region of nonsignificance). Consequently, this strategy is similarly as conservative (or liberal) as the procedure that we suggested for the strict asymmetric congruence model, in which data points are allowed to exist behind E_2 as long as they do not have a statistically significantly higher predicted outcome than the points on E_2 .

region of negative significance, whereas if a region of positive significance exists, there should be no data in it (Figure 6b). The respective conditions for all variants of the strict level-dependent congruence hypothesis can be found in Table 5.

"Rising Ridge" Versions of the Hypotheses: Investigating Simple and Complex Congruence Effects Combined with Main Effects

In many psychological domains, not only is the degree of congruence expected to affect the outcome, but the mean level of the predictor variables is also expected to do so. For example, when considering self-views and reputation concerning intelligence, it might be reasonable to expect that congruence is beneficial for psychological adjustment and that, in addition, out of two people with the same discrepancy between self-view and reputation, the person with the higher mean predictor level should be happier.

We use the labels *rising ridge simple congruence hypothesis*, *rising ridge asymmetric congruence hypothesis*, and *rising ridge level-dependent congruence hypothesis* for the hypotheses that posit such a linear level effect in addition to a simple, asymmetric, or level-dependent congruence effect, respectively (see Table 2 for formal definitions of these hypotheses). To obtain statistical models for their representation, the strict simple congruence model (Equation 1), the strict asymmetric congruence model (Equation 5), and the strict level-dependent congruence model (Equation 13) can be extended by adding $(x + y)$ as a linear predictor. For example, the model that allows a linear level effect in addition to a simple congruence effect is defined as follows:

Rising ridge simple congruence model:

$$\begin{aligned} z &= c_0 + c_{level}(x + y) + c_1(x - y)^2 + \varepsilon \\ &= c_0 + c_{level}x + c_{level}y + c_1x^2 - 2c_1xy + c_1y^2 + \varepsilon \end{aligned} \tag{20}$$

Models that represent the rising ridge versions of the asymmetric and the level-dependent congruence hypotheses can be obtained in an analogous way.⁸ Figure 8 shows example graphs of the resulting models with positive linear level effects c_{level} . Owing to this level effect, the surfaces rise toward the back corner of the coordinate cube.

The only difference between the rising ridge and the strict variants of the models (e.g., compare Equation 20 with Equation 1) is that the two linear predictors x and y have equal

⁸ Note that depending on the direction of the congruence effect and of the linear level effect, the "rising ridge" models can also have "falling" ridges (when $c_{level} < 0$) or rising/falling troughs (e.g., when $c_1 > 0$, see also Figure 1b).

coefficients. Accordingly, the Step 1 constraints presented in Tables 3, 4, and 5 must be adapted: The two conditions “ $b_1 = 0$ and $b_2 = 0$ ” must be replaced by the condition $b_1 = b_2$ (see OSF Material D at osf.io/drv3a for the complete sets of constraints). The conditions to be tested in Step 2 for the rising ridge versions of the hypotheses are almost analogous to the rules presented in Tables 3, 4, and 5, respectively (see also OSF Materials D to G at osf.io/drv3a). There is only one exception:

For the rising ridge asymmetric congruence model, when investigating whether the estimated model is in line with the hypothesis for the whole range of realistic predictor variables, we must again consider every data point behind the second extremum line E_2 and test whether it has a significantly higher predicted outcome than the more congruent points on E_2 . Here, however, it is not sufficient to compare the outcome predictions with the prediction at the intersection point of the LOIC and E_2 because the predicted outcome value now differs for different points on E_2 because of the linear level effect. Instead, for each data point (x_i, y_i) behind E_2 , we must identify the point on E_2 that has the same level. It can be computed as the point of intersection of E_2 and g_k , where $k = (x_i + y_i)/2$ is the level of (x_i, y_i) . In the R package *RSA*, we provide a function that, for each data point behind E_2 , computes the one-sided confidence interval of the predicted outcome at the intersection point of E_2 and g_k and determines whether the predicted outcome at (x_i, y_i) is included in this interval. The interpretation is analogous to the one from the strict asymmetric congruence model (see Table 4). No adaptations are necessary for the rising ridge level-dependent congruence model because the curvature $\hat{\eta}_k$ above g_k , the crucial element in the computation of the regions of significance, is not affected by the linear main effects.

Finally, the linear level effect must be tested in addition to the conditions presented in Tables 3, 4, and 5. This effect is reflected in the parameter $\hat{u}_1 = \hat{b}_1 + \hat{b}_2$.⁹ Graphically, $\hat{u}_1$ equals the slope of the surface above the LOC and above each line that is parallel to it. When $\hat{u}_1$ is significantly positive (as in Figure 8), this indicates a positive level effect, and when $\hat{u}_1$ is significantly negative, this is in line with a negative level effect (see also OSF Materials E, F, and G at osf.io/drv3a).

⁹ For the rising ridge *simple* congruence model, the linear level effect is usually denoted $\hat{a}_1$ instead of $\hat{u}_1$ (e.g., in the R package *RSA*).

(a) Rising ridge simple congruence model:
 $\hat{z} = \hat{c}_0 + \hat{c}_{level}(x + y) + \hat{c}_1(x - y)^2$.

(b) Rising ridge asymmetric congruence model:
 $\hat{z} = \hat{c}_0 + \hat{c}_{level}(x + y) + \hat{c}_1(x - y)^2 + \hat{c}_2(x - y)^3$.

(c) Rising ridge level-dependent congruence model:
 $\hat{z} = \hat{c}_0 + \hat{c}_{level}(x + y) + \hat{c}_1(x - y)^2 + \hat{c}_3(x + y)(x - y)^2$.

Figure 8. Prototypical rising ridge congruence models.

Case Study 1: Strict Asymmetric Congruence Hypothesis

In the following, we describe a research objective in which it might be realistic to assume asymmetric congruence effects. We use simulated data to demonstrate how cubic RSA can be applied, interpreted, and reported in an empirical article (see osf.io/drv3a for the data and R code).

Research Objective and Hypothesis

Do people maintain better working social relationships when their self-views about their own virtues or attributes are in line with the views that other people hold about them? It was suggested that self-other agreement is interpersonally adaptive because, for example, it benefits the quality of social interactions and reduces feelings of cognitive dissonance related to peer feedback (e.g., see Human et al., 2018).

Here, we focus on the specific research question that addresses whether congruence

between self- and friend-viewed intelligence affects harmony in social interactions. As an example, consider Kate and her friend Matt. Potentially, the more discrepant Kate’s self-view of her own intelligence is from Matt’s view of Kate’s intelligence, the more often Matt will get the impression that Kate is trying to oversell (if Kate’s self-view $>$ Matt’s view) or undersell (if Kate’s self-view $<$ Matt’s view) herself. Subsequently, the more often Matt will provide (verbal or nonverbal) feedback that Kate will perceive as offensive or at least as dissonant with her self-view and, as a consequence, the more disregard and conflict Kate will experience during social interactions with Matt. We thus hypothesize that congruence between self-viewed intelligence x and friend-viewed intelligence y is positively related to the average harmony z that the target person perceives during his or her social interactions with the respective friend.

Furthermore, we hypothesize that this congruence effect is asymmetric. It seems reasonable to expect that overselling will be evaluated as more annoying than underselling. Moreover, getting feedback about one’s intelligence that is more positive than expected might evoke feelings of dissonance and thus lead to some discussion, but feedback that a person perceives as unjustifiably *negative* carries much greater potential for social conflict. Consequently, we hypothesize that in addition to the suggested beneficial effect of congruence, people whose self-view exceeds the friend-view will have less harmonic interactions with the respective friend than people whose self-view lags behind the friend-view by the same amount (first row of Table 4; Figure 4a).

Data, Data Preparation, and Pre-Analyses

To subject the suggested asymmetric congruence hypothesis to an empirical investigation, we would need to assess self-viewed intelligence x , friend-rated intelligence y , and interaction harmony z . A suitable study procedure could be to assess participants’ self-viewed intelligence with an online questionnaire (x), let them each nominate one friend who is then asked to rate the target’s intelligence (y), and additionally ask the target to rate the average harmony z of his or her interactions with this specific friend.

When planning the data assessment, one should be aware that cubic RSA requires quite a large sample to achieve sufficient power and enough precision in estimating the complex third-order polynomial models. There are yet no tools to determine precise sample size requirements for RSA and we are hesitant to provide a heuristic because sample size planning depends on various specifics of the research situation (see also the Discussion section). As a

very rough guideline, we note that the sample size should by no means be smaller, rather substantively larger than the size needed to detect small to medium second-order effects (i.e., interaction and quadratic effects). This provisional guideline should be replaced by a more precise method as soon as a respective tool has been developed for the context of RSA.

When planning the measurement instruments, special attention must be placed on the commensurable assessment of the predictor variables x and y (Edwards & Shipp, 2007). An investigation of congruence hypotheses requires that participants' x and y values can be reasonably compared, as all (simple and complex) congruence models are based on the assumption that the predictor combinations on the LOC (i.e., that satisfy $x = y$) are the combinations that are understood as “congruent” from a conceptual point of view (e.g., “self-view congruent with friend-view”). Therefore, the predictor variables should be assessed on the same numerical scale.¹⁰ With regard to the simulated example data, we assume that self- and friend-estimated intelligence were assessed on an IQ point scale and that interaction harmony was rated by the target persons on a Likert scale ranging from 1 (*very unharmonic*) to 7 (*very harmonic*). Table 6 shows descriptive statistics for the simulated data.

To prepare model estimation, we centered the predictors x and y on their grand mean (i.e., the mean of the combined x and y variables). It is important that the same number (e.g., the grand mean) be used for centering the two variables, rather than centering the variables on their individual means, because only by subtracting the same value from people's scores on x and y can we preserve their original discrepancy values $x - y$.¹¹ Grand mean centering will not change the conclusions drawn from a respective cubic RSA, but it can avoid estimation problems resulting from linear dependencies of the predictor variables in the third-order

¹⁰ In case that the predictor variables were not assessed on the same scale this can, in some cases, be solved by transforming the variables in a suitable way after data assessment (e.g., within-variable standardization, computation of rank-order positions, or POMS transformations, see Little, 2013). However, such transformations change the meaning of the two predictor variables and lead to different individuals having congruent values of x and y . The choice of the transformation method can thereby affect the conclusions drawn from an investigation of congruence effects. Therefore, one should think very carefully about this analytical decision. Transforming the predictor variables with a selected transformation method is only valid if one can justify that for each numerical value that the transformed predictor variables can take, the individuals who have this value on both predictor variables are “congruent” in the light of the respective research objective and the corresponding theoretical accounts.

¹¹ Instead of using the grand mean/grand SD for centering/scaling, any other meaningful number can be used (e.g., the midpoint of the predictors' response scale for centering), as long as the same value is used for both variables. In some specific applications, it can make sense to center and/or scale the predictors variable-wise instead of using the grand mean and grand SD, thereby intentionally changing the meaning of congruent predictor combinations in the data (see also Footnote 10). However, we believe that this should be the exception rather than the norm, and application of within-variable scaling requires justification why congruent values of the transformed scales match the concept of congruence implied by one's theory.

regression models. However, some specific interpretations might be a bit more difficult to describe when the scales were centered (e.g., the x and y axes in RSA plots will show values roughly around zero instead of the original scales). In sum, we thus recommend to apply (grand mean) centering if (a) some of the required models do not converge when the raw scales are used or if (b) one does not care about or even prefer to base interpretation on centered instead of raw scales.

In a similar vein, we scaled x and y through dividing both variables by the *grand* standard deviation of x and y , and we scaled z by division through its standard deviation. Again, it is important that the predictor variables x and y are divided by the same value (in this case, their grand SD). This ensures that the theoretical meaning of (in)congruent predictor combinations stays the same. Scaling the three variables can lead to better convergence properties of the cubic models (e.g., when the predictor scales versus the outcome scale have very different ranges, as in the present example). Conclusions of cubic RSA based on (grand SD) scaled versus unscaled variables will be the same. Only the size of the estimated coefficients (but not their p-values) will be different, as they will reflect either standardized or unstandardized regression coefficients. In sum, we recommend to scale the variables if (a) some of the required models do not converge when the raw scales are used or if (b) one prefers to report standardized instead of raw coefficients.

Furthermore, one should inspect the data before beginning with the RSA analysis to get an impression of whether there are individual differences in the participants' levels of congruence (Edwards, 2002). An auxiliary strategy that can support this step is to categorize a predictor combination as "incongruent with $x \gg y$ " ($x \gg y$ is short for " x much greater than y " in mathematical notation) when x exceeds y by at least half a grand standard deviation of x and y , as "incongruent with $x \ll y$ " (" x much less than y ") for the other way around, as "roughly congruent" when x and y are within half a grand SD from one another, and to count how many data points fall within each of the three categories. This strategy should be considered only a tool that supports eyeballing the data, but not an objective test. For example, the "half a grand SD" rule for categorizing the data is an arbitrary choice (made by Fleenor et al., 1996) and no universal rule exists for the percentages of data points that should fall within each category. In the example data, 33% of the predictor combinations in the data were categorized as incongruent with $x \gg y$, 31% were categorized as incongruent with $x \ll y$, and 36% were categorized as roughly congruent. That is, the data were nearly equally

distributed across the three categories, which we subjectively interpret as a sign that the predictor distributions were sufficient for investigating congruence effects. Had we instead found that 100% of the predictor combinations fell within the " $x \gg y$ " category, the data would have been clearly insufficient to investigate congruence effects. Oftentimes, one is faced with data somewhere between these extremes (e.g., 10% of the data in one category). Our advice for such situations is to transparently report these numbers, evaluate them in the light of the whole study situation (especially the sample size), and consider integrating them into the conclusions drawn from RSA (e.g., conclusions drawn for people in a sparsely populated category should be considered as more tentative than those for densely populated categories).

Table 6
Descriptive Statistics for Simulated Example Data

Variable	Mean	SD	Correlations	
			1.	2.
1. Self-rating x	114.95	5.37		
2. Friend-rating y	115.01	4.95	.40	
3. Interaction harmony z	3.57	1.04	-.07	.07

Analytical Strategy

In the OSF project that accompanies this article (osf.io/drv3a), we provide the simulated data and the R script that goes through the whole process of investigating the hypothesis (see the file “illustration.R” at osf.io/drv3a). We also provide a detailed step-by-step description of the R code (see OSF Material H at osf.io/drv3a). All analytical steps were conducted with the R package *RSA* (Schönbrodt & Humberg, 2020). To accompany this article, we implemented the four cubic models (asymmetric and level-dependent, strict and rising ridge versions, respectively) into the package, along with additional functions for investigating whether a hypothesis is supported in the area of realistic predictor combinations. Plotting was implemented with the `plot()` function from the *RSA* package.

In the first step, the strict asymmetric congruence model (labeled the CA [“cubic asymmetry”] model in the *RSA* package) and the full third-order model (`cubic`) were estimated with the function `RSA()`.¹² This function estimates the two models in *lavaan*. Our

¹² To investigate the rising ridge instead of the strict variant of the asymmetric congruence hypothesis, the model `RRCA` (“rising ridge cubic asymmetry”) must be specified in all functions of the *RSA* package (see the

main reason for choosing *lavaan* was that the *lavaan* model syntax is very convenient when it comes to implementing model constraints; it would, however, also be possible to estimate the models with the `lm()` function. When estimating complex regression models, the data should be screened for influential cases to rule out the possibility that single data points are strongly influencing the results. By default, the `RSA()` function identifies and removes influential cases according to a strategy that Edwards (2002) recommended for RSA.¹³ This strategy can be modified by using the `out.rm` option. To compare the estimated models in terms of their model fit, we applied the `compare2()` function, which provides results from a χ^2 likelihood ratio test. We also report the two models' R^2 values as complementary information to the relative model test.

In the second analytical step, the estimated model coefficients were extracted from the model estimates by using the `getPar()` function. To account for inflated standard errors due to the model selection procedure and to account for the simultaneous test of two coefficients ($\hat{c}_1$ and $\hat{c}_2$), we chose a more conservative α level of $\alpha = .01$ for the tests of the coefficients (see the Discussion section for considerations of the issue of post-selection inference).

Finally, we inspected whether the effect was present for the whole range of realistic predictor combinations (see the last column of Table 4) by applying the function `caRange()` to the `RSA()` output object. When testing each of the data points behind E_2 , we again used an α level of $\alpha = 0.01$. We also applied a Bonferroni correction (implemented as the option `alphacorrection='Bonferroni'` in the `caRange()` function). We thereby controlled the Type I error rate of falsely rejecting the universal null hypothesis, which states that no data point contradicts the hypothesis' predictions (see also Armstrong, 2014). 16 data points' outcome predictions needed to be tested, so that the de facto α level was $\alpha = .000625$. One should be aware that these corrections rendered the investigation more liberal than if we had used $\alpha = .05$ or refrained from applying a correction because the data are in line with our hypothesis when we do *not* reject the universal null hypothesis. The α level and the correction method can be adapted with the options `alpha` and `alphacorrection` of the `caRange()` function.

illustrative R code at osf.io/drv3a). In addition, one would need to inspect the linear level effect $\hat{u}_1$, which is provided in the `getPar()` output of the model.

¹³ That is, a data point is categorized as an influential case if the following three conditions hold for $dfFit$, Cook's distance D , and the *hat* value (Edwards, 2002): (a) $|dfFit| > 3\sqrt{(k/(n-k))}$, (b) $D > 50\text{th percentile of } F(k, n-k)$, and (c) $hat > 3k/n$, where k is the number of estimated parameters in the respective model, and n denotes the sample size.

Results

Results for the simulated example data are presented in Table 7.

Table 7

Cubic RSA Results for Simulated Example Data

Model	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$	$\hat{b}_4$	$\hat{b}_5$	$\hat{b}_6$	$\hat{b}_7$	$\hat{b}_8$	$\hat{b}_9$	$\Delta\chi^2$	R^2
CA model	3.51	0	0	-0.06	0.13	-0.06	-0.02	0.06	-0.06	0.02	7.06	0.035
p-values	<.001			<.001	<.001	<.001	.001	.001	.001	.001	.422	
full model	3.53	-0.13	0.06	-0.06	0.13	-0.11	0	0.06	-0.02	0		0.043
p-values	<.001	.061	.434	.032	.003	<.001	.815	.035	.579	.842		

Note. CA model = strict asymmetric congruence model. full model = full third-order polynomial model.

The coefficient estimates $\hat{b}_0$ to $\hat{b}_9$ refer to the full third-order polynomial model

$$z = b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + b_6x^3 + b_7x^2y + b_8xy^2 + b_9y^3.$$

$\Delta\chi^2$ = difference between the χ^2 values of the two models.

Step 1: Are the model constraints justified? The strict asymmetric congruence model did not fit the data significantly worse than the full model ($\chi^2(7) = 7.06, p = .422$; see Table 7). That is, the data supported the constraints imposed by the strict asymmetric congruence model, which allowed us to proceed to the second analytical step.

Step 2: Does the estimated model reflect the suggested effect? Figure 9 and Table 7 show the graph and the coefficient estimates, respectively, of the strict asymmetric congruence model.

In Figure 9, a bagplot (a bivariate extension of a boxplot; see Rousseeuw et al., 1999) and the model-predicted outcome values for the data points are shown on the regression surface. They visually represent the regions of the surface that make predictions for realistic predictor combinations. When interpreting the results of a quadratic or cubic RSA, one should be sure to interpret the model predictions for the whole range of predictor combinations that are realistic (hence the tools that we suggest should be used for this aim in Step 2 of investigating a complex congruence hypothesis), but one should also be sure not to interpret regions of the surface that are extrapolations beyond the actual observations (see also Schönbrodt, 2016).

Figure 9. Graph of the estimated strict asymmetric congruence model for the simulated example data.

The coefficient $\hat{c}_1 = \hat{b}_3$ that reflects the congruence effect (see Table 4) was significantly negative ($\hat{b}_3 = -0.06, p < .001$). That is, when considering a small area around the LOC, the regression surface falls to both sides of the LOC (see Figure 9). Note, however, that we cannot yet draw a conclusion about whether congruence is *in general* associated with greater harmony as long as we have not implemented the two remaining steps.

The coefficient $\hat{c}_2 = \hat{b}_6$, which reflects the asymmetry effect, was significantly negative ($\hat{b}_6 = -0.02, p = .001$). That is, beginning at the LOC, the surface falls faster in the direction of incongruence where $x > y$ than in the direction of $x < y$ (see Figure 9).

Finally, an inspection of the range of realistic predictor combinations showed that 2% of the predictor combinations in the data were positioned behind the second extremum line E_2 (i.e., left of the pink line in Figure 9) and that 0% of the outcome predictions at these points fall outside the confidence interval $(-\infty, 3.78]$ of $\hat{z}_r$ (i.e., of the outcome prediction for points on E_2).

Conclusion. Overall, the data support the strict asymmetric congruence hypothesis. The simulated example persons reported more harmonic interactions with their friends the more congruent their self-estimated intelligence was to their friends' estimates, and in addition, self-views that exceeded friend-views were associated with less harmony than self-views that trailed behind friend-views.

Case Study 2: Strict Level-Dependent Congruence Hypothesis

We now describe a different research question to demonstrate the investigation of the level-dependent congruence hypothesis. Again, we use simulated data for demonstration purposes (see osf.io/drv3a for the data and R code).

Research Objective and Hypothesis

As another potential research objective concerning the adaptiveness of self-other agreement, we consider the question of whether people feel emotionally distant from their romantic partner when the partner has misconceptions about the target's personal values. Here, we focus on the "personal security" value as defined in the Theory of Basic Human Values (Schwartz et al., 2012), which refers to the importance that people place on achieving harmony and relationship stability. Potentially, romantic partners who have a misconception about the importance that the target person places on personal security show misfitting (as perceived by the target) amounts of security-related support, which might lead to feelings of emotional distance in the target person. For example, imagine that Kim's partner Tom underestimates the importance that Kim places on relationship safety; he might show behavior that leads Kim to doubt his commitment to the relationship (e.g., give her little somethings too seldom; not pay attention to reciprocating favors). As another example, imagine that Sam *overestimates* the value that his girlfriend Susan places on personal security; he might show behavior that annoys Susan or makes her feel constricted (e.g., overwhelm her with assurances of his love). In both scenarios, the target persons (Kim and Susan) might feel emotionally distant from their respective spouses. By contrast, partners who accurately estimate the target's security importance might show the right amount (as evaluated by the target) of security-supporting behavior and thus give the targets no reason to feel emotionally distant.

Finally, we expect this congruence effect to be level-dependent: The more importance a person places on personal security, the more attention he or she will pay to the spouse's degree of security-supportive behavior, and the more impact should the (in)accuracy of the spouse's estimate have. Consequently, we expect the congruence effect to be stronger for higher mean levels of self- and partner-reported importance levels.

We thus posit a strict level-dependent congruence hypothesis: Congruence between targets' self-reported security importance (x) and their partners' estimates of the targets' security importance (y) are expected to be associated with the lowest feelings of emotional distance in the targets (z), and this effect is expected to be weaker or nonexistent for lower

levels of self- and partner-reported value importance than for higher levels (fourth row of Table 5; Figure 6d).

Data, Data Preparation, and Pre-Analyses

Self-reported (x) and partner-reported (y) importance of personal security could be assessed in a commensurable way by using a self- and a peer version of the Schwartz Value Survey (SVS; Schwartz et al., 2012). For example, the targets and their respective romantic partners could indicate the importance that the target places on aspects of personal security with a prototypical item from the SVS (e.g., “sense of belonging/feeling others care about me”) that is rated on a scale ranging from -1 (*opposed to my values/to my spouse’s values*) to 7 (*supreme importance*). Finally, the target would indicate his or her feelings of emotional distance from his or her partner, for example, by using a respective item that is rated on a Likert scale ranging from 1 (*very low emotional distance*) to 7 (*very high emotional distance*). As discussed in Case Study 1, we should assess a sample that is large enough to estimate the third-order models and detect complex congruence effects. Table 8 shows descriptive statistics for the respectively simulated data.

Table 8
Descriptive Statistics for Simulated Example Data

Variable	Mean	SD	Correlations	
			1.	2.
1. Self-reported value x	4.02	0.98		
2. Partner-reported value y	3.98	0.94	.36	
3. Emotional distance z	4.13	1.06	.08	.11

Data preparation and pre-analyses were conducted similar as for Case Study 1: We centered x and y on their grand mean, and we inspected the individual differences in the participants’ levels of congruence. 30% of the predictor combinations in the data were categorized as incongruent with $x \gg y$, 34% were categorized as incongruent with $x \ll y$, and 37% were categorized as roughly congruent.

Analytical Strategy

The analytical procedure for investigating the hypothesis by using the *RSA* package was very similar to the procedure in Case Study 1 (see the file “illustration.R” and OSF Material

H at osf.io/drv3a for reproducible R code and a step-by-step description of the code). In the first and second analytical steps, we estimated the strict level-dependent congruence model (labeled CL [“cubic level”] model in the *RSA* package) and inspected its coefficients by following the same procedure as described for the strict asymmetric congruence model in Case Study 1.¹⁴ In the second step, we computed the regions of significance by using the function `clRange()`. Following the same rationale as described for Case Study 1, we specified an α level of $\alpha = 0.01$ for the tests of the coefficients and for computing the regions of significance.

Results

Results for the simulated example data are presented in Table 9.

Table 9

Cubic RSA Results for Simulated Example Data

Model	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$	$\hat{b}_4$	$\hat{b}_5$	$\hat{b}_6$	$\hat{b}_7$	$\hat{b}_8$	$\hat{b}_9$	$\Delta\chi^2$	R^2
CL model	4.02	0	0	0.09	-0.19	0.09	0.06	-0.06	-0.06	0.06	3.02	0.043
p-values	<.001			.005	.005	.005	<.001	<.001	<.001	<.001	.883	
full model	4.01	-0.07	0.14	0.1	-0.16	0.1	0.07	-0.08	-0.04	0.03		0.049
p-values	<.001	.478	.182	.025	.024	.037	.005	.19	.571	.428		

Note. CL model = strict level-dependent congruence model. full model = full third-order polynomial model.

The coefficient estimates $\hat{b}_0$ to $\hat{b}_9$ refer to the full third-order polynomial model

$$z = b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + b_6x^3 + b_7x^2y + b_8xy^2 + b_9y^3.$$

$\Delta\chi^2$ = difference in the χ^2 values between the two models.

Step 1: Are the model constraints justified? The constraints of the strict level-dependent congruence model were supported because this model did not fit the data significantly worse than the full third-order model ($\chi^2(7) = 3.02, p = .883$; see Table 9). That is, it is justifiable to inspect the estimated strict level-dependent congruence model in the second step. The graph of this model is shown in Figure 10.

¹⁴ The rising ridge version of the level-dependent congruence hypothesis can be investigated by specifying `RRCL` (“rising ridge cubic level”) in the applied functions and by extracting the linear level effect $\hat{u}_1$ with the `getPar()` function (see the illustrative R code).

Figure 10. Graph of the estimated strict level-dependent congruence model for the simulated example data. The data points are jittered to alleviate overplotting.

Step 2: Does the estimated model reflect the suggested effect? When inspecting the coefficients of the strict level-dependent congruence model, we find that $\hat{c}_3 = \hat{b}_6$ is significantly positive ($\hat{b}_6 = 0.06, p < .001$), which is in line with the expected level-dependency effect (see Table 9; see the fourth row of Table 5 for the conditions). That is, the curvature of the surface above the lines g_k increases (becomes more positive) with an increasing predictor level k . This can also be seen in Figure 10: The surface seems nearly flat in the front corner of the coordinate cube, whereas it has a strong U-shape (i.e., positive curvature) in the back area of the cube.

A consideration of the regions of significance shows that this visual impression is also backed up by the data. As can be seen in Table 10, the curvature of the surface is significantly negative above all g_k with $k \in (-\infty, -2.33]$ (i.e., in front of the foremost pink line in Figure 10), and this region should not be interpreted because 0% of the data points' mean predictor levels k lie in this region (see the last column of Table 10). The surface curvature is nonsignificant for $k \in [-2.33, -0.08]$ (between the two pink lines), and 35% of the data points lie in this area; the emotional distance reported by people who place little importance on personal security does not depend on their partners' accuracy in estimating this importance. Finally, the surface has a significantly positive curvature above all g_k with $k \in [-0.08, \infty)$

(behind the backmost pink line), and 65% of the data points lie in this region; for people who place rather high importance on personal security, the accuracy of partner-reports is associated with lower feelings of emotional distance. The findings are thus in line with the hypothesis (see Table 5).

Table 10

Output of the `clRange()` Function for Simulated Example Data

region	lower_bound	upper_bound	data_points	percent_data
neg. sign.	-Inf	-2.33	0	0.00
nonsign.	-2.33	-0.08	140	35.00
pos. sign.	-0.08	Inf	260	65.00

Note. neg. sign. = region of negative significance. nonsign. = region of nonsignificance. pos. sign. = region of positive significance. lower_bound / upper_bound = k value of the lower/upper boundary of the respective interval. data_points = number of data points in the respective interval. percent_data = percentage of data points in the respective interval. Inf = infinity.

Conclusion. Overall, the example data supported the suggested strict level-dependent congruence hypothesis. For low importance levels, the congruence between the importance of self- and partner-reported personal security was not associated with perceived emotional distance, but for high levels, people perceived their relationship as less emotionally distant the more accurately their partner estimated how important personal security was for them.

Case Study 3: Rising ridge Asymmetric Congruence Hypothesis

We now demonstrate cubic RSA with real data. Because we screened the data set to find variables that can serve as an informative illustration, our analyses must not be understood as a confirmatory investigation of the hypothesis. They rather show that cubic RSA can be applied in a more exploratory mode to motivate working hypotheses, which would then need to be cross-validated in independent data. In a similar vein, we note that the sample ($N = 172$) might in fact be too small to have a high enough power to detect complex congruence effects.

Research Objective and Hypothesis

A prominent question in organization psychology refers to the consequences of person-job fit. For example, are people more satisfied with an activity domain of their job when they can spend around the same amount of time on the activity as they would like to

spend on it (e.g., Locke, 1976)? For job activities that refer to motivating or rewarding other people, Edwards (1994) reported results of a quadratic RSA that pointed towards a rising ridge simple congruence effect: People whose actual and preferred amount of "motivating/rewarding" activities were more congruent were more satisfied with this aspect of their job, and in addition, higher mean levels of actual and preferred amount were associated with higher satisfaction.

However, a common theoretical notion in research on person-job fit is that different mental processes should occur for people whose needs fall behind their jobs' supplies versus those whose needs exceed supplies (Edwards & Shipp, 2007). Following this idea, we would not expect the detrimental impact of incongruence to be equally strong at both "directions" of congruence, but to be asymmetric. Whereas deviations of a job's actual attributes (e.g., how much time one typically spends with motivating or rewarding others) from one's preference (e.g., how much time one would prefer to spend with motivating or rewarding others) might generally lead to lower satisfaction, this effect might, for example, be more pronounced if demands exceed preferences (e.g., if someone has to motivate or reward others more than he or she prefers) than if demands fall behind preferences (e.g., if someone has to motivate or reward others less than preferred). We can explore such nuanced congruence effects in the data from Edwards (1994) with cubic RSA.

Data, Data Preparation, and Pre-Analyses

The data that we used is publicly available on J. Edwards' website (Edwards, n.d.-a). A sample of 172 MBA students completed a questionnaire in which they indicated, for their most recent job, the amount of time that they spent on behaviors referring to motivating/rewarding others ("actual amount" x), as well as the amount of time they would in fact like to spend on motivating/rewarding activities ("preferred amount" y). In addition, participants were asked how satisfied they were with this aspect of the job (z). Table 11 shows descriptive statistics for the three variables.

Table 11

Descriptive Statistics for the "Motivating/Rewarding" Data from Edwards (1994)

Variable	Mean	SD	Correlations	
			1.	2.
1. Actual amount x	14.87	9.88		
2. Preferred amount y	22.22	8.38	.56	
3. Satisfaction z	20.35	8.36	.54	.17

The actual amount (x) and preferred amount (y) variables were centered on their grand mean. There were only few participants (4%) who reported an actual amount x that exceeded their preferred amount y by at least half a grand standard deviation of x and y , so results for the $x > y$ part of the surface will need to be viewed with reservations. If the aim was not of an exploratory kind but rather a confirmatory investigation of the hypothesis, we would recommend careful interpretation and cross-validation of the results in data that involves more people with $x > y$ (e.g., a non-student sample). 58% of the participants were categorized as incongruent with $x \ll y$ and 38% were categorized as roughly congruent.

Analytical Strategy

The analytical procedure was analogous to the procedure in Case Study 1, only that we specified the model RRCA (“rising ridge cubic asymmetry”) to allow for the linear level effect (see the file “illustration_Edwards1994_data.R” at osf.io/drv3a for reproducible R code; see also OSF Material D).

Results

Results are presented in Table 12. As can be seen in this table, the model constraints of the rising ridge asymmetric congruence model were supported by the data, which allows us to inspect the rising ridge asymmetric congruence model (see Figure 11 for the graph).

Table 12

Cubic RSA Results for the "Motivating/Rewarding" Data from Edwards (1994)

Model	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$	$\hat{b}_4$	$\hat{b}_5$	$\hat{b}_6$	$\hat{b}_7$	$\hat{b}_8$	$\hat{b}_9$	$\hat{u}_1$	$\Delta\chi^2$	R^2
RRCA model	23.28	0.19	0.19	-0.03	0.06	-0.03	-0.0004	0.0013	-0.0013	0.0004	0.37	8.56	0.385
p-values	<.001	<.001	<.001	<.001	<.001	<.001	.065	.065	.065	.065	<.001	.2	
full model	22.94	0.11	-0.01	-0.03	0.05	-0.01	0	0.00122	-0.0011	0.0008			0.414
p-values	<.001	.422	.945	.005	.004	.404	.913	.222	.295	.478			

Note. RRCA model = broad asymmetric congruence model. full model = full third-order polynomial model.

The coefficient estimates $\hat{b}_0$ to $\hat{b}_9$ refer to the full third-order polynomial model $z = b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + b_6x^3 + b_7x^2y + b_8xy^2 + b_9y^3$, where x = actual amount of time spent on motivating/rewarding others, y = preferred amount of time spent on motivating/rewarding others, z = satisfaction with the "motivating/rewarding others" aspect of the job.

$\hat{u}_1$ = coefficient of the linear level effect in the RRCA model, computed as $\hat{u}_1 = \hat{b}_1 + \hat{b}_2$.

$\Delta\chi^2$ = difference between the χ^2 values of the two models.

Figure 11. Graph of the estimated rising ridge asymmetric congruence model for the "motivating/rewarding" data from Edwards (1994).

The coefficient $\hat{c}_1 = \hat{b}_3$ that reflects the congruence effect was significantly negative and the coefficient of the linear level effect $\hat{u}_1$ was significantly positive (see Table 12). The coefficient $\hat{c}_2 = \hat{b}_6$, which reflects the asymmetry effect, was negative ($\hat{b}_6 = -0.0004$) with a p-value of $p = .065$ (i.e., the asymmetric congruence hypothesis would be rejected at $\alpha = 0.01$ if this were a confirmatory investigation). The negative coefficient means that, beginning at the LOC, the surface falls faster in the direction of incongruence where "actual > preferred" than in the direction of "actual < preferred" (see Figure 11). Finally, there were no predictor combinations that were positioned behind the second extremum line E_2 .

Conclusion. Altogether, thus, the results indicate that congruence between actual and preferred amounts of "motivating/rewarding" job contents is associated with greater satisfaction with this attribute of the job, and that in addition, people who spend and prefer higher amounts of such activities are more satisfied; these results are basically a reproduction of the findings reported by Edwards (1994). In addition, cubic RSA allowed us to reveal that the congruence effect might be of an asymmetric nature: Participants who spent *less* time with motivating/rewarding others than they actually want seemed to be more satisfied than individuals who spent the same amount *more* time as compared to their preference.

Discussion

Congruence hypotheses are important in many research fields, and conceptual expectations regarding congruence effects are becoming increasingly more refined. Specifically, congruence effects are often assumed to be asymmetric or level-dependent. This has led to advanced hypotheses on, for example, the consequences of person-environment fit, intrapersonal consistency, self-other agreement, or dyadic similarity in families, therapies, romantic encounters, or peer networks (see also Table 1).

Empirical investigations of such precise expectations with respect to congruence effects promise to essentially increase theoretical precision and to speed up scientific progress in the respective fields (Edwards & Berry, 2010). To date, however, these complex hypotheses have not yet been explored with suitable statistical approaches. They were not statistically investigated at all or tackled with quadratic RSA, which is not appropriate for this purpose and can easily lead to misconclusions. In the present article, we explained why complex congruence hypotheses cannot be investigated with quadratic RSA, and we presented cubic models and corresponding analytical tools as a valid statistical approach.

We hope that the application of cubic RSA helps to increase knowledge about complex

congruence effects. To do so, it is necessary to take some theoretical and methodological considerations into account. Also, it is possible (and sometimes necessary) to adapt cubic RSA to fit special research situations or objectives.

Theoretical and Verbal Precision

Quadratic and cubic RSA can be applied to explore complex congruence hypotheses (as listed in Table 2). It is important to distinguish between the hypotheses that RSA can be applied to investigate and verbally similar hypotheses that would require different statistical representations. For example, congruence hypotheses that assume an (inverse) V-shaped association between $(x - y)$ and z instead of a smooth (inverted) U-shape can be investigated with absolute difference or linear spline models (Edwards, 2002; Edwards & Parry, 2018). The decision about whether to apply these models or RSA should be made on theoretical grounds, for example, depending on whether theory suggests a smooth change in direction at the point of congruence (RSA) or not (absolute-difference-based models). If the choice between the two models is inconclusive on the basis of theory, a promising idea could be to fit and compare the two models, aiming to derive an assumption about the shape of the association in an exploratory way. However, this would first require the development of a test that can be used to compare the two models (see also Edwards & Parry, 2018).

As another example, the asymmetric congruence hypothesis must be distinguished from a hypothesis that posits an asymptotic association between $(x - y)$ and z . The asymmetric congruence hypothesis posits that z is maximized (or minimized for the opposite version of the hypothesis) when $x = y$ (e.g., see Figure 4b). An asymptotic hypothesis would instead posit, for example, a monotonously positive but diminishing association between $(x - y)$ and z . To investigate such an asymptotic hypothesis, one would need to identify a suitable asymptotic function and fit the model that relates z to $(x - y)$ via this function. Finally, as was the case for the strict simple congruence model in RSA, one would need to find a way to test the implicit model constraints.

It is unarguably a nontrivial task to correctly identify and describe the statistical hypothesis (be it testable with RSA or not) that reflects the “three-dimensional” expectations implied by one’s theory. In our view, one major aid can be the use of the graphs of bivariate polynomial models (e.g., as in Figures 1, 4, 6, and 8) as guides in theory development. Such graphs provide visual guidance for deriving or discussing outcome expectations for all possible combinations of the two predictor variables (i.e., all points on the bottom of the coordinate

cube). Moreover, plotting a prototypical graph that reflects one's hypothesis can also help to communicate complex congruence hypotheses to readers (e.g., see Humberg, Dufner, et al., 2018; Weidmann et al., 2017; Wurst et al., 2017a, for examples in the case of quadratic RSA).

Model Selection

In the first step of investigating a complex congruence hypothesis, the respective hypothesis-conforming model must be tested against the full third-order polynomial model to determine whether the model constraints are justified. We referred to likelihood ratio testing as one possible strategy for meeting this aim. Another class of approaches for model selection is the application of information criteria such as the Akaike Information Criterion (AICc; Akaike, 1973; Burnham & Anderson, 2002; Sugiura, 1978) or the Bayesian Information Criterion (*BIC*; Schwarz, 1978). The choice of a model selection strategy should depend on the specific research situation and should incorporate theoretical, practical, and philosophical considerations (see also Burnham & Anderson, 2004; Wagenmakers & Farrell, 2004; Ward, 2008).

Significance Testing after Model Selection

Assume that the first step of investigating a (simple or complex) congruence hypothesis revealed that the respective hypothesis-conforming model did not fit the data significantly worse than the full third-order model. Then the coefficients of this model must be tested for significance in a second step, to determine whether the suggested effects are significant and in the expected direction. One open methodological issue refers to the question of how the model selection uncertainty from the first step can be considered when testing the coefficients in the second step, in order to avoid potential inflation of Type I error rates (Berk et al., 2010; Leeb & Pötscher, 2006). Different strategies have been suggested to account for such post-selection inference (e.g., splitting the data into a model selection sample and a coefficient testing sample, Berk et al., 2010; computing confidence intervals that consider all other models that might have been chosen by the model selection procedure, besides the one that was in fact chosen, Berk et al., 2013; or computing a confidence interval whose coverage probability is secured given the selected model, e.g., Lee et al., 2016; Rügamer & Greven, 2018). However, there seems to be no consensus yet on which method is the "most valid" one. Thus, we cannot yet make a well-informed recommendation on how to account for post-selection inference in cubic RSA. In the illustrative R Code, we decided to reduce the alpha level to .01, but this

approach should be seen as preliminary and should be adapted on the basis of future methodological work on the question of post-selection inference.

Note that an analogous challenge occurs when considering the range of the data for the strict asymmetric congruence hypothesis. Here, for each data point that lies behind the line E_2 , we recommend that researchers test whether the prediction of the model at the respective points differs significantly from the prediction at the points that lie on E_2 . Similar to tests of regression coefficients, judging the accuracy of a model's *prediction* should take the model selection step into account. Again, different approaches have been suggested for incorporating the selection step, including, for instance, bootstrapping procedures (Efron, 2014). However, it is at present unclear which method performs best in a certain application, and so we recommend that the approach used here be adapted once a methodological consensus has been reached.

Effect Sizes

Another area for future research concerns the determination and evaluation of effect sizes in the context of (quadratic or cubic) RSA. For example, in Case Study 2, the strict level-dependent congruence effect explained 4.3% of the variance of the outcome variable (see Table 9). To evaluate the magnitude of specific components of this effect, we think that one possible effect-size index is the R^2 -difference between the respective model and a restricted model in which the effect of interest is constrained to zero (e.g., strict level-dependent congruence model versus strict simple congruence model for an effect size of the level-dependency effect alone; Aguinis et al., 2005; Aiken & West, 1991; Jaccard & Turrisi, 2011). However, future methodological research should examine and compare this and other effect size measures in the context of RSA. In addition, there is a need for applied research that helps to judge the size of the effects in terms of their theoretical or practical relevance.

Power Analysis and Sample Size Planning

Given the complexity of effects that can be investigated with cubic RSA, it is clear that quite large samples are needed to apply this method. However, it is yet unclear how large exactly a sample must be to achieve sufficient power for detecting complex congruence effects. The rule of thumb that we provide in Case Study 1 should be seen as a temporary solution in the absence of a more precise method. Future methodological research should aim to equip applicants of (quadratic and cubic) RSA with tools for power estimation and sample size

planning that incorporate the specifics of the respective research situation (e.g., expected predictor correlations, reliabilities, expected effect sizes). This task will not be trivial, as quite a few complexities of RSA will need to be incorporated (e.g., the decision for appropriate effect size measures; the test "for" the null hypothesis in Step 1; the auxiliary tools to prevent misinterpretation in Step 2 of cubic RSA).

Why not Simply Interpret the Full Model?

The analytical approach that we described in this article is based on the strategy of interpreting a model that is constrained in accordance with a respective hypothesis. An alternative RSA strategy that has often been applied in the psychological literature involves basing inferences on only the estimated full second-order model, where the graph of the estimated model and several auxiliary parameters are used to guide interpretation. For example, when simple congruence effects are of interest, the relevant auxiliary parameters refer to the shape of the regression surface above the LOC and above g_0 (which has sometimes been labeled the "line of incongruence") and the positions of the principal axes of the surface (Edwards, 2002; Edwards & Parry, 1993; see also Humberg, Nestler, et al., 2018).

When the aim of the research is confirmatory in nature, as was assumed in this article, and when a simple congruence hypothesis is of interest, either of the two strategies (model comparison or full model interpretation) can be applied; they are equivalent with regard to the underlying mathematical models. However, the full model strategy requires that at least six auxiliary parameters that each describe one specific mathematical property of the three-dimensional surface must be considered and interpreted simultaneously. We believe that this strategy is therefore prone to misinterpretations (see Humberg, Nestler, et al., 2018, for clarifications of typical misconceptions about the interpretation of the full second-order model). Given that the full third-order polynomial model is even more complex than the second-order model, we recommend against its free interpretation whenever one can avoid it (e.g., when in a confirmatory research setting), although it would be theoretically possible to derive respective auxiliary parameters that could guide the interpretation. In the present article, we therefore focused on the model comparison strategy and extended it to cover the complex hypotheses.

What if the Research Objective is Exploratory in Nature?

This article was concerned with a confirmatory research situation in which the aim is to investigate a complex congruence hypothesis that is implied by one's theory. An alternative situation would be present if the research aim is instead exploratory, aiming at generating working hypotheses about how the interplay of x and y is related to z , while allowing for the possibility to observe asymmetric or level-dependent effect patterns. In this case, one would need to fit and interpret the full third-order polynomial model, for example, by using respective auxiliary parameters, or one might go a step further and fit a penalized spline model that is even more flexible in approaching the data. In either case, analyses that involve free interpretations of such highly flexible models must be preceded by extensive methodological considerations with regard to the model's correct interpretation in terms of the complex three-dimensional hypotheses.

What if the Model's Prediction of Only One Data Point Contradicts the Hypothesis?

To investigate complex congruence hypotheses (asymmetric, level-dependent), we suggested that users examine whether the data contain predictor combinations whose model prediction significantly contradicts the hypothesis. When no such data points exist (and all other conditions considered in Steps 1 and 2 are satisfied), then the data support the respective hypothesis in the range of realistic predictor combinations. But what if only a tiny fraction of data points, or even only a single point, fail to meet this criterion? One way to deal with such a situation is to transparently report this finding and interpret the strength of evidence for the hypothesis in the light of it (e.g., by stating that results provide tentative but inconclusive support for the hypothesis, calling for an attempt at replication). The observation can also be used to derive exploratory working hypotheses: Could it make sense to assume that a beneficial effect of congruence occurs only for a restricted range of predictor combinations, whereas at extreme combinations, processes that were not initially expected come into play? In either case, further studies would be needed to obtain more conclusive insights.

Future Directions

Cubic RSA can be adapted or extended to a variety of interesting research objectives. First, the cubic RSA models can be adapted to fit complex data structures if needed for a specific research context. For example, the models can be adapted to accommodate multilevel data (e.g., students nested in classes; employees nested in companies; scholars nested in

universities) in a manner that is analogous to how quadratic RSA was recently adapted to be appropriate for such data (Nestler et al., 2019). This would also allow users to investigate potential Level 2 covariates for an asymmetry or a level-dependency effect. As another example, when assessing data in dyads (e.g., couples, friends, boss-employee dyads, therapist-patient dyads) instead of single individuals as the entity of interest, users might aim to model *two* instead of one outcome value per dyad (e.g., men's and women's relationship satisfaction as a potential outcome of couple similarity; Weidmann et al., 2017). To investigate asymmetric or level-dependent congruence hypotheses in this setting, the third-order models introduced above can be combined with the Actor-Partner-Interdependence Model (APIM; Kenny et al., 2006). This adaptation would follow the same idea as recently suggested for quadratic RSA (Schönbrodt et al., 2017), and it would also allow users to explore whether the degree of congruence affects the two outcome variables (e.g., women's vs. men's satisfaction) differently.

Second, another interesting topic for future research would be to extend the cubic RSA so that moderation or mediation effects can be investigated in combination with congruence effects. For example, does the strength of a (simple or complex) congruence effect concerning self-views and reputation depend on the degree of awareness that people have about their reputation (moderation effect)? Is there a (simple or complex) congruence effect of self- versus friend-estimated intelligence on interaction harmony that is mediated by friends' impressions of the targets' under- or overselling tendencies (mediation effect)? For the standard quadratic RSA, Jeffrey R. Edwards presented rationales for how one could combine quadratic RSA with moderation (Edwards, n.d.-c) or mediation (Edwards, n.d.-b) analyses. Pursuing these approaches could also be fruitful avenues for cubic RSA.

Third, the cubic RSA can also be extended to cover hypotheses beyond congruence effects. Most applications of quadratic RSA in the psychological literature have been aimed at investigating congruence hypotheses. Quadratic RSA, however, can be used to assess a variety of other effects (e.g., see Humberg, Dufner, et al., 2018). For example, *optimal margin hypotheses* state that an outcome should be maximized (or minimized) when the predictors x and y differ by some specific amount (e.g., self-view slightly higher than reputation; see also Baumeister, 1989). Although we know of no theoretical accounts in this direction, it would be possible to come up with complex versions of this hypothesis, stating an *asymmetric optimal margin hypothesis* (e.g., outcome falls faster when difference of x and y is higher than optimal

than when it is lower than optimal) or a *level-dependent optimal margin hypothesis* (e.g., the strength of the optimal margin effect depends on the predictor level). These hypotheses could be investigated by extending the quadratic optimal margin model (“shifted squared difference model”, see Schönbrodt, 2016) in the same way as we extended the simple congruence model and by adapting the tools that we presented to explore whether the model predictions are in line with the hypothesis for the whole range of realistic predictor combinations.

Finally, cubic RSA models can be used in a framework of multiple hypothesis evaluation. Many fields of research provide a rich pool of hypotheses that posit contradictory expectations of how the interplay between two predictor variables should be related to the values in an outcome variable (e.g., congruence effect vs. optimal margin effect vs. mere linear main effects). In such a situation, the strongest inference will result not from the selective investigation of one hypothesis but from comparing the empirical evidence of *all* suggested alternative hypotheses (Chamberlin, 1890; Edwards & Berry, 2010; Platt, 1964). To this aim, the quadratic and cubic RSA models that were described in the literature and in this article can be used to identify a statistical representation for each of the hypotheses. The models’ support in empirical data can then, for example, be compared by applying an information-theoretic approach (Burnham & Anderson, 2002; Humberg, Dufner, et al., 2018).

For all these possibilities, it is important to note that, because cubic RSA is based on models that are already quite complex, the suggested extensions for complex data structures or research situations will not be trivial. We therefore call for methodological work that derives these methods, carefully evaluates potential pitfalls and necessary considerations implied by the increased complexity, and prepares these contents in the form of tutorials that are accessible to readers and at the same time ensure correct application of the methods.

Conclusion

Cubic RSA allows users to investigate asymmetric and level-dependent congruence hypotheses. In this article, we presented statistical models that reflect these hypotheses, described the analytical procedure that is needed to apply them, and introduced further tools that are necessary to reveal whether the data support a respective hypothesis for the whole range of realistic predictor combinations. All models and tools presented in this article were implemented in the R package RSA, which allows a user-friendly application of cubic RSA, which we also demonstrated in illustrative analyses. We hope that our developments will help interested researchers empirically investigate advanced hypotheses on complex congruence effects.

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