Is thirty-two three tens and two ones? The embedded structure of cardinal numbers

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Abstract

The acquisition and representation of natural numbers have been a central topic in cognitive science. However, a key question in this topic about how humans acquire the capacity to understand that numbers make ‘infinite use of finite means’ (or that numbers are generative) has been left unanswered. Here, we test the hypothesis that children’s understanding of the syntactic rules for building complex numerals—or numerical syntax—is a crucial foundation for the acquisition of number concepts. In two independent studies, we assessed children’s understanding of numerical syntax by probing their knowledge about the embedded structure of cardinal numbers using a novel task called Give-a-number Base-10 (Give-N10). In Give-N10, children were asked to give a large number of items (e.g., 32 items) from a pool that is organized in sets of ten items. Children’s knowledge about the embedded structure of numbers (e.g., knowing that thirty-two items are composed of three tens and two ones) was assessed from their ability to use those sets. Study 1 tested English-speaking 5- to 10-year-olds and revealed that children’s understanding of the embedded structure of numbers emerges relatively late in development (several months into kindergarten), beyond when they are capable of making a semantic induction over a local sequence of numbers. Moreover, performance in Give-N10 was predicted by children’s counting fluency even when school grade was controlled for, suggesting that children’s understanding of the embedded structure of numbers is founded on their earlier knowledge about the syntactic regularities in the counting sequence. In Study 2, this association was tested again in monolingual Korean kindergarteners (5-6 years), as we aimed to test the same effect in a language with a highly regular numeral system. It replicated the association between Give-N10 performance and counting fluency, and it also demonstrated that Korean-speaking children understand the embedded structure of cardinal numbers earlier in the acquisition path than English-speaking peers, suggesting that the knowledge of numerical syntax governs children’s understanding of the generative properties of numbers. Based on these observations and our theoretical analysis of the literature, we propose that the syntax for building complex numerals represents the structure of numerical thinking.

Keywords

Number acquisition; numerical syntax; counting; recursion; successor principle

1 Introduction

The mental representation of natural numbers has been studied for decades, with children's acquisition of number concepts at the center of scientific inquiry. The acquisition of number concepts—notably the meaning of number words and the generative properties of natural numbers—is not trivial indeed. It takes years for a developing child to understand the meaning of numbers and their relations (Carey, 2004), and the concept of precise numerical value may never be acquired (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). Most if not all psychological theories assert that the acquisition of number concepts culminates when children understand the *successor principle* that refers to the recursive rule according to which any natural number has a successor that is also a natural number. In this paper, we will argue that the successor principle is not the ideal representation for explaining the cognitive mechanism by which children conceptualize number. Instead, we develop alternative forms of mental representations for natural numbers based on the linguistic structure of numerals which we believe capture psychological reality.

1.1 Theories about children's acquisition of the successor principle

An earlier theory about children's acquisition of number concepts asserts that the symbolic number system, represented by number words organized in a counting sequence, share the same abstract structure with the approximate number system, a nonverbal mechanism for representing approximate numerical quantities, and that the isomorphism between the two systems drives children's acquisition of the meaning of number words (Dehaene, 2001; Gallistel & Gelman, 1992, 2005). Some of these authors further argue for an innate nonlinguistic recursive mechanism, the integer symbol generator, that generates mental symbols to represent discrete quantities which calibrates approximate values of the approximate number system to the precise values represented by number words (Leslie, Gelman, & Gallistel, 2008). According to Leslie and colleagues (2008), this mechanism includes a representation for the symbol *one* and the recursive rule of succession. At least two developmental predictions can be drawn from this integer symbol generator theory: 1) If this integer generator includes the representation of one, then children should acquire the concept of the cardinal number one without difficulties; 2) If the successor rule is embedded in the integer generator, children should be able to deduce the next cardinal value to any number, potentially infinitely. There is evidence against both predictions. First, children usually spend two or three years to acquire the cardinal number one, after which they spend another 6 to 9 months to understand the cardinal number two (Wynn, 1990, 1992). These results suggest that children do not have early access to the cardinal meaning of one. Second, recent studies show that children who understand the counting principles do not know what the next cardinal number is even for small numbers within their counting range (Spaepen, Gunderson, Gibson, Goldin-Meadow, & Levine, 2018). Additionally, cross-linguistic analyses show that some natural languages use words to represent sets of one, two and three but lack symbols to represent higher quantities (Corbett, 2000). Such findings suggest that the knowledge of the next number is available neither to all children nor all languages.

Carey (2004) states that the meaning of number words is acquired through the interaction between a perceptual/attentional mechanism for representing a small number of objects (the so-called object file system) and children's exposure to utterances of quantifiers, plural and singular expressions, and the counting sequence (Le Corre & Carey, 2007, 2008). According to this account, the successor rule is acquired by an inductive inference in which children realize that an

increment of one item in the environment or in working memory is equivalent to moving one position in the counting sequence. Children at first apply this rule gradually in a smaller set size (up to four or five items) but become capable of generalizing the rule at least within their count list, a state known as having the cardinal principle (CP) knowledge (Wynn, 1992). A study by Sarnecka and Carey (2008) indeed demonstrated that only CP knowers reliably understand that adding exactly one item means moving exactly one position forward in the counting sequence, evaluated by the so-called Unit task. More recent studies, however, have shown results inconsistent with that previous conclusion, as many CP knowers were found to fail the Unit task even within the range of their count list (Cheung, Rubenson, & Barner, 2017; Davidson, Eng, & Barner, 2012; Spaepen et al., 2018). These newer data thus question the theoretical argument that CP knowledge is derived from the acquisition of the successor rule. It should be noted, however, that the interpretation of children's performance in the Unit task requires caution because in theory this task can be solved successfully as long as a child has a local successive knowledge of cardinal numbers between one and nine. Thus, this task may not inform anything about the child's knowledge about the successor, or the next number, in any abstract sense. We return to this point later.

More recently, children's acquisition of the meaning of number words and the successor principle has been viewed from a perspective of linguistic theories, namely that the same logical operators for language acquisition are used to acquire numerical concepts. For example, Barner (2017) states that children acquire the successor rule when they discover the recursive nature of numerals in a base system, and that one of the end states of this knowledge is the understanding of infinity. Consistent with this idea, Cheung et al. (2017) found that only CP knowers who can count up to a high number (close to one hundred) without an error have an explicit knowledge of infinity. These findings support the view that understanding the linguistic structure of the counting sequence is an important step towards acquiring the concept of infinity; however, they themselves do not yet provide clear explanations for the acquisition process.

1.2 The syntactic rules of complex numerals

A potentially effective approach to studying the mental representation of generative properties of natural numbers is investigating the syntactic rules that form complex numerals. After all, natural numbers are expressed in numerals, and children acquire the meaning of natural numbers by learning number words in a base-10 number system (i.e., not by abstract rules such as $S(n) = n + 1$ in the Peano's axioms). This base system represents numbers using an arithmetic algorithm which defines the cardinal value using the operations of addition and multiplication (i.e., 206 is equivalent to $2 \times 10^2 + 0 \times 10^1 + 6 \times 10^0$) but does not require the use of the successor function.

Hurford (1975, 1987) in his seminal works first formalized the idea that recursion in number is enabled by the syntactic rules of complex numerals. He states that cardinal numbers are divided into two types: lexical primitives and complex numerals. The former category consists of unique words that must be individually memorized (e.g., the numbers from one to nine and some multipliers). The latter are the product of the combination of the lexical primitives using Merge, a fundamental linguistic operation in charge of building linguistic structures (Chomsky, 2014; Ohta, Fukui, & Sakai, 2013). In most languages, complex numerals are represented using additive and multiplicative bases (most typically in a power of ten or base-10) and they share a set of phrase structure rules, which for simplicity we refer to as *numerical syntax* in the rest of this paper. According to Hurford (2007), these rules operate on the lexical classes of Digits (e.g.,

one, two, ..., nine) and Multipliers (e.g., -ty, hundred, thousand), resulting in the lexical category of either Number or Phrase. A Phrase is either a bare Multiplier (e.g., mil for 1000 in Spanish) or a result of a Merge operation between a Number and a Multiplier as in two hundred for 200 in English. A Number is a bare Digit (e.g., Six), a Phrase (e.g., [Two Hundred]), or a Merge of a Phrase and a Number (e.g., [[Two Hundred] [Six]])¹. These categories have a formal arithmetic interpretation that reflects the additive and multiplicative operations in a base number system. The Merge of a Number and a Multiplier represents a multiplicative composition (e.g., $\text{Phrase}[\text{Number}[\text{Two}] \text{M}[\text{Hundred}]]$ for 2×10^2) and the Merge of a Phrase and a Number represents an additive composition (e.g., $\text{Number}[\text{Phrase}[\text{Two Hundred}] \text{Number}[\text{Six}]]$ for $200+6$). According to Hurford (2007), one of the most important features of this system is that the category Number is recursive. That is, a Number can be embedded in another Number (see Fig. 1A). Under this theoretical framework, the generative properties of natural numbers are enabled by the recursive rules of numerical syntax and not by the knowledge of the succeeding number, making the successor principle unnecessary.

There are variations in how developed numeral systems reflect arithmetic operations, even within the base-10 number system. Some numeral systems are considered more regular because they clearly reflect the additive and multiplicative operations. In Korean, the numbers 12 and 20 are represented as *sib-ee* (i.e., ten-two) and *ee-sib* (i.e., two-ten) in which the same digit and multiplier are merged in two different ways: Phrase-Number (i.e., that represents the additive composition $10+2$) and Number-Multiplier (i.e., that represents the multiplicative composition 2×10), respectively. In English, the cardinal numbers twelve and twenty are much less or non-transparent lexically, being essentially holistic, along with thousand, million. The influence of the transparency of cardinal numbers in their production has been shown in cross-linguistic studies (Lefevre, Clarke, & Stringer, 2002; Leybaert & Van Cutsem, 2002; Miller & Stigler, 1987) and a longitudinal study (Gould, 2017). Miller and Stigler (1987) found that Chinese children count significantly higher than American children of the same age. Also, Lefevre, Clarke, and Stringer (2002) showed that Canadian English-speaking children master the counting sequence before their French-speaking peers do and have attributed the results to greater irregularities in the French numeral system. Leybaert and van Cutsem (2002) compared an additive base-5 system used by deaf Belgian children and a base-10 system used by hearing Belgian children and found that only deaf children commit more errors when counting to the number 6, which is the first position where the additive rule applies in Belgium sign language. Gould (2017) found that Australian kindergarteners show difficulties in abstract counting beyond certain numbers, including twelve, thirteen, twenty-nine, thirty-nine, and one hundred nine. In English, these “stopping points” represent the cardinal numbers prior to the Phrase that represents addition (i.e., twelve, thir[teen], four[teen]), prior to the Phrases that represent multiplication (i.e., thir[ty] and for[ty]), or the first Phrase-Number in which the Number category is a bare Multiplier (i.e., one hundred ten). These studies suggest that syntactic regularities influence the acquisition of the rules to correctly produce the counting sequence. Nevertheless, they do not address the conditions in which syntactic rules are acquired nor the relationship between the syntactical knowledge and the conceptual representation of cardinal values.

¹ This analysis of numerical syntax is only partially articulated from the perspective of modern linguistic theories, for example as can be seen in the unique use of the notion Phrase in this analysis.

1.3 The present study

As a step towards understanding children's acquisition of the generative properties of natural numbers, we investigated children's cardinal understanding² of numerical syntax in two independent studies. We developed a novel task to assess children's understanding of the embedded structure of cardinal numbers. We then examined the age at which children acquire this knowledge and tested how this knowledge relates to other types of numerical knowledge. This novel task (which we call Give-a-Number-Base-10 or Give-N10) is much alike the well-known Give-a-Number task developed by Wynn (1990) as children are asked to give a specific number of items to the experimenter. Critically, however, the task utilizes four spatially distinct groups of ten items, and the children are asked to give a large number of items expressed in complex numerals such as sixteen, twenty-seven, and thirty-two (see Fig. 1B). Importantly, prior to testing, the experimenter models a case of retrieving 31 items by grabbing three groups of ten items one at a time and then grabbing a single item from the last group, illustrating the syntactic nature of the numeral thirty-one, [[three -ty] one]. We reasoned that if children understand the syntactic structure of cardinal numbers and its meaning, they would produce the inquired number maximally using the groups of ten items. We hypothesized that this cardinal understanding of the embedded structure of complex numerals is one of the last stages towards children's understanding of the generative properties of numbers and is nevertheless founded on children's earlier-developing abilities to process syntactic rules of complex numerals (such as rote counting). We tested this hypothesis in two different languages in two independent studies.

2 Study 1

2.1 Methods

2.1.1 Participants

Thirty-three English-speaking children participated in the study (mean age = 6 years 8 months; range = 5;1-10;2; in school grades between preschool and 4th grade). Because the study spanned over multiple months into a school year, children's grade which we used in our analysis was adjusted by the date of children's participation in the study. For example, a child tested in September was given an additional 1/12 of a grade and a child tested in February was given an additional 6/12 of a school grade. All were fluent native English speakers and had no history of developmental disability. Participants were recruited from the Western Massachusetts Area and were tested at the University of Massachusetts Amherst in the Cognitive and Developmental Neuroscience laboratory. The majority of the children belonged to middle-class families. Thirty children were identified as White or Caucasian, one child as Asian, one as Native Hawaiian, and the other did not provide information. Children received stickers and a small gift for their participation, and the family received \$10 for travel compensation. Three additional children were originally enrolled in the study but were not included in data analysis: two of the three did not complete the study and the third had a very short count list (being able to count only up to

² We use the term *cardinal* to refer to the knowledge about the number of elements in a set. This is to be distinguished from a purely verbal or syntactic knowledge by which a child can rote count but may not have an understanding of the meaning of that number; it would be like children's knowledge of a sequence in knowing the alphabet.

18). All procedures were approved by the University of Massachusetts Institutional Review Board.

2.1.2 Procedure and Task

Children played the following five interactive games/tasks with the experimenter. The *Give-a-Number* task (Wynn, 1990; 1992; Cheung et al., 2017) and the *Unit task* (Sarnecka & Carey, 2008) evaluated children's understanding of the cardinal principle and sequential cardinal knowledge, respectively. The *Interval-Counting* task and the *Number-Multiplier-Syntax* task evaluated children's knowledge about the syntactic rules that govern numerals. Finally, the *Give-a-Number-Base-10* task evaluated children's cardinal knowledge about the syntactic structure of complex numerals. The five tasks were tested in the following order in all children: Interval-Counting, Unit, Give-a-Number-Base-10, Number-Multiplier-Syntax, and Give-a-Number. The five tasks took approximately 25 minutes.

Interval-Counting. This task evaluated children's knowledge of the abstract counting sequence, similar to what was used in previous studies (Davidson et al., 2012; Gervasoni, 2003; Gould, 2017; Miller & Stigler, 1987; Siegler & Robinson, 1982; Song & Ginsburg, 1988). Unlike previous studies, however, we asked children to count within given intervals: from 15 to 24, from 37 to 51, and from 95 to 111. For example, for the second interval the experimenter asked "Can you count from 37 for me?" If the child failed to respond, the experimenter encouraged the child by saying the three consecutive numbers that come before the starting number. In the above example, the experimenter would say "Let's count together. 34, 35, 36" with a rising intonation encouraging the child to continue counting alone. When the child reached the upper bound of the interval, say 51, the experimenter stopped the child and said "Thank you." We used intervals because many children found reciting the entire count list (from 1) tedious and boring. The three intervals were chosen based on previous observations (e.g., Gould, 2017), which demonstrated that children show difficulties counting beyond certain numbers. In particular, children who do not have an understanding of the regularity of the syntactic structure of number words fail to make a transition at a decade boundary, for example, going from 29 to 30, from 39 to 40, and from 99 to 100.

In order to quantify children's knowledge of the regularities in the counting sequence, the Decades Transition (DT) score was defined. Each child's DT score was computed as the number of times that the child passed the decade boundary. The points of decade transition were 19, 39, 49, 99, 109, and each transition was marked 0 (fail) or 1 (pass) making a maximum of 5 points across all the intervals. One prevalent idea is that children learn the count list progressively (e.g., they learn the 39-to-40 transition before the 49-to-50 transition). Consistent with this view, most of our child participants who passed the higher decade passed the lower decade also, with a single exception (one child passed 99 but not 49). Thus, we reasoned that the overall competence to produce decade names can be unbiasedly quantified if each decade transition is treated as an independent item in a test.

Unit. This task was originally developed by Sarnecka and Carey (2008) and was later adopted by other authors including Davidson et al. (2012) and Cheung et al. (2017), in which they call it the successor task. For Sarnecka and Carey (2008), this task allows to infer children's acquisition of the successor function by testing how children associate the next number in the count list with the next cardinal value of a set. While the original version used by Sarnecka and Carey (2008) used numbers between 4 and 7, later versions by Cheung et al. (2017) used much higher range up

to 78. We followed these later versions and used the following eight numbers: 5, 6, 12, 17, 24, 36, 57, and 75. The materials used in this task were an opaque plastic cylinder and small plastic bears. At the beginning of the task the experimenter said to the child, "Now, we are going to play with this container by putting bears inside." On each trial, the experimenter presented the empty container and guided the child's attention to the container and said, "I am going to put [N] bears in the container," where N was equal to the number of objects tested on the trial. To ensure that the child remembered the number of items in the container, the experimenter asked, "How many bears are there in the container?" If the child answered incorrectly or failed to respond, the experimenter said, "Oops, let's try again! Remember, I put [N] bears in the container." The trial was repeated until the child responded correctly to this memory-check question. We had a protocol to skip a number if the child gave a wrong answer in the memory-check questions for three times; however, none of the children failed to repeat the correct number. After a correct answer to the memory-check question, the experimenter said, "Right! Now watch." Simultaneously, the experimenter added one bear to the container and elicited a choice between $N + 1$ and $N + 2$ by saying, "Are there [N+1] bears or [N+2] bears in the container?" The order of the numbers (i.e., $N+1$ and $N+2$) in the questions was counterbalanced within and between children. This task consisted of eight two-alternative forced-choice trials, and the accuracy that corresponds to the number of correct answers divided by the total number of trials was computed for each child's performance measure.

Give-a-Number-Base-10 (Give-N10). This task evaluated children's cardinal knowledge of the syntactic structure of complex numerals, especially the comprehension of the additive and multiplicative operations in a base-10 number system. Simply put, it tested whether children understand that, say, a cardinal number "twenty-seven" is composed of two tens and a seven (Fig. 1A). The materials for this task were 40 blue squares of 1 inch per side, and a template of polished clear vinyl with five rectangular spots. Four of these spots were used for the researcher to locate sets of squares, and the fifth was used for the child to give the answer (Fig. 1B). At the beginning of the task, the experimenter said to the child, "Let's play a new game with the squares now. Every time I'm going to ask you to give me some squares. All right, let's play!" The researcher showed to the child 40 individual squares organized in 4 sets of 10 squares and said, "Look, I have 10 squares here." Then the researcher counted out loud one set of ten squares with the child. After counting, the researcher pointed each one of the remaining three sets and said, "There are also 10 squares here, 10 squares here, and 10 squares here." Then, the experimenter modeled a trial of acquiring 31 squares by using sets of 10 squares representing the base-10 system. That is, the experimenter said "Now, I am going to take 31 squares. Look, ten, twenty, thirty, thirty-one." While saying this, the researcher first took three sets of ten squares one set by one set and then grabbed one individual square. The experimenter modeled this activity of retrieving squares using the base-10 knowledge in order to facilitate the use of that more sophisticated strategy in children, inspired by a previous study which showed that children easily adopt a modeled strategy when they have the conceptual knowledge for using it (Miura, Okamoto, Kim, Steere, & Fayol, 1993). After this modeling, the task started when the experimenter asked, "Can you please give me N squares?" Each child was asked to give the following six numbers in that order: $N = 3, 7, 10, 16, 27$ and 32 .

The performance and success in this task was analyzed based on the pattern of children's response to the numbers 16, 27, and 32. Children earned 0, 0.5, or 1 point for each of the three trials. The following two criteria were applied for a child to receive the score of 1. First, the child

must retrieve the correct number of squares. Second, the child must use the strategy of retrieving the sets of 10 squares and maximize the use of that strategy. That is, to give 16 squares, the child must retrieve one set of ten squares; for 27 squares, the child must retrieve two sets of ten squares; and for 32 tiles, the child must retrieve three sets of ten squares. If a child used at least one set of 10 squares but additionally retrieved more than 10 squares without using sets of 10, the child received the score of 0.5. Cases other than these patterns were given the score of 0.

Figure 1. A graphical illustration of numerical syntax and the Give-N10 task. A. Syntax tree for the numeral twenty-seven according to the phrase structure rule developed by Hurford (1975). Note that Number2 and Number3 are recursively embedded in Number1. B. In the Give-a-number base-10 task (Give-N10), the participant is given four sets of 10 squares and is asked to give, for example, 16, 27, and 32 squares. Importantly, the experimenter models a trial of acquiring a large number of squares maximally using the base-10 knowledge.

Number-Multiplier Syntax. This task evaluated children's knowledge about the syntactical structure of verbal numbers. An open abacus with three columns were used in this task. The three columns each represented the ones, tens, and hundreds unit (i.e., multipliers). Round beads were used to represent the number in each of the units. Children were tested on their knowledge about how the number of beads in each of the three units all together makes up a whole number. No beads were present in the abacus at the beginning of each trial, and only the experimenter added or removed the beads from it. There were a total of eight trials, four of which tested the number in the tens and the other four tested the number in the hundreds.

At the beginning of the task, the researcher said while pointing to an open abacus, "We can use this toy to make numbers. I am going to show you how it works. If I put one bead here, we have the number one. If I put another bead here, we have the number two." The researcher put the beads in the rightmost column (ones unit) of the abacus and asked, "Can you please make the number three for me? After the child gave the correct answer, the researcher retrieved the three beads to empty the abacus. To explain the value of the middle column (tens unit), the researcher said, "If I put one bead here, we have the number ten. If I put another bead here, we have the number twenty." While saying so, the researcher put two beads in the middle column and said, "Can you please make the number thirty for me?" Then, the test began with the researcher putting the equal number the beads in the ones and tens columns and saying, "Now, I have the number {[N-ty] [N]}." To ensure that the child remembered the number, the experimenter asked, "What number do I have?" The trial was repeated until the child responded correctly to the memory-check question. Then, the experimenter put one bead in the position of ones or tens

columns and asked, “Is the new number {[N-ty] [N+1]} or {[N+1-ty] [N]}?” The numbers represented in the abacus were 23, 43, 65, and 45, and numbers in the question were 23/32, 34/43, 56/65, and 45/54, respectively.

After four trials in tens were tested, another four trials in hundreds were given to the children. The experimenter while putting two beads in the leftmost column said, “If I put one over here, we have the number one hundred. If I put another bead, it is two hundred.” Then, the experimenter asked “Can you please make the number three hundred for me?” After the child gave the correct answer, the researcher retrieved the three beads to empty the abacus. For the first two of the four trials, the researcher put one bead in the leftmost column (hundreds unit) and the equal number the beads in the tens and ones columns and said, “Now, I have the number {[one-hundred] [N-ty] [N]}.” To ensure that the child remembered the number, the experimenter asked, “What number do I have?” The trial was repeated until the child responded correctly to the memory-check question. Then, the experimenter put one bead in the position of ones or tens and asked, “Is the new number {[one-hundred] [N-ty] [N+1]} or {[one-hundred] [N+1-ty] [N]}?” For the last two trials, the researcher put the same number the beads in all three columns and said, “Now, I have the number {[N-hundred] [N-ty] [N]}.” To ensure that the child remembered the number, the experimenter asked, “What number do I have?” Then, the experimenter grabbed two beads and put one bead in the ones column and the other either the tens or the hundreds column and asked, “Is the new number {[N+1-hundred] [N-ty] [N+1]} or {[N-hundred] [N+1-ty] [N+1]}?” The numbers represented in the abacus were 154, 167, 323 and 566, and the numbers in the questions were 145/154, 167/176, 233/323 and 665/566, respectively. The order of the alternatives in the questions was counterbalanced within and between children.

This task consisted of eight two-alternative forced-choice trials, and the accuracy that corresponds to the number of correct answers divided by the total number of trials was computed for each child’s performance measure.

Give-a-Number (Give-N). This task evaluated children’s understanding of the counting principles, which was first introduced in Wynn (1990). Here, we used a variant of this task used by Cheung et al. (2017) to test whether or not a child was a cardinal-principle (CP) knower. We used this simpler version because this task only served the purpose of ensuring that our participants were CP-knowers. The material used in this task was a set of 12 small plastic bears of the same kind. On each trial, the researcher asked the child, “Can you give me N?” After the child’s response, the experimenter asked, “Is that N? Can you count and make sure?” and motivated the child to count the bears. If the child realized an error, the experimenter allowed the child to change his or her response. Children were tested on four trials and asked to give the following number in that order: 7, 8, 7, and 8. Only those who successfully retrieved correct number of items in all four trials were considered as CP-knowers. Note that this task made an explicit demand on the children to count items one by one. Thus, if this task were given prior to the give-N10 task, it could have influenced children’s behavior when the strategy of counting one by one is not optimal in the give-N10 task. Therefore, this task was administered last.

The data are available in a public repository, doi:10.17605/OSF.IO/6GDC8.

2.2 Results

2.2.1 Give-a-Number task

All the children correctly gave 7 and 8 objects in the Give-N task. Thus, according to the criteria used in previous studies (Cheung et al., 2017; Davidson et al., 2012), all were considered cardinal principle (CP) knowers.

2.2.2 Unit task

The Unit task measured children's ability to infer the cardinality of the next number within local counting sequences (Cheung et al., 2017; Davidson et al., 2012; Sarnecka & Carey, 2008). The majority of the children performed successfully in this task. Specifically, 30 out of 33 children got at least seven out of eight total trials correct. Considering the binomial distribution $B(8, .5)$ with $\alpha = .05$, these 30 children were considered to have passed the Unit task. Only three children (a 5 year-old preschooler, a 5 year-old kindergartener, and a 6 year-old kindergartener) fell below this criterion. Percentage correct in the Unit task was strongly associated with school grade ($\beta = 0.04$, $t(31) = 3.061$, $p = .004$).

2.2.3 Give-a-Number-Base-10 (Give-N10) task

This task evaluated children's cardinal knowledge of the syntactic structure of complex numerals. As shown in Figure 2, approximately half (18 out of 33) of the children received a perfect score in this task, meaning that those children when asked to give 16, 27, and 32 items gave the correct numbers while maximizing the strategy of retrieving the sets of 10 items (see Methods for details). 9 children failed to use such a strategy completely. The rest (6) of the children laid in between the two end points.

Figure 2. Success in the Give-N10 task as a function of school grade, with the best-fitting sigmoid function. School grade is adjusted by taking the study date into account—in other words by considering the number of days into each child's current grade.

Note that surprisingly a fourth grader in our sample did not receive the perfect score under our scoring criteria (note the right most data point in Figure 2). That child did not receive any credit for the 16-item question according to our scoring criteria (but full credits for all other numbers) because instead of retrieving a set of ten items and then retrieving six individual items, she retrieved two groups of three items and then counted ten more items. In fact, that pattern of

behavior demonstrates a unique composition of the word *six[teen]* as well as an understanding of the decompositionality of numbers (in this case six). We speculate that, while this child did not follow the rather strict scoring criteria we set, she has sophisticated knowledge in numerical syntax. Based on such rationality, we treated this child's performance as a perfect score in subsequent statistical analyses, but it should be noted that the inferential results were identical regardless of whether the child is given a full or partial credit on this task.

In order to characterize the developmental trajectory of the performance in the Give-N10 task, we fitted the following sigmoid function assuming that the child transitions from having no knowledge of the syntactic structure of cardinal numbers to having full knowledge of it (minimum and maximum values set to 0 and 1, respectively):

$$f(x) = (1 / (1 + \exp(-b \cdot (x - a))))$$

The result indicated the estimated midpoint (*a*) of this transition at around 8 months into kindergarten with the estimated slope (*b*) of 3.82. The midpoint in terms of age in our sample was 6 years and 2 months.

Underlying our central hypothesis that children's cardinal understanding of numerical syntax is a critical condition for the acquisition of number concepts lies our implicit assumption that the performance in the Unit task is not a sufficient measure for assessing children's knowledge about the successor principle, unlike some authors have previously argued. This is because the Unit task after all requires minimal knowledge about the syntactic structure of complex numerals. We thus evaluated the relationship between children's performance in the Unit and Give-N10 tasks. If our reasoning that passing the Unit task is not sufficient for understanding the syntactic structure of cardinal numbers, then children who pass the Give-N10 task should pass the Unit task, but not vice versa. This pattern was indeed evident. All of the 20 children who passed the Give-N10 task also passed the Unit task. Of 13 children who failed the Give-N10 task, 10 passed and 3 failed the Unit task. The mid-P McNemar test for the two tasks showed that the marginal proportions were significantly different from each other ($p < 0.001$). These results are consistent with the idea that passing the Unit task is only a necessary but not a sufficient condition for understanding the syntactic structure of cardinal numbers.

There have been previous approaches to evaluate children's understanding of the syntactic regularities in number words (such as rote counting). However, Give-N10 goes beyond these previous approaches, as this task assesses the *cardinal* understanding of complex numerals by asking children to produce a set of its cardinal value. Nevertheless, given the novelty of the Give-N10 task, it is yet unclear whether the performance in this task is indeed founded on children's knowledge of syntactic regularities in number words. Thus, to better understand what Give-N10 is assessing, we tested how two other tasks requiring numerical syntactic knowledge (Interval-Counting and Number-Multiplier-Syntax) relate to children's Give-N10 performance.

2.2.4 Interval-Counting task

Observing children's acquisition of the counting sequence is a unique way to peek into their understanding of the syntactic regularities (and irregularities) of complex numerals (Gould, 2017). In the Interval-Counting task, we asked children to rote count within three intervals (from 15 to 24, from 37 to 51, and from 95 to 111; see Methods for details) each of which containing some "hurdle" points according to previous work (e.g., Gould, 2017). In the first interval, almost all the children reach the upper limit of 24 (31 of 33), although one stopped at 19 and the other at

20. In the interval from 37 to 51, 26 out of 33 children successfully counted up to the number 51, while five children stopped at the numbers before a new decade (e.g., the numbers 39 or 49), one child stopped at the number 47, and the other child did not start to count by himself (which we treated as missing value). In the interval from 95 to 111, only about half of the children (17 out of 33) successfully counted up to the number 111. Of those who failed to reach 111, eight children stopped at the number 109, one child stopped at the number 100, and six children stopped at the numbers before 100 (see Fig. 3). All the children who were able to count beyond 109 in the second interval successfully counted up to the last number (the number 51) in the first interval, but only 18 of 26 who counted up to the number 51 counted beyond 109 in the second interval. Interestingly, four of those who failed to reach 51 passed the Unit task, correctly solving the questions asking for $57 + 1$ or $75 + 1$ in the Unit task. This result confirms our intuition that a child's performance in the Unit task is not bound by her count list, and indeed suggests that sequential knowledge at the *ones* level is enough to succeed in the Unit task.

Figure 3. Highest number reached within the interval from 95 to 111 in the Interval-Counting task. Each data point represents one child. 17 of the 33 children who counted up to 111 were stopped by the experimenter as that was the last number in that interval.

The pattern in which children stop at the last number before some of the decades (39, 49, or 109) replicates and confirms previous reports (Gould, 2017; Siegler & Robinson; 1982; (Fuson, 1988), and illustrates children's understanding of the syntactic rule (or lack thereof) for building complex numerals. In subsequent analyses, we used the Decade Transition (DT) score as a summary measure for each child's counting fluency especially concerning the child's knowledge of numerical syntax. Each of the decade transitions in all the intervals (19, 39, 49, 99, 109) was marked 0 (fail) or 1 (pass), resulting in a maximum of 5 points in each child participant (see Methods for more details). The DT score was significantly predicted by school grade ($\beta = 0.62$, $t(31) = 3.70$, $p < .001$).

2.2.5 Number-Multiplier Syntax task

This task also evaluated children's understanding of the syntactic regularities in complex numerals. More specifically, it assessed how children associate the sequence of Digits (from one to nine) with Multipliers used in various levels. Children's performance varied widely, but in

general was predicted, again not surprisingly, by school grade ($\beta = 0.11$, $t(31) = 3.56$, $p = .001$). Twenty out of 33 children got at least seven out of eight total trials correct. Considering the binomial distribution $B(8, .5)$ with $\alpha = .05$, these 20 children were considered to have passed the Number-Multiplier Syntax task, meaning that they are capable of associating numerical sequence correctly with each of the multipliers.

2.2.6 Regression Analyses

performance in the Interval-Counting and the Number-Multiplier Syntax tasks, designed to assess their numerical syntactic knowledge, predicted their performance in Give-N10. First, in two independent regression models, the Give-N10 performance was predicted by the Interval-Counting performance (see Table 1, Model 1: Decade Transition score) and by the Number-Multiplier Syntax performance (Model 2: dichotomous coding of whether or not the child was able to pass the task), while controlling for school grade. Children's DT score was a significant predictor of the Give-N10 performance even after controlling for school grade ($\beta_1 = 0.13$, $t(29) = 2.99$, $p < .01$). Likewise, children's performance in the Number-Multiplier Syntax task was a significant predictor of the Give-N10 performance even when the school grade was controlled for ($\beta_1 = 0.98$, $t(29) = 2.56$, $p = .015$).

Table 1. Regression models with the Give-N10 score as the dependent variable

	<i>Estimate</i>	<i>S.E.</i>	<i>t</i>	<i>P</i>
<i>Model 1 (Adjusted R-square = 0.61)</i>				
β_0 (Intercept)	-0.46	0.16	-2.83	.008**
β_1 (DT)	0.13	0.04	2.99	.005**
β_2 (School Grade)	0.18	0.05	3.82	.000***
<i>Model 2 (Adjusted R-square = 0.61)</i>				
β_0 (Intercept)	-0.67	0.32	-2.12	.043*
β_1 (Number – Multiplier Syntax)	0.98	0.38	2.56	.015*
β_2 (School Grade)	0.40	0.13	3.15	.004*
β_3 (School Grade) $\times$ (Number – Multiplier Syntax)	-0.25	0.14	-1.82	.078
<i>Model 3 (Adjusted R-square = 0.62)</i>				
β_0 (Intercept)	-0.37	0.17	-2.14	.041*
β_1 (DT)	0.10	0.04	2.08	.046*
β_2 (Number – Multiplier Syntax)	0.19	0.13	1.47	.153
β_3 (School Grade)	0.15	0.05	3.10	.004**

* $p < .05$; ** $p < .01$; *** $p < .001$

In the third regression model, performance measures from both Interval-Counting and Number-Multiplier Syntax were entered simultaneously into one model with school grade as a covariate (see Model 3 in Table 1) to predict the Give-N10 performance. The Interval-Counting

performance ($\beta_1 = 0.10$, $t(29) = 2.08$, $p = .046$) but not the Number-Multiplier Syntax performance ($\beta_2 = 0.19$, $t(29) = 0.13$, $p = .153$) remained significant. This pattern likely means that the two regressors of interest are correlated and that one (Interval-Counting) of the two is taking most of the variance in explaining the variability of the Give-N10 performance. Regardless, these results confirm that what was measured in Give-N10 was certainly associated with children's understanding of the syntactic structure and the syntactic regularities of complex numerals. Critically, however, note that Give-N10 involves the representation of items in a set (cardinality) while such a representation is not involved in the Interval-Counting or Number-Multiplier Syntax tasks.

2.2.7 Decade Transitions and Performance in Give-N10 task

The results so far indicate that the DT score is a significant predictor of children's Give-N10 performance. However, the DT score is a summarized measure of children's knowledge about the regularities in the counting sequence, so it is yet unclear specifically which kind of syntactic rules for forming complex numerals give rise to children's understanding of the embedded structure of cardinal numbers. In fact, it is conceivable that a success in some particular decade transition may be more informative for predicting the Give-N10 performance.

To address this question, we performed a post hoc analysis examining the association between whether or not a child passed Give-N10 (0.66 or above) and whether or not the child successfully produced the decade name in each of the decade transitions in the Interval-Counting task in a cross tabulation. The chi-square values of these associations are plotted in Figure 4. The highest association was found between counting past one hundred nine and passing Give N10 ($\chi^2_{(1)} = 16.00$, $p < .001$). Two other transitions (counting past forty nine and ninety nine) resulted in a lower chi-square value ($\chi^2_{(1)} = 4.41$, $p = .035$).

Figure 4. Association between successful decade transition and Give-N10 performance in Study 1. For each decade transition, a chi-square test for independence was performed by cross-tabulating two dichotomous variables: pass/fail decade transition and pass/fail Give-N10. The greatest association was found at the decade 110, which indicates that children who can count beyond 109 reliably solved the Give-N10 task.

These results suggest that counting beyond one hundred nine is a unique developmental milestone. Gould (2017) suggests that children's knowledge of English grammar for tens inhibit the production of "ten" in "one hundred ten." In tens, children use the sequence between one and

nine (e.g., from thirty-one to thirty-nine) but the use of “ten” after the digit “nine” is not allowed. Indeed, when children produce an ungrammatical cardinal number as “thirty-ten,” they are often corrected. According to this idea, this hesitation to say “one hundred ten” implies that those children do not have a correct cardinal understanding of the word “one hundred.” One possibility is that they may be treating it as something like “ten-ty” that follows “nine-ty,” which is especially possible if children do not have a firm understanding that the same one-through-nine sequence is recursively used in the ones level and in the decades level (and beyond of course). In addition, much of the number system below one hundred has idiosyncratic, non-analytical properties that appear to obscure or block the child’s capacity to realize the system which is systematic when several levels of embedding are entailed, as with numbers that embed tens, hundreds, and even thousands within a single number.

2.3 Discussion

Despite the theoretical importance of and scholarly interest in the successor principle, little empirical work has directly investigated this logical foundation of natural numbers. To our knowledge, only two types of empirical approaches have directly investigated the acquisition of the successor principle. The first type is via the Unit task, first introduced by Sarnecka and Carey (2008) and used in other following studies (Cheung et al., 2017; Davidson et al., 2012), including the current study. The structure of this task mimics how the successor principle is defined (i.e., making a semantic induction from N to $N+1$), and thus this task is claimed to measure children’s knowledge of the successor principle (Sarnecka & Carey, 2008; Le Corre & Carey, 2007; 2008). However, this task by design does not require children’s ability to make a semantic induction over all possible numbers nor over the children’s own count list (Cheung et al., 2016; Davidson, et al., 2012). This is because knowing how to make an induction over a *local* sequence of numbers from one to nine is sufficient to solve this task successfully as long as the child has some knowledge about the syntactic structure of complex numerals. In other words, a child can solve the Unit task with a minimal understanding that, say, the numeral twenty-five contains the word five (along with some other word) and that an increment of one item in the environment is associated with the next number (i.e., six) in the local numerical sequence. In fact, our results show that some children who pass the Unit task (with numbers as high as 75) are still not able to count fluently. Two of those children stopped at 39 and two other children stopped at 49 in the Interval-Counting task, suggesting that the Unit task can be solved in intervals where the counting sequence is not mastered yet. These results make it difficult to argue that the Unit task measures any abstract knowledge about the successor principle nor the recursive operations of numerals. Moreover, this task cannot explain how children come to understand that any number has a successor.

The second type of work targets children’s declarative knowledge of infinity, exploiting the idea that one of the end states of the acquisition of the successor principle is the comprehension of infinity. Cheung et al. (2017), for instance, inquired children’s knowledge about infinity by asking them about the biggest number they know and about adding one to that biggest number. Falk (2010) also devised a similar approach to assess whether and when children grasp the concept of infinity. While these studies have provided meaningful results, the interpretation requires some caution. In some items of the infinity task, the experimenter explicitly asks the children about an arithmetic operation (i.e., adding one to a number). Even if the children answered this type of question correctly and they understand that adding one to a big number generates a higher number, there is a difference between knowing the successor rule and

knowing the result of an arithmetic operation. In addition, the introspective nature of this task restricts the interpretation about children's precise knowledge level. Most importantly, even if this approach may allow us to identify the final state of the acquisition of number concepts it is limited in explaining how that state is achieved.

As described above, the prevalent assumption in the field has been that children's concept of nature numbers comes from the understanding of the successor principle or, in an empirical sense, from understanding the next number (see also Chomsky, 2005; Rips, Bloomfield, & Asmuth, 2008; Barner, 2017). However, tasks that inquire children about the next number (e.g., the Unit task) fail to show that the knowledge of the successor is directly related to children's understanding of natural numbers. Such results invite us to rethink the appropriateness of the successor principle for explaining children's acquisition of number concepts. In fact, the idea of the successor principle in its abstract form, as described in the Peano axioms, is derived from a formal mathematical definition that has little to do the structure of the symbolic system used to learn and mentally represent numbers. Thus, the successor principle may not be the right framework to investigate the mental representation of number. Instead, we argue that a more plausible candidate for the mechanism underlying number concept acquisition is the comprehension of the syntactic structure of cardinal numbers. After all, the analysis of recursive structure of numerals is only possible after the analysis of its syntax.

Our study is designed to assess children's understanding of numerical syntax, as an important step towards understanding the state of their number concepts. In particular, we devised and ran a task (Give-N10) that evaluates children's knowledge of the base-10 number system. Specifically, to succeed in this task, children must parse a complex numeral and understand the cardinal meaning of each element in that numeral. Our results demonstrate that relatively older children, first graders and above in our sample, are able to perform this task maximally using the base-10 number knowledge. It is worth reiterating that everyone who passed this Give-N10 task passed the Unit task, but the reverse was not true. This pattern again speaks to the idea that the Unit task is not sufficient to demonstrate children's number concepts.

Considering that a substantial amount of formal math education begins in kindergarten, it is not surprising to see that the performance in Give-N10 ramps up rapidly in kindergarten and first grade (Fig. 2). What is critical, however, is that the performance in this task is highly predicted by children's counting performance even when school grade was controlled for (Table 1). These results indicate that even after removing the effect of schooling, the comprehension of numerical syntax and its associated cardinality is founded on the understanding of the syntactic regularities in the counting sequence. Thus, the results further demonstrate that counting is not a mere recitation of a memorized sequence. Rather, it is a demonstration of children's knowledge about the syntactic regularities of numerals.

The results so far suggest that children's understanding of the embedded structure of cardinal numbers can be measured empirically using Give-N10, which seems to be founded on their knowledge about the syntactic regularities of complex numerals. In our sample of English-speaking children, it was not until the age of 6 years on average (several months into kindergarten) that the majority of the children passed the Give-N10 task. These findings suggest that a syntactic understanding of the meaning of number words (even within the ranges of children's count list) comes much later in development than previously thought, and that understanding the embedded structure of numbers may be a critical cornerstone for the

acquisition of number concepts. However, as novel as the task was, we conducted a second independent study to test whether Give-N10 provides a meaningful, interpretable results in another sample of children using another language.

3 Study 2

A follow-up replication study was conducted to test whether Give-N10 provides a meaningful and interpretable measure of children's understanding of the embedded structure of cardinal numbers. In particular, we focused on capturing the Give-N10 performance in Korean-speaking children around the age of five and six and tested its association with their counting ability measured by the decade transition score³. We ran this replication study in Korea in part as it was a unique opportunity to exploit the regularity of the Korean number system.

The Korean language has two sets of numerals, both of which used regularly in real life although they are used in slightly different circumstances. While the native Korean system contains irregular decade names, the Sino-Korean system (derived from Chinese) is extremely regular in terms of the base-10 structure. For example, unlike English, the Sino-Korean numerals are regular in both teens and decades: teens are simply a combination of the word for ten (i.e., sib) and a word in the ones class (e.g., seventeen is ten-seven which is sib-chil in Korean) and decades are simply a combination of a word in the ones class and the word for ten (e.g., seventy is seven-ten or chil-sib). In other words, the arithmetic operations making up the base-10 number system is transparent in Korean.

By evaluating children's performance in Give-N10 and its association with their counting fluency in Korean-speaking children, we aimed not only to validate the original results but also to better understand the influence of the regularity of the numeral system on children's acquisition of the embedded structure of cardinal numbers.

3.1 Methods

3.1.1 Participants

Thirty-one kindergarteners (19 boys and 12 girls; age range=5;3-6;5) participated in this study. Children were recruited from three different public kindergartens in Seoul, South Korea with parental consent, and the study was conducted in the schools. In South Korea, the academic calendar runs from January to December. Children who turn seven in the upcoming academic year enter first grade in elementary school, while younger children are eligible to enroll in public kindergartens. We aimed to recruit and run kindergarteners because the age range of these children in Korea best matched the age range in which we observed the widest variability in English-speaking children's Give-N10 performance (see Fig. 2). Children completed three tasks in the following order: Interval-Counting, Give-N10, and Give-N. All procedures were approved by the University of Massachusetts Institutional Review Board. The way that the three tasks were run was identical to that of Study 1, with minor differences. In the following section, we limit our description of the procedure and tasks to those differences. Some of the children were given another task after Give-N10, and some 5-year-olds were given yet another task prior to Give-N10.

³ As we ran the study in local kindergartens, our time with each child was limited. Thus, we chose to test the most critical association found in Study 1

These tasks are not discussed here as they are outside the scope of the current paper, but we note that none of these other tasks involved counting objects.

3.1.2 Procedure and Tasks

Interval-Counting. Children were asked to count the following intervals in that order using the Sino-Korean system: from 1 to 11, from 16 to 32, from 37 to 51, from 95 to 111, and from 285 to 311. Children were asked to count all the intervals regardless of whether or not they made errors in any of the intervals. Unlike in the case of the first interval (1-11) where the first three numbers were introduced by the experimenter, only the very first number was introduced in the rest of the intervals. When the child counted all the numbers in a given interval without an error, he/she proceeded to the next interval. When the child made an error in a given interval, the highest number (N) that the child counted up to before the error was first recorded. In that case, we asked “What number comes after N?” without giving any hint about the next number after N. If the child did not know the next number, we stopped the counting activity for that interval and proceeded to the next interval. If the child knew the next number and resumed counting the numbers, we let him or her continue until the second error, if there was any. We prodded the child the same way at the second error. At the third error, if it existed, we stopped the interval and proceeded to the next interval by saying for example “Okay, then let’s now count from [ten-six] (which is a literal translation of the word sixteen in Korean).” As in Study 1, the decade transition (DT) scores were quantified as a measure of children’s understanding of the regularities of the numerical syntax. The points of decade transition were 9, 19, 29, 39, 49, 99, 109, 289, 299, and 309. Thus, the highest score that a child can reach was 10.

Give-a-Number-Base-10 (Give-N10). The procedure for Give-N10 was identical to that of Study 1 except a different set of cardinal numbers was inquired. Instead of using four distinct groups of ten squares, Study 2 incorporated five distinct groups of ten squares, and asked children to give 27, 32, and 45 squares. To be specific, the experimenter first modeled the case of retrieving 31 squares, and the children were asked to give 1, 7, and 10 item(s) so that they have an opportunity to retrieve individual items from a group of ten as well as to retrieve ten items at once (for those who can). Then, as the experimental trials, the experimenter asked the children to move 27, 32, or 45 squares to a designated spot.

We used the number 45 instead of the teen number used in Study 1 for two reasons. First, native Korean numerals are typically preferred when it comes to counting items below twenty objects. Because we used Sino-Korean numerals, we avoided the numbers below twenty. Second, we reasoned that Korean-speaking children would have a better understanding of the numerical syntax owing to the regularity of its numerals. Thus, we reasoned that Korean-speaking children would be able to handle an even higher number like 45 compared to the highest number 32 used in Study 1. The coding scheme for the performance in this task was identical to that in Study 1.

Give-a-Number (Give-N). The procedure of the Give-N task was identical to that in Study 1. While counting objects within a small range of numbers (1-20 objects) is most natural using the native Korean numerals, we used the Sino-Korean numerals to be consistent with the rest of the tasks. While this may have sounded somewhat unconventional, it should not have interfered with our primary purpose which is to assess children’s knowledge about the counting principles.

3.2 Results

3.2.1 Give-a-Number task

All but three children successfully gave 7 and 8 objects and were categorized as CP-knowers.

3.2.2 Give-a-Number-Base-10 (Give-N10) task

Children's Give-N10 performance ranged all over the scale. This was expected as the age range of the Korean kindergarteners tested was narrow (only 5- and 6-year-olds), which we learned from Study 1 is the range when there is the greatest variability in the Give-N10 performance. Seven out of 31 children failed to use the base-10 structure in all three trials, ten children succeeded in all the trials, and the rest fell in between the two end points. We then addressed the more interesting question of whether the individual differences in this Give-N10 performance can be explained by children's counting ability as in Study 1.

3.2.3 Interval Counting task

We first validated the stopping points in the Interval-Counting task that we have observed in Study 1. In the first interval (1-11), all but one child counted up to 11. In the second interval (16-32), 8 out of 31 children did not reach the upper limit, half of them stopping at numbers ending with 8 or 9. In the third interval (37-51), 6 out of 31 children did not reach the upper limit, most of them stopping at 39. The highest number reached within the fourth (95-111) and fifth (285-311) intervals are plotted in Figure 5. Across both intervals, the prominent stopping points were 99, 109, and 289, consistent with the idea that children find decade transitions most difficult when rote counting.

Figure 5. Highest number reached within the interval from 95 to 111 and from 285 to 311 in the Interval-Counting task in Study 2. Each data point represents one child. In the first interval, 10 of the 31 children who counted up to 111 were stopped by the experimenter as that was the last number in that interval. In the second interval, 3 of the 31 children who counted up to 311 were stopped by the experimenter as that was the last number in that interval.

Children's counting fluency was quantified using the Decade Transition (DT) score. The average DT score across all children was 5.6. If we are to assume the case that children who pass the higher DT always pass the lower DT, the DT score of 5.6 can be interpreted as having a highest count somewhere between 49 and 99. More importantly, to follow-up our results in Study 1, we tested whether DT score predicted children's Give-N10 performance. It did indeed. DT score

was a significant predictor of Give-N10 both with ($\beta = 0.26$, $t(28) = 3.10$, $p = .004$) and without ($\beta = 0.29$, $t(28) = 3.66$, $p < .001$) using age as a covariate.

As in Study 1, we examined the association between children's Give-N10 performance and successful decade transition in each of the decades in the Interval-Counting task. Figure 6 illustrates that the highest association was found between counting past ninety nine and passing Give N10 ($\chi^2_{(1)} = 5.67$, $p = .017$). The chi-square values for passing thirty nine and forty nine were 4.84 ($p = .028$).

Figure 6. Association between successful decade transition and Give-N10 performance in Study 2. For each decade transition, a chi-square test for independence was performed by cross-tabulating two dichotomous variables: pass/fail decade transition and pass/fail Give-N10. The greatest association was found at the decade 100, which indicates that children who can count beyond 99 reliably solved the Give-N10 task.

3.3 Discussion

The core finding in Study 1 was replicated in Study 2. That is, the Give-N10 performance was predicted by their understanding of the syntactic regularities in the counting sequence measured by the decade transition score in rote counting. This positive relationship found in the speakers of two languages bolsters the idea that Give-N10 is indeed a valid way of assessing children's understanding of the embedded structure of cardinal numbers and that this understanding comes from the recursive structure of the counting sequence.

At the same time, results from Study 2 provide some evidence that the regularity of a language's number system facilitates children's cardinal understanding of numerical syntax. Indeed, previous studies have demonstrated that the regularity of the numerals influences rote children's counting ability. For example, Korean preschoolers outperform English-speaking preschoolers in counting (Miller & Stigler, 1987) and in solving single digit arithmetic problems (Fuson & Kwon, 1992). In another previous study, Miura, Kim, Chang, and Okamoto (1988) compared the preference of the child speakers of English and Asian languages for using a set of tens and units when they are asked to show double digit numbers such as 11, 13, 28, 30, and 42. Asian children

preferred to compose the numbers with both tens and units blocks, whereas, English speaking children preferred to compose the numbers primarily with units blocks.

Our current results suggest that the point at which children understand the embedded structure of cardinal numbers may differ slightly depending on the regularity of the numerical syntax they are using. In English-speaking children (Study 1), the most prominent point in the Interval Counting task was one hundred nine, in a sense that those who were able to count beyond one hundred nine were likely to succeed in the Give-N10 task. In Korean-speaking children (Study 2), it was the number ninety nine that best distinguished children who passed or failed the Give-N10 task. This pattern supports the idea that the regularity of the numerical syntax facilitates children's learning of the recursive and embedded structure of cardinal numbers. For instance, because it is so apparent in Sino-Korean numerals that the same lexical element from one to nine is used to express the decade numbers, it may be more straightforward for a child to think that the numbers that come after ninety nine (goo-sib-goo)—after being able to count past chil-sib (seven-ten), pal-sib (eight-ten), and goo-sib (nine-ten)—must be reaching another level of numerical expression, thus eventually understanding the recursive structure. There is another feature that is present in Sino-Korean but not in English cardinal numbers, at least in Hurford's framework. The multiplier for 10 (i.e., sib) is used both as a bare multiplier in the range from 11 to 19 (e.g., Number[Phrase[Multiplier[Sib]]-Number[Digit[ee]]]) for the number 12) and as a multiplier in the range from 20 to 99 (e.g., Number[Phrase[Number[Digit[ee]]-Multiplier[sib]]] for the number 20). The consistent use of the same word in two different structures may facilitate children's understanding about the idea that the lexical elements expressing cardinal numbers are combinatorial.

4 General Discussion

In two independent studies in two other languages (English and Korean), we evaluated children's cardinal understanding of the syntactic structure of complex numerals using a novel task, Give-a-Number Base-10. This task goes beyond children's ability to infer the next number in a local counting sequence (assessed by the Unit task). This task also goes beyond children's mere knowledge about numerical syntax (assessed by the Interval Counting and the Number Multiplier Syntax tasks), as it involves their understanding of cardinality, yet the performance in this task is highly associated with children's understanding of syntactic regularities in the counting sequence in both languages. These results highlight the idea that the linguistic structure of numerals is key to understanding the principles of natural numbers.

Give-N10 was designed to assess children's understanding of or sensitivity to the arithmetic operations of the base-10 number system. It should be noted that there is a large literature on children's understanding of the base-10 knowledge using place value in multi-digit Arabic numerals (Kamii, 1986; Miura & Okamoto, 1989; Ross, 1989; Fuson, 1990). While both the place value systems and the cardinal numbers are an external representation of the formal numerical concept, the former is fundamentally different from the latter. For instance, although related, seeing 109 is entirely different from saying or hearing one hundred nine. More precisely, in place value systems, space (or place) is central to expressing the power of the base, whereas in cardinal numbers, the power is represented by lexical elements (i.e., multipliers). Moreover, the structure of the place value systems does not incorporate a sequence, whereas numerical sequence is one of the defining characteristics of cardinal numbers. Unlike how much is studied about children's learning of the place value system, relatively little has been known about how

children acquire the base number knowledge from cardinal numbers. Our work paves the way for this venture.

In a way, our approach and conceptualization of children's knowledge about the embedded structure of cardinal numbers is similar to how Fuson and colleagues (1997) have theorized the development children's understanding of multi-digit numbers. In particular, Fuson et al., (1997) claims that children go through a phase from a "unitary" understanding of multi-digit numbers to a "decade and ones" phase, and eventually to a phase where the tens and the ones are represented independently ("sequence tens and ones" or "separate tens and ones" phases). In that framework, the state of passing the Give-N10 task is likely to be categorized by this very last stage. Nevertheless, Fuson and colleague's model has been proposed to explain the comprehension of multi-digit numerals using three external representations, Arabic numerals, numbers words, and blocks, and this model provides little basis for how the conceptual representations for complex numerals develop. Our work contributes to this literature by suggesting the importance of numerical syntax for a comprehensive understanding of cardinal numbers. More importantly, we argue that such an understanding is the basis of children's understanding of the generativity of natural numbers.

Then, how is the cardinal understanding of numerical syntax related to the acquisition of number concepts? One possibility is that the recursive properties of syntactic elements of complex numerals could directly give rise to the understanding of the successor principle as previously alluded to (e.g., in Hurford, 1975 and Rips et al., 2008). However, this scenario requires a precise explanation for how a recursive hierarchical number structure results in an abstract principle. In fact, Relaford-Doyle and Núñez (2018) criticize the idea of successor principle as a cognitive mechanism by providing evidence that mathematical functions for the successor principle of natural numbers (such as in the Peano's axioms) is learned only by formal training and that most adults without formal training never develop this abstract function. According to this view, it is unlikely that young children come to understand the abstract nature of the successor principle.

Instead, a plausible explanation for children's acquisition of number concepts concerns their understanding of the additive and multiplicative compositions of complex numerals that represent cardinal numbers (Hurford, 2007). For example, in order to understand the cardinal number seventy-two, a child must understand that seventy is composed of seven (Number) of the tens (-ty; Multiplier) and that seventy two is composed of the addition of seventy (Phrase) and two (Number). Potentially, there are several developmental stages before a full comprehension of the cardinal number and its relationship with the arithmetic algorithm of the base-10 number system. Young children at first may not understand that the elements in a complex numeral resemble a hierarchical construction. These children would produce numbers, for example in the current Give-N10 task, by counting one by one until the target number. Children then may come to understand that the constituents in complex numerals represent different categories but without fully knowing the arithmetic operations that the numerals resemble. These children would be capable of using the counting procedure as an iterative operation in different units. For example, they would count by tens (e.g., 10, 20, 30, ...) to reach the target decade and by ones (e.g., 1, 2, 3, ...) to reach the final number. Finally, children may come to understand the arithmetic operations that complex numerals represent, although in younger children, it may be possible that the category Phrase is processed as a repeated addition of tens (for seven times to indicate seventy), as Carraher, Carraher, and Schliemann (1985) showed that repeated addition is typically in place of multiplication in everyday use of mathematics. After this stage, children

may come to understand the multiplicative operations represented by the Merge of the Number and Multiplier categories (e.g., [three-ty]). Importantly, our crosslinguistic data suggest that the regularity of numerals plays a critical role in children's learning of these additive and multiplicative compositions of complex numerals.

Our data from the Give-N10 task currently cannot distinguish between a completely multiplicative account and a repeated addition account. Nevertheless, the critical element for the acquisition of number concepts, as we propose, is the understanding of the meaning (i.e., cardinal value) of each of the recursively embedded number words in a complex numeral and their operational relations. It is worth noting that Spelke (2017) develops a similar hypothesis, where it is proposed that the grammar of conjunctive noun phrases (e.g., two cats and three dogs) and prepositional phrases (e.g., three piles of two waffles) facilitate the acquisition of new cardinal numbers. However, there is no empirical support behind the idea that children learn, for example, the number five from the expression "two and three" or the number six from the expression "three of two." Our proposal, on the other hand, hinges on the base-10 numeral system in that the cognitive processes for multiplicative (or repeated additive) and additive compositions arise from the syntactic structure of complex numerals.

5 Conclusion

We conclude by proposing a theoretical framework for children's acquisition of number concepts, in particular the generative properties of natural numbers. The core research question at stake is how children understand, in a philosophical sense, the infinite use of finite means. The most dominant idea in the literature is that recursion, particularly in the form of the successor function, drives the acquisition of number concepts (Chomsky, 2005; Rips, Bloomfield, & Asmuth, 2008; Barner, 2017). However, our theoretical analysis and empirical observations lead us to challenge that view.

It is first important to note that number is different from numeral, or the external representation of number. The successor principle provides a theoretical explanation for natural numbers, and it has the capacity to generate any natural number regardless of its external representation. In reality, however, children acquire the meaning of numbers through the exposure and the use of numerals, instead of having direct access to the abstract representation of number. In addition, as mentioned earlier, there is empirical evidence to suggest that even adults may not have a genuine understanding of the successor rule unless they receive a formal training in mathematical induction (Relaford-Dolye & Nunez, 2018). Observations from cross-linguistic studies also speak against the idea of the successor principle. If the successor principle is the core mental mechanism for representing natural numbers regardless of the form of their external representations, it must have been invented/discovered in all languages that have numerals, and consequently these languages must produce numerals that exhibit the successive properties of natural numbers. However, this is not true, as there exist many languages with a finite set of number words (Beller & Bender, 2008; Corbett, 2000; Saxe, 1981). This could mean that these populations have the mental capacity for the successor principle but their languages do not provide a rule to express large numbers, in which case the successor principle is not sufficient for representing natural numbers. In contrast, it could mean that these populations do not have the mental mechanism for the successor principle because their external representations are bounded. In that case, their concept of numbers is bounded within the external representations, again making the successor principle unnecessary for explaining their number concepts.

All mental knowledge requires mental representations. If we can comprehend an overt representation of a phenomenon, then it is most likely that the overt representation must have a definable relation to that mental representation. Thus, when it comes to the acquisition of number concepts, it is reasonable to posit that the acquisition is directly based on the structure of the external representations of number. In the current work, we have demonstrated that children's cardinal understanding of this embedded structure of numbers—or numerical syntax—can be empirically measured by the Give-a-Number Base-10 task and that this understanding develops most prominently in kindergarten-age children. Moreover, this understanding is tightly associated with children's counting fluency, and this association is influenced by the regularity of a language's numeral system. Based on these observations, we propose that children's comprehension and production of the recursive syntactic rules that represent arithmetic operations in a base system is the foundation of their understanding of the generative properties of natural numbers. Further, we propose that the cardinal representation of numerical syntax *is* the structure of numerical thinking.

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