Constructing the Objective World from Subjective Perceptions

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Abstract

Starting from two propositions, 1. The observer is part of reality and 2. Observers are independent of each other, we develop an approach to Hume's problem of scepticism. We show that despite all knowledge coming through subjective perceptions we can nevertheless construct objective or transitive knowledge through relationships between entities, rather than the entities themselves. We posited that some phenomena r_i of the real world are transitive and some are not. Transitive phenomena are transferable between observers without change and are thus independent of observation. Transitive phenomena are relative quantities (r_i, r_j) Relative quantities are observational invariants and form the building blocks of our objective world. We show objective knowledge is developed from relative quantities whereas subjective knowledge is rooted in absolute quantities. The recognition that transitive quantities are invariants of observation gives them an independence from which we can construct an objective world.

Hume's Problem

David Hume (1711-1776) is one of the most important among philosophers because he developed to its logical conclusion the empirical philosophy of Locke and Berkeley, and by making it self-consistent made it incredible.

Bertrand Russell [1]

John Locke is credited with being the father of empiricism and set forth the idea that “the only source of knowledge is experience”. [2] Picking up empiricism from Locke, David Hume pushed the idea as far as logic would take it and concluded that we cannot know anything about the world. This seemingly ‘incredible’ conclusion is reached from the following argument. If our knowledge is limited to our experiences, all our experiences come to us from our senses and nowhere else. We have every understanding of our senses but no understanding of what is beyond them. “*I never catch myself at any time without a perception and never can observe anything but the perception*” [3] - a statement with which it is hard to disagree. Hence, we have no knowledge of the external world and cannot in principle know anything about that external world – an idea which has been called “*external scepticism*”. Hume then went a step further and drew a second conclusion, namely that we have no reason to suppose the patterns of past experience will hold for future experience. The argument he presented for this seemed similarly watertight: We have no ground for believing that one event can cause another event, since there are no events, only sense impressions. We have no reason at all for believing that events are connected, only that one perception succeeds another perception. We cannot link events together and we cannot expect past links to be repeated in the future. This has been called “*inductive scepticism*”. External scepticism banishes everything into a subjective world while inductive scepticism banishes even the possibility of theorising about the connections in that world. We stand like Dante, imprisoned in a subjective world.

My mind became, at this point, so withdrawn into itself that the reality of things outside could not have entered there.

Dante, The Divine Comedy, Volume 2: Purgatory, Canto 17, line 22 [4]

To agree with Hume forces us into a position which is unwelcome. If all we know is perception, then we know nothing about the world. We are locked inside a subjective world the knowledge of which is not sharable with anyone else. The objective world is beyond our grasp and everything objective – science and the scientific method, progress in knowledge, objective truth and even communication with others - is out of reach.

The first to try to address Hume's scepticism was Kant. Hume's argument was so disconcerting to Kant that it famously 'shocked him from his dogmatic slumbers'. [5] Kant took the view that knowledge from experience is not enough by itself, you also need knowledge from reason. Knowledge from experience he called *a posteriori* knowledge (i.e. This man is 24 years old). Knowledge from reason he called *a priori* knowledge (i.e. This man is male). The first is true from experience, the second is true independent of experience. So far Hume would have agreed and dismissed *a priori* knowledge as telling us nothing. But then, Kant draws out a subtlety. It is not true that all *a priori* knowledge is redundant. Some *a priori* knowledge can give us new information not dependent on our experience. To explain this Kant makes another distinction and says that knowledge from reason (*a priori* knowledge) is of two kinds: true statements which add no new information and come from the meaning of the words alone, which he called analytic (i.e. This man is male) and true statements which do add new information which comes from reason, which he called synthetic (i.e. $5+7=12$). This is simply a way of saying that not all our knowledge comes from experience and consequently we are not caught in Hume's trap. But of course, the issue now becomes, is synthetic *a priori* knowledge possible? Can we gain new knowledge about the world through reason, as Kant says?

What Kant would admit is that *a posteriori* knowledge is still empirical knowledge and therefore is still subject to Hume's critique. Only *a priori* knowledge provides a possible loophole to escape Hume's scepticism because this knowledge does not come through our senses. But can it really tell us anything new? Not by itself claims Kant, but it can tell you new things about the universe you experience through your senses. "*A priori knowledge provides support for, but also derives its content from, empirical discovery.*" [6] In short, knowledge requires more than content, it also needs form. The content comes from empiricism and the form comes from *a priori* reasoning. The combination of the two is objective knowledge, claims Kant.

But it is not at all clear that empirical knowledge becomes objective merely because it is given an objective wrapper. The wrapper may be independent of our senses, but the content is not. We can only treat such knowledge as objective if we do not take it out of its wrapper and examine it. Just putting subjective knowledge into an *a priori* form does not make it objective knowledge. It makes it subjective knowledge arranged in a rational way. It seems to me we need a better solution than that. What we really need is objective content to go with the reasoned wrapper.

Furthermore, it is not clear that synthetic *a priori* knowledge exists. Is ' $5+7=12$ ' synthetic or analytic? It has been argued that statements like ' $5+7=12$ ' are not purely analytic but are derivable from the meaning of the numbers themselves. ' $5+7=12$ ' is true only because it is derived via logical principles and it cannot be any other way. That is what makes it *a priori*. For it to be new (synthetic) knowledge '12' must not be contained in the phrase ' $5+7$ '. Bernard Bolzano rejected Immanuel Kant's claim that arithmetic and geometry were grounded on synthetic *a priori* judgements based on pure intuition. [7] On the other hand Charles Sanders Peirce, a great admirer of Kant, believes that mathematics can provide a knowledge which is both general and "ampliative". [8] With such contention the best that can be said is that the existence of synthetic *a priori* statements is open to question and not a conclusive answer to Hume. Something else is needed.

Approaching Hume's Problem

To present an argument against Hume's position we start with two principles, each of which might be considered straightforward and unobjectionable.

Principle 1. The observer is a part of reality.

The observer does not stand outside the universe which is being observed. The observer is a part of the universe and subject to the same representation as the universe. This is a principle which is not only reasonable, but which has been emphasized by Quantum Theory and incorporated into that theory at a fundamental level. The observer O is a part of the universe $\mathbb{R}$ even to the point that the observer affects the thing being observed.

$$O \in \mathbb{R}$$

The consequence of the idea that the observer does not stand apart from the universe is that observers need to be included in the representation. The same descriptive language which applies to the thing being observed must also apply to the observer, who is included with the same tools of representation that apply to the phenomena. For example, if we were to represent facts or elements of reality by vectors in a representation space, then Principle 1 means that the observer O is also represented in the vector space. In this way we ensure that subjectivity is representable within the system.

Principle 2. Observers are independent of each other.

If each observer has their own perceptions, how can we be sure that the perception I have is the same as the perception you have? If you look at a yellow flower how do I know that you see the same yellow that I see? Since we do not know how you perceive the sensation of yellow, we can't be sure you are seeing the same yellow I am. It's possible that what you experience as yellow, I may experience as red. This problem was discussed by John Locke.

"That the same Object should produce in several Men's Minds different Ideas at the same time; v.g. if the Idea, that a Violet produced in one Man's Mind by his Eyes, were the same that a Marigold produces in another Man's, and vice versa. For since this could never be known because one Man's Mind could not pass into another Man's Body, to perceive, what Appearances were produced by those Organs; neither the Ideas hereby, nor the Names, would be at all confounded, or any Falsehood be in either." [9]

Locke is making the point that you do not know if someone sees the same yellow that you do. But it's more than that; you cannot know, not now, not ever. It is not possible any more than it is to put your mind inside another person's head, because that is what you would have to do to find out. One observer cannot get inside the brain of another observer and see through the other's eyes. This is Principle 2, the independence of observers. In representational terms it means that there is no function which will take you from one observer O_1 to another O_2 . This is the mathematical constraint which embodies the idea that "*one Man's Mind could not pass into another Man's Body*". Again, if we were to use a vector representation of observers, we would say Principle 2 means there exists no function which takes you from O_1 to O_2 and vice versa¹. We express this as:

$$\{\nexists f \mid f(O_1) = O_2\}$$

$$\{\nexists f \mid f(O_2) = O_1\}$$

From these two principles it is possible to demonstrate two things: firstly that objectivity can be constructed from subjective perceptions, and secondly that objective knowledge is relative knowledge. From these two conclusions several results follow (such as the hard problem of consciousness can never be solved). However first we need to turn to the concept of objectivity and make the definition of it operationally precise.

Objectivity and Transitivity

Objectivity is a slippery term. It means many things to many people. It can mean unbiased, independent, impartial, but it can also mean an aim or goal and it is not always clear whether the speaker is moving silently between the different meanings. Neither is it straightforward to represent objectivity in a mathematical way so that it can be measured and discussed. Instead, I propose to use the alternative term 'transitivity'² which means that knowledge can be transmitted from one observer to another unchanged. This provides a definition which is a working equivalent of 'objective', but which also has a precise operational meaning which can be expressed mathematically.

To be clear, by 'transitive' I mean that property which can be shared unambiguously between observers. It is that which can be passed from observer to observer and which remains unchanged. It is not therefore dependent on the viewpoint of any observer and carries with it the invariants of the object itself. Transitive literally means 'to go across' so we will use the term strictly to mean that quantity which can 'go across' from one observer to another unchanged. This is, I maintain, the essential quality of objectivity that is required to build scientific knowledge, for without transitivity there can be no transferable knowledge and consequently no building of knowledge and no repository of knowledge common to all. It is in this transitive sense we talk about objective knowledge.

We posit that some phenomena r_i of the real world are transitive and some are not. If the phenomena r_i are transitive it means that all observers would see and agree they see the same thing. If the property of r_i is non-transitive they would be unable to agree even if in principle, they see the same thing.

Consider first, non-transitive phenomena. Suppose Observer 1 (O_1) experiences a phenomenon r_i , for example they perceive the colour yellow. When Observer 2 (O_2) experiences the same phenomena r_i they have no way of

¹ Or there is no way of knowing what the vector is which takes you from O_1 to O_2 which is equivalent.

² Transitivity as used here should not be confused with transitive relations such as greater than $>$. In this sense, transitive is a mathematical term which means that, for example, if $A > B$ and $B > C$ then transitively $A > C$.

knowing whether the yellow that the first observer sees is the same as the yellow that they themselves see according to Locke. [9] Neither can they communicate their non-transitive experience to another. They are locked into their own non-transitive world which cannot be shared with anyone else even in principle. We express this for all observers $O_1, O_2 \dots O_i$ as:

Non-Transitive: $\{ \forall O_j, O_k | O_j(r_i) \neq O_k(r_i) \}$

Suppose on the other hand, there were some phenomena r_i which were transitive. These transitive properties by definition would be the same for all observers $O_1, O_2 \dots O_i$ and would be expressed:

Transitive: $\{ \forall O_j, O_k | O_j(r_i) = O_k(r_i) \}$

For transitive phenomena r_i which has a value which is independent of all O_j the role of the observer is irrelevant and can be removed, allowing r_i to stand alone, indicating that perception is divorced from the representation of transitive properties. Consequently, we can write:

Transitive: $O_j(r_i) \rightarrow r_i$

The question we now turn to is; are there any phenomena r_i which are fully transitive such that $O_j(r_i) = O_k(r_i)$? Does any such r_i independent of perceptive experience actually exist? Hume says that such a thing is not possible and if he is right then that leads us down to scepticism. We show here that in general, for any two distinct observers O_1 and O_2 observations of single quantities r_i are non-transitive. But this does not hold true for relative quantities (r_i, r_j) which are transitive and so provide a basis for objective knowledge.

Non-Transitive : $O_1(r_i) \neq O_2(r_i)$

Transitive: $O_1(r_i, r_j) = O_2(r_i, r_j)$

The perception of absolute quantities and subjective experience

We take a realist viewpoint, by which we mean the world $\mathbb{R}$ is populated with real entities r_i which exist independently of our observation of them. We might represent the properties of these entities as points in an n-dimensional space R_n . Many such representations are possible. We might for instance assume that each dimension in the n-dimensional space represents an independent property whose measurement is a point along a line.

Let us represent colour in such a vector space C . There are many ways to do this and it doesn't matter which representation we choose to illustrate the point. For convenience we choose a simple RGB space [10]. Let us further represent the perception of colour in a set of disjoint vector spaces O_i where each observer has their own perception space which has an unknown relation to the other. Thus for Observers 1 and 2 we have associated perception spaces O_1 and O_2 and C represents the colour space.

The joint vector space $R_n = C \oplus O_1 \oplus O_2$

We now propose that an observer O_1 who, according to Principle 1, is part of the world $\mathbb{R}$ and has observed a property in $\mathbb{R}$ which for example might be a colour C . We will say that the colour yellow is represented by a point in C by the vector $\vec{r}_C$. The perception of the colour by O_1 we will represent by a vector $\vec{r}_1$ which links the property C to the observer O_1 .

Perception by Observer 1 of Colour C : $O_1(C) = \vec{r}_1$

The perception vector $\vec{r}_1$ is unique to Observer 1, according to Principle 2, and cannot be shared by any other observer because each observer is unique and represented by a unique point in R_n . A second observer O_2 occupies a different position in R_n . and their perception of Colour C will be correspondingly different:

Perception by Observer 2 of Colour C : $O_2(C) = \vec{r}_2$

Two observers O_1 and O_2 independently perceive the colour yellow and their individual perceptions are represented by $\vec{r}_1$ and $\vec{r}_2$ respectively. Each observer has their own perception of Colour C and it is impossible to compare the perceptions of the two observers because they inhabit their own perception subspace of R_n which are, from Principle 2, unconnectable.

Figure 1 Two observers O_1 and O_2 perceive the same colour C

If the two subspaces O_1 and O_2 are related by the vector, $\vec{R}_{12}$ then we are only in a position to express one perception in terms of another provided that we know $\vec{R}_{12}$. However the only way that we can know if two observers experience similar or comparable perceptions would be by finding a relation between $\vec{r}_1$ and $\vec{r}_2$. Such a relation between the vectors can be seen from Figure 1 to be:

$$\vec{r}_2 = \vec{r}_1 - \vec{R}_{12}$$

This is equivalent to asking observer 1 to experience what observer 2 saw. However, the existence of $\vec{R}_{12}$ which represents the relationship between O_1 and O_2 , is indeterminate according to Principle 2. From our Lockean position expressed in Principle 2 we see that $\vec{R}_{12}$ is forever unknowable for to know $\vec{R}_{12}$ would mean being able to pass “one Man's Mind....into another Man's Body”. $\vec{R}_{12}$ can never be known and that will always be the case so we can never express $\vec{r}_1$ in terms of $\vec{r}_2$ or vice versa. The consequence is that we can never know if the perception of $\vec{r}_1$ is the same as the perception of $\vec{r}_2$, nor can we know how they are related. In other words, the yellow that I see might be the same as the yellow that you see but we can never be sure whether it is or not. The direct perception of colour is the perception of an absolute quality impinging upon our senses directly and absolute qualities give rise only to subjective experiences. That means that each observer is locked inside their own perceptions and cannot know the perceptions of another – just as Locke said. This creates a subjective world of experience for each observer which cannot be directly shared with another observer except by analogy and description. Experiences are not transferable or translatable (transitive) from one observer to another. If someone perceives or experiences the sensations of yellow or pain or happiness, then we cannot verify that directly and can only take the person's word for it or infer it from secondary evidence. Perception is a non-transitive, un-sharable and uncheckable experience. All of that follows from the fact that $\vec{R}_{12}$ is indeterminate. It is this which forges the prison of Hume's subjective world from which he concluded there is no escape. However, this is not the case with relative quantities.

Moving from the subjective to the objective

We distinguish between the subjective world (which we directly perceive ourselves and cannot be shared with anyone else in the Lockean sense) and the objective world which is perceived by all and can be shared with others and which can be communicated unambiguously to others. This is the transitive property that is the essential feature of objectivity.

Since all our knowledge starts from our perceptions and because, as Hume says, the only things we know come from our senses, [11] it might, at first sight, seem that we are locked into a subjective world from which we can never escape. George Berkeley indicated there was no escape from the subjective world. Unaided we could never pull ourselves out by our own bootstraps. However he added that it would take God himself to release us from a solipsistic prison for God's perceptions are all encompassing and all-knowing and in this way we can rely on the created world that God perceives at every moment. [12] God's perceptions alone provide objective reality to the world while our own are merely subjective. But Berkeley was wrong, we do not require that divine hypothesis. It is quite possible to construct a transitive, objective world out of the non-transitive subjective world which can be shared, agreed, and talked about in an unambiguous way. The real things which are independent of our perceptions and which can be shared transitively are not in the things themselves but the relation of things to each other. It is relative quantities rather than absolute quantities which are independent of our perceptions.

Henri Poincare was one of the first to point out that we cannot measure absolute quantities. We can only know, understand and measure relative quantities.

The aim of science is not things themselves, as the dogmatists in their simplicity imagine, but the relations between things; outside these relations there is no reality knowable³. [13]

Things are subjective and relationships between things are objective. We use relative quantities to construct the object world from our direct perceptions. It is relative quantities which are transitive or sharable with others and it is relative quantities which have an invariance under observation. It is that invariance which gives relative quantities independence and makes them suitable tools to build an objective view.

The perception of relative quantities and objective experience

When we move from absolute to relative quantities we move from the subjective to the objective. For instance, we can weigh something by holding it in our hand and considering the weight, whether it be light or heavy. But the problem with this method is that each person may come to a different judgement. That is because it is a subjective assessment – made by the perception of weight directly through our senses. We can see this also in the case of our hard 'yellow problem'. But the difference between two weights is a different matter. When comparing one weight against another it is possible to determine which is the heaviest unless they are very close. Consequently, relative quantities give rise to objective states whereas absolute quantities give rise to subjective states. It is possible to share relative values, but impossible to share absolute values.

Relative quantities are fundamentally different from absolute quantities in that relative quantities are invariant under the action of a constant. Absolute quantities may vary with each individual but relative quantities are fixed by their relation to each other. For example, absolute quantities vary under multiplication by a constant, but relative quantities are invariant under multiplication by a constant. You can multiply any relative value by any quantity and the relative difference will remain unchanged.⁴

We can see this more clearly in our vector representation above. In Figure 2 the relative distance between two points given by the displacement vector $\vec{S}$ is independent of the perception space coordinate system.

Each of the observers will have their own perception of $\vec{S}$. From the perception of observer 1 at O_1 we find that $\vec{S}$ is given by:

$$O_1(\vec{S}) = \vec{r}'_1 - \vec{r}_1$$

From the perception of observer 2 at O_2 we find that $\vec{S}$ is given by:

³ When Poincare said that "outside these relations there is no reality knowable" this is not strictly true. While relative quantities give us objective measures, absolute quantities, known through direct perception, allow us to know personal information known only to us such as the sensation of the colour yellow. For instance, we might use our hands to measure the weight of something by picking it up or we might use our eyes to determine colour. But the price we pay for this direct measure is that we cannot transmit this knowledge to others. Absolute quantities are subjective and remain locked up inside our senses. But absolute quantities give rise to our experience of yellow, and pain and happiness and these are no less knowable for being non-transitive, just personal. And on that small point, Poincare didn't quite get it right.

⁴ Invariants exist in many forms in mathematics and relative quantities are one of the simplest forms of invariants. For example Absolute quantities change under multiplication by a constant ($A \times \text{constant} = B$) whereas relative quantities A/B are invariant ($(A \times \text{constant}) / (B \times \text{constant}) = A/B$). Similarly absolute quantities vary under addition of a constant ($A + \text{constant} = B$) while relative differences are invariant ($(A - B) = (A + \text{constant}) - (B + \text{constant}) = A - B$).

$$O_2(\vec{S}) = \vec{r}'_2 - \vec{r}_2$$

Figure 2 Perception of a relative quantity S by two observers O_1 and O_2

As before we can relate the observations of O_1 and O_2 if we know the value of $\vec{R}_{12}$ since from Figure 2 it is clear that:

$$\vec{r}'_1 = \vec{r}'_2 + \vec{R}_{12}$$

$$\vec{r}_1 = \vec{r}_2 + \vec{R}_{12}$$

Substituting into the expression for $O_1(\vec{S})$ we find:

$$O_1(\vec{S}) = \vec{r}'_1 - \vec{r}_1 = (\vec{r}'_2 + \vec{R}_{12}) - (\vec{r}_2 + \vec{R}_{12}) = \vec{r}'_2 - \vec{r}_2 = O_2(\vec{S})$$

Even though $\vec{R}_{12}$ is completely unknown, it makes no difference to relative quantities as the factor cancels out. This means that the relative quantities observed by O_1 and O_2 are the same. In other words, relative quantities $\vec{S}$ are transitive in contrast to absolute quantities.

$$\text{Transitive } O_1(r_i, r_j) = O_2(r_i, r_j)$$

Relative quantities can be shared unambiguously between observers and remain unchanged. They are independent of the viewpoint of any observer and carry with it, invariants which are properties of the object itself and not of the perception of the object. These are the properties of independent reality, not of our senses and can therefore be used as the building blocks of objective scientific knowledge. Transitivity is found in relationships and all transferable knowledge is the knowledge of relationships between things.

What does this mean for our perception of phenomena like colour? The relative colour displacement vector ($\vec{S}$) is the same for all observers while the direct observation of colour is not. The implication is that relative changes in colour are objective while absolute colour is subjective. Thus, the difference between two colours can be agreed by two equal observers⁵ while the perception of colour itself cannot. How is a relative difference in colour perceived? It means that two different colours next to each other will be seen as different by equal

⁵ The notion of 'equal observers' is important here. If two observers have different capabilities, then their observations will not necessarily be equal but that has nothing to do with the property under observation. For instance, two observers, one of whom is colour-blind, looking at the same colour difference may not agree on what they see but that is due to the deficiency of the observation, not the true difference in colour.

observers while the colours themselves cannot be determined as different or the same. The relative difference between colours measured by ($\vec{S}$) is perceived the same by all. If this represents change in colour, then change in colour is perceived the same for all. Relative quantities are able to pass from “one Man's Mind... into another Man's Body” – not literally but effectively, because we know that what appears in one man's mind is the same as what appears in another man's mind. These ‘transitive’ quantities are independent of perception and the elements from which we construct the objective world.

Some Implications

A shift from absolute values to relative values is a fundamental change in description and has implications for a wide range of problems. First and most obviously it has implications for the scepticism of Hume without resorting to the a priori knowledge of Kant. There are also implications for the representation of causality which may have an impact on dynamics. The Hard Problem of Consciousness described eloquently by David Chalmers may also be shown to be non-transitive and hence intractable. And lastly these ideas may have some implications for the dependence of Newtonian mechanics on the supposition of absolute space and time.

Implications for Empiricism and Scepticism

Kant was the most perturbed by Hume's sceptical conclusions. He famously commented that “*it was my recollection of the thought of David Hume that broke into my dogmatic slumber and pointed my work in speculative philosophy in a completely new direction. I was nowhere near accepting his conclusions.*” [5] Kant solved the problem to his satisfaction by finding a gap between different types of a priori knowledge.

That all our knowledge begins with experience there can be no doubt.... But, though all our knowledge begins with experience, it by no means follows that all arises out of experience... our empirical knowledge is a compound of that which we receive through impressions and that which the faculty of cognition supplies from itself. [14]

Thus, Kant proposes that knowledge comes from two sources: empirical knowledge from our senses (*a posteriori*) and knowledge independent of experience from our reason (*a priori*) In this way he seeks to escape the idea that all knowledge comes from experience.⁶ And if genuine new knowledge comes from something other than our senses then says Kant, we can build a knowledge system from that which does not come only from our senses. However, a possibility does not constitute proof of a certainty, and a gap that *can be* moved through does not mean that it *is* moved through; only that it is possible. Kant opens the gap but does not quite go through it.

The approach here has been different to that of Kant. (See Figure 3 Bridging the gap between subjective and objective knowledge.) The relational knowledge which is presented here is not the same as the synthetic *a priori* knowledge of Kant. Relational knowledge is purely empirical and only comes from perception, and yet because it is relational it is independent of perception. We have no need to suppose there is such a thing as *a priori* synthetic knowledge and can stand with Hume and say that all knowledge arises out of experience. To paraphrase Kant, *though all our knowledge arises out of experience, it follows that all is dependent on experience.* Relative knowledge is independent of experience because it is an invariant of observation. It is not just the categories of synthetic and analytic which are important here; we must add the further categories of relative and absolute. Both relative and absolute knowledge are *a posteriori* and synthetic in the sense that Kant would understand, but he would claim that this would be unhelpful in solving his problem.

What has not been recognised by Kant or Hume is there are two forms of a *a posteriori* synthetic knowledge. We distinguish between the relative, a *a posteriori*, synthetic and the absolute, a *a posteriori*, synthetic. It is only the latter which Hume and Kant understood and only the latter which leads to scepticism. But it is the former, the relative, a *a posteriori*, synthetic knowledge, which provides us with a basis for stepping outside the subjective world.

⁶ To use Kant's terminology not all synthetic judgements are *a posteriori*. But there are also synthetic *a priori* judgements such as $12 + 7 = 19$. This is where he differs from Hume who would have maintained that all synthetic judgements are *a posteriori*. And through this gap Kant escapes from the trap of scepticism.

Figure 3 Bridging the gap between subjective and objective knowledge.

Implications for Causality

David Hume was the first modern philosopher to ask the question “How can a thing A act on another thing B?” [15] He concluded that there is no such thing as causality, only a succession of events in time. Hume reasoned that if all we know is perception then we don’t know facts or events. Two seemingly related events are nothing more than two unrelated perceptions. They are not about the facts at all. Since all causality is about the relationship of facts and we only know perceptions, we can know nothing about causality [16]. Nothing can be said about causality at all⁷. That, in short, is Hume’s argument against it.

What this amounts to is a denial of the idea that one event depends on another event and that events unfold in time and space according to those dependencies. But this view removes not only causation but also all scientific method according to Kant. Whenever we observe an event, we assume there is some prior event from which both are connected by some rule. It doesn’t matter if you can find the rule or not because if you can’t find it then science simply encourages us to look for it. Which means, as Heisenberg reminds us, that the law of causation is deeply embedded in the scientific method itself. [17] If there is no causality, there is no science.

“The law of causality simply asserts that the phenomena of nature are dependent on one another. The special emphasis put on space and time in the expression of the law of causality is unnecessary, since the relations of space and time themselves explicitly express that phenomena are dependent on one another.” Ernst Mach [15]

Causality may be expressed then as the necessary relation R between two events A and B which can be represented as

Causal Relation R between A and B = $(A|R|B)$

There are two kinds of events which may be considered having necessary connection. The first kind is where the connection cannot be observed directly but needs to be inferred. This kind of connection comes from patterns gained from repeated observations. In this instance we rely on coincidence and induction, but as Hume pointed out induction is not reliable. If A is a lightning strike and B is the sound of thunder, we have two events, but

⁷ It would of course be possible to redefine causality as the relationship between perceptions rather than facts. We can then just proceed as normal. But some would think this too cheap a solution to the problem, even though it may be the best.

what we do not have is a direct experience of the relation R between them. Just because lightening has always been followed by thunder in the past, does not mean that it will be followed by thunder in the future. Through observation all we have is a psychological association of A and B produced by the senses. What we do not have is proof of necessary connection R. We call this *inference causality*.

The second kind is where the connection R is seen directly, as in the case of one billiard ball hitting another. This is not causality inferred by repeated observations but by single direct observation where the mechanism of cause or the relation R is also observed. In this case a single instance is enough to establish causality. When we see one billiard ball hit another and the other scatters off as a result, we do not need induction to deduce causality, for the mechanism is observed. We only need induction when the mechanism is not observed. Thus, there are two ways of arriving at causal relationships; first by inference and second by direct observation of the mechanism or cause. *Directly observed causality* comes from relationships.

With the absolute approach we have a problem, for we would need to observe the ball A, then observe the relation R and then observe the effect on B. These are three unrelated events according to Hume which we can infer a connection between but cannot prove.

$$A | R | B \neq (A|R|B)$$

However with the relational approach, all the invariants of observation are relationships. It is from the relational approach we present here that direct causality can be constructed. Suppose that billiard ball A connects with billiard ball B and they relate in an interaction R. The observation of the first part of the interaction will be expressed in the relationship.

$$(A|R)$$

The second part of the directly observed interaction is represented by the relation

$$(R|B)$$

Thus, from the two objective relational observations we can combine the invariants⁸ so that

$$(A|R) + (R|B) = (A|R|B) \rightarrow (A|B)$$

From this, further interactions can be combined in nested relations, and in this way a causal chain can be generated from a succession of relational observations:

$$(A_1|R_1) + (A_2|R_2) + \dots (A_n|R_n) \rightarrow (A_1|A_n)$$

The laws of nature are equations which express these causal relationships. But by using transitive quantities it is possible to build causal systems using relative variables which we can generalise as pairs of (r_i, r_j) and consequently causal relations can be expressed.

Implications for The Hard Problem of Consciousness

The hard problem of consciousness, says Chalmers, is the problem of experience. [18] The experience of consciousness is as mysterious to us as it was to the ancient Greeks, so little understanding do we have of personal awareness. The inner experience of consciousness presents us with a number of unanswered questions such as, “Where does the quality of deep blue that I see, come from?”, “How do we explain the sensation of hearing middle c?” and “Where does the rich experience of inner life come from?”

The division of knowledge into the subjective/absolute and objective/relative provides some insight into the intractability of the ‘hard problem’. We have discussed this issue in more detail elsewhere [19] where we summarise the argument as (1) the hard problem of consciousness is concerned with subjective experience; (2) subjective experience arises from the measure of absolute quantities directly by our senses; (3) objective experience, on the other hand, arises from the measure of relative quantities which are invariant to perception; (4) only objective experience can be shared, discussed or transmitted to others – i.e. transitive; (5) consequently

⁸ Where we have made use of the relational calculus $(a|B) + (B|c) = (a|B|c)$ and consequently the observed relations combine as $(a|B) + (B|c) + (c|D) + (D|e) = (a|B|c) + (c|D|e) = (a|B|c|D|e)$

the hard problem is forever locked inside the head of each individual and can never be discussed, explained or shared with another. Accordingly, no theory can, in principle, be formulated to explain it.

Implications for Newtonian Mechanics

We have come to a conclusion about absolutes; that they cannot be used for an objective description of the real world and that must include concepts in physics such as absolute space and absolute time. Only relational quantities are transitive and therefore theories should be formulated only in terms of relational quantities. In his formulation of mechanics Newton was wrong to introduce the concepts of absolute space, absolute time, and absolute motion, and indeed any quantities which depend upon the observer. This approach led to the need to introduce complexities like frames of reference whereas values in a relational mechanics have the same values in all frames of reference. [20]

Criticism of Newton is not new when it comes to absolutes. Newton formulated his theory of mechanics against a background of absolute space, absolute time and absolute motion. Even in his own day these ideas were strongly criticised by Leibniz and Berkeley and later by Mach and Einstein. Leibniz held that space is “*something merely relative, as time is, that I hold it to be an order of coexistences, as time is an order of successions*”. Berkeley also held that absolute space is “*that phantom of the mechanical and geometrical philosophers, ... that it is neither perceived by our sense nor proved by our reason*” [21] He further added that, “*it suffices to replace their ‘absolute space’ by a relative space determined by the heavens of the fixed stars*” and “*Motion and rest defined by this relative space can be conveniently used instead of the absolutes*”. [21]

Leibniz was suspicious of an absolute space which entails the idea that the world oriented in one way with respect to absolute space would have to be distinct from the world oriented in another way with respect to absolute space. But, according to Leibniz both these would be indistinguishable and not even God could recognize any difference between them. Hence absolute space must be rejected, he said. “*If two things are indistinguishable then we should view them as the same thing under two names*”. [22] What was then the minority view of Berkeley and Leibniz would some two hundred years later become more widely accepted. In 1912 Mach comments, “*the view that the notion of ‘absolute motion’ is meaningless without any empirical content, and scientifically without use... is upheld by many well-known investigators*” [15]

Moving to a mechanics based on relational variables means that there is no need to introduce the concepts of inertia, inertial mass, inertial frames or absolute space, absolute time and absolute motion. Only relational forces $\vec{F}_{AB}$ between two entities A and B have any meaning. [17] Thus Newton’s third law, for instance, that every action has an equal and opposite reaction can be simply expressed as

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Furthermore relational mechanics implies that the sum of all the forces on a body is always zero in all frames of reference, for all time in all instants. [20] This is simply expressed as:

$$\sum_{\substack{\text{all } i \text{ and } j \\ i \neq j}} F_{ij} = 0$$

Consequently, multiplying all the forces by the same constant has no effect on the results because only the ratio of forces has meaning. “*We never know the absolute value of any force only how much one force is larger or smaller than another force.*” [20]. The sum of all the relative forces between all the particles in a universe is equal to zero. It is like relativistic dynamics where everything is based on the ratio of quantities not on their absolute values. The ratios or differences don’t change and because they don’t change, they provide us with the tools to create an objective world.

Summary and Conclusion

We posited that some phenomena r_i of the real world are transitive and some are not. Transitive phenomena are transferable between observers without change and are thus independent of observation. Non-transitive phenomena are dependent on the observer and cannot be transferred. We have shown that transitive phenomena are relative quantities (r_i, r_j) and non-transitive are absolute quantities (r_i). The recognition that transitive quantities are invariants of observation gives them an independence from which we can construct an objective world. This solves the problem that Hume posed without resorting to Kant’s synthetic *a priori* statements. The implication is that we need to cast all theoretic representations of the world into relative relationships. This has some implications for a wide range of issues from the hard problem of consciousness to Newtonian physics.

Wittgenstein expressed the view that “*Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge*” or The world is the totality of facts, not of things. [23] In this paper we have shown that the objective, transitive world is the totality of facts, the subjective non-transitive world is the totality of things. Facts are the relationship between things. It is possible to have a world without things but not a world without facts. Things are perceived directly by our senses and we have a subjective experience of them. Facts are constructed from the relationships between things and, being the invariants of observation, have an objective existence. We cannot communicate with someone else unless we use relative quantities. Facts arising from relationships we can share with others. Things which are absolutes forever remain private to us.

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